Improving the Diversity of Spatial Modulation in MISO Channels by Phase Alignment

Christos Masouros, *Senior Member, IEEE*

***Abstract*—The performance of spatial modulation (SM) is known to depend on the minimum Euclidean distance in the received SM constellation. In this letter, a symbol scaling technique is proposed for spatial modulation in the multiple-input-single-output (MISO) channel that enhances this minimum distance. It achieves this by aligning the phase of the relevant channels so that the received symbol phases are distributed in uniformly spaced angles in the received SM constellation. In contrast to existing amplitude-phase scaling schemes that are data-dependent and involve an increase in the transmitted signal power for ill conditioned channels, here a phase-only shift is applied. This allows for data-independent, fixed per-antenna scaling and leaves the symbol power unchanged. The results show an improved SM performance and diversity for the proposed scheme compared to existing amplitude-phase scaling techniques.**

***Index Terms*—Spatial modulation, multiple-input-single-output, pre-scaling.**

I. Introduction

TRADITIONAL spatial multiplexing has been shown to improve the capacity of the wireless channel by exploiting multi-antenna transmitters [1]. More recently, Spatial Modulation (SM) has been explored as a means of encoding information in the index of the antenna used for transmission, offering a low complexity alternative [2]. It's two central benefits include the absence of inter-channel interference (ICI) and the fact that it only requires a subset (down to one) of Radio Frequency (RF) chains compared to spatial multiplexing, for the transmission of data. Early work has focused on the design of receiver algorithms for SM aiming at minimum error performance and complexity. The work has looked at several approaches, from low-complexity matched filtering, to maximum likelihood (ML) detection and reduced-space sphere detection [2]-[6].

Recent work has focused on constellation shaping for SM by means of symbol pre-scaling at the transmitter, aiming to maximize the minimum Euclidean distance in the received SM constellation [7], [8], [9]. This is known to be central to the performance of SM detection. In [10] the transmit diversity of coded SM is analyzed for different *spatial constellations*, denoting the possible sets of active antennas at the transmitter. While spatial constellation shaping is discussed in [10], we note that this is distinctly different to the focus of this paper and [7], [8] which is on the design of the received SM constellation which combines the choice of the transmit antenna as well as the constellation of the transmit symbol. The constellation shaping approach in [7], [8] has been to fit the receive SM constellation to one of the existing optimal constellation formats in terms of minimum distance, such as e.g. quadrature amplitude modulation (QAM). The strict constellation fitting requirement in both amplitude and phase tens to reduce the received signal to noise ratio (SNR).

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C. Masouros is with the Dept. of Electrical & Electronic Eng., University College London, Torrington Place, London, WC1E 7JE, UK (e-mail: chris.masouros@ieee.org).

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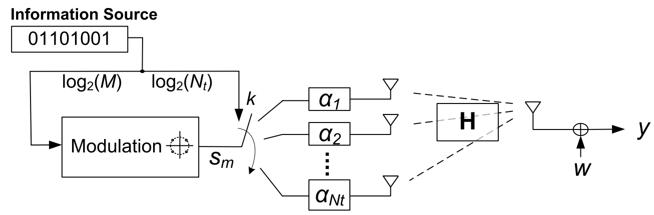

Fig. 1. Block diagram of Spatial Modulation transmitter with per-antenna symbol scaling.

To alleviate this shortcoming, in this letter we introduce a new pre-scaling scheme which is based on unit-power factors that only influence the phase of the transmit signals. A similar phase-only approach is shown in [9] for Space Shift Keying (SSK) transmisison. We note however that the proposed technique applies to the more generic SM scenarios, while it is also accompanied by a mathematical diversity analysis. Using the existing amplitude-phase structure of the MISO channel, here the aim is to shape the receive SM symbols so that the angles of the constellation points are uniformly spaced within the $[0, 2\pi)$ spread in the received constellation. This therefore statistically enhances the minimum Euclidean distance in the constellation. In comparison to existing amplitude-phase prescaling techniques, the key benefits are that a) it is channel-only dependent as opposed to the data- and channel- dependent scaling of [7], [8], b) it does not alter the transmit power and no power scaling (with the associated SNR loss) is further required and c) the transmit power per symbol time is normalized to the power budget on an instantaneous basis, which facilitates the use of a low-cost power amplifier. Accordingly, the proposed pre-scaling provides better performance than the existing approaches for an equal transmit power.

II. Spatial Modulation with Phase Alignment

A. System Model

Consider a multiple-input-single-output (MISO) system where the transmitter is equipped with N_t antennas. For simplicity, in this paper we assume that the transmit power budget follows $P = 1$. We focus on the single RF chain SM approach where the transmit vector is in the all-but-one zero form $\mathbf{s} = [0, \ldots, s_m^k, \ldots, 0]^T$ where the notation $[.]^T$ denotes the transpose operator. Here, $s_m, m \in \{1, \ldots, M\}$ is a symbol taken form an M-order modulation that represents the transmitted waveform in baseband domain conveying $log_2(M)$ bits and k represents the index of the antenna used for transmission conveying $log_2(N_t)$ bits in the spatial domain. Clearly, since $\mathbf{s}$ is an all zero vector apart from s_m^k there is no inter-channel interference.

The pre-scaling approach proposed here is shown in Fig. 1. The signal fed to each antenna is scaled by a complex coefficient $\alpha_k, k \in \{1, \ldots, N_t\}$ for which $|\alpha_k| = 1$, where $|x|$ denotes the amplitude of a complex number x. Defining the MISO channel vector as $\mathbf{h} = [h_1, h_2, \ldots, h_{N_t}]$ where h_k denotes the complex channel coefficient from the k-th transmit antenna to the receiver, the received symbol can be written as

$$y = \mathbf{hAs} + w \tag{1}$$

where $w \sim \mathcal{CN}(0, \sigma^2)$ is the additive white Gaussian noise (AWGN) component with variance σ^2 at the receiver and $\mathbf{A} = \text{diag}(\mathbf{a})$ is the pre-scaling matrix with $\mathbf{a} = [\alpha_1, \alpha_2, \ldots, \alpha_{N_t}]$. $\text{diag}(\mathbf{x})$ denotes the diagonal matrix with its diagonal elements taken from vector $\mathbf{x}$. Note that the diagonal structure of $\mathbf{A}$ guarantees an all-but-one zero transmit vector $\mathbf{t} = \mathbf{As}$, so that the single RF chain aspect of SM is preserved.

Existing pre-scaling schemes for SM aim at maximizing the Euclidean distance between adjacent points $\mathbf{y}_i, \mathbf{y}_j$ in the receive SM constellation so that

$$\mathbf{A}^* = \arg\max_{\mathbf{A}} \min_{i,j} ||\mathbf{y}_i - \mathbf{y}_j||^2, \; i \neq j \tag{2}$$

subject to $\mathbf{A}^*$ having a diagonal structure.

B. Amplitude-Phase Pre-scaling (SM-AP)

Accordingly, in [7] the receive SM constellation is fitted to a Q-QAM constellation with $Q = N_t M$ by choosing

$$\tilde{\alpha}_m^k = \frac{q_{(m-1)M+k}||\mathbf{h}||}{h_k s_m \sqrt{N_t}} \tag{3}$$

where q_n is the n-th constellation point in the Q-QAM constellation, $||\mathbf{x}||$ denotes the norm of vector $\mathbf{x}$ and the factor $\frac{||\mathbf{h}||}{\sqrt{N_t}}$ is used to normalize the receive constellation so that $E\{q\} = 1$.

We note that, while the scaling in (3) normalizes the receive constellation, it does not normalize the transmit power. To illustrate this, Fig. 2(a) shows the probability distribution of the transmit power for $E\{|s_m|^2\} = 1$ using the factors $\tilde{\alpha}_m^k$ proposed in [7]. It is clear that the transmit power is not constrained and its average is above one. Therefore, power-normalized scaling coefficients should be used in the form

$$\alpha_m^k = \frac{\tilde{\alpha}_m^k}{||\tilde{\mathbf{a}}||}. \tag{4}$$

These result in the power distribution shown in Fig. 2(b). Still, it is observed that the distribution spreads up to multiples of the average power (more than 5 times the average power for the example shown). This makes the design of the power amplifier at the transmitter challenging. It can also be seen that for badly conditioned channel coefficients, even for just one of the transmit antennas, this leads to low power-scaling factors

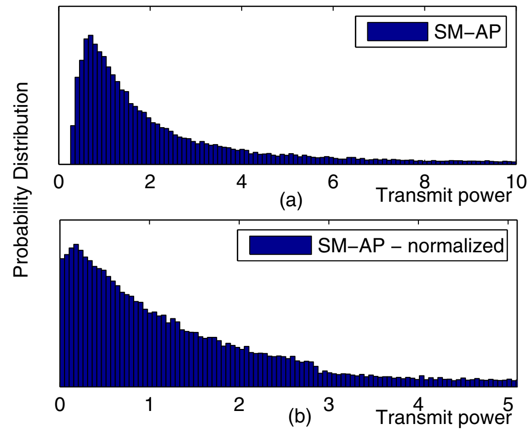

Fig. 2. Probability distribution of the transmitted power of amplitude-phase scaling [7] with and without power normalization.

$f = 1/||\tilde{\mathbf{a}}||$ which limits the performance of transmission for all transmit antennas. Finally, note that α_m^k are data dependent as evidenced by the index m, which does not allow for a fixed per-antenna scaling coefficient as in Fig. 1.

C. Proposed Phase-only Pre-scaling (SM-P)

To alleviate the the drawbacks of the above technique, a phase-only scaling is proposed which is does not change the power of the transmit symbols and therefore does not require power scaling that introduces performance losses. Moreover it preserves normalized instantaneous power which allows simple power amplifiers at the transmitter and facilitates fixed, data-independent scaling at each transmit antenna. The relevant coefficient for the k-th antenna is defined by the expressions

$$\alpha_k = e^{j\varphi_k} \tag{5}$$

$$\varphi_k = \theta_i - \vartheta_k \tag{6}$$

where ϑ_k is the phase of h_k and θ_i is the i-th angle taken form an equally spaced angle arrangement within $[0, 2\pi)$ in the form

$$\theta_i = \frac{2\pi}{N_t M}(i-1), i \in \{1, \ldots, N_t\}. \tag{7}$$

Since the channel coefficients are estimated at the receiver for detection [2]-[6], (5)-(7) can be used to derive the above factors independently at the receiver. Therefore no feed forwarding of α_k is required. With the proposed scaling, while the amplitudes of the symbols in the original received constellation are preserved in the modified constellation, the received phases are arranged such that a constant phase difference appears between adjacent constellation points. A characteristic example of this operation is illustrated in Fig. 3 showing the original and scaled received constellation for a 4×1 SM MISO system. It can be seen that, while suboptimal in the constellation design sense, the proposed pre-scaling enhances the minimum distance in the constellation with respect to conventional SM. In addition, it will be shown in the results section that the absence of power scaling also improves the error rate of the system compared to conventional SM and the SM-PA of [7]. First, we analytically characterize the diversity gains of the proposed approach in the following section.

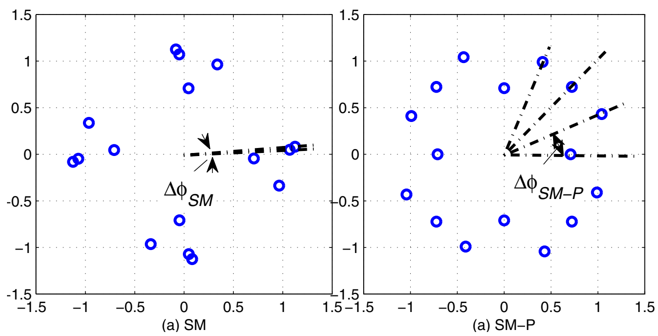

Fig. 3. An example of received constellation for conventional SM and proposed pre-scaled SM in a 4×1 MISO channel using 4QAM.

III. Diversity Analysis

The proposed constellation shaping by means of phase alignment as shown above leads to an increase in the transmit diversity. That is, while the transmit diversity of the single-RF SM is known to be one, the proposed pre-scaling introduces a phase diversity in the transmission. For the single receive antenna scenario, the system is said to have diversity order d if the bit error probability decays with γ^{-d} in the high-SNR region, with γ being the SNR. To analyze diversity we note that the pairwise error probability (PEP) for SM scales with the Euclidean distance between constellation points as [10]

$$PEP(\mathbf{y}_i, \mathbf{y}_j) = \mathcal{Q}\left(\sqrt{\frac{||\mathbf{y}_i - \mathbf{y}_j||^2}{2\sigma^2}}\right) \tag{8}$$

where $\mathcal{Q}(x)$ denotes the Q-function and

$$\begin{aligned} ||\mathbf{y}_i - \mathbf{y}_j|| &= \sqrt{||\mathbf{y}_i||^2 + ||\mathbf{y}_j||^2 - 2\mathbf{y}_i \bullet \mathbf{y}_j} \quad (9) \\ &= \sqrt{||\mathbf{y}_i||^2 + ||\mathbf{y}_j||^2 - 2||\mathbf{y}_i||||\mathbf{y}_j|| \cos(\Delta\phi)} \end{aligned}$$

where $\mathbf{a} \bullet \mathbf{b}$ denotes the dot product of vectors and $\Delta\phi$ denotes the phase difference between the two points. The proposed technique leaves the amplitudes and norms in (9) unchanged and improves diversity by increasing the minimum $\Delta\phi$ with respect to conventional SM. Accordingly, since the PEP is dominated by the minimum Euclidian distance and here the amplitudes of the received constellation points are left unchanged, we focus on the gain in minimum phase separation between adjacent constellation points for the proposed SM-P, defined as

$$G = \frac{\Delta\phi_{SM-P}}{E\{\min \Delta\phi_{SM}\}} \tag{10}$$

where $\Delta\phi_{SM-P}$ is the constant phase separation obtained with SM-P and $\min \Delta\phi_{SM}$ is the minimum phase separation with conventional SM, both shown in Fig. 3. By (9) and (10) and the associated pairwise error probability analysis it can be seen that the diversity gain of SM-P can be approximated by $d \approx \sqrt{G}$, which we verify empirically in the results section. With respect to (10), firstly, it is clear from (7) that

$$\Delta\phi_{SM-P} = \frac{2\pi}{N_t M}. \tag{11}$$

To quantify G we derive $\min \Delta\phi_{SM}$ for SM in a channel where the phases of the channel coefficients h_k are uniformly distributed within $[0, \phi)$. Under this condition, the phase separation $\Delta\phi_{SM} = \theta_i - \theta_j$ between two receive SM constellation points follows a triangular distribution [13] in $[0, \phi)$ with its probability density (PDF) and cumulative distribution functions (CDF) expressed respectively as

$$f_{\Delta\phi_{SM}}(x) = \frac{2}{\phi} - x\frac{2}{\phi^2}, \tag{12}$$

$$F_{\Delta\phi_{SM}}(x) = 1 - \frac{(\phi - x)^2}{\phi^2}. \tag{13}$$

For the receive SM constellation with N_t channels and M-order modulation this yields $n = N_t M$ constellation points. For the minimum phase separation in the constellation we use the results from order statistics [14], [15] by which

$$f_{\min \Delta\phi_{SM}}(x) = n\left(1 - F_{\Delta\phi_{SM}}(x)\right)^{n-1} f_{\Delta\phi_{SM}}(x), \tag{14}$$

to obtain its PDF as

$$f_{\min \Delta\phi_{SM}}(x) = N_t M \left(\frac{\phi - x}{\phi}\right)^{2N_t M - 1} \frac{2}{\phi}. \tag{15}$$

Hence, the average minimum phase separation with conventional SM is given as

$$E\{\min \Delta\phi_{SM}\} = \int_0^{\phi} x f_{\min \Delta\phi_{SM}}(x) dx = \frac{\phi}{1 + 2N_t M}. \tag{16}$$

For the uncorrelated Rayleigh fading the phase spread is $\phi = 2\pi$. Combining (16) with the expressions for ϕ and $\Delta\phi_{SM-P}$ from above we have

$$G = \frac{\frac{2\pi}{N_t M}}{\frac{2\pi}{2N_t M + 1}} = \frac{2N_t M + 1}{N_t M}. \tag{17}$$

The above phase separation gain reflects a diversity gain for SM while it is known that the transmit diversity for conventional SM is one. The simulation results in the following show that the diversity of the system can be empirically approximated as

$$d \approx \sqrt{\frac{2N_t M + 1}{N_t M}} \tag{18}$$

with $\lim_{n\to\infty} d = \sqrt{2}$ for high dimensional (large N_t, M) transmission.

IV. Simulation Results

To evaluate the benefits of the proposed technique, this section presents numerical results based on Monte Carlo simulations of conventional SM without scaling (termed as SM in the figures), SM with amplitude-phase scaling (SM-AP) and the proposed phase only pre-scaled SM (SM-P). A Rayleigh fading channel is used whose impulse response is assumed perfectly known at the transmitter. Without loss of generality we assume the transmit power restricted to $P = 1$. 4×1 and 8×1 MISO systems employing 4QAM and 16QAM modulation are explored while it is clear that the benefits of the proposed technique extend to larger scale systems and higher order modulation.

First, we show the bit error rate (BER) performance with increasing transmit SNR for the three schemes for a 4×1 MISO employing 4QAM in Fig. 4. The graph includes both

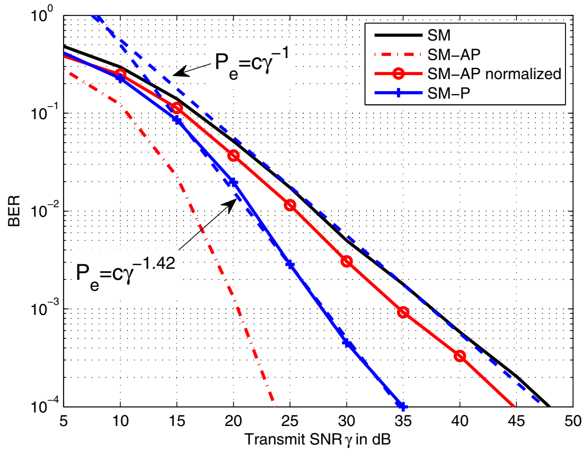

Fig. 4. BER vs. SNR for a 4×1 MISO with SM, SM-AP, SM-P, 4QAM, including diversity trend where P_e denotes error probability and c is a generic constant.

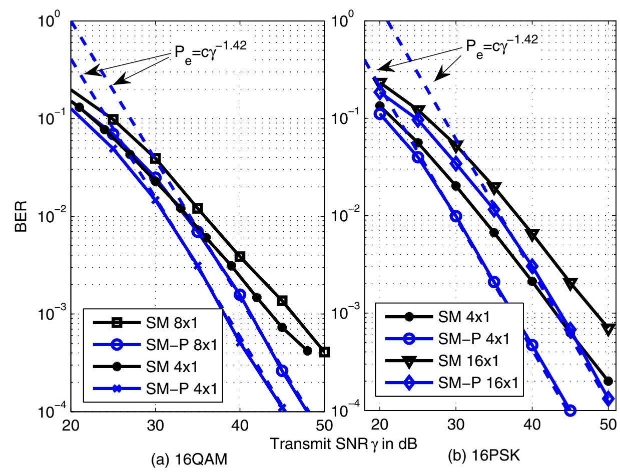

Fig. 5. BER vs. SNR for 4×1, 8×1 and 16×1 MISO with SM, SM-P, 16QAM and 16PSK, including diversity trend where P_e denotes error probability and c is a generic constant.

versions of SM-AP with (4) and without (3) power normalization, to illustrate the losses introduced by power scaling for the technique in [7]. Clearly, while SM-AP without power normalization shows vast performance gains, once the transmit power is normalized back to P, the BER performance moves much closer to the once for SM. As mentioned, this is because (3) involves a channel inversion which for badly conditioned channels leads to large symbol scaling factors $\tilde{\alpha}_m^k$ and low power scaling factors and received SNRs. On the contrary, the proposed scheme applying only a phase change, avoids this shortcoming. A significant SNR gain of almost 10dB can be observed between the proposed SM-P and conventional SM at the BER of 10^{-3}. This is due to the enhancement of the minimum Euclidean distance on the received SM constellation, as illustrated in Fig. 3. The relevant SNR gain with respect to SM-AP is around 7dB.

Fig. 5 explores systems with more antennas and higher order modulation. Three systems with 4, 8 and 16 transmit antennas are shown for both 16QAM and 16PSK. It can be seen that the performance gains of the proposed SM-P approach compared to SM persist. To verify the above diversity analysis, both Figs. 4,5 include the theoretical curves illustrating diversity, represented by the dashed lines. For the parameters used in the figures we have $d = \sqrt{\frac{2N_t M+1}{N_t M}} \approx 1.42$ for SM-P for all systems, while clearly SM provides transmit diversity equal to one. The graphs show that indeed the BER follows the diversity trend as analyzed above, and thus SM-P provides important diversity gains with respect to conventional SM.

Conclusion

A new constellation shaping approach has been introduced for spatial modulation in the MISO channel. Conventional constellation shaping techniques offer limited gains compared to SM due to the strict fitting to a fixed constellation, which often requires the inversion of ill conditioned channel coefficients. It has been shown that, by influencing only the phase of the received constellation points, the proposed algorithm offers significant performance and diversity gains to existing SM constellation shaping approaches.

References

[1] D. Gesbert, M. Kountouris, R. Heath, C.-B. Chae, and T. Salzer, "Shifting the MIMO paradigm," *IEEE Signal Process. Mag.*, vol. 24, no. 5, pp. 36–46, Sept. 2007.

[2] R. Mesleh, H. Haas, S. Sinanovic, C. W. Ahn, and S. Yun, "Spatial modulation," *IEEE Trans. Veh. Technol.*, vol. 57, no. 4, pp. 2228–2241, July 2008.

[3] M. Di Renzo and H. Haas, "Bit error probability of SM-MIMO over generalized fading channels," *IEEE Trans. Veh. Technol.*, vol. 61, no. 3, pp. 1124–1144, Mar. 2012.

[4] M. Di Renzo, H. Haas, A. Ghrayeb, S. Sugiura, and L. Hanzo, "Spatial modulation for generalized MIMO: challenges, opportunities and implementation," *Proc. IEEE,* Jan. 2014.

[5] J. Jeganathan, A. Ghrayeb, and L. Szczecinski, "Spatial modulation: optimal detection and performance analysis," *IEEE Commun. Lett.*, vol. 12, no. 8, pp. 545–547, Aug. 2008.

[6] A. Younis, S. Sinanovic, M. Di Renzo, R. Mesleh, and H. Haas, "Generalised sphere decoding for spatial modulation," *IEEE Trans. Commun.* , vol. 61, no. 7, pp. 2805–2815, July 2013.

[7] X. Guan, Y. Cai, and W. Yang, "On the mutual information and precoding for spatial modulation with finite alphabet," *IEEE Wireless Commun. Lett.,* vol. 2, no. 4, pp. 383–386, Aug. 2013.

[8] J. M. Luna-Rivera, D. U. Campos-Delgado, and M. G. Gonzalez-Perez, "Constellation design for spatial modulation," *Procedia Technol.,* vol. 7, pp. 71–78, 2013.

[9] K. Ntontin, M. Di Renzo, A. Perez-Neira, and C. Verikoukis, "Adaptive generalized space shift keying," *EURASIP J. Wireless Commun. and Networking*, vol. 2013, Feb. 2013.

[10] M. Di Renzo and H. Haas, "On transmit diversity for spatial modulation MIMO: impact of spatial constellation diagram and shaping filters at the transmitter," *IEEE Trans. Veh. Technol.*, vol. 62, no. 6, pp. 2507–2531, July 2013.

[11] J. Jeganathan, A. Ghrayeb, L. Szczecinski, and A. Ceron, "Space shift keying modulation for MIMO channels," *IEEE Trans. Wireless Commun.,* , vol. 8, no. 7, pp. 3692–3703, July 2009.

[12] M. Di Renzo and H. Haas, "Bit error probability of space modulation over Nakagami-m fading: asymptotic analysis," *IEEE Commun. Lett.*, vol. 15, no. 10, pp. 1026–1028, Oct. 2011.

[13] M. Abramowitz and I. Stegun, *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, 1972.

[14] T. Ratnarajah, C. Masouros, F. Khan, and M. Sellathurai, "On the multiuser diversity gains of opportunistic spectrum sharing in cognitive radio systems," *IEEE Trans. Commun.*, vol. 61, no. 7, pp. 2664–2677, July 2013.

[15] H. A. David and H. N. Nagaraja, *Order Statistics*, 3rd ed. Wiley, 2003.