THE DISCOVERY OF A FRAGMENT OF ISAAC HA-ISRAELI'S *YESOD ʿOLAM* IN THE CAIRO GENIZAH

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Abstract

This article presents the discovery of a fragment that belongs to Isaac ha-Israeli's scientific book *Yesod Olam* in the Cairo Genizah thanks to one trigonometric expression typical of this author. The discovery is discussed within the larger context of the evolution of Hebrew, Latin and Arabic trigonometric terminology and the role of *Yesod Olam*.

Keywords

Medieval Hebrew science; mathematical terminology, Isaac ha-Israeli, *Yesod Olam*, the Cairo Genizah

Introduction[1]

As most, or perhaps, all discoveries, this one happened to me by accident. While I was browsing Genizah fragments on science on the Friedberg Genizah site,[2] fragment ENA 3745.9 from the Jewish Theological Seminary in New York emerged. I noticed the mathematical term *beqaʿ qeshet* (בקע קשת) literally 'half of an arc'. Then I remembered reading that this term, which in modern mathematical terminology designates the trigonometric function 'sine,' is distinctive of Isaac ha-Israeli's mathematical language in his *Yesod Olam* (The Foundations of the World), a

[1] I wish to thank Malachi Beit-Arié and Sacha Stern for their advice.

[2] The URL is http://www.genizah.org/.

major scientific treatise from fourteenth-century Toledo.[3] A perusal of the second printed edition of this text from 1848 has confirmed that this fragment indeed belongs to *Yesod Olam*.[4]

Yesod Olam

Composed in 1310 in Toledo by Isaac ben Joseph ha-Israeli, *Yesod Olam* was dedicated to Rabbi Asher ben Yehiel, the chief Rabbi of Toledo. A priori, this tract could have been regarded as a Hebrew scientific encyclopaedia because it covers numerous themes in mathematics, cosmology, astronomy and the reckoning of the Jewish calendar, but it does not include *all* known mathematical, cosmological and astronomical fields at the time, only those which are pertinent to the calculation of the Jewish fixed calendar. It is thus more exact to perceive *Yesod Olam* as a calendrical treatise. The first three discourses on mathematics, cosmology and astronomy pave the way to understanding the calendrical fundaments in the fourth and fifth discourses of this tract. The calendrical essence of the text is also evident from the author's introduction: …הוא חיבור בסוד עיבור יעורר לבבכם, 'It is a composition on the calculation of the calendar, it will awaken your hearts…'

The Fragment ENA 3745.9

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[3] G.B.A. Sarfatti, *Mathematical Terminology in Hebrew Scientific Literature of the Middle Ages* [Heb.] (Jerusalem 1968), 218–220.

[4] *Yesod Olam* was first published in Berlin in 1777 by Jacob Shklower. A more complete edition, with a preface by David Cassel, was published by B. Goldberg and L. Rosenkranz in Berlin in 1848. It is interesting to note that from a purely probabilistic perspective, the a-priori chances of the fragment being one that includes the term *beqa'* were less than 3 percent, to judge from the distribution of the term in the entire text (my analysis is based on the 1848 edition).

JTS ENA 3745.9 is written in semi-cursive Byzantine hand and it dates from the fifteenth century. Much of it is hardly legible due to erasure, stains and oxidation of the brown ink. Its Byzantine provenance (or at least the scribe's) is significant because only a fraction of the known surviving copies of *Yesod Olam* are of Byzantine origin; the bulk of the remaining copies are of Ashkenazi, Sephardic and Italian origins.[5] One has to wonder how exactly our fragment has made its way to the Cairo Genizah. Was it copied in Fustat by someone of Byzantine origin or was the text sent to Egypt from afar? In any case, the Byzantine origin of this fragment serves as palpable proof to a strong personal or textual link between the Jewish communities in Byzantine and Cairo.

Regarding the content of the fragment, it shows examples demonstrating three theorems on the sine of arcs belonging to spherical triangles, which are the intersection of great circles, i.e., circles on a sphere whose radii equal the radius of the sphere. This material on spherical trigonometry, which is the basis to the astronomical models following in the text, appears towards the end of the first book.[6]

As for the exact location of the fragment within *Yesod Olam*, the recto side[7] corresponds approximately to the last thirteen lines in the left column on page 13a in the 1848 edition, probably going up to the first, or even second lines on page 13b.[8] The text in the verso side of the fragment is found on line 6 of the right column on page 13b, which continues approximately

[5] Only four copies out of fifty-three are Byzantine and they date from the fifteenth and sixteenth centuries This data is based on catalogue information, as found in the site of the Institute of Microfilmed Hebrew Manuscripts at the National Library of Israel, and the URL is http://jnul.huji.ac.il/imhm/.

[6] An analysis of the details within the fragment should be part of an examination of the entire tract, or at least of the entirety of the first book on mathematics.

[7] See Figure 1.

[8] Given the very poor legibility of the first and last lines of the fragment, my estimate is based on the ratio between the length of lines in the fragment and those in the 1848 printed edition.

until line 26 in the same column.[9] There is only a gap of 4 or 5 lines of missing text both sides of the fragment. This means that the recto part was written very close to the bottom of the page and the verso very close to the top, or perhaps even at the very top.[10] Furthermore, the folios in the original manuscript from which this fragment originates must have been relatively small (each containing about 20 lines on each side), unless a diagram was inserted above the contents of the verso side. The latter option, however, is not highly likely because there are traces of diagrams in the margins: a tip of a spherical triangle in the verso side and an arc in the recto side but we cannot rule out the option of diagrams being inserted both within the text and in the margins.

[9] See Figure 2.

[10] The right side of the top part of the fragment is not torn like its left side, and even though it is not straight it seems to have been cut out.

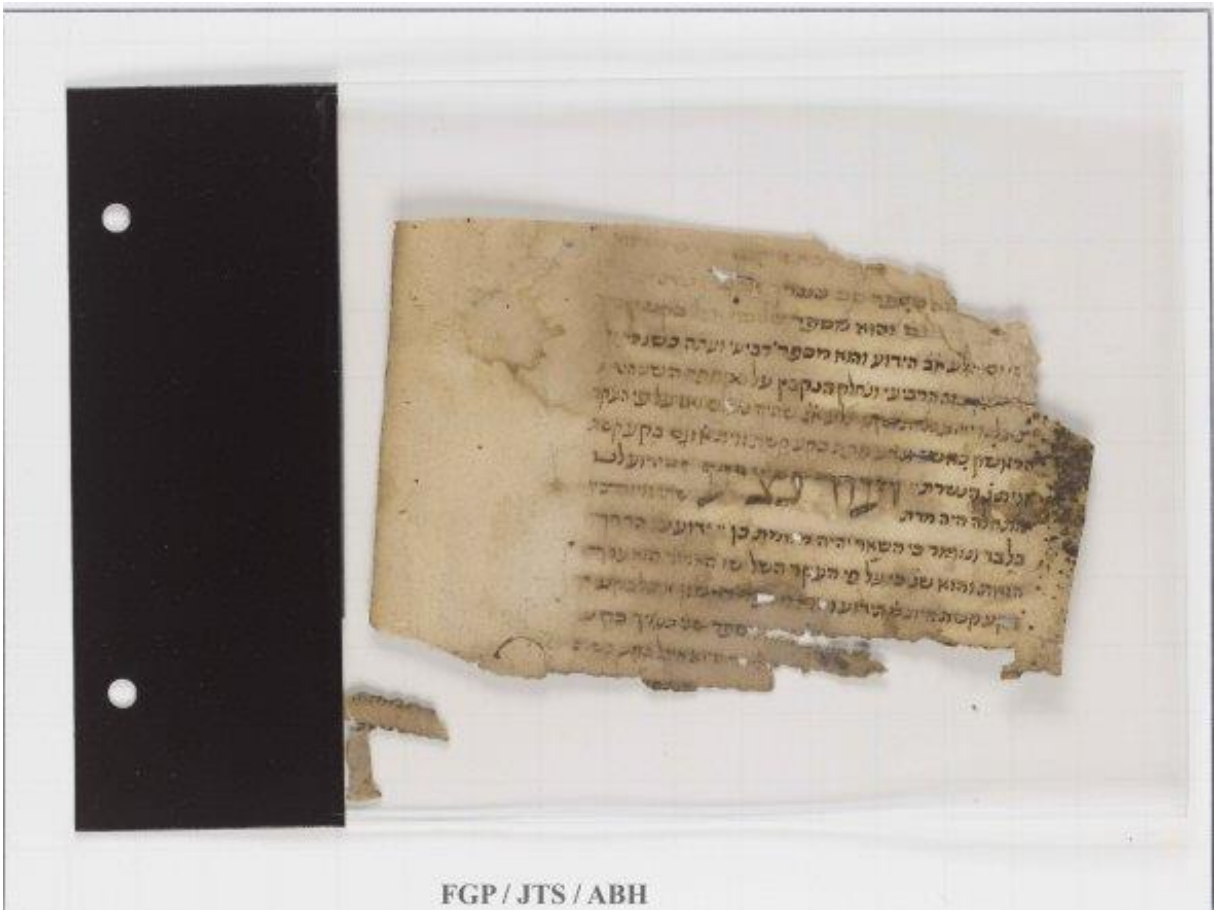

FIGURE 1

JTS ENA 3745.9r

Courtesy of the Jewish Theological Seminary

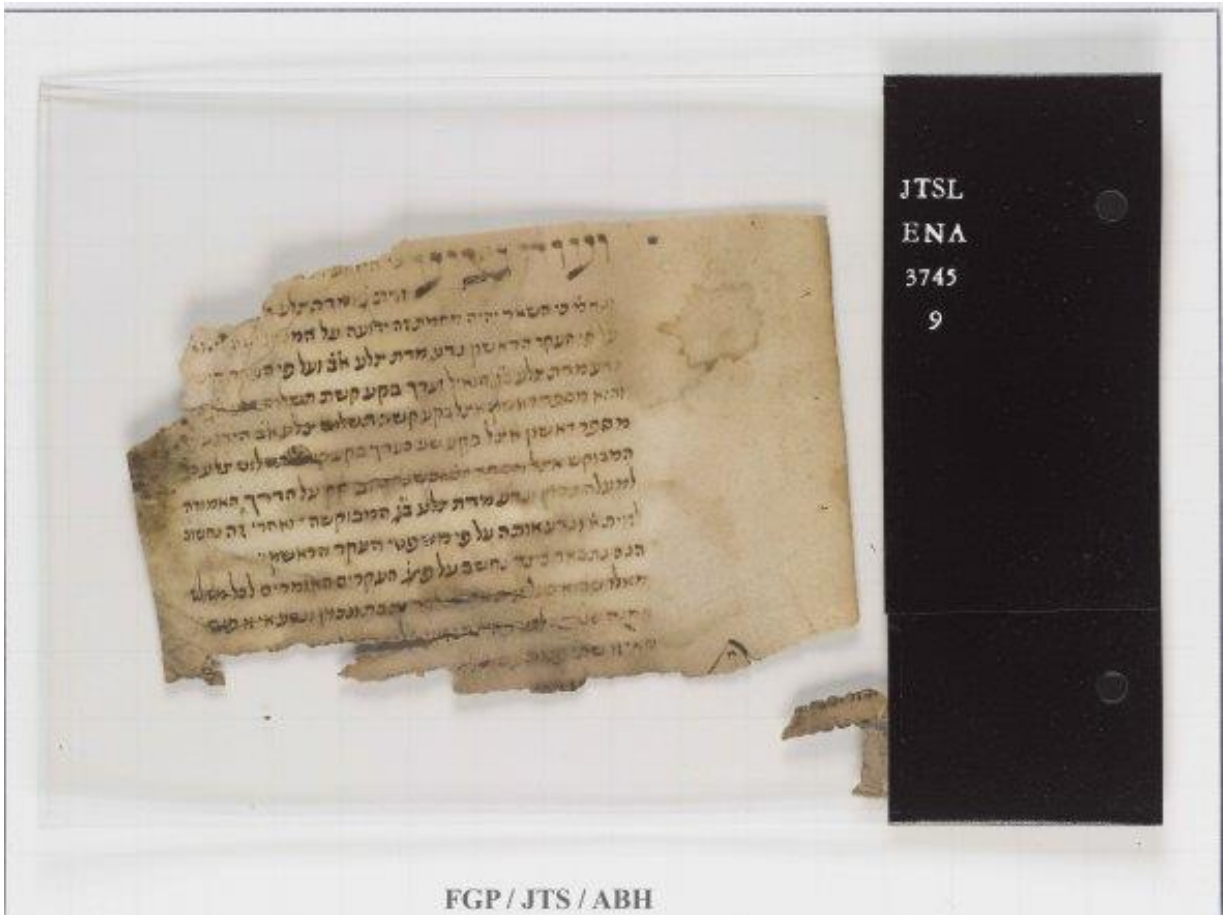

FIGURE 2

JTS ENA 3745.9v

Courtesy of the Jewish Theological Seminary

Isaac ha-Israeli's Definition of Beqaʿ Qeshet

A few folios before the section in which the contents of both sides of our fragment are found Isaac ha-Israeli defines the term *beqaʿ*:

אחוג עתה עגולה קוית סביב מרכז ה ואפריש ממנה קשת אב פחות מחציה בכדי שארצה ואוציא מנקודה ב שהוא הקצה האחד שלה ואמשיך אלכסון בהז ואוציא מנקודה א שהוא הקצה השני שלה ואמשיך קו אג הנופל עמוד על האלכסון בהז ואומר כי על קו אג יאמר כי הוא בקע קשת אב זאת המוצעת כמו שקו אב הישר הוא יתרה וכן הוא אג כמו כן בקע קשת אז שהוא תשלום קשת אב לכדי חצי העגולה ועל דרך כלל אומר כי בקע כל קשת כמו כן הוא הקו הישר היוצא מהקצה האחד שלה ונופל עמוד על אלכסון היוצא מהקצה השני שלה ובקע כל קשת כמו כן הוא כדי חצי יתר כפלה שהרי קו אג חצי קו אד שהוא יתר קשת אבד שהוא כפל קשת אב.

> I shall now draw a circle around the centre E[11] and demarcate any arc AB whose length is less than half the circle. I will start in point B which is its one end and outline the diameter BEG. I will start from point A, its other end, and outline the line [i.e., segment] which is perpendicular to the diameter BEG. I will say that line AC is said to be the *beqaʿ* of the assigned arc AB such as the straight line AB is its hypotenuse. Also AC is the *beqaʿ* of arc AG, which is the complement of arc AB to half a circle. In general, I will say that the *beqaʿ* of every arc [AB] is the straight line emerging from its one end [A] and falling perpendicularly on the diameter emerging from its other end [B]. The *beqaʿ* of every arc is also half the chord pertaining to its double arc since the line AC is half the length of line AD, the hypotenuse of arc ABD, which is double the arc AB.[12]

[11] See Figure 3.

[12] B. Goldberg and L. Rosenkranz, *Yesod Olam* (Berlin 1848) 9a.

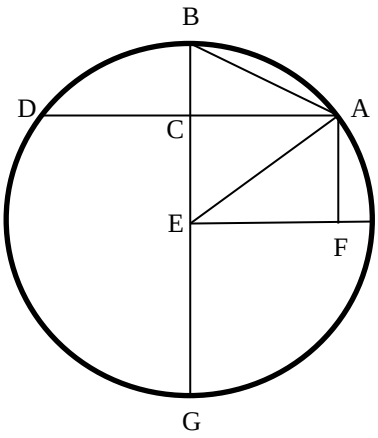

FIGURE 3

The readers will surely notice the equivalence between the medieval definition of *beqa'* and the modern trigonometric definition of the sine of the central angle which leans on chord AB; the ratio between the opposite side AC and the hypotenuse AE, which without loss of generality, can be assumed to be of length 1 (a unit circle). Similarly, it is easy to see that Isaac ha-Israeli's definition of the *beqa'* of the complementary arc (בקע קשת תשלום) is equivalent to the modern definition of the cosine of the central angle which leans on chord AB, i.e., the ratio between the adjacent side EC and the hypotenuse AE in a unit circle.

The Etymologies of Beqa' and Sine

During the Renaissance of Hebrew of the twelfth century, Hebrew gradually became a scientific *lingua franca* for Jews.[13] Greek and Arabic science was transmitted into Hebrew and new scientific Hebrew vocabulary emerged. Various mechanisms were implemented in order to create scientific jargon that was lacking in the Hebrew vocabulary of the time. The method relevant to the formation of the term *beqa'* is the adoption of part of the meaning of the biblical

[13] T. Lévy, 'Hebrew Mathematics in the Middle Ages: an Assessment,' in F. Rajep and S. Rajep, eds, *Tradition, Transmission, Transformation* (Leiden 1996) 71–88.

word *beqaʿ* and bestowing it with mathematical meaning when combined with *qeshet* (arc), resulting in the meaning 'sine.'[14]

The word *beqaʿ* appears in the Bible twice, the first time in Genesis 24:22:[15]

> When the camels had finished drinking, the man took out a gold nose ring weighing a *beqaʿ* and two gold bracelets weighing ten [gold] shekels.

וַיְהִי, כַּאֲשֶׁר כִּלּוּ הַגְּמַלִּים לִשְׁתּוֹת, וַיִּקַּח הָאִישׁ נֶזֶם זָהָב, בֶּקַע מִשְׁקָלוֹ--וּשְׁנֵי צְמִידִים עַל-יָדֶיהָ, עֲשָׂרָה זָהָב מִשְׁקָלָם.

Thus we learn that *beqaʿ* is a measure of weight. In Exodus 38:26 we find its precise definition:

> One *beqaʿ* per person, that is, half a shekel,[16] according to the sanctuary shekel, from everyone who had crossed over to those counted, twenty years old or more, a total of 603,550 men.

בֶּקַע, לַגֻּלְגֹּלֶת, מַחֲצִית הַשֶּׁקֶל, בְּשֶׁקֶל הַקֹּדֶשׁ--לְכֹל הָעֹבֵר עַל-הַפְּקֻדִים, מִבֶּן עֶשְׂרִים שָׁנָה וָמַעְלָה, לְשֵׁשׁ-מֵאוֹת אֶלֶף וּשְׁלֹשֶׁת אֲלָפִים, וַחֲמֵשׁ מֵאוֹת וַחֲמִשִּׁים.

Isaac ha-Israeli was clearly inspired by the connotation 'half' of the biblical *beqaʿ* to apply it in a purely trigonometric framework and create the term 'sine.' As far as we know, in the medieval mathematical Hebrew context, *Yesod Olam* is the first and last known medieval text in which

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[14] Other techniques include the extension of the semantic field of words found in Rabbinic literature, or the use of Arabic (loan translation and phono-semantic matching), see Sarfatti, *Mathematical Terminology*.

[15] The translation of the biblical verses is based on the New International Version (NIV).

[16] About 1/5 ounce or about 5.7 grams, see, for example, the NIV translation.

beqa' is used.[17] Among his mathematical predecessors, Abraham bar Hiyya (c. 1065–c. 1136) created the term 'a halved chord' (מיתר מחוצה) whereas Abraham ibn Ezra (c. 1092- c. 1167) coined the expression 'half a chord' (חצי יתר).[18] Isaac ha-Israeli either did not know the mathematical works of the two Abrahams, or deliberately chose to create his own term for 'sine.'[19]

None of the medieval terms for 'sine' was revived in Modern Hebrew.[20] Instead, one uses the transliteration of the Latin 'sinus' (סינוס). This is linguistically unfortunate because all three Hebrew terms we have seen intrinsically convey the mathematical idea that 'sine' is 'half the chord,' whereas the Latin term 'sinus' was born in error. In Ancient Greece astronomers used the relation between a chord and the arc which pertains to it. However, mathematicians in India, followed by the Arabs, replaced the chord of a certain arc with half the arc of the double arc. The Arabs used the term 'half a chord' (نصف وتر) but also borrowed the Sanskrit *jiva* (chord). Robert of Chester, in his twelfth-century Latin translation of al-Khwarizmi's astronomical tables from the ninth century, confused the Arabic term *jiba* with *jaib* (inlet, bay), probably because the vowels were omitted (جيب) and this resulted in the false translation 'sinus.'[21]

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[17] Joseph Solomon Qandia Delmedigo in his *Sefer Elim* (Italy, 17th century) used *beqa'*, referring to Isaac ha-Israeli.

[18] From the Arabic نصف وتر.

[19] Medieval Jewish scientific authors coined mathematical terms either for the obvious reason of wanting words in Hebrew or not being aware of existing ones. However, in other cases, a novel scientific term served as a 'lexical signature' of the author. I suspect that this is the case here because the mathematical works of 'the two Abrahams' were wide-spread, and Isaac ha-Israeli analysed parts of Abraham bar Hiyya's calendrical work in *Yesod Olam*.

[20] In Modern Hebrew the term *beqa'* means either 'a crack' or, in the medical jargon, hernia(!)

[21] Sarfatti, *Mathematical Terminology*, 107–108, and C.B. Boyer, *A History of Mathematics* (New York 1989) 252.