Applying an Entropy Maximising Model for Understanding the Rise of Urbanism

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Abstract

The chapter presents a spatial interaction entropy model that addresses the dynamics of urban growth using sites from the Late Uruk period in southern Mesopotamia as examples. The model addresses to what extent geography, transportation, and factors that make locations attractive for trade and settlement affect why some settlements grow while others stabilize or diminish in size through time. The results show that geographic and transport factors can enable some regions, such as the northern and central alluvium in southern Mesopotamia, to have some initially favourable advantages for urban growth. In contrast, greater attraction to specific centres and decreasing mobility of goods and people to many towns enable sites such as Uruk to rapidly grow through positive feedback effects without natural population increase. This growth also influences other settlements' populations and use of the transport infrastructure, where aggregation of population to few centres leads to a large number of small sites or even near abandonment of sites. Other results demonstrate how external trade and contacts enable towns to prosper at the expense of other settlements as well as how settlements could become relatively resilient to changing conditions that diminish their populations by having effective links to sites and transport infrastructure. Overall, the results demonstrate a quantitative model that is useful in explaining periods of rapid urban growth and regional urban layout transformations without necessarily having full knowledge of the archaeological data.

Keywords: spatial interaction, retail modelling, entropy, urbanisation, Uruk
Supplementary data: http://discovery.ucl.ac.uk/1355838/

Introduction

This chapter introduces a spatial interaction model for investigating the development of preindustrial urbanism and regional settlement patterns in southern Mesopotamia. The chapter uses sites from the Late Uruk period as examples of how the geographic setting and attractiveness of sites for trade and settlement, whether because of social or environmental reasons, could have influenced urban growth or decline. Southern Mesopotamia serves as a natural setting for the study of early preindustrial urban change, in part because debates and discussions have focused on this region particularly during the Uruk period (Adams 1981, Pollock 2001, Algaze 2008) when some of the first cities arose. While theories such as Complexity Theory (Adams 2001) have been increasingly used to discuss how urbanism developed in southern Mesopotamia, relatively few attempts have advanced quantitative techniques that could provide an explanation of relevant processes that propelled urban transformations.

Spatial entropy maximising models (Harris and Wilson 1978; Wilson 2012; Dearden and Wilson 2012), or spatial interaction models, have the potential to provide explanations that address how settlement expansion or contraction under given geographic settings occur as well as how settlements affect each others growth. Such models are particularly attractive because specific causal factors (e.g., avulsion, ideology, population pressures, etc.) for urban change are difficult to isolate or quantify from the archaeological record. However, such transformative vectors' effects can be represented or applied within general models, such as entropy models, for understanding how they affect urban growth. In summary, entropy models allow the incorporation of spatial factors and feedback effects and interactions over a given time that enable urban patterns to develop across a study region while avoiding a focus on specific factors that created initial conditions for urban transformations to develop. This chapter explores the utility of such modelling and provides the potential insights that might be gained even when the data are limited. The goal is to present a simple simulation model that explores how the spatial setting and factors that affect the flow of goods and people can influence urban transformations and settlement layouts. The chapter utilizes the pattern of Late Uruk sites (LC 5: ca. 3400-3000BC) first surveyed and compiled by Adams and Nissen (1972) and Adams' (1981) in southern Mesopotamia. While using empirical survey data, the chapter also uses hypothetical data to explore how sites that initially seem to be equal in size and population may quickly become differentiated and grow or diminish in size as a result of feedback and interaction. While one cannot recreate what exactly happened in the Uruk period to contribute to urban transformations, the model applied demonstrates a formal approach that suggests explanations for evolutionary processes that affect settlement within a geographic setting.

The paper begins by introducing background information on the case study and discussions on early urbanism and relevant theoretical perspectives. The modelling applied is then presented in order to demonstrate how the concepts discussed and the associated background information can be applied in quantitative form. After this, the modelling results, including variations from the initial scenario, are provided. These results explore different factors that may catalyze or diminish urban population growth. The significance of the modelling results to the understanding of the development of early urbanism in southern Mesopotamia is then discussed. Finally conclusions are provided with regard to the methodology and its potential for general ancient settlement system research.

Background to the Case Study

The data applied for this case study derive from surveys conducted by Adams and Nissen (1972) and Adams (1981) in southern Mesopotamia between the Tigris and Euphrates south of Hillah and to the region just south of Samawah (Figure 1). While the modelling uses sites that date to the Late Uruk period, which corresponds to a period when the city of Uruk and other settlements in southern Mesopotamia had become established as major centres (Algaze 2008), the intent here is to demonstrate how major sites could arise while also affecting the surrounding settlement layout. Therefore, the goal is not to thoroughly present each site and explain why it may have been settled or reached a given size or population by the Late Uruk; rather, the goal is to show how processes associated with urban transformations could have developed once given conditions, such as environmental or social advantages, are in place. Although some site size estimates from archaeological surveys can be problematic, the results, in general, demonstrate that in the Late Uruk period relatively few sites appear to have reached a large size, whereas the vast majority were very small. Certainly by the Late Uruk many aspects of southern Mesopotamian urbanism, such as large temple complexes, monumental architecture, and political institutions, were well established, indicating a level of social complexity not witnessed during the Ubaid and other prehistoric periods (Algaze 2005). Although it is very likely that by the Early and Middle Uruk such urban characteristics were already established (Wright 2006), sites from the Late Uruk are focused on because it is clear that this period represents a mature stage of early urbanism; the site of Uruk, for example, may have reached a size of approximately 250 ha by this period (Finkbeiner 1991).

Not only has there been considerable scholarly interest in the Uruk period, contemporary archaeological theory has also tended to focus on the economic and environmental circumstances that made southern Mesopotamia attractive for settlement and urban growth in the 4th millennium BC (Hole 1994; Algaze 2005). This was a period when the regional climate was probably wetter, and provided relatively ample water supplies (Aqrawi 2001). The marshes and hinterlands must have provided ideal food sources for early centres to develop (Pournelle 2003). Such circumstances provided the potential for southern Mesopotamia to develop an urban society at an early date. Nevertheless, such information does not fully explain how the process of urbanism, with its associated feedbacks and interactions between different social and environmental components, developed over decades or hundreds of years. In addition, it is unclear from such data how initial advantages for some population centres may affect surrounding urban regions.

Theories on Urban Growth

Algaze (2008) has summarized the economic and environmental reasons that underlie the early development of southern Mesopotamia urban growth. Economic advantages include the development of low-cost transport, via canals and boats, increased trade activity, and geographic location. Together, these not only help centres to initially develop, but also enable positive feedbacks to form that reinforce the advantages gained with the result that towns grow even further. The effects of positive feedback systems on urbanization have certainly been noticed in the context of modern trade and the global economy. In such cases, larger or more significant centres took advantage of their economic and social position and grew to unprecedented scales, whereas other regions diminished in economic strength due to processes of negative feedback (Krugman 1995). Feedback systems essentially create new economic and social opportunities, such as facilitating innovations and entrepreneurship (Lane et al. 2009), that enable major cities and regions to grow to a greater extent. These then have secondary and interactive effects between settlements that result in reduced or increased population and economic growth in the entire settlement structure within regions (Jacobs 2000). The system of growth and decline may then stabilize, that is reach homeostasis, until new events disrupt a given state, once again changing patterns of settlement growth and decline. Similar reasons have been given for processes that underlie the development of states and empires, urban development, and changing social or political complexity in various periods and parts of the world. For example, Braudel (1995), Fox (1971), and Desrochers (2001) all incorporated geography, environment, and transport as factors that shaped social developments that transformed major urban centres so that they possessed significant advantages relative to other regions. While such factors probably played a major role in urban growth, ideology, government, and social institutions that concurrently developed in association with settlements also affected social norms (Wheatley 1971), which can act to reinforce the status of major centres (Collins 2000). Ideology and institutions could, therefore, contribute to positive feedback, enabling settlements to expand further and simultaneously potentially diminishing the population and economic potential of other urban settings. In fact, for early preindustrial societies, one can likely assume that low rates of natural population growth (McNeil 2000) prevented rapid urban growth, whereas “pull” factors, such as trade, economic incentive, or ideology, are more likely to have increased population and size of settlements more rapidly while at the same time diminishing other places (Persson 2010).

Cronon (1991) demonstrates a specific example of feedback and associated growth in his presentation on the reasons behind the rapid growth of Chicago in the 19th century. Despite Chicago’s initially marginal setting, the geography of Chicago, with its access to large inland lakes and rivers and its central location within the burgeoning railroad networks of the 19th century, soon provided the city with significant advantages over other urban areas. These early advantages eventually contributed to the development of industries and economic innovations, which, in turn, attracted many more people to the city. Thus, while the city grew naturally from population growth, the vast majority of Chicago’s early population growth is attributed to the pull and attractiveness that the city had for many migrants and goods. As the city grew, the surrounding geography and smaller urban centres developed in relation to the larger city of Chicago, so that some areas benefited while others lost prominence in relation to Chicago.

Such examples of Chicago’s rapid growth and reasons as to why preindustrial cities grew or diminished in population lend themselves well to Complexity Theory, which incorporates concepts of feedback interactions as well as the role that socio-environmental systems play in influencing societal transformations and urban development (Adams 2001; Batty 2005; Bentley and Maschner 2008). Complexity Theory essentially provides an analytical framework that allows systems such as the urban setting, environment, and cultural practices, to interact through feedbacks that shape how cities transform. However, some factors have more relevance in contributing to transformations, and it is these interactions that enable a settlement to grow, decline, or stabilise, sometimes at a very rapid rate.

Discussions of the economic, geographic, and ecological factors contributing to urban growth are well articulated in the field of urban geography. Huff (1963) had conceptualized these factors in work describing trade and urban areas for modern systems. Later, more formal, quantitative methods were developed in order to analyze spatial relationships, population, markets, and the significance of centres in attracting urban consumers together with population growth (Wilson 1967; Wilson 1970). This type of modelling has been used to describe how cities develop in a variety of different settings and circumstances, both in the present (Birkin and Heppenstall 2011) and past (Wilson 2012). Given advancements in quantitative methods that explain spatially bounded urban transformations affected by different social and environmental vectors, one can begin to apply approaches such as Complexity Theory in a quantitative form to demonstrate and explain urban transformations such as those evident by the Late Uruk period in southern Mesopotamia.

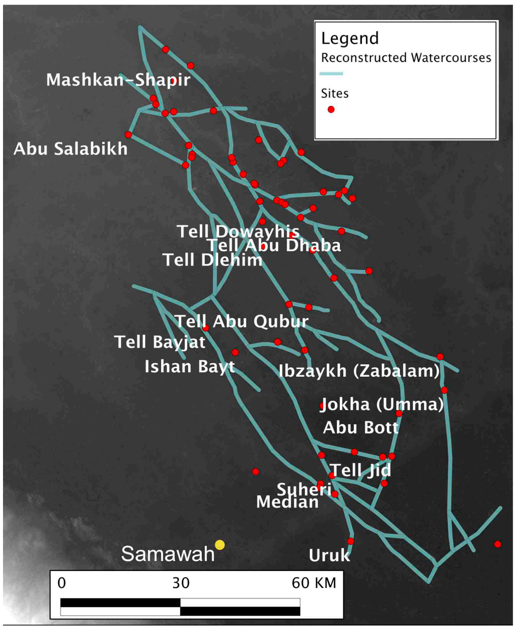

Figure 1. Late Uruk sites investigated in southern Iraq (site shapefiles provided by Carrie Hritz) and ancient watercourses reconstructed from ASTER (2012) elevation data.

Modelling Approach

Among the most widely used urban economic or population growth models have been entropy maximisation models. These have been used to describe, not only urban growth within regional contexts, but also smaller scale settings such as the locations of individual stores and areas within cities that have experienced different economic fluctuations (Heppenstall et al. 2006; Birkin et al. 2010; Birkin and Heppenstall 2011). These models have traditionally taken a form of Lotka-Volterra equations, which have been used to model such dynamics as predator-prey relationships. In the case of urban and spatial modelling, these have been applied in the form of Boltzmann-Lotka-Volterra equations (Wilson 2008). Entropy maximisation essentially allows estimates of likely areas of population growth or decline in geographic locations under conditions of uncertainty. In this case, factors of distance, economic or social relevance, and movement capability become the generalized variables that account for urban transformations. These generalized variables allow one to capture a wide range of phenomena, including social and environmental conditions that place some regions or urban settings at a relative advantage to others. Such models also allow feedbacks and interactions between settlements to develop such that sites evolve based on how other sites in the given region change whereby a level of stability can be reached or conditions within the system could change once again, causing further evolution in settlement growth. More recently these models have incorporated bottom-up perspectives, specifically via the application of agent-based modelling (Dearden and Wilson 2012). Nevertheless, the underlying entropy model has continued to be an effective tool for articulating how population growth or decline in cities and markets takes place over a given period and region.

For the purposes of this paper, a spatial entropy model, such as that advanced by Wilson (1967; 1970) and Harris and Wilson (1978), is very useful for applying to preindustrial urban contexts where rapid population growth is less likely to have been driven by natural population growth. In other words, to understand how preindustrial cities grew one needs to understand non-biological growth that created opportunities for some centres to thrive whereas others grew at only marginal rates or even declined. Because this is the pattern apparently noticed for the Late Uruk period (Adams 1981), where few centres exceeded 20 hectares and a vast majority of sites were likely to be very small (on the order of 1 hectare or less), the model presented is used to describe this process in quantitative form. The first step as the model begins is stated as:

$$S_{ij} = e_i P_i W_j^{\alpha_j q_j} \exp(-\beta d_{ij}) \tag{1}$$

where S is flow (or movement of goods or people) for settlement i in relation to another settlement j; e_i is a scalar used for settlement i; P is the population of site i; W is the size of settlement j, in this case the variable is a relative and notional value; α represents the "attractiveness" of site j's location and size, which can include social or environmental factors that make specific settlements more attractive than they otherwise would be; q represents external contacts, namely those outside the modelling region, such as trade that make j more or less attractive; β represents the willingness or ability of individuals to travel a given distance to a settlement, (i.e., as β increases, an individual's preference to travel short distances increases for any given reason); and d is distance. In summary, this step determines how much flow or relative rate of movement of goods or people to a given site based on attractiveness (α), including external links that make a site attractive (q), willingness or ability to travel (β), and distance (d) in relation to the size (W) and population (P) of a given settlement.

For outputs of S_{ij}, a base-10 logarithm is adopted to scale the value. In this model, distance (d) between towns is determined by an A* algorithm (Hart 1968) that uses distance based on watercourses connecting sites (see Figure 1). In other words, in the simulation people and goods from within sites access the nearest watercourse to them and move to sites using the shortest distance along any watercourse. This simulates the effect of river-based traffic as a major factor in shaping urban growth in southern Mesopotamia, and it also calculates the distance using relative topographic changes in the region. This topography, which represents the levees of channels and rivers, is based on ASTER elevation data (ASTER 2012).

Other ways of determining d include the use of a probabilistic framework for route selection or social factors (e.g., political or territorial divisions) that constrain route selection between cities. In terms of modern economics, this simple model could make it possible to determine likely profits for any given location based on "capital" flow from consumers as mediated by distance, accessibility, and overall attractiveness to consumers. For application to ancient societies, flow signifies the flow of goods and people through migration to different settlements. In the next step of the model, overall flow for a given site j is then determined:

$$D_j = \sum_i S_{ij} \tag{2}$$

where the summation of S is D (overall or cumulative flow); in this case all cities are considered to be residential zones with some market or social functions that have flow or movement of goods and people. The next step determines the size of an urban area based on overall flow calculated:

$$W'_j = W_j + \varepsilon(D_j - kW_j) \tag{3}$$

where W' is the next increment of site size (i.e., in the next time iteration) based on ε that controls the pace of change, overall flow (D), a constant (k), a notional value representing the cost of operating a city of a given size, and the current city size (W). This step simply looks at revenues and costs and how these either negatively or positively influence urban growth. While k is difficult to determine for ancient urban areas, it is possible to measure the growth of cities relative to each other. Absolute growth is not the main focus; rather, because the results measure sites against each other, a relative scale is used to determine where urban growth is more pronounced in relation to other sites. This allows us to account for k without necessarily requiring the specific knowledge of this variable. Therefore, k is constant for all sites and simulation scenarios. In the model applied, the next step is then to determine the population and where population is drawn towards as a function of settlement size. In other words, the next step applies population migration or flux to settlements in the model based on the following function:

$$P'_j = \| W_j * \sum_j P_j \Big/ \sum_j W_j \| \tag{4}$$

This takes the nearest integer of the result in order to modify P. Essentially, this proportionally adjusts j's population based on site size calculated in (3). Although it might be difficult to determine the exact population for settlements, in a manner similar to W, P provides a notional value that indicates where population is likely to cluster and at what relative scale. For the simulated cases, there is no population growth and people are assumed to migrate based on which city is growing in size. Given that W and P are used hypothetically to enable comparisons between sites, raw outputs are not intended to replicate empirical values.

Modelling Results

Based on the above model, four scenarios are represented in the following results. The scenario data used and discussed for the simulations can be found at: http://discovery.ucl.ac.uk/1355838/. Whereas sites sampled in the modelling date to the Late Uruk period, the model uses demonstrative values to test how trends in urban transformations may develop. This has the goal of determining how a situation like that of the Late Uruk, where few urban centres thrive, is possible even if initially populations and site sizes are the same. Results are then able to provide insights into how centres emerge, stabilize, or decline, assuming that such dynamics occur without biological population change. Given these assumptions, the first example provides a baseline case where all values are equal for all locations; this tests the role of geography and transport in shaping urban growth. In the sub-scenarios for this case, α (attractiveness) and β (movement willingness or ability) are the same for all sites, but are varied in each sub-scenario. The next scenario tests variations on α for specific sites, together with changes to β, which demonstrate how differential values representing each settlement's importance can be used to create urban layouts similar to those of the Late Uruk. The third case investigates the role of foreign trade and contacts in affecting urban change. Finally, a scenario is applied which investigates how shifts in urban importance and the flow of goods can influence the resilience of populations in surrounding settlements.

Scenario 1: Baseline Scenario

For this case, the intent is to identify which sites may have geographic and transport advantages that allow them to potentially emerge as relatively larger sites and to what extent site attractiveness (α) and willingness to travel and move goods (β) affect migration of people to settlements. As α increases, attractiveness to any given site is considered to increase, whereas the increase of β indicates there is less willingness to travel far to sites. Initial values used for this scenario are listed in Table 1, specifically for Scenario 1a, with sub-scenarios b-i varying α and β. Therefore, in all sub-scenarios site size and population are equal at the beginning of the simulations. The scenario is run until the population and size results are considered stable or the simulation ends, resulting in runs being 120 simulation ticks long.

Figure 2 shows the end result of this scenario; the outputs indicate the point locations and the populations of the settlements relative to each other. The point colours differentiate where population concentrates in settlements. When comparing sub-scenarios, it is evident that Figure 2c has the greatest variation in site population values. In contrast, Figure 2g has the least variation, indicating that population is more evenly distributed. While α increases (i.e., Figure 2d-i), resulting in the increase of the size variable (W), proportionally, the population of settlements does not increase because all sites are increasing in size together. In other words, when α alone increases for all sites, no settlement becomes pre-eminent over other sites, thus creating a relatively even distribution of population. On the other hand, β has a greater effect in the differentiation of population (Figure 2b-c, e-f, h-i), so that more distant and less centrally located sites become less attractive as β increases. Figure 3 shows population differences in three of the sub-scenarios modelled, showing that low α and greater β result in populations becoming more differentiated. In essence, equally high α values do not allow any particular settlement to increase dramatically in population, whereas changes to β have a more pronounced effect on population differences.

This results in Figure 2c exhibiting the greatest differentiation in population because the more north central sites in the model region have more nearby settlements, and therefore draw on greater access to flow as well as movement of people because there is less incentive in going to more distant settlements. This results in the north central sites (e.g., Sites 1129, 1069, and Tell Dlehim) increasing their populations relative to surrounding sites. The results imply that areas to the north of Uruk and in the region of Nippur are more likely to attract a greater portion of people if all others factors are equal. Another pattern to note is that Uruk is among the smallest sites in all the results, reflecting that it does not attract as much flow of goods and people despite its spatial location and accessibility along watercourses.

Population *(P)*	**Size** *(W)*	**Scalar** *(e)*	**Attractiveness** *(α)*	**Travel** *(β)*	**ε**	**Size Cost** *(k)*	**External** *(q)*
1000	20	1	1	0.2	0.05	1	1

Table 1. Initial values used in Scenario 1a.

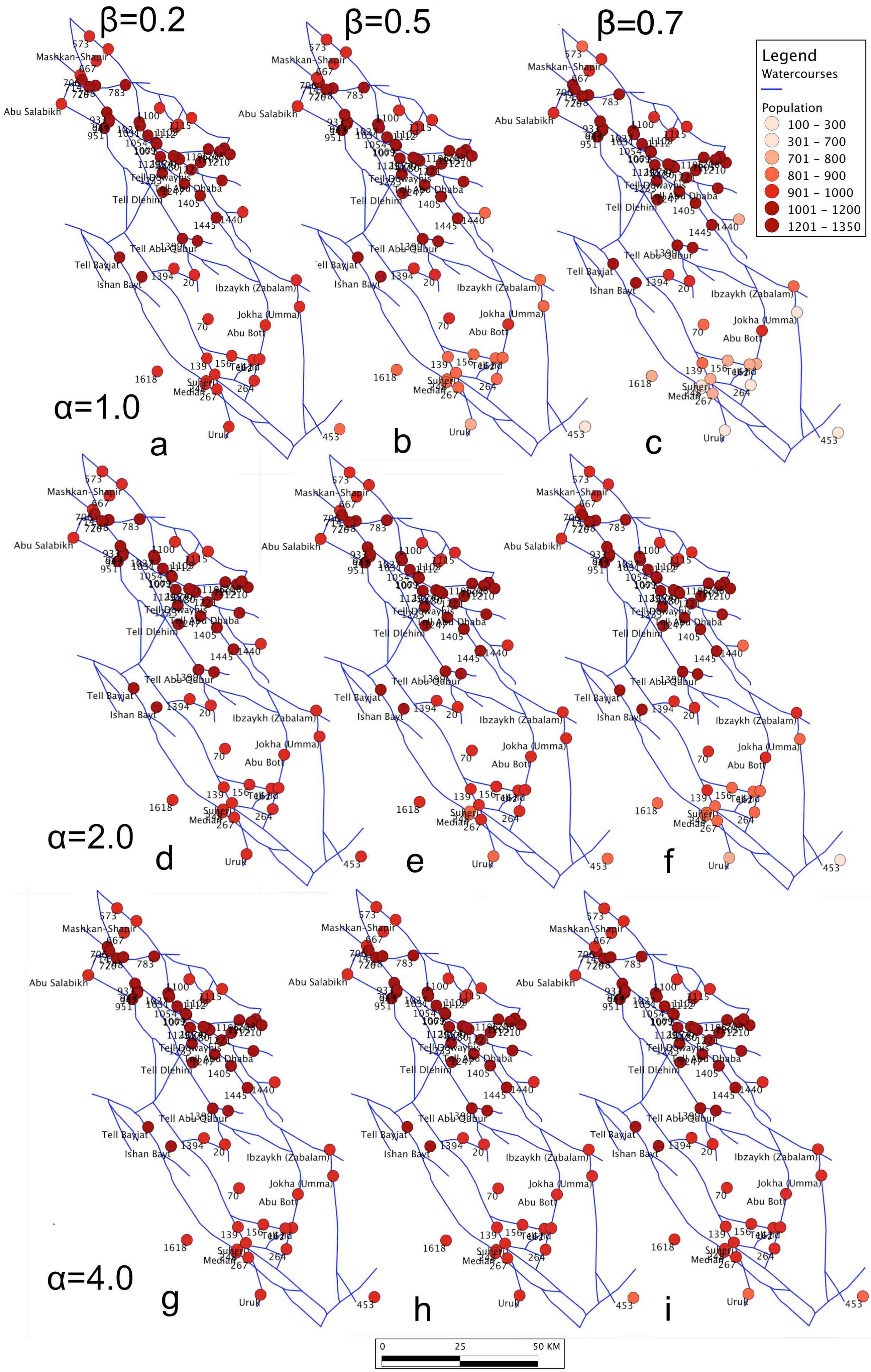
β=0.2
β=0.5
β=0.7
α=1.0
α=2.0
α=4.0
a
b
c
d
e
f
g
h
i
Legend
Watercourses
Population
100 – 300
301 – 700
701 – 800
801 – 900
901 – 1000
1001 – 1200
1201 – 1350
0
25
50 KM
573
Mashkan-Shapir
667
783
Abu Salabikh
1100
1115
951
1054
1112
1210
Tell Dowaybis
Tell Abu Dhaba
Tell Dlehim
1405
1445
1440
Tell Abu Qubur
Tell Bayjat
Ishan Bayt
1394
20
Ibzaykh (Zabalam)
Jokha (Umma)
70
Abu Bott
1618
139
156
Suheri
Mediab
267
264
Uruk
453

Figure 2. Final simulation results from the baseline scenario (a-i) showing variations of α and β and their effects. Settlement population (*P)* variations are displayed using colour and point size.

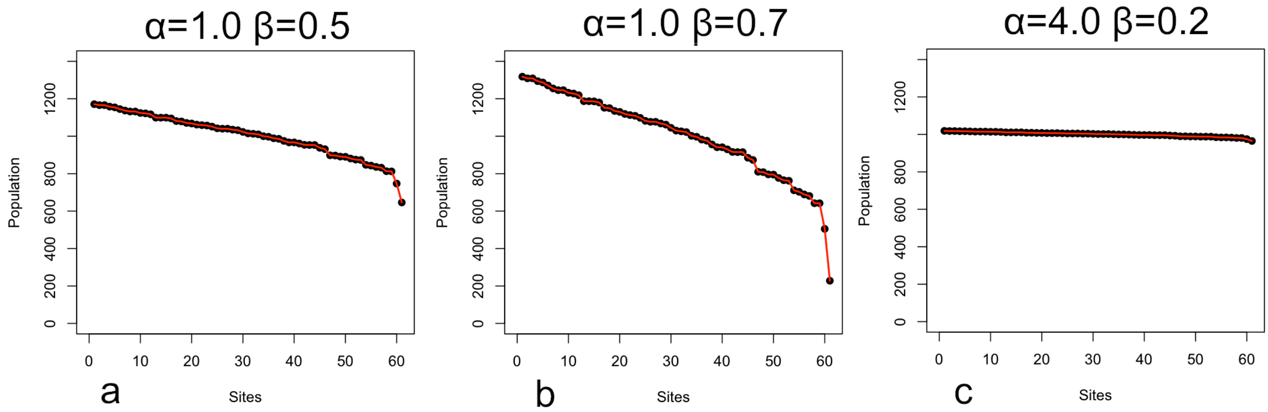

Figure 3. End population results for three (Figure 2b, c, & g respectively) of the nine subscenarios modelled in Scenario 1; line indicates a lowess regression for site populations.

Scenario 2: Variation of Site Attractiveness and Willingness to Travel

The previous scenario simply examined the geographic and transport advantages within the region being modelled. However, it is known that Uruk, a relatively southern site, became the largest site during the Late Uruk period. In Scenario 2, we investigate reasons as to how this may have developed specifically based on regional interactions. This scenario conducts parameter sweeps, or incremental and iterative variation of parameter values, (North and Macal 2007) and tests, specifically for α, for each site, and β as a global variable, with all other values remaining the same as in Scenario 1; these settings can be found in the data hyperlink provided. The intent is to find α and β settings that produce resultant populations that at least qualitatively resemble those perceived for the Late Uruk (Adams 1981). Two sub-scenarios (a & b) in Scenario 2 are used to illustrate contrasting results. As before, the outcome of the simulation focuses on population variation (Figure 4), with point size derived by dividing a settlement's population by the median population value for the region. The scenario has been run until the results appear relatively stable (120 ticks in Scenario 2a) or when sites completely lose their population (78 ticks in Scenario 2b).

Figure 4a shows the population outputs for Scenario 2a, whereby Uruk, Umma, Tell Dlehim, and Abu Salabikh are the largest sites. Uruk has now become nearly 62 times larger in population than the median site in Scenario 2a, while Tell Dlehim (~12 times larger), Umma (~12), and Abu Salabikh (~6) have become the next largest sites. On the other hand, the majority of sites are near 1.0 or lower and appear in Figure 4 as very small sites or not at all. For Scenario 2b, β has been changed from 0.2 to 0.44; this increase was chosen, rather than even higher values, because higher β causes the simulation to end very quickly, as sites reach 0 size and population. In Figure 4b (i.e., Scenario 2b), the results are even more dramatic than in Scenario 2a, as Uruk increases to ~81 times the population of the median site. The next largest sites remain roughly the same as before except that their sizes are now ~15, ~15, and ~7 times the median population value respectively. These have not increased as much as Uruk but nonetheless they represent an increase from Scenario 2a. As for the smaller sites, they have become even smaller than in Scenario 2b. This demonstrates the effect of β once again, whereby travel to the larger α sites is maintained, but increasing β provides less reason to travel or maintain relative flow to sites with a lower value of α. This results not only in greater relative accumulation of cumulative flow to the larger centres, but also that sites near those with larger α values and farther away from other sites, decrease in population more dramatically. The settlement populations for the two subscenarios are indicated in Figure 5a&b.

Overall, the median population values for the region are 388 and 329, which represent a decrease of 15%, for Scenarios 2a&b respectively; standard deviation are 3155 and 3479, indicating a greater variation in settlement population in Scenario 2b. This result indicates that a larger percentage of the population is concentrated in fewer larger centres in Scenario 2b, resulting in larger centres. In contrast, in Scenario 2a smaller settlements are better able to maintain their population. Figure 5c&d illustrate this result and show settlement size for two contrasting sites as well as what happens to larger and smaller settlements. W, (settlement size), is always increasing or stabilizing, although not at the same rate, throughout Scenario 2a (Figure 5c&d), resulting in slight growth or a stabilizing population for Site 453. However, this is not the case in Scenario 2b. In this Scenario, Site 453 (Figure 5d) has negative population flow and eventually reaches zero, whereas Abu Salabikh maintains a slightly higher size. This indicates how a decreased incentive for travel to smaller sites contributes to the raw size of a site with population reaching 0 for Site 453 at the end of the simulation run. In the case of Site 453, this is located near Uruk, therefore a greater portion of its population is pulled to Uruk, whereas at the same time few sites near Site 453 are able to contribute population. As a result, there is a disproportionate decline in population for any settlement that is more distant from other sites and at the same time is affected by a nearby, larger site's attraction.

To emphasize the contrast in the sub-scenarios, Figure 6 shows aggregate flow (S_{ij}) between sites for sub-scenarios a & b. As indicated in (1), flow relates to how sites receive goods and people from surrounding towns, ultimately affecting site size (W) and population (P). In other words flow measures how much pull a site may have on the surrounding settlements, because flow helps to determine how large a site will become. In Figure 6, which looks at the largest and smallest two sites in sub-scenarios a & b (Uruk and Site 453 respectively), the results indicate the underlying dynamics that affect differences in the sub-scenarios. In Figure 6, Uruk always receives a larger share of the flow between the two towns. However, there is a distinction between Scenario 2a&b. Whereas there is positive flow in Scenario 2a for both Uruk and Site 453 (Figure 6a), in Scenario 2b Site 453 (Figure 6b) experiences mostly negative flow. This negative flow causes the outcome seen in Figure 5d, in which size and ultimately population decrease toward 0. While the results in Figures 4b and 5b show that Uruk increases in size in Scenario 2b; however, Figure 6b makes it clear that flow is not actually increasing but is sharply decreasing at smaller sites such as Site 453. This enables Uruk to be relatively larger in Scenario 2b. The relative percentage flow to Uruk, therefore, increases due to decreases in flow at the smaller sites.

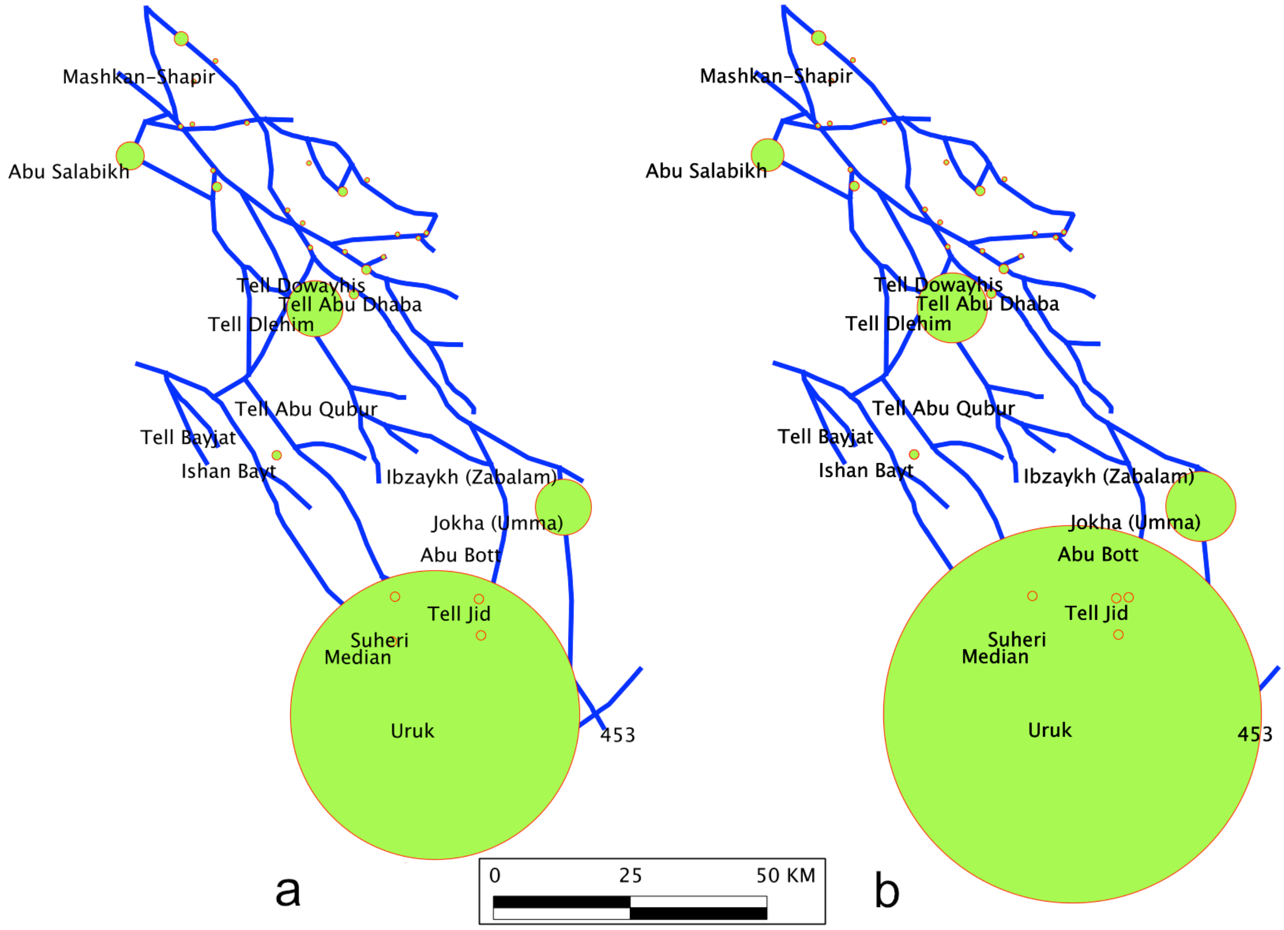

Figure 4. Final output results from Scenario 2a&b investigating α variations for sites and changes to β. Settlement size indicates proportional population calculated by dividing each settlement's population by the median population.

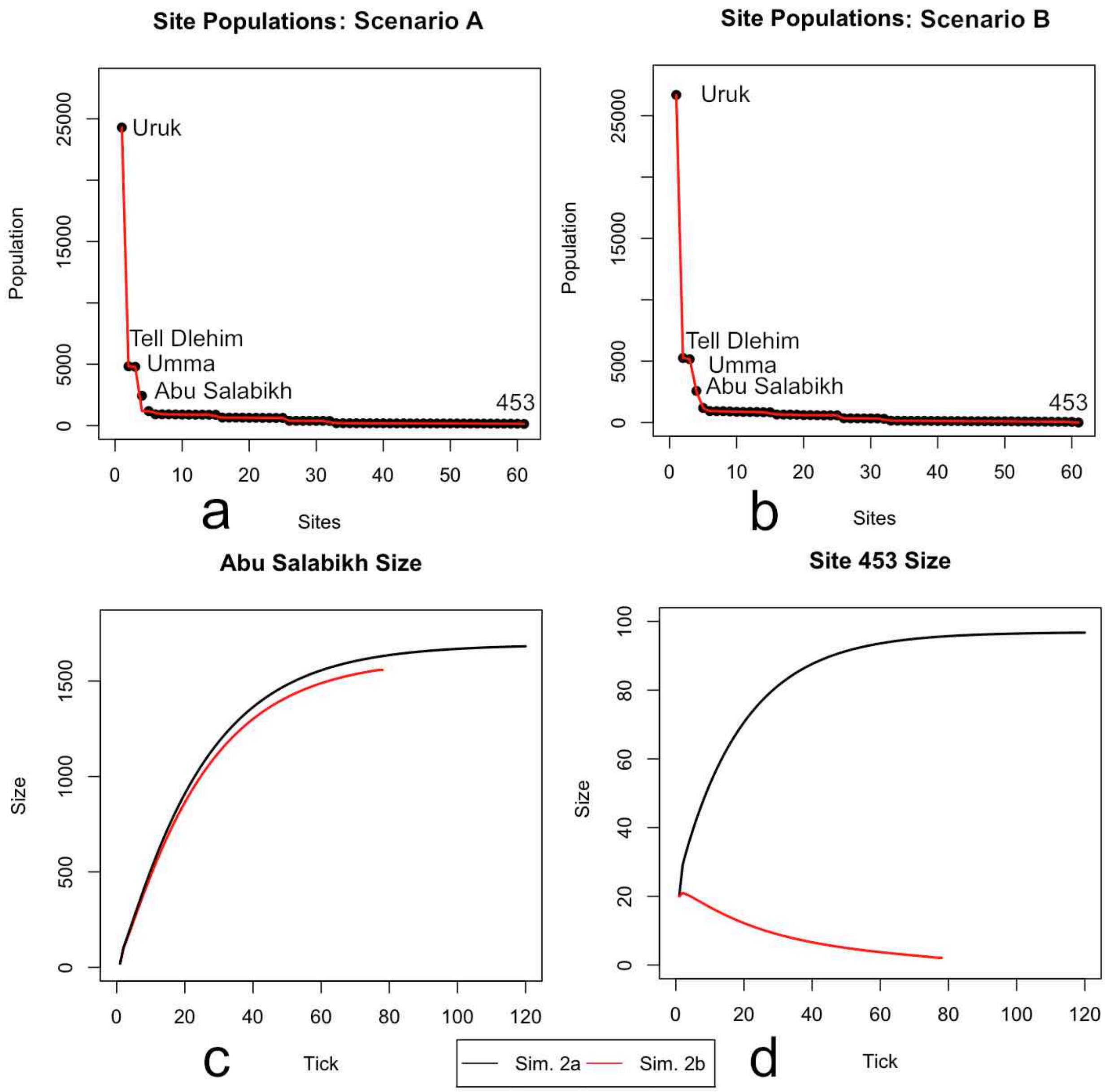

Figure 5. Population for Scenario 2a&b (a & b respectively) at the end of simulations and the evolution of the size variable (*W*) in two of Scenario 2's sites (c & d).

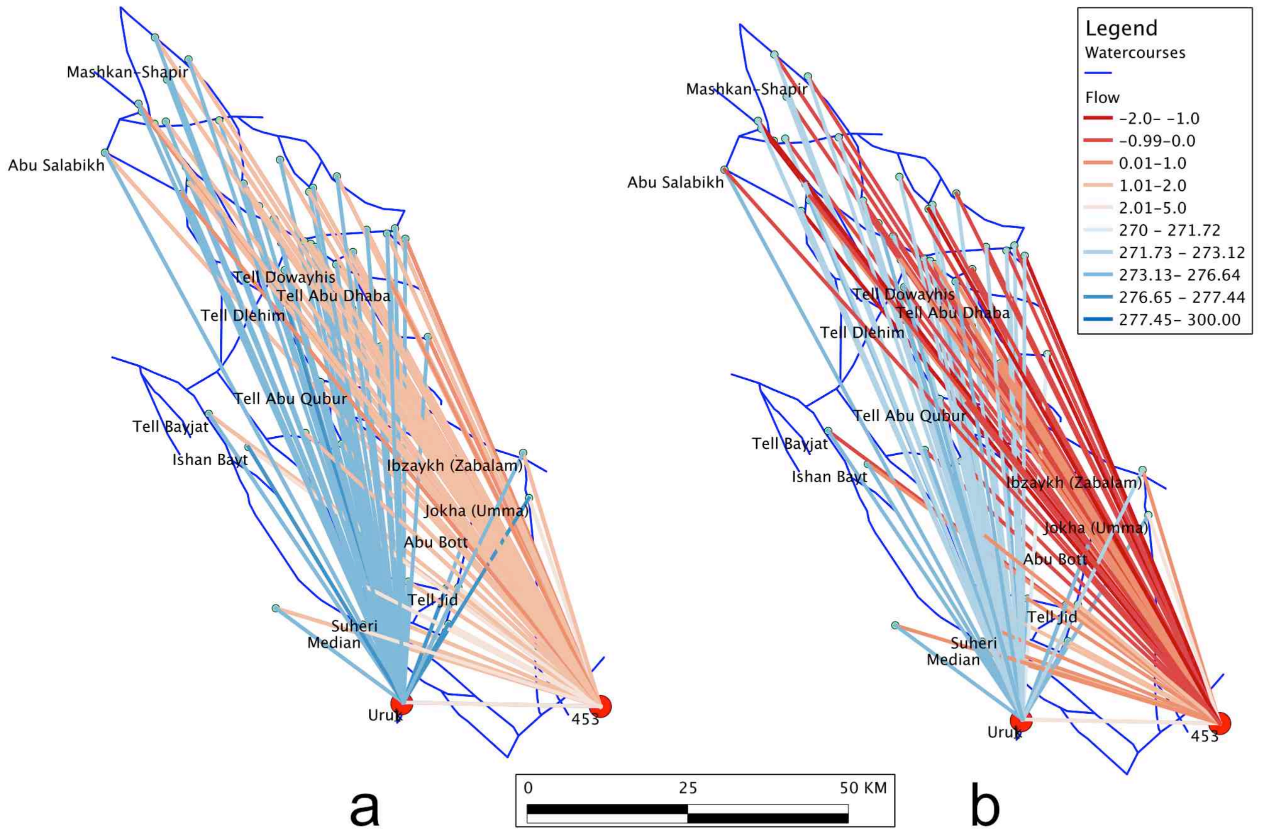

Figure 6. Aggregate flow (S_{ij}) for Uruk and Site 453 at the end of Scenarios 2a&b (a & b respectively).

Scenario 3: The Role of External Contacts

The previous results have ignored the role of settlements or regions outside the model area have played on the development of settlements in southern Mesopotamia. Scenario 3 is similar to the previous cases in that site size hierarchies are comparable to those reconstructed for the Late Uruk period; however, here the role of external contacts (q) is also considered. In this case, q represents trade or interactions that increase or decrease the importance of a site based on links outside the modelled region. The variable captures exogenous settlements and regions that affect how an settlements within the region might evolve. Therefore, exogenous areas could negatively or positively affect flow and ultimately population change.

As before, the parameter settings used are in the data link provided. In Scenario 3, two sub-scenarios (a & b) are modelled. The difference between these sub-scenarios is that Scenario 3a applies β at the level of 0.2, while in Scenario 3b β is 0.65, for similar reasons as stated for Scenario 2b. Scenario 3a is run to 120 ticks, whereas Scenario 3b is 78 ticks. Figure 7 shows the relative populations based on point size using the same type of output as discussed previously. As for Scenario 2, results indicate varying degrees to which the population of a settlement changes as β is changed. In Scenario 3a (Figure 7a), Uruk reaches a population ~64 times (24,792 individuals) the size of the median population, whereas for Scenario 3b (Figure 7b) it is ~92 (29,236). The reasons for this discrepancy in size are the same as for Scenario 2; the key focus here, however, is how q (i.e. external links) influence the results. In both Scenarios 3a&b, q is set to 7 for both Uruk and Mashkan-Shapir. Alpha is 10 for Uruk, whereas it is 1 for Mashkan-Shapir as well as for the majority of the other sites. The results show how a settlement such as Uruk could have made up for its diminished local importance by increasing its relevance outside of the southern alluvium. For example, the well-known Late Uruk trade colonies could be an example of how external linkages

(q) affect the model results such as those shown in Figure 7 (Stein 1999). In addition, in both Figure 7a&b the results demonstrate how a site such as Mashkan-Shapir could become a relatively large town despite the few attributes that make it more attractive locally (i.e., a relatively low α value). In Figure 7, Mashkan-Shapir's population is closer to those sites with greater α values such as Umma and Tell Dlehim.

Another relevant outcome of this scenario, and largely ignored in the discussion so far, is which routes people use to migrate or to bring goods to settlements. There are many ways of dividing up route usage based on flow outputs (i.e., S_{ij}). For simplicity, the approach taken here adopts the least cost path and applies the route in proportion to the flow. This indicates a potential path that could be taken in the flow calculation. Figure 8a&b show route usage for the last scenario, Scenario 2a&b, respectively. This output is contrasted with Figure 8c&d that shows Scenario 3a&b respectively. Not surprisingly, those routes in closest proximity to other settlements, that is in the north central parts of the alluvium, and those that lead to Uruk and other larger centres, are the most used in Figure 8a&b. As the larger centres gain a greater percentage of the flow, routes connecting these larger towns become more heavily used (Figure 8b), which follows conceptually the point made in Figure 6. In Scenario 3b (Figure 8d), the results show how as β is increased, population (expressed by standard deviation), accumulates in fewer centres than in Scenario 3a (Figure 8c). This then concentrates flow to watercourse routes that connect the fewer relatively large centres. Sites such as Uruk, Mashkan-Shapir, Umma, and Abu Salabikh, for example, all have nearby watercourses that are more heavily travelled, while in Scenario 3a there is more dispersion of flow to other settlements. The watercourses between Umma and Uruk, for instance, are relatively more heavily used in Scenario 3b than in Scenario 3a. In short, Figure 8 demonstrates the effect of constraining relative travel (that is increasing β), upon those routes that have access to the towns with larger values of α and q, with the result that these towns take an increased percentage of overall flow.

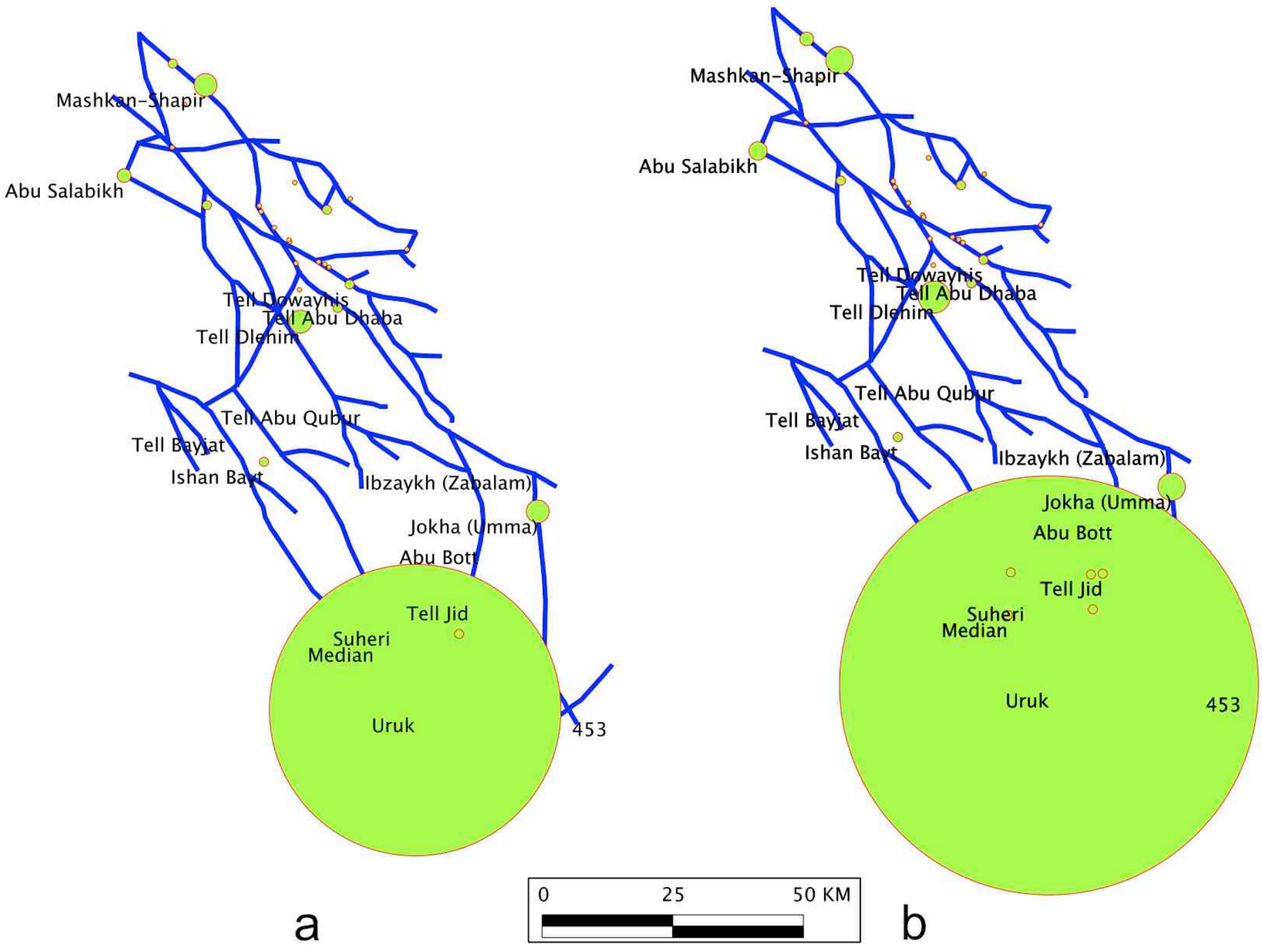

Figure 7. Settlement population for sites indicated by relative size of settlements in Scenario 3a&b

(a & b respectively).

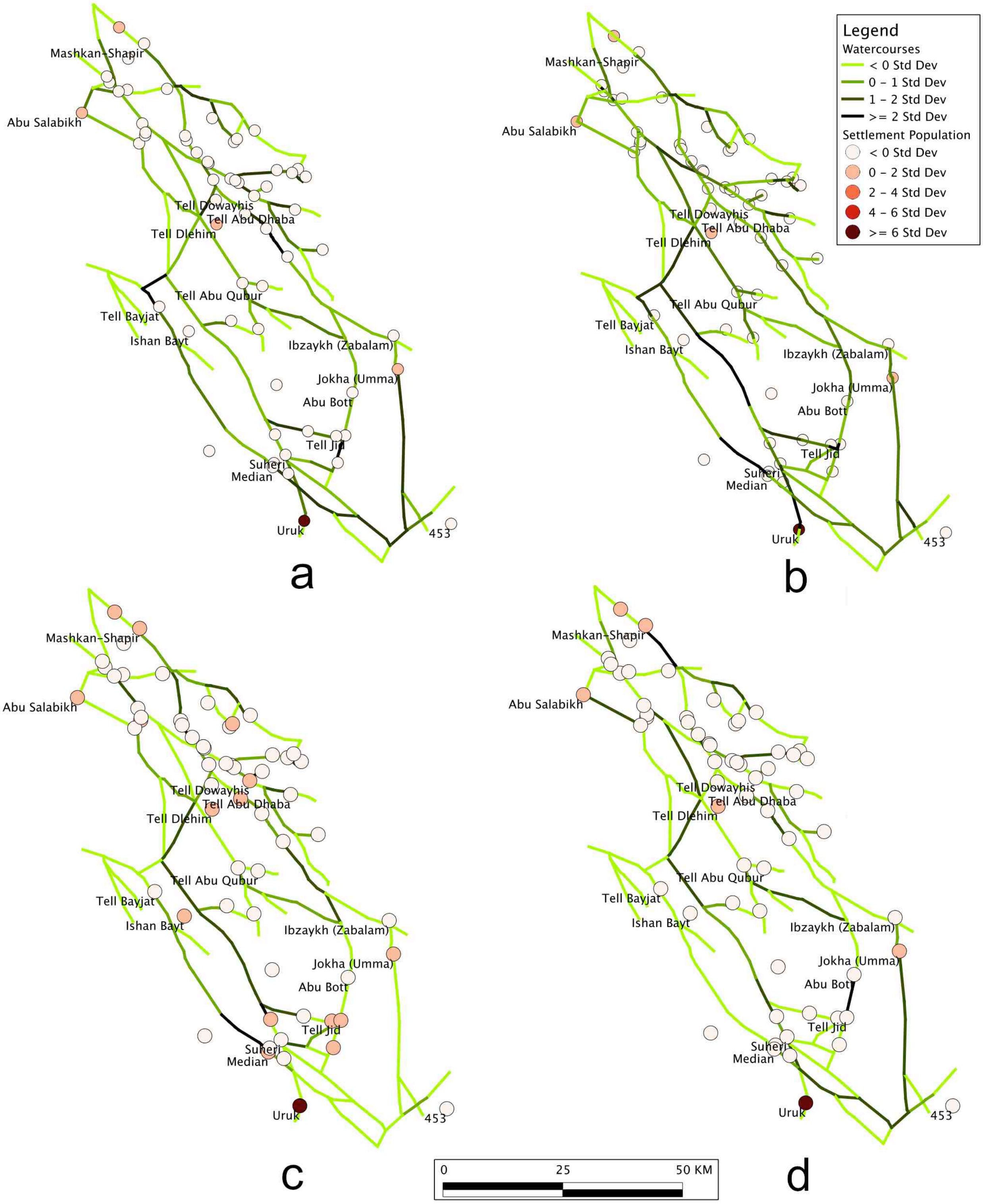

Figure 8. Watercourses travelled based on proportional division of settlement flow (S_{ij}) between sites in Scenarios 2a&b (a & b) and 3a&b (c & d).

Scenario 4: Measuring Resilience

In previous scenarios, the results attempt to measure some characteristics of the Late Uruk settlement pattern. These include site geography, transport, trade, and external contacts that are relevant to the development of urbanism during that period. Scenario 4 conducts simulations to determine how significant transformations, such as changes of α during a simulation, might suggest which settlements could be more resilient, and thus maintain a greater population, under conditions of changing α. Such changes are intended to replicate political, social, or environmental factors that may make some regions more or less attractive than others. In other words, if some circumstance occurs that makes a site less attractive, such as environmental change or diminished political significance, then this scenario tests to see if the diminished α site is still able to retain some percentage of its population above what might be expected.

In this scenario Abu Salabikh has initial values of α=5 and q=2, whereas all other settlements have both values set to 1.0. This, as expected, makes Abu Salabikh the dominant settlement at the beginning of the scenario. However, after every 20 ticks, starting at tick 10, α=5 and q=2 are applied to a random set of settlements chosen. For example, in Figure 9, by tick 60 several settlements are randomly chosen to become more dominant, specifically Tell Jid, Site 1115, Site 1445, and others indicated by darker colours. However, by tick 70 this has shifted once again as new sites have been chosen. The key point in this situation is how some sites are able to retain a higher level of population than other sites, despite having α and q values equal to most other sites. The results at tick 80 show that Site 1445 maintains a greater population than most surrounding settlements, even while other former high α and q sites have already become more like some of the other lower α and q settlements. This is because transport access and location to nearby settlements gives advantages to some settlements to maintain some higher portion of their population even if their attractiveness has declined. The north central regions of the alluvium are more likely to attract larger settlements despite changes to importance of a site, as a greater concentration of potential sites and access to nearby watercourses facilitates movements in these areas more and, therefore, higher levels of population flow to sites than other areas with equal attractiveness values. By tick 100, Site 1445 and Tell Dlehim attain high α and q values, with these sites also chosen randomly, while other sites are set to 1.0 for these parameters. At this time, the former higher α and q value sites have a similar population to other surrounding settlements. In tick 120, Site 1445, once again, and Tell Dlehim, are able to retain relatively greater populations, although they had lost their higher α and q values.

As stated, these results suggest that transport links and geography are capable of maintaining some level of resilience to decreases in α and q that results in α and q being more similar to other sites. Access to a large number of settlements enables sites to have easier access to some share of trade and population flow. Such results largely highlight similar results to Scenario 1, where advantages in the north central regions of the alluvium are seen for some of the sub-scenarios based on similar reasons as that given for Scenario 4. Figure 10 illustrates this point for Abu Salabikh and Tell Dlehim, by showing how Tell Dlehim is more robust with greater flow values after both sites had lost their greater α and q values by tick 110.

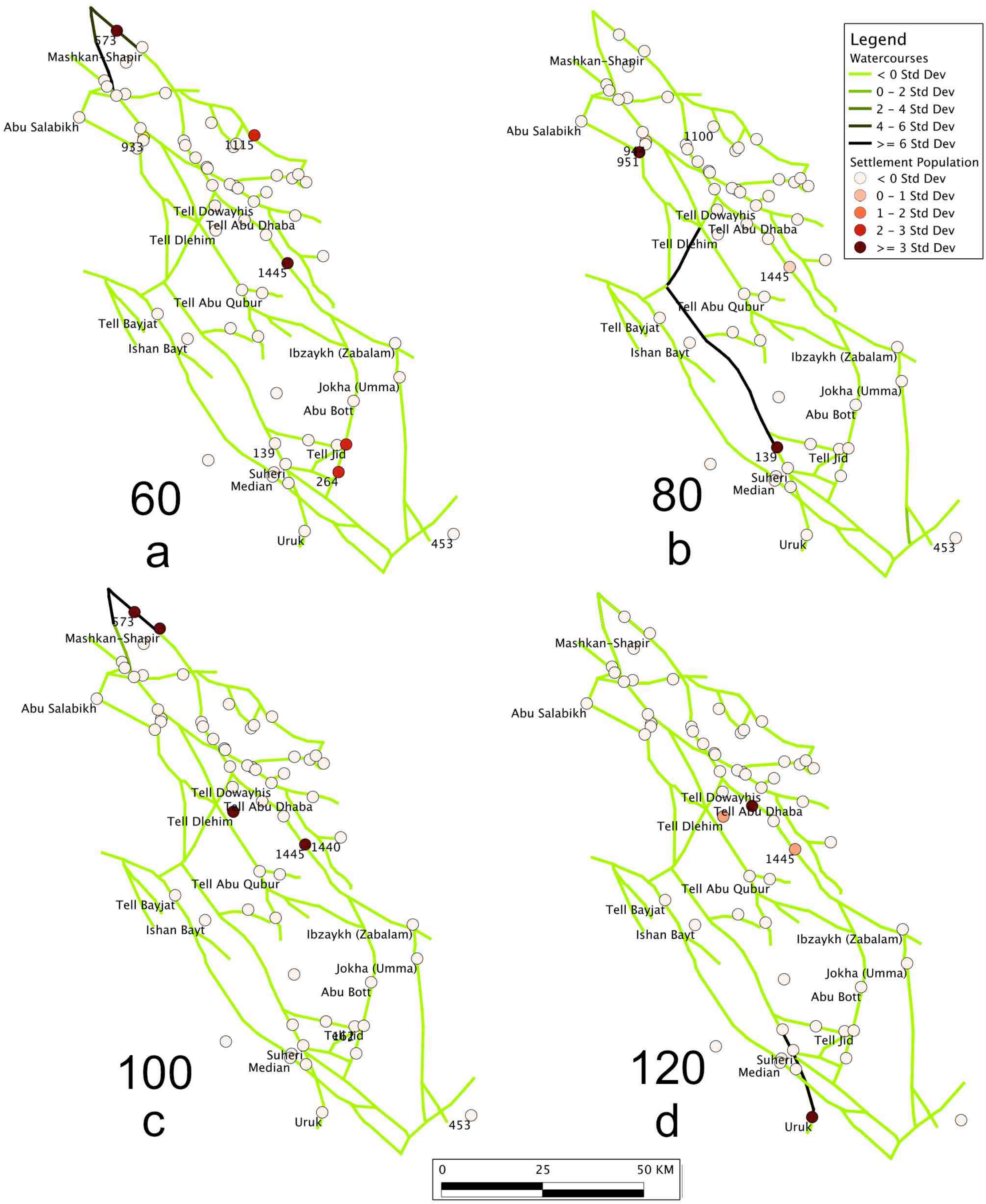

Figure 9. Watercourses travelled and population (in standard deviation) in Scenario 4 with tick counts indicated above the figure letters.

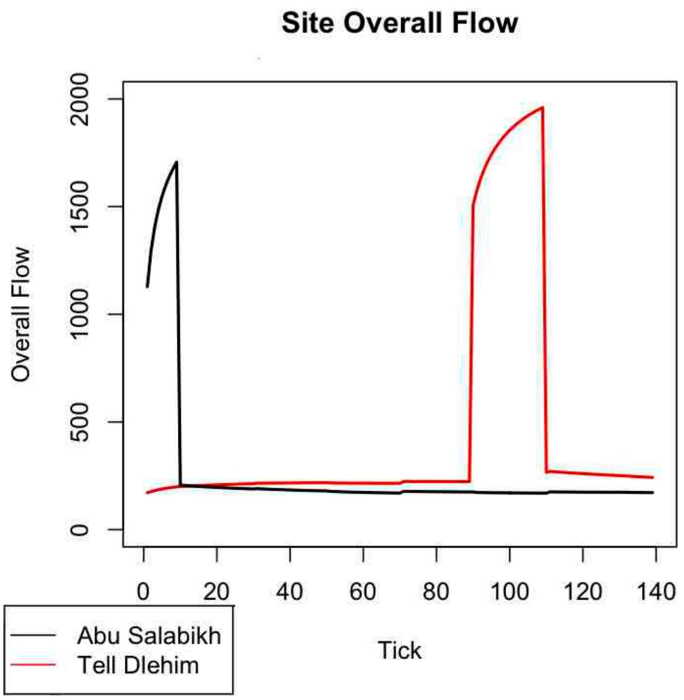

Figure 10. Overall flow (D_j) of Abu Salabikh and Tell Dlehim in Scenario 4. In this case, Tell Dlehim is able to maintain relatively higher overall flow than Abu Salabikh after both sites have lost their greater α and *q* values. The sharp increases in overall flow indicate when these sites have greater α and *q* values, while sharp decreases indicate when these values are equal to most other sites.

Discussion

Scholars have debated how growth and population shifts can affect urban systems for a variety of periods and under very different circumstances (Chandler 1987; Turchin 2003). For early urban Mesopotamia, debates have centred around the themes of economic, trade, ideological, environmental, and transport developments that enabled some centres to become much larger than surrounding settlements (Algaze 2008). While models such as those presented above cannot determine with certainty what outputs match past events, entropy models do provide a relatively simple way of showing heuristically how observed phenomenon may have developed. Each scenario demonstrates a hypothetical case that displays how different factors might affect urban transformations. While it is difficult to know if specific circumstances such as river avulsion, soil salinisation, ideology, or political power may have propelled cities such as Uruk into a position of rapid growth, the methods presented here demonstrate that if such factors are evident, it is therefore possible to explain how such factors may shape not only large settlements such as Uruk but also affect population change in surrounding settlements. As an example, in the model we apply α as a variable which represents the attractiveness of any city. However, this variable does not indicate what specifically made a site relatively attractive (e.g., importance could be based on social and/or environmental reasons). Similarly, β attempts to represent the abilities or desires of a population and goods to be mobile, but it does not address the specific reasons that cause greater or diminished levels of mobility (e.g., political facilitation of movement, lower/higher transport friction, etc.) at distant locations. This signifies that in the absence of specific social or ecological reasons as to why settlements grow or decline, spatial entropy models can determine how different levels of importance, incentives for mobility, and other variables affect settlement interactions and transformations in space and time. This allows us to understand how regions transform in their population once initial conditions for settlement growth are in place.

Scenario 1, shows how the advantages of geography and access to watercourses give some sites initial benefits over others. This is evident in the north central parts of the southern Mesopotamian plain where settlements grow larger because of their proximity to other settlements and access to different watercourses. The results show how diminished incentive for distant travel, that is increased β, results in a greater differentiation of site population, as more people and flow of goods concentrate in the most accessible sites. On the other hand, increased α increases the overall flows to settlements, but those settlements do not become more distinguished in terms of relative population from other sites.

Whereas Scenario 1 demonstrates which regions may have initial advantages, Scenario 2 attempts to understand how internal interactions between sites could propel a settlement such as Uruk to become very large relative to other sites. In addition, Scenario 2a shows that settlement size remains stable when trade flow is maintained, even for the smaller sites. On the other hand, Scenario 2b indicates how flows, which replicate trade and the movement of people, enable sites to grow and diminish at more extreme scales. In Scenario 2, a greater value of α, or attractiveness, enables Uruk to attract more flow and increase its size, so that overall population increased nearly 24-27 times the initial population of 1000. In addition, increasing β creates a negative effect on smaller sites, as people and goods are less inclined to move to lower α sites with the result that overall flow even becomes negative. This indicates that as attractiveness and mobility decrease and the sites with smaller α values are less able to attract goods and people, population became concentrated at fewer settlements, resulting in the majority of settlements becoming very small or even being abandoned. This phenomenon, where growth concentrates in few centres, has been proposed for early Mesopotamian cities by Falconer and Savage (1995). This pattern is most clearly demonstrated for small sites near a major settlement such as Uruk. In this case, Site 453's population became subsumed within the larger centres because this site had fewer nearby connections that were capable of attracting migration and flow to maintain its population. In other words, for sites with a low value of α, the existence of few connections to other sites and its proximity to a high α site leads to decreasing flow and population as the incentive for travel decreases (i.e., β increases). This relatively benefits the growth of the larger sites as flow concentrates in these areas.

Scenario 3 largely replicates the results of Scenario 2, but this scenario suggests how settlements could depend on external contacts and links to compensate for shortfalls in α. This demonstrates that settlements with powerful ties to trade networks and external contacts could become relatively large. This suggests how trading colonies of the Uruk period (Stein 1999) could have shaped settlement growth. Whereas settlements may have become transformed as α and β varied, it is also evident that these variables affect transport as the greater value of α increases flow to sites and the greater β values direct more transport to routes leading to the larger settlements. This has the effect of making some transport routes more significant than others in how trade or migration is conducted.

The final scenario, Scenario 4, demonstrates how resilience could enable some sites to persist at relatively higher population levels even after their attractiveness for trade and migration has declined. This demonstrates how geography and transport can increase resilience by increasing access to beneficial links to surrounding communities, providing some relatively higher level of flow even after a site looses its attractiveness.

The models and simulations demonstrate how the survey results of Adams (1981) can be understood from the context of populations and settlement dynamics. Without any major change in natural population growth, the populations of settlements can shift as sites gain or lose advantages (McNeil 2000). These exploratory outputs enable the Late Uruk phenomenon to be succinctly expressed in a relatively simple quantitative form that shows how the urban process may have shaped settlement patters across the southern alluvium in Mesopotamia. Whereas the results from the surveys and simulations may not be comprehensive, they can be applied together to show how geography, transport, attractiveness of sites, both locally and external to the region, as well as the flow of goods and people could shape the urbanisation process in southern Mesopotamia.

Conclusion

The model outputs demonstrate how feedbacks of growth and population change can be studied for a range of settlement contexts and for a variety of settings using different types of values. These include, but are not limited to, values that demonstrate the flow of goods and economic factors that represent a core-periphery dynamic in which some areas prosper while others do not (e.g., Fujita and Krugman 1995). This reflects the work of scholars such as Adams (2001) and Algaze (2008) who have identified Complexity and Complex System dynamics as explanations for urban transformations in certain periods. According to these approaches, the presence of cultural, economic, and environmental interactions as well as associated feedbacks and interactions are likely to have operated. The model presented here, therefore, offers a useful technique to explain fairly complex processes without being empirical specific for any given period. In other words, the few variables employed represent generalizing phenomena which make it possible to test the implications of a site becoming, for example, more attractive. In summary, simple entropy models offer a way to generalize theories such as Complexity Theory and present them in a quantitative form within a specific spatiotemporal setting. Such modelling contributes to the discussion of, for example, the Late Uruk expansion because it makes it possible to apply very different parameters to determine how different vectors may affect change in urban systems even in conditions of uncertainty. Archaeological surveys frequently provide a limited spatiotemporal perspective, because the data can never be completely recovered. The methods applied here, therefore, enable scholars to account for missing empirical data and to explore and generalize socio-environmental effects on settlement dynamics, such as changes in settlement size or transportation between cities. At the same time it is possible to assess the modelling outputs to see how well they match archaeological data. In summary, a spatial entropy model makes it possible to draw conclusions about past urban transformations and processes for sites and settlement regions and provide succinct explanations for at least some empirical observations.

Acknowledgments

This research was supported financially by grants from University College London's Strategic Development Fund in the Humanities and the National Science Foundation’s Biocomplexity in the Environment program (NSF Grant # 0216548).

References

Adams, R.M., Nissen, H.J. 1972. *The Uruk Countryside: The Natural Setting of Urban Societies*. The University of Chicago Press, Chicago.

Adams, R.M. 1981. *Heartland of Cities: Surveys of Ancient Settlement and Land Use on the Central Floodplain of the Euphrates*. University of Chicago Press, Chicago.

Adams, R.M. 2001. Complexity in archaic states. *Journal of Anthropological Archaeology* 20:345-360.

Algaze, G. 2005. The Sumerian takeoff. *Structure and Dynamics: eJournal for the Anthropological and Related Sciences* 1(1). http://escholarship.org/uc/item/76r673km. Accessed July 3, 2012.

Algaze, G. 2008. *Ancient Mesopotamia at the Dawn of Civilization: The Evolution of an Urban Landscape*. The University of Chicago Press, London.

Aqrawi, A.A.M. 2001. Strategraphic signatures of climatic change during the Holocene evolution of the Tigris-Euphrates Delta, Lower Mesopotamia. *Global and Planetary Change* 28:267-283.

ASTER. 2012. http://asterweb.jpl.nasa.gov/gdem.asp. Accessed July 3, 2012.

Batty, M. 2005. *Cities and Complexity: Understanding Cities with Cellular Automata, Agent-Based Models, and Fractals*. MIT Press, Cambridge, MA.

Bentley, R.A., Maschner, H.D.G. 2008. Complexity theory, in: Bentley, R.A., Maschner, H.D.G. (Eds.): *Handbook of Archaeological Theories*, AltiMira Press, Lanham, pp. 245-272.

Birkin, M., Clarke, G., Clarke, M. 2010. Refining and operationalizing entropy-maximizing models for business applications. *Geographical Analysis* 42(4):422-445.

Birkin, M., Heppenstall, A. 2011. Extending spatial interaction models with agents for understanding relationships in a dynamic retail market. *Urban Studies Research*. DOI:10.1155/2011/403969.

Braudel, F. 1995. *The Mediterranean and the Mediterranean World in the Age of Phillip II*. University of California Press, London.

Chandler, T. 1987. *Four Thousand Years of Urban Growth: An Historical Census*. Edward Mellon, Lampeter, UK.

Collins, P. 2000. *The Uruk Phenomenon: The Role of Social Ideology in the Expansion of Uruk Culture during the Fourth Millennium BC*. BAR International Series, 900. Archaeopress, Oxford.

Cronon, W. 1991. *Nature's Metropolis: Chicago and the Great West*. W. W. Norton, Chicago.

Dearden, J., Wilson, A.G. 2012. The relationship of dynamic entropy maximising and agent-based approaches in urban modelling, in: Heppenstall, A., Crooks, A.T., See, L., Batty, M. (Eds.): *Agent-Based Models of Geographic Systems*, Springer, London, pp. 705-720.

Desrochers, P. 2001. Geographic proximity and the transmission of tacit knowledge. R*eview of Austrian Economics* 14:25-46.

Falconer, S. and Savage, S. 1995. Heartlands and hinterlands: Alternative trajectories of early urbanization in Mesopotamia and the Southern Levant. *American Antiquity* 60(1):37-58.

Finkbeiner, U. 1991. *Uruk: Kampagne 35-37, 1982-1984: Die archäologische Oberfldchenuntersuchung*. Ausgrabungenin Uruk-Warka 4. P. von Zabern, Mainz.

Fox, E.Q. 1971. *History in Geographic Perspective: The Other France*. W.W. Norton, New York.

Fujita M., Krugman, P. 1995. When is the economy monocentric? von Thünen and Chamberlin unified. *Regional Science and Urban Economics* 25:505–528.

Harris, B., Wilson, A.G. 1978. Equilibrium values and dynamics of attractiveness terms in production-constrained spatial interaction models. *Environment and Planning* A 10:371-388.

Hart, P. E.; Nilsson, N. J.; Raphael, B. 1968. A formal basis for the heuristic determination of minimum cost paths. *IEEE Transactions on Systems Science and Cybernetics* 4(2):100–107.

Heppenstall, A.J., Evans, A.J., Birkin, M.H. 2006. Application of multi-agent systems to modelling a dynamic, locally interacting retail market. *Journal of Artificial Societies and Social Simulation* 9(3). http://jasss.soc.surrey.ac.uk/9/3/2.html. Accessed July 3, 2012.

Hole, F. 1994. Environmental instabilities and urban origins, in: Stein, G., Rothman, M.S. (Eds.): *Chiefdoms and Early States in the Near East*, Prehistory Press, Madison, pp. 121–152.

Huff, D. 1963. Defining and estimating a trading area. *Journal of Marketing* 28:34–38.

Jacobs, J. 2000. *The Nature of Economies*. Random House, New York.

Krugman, P. 1995. *Development, Geography, and Economic Theory*. Cambridge, MIT Press.

Lane, D., Pumain, D., Leeuw, S.v.d. 2009. Introduction, in: D. Lane, D. Pumain, S.v.d. Leeuw, and G. West (Eds.): *Complexity Perspectives in Innovation and Social Change*. Springer, Berlin, pp. 1-7.

McNeil, W.H. 2000. Information and transportation nets in world history, in: Denemark, R.A., Friedman, J., Gills, B.K., and Modelski, G. (Eds.): *World System History*, Routledge, London, pp. 201-215.

Denemark, R.A., Friedman, J., Gills, B.K., and Modelski, G. (Eds.): *World System History*, Routledge, London, pp. 201-215.

North, M. J., C. M. Macal. 2007. *Managing Business Complexity: Discovering Strategic Solutions with Agent-Based Modeling and Simulation*. Oxford University Press, Oxford.

Persson, K.G. 2010. *An Economic History of Europe: Knowledge, Institutions and Growth, 600 to the Present*. Cambridge University Press, Cambridge, UK.

Pollock, S. 2001. The Uruk period in Southern Mesopotamia, in: Rothman, M. (Ed.): *Uruk Mesopotamia and Its Neighbors*, SAR Press, Santa Fe, pp. 191-232.

Pournelle, J. 2003. *Marshland of Cities: Deltaic Landscape and the Evolution of Early Mesopotamian Civilization*. PhD. Dissertation. University of California, San Diego.

Stein, G. 1999. *Rethinking World-Systems: Diasporas, Colonies, and Interaction in Uruk Mesopotamia*. The University of Arizona Press, Tucson.

Turchin, P. 2003. *Historical Dynamics: Why States Rise and Fall.* Princeton University Press, Princeton, New Jersey.

Wheatley, Paul. 1971. *The Pivot of the Four Quarters*. Aldine, Chicago.

Wilson, A.G. 1967. A statistical theory of spatial distribution models. *Transportation Research* 1:253-269.

Wilson, A. G. 1970. *Entropy in Urban and Regional Modelling*. Pion, London.

Wilson, A.G. 2008. Boltzmann, Lotka and Volterra and spatial structural evolution: an integrated methodology for some dynamical systems. *Journal of the Royal Society Interface* 8:865-871.

Wilson, A.G. 2012. Geographic modeling for archaeology and history: Two case studies. *Advances in Complex Systems* 15(1-2). DOI: 10.1142/S0219525911003384.

Wright, H. 2006. State dynamics as political experiment. *Journal of Anthropological Research* 62(3): 305-319.