Pumping vs. Iron

Adaptive Structures for whole life energy savings

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***Abstract* — The design methodology explained in this paper[1] takes a substantial shift from conventional methods where sizing is based on a single load case i.e. the maximum expected load. The difference from a conventional passive approach is that strategically located elements of the system provide controlled output energy (actuators) in order to manipulate actively the internal flow of forces and stresses. In this way stresses can be homogenized and deflections kept within desired limits. The alternative we are proposing offer a way to actively counteract loads when needed. Two dimensional pin-jointed trusses designed using this methodology show that substantial weight savings can be achieved respect to optimised "passive" structures (designed using Fully Utilised Design method).**

While the decrease in mass through actuation leads to reduction of embodied energy, it increases the operating energy that the active elements need to provide. Whole life energy analysis, implemented as coupled optimization between embodied and operating energy, reveals that an optimal trade-off exists. Results show that energy savings remain significant even considering the operating energy of the actuators for the entire life-cycle of the structure.

Keywords: adaptive structures, whole life energy, multi-objective optimisation

1.0 INTRO

Adaptive Structures can be thought of as spatial structures with embedded sensors, actuation and control intelligence. The main components (fig. 1) of an adaptive structural system can be classified in:

- **sensing**, which is the capacity of the system to reach an awareness of its state (stress, strain, relative positions of its elements);
- **actuation;** which involves transformation of stored (chemical) or supplied (electrical, magnetic) energy to the system into mechanical energy. During actuation the system modifies its properties (i.e. stiffness, phase change, chemical composition) or it restructures the interactions between its elements (i.e. change of its shape);
- **control intelligence** which processes the information gathered by sensors and provide appropriate input for the actuators in order to keep the system within desired boundaries (closed loop feedback). The importance of a closed feedback loop is paramount to achieve adaptive response since the relation between data gathered by sensors and input to actuators must be determined on-line on the base of the current state of the system. Machine learning techniques can enable adaptive structures to learn from data gathering in order to predict recurrent patterns of load for maximising control efficiency.
- **load-bearing capacity** in order to withstand static/dynamic, external/internal (the actuators themselves) loads;

Sensing, actuation and control intelligence can be designed to improve the load bearing capacity of the structure which is enabled to counteract severe loads at occurrence and to monitor continuously its state of stress and deflections.

However, since active elements require input energy, adaptive systems must be designed so that the benefits brought by new acquired functionalities outweigh the consumption of energy needed.

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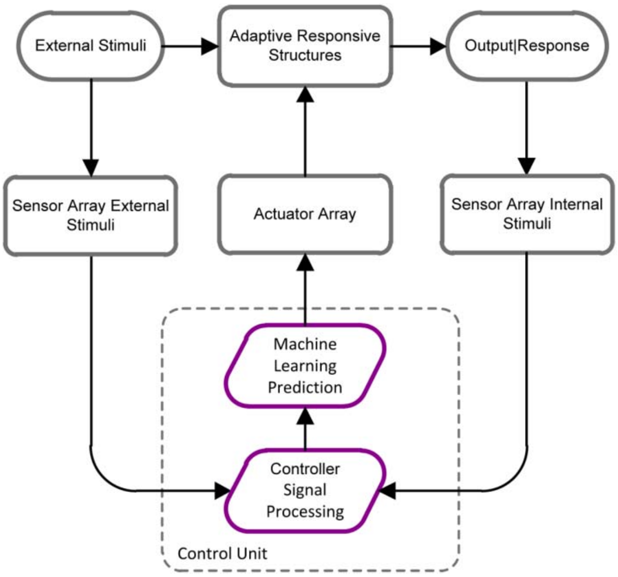

Fig. 1. Adaptive structural system schematic flow-chart

1.1 The Scope of Adaptive Building Structures

The scope for adaptive structures can be framed in two main categories:

- **response control**;
- **shape morphing control**;

Response control aims at controlling the structural state of the system by varying its stiffness locally and globally on the occurrence of unusual or unexpected loads. This type of control would apply to situations in which forces are generated by external agents such as wind, snow and vibrations caused by earthquakes or aerodynamic forces (i.e. tall buildings and bridges).

The scope of shape morphing control can be divided in two main levels, global and local. The first aims at restructuring the shape of buildings in order to minimize/maximize exposure to external agents such as wind, solar radiation or snow or others. The second aims at controlling smaller regions in order to maintain ambient conditions within desired boundaries. This involves modulation of direct and diffuse light (shading), control and enhancement of buoyancy effects (natural ventilation).

Both of them present a radical change of perspective regarding design criteria:

- The former aims at controlling the structural state of the system by redistribution of internal forces through the action of active elements (change in stiffness or change in length).
- The latter sees as primary design targets a set of optimal deformed shapes in which the structure, or part of it, should be able to morph. These "modal shapes" represent the physical embodiment of desired performances that the building structure will have to provide. In other words, the "modal shapes" are the result of a mapping between external stimuli and desired building performances.

1.2 *Aim*

This paper investigates the potential whole-life energy savings for adaptive structures applied on a simple 2-dimensional truss subjected to time-varying loads taken as case study.

Teuffel (1) showed how appropriate substitution of some of the elements of a truss structure with actuators can lead to significant weight savings and high control of deflections. This is mainly due to changes in length of the active elements that redistribute the internal load-path and keep deflections within desired limits.

Since the majority of building structures are subjected to extreme conditions rarely, actuators activation should only be necessary for a small percentage of the lifetime of a structure thus keeping control/actuation energy consumption to a minimum. A Coupled-optimization process is here formulated in order to derive the optimal configuration of sizes of passive members, actuators position and their length change that minimizes both mass (embodied energy) and operating energy (actuators work).

1.3 Intro to Methodology

The design process presented in this paper aims at finding the least-weight adaptive structural configuration that satisfies ultimate and serviceability limit state as well as requires minimum operating energy from the actuators.

The structure we consider is a pin-jointed truss of a given geometry. The variables to be determined are cross-sectional areas and the number and position of the actuators. Active load-bearing capacity is provided by actuators that replace some of the elements and whose controlled length change allows the pattern of internal forces to be modified, "load path management" (3). In so doing stresses can be minimized and homogenized while displacements are kept within desired limits.

To avoid large operating energy consumption a load threshold, above which the active elements should be activated and below which the structure is able to perform adequately without active control, is introduced as one of the outcome of the design process. This can be explained with the conceptual graph in fig.2 where embodied energy (mass) and operating energy (actuators work) are expressed as function of this parameter which can be considered as a percentage of the maximum expected load. The minimum of the sum of the functions corresponds to a solution which features both minimal mass and minimum operating energy (actuators' work).

Fig. 3 shows the cumulative frequencies of occurrence for a generic load with a certain time-history. The dotted line represents the activation threshold which traces the difference between two zones of the load-history. On the left there all loads and corresponding frequencies with which the active elements will not deal directly and that passive load-bearing capacity of the structure will be able to withstand. On the right there are the loads with higher magnitude but less probability of occurrence which the adaptive structure will be able to withstand using both passive and active (actuators length change) load-bearing capacity.

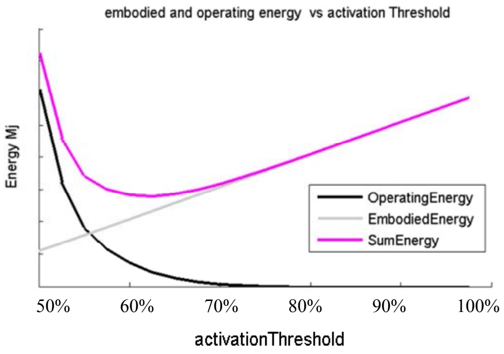

Fig. 2. Embodied/operating energy vs. activation threshold

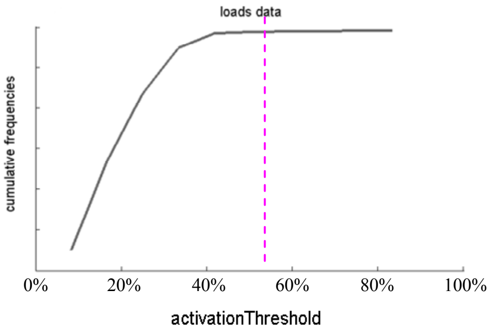

Fig. 3. Load cumulative frequencies of occurrence vs. activation threshold

1.4 Case Study | The Setting

The case study aims at analysing the potential of adaptive structures for a typical roof-type structural configuration made by a series of trusses arrayed along one direction (with span=10 m). Fig. 5 shows one of the trusses indicating with black text the enumeration of the degrees of freedom of its nodes while with red text the enumeration of its elements.

As far as load cases go, wind is often the main cause of concern of this typology of structures and is taken here as example of time-varying load. We consider wind loads for prevailing wind directions as recorded by the weather station in Heathrow, London South East. Fig. 4(left) plots the frequencies in hours for recorded velocities for 360 degree directions. It is clear that the highest values for the velocities (up to 50 km/h) occurs only for very few hours (less than 60) while velocities with higher frequencies of occurrence are those indicated by darker pixels (around 10-15 Km/hours). Fig. 4(right) represents the wind velocity landscape for the entire year where the x axis is weeks, y is hours within a day and the z axis is speed (km/h). The values of wind velocities are factored as indicated by the EuroCode 1 (4) considering a height above sea level of 10 m and probability of storm occurrence of 1 in 50 years. Being the roof-like structure flat, it is possible to assume a pressure coefficient (cpe) of -1 (suction) such that:

$$F = \frac{vel^{\,2}}{2} \times \rho_{\,air} \times cpe \times Area \qquad (1)$$

where Area measures the planar extension of the surface described by to consecutive truss modules (4x10=40 m^2). The sum of these forces at each node of a single truss is showed in fig. 5 (the truss is considered to be between two other trusses). This load case is obtained by considering wind velocity at its max value. The structure to be designed must still satisfies stress and deflections limits for the maximum expected load. However, it can also rely on active elements and not only on passive load-bearing capacity.

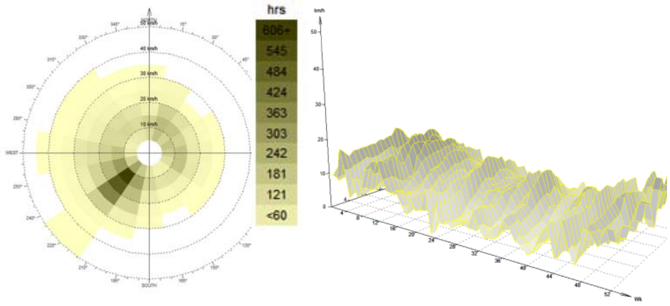

Fig. 4. Wind velocities and frequencies (hours)

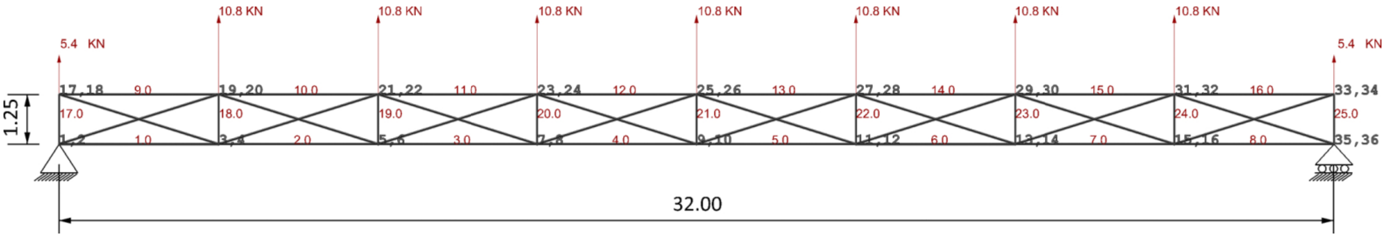

Fig. 5. Wind load case considering max expected velocity in 50 years

2.0 ADAPTIVE STRUCTURE | DESIGN PROCESS

2.1 Linear Programming

Following Teuffel (1) the design methodology starts with finding the optimal distribution of section areas A_i and axial forces N_i as a linear optimization routine (Linear Programming). The objective function to be minimised is:

$$\min V = \sum_{i=1}^{ne} A_i * l_i \tag{2}$$

where V is the total volume, A_i the cross-sectional area of each element and l_i their length. This function is subjected to a set of equality and inequality constrains given by:

$$C \cdot N_k - P_k = 0 \tag{3}$$

$$\frac{N_{ik}}{A_i} \le \frac{\sigma_T}{\gamma}; \quad -\frac{N_{ik}}{A_i} \le \frac{\sigma_C}{\gamma} \tag{4}$$

where C is a matrix containing information on the cosines directions of the elements and constrained degree of freedom; N_k and N_{ik} are the vector of axial forces for each load case $P_{k,}$ and the force in the i_{th} member respectively. Admissible stress limits in tension and compression are σ_T and σ_C which can be factored with the parameter γ eq. (4).

Further boundary conditions can be assigned for taking into account a minimum value of area in order to match commercially available bar sections. Buckling conditions are not considered explicitly at this stage. In order to include instability we increase (double) the parameter γ for compression stresses in the inequality constraints eq. (4). Inequality constraints on buckling would make the problem non-linear requiring other form of optimization algorithms (non-linear programming, sequential quadratic programming).

The optimization routine is able to find an absolute minimum of the cross sectional area whereby the members of the truss are 100% utilised (i.e. stress equals to ultimate stress limit). Fig. 6 and fig. 7 show the optimal distribution of axial forces and cross sectional areas for the structure taken into exam.

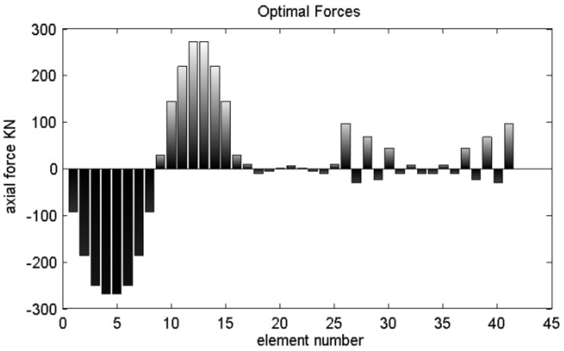

Fig. 6. Optimal Axial Forces

It is worth pointing out that constraints for geometrical compatibility are not included because they are going to be satisfied by appropriate length changes of the active elements obtained through a least square optimization routine explained in next section. The minimum number of actuators is equal to the number of indeterminacy of the system plus the number of desired controlled displacements (one for each DOF). This is the minimum number of actuators to turn a hyper static structure into a controlled mechanism.

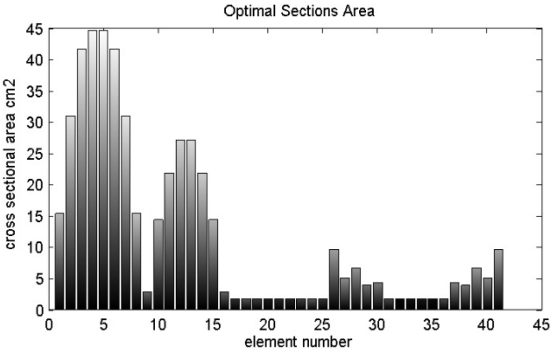

Fig. 7. Optimal distribution of sections

2.2 Controlled Displacements

The next step in the design process is to analyse the structure using the optimal section areas obtained from the linear programming optimization routine. Since geometrical compatibility constraints and inequality constraints on displacements are not included in the first step, this FEM analysis reveals what would be the state of stress and displacements (fig. 9) of the optimised structure considered without active elements.

Fig. 8 shows the difference (ΔN) between the "optimal Forces" (not compatible) resulting from the first optimization routine and the compatible axial forces derived from the FEM analysis. These force differences is what the actuators must be able to provide in order to satisfy compatibility conditions.

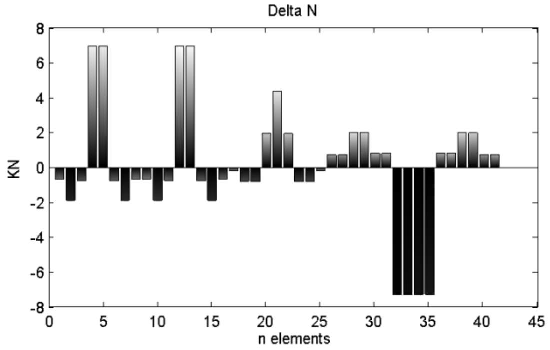

Fig. 8. △N=optimal Forces - compatible Forces

As part of the design process, the displacements of selected nodes can be kept within desired limits. Considering that the max displacement of the optimised structure in middle span with no active elements is more than double than the desired limit (span/1000), we choose all the vertical DOFs of the top cord to be controlled. This increases the minimum number of actuators to 15.

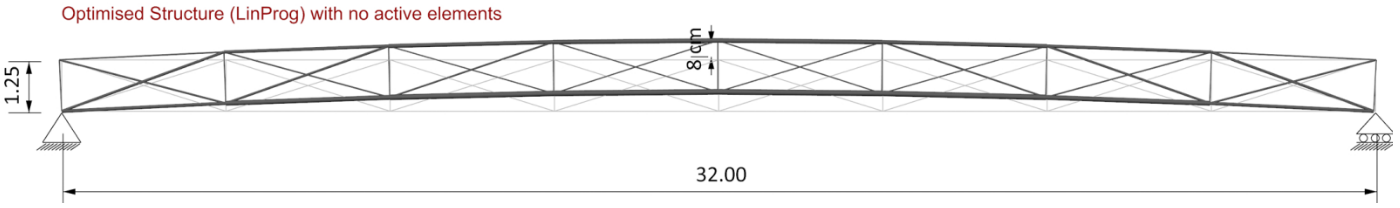

Fig. 9. Compatible displacements

Having set the max allowed displacement in middle span to span/1000 (3 cm), it is possible to compute a Δu between the desired displacements and the so derived compatible displacements computed by the FEM analysis.

2.3 Sensitivity Analysis

The most efficient actuator positions are the configuration where the active elements have the largest effect on both axial forces and controlled displacements. A computation of all possible computation would be infeasible since it would involve a number of FEM analyses equal to:

$$\frac{ne!}{na!\,(ne-na)!} = 3{,}268{,}760 \tag{5}$$

where *ne* is the total number of elements and *na* is the number of actuators.

This problem can be formulated as a least square optimization routine starting with the computation of the sensitivity matrices S_U and S_N for displacements and axial forces. These matrices record the effect of a unit length change for each element on nodal displacements (S_U) and axial forces (S_N) of the other elements. Using the principle of virtual works, each element length is increased by one unit and a FEM analysis derives the vectors ΔN_{ij} and ΔU_{ij}, which are the resulting axial forces in all the other elements and nodal displacements i caused by element j. Once the sensitivity matrices are computed it is possible to find, by means of least square minimization, the minimum elements length change ΔL that satisfies compatibility conditions:

$$\min \| S_U \cdot \Delta L - \Delta u \|^2 \tag{6}$$

subjected to equality constrains:

$$S_N \cdot \Delta L = \Delta N \tag{7}$$

In this way, both compatibility constraints and desired controlled displacements are satisfied.

Note that ΔL is obtained considering all elements being active. In order to derive the set of most efficient elements we compute the efficiency of each member as:

$$e_{ik} = \frac{S_N \cdot \Delta\widetilde{L_{ik}}}{\Delta N_k} \tag{8}$$

where $\Delta\widetilde{L_{ik}}$ is the vector composed of the length change of element i for the load case k having all the others components set to 0. The global efficiency E_i of each member is obtained as (2):

$$E_i = \frac{\sum_{k=1}^{nLCases} \sum_{i=1}^{ne} e_{ik}}{\sum_{i=1}^{ne} \sum_{k=1}^{nLCases} \sum_{i=1}^{ne} e_{ik}} \tag{9}$$

For the structural configuration taken into consideration, the elements positions efficiency is plotted in fig. 10 and those chosen to be active, following the so described sensitivity analysis, are represented in fig 11 with dashed-magenta lines. The black circles highlight the controlled DOFs (vertical displacement).

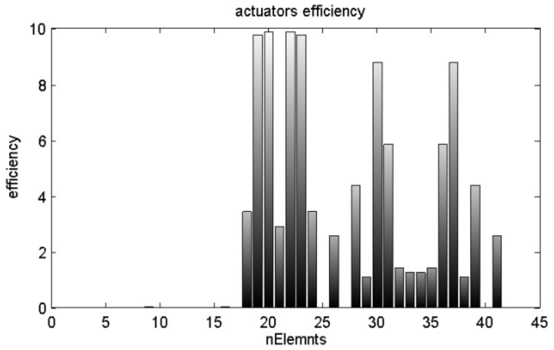

Fig. 10. Actuators efficiency for controlled DOFs

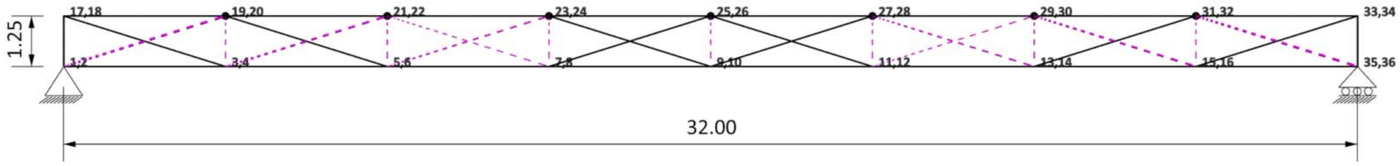

Fig. 11 Most efficient position for the actuators (nActuators=indeterminacy + nControlledDOFs)

2.4 Adaptive Structure

When considering the structure with only the most efficient active elements in place, it is necessary to run a least square minimization to find the vector of length changes ΔL for such configuration of actuators that satisfies compatibility conditions. The sensitivity matrices S_N and S_U are reduced by taking only the columns corresponding to the chosen elements such that:

$$\min \| S_{Ured} \cdot \Delta L - \Delta u \ \|^2 \qquad (10)$$

Subjected to equality constraints:

$$S_{Nred} \cdot \Delta L = \Delta N \qquad (11)$$

Finally, a FEM analysis in performed on the structure imposing these length changes (ΔL_i for each actuator using principle of virtual works) verifying that the displacements of the controlled DOF are within the desired limits. Fig. 12 shows the set of axial forces when the active elements are activated and in deployed state.

Comparison between the adaptive solution and a passive optimised structure obtained through FUD (5) optimization (Fully Utilised Design) reveals significant weight savings (fig. 14). The mass of the former is 8 times lower than that of the latter. Fig. 13 shows the comparison between the section areas of the elements of the two systems.

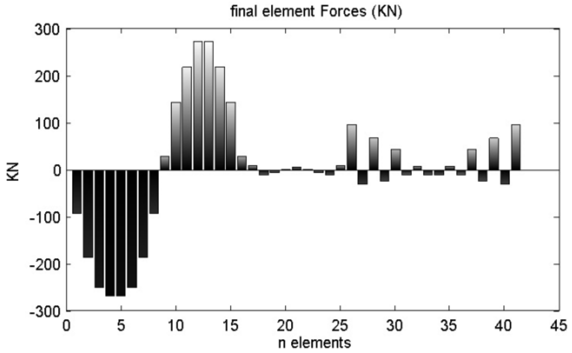

Fig. 12. Axial forces with actuators activated

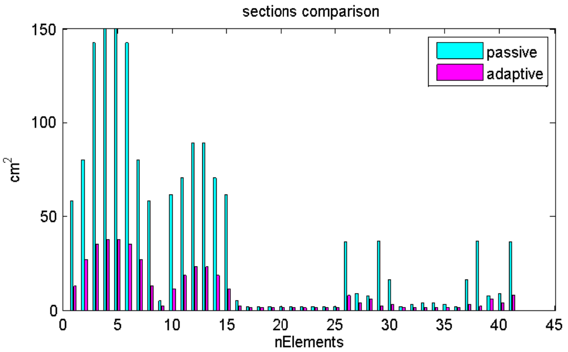

Fig. 13. Comparison section area passive/adaptive

3.0 ENERGY ASSESSMENT | DESIGN PROCESS EXTENDED

3.1 Coupled optimization embodied-operating energy

In order to compute the operating energy of the adaptive structure, we have to make some initial assumptions about the operating frequency of the actuators as well as the time of occurrence of the load cases (i.e. wind velocity). It is sensible to assume that a transient load such as wind is able to excite structures up to 5 Hz at worst (high velocity and vortex shedding). The work done by an active element in order to satisfy compatibility conditions and desired control displacements can be formulated as:

$$W_{\,ik} = \frac{N_{\,ik} * \Delta L_{\,ik} * freq.* time_k}{actuatorEfficiency} \qquad (12)$$

where N_{ik} is the axial force that the actuator i_{th} has to exert at load case k; ΔL_i is its corresponding length change; *freq.,* the operating frequency of the actuators, is assumed to be constant and set to 2.5 HZ (which can considered a rather conservative assumption); $time_k$ is the number of hours for each load case k the i_{th} actuator has to provide N_{ik} and corresponding ΔL_{ik} and *actuatorEfficiency* is the working efficiency of the actuator. Since we do not take explicitly into account the typology of the actuators, their efficiency is set as 50% as a conservative assumption.

As far as the occurrence of each load case is concerned, we assume that wind velocities and frequencies stay the same as those retrieved from the weather file for the entire life-cycle of the structure (here taken as 50 years).

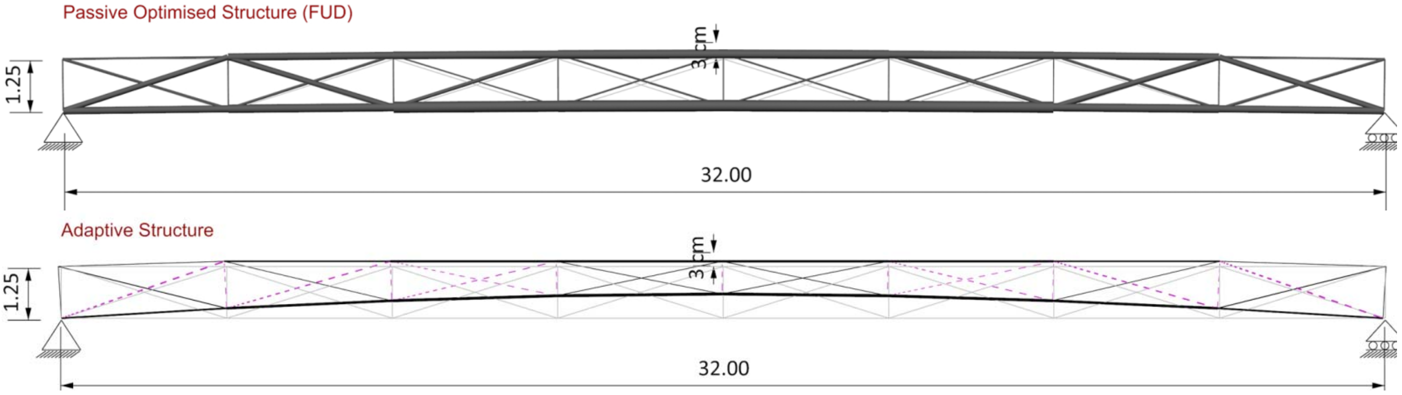

Fig. 14. Comparison between deformed shapes of passive optimized (FUD) and adaptive structure

In order to take into account the minimization of the operating energy, the design process described above is repeated iteratively. It could be considered as an inner step within an outer loop. The main variable of the outer loop is the parameter γ that is used to derive the maximum allowable stress utilised in the inequality constraints eq. (4). The parameter γ is also associated to the degree of redundancy of the system in case of failure of one of more actuators (1) and it would be equivalent to factoring the value of the maximum expected load. Higher values of γ are therefore associated with less probable occurrences of load. By increasing this factor, the optimization routine returns structures with bigger sections and consequently smaller ΔL for the actuators. In other words, by varying γ one can vary from least-weight structures with large operating energy (actuators work) to structures with bigger sections and smaller operating energy consumption. In this way, it is possible to obtain different values for the activation threshold which is the load (here given as design wind velocity) above which the actuators must be activated to satisfy imposed displacement constraints.

The design process can be subdivided in the following steps:

- Define a range of γ;
- For each γ_i repeat steps 2.0 to 2.4 in order to size passive members and find optimal position and length changes of the actuators;
- Analyse (FEM) each solution without active elements as many times as there are load cases in order to find the threshold (activation threshold) below which the structure works adequately (ULS, SLS and controlled DOF displacements respected) even without actuation;
- Compute the operating energy eq. (12) and the embodied energy for each γ_i. The embodied energy is calculated using conversion coefficient for the energy intensity of steel in form of bar-rod taken from the Inventory of carbon and energy - Bath University (6);
- Find the $\gamma_{optimal}$. This is where the minimum of the function given by the sum of embodied and operating energy occurs (fig. 15);
- Repeat optimization (steps 2.0 to 2.4) with $\gamma_{optimal}$ to obtain the optimal configuration and corresponding activation threshold (fig 16);

Fig 17 shows the comparison between the embodied energy of an optimised passive structure with identical topology (using FUD method (5)) and the sum of embodied and operating energy for the equivalent adaptive structure configuration.

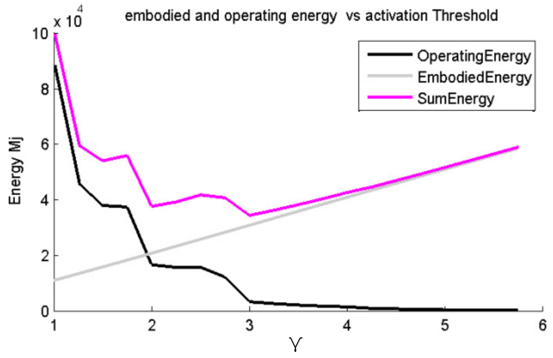

Fig. 15. Embodied energy – operating energy vs. γ

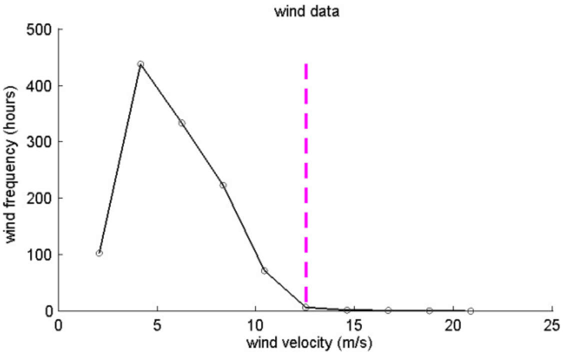

Fig. 16. Activation threshold for optimal configuration

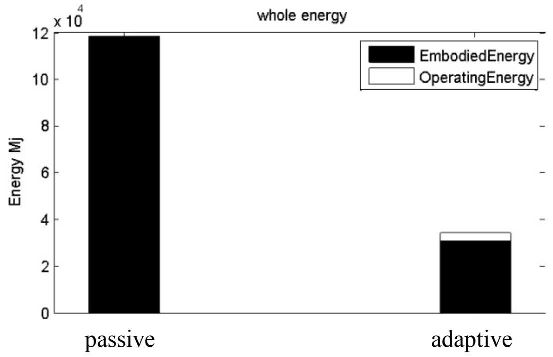

Fig. 17. Comparison passive/adaptive energy demand

The balance is in clear favour of the adaptive structure. Fig. 18 shows the optimal configuration with active elements in deployed state (whose mass is 4 times smaller respect to the configuration obtained with FUD). Note that the size of the sections is larger (bigger γ, fig. 15) respect to previous case (fig. 14) since the optimization process takes into account both embodied and operating energy.

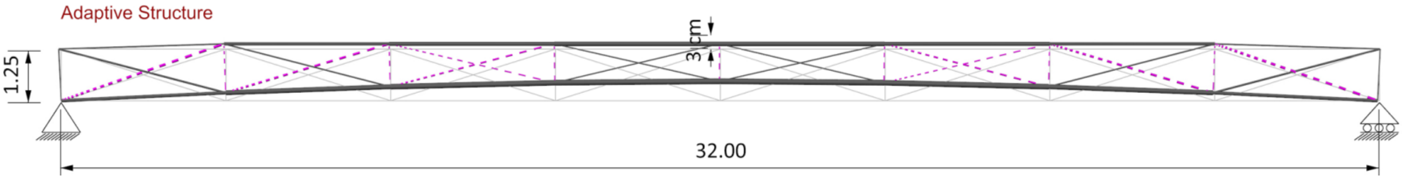

Fig. 18. Optimal configuration

Finally by increasing the number of actuators, it is possible to achieve higher level of control on the nodal displacements of the top cord (controlled DOFs) at the expense of operating energy consumption. However, such an increase is still very small compared to the embodied energy of the passive structure (fig 19).

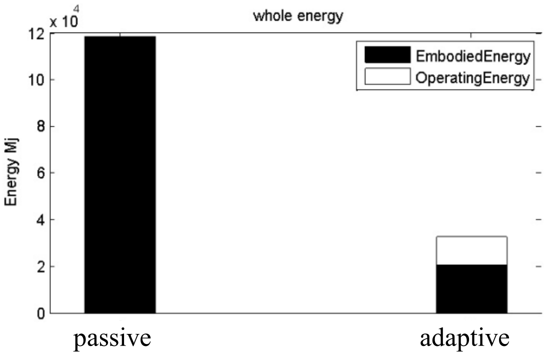

Fig. 19. Operating energy with +6 actuators remains small

Fig. 20 shows the schematic flowchart of the entire design process which starts by defining the objective criteria and constraints. The central block shows how the steps described in this section are executed in iteration. As result of this process the optimal value for the parameter γ is found. Repeating once more steps 1 to 6 with γ_{optimal} gives back the optimal solution (embodied & operating energy minimised) and its corresponding activation threshold for the load.

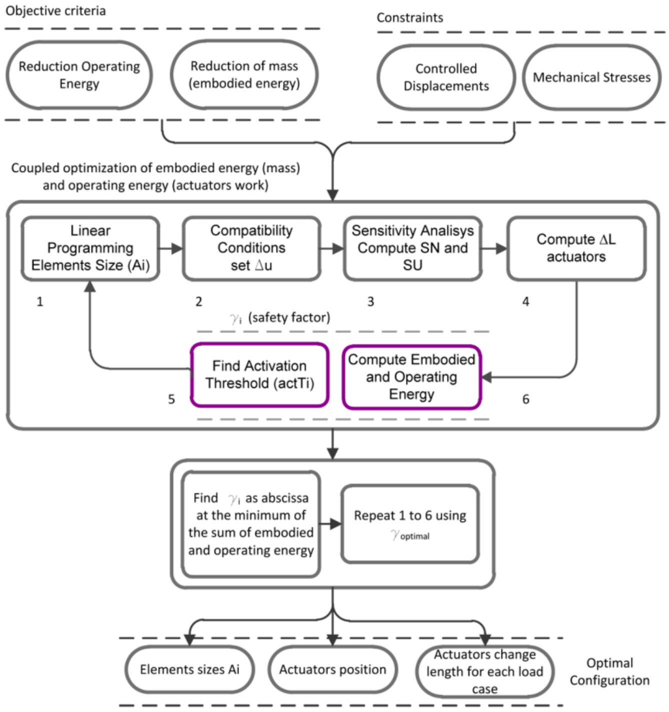

Fig. 19. Coupled optimization process embodied – operating energy

CONCLUSIONS

This paper provides a framework for evaluating the benefit of response control strategies applied to the built environment. The presented design process based on coupled optimization of embodied and operating energy shows that adaptive structures are a viable way to widen and improve building structures performances. The presence of active elements and the possibility of counteracting severe loads at occurrence lead to least weight structures with minimum operating energy consumption. In addition, the derivation of the activation threshold, which is the load above which the active elements are activated, shows how passive and active load bearing capacity can be combined in order to reach higher level of efficiency respect to passive structure.

Further analysis is needed to validate the results presented in the energy assessment which are dependent on a set of rather conservative initial assumptions. Next steps will look at including in the design process non-linear constraints (buckling) and criteria that take into account possible failure of the actuators. Along with this, a feasibility analysis will be carried out in order to understand what technologies at hand can be used for building adaptive structural configurations.

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