Duality in Entanglement Enabling a Test of Quantum Indistinguishability Unaffected by Interactions

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We point out an earlier unnoticed implication of quantum indistinguishability, namely, a property which we call "dualism" that characterizes the entanglement of two identical particles (say, two ions of the same species)—a feature which is absent in the entanglement of two nonidentical particles (say, two ions of different species). A crucial application of this property is that it can be used to test quantum indistinguishability without bringing the relevant particles together, thereby avoiding the effects of mutual interaction. This is in contrast to the existing tests of quantum indistinguishability. Such a scheme, being independent of the nature and strength of mutual interactions of the identical particles involved, has potential applications, including the probing of the transition from quantum indistinguishability to classical distinguishability.

A profound feature of the quantum world is the indistinguishability of various copies of a given particle—a property which has been verified to hold for photons [1,2], mesons [3], electrons [4], neutrons [5], and recently for He and Rb atoms [6–8]. The last set of experiments are significant attempts in testing quantum indistinguishability (QI) for increasingly massive objects. We may stress that extending the verification of QI to objects more complex than atoms is of fundamental importance since this will probe the limits of the quantum world [9] from a perspective which is distinct from testing the superposition principle for macroscopic systems—a widely pursued program [9,10] that notably advanced recently [11]. A further motivation comes from a recent study of tunably indistinguishable photons [2] that leads to the question of whether similar tunings of QI can occur while verifying it with increasingly macroscopic objects. A key condition for such tests [2,4,6,12] is to ensure that the observed statistical effects arise *solely* from QI. This requirement is hard to satisfy for macroscopic or any other type of strongly interacting objects, such as while testing the QI of large molecules by bringing them together at a beam splitter. For example, for mutually repelling bosonic objects, fermionic behavior may be seen [13]. In this context, the very recent photonic simulations of the effects of interactions on the tests of QI [14] underscore the topical interest of this issue [7,8]. Thus, the question arises of whether QI can be tested in a way that is *unaffected* by mutual interactions of the objects involved. That such an "interaction-independent test" of QI is indeed possible is revealed by the present work. This possibility arises from a hitherto unexplored property of an entangled state of identical particles (IPs) which we call "dualism."

The above mentioned dualism can be stated as follows: If two IPs, distinctly labeled by a dynamical variable $\mathcal{A}$, are entangled in terms of a different dynamical variable $\mathcal{B}$, then these particles can also be regarded as being entangled in the variable $\mathcal{A}$ when labeled by the other variable $\mathcal{B}$. Interestingly, this feature provides a testable difference between an entangled state of IPs, say, two ions of the same species localized in traps at distinct locations (as in Ref. [15]), and an entangled state of nonidentical particles, say, an ion and a photon (as in Ref. [16]). Here we formulate a generic scheme to test this dualism that is implementable with any pair of distinctly labeled IPs, provided they can be entangled. While such entangled states are routinely produced for photons [17] and trapped ions [15], very recently, the productions of entangled mobile electrons [18] and trapped atoms [19], along with notable advances in entangling mobile atoms [20] and distant molecules [2], have been achieved. Here an important point is that if QI of given IPs is to be tested, this property should not in itself be invoked to generate the required entanglement. From this point of view, the entangling mechanisms of Refs. [2,15,19,21–23] are particularly apt. Further, we may note that the scheme proposed here could be practically useful. For example, it implies that a given entangled state of spin or internal degrees of freedom [15,17–19,21–23] can also function as a momentum entangled state. It may thus allow the flexibility of invoking the *same* entangled state as a resource for processing quantum information using either the internal or the motional variables.

While this Letter will be couched througout in terms of entangled "particles," the treatment will be equally valid in terms of entangled "modes" [24] when each mode has *exactly one* particle [25]. Situations abound in which two identical particles can be distinctly labeled using a suitable dynamical variable—say, the EPR-Bohm—type states of two identical particles where the terminology particle 1

and 2 is widely used. Such distinct identification may be made through a difference in spatial locations of the particles (such as ions in distinct traps) or in their momenta (such as photons flying in different directions). These types of entangled states are crucial in applications of quantum information, where the terminology such as ''a local operation on particle 1'' (say, belonging to a party Alice) and ''a local operation on particle 2'' (say, belonging to a party Bob) is frequently used even if the particles under consideration (e.g., two photons or two electrons) are identical. On the other hand, the two correlated electrons in a helium atom exhibiting quantum indistinguishability cannot be distinctly labeled; however, it is important to stress that in our Letter we are *not* considering such a situation. In this Letter, we proceed to show that in the former EPR-Bohm—type situation, by formulating a suitable example, quantum indistinguishability can be made to manifest for identical particles that are distinctly labeled. This becomes possible because of the way the choice of the dynamical variable for labeling the particles is appropriately varied in the course of our experimental scheme.

Let us consider the EPR-Bohm entangled state of two spin-$\frac{1}{2}$ IPs (e.g., electrons) written as

$$|\Psi\rangle_{12} = \alpha|\uparrow\rangle_1|\downarrow\rangle_2 + \beta|\downarrow\rangle_1|\uparrow\rangle_2, \qquad (1)$$

where α and β are nonzero amplitudes. In writing Eq. (1), the labels 1 and 2 need to correspond to different values of dynamical variables because the identical particles cannot be distinguished in terms of their innate attributes such as rest mass or charge. Although Eq. (1) is widely used, an alternative description of the above EPR-Bohm entangled state of IPs is given in the usual second quantized notation as $|\Psi\rangle_{12} = (\alpha c^\dagger_{k_1,\uparrow}c^\dagger_{k_2,\downarrow} + \beta c^\dagger_{k_1,\downarrow}c^\dagger_{k_2,\uparrow})|0\rangle$, where $c^\dagger_{k_i,\sigma}$ creates a particle in momenta state k_i and spin state $\sigma = \uparrow, \downarrow$ and $|0\rangle$ is the vacuum state. This second quantized representation clarifies that, in order to meaningfully describe an EPR-Bohm state of two identical particles, we need at least two variables: one variable $\mathcal{A}$ (e.g., momentum in the above example) to label the particles, and another variable $\mathcal{B}$ (e.g., spin) which is entangled, where $[\mathcal{A}, \mathcal{B}] = 0$. In terms of distinct eigenvalues A_1, A_2 and B_1, B_2 of the variables $\mathcal{A}$ and $\mathcal{B}$, respectively, one may thus write an EPR-Bohm state as

$$|\Psi(A_1, A_2, B_1, B_2)\rangle = (\alpha c^\dagger_{A_1,B_1} c^\dagger_{A_2,B_2} + \beta c^\dagger_{A_1,B_2} c^\dagger_{A_2,B_1})|0\rangle, \qquad (2)$$

where $c^\dagger_{A_i,B_j}$ creates a particle in the simultaneous eigenstate $|A_i, B_j\rangle$ of the variables $\mathcal{A}$ and $\mathcal{B}$. In order to put Eq. (2) in the form of Eq. (1) we rewrite $c^\dagger_{A_i,B_j}|0\rangle$ as $|B_j\rangle_{A_i}$, where A_i is taken as the ''which particle'' label (thus, $|B_j\rangle_{A_i}$ is a second quantized notation), whence

$$|\Psi(A_1, A_2, B_1, B_2)\rangle = \alpha|B_1\rangle_{A_1}|B_2\rangle_{A_2} + \beta|B_2\rangle_{A_1}|B_1\rangle_{A_2}. \qquad (3)$$

The above form of rewriting is, in fact, standard and is widely used in describing the entangled states of IPs generated in actual or proposed experiments [12,17,26,27]. For example, in the routinely used two-photon entangled state $\frac{1}{\sqrt{2}}(|H\rangle_1|V\rangle_2 + |V\rangle_1|H\rangle_2)$, the symbols $|H\rangle_i$ and $|V\rangle_j$ are, in fact, rewritten forms for $c^\dagger_{k_i,H}|0\rangle$ and $c^\dagger_{k_j,V}|0\rangle$, where k_i and k_j are labels for distinct momenta directions. Considering in the same sense as Eq. (1), the above Eq. (3) is an entangled state of the variable $\mathcal{B}$ (say, polarization or spin) with the variable $\mathcal{A}$ (say, position or momentum) being the ''which particle'' label. Alternatively, we may use the eigenvalues of the variable $\mathcal{B}$ to label the particles and replace $c^\dagger_{A_i,B_j}|0\rangle$ in Eq. (2) by $|A_i\rangle_{B_j}$ to rewrite Eq. (2) as

$$|\Psi(A_1, A_2, B_1, B_2)\rangle = \alpha|A_1\rangle_{B_1}|A_2\rangle_{B_2} \pm \beta|A_2\rangle_{B_1}|A_1\rangle_{B_2}, \qquad (4)$$

where, in the last step, $|A_2\rangle_{B_1} \equiv c^\dagger_{A_2,B_1}$ and $|A_1\rangle_{B_2} \equiv c^\dagger_{A_1,B_2}$ have been exchanged to bring Eq. (4) to the same form as Eq. (1) (i.e., the ''which particle'' label B_1 preceding the ''which particle'' label B_2 in both terms of the superposition). The upper and lower signs of $\pm$ in Eq. (4) correspond to bosons and fermions, respectively, and arise from the above exchange of creation operators. The two equivalent representations of the state $|\Psi(A_1, A_2, B_1, B_2)\rangle$ given by Eqs. (3) and (4) bring out the property of dualism. This means that a class of states of two identical particles can *equally* well be regarded as entangled in *either* the variable $\mathcal{A}$ *or* the variable $\mathcal{B}$, depending upon whether the variable used for distinguishing (labeling) the particles is $\mathcal{B}$ or $\mathcal{A}$, respectively. The way this property of dualism arises can also be seen clearly through the derivation given in the Supplemental Material [28] in terms of a first quantized formulation based on appropriate symmetrization or antisymmetrization using pseudolabels.

That the above property of dualism holds essentially for IPs can be seen by replacing for nonidentical particles the right-hand side of Eq. (2) by $(\alpha c^\dagger_{A_1,B_1} d^\dagger_{A_2,B_2} + \beta c^\dagger_{A_1,B_2} d^\dagger_{A_2,B_1})|0\rangle$, where $c^\dagger$ and $d^\dagger$ create different species of particles. While the above state can be written in the analogue of Eq. (3), $\alpha|B_1\rangle_{A_1,C}|B_2\rangle_{A_2,D} + \beta|B_2\rangle_{A_1,C}|B_1\rangle_{A_2,D}$, where C and D stand for distinct particle attributes such as mass or charge, it cannot be written in the analogue of Eq. (4), as that would entail superposing states $|C\rangle$ and $|D\rangle$, which is not allowed.

There is a *complementarity* in the dualism in the sense that one cannot observe simultaneously the entanglement in *both* the variables. This complementarity makes evident the way $|\Psi(A_1, A_2, B_1, B_2)\rangle$ differs from the hyperentangled states [29] in which more than one variable is simultaneously entangled. We may also stress that this property of dualism, stemming from the *interchangeability* of two different dynamical variables that are used for labeling the concerned particles, is a manifestation of quantum indistinguishability that is different from its other

manifestations, e.g., the behavior of IPs on simultaneous incidence on a beam splitter [17].

Next, we discuss how the above dualism can be tested. Since an entangled state is required for this purpose, we first consider the readily available polarization entangled states of photons. Such a state can be written as in Eq. (1) with the particle indices 1 and 2 corresponding to momenta labels $-\mathbf{k}$ and $\mathbf{k}$, respectively, and $\uparrow$ and $\downarrow$ representing polarization states H and V, respectively. The dualism can then be expressed as

$$|H\rangle_{-\mathbf{k}}|V\rangle_{\mathbf{k}} + |V\rangle_{-\mathbf{k}}|H\rangle_{\mathbf{k}} \equiv |-\mathbf{k}\rangle_H|\mathbf{k}\rangle_V + |\mathbf{k}\rangle_H|-\mathbf{k}\rangle_V. \tag{5}$$

Let $\mathbf{k}$ be chosen to be along the x axis. Then the polarization entanglement implied by the left-hand side of Eq. (5) can be tested in the usual manner by Alice and Bob on opposite locations along the x axis. For testing its dual, we separate the H and V components of the state along the y axis, as shown in Fig. 1 with the aid of a polarization beam splitter (PBS). Then the labels H and V on the right-hand side of Eq. (5) become identifiable with distinct momenta along the y axis, and the particle labeled H reaches Charlie, while the particle labeled V reaches Diana, as shown in Fig. 1. Charlie and Diana are thus in possession of the entangled state $|-\mathbf{k}\rangle_C|\mathbf{k}\rangle_D + |\mathbf{k}\rangle_C|-\mathbf{k}\rangle_D$, where C stands for the particle possessed by Charlie and D for the particle possessed by Diana. In this entangled state, the momenta component along the x axis appears as a simple dichotomic variable on which Charlie and Diana can perform a Bell's inequality experiment using a beam splitter and detectors using the procedure described in detail in the caption of Fig. 1. We stress that the violation of Bell's inequality by the dual forms of entanglement is merely a convenient tool to verify the property of dualism (an entanglement witness can also be used [30]). Importantly, the same test is also possible with other entangled IPs such as ions [15,31], where efficient detectors make the study of Bell's inequality free of the detection loophole [31]. Moreover, in this context, the necessity of ensuring spacelike separation does not arise at all.

The predicted sign difference between the dual forms of entanglement Eqs. (3) and (4) in the case of fermions should also be testable. The presence of such sign difference is reinforced through an alternative derivation of the dualism in terms of symmetrization or antisymmetrization in the first quantized notation, given in the Supplemental Material [28]. If separate experiments measuring the expectation value of the Bell operator (i.e., the expectation value of the linear combination of four correlators occurring on the left-hand side of the Bell inequality, without taking the modulus) are performed using entanglements in the variables $\mathcal{A}$ and $\mathcal{B}$, respectively, the expectation values in the two cases will have an opposite sign for fermions. Hence, the testing of such dualism can enable verifying the bosonic or fermionic nature of the particles.
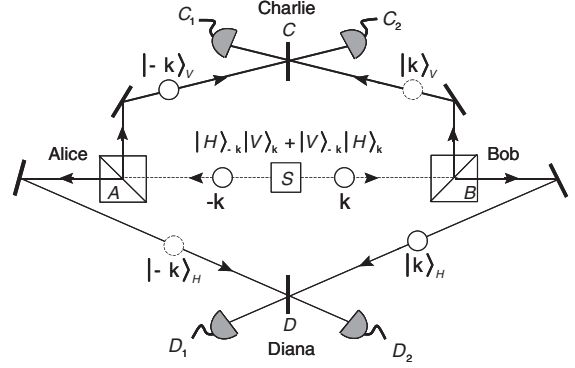

FIG. 1. Scheme to test the dualism in entanglement of identical particles. Two photons of the same frequency in the polarization entangled state $|\Psi\rangle_{12} = \frac{1}{\sqrt{2}}(|H\rangle_1|V\rangle_2 + |V\rangle_1|H\rangle_2)$ are emitted in opposite directions by the source S. The labels 1 and 2 stand for their momenta $-\mathbf{k}$ and $\mathbf{k}$, respectively, along the x axis. The $-\mathbf{k}$ and $\mathbf{k}$ photons fly towards Alice and Bob, respectively. If they conducted polarization correlation measurements on these photons, they could verify their polarization entanglement. Instead, in order to verify the *dual* momentum entanglement implied by the right-hand side of Eq. (5), they put polarization beam splitters A and B in the paths of the photons which deflects $|V\rangle$ photons in the $+y$ direction (towards Charlie), and $|H\rangle$ photons in the $-y$ direction (towards Diana). It is because the photon pair is emitted in the state $|\Psi\rangle_{12}$ that only one photon reaches Charlie, while its partner reaches Diana. Then, by the dualism of Eq. (5), the x component of their momenta is entangled. Now, since there are only two possible values of the x component of the momentum, namely, $-\mathbf{k}$ and $\mathbf{k}$, one can associate a dichotomic pseudospin observable [40] with this momentum using operators $\sigma_x = |-\mathbf{k}\rangle\langle\mathbf{k}| + |\mathbf{k}\rangle\langle-\mathbf{k}|$, $\sigma_y = i(|-\mathbf{k}\rangle\langle\mathbf{k}| - |\mathbf{k}\rangle\langle-\mathbf{k}|)$, and $\sigma_z = |\mathbf{k}\rangle\langle\mathbf{k}| - |-\mathbf{k}\rangle\langle-\mathbf{k}|$. These pseudospin operators and their linear combinations are measurable by a beam splitter with a tunable reflectivity and two detectors if exactly one photon is incident on the beam splitter at a time. By noting the coincidence of clicks in their detectors, Charlie and Diana can verify whether a Bell-type inequality is violated by the momentum pseudospin correlation measurements on their photons, thereby testing the property of dualism.

Further, note that the PBS in the above scheme is merely used to separate the photons according to their polarization and enable the verification of the dual entanglement (the PBS has no role in creating the dual entanglement). A practical application of the above scheme would thus be to use spin or polarization entangled states (if they are easier to produce) as a resource for obtaining momentum entangled states. For example, spin entangled mobile electrons have just been realized [18], where the state $|\uparrow\rangle_{\mathbf{k}_1}|\downarrow\rangle_{\mathbf{k}_2} + |\downarrow\rangle_{\mathbf{k}_1}|\uparrow\rangle_{\mathbf{k}_2}$ is produced with $\mathbf{k}_1$ and $\mathbf{k}_2$ being momenta states in two distinct one-dimensional conducting channels. When such a state is generated, the dualism pointed out will enable one to easily obtain a momentum entangled state (electronic waveguides have been fabricated [32] and spin analogues of a PBS have been proposed [33]). Almost any other method for obtaining momenta

entangled states from spin entangled states will be more complicated (involving either delocalized spin measurements or additional momenta dependent spin flips). Next, we discuss a foundational application of the property of dualism.

Existing tests of QI involve bringing two IPs together [1,4,12] to exhibit, for example, bunching and antibunching, whose results would be modified by particle interactions [13]. As the identical objects get complex, the outcomes of such tests could increasingly deviate from the ideal noninteracting case in view of complicated scatterings, including inelastic collisions, fragmentations, or chemical reactions. However, in testing our dualism as above, the objects in the $|\mathbf{k}\rangle$ and the $|-\mathbf{k}\rangle$ state are *never* present concomitantly. Consequently, the outcomes of our proposed experiment should be the same whatever the mutual interactions of the IPs, thereby providing an interaction-independent test of QI.

The caveat is that the IPs will have to be in an entangled state *before* the experiment. Preparing such a state is generically challenging, for which invoking QI could be required [12]. However, there are several schemes which do not invoke QIs of the IPs involved; e.g., in a recent breakthrough, atoms held in distinct tweezers were entangled [19]. The internal levels of two identical ions in distant traps have already been entangled by photodetection [15], whose feasibility for molecules was also demonstrated recently [2]. In fact, the generation of entangled molecules is being explored so actively that testing Bell's inequalities with molecules is just a matter of time [23]. Polar molecules (e.g., CO, ND_3, OH, YbF) cooled to a ground state and trapped [34] can be entangled either by a direct interaction [21] or through a mediating resonator [22]. Although the interactions are used here to generate a entangled state which is used to test QI through our dualism, the test of QI in itself remains unaffected by interactions, provided the entangled particles are kept well separated during the test.

The entangling methods of the previous paragraph generate the state $|\uparrow\rangle_{\psi_1}|\downarrow\rangle_{\psi_2} + |\downarrow\rangle_{\psi_1}|\uparrow\rangle_{\psi_2}$, where $\uparrow/\downarrow$ stand for ionic (molecular) internal states (spins), ψ_1 and ψ_2 label the center of mass wave function of ions or molecules in the traps 1 and 2, respectively, and $|\uparrow\rangle_{\psi_j} \equiv c^{\dagger}_{\uparrow,\psi_j}|0\rangle$ and $|\downarrow\rangle_{\psi_j} \equiv c^{\dagger}_{\downarrow,\psi_j}|0\rangle$ (the rotational-vibrational modes of the molecules are taken to be cooled to their ground states [21,34]). To verify the dualism using our scheme (Fig. 1), one needs to transfer the entangled ions or molecules from their traps to matter waveguides [35], thereby converting their trap states to momenta states: $|\psi_1\rangle \rightarrow |-\mathbf{k}\rangle$, $|\psi_2\rangle \rightarrow |\mathbf{k}\rangle$. For the subsequent interferometric procedure, beam splitters, waveguides, or PBS are available (using atom chips [13,35], molecular waveguides [36,37], and molecule chips [38]). This opens up a way to reveal the true indistinguishability of those mutually repulsive bosonic ions and polar molecules which show deceptive antibunching [13].

We now consider the role in the dualism experiment of those degrees of freedom which are not involved directly in any of the dual forms of entanglement. These degrees of freedom are assumed to be in the *same* collective state $|\chi_0\rangle$ for both the trapped IPs when they are first entangled. We thus write the initial state as $|\uparrow\rangle_{\psi_1,\chi_0}|\downarrow\rangle_{\psi_2,\chi_0} + |\downarrow\rangle_{\psi_1,\chi_0}|\uparrow\rangle_{\psi_2,\chi_0}$. Now, suppose *after* the preparation of the above state and the switching off of any interaction between the IPs (say, by pulling their traps far apart) they are held in their respective traps for a time t over which neither the c.m. motion nor the entangled spinlike variable is significantly affected by the environment. This is possible, since the spinlike variables considered by us can have long coherence times [21], and stable superpositions of motional states of the c.m. have been demonstrated for large molecules [10]. However, with increasing molecular complexity, the number of degrees of freedom involved in $|\chi_0\rangle$ becomes larger with their collective state more influenced by the environment. Then, $|\chi_0\rangle$ would evolve *differently* to $|\chi_1(t)\rangle$ and $|\chi_2(t)\rangle$ in the respective traps. Although this does not affect the violation of Bell's inequality for the spin entanglement, it *suppresses* Bell's inequality violation for the dual momentum entanglement by a factor $|\langle\chi_1(t)|\chi_2(t)\rangle|^2$—a form of decoherence relevant to the transition from QI to classical distinguishability of IPs. In the classical limit, $\chi_1(t \rightarrow \infty)$ and $\chi_2(t \rightarrow \infty)$ emerge as intrinsic labels for IPs as $|\langle\chi_1(t\rightarrow\infty)|\chi_2(t\rightarrow\infty)\rangle|^2 \rightarrow 0$. Such a quantum to classical transition complements the widely studied quantum to classical transition through the decoherence of superpositions [10,39].

Finally, the very feature, as we have shown, that there is a scheme *unaffected* by interactions whereby one can test whether strongly interacting identical complex particles can "justly be regarded as being created from the same vacuum" should be interestig in itself. Further, that this stems from a hitherto unnoticed dualism in the entanglement of IPs enhances the need for its experimental verification even for photons, ions, atoms, or electrons. Also, importantly, as we have argued, if tested with more complex objects, this dualism has the potentiality to provide a fruitful way of studying the transition from QI to classical distinguishability.

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