Numerical modeling method for the dispersion characteristics of single-mode and multimode weakly-guiding optical fibers with arbitrary radial refractive index profiles

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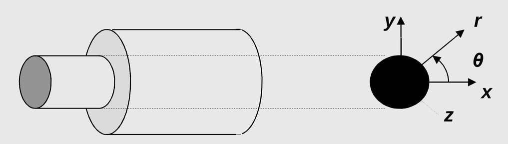
Single-mode Optical Fiber



- cylindrical waveguide
- circular symmetry
- radially inhomogeneous refractive index profile
- infinite uniform cladding

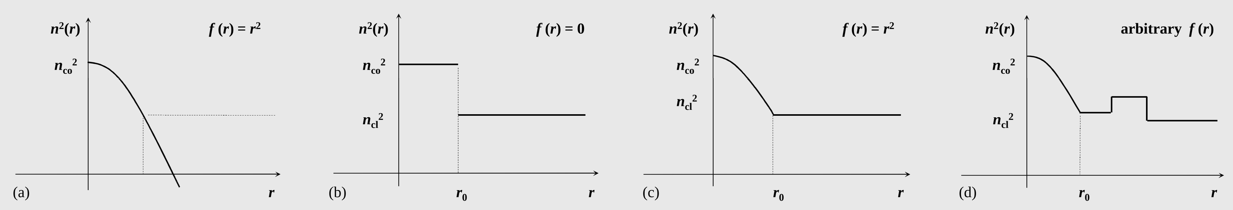


Figure 1. Schematic refractive index profiles:
(a) infinitely extended parabolic profile; (b) step-index profile; (c) truncated parabolic profile; (d) arbitrary profile

Optical Fiber Model

Scalar wave equation

$$\Delta\Psi + k_0^2 n^2(r)\Psi = 0$$

$$\left[\nabla_t^2 + k_0^2 n^2(r)\right]\psi(r,\theta) = \beta^2\psi(r,\theta)$$

$$\left[\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} + k_0^2 n^2(r) - \frac{m^2}{r^2} - \beta^2\right] R_{mn}(r) = 0$$

- time harmonic fields (with angular frequency ω)
- arbitrary radial refractive index profile $n(r)$
- direction of propagation is along the z-axis
- β is the propagation coefficient
- k_0 is the free-space wavenumber

- m is the azimuthal mode number
- n is the radial mode number

Boundary conditions:

- fields are finite at the core center and
- decay to zero as $r \to \infty$

Galerkin Method

Expansion in terms of basis functions [1]

$$\psi(r,\theta) = \sum_{i=1}^{N} c_i b_i(r)\exp(-im\theta)$$

$$\left[\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} + k_0^2 n^2(r) - \frac{m^2}{r^2} - \beta^2\right] R_{mn}(r) = 0$$

$$\mathbf{Ac} = \beta^2\mathbf{c}$$

Basis functions: Laguerre-Gauss polynomials

$$b_{mn}(x(r)) = \left[\frac{V}{\pi r_0^2}\frac{n!}{(n+m)!}\right]^{1/2}\exp(-x(r)/2)\,x(r)^{m/2}L_n^m(x(r))$$

$L_n^m(x)$ are the generalized Laguerre polynomials

V is the normalized frequency and $x(r) = V(r/r_0)^2$

Laguerre-Gauss polynomials

- form a complete discrete set of orthonormal functions
- satisfy the boundary conditions
- represent the modal eigenfunctions for the infinitely extended parabolic profile in circular waveguides (Figure 1a)

Matrix eigenvalue problem

- A is symmetric and
- has purely discrete real eigenvalue spectrum

- the eigenvalues provide the propagation coefficients β for the given value of m
- the components of the corresponding eigenvector represent the expansion coefficients c_i

The technique is also valid for multimode fibers

Dispersion Characteristics

- the group slowness, τ_g, dispersion, D, and dispersion slope, DS, are proportional to the first, second and third order derivatives of the propagation coefficient, β with respect to frequency (or wavelength) correspondingly
- to define the derivatives of the propagation coefficient, the matrix equation is differentiated analytically repetitively

$$\tau_g[s/km] = \frac{d\beta}{d\omega}$$

$$D[ps/(nm\cdot km)] = -\frac{2\pi c}{\lambda^2}\frac{d^2\beta}{d\omega^2}$$

$$DS[ps/(nm^2\cdot km)] = \frac{dD}{d\lambda}$$

$$\frac{d\beta^2}{d\omega} = \mathbf{c}^{\mathrm{T}}\frac{d\mathbf{A}}{d\omega}\mathbf{c}$$

$$\frac{d^2\beta^2}{d\omega^2} = \mathbf{c}^{\mathrm{T}}\left(\frac{d^2\mathbf{A}}{d\omega^2}\right)\mathbf{c} + 2\mathbf{c}^{\mathrm{T}}\frac{d\mathbf{A}}{d\omega}\frac{d\mathbf{c}}{d\omega}$$

$$\frac{d^3\beta^2}{d\omega^3} = \mathbf{c}^{\mathrm{T}}\left(\frac{d^3\mathbf{A}}{d\omega^3}\right)\mathbf{c} + 3\mathbf{c}^{\mathrm{T}}\left(\frac{d^2\mathbf{A}}{d\omega^2}\right)\frac{d\mathbf{c}}{d\omega} + 3\mathbf{c}^{\mathrm{T}}\left(\frac{d\mathbf{A}}{d\omega} - \frac{d\beta^2}{d\omega}\mathbf{I}\right)\frac{d^2\mathbf{c}}{d\omega^2}$$

- simplicity of Laguerre-Gauss basis functions allows to analytically determine $\frac{d\mathbf{A}}{d\omega}$, $\frac{d^2\mathbf{A}}{d\omega^2}$ and $\frac{d^3\mathbf{A}}{d\omega^3}$
- $\frac{d\mathbf{c}}{d\omega}$ and $\frac{d^2\mathbf{c}}{d\omega^2}$ are the first and second derivatives of the eigenvector
- rather laborious at programming stage
- the reward is more accurate and faster evaluation of the dispersion characteristics

Numerical Results

Conclusions

- the approach provides more accurate results compared to approximation methods
- the number of basis functions in the range 20 to 28 was found a good compromise between accuracy and computation time
- excellent computation time reduction for fiber characteristics, especially for the dispersion and its slope
- the computation times for the calculation of the propagation coefficient, group delay, dispersion and dispersion slope for 25, 35 and 45 basis functions, Figure 4, are 0.059s, 0.195s and 0.4 s respectively (Intel Pentium(R) CPU 2 GHz)
  For comparison: the time required to calculate a single dispersion value at a fixed wavelength in [2] is about 5 min
- can be used in the case of any arbitrary radial refractive index profile and few-mode fibers

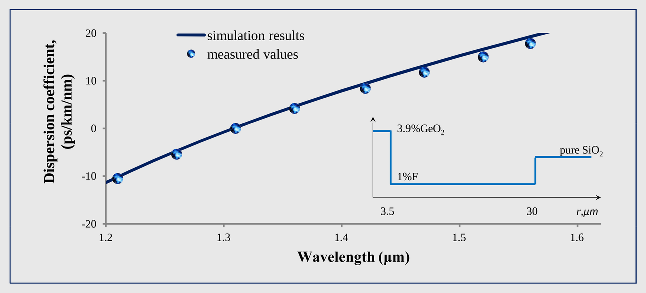


Figure 2. Dispersion graph for the depressed cladding fiber from [4] compared to measured values

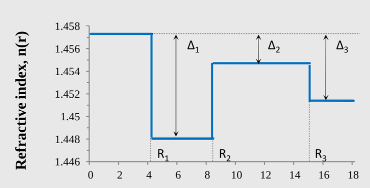


Figure 3. Refractive index profile for the triple-clad fibre [3]

FIBER PARAMETERS			
	Core material	Core and cladding radii	Dopants used in cladding with the doping levels
Single-clad fiber	pure SiO_2	3.5 µm	Fluorine, 1.782%
Double-clad fiber	pure SiO_3	4.2 µm, 5.2 µm	Fluorine, 4.5%, 1.08%
Triple-clad fiber	pure SiO_4	4.2 µm, 8.25 µm, and 15 µm	Fluorine, 1.782%, 0.509%, and 1.131%

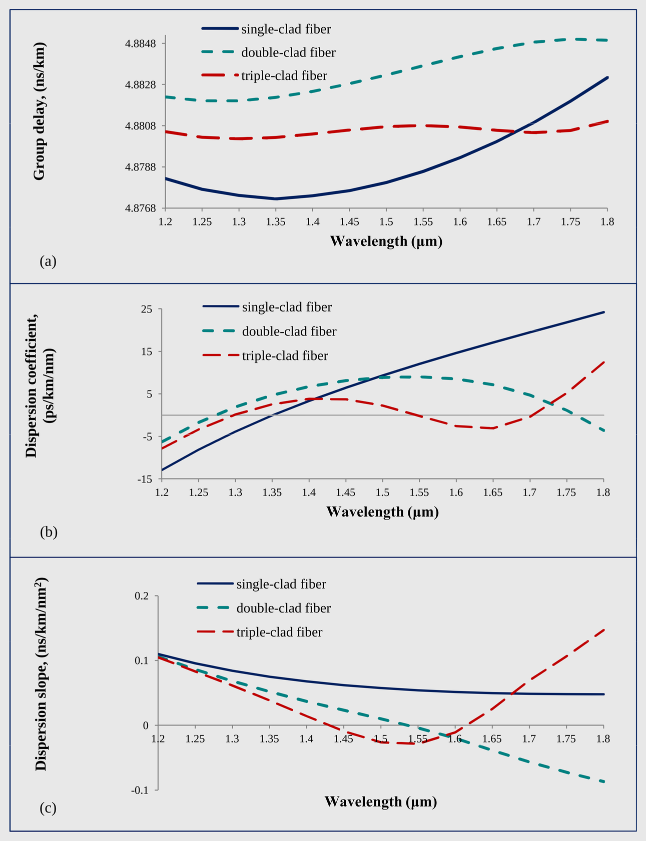


Figure 4. Comparative dispersion characteristics of exemplary single-, double- and triple-clad fibers: (a) group delay; (b) chromatic dispersion; (c) dispersion slope; the parameters of the fiber designs are given in the table; the triple-clad fiber was designed in [3]

References

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[2] Silvestre E., Pinheiro-Ortega T., Andrres P., Miret J. J., and Ortigosa-Blanch A., "Analytical evaluation of chromatic dispersion in photonic crystal fibers," Opt. Lett. 30(5), 453–455 (2005).

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