

Robust One-Step Catalytic Machine for High Fidelity Anticloneing and W -State Generation in a Multiqubit System

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We propose a physically realizable machine which can either generate multiparticle W -like states, or implement high-fidelity $1 \rightarrow M$ ($M = 1, 2, \dots, \infty$) anticloning of an arbitrary qubit state, in a single step. This universal machine acts as a catalyst in that it is unchanged after either procedure, effectively resetting itself for its next operation. It possesses an inherent *immunity* to decoherence. Most importantly in terms of practical multiparty quantum communication, the machine's robustness in the presence of decoherence actually *increases* as the number of qubits M increases.

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Quantum mechanics provides two remarkable “laws” concerning what is and is not possible in our Universe. The *linearity* of quantum mechanics implies it is impossible to copy an arbitrary quantum (qubit) state [1], no matter how ingenious the experimental scheme. The *unitarity* of quantum mechanics implies that there is no quantum device, no matter how well built, which can perfectly transform an arbitrary qubit state into its orthogonal complement [2,3]. In practice, however, it is possible to clone and complement qubits with reasonably high fidelity [3,4]. Indeed these two processes appear to be closely related [3]: optimal $1 \rightarrow 2$ cloning (i.e., partial copying from one to two qubits) and the universal NOT of photon polarization states, can both be performed using the same unitary transformation [5,6]. A combination of copying and complementing might even lead to optimal entangling transformations [7]. The connections between these two quantum processes are, however, not well understood either in theoretical or practical terms. This Letter provides a concrete connection between quantum copying, complementing, and entangling operations, by proposing a specific multiqubit-cavity scheme in which the *same* unitary transformation can be used to produce both multiqubit W -entangled states and high- (in some cases optimal) fidelity $1 \rightarrow M$ anticloning. Our results and conclusions are valid for *any* number of qubits M [8].

The physical implementation of our scheme offers several important advantages and features. First, the cavity acts as a *catalyst* in that its state is unchanged after either procedure—in short, our machine acts as its own reset button. Second, the machine has an inherent immunity to decoherence effects: the entangling and anticloning operations become increasingly robust as the number of qubits M increases, in contrast to typical quantum information schemes whose performance would deteriorate as the number of degrees of freedom increases. Third, our machine avoids the need for carefully engineered nearest-neighbor interactions [9], multiple cavities and/or gate operations

[10,11]. Our multiqubit-cavity machine can be built using current atom- or ion-cavity technology [12,13], or next-generation solid-state-qubit-cavity technology [14].

The Hamiltonian for the M -qubit-plus-cavity system in the interaction picture and rotating-wave approximation ($\hbar = 1$) is

$$H_I = \sum_{j=1}^M \gamma_j \{a^\dagger \sigma_j^- + \sigma_j^+ a\}, \quad (1)$$

where $\sigma_j^+ = |1_j\rangle\langle 0_j|$, $\sigma_j^- = |0_j\rangle\langle 1_j|$ with $|1_j\rangle$ and $|0_j\rangle$ being the excited and ground states of the j 'th qubit. a^\dagger and a are cavity-photon creation and annihilation operators while $\{\gamma_j\}$ are the set of (in general unequal) qubit-cavity couplings. Since $[H_I, \mathcal{N}] = 0$ where $\mathcal{N} = a^\dagger a + \sum_{i=1}^M \sigma_i^+ \sigma_i^-$ is the excitation number operator, the dynamics is separable into subspaces having a prescribed eigenvalue N of \mathcal{N} . In the subspace with $N = 0$ there is only one state $|\phi_0\rangle = |0_1, 0_2, 0_3 \dots 0_M; 0\rangle$ while in the $N = 1$ subspace, the basis states are

$$\begin{aligned} |\phi_1\rangle &= |1_1, 0_2 \dots 0_M; 0\rangle = |Q_1\rangle \otimes |0\rangle \\ |\phi_2\rangle &= |0_1, 1_2 \dots 0_M; 0\rangle = |Q_2\rangle \otimes |0\rangle \\ &\vdots \\ |\phi_j\rangle &= |0_1, 0_2 \dots 1_j, \dots 0_M; 0\rangle = |Q_j\rangle \otimes |0\rangle \\ |\phi_{M+1}\rangle &= |0_1, 0_2 \dots 0_M, 1\rangle \end{aligned} \quad (2)$$

where the last label in each ket denotes the photon number in the cavity. In this $N = 1$ subspace, the system's state at time t is $|\Psi(t)\rangle = \hat{U}(t, 0)|\Psi(0)\rangle$ where $\hat{U}(t, 0)$ in the basis of states $\{|\phi_1\rangle \dots |\phi_{M+1}\rangle\}$ is given by Eq. (3). Here, we define the effective collective Rabi frequency of the M qubits as $\omega^2 = \sum_{j=1}^M \gamma_j^2$, and $\beta = \sin^2(\omega t/2)/\omega^2$. We proceed to show how to build our machine using this temporal evolution:

$$\hat{U}(t, 0) = \begin{pmatrix} 1 - 2\gamma_1^2\beta & -2\gamma_1\gamma_2\beta & \cdots & -2\gamma_1\gamma_M\beta & -i\gamma_1 \sin(\omega t)/\omega \\ -2\gamma_2\gamma_1\beta & 1 - 2\gamma_2^2\beta & \cdots & -2\gamma_2\gamma_M\beta & -i\gamma_2 \sin(\omega t)/\omega \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ -2\gamma_M\gamma_1\beta & -2\gamma_M\gamma_2\beta & \cdots & 1 - 2\gamma_M^2\beta & -i\gamma_M \sin(\omega t)/\omega \\ -i\gamma_1 \sin(\omega t)/\omega & -i\gamma_2 \sin(\omega t)/\omega & \cdots & -i\gamma_M \sin(\omega t)/\omega & \cos(\omega t) \end{pmatrix} \quad (3)$$

Consider an initial product state where one of the qubits (e.g., $j = 1$) is in a coherent superposition $|q_1(0)\rangle = \sin(\theta/2)|0\rangle + e^{i\alpha} \cos(\theta/2)|1\rangle$ with the others unexcited:

$$\begin{aligned} |\Psi(0)\rangle &= |q_1(0)\rangle \otimes |0_2, \dots, 0_M; 0\rangle \\ &= \sin(\theta/2)|\phi_0\rangle + e^{i\alpha} \cos(\theta/2)|\phi_1\rangle. \end{aligned} \quad (4)$$

Using Eq. (3) yields

$$|\Psi(t)\rangle = \sin(\theta/2)|\phi_0\rangle + e^{i\alpha} \cos(\theta/2)|\phi_1(t)\rangle \quad (5)$$

with

$$|\phi_1(t)\rangle = \sum_{j=1}^M U_{j1}(t)|Q_j\rangle \otimes |0\rangle - i \frac{\gamma_1 \sin(\omega t)}{\omega} |\phi_{M+1}\rangle. \quad (6)$$

When $\omega t = m\pi \equiv \omega\tau^*$ (m odd) a vacuum trapping state condition is achieved: the cavity state is unchanged overall and becomes fully separable from the multiqubit subsystem. However, its catalytic action has induced entanglement into the initially unentangled multiqubit subsystem. Because of the cavity's inertness at $t = \tau^*$, we will drop the cavity state notation. Consider the following examples: (i) $\theta = 0$, which will yield one-step W -state generation; (ii) $\theta = \pi/2$, which will yield optimal quantum anticloning.

(i) Using $\theta = 0$ yields

$$|\Psi(\tau^*)\rangle = (1 - 2\gamma_1^2/\omega^2)|Q_1\rangle - (2\gamma_1/\omega^2) \sum_{j=2}^M \gamma_j |Q_j\rangle. \quad (7)$$

In general, an M -qubit W state cannot be generated using identical couplings $\gamma_i \equiv \gamma$. However for nonidentical couplings, the qubit-exchange symmetry is broken, thereby allowing control over the degree of entanglement and the final state symmetry. Suppose $\gamma_1 \neq \gamma_j = \gamma$ for all $j > 1$, and define $r = \gamma_1/\gamma$. The collective qubit frequency is $\omega = \gamma(r^2 + M - 1)^{1/2}$ and

$$|\Psi(\tau^*)\rangle = a_1(\tau^*)|Q_1\rangle + a(\tau^*) \sum_{j=2}^M |Q_j\rangle \quad (8)$$

where

$$a_1(\tau^*) = \frac{M - 1 - r^2}{M - 1 + r^2}, \quad a(\tau^*) = \frac{-2r}{M - 1 + r^2}. \quad (9)$$

Two W states of M qubits can now be generated for $a_1(\tau^*) = \pm a(\tau^*)$, yielding an optimal coupling ratio $r_W^\pm = \sqrt{M} \pm 1$. Here r_W^\pm correspond to symmetric and antisymmetric states with respect to exchange of qubit 1 with any other. The corresponding state is

$$|\Psi(\tau^*)\rangle = |W_M^\pm\rangle = \frac{e^{i\pi}}{\sqrt{M}} \left[\pm |Q_1\rangle + \sum_{j=2}^M |Q_j\rangle \right]. \quad (10)$$

For $M = 4$, both $r = 1$ and $r = 3$ produce a fully symmetric W state. However in the many-qubit limit $M \rightarrow \infty$, it is only for nonidentical couplings ($r_W \approx \sqrt{M}$) that we can generate a multiqubit W entangled state. This state is important for quantum information protocols since the excitation has equal probability of being found on any qubit. For $M \geq 3$, a fully symmetric W state of $M - 1$ qubits can also be obtained when $a_1(\tau^*) = 0$, yielding $r_W = \sqrt{M - 1}$. The initial excitation gets transferred to, and shared among, the remaining $M - 1$ qubits.

(ii) Using $\theta = \pi/2$ enables us to anticloning (i.e., copy the orthogonal complement) of the state of qubit 1 (i.e., the input qubit) to the target qubits (i.e., the $M - 1$ qubits initially in state $|0\rangle$). Since the initial state of qubit 1 is in the equatorial plane of the Bloch sphere [see Eq. (4)], we call this process phase-covariant anticloning in analogy with phase-covariant cloning (PCC) [15]. An ideal anticloning process is defined as [2]

$$|q\rangle_a |0\rangle_b |D\rangle_{in} \rightarrow |q^\perp\rangle_a |q^\perp\rangle_b |\tilde{D}\rangle_{out} \quad (11)$$

where $|q\rangle_a$ is the initial state of the input qubit, $|0\rangle_a$ is the initial state of a target qubit, $\langle q|q^\perp\rangle = 0$, and $|D\rangle_{in}$ and $|\tilde{D}\rangle_{out}$ are the input and output states of the copying device. Ref. [15] showed that the optimal fidelity for $1 \rightarrow 2$ PCC is $\mathcal{F}^{opt} = \frac{1}{2} [1 + \frac{1}{\sqrt{2}}]$. Here we demonstrate that the fidelity of our $1 \rightarrow 2$ anticloning equals this optimal value. We also show that there are two protocols to achieve this process, the main difference being the final state of the input qubit. For an arbitrary number of output qubits M with asymmetric couplings, we find that the fidelity of the anticloning operation is comparable to that obtained for a XY spin star network [9] and reaches larger values than for the case of identical couplings. The state of the system is now $|\Psi(t)\rangle = \frac{1}{\sqrt{2}} [|\phi_0\rangle + e^{i\alpha} |\phi_1(t)\rangle]$ and the reduced density matrix of the j 'th qubit reads

$$\begin{aligned} \rho_j(t) &= \frac{1}{2} [(2 - |U_{j1}(t)|^2) |0_j\rangle\langle 0_j| + |U_{j1}(t)|^2 |1_j\rangle\langle 1_j| + U_{j1}(t) \\ &\quad \times (e^{-i\alpha} |0_j\rangle\langle 1_j| + e^{i\alpha} |1_j\rangle\langle 0_j|)]. \end{aligned} \quad (12)$$

The fidelity of copying $|\tilde{q}\rangle = (1/\sqrt{2})[|0\rangle + e^{i\mu}|1\rangle]$ to the j 'th qubit, is $\mathcal{F}_j(t) = \langle \tilde{q} | \rho_j | \tilde{q} \rangle = \frac{1}{2} \{1 + U_{j1}(t) \times \cos(\alpha - \mu)\}$. For a target qubit, at $t = \tau^*$,

$$\mathcal{F}_{j>1}(\tau^*) = \frac{1}{2} \{1 - 2\gamma_1 \gamma_j \cos(\alpha - \mu) / \omega^2\}, \quad (13)$$

hence the fidelity is greater than 1/2 when the state that has been copied corresponds to the orthogonal complement of the input state (anticloning), i.e., $\alpha - \mu = \pi$. Figure 1 shows the fidelity of a target qubit and the input qubit (inset) as a function of the number of qubits. For coupling ratio r_W^+ , the input qubit finishes entangled with the target qubits, i.e., $|\Psi(\tau^*)\rangle = \frac{1}{\sqrt{2}}[|\phi_0\rangle + e^{i\alpha}|W_M^+\rangle]$ such that the fidelity of the input qubit (see inset) equals the fidelity of the target qubits. Hence we obtain M outputs (including the input qubit) with fidelity $\mathcal{F}^+ = \frac{1}{2}[1 + \frac{1}{\sqrt{M}}] = \mathcal{F}_1^+$. For $M = 2$, we obtain $\mathcal{F}_{M=2}^+ = \frac{1}{2}[1 + \frac{1}{\sqrt{2}}]$, which equals the optimal value for the $1 \rightarrow 2$ PCC [15]. Remarkably, such *optimal* transformation combines two operations in one step: complementing the original qubit's state and copying. We also note that this optimal fidelity is achieved for the same conditions (i.e., coupling ratio and time) under which two-qubit maximally entangled states were found [8], hence establishing a direct connection between optimal anticloning and maximal entanglement. For r_W^- , the fidelity of the target qubits equals \mathcal{F}^+ but the fidelity of the input qubit is always less than 1/2, i.e., $\mathcal{F}_1^- = \frac{1}{2}[1 - \frac{1}{\sqrt{M}}]$ which is undesirable for a single qubit NOT operation. For $r_{W'} = \sqrt{M-1}$, we obtain $M-1$ outputs with fidelity $\mathcal{F}^{\text{sep}} = \frac{1}{2}[1 + \frac{1}{\sqrt{M-1}}]$ while the fidelity of the input qubit equals 1/2 irrespective of the number of qubits (see inset). This is because the input qubit ends in its ground state and sepa-

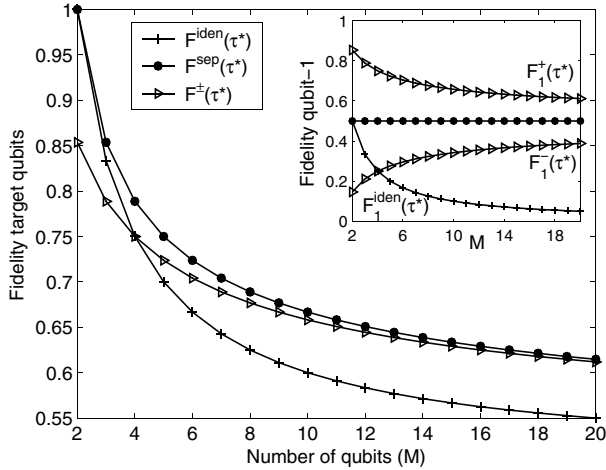


FIG. 1. Anticloning fidelity as a function of the number of qubits M . For the target qubits ($j > 1$), $\mathcal{F}^{\text{iden}}(\tau^*) = \frac{1}{2}[1 + \frac{2}{M}]$ denotes the case of identical couplings, $\mathcal{F}^{\pm}(\tau^*) = \frac{1}{2}[1 + \frac{1}{\sqrt{M}}]$ denotes the case where $r_W^{\pm} = \sqrt{M} \pm 1$, and $\mathcal{F}^{\text{sep}}(\tau^*) = \frac{1}{2}[1 + \frac{1}{\sqrt{M-1}}]$ denotes the case where $r_{W'} = \sqrt{M-1}$. Inset: Fidelity of original qubit \mathcal{F}_1 for $r = 1$: $\mathcal{F}_1^{\text{iden}}(\tau^*) = 1/M$, r_W^+ : $\mathcal{F}_1^+(\tau^*) = \frac{1}{2}[1 \pm \frac{1}{\sqrt{M}}]$, and $r_{W'}$: $\mathcal{F}_1^{\text{sep}} = 1/2$. All results are evaluated at the trapping time $t = \tau^*$.

rated from the rest, i.e., $|\Psi(\tau^*)\rangle = |0_1\rangle \otimes \frac{1}{\sqrt{2}} \times [|\tilde{\phi}_0\rangle + e^{i\alpha}|W_{M-1}\rangle]$ with $|\tilde{\phi}_0\rangle = |0_2, 0_3, \dots, 0_M\rangle$. For $M = 3$ we obtain $1 \rightarrow 2$ anticloning with optimal fidelity \mathcal{F}^{opt} for the target qubits. In general, $\mathcal{F}^+(M) = \mathcal{F}^{\text{sep}}(M+1)$ which means that there exist two protocols for obtaining M outputs with high fidelity: (i) M qubits and $r = r_W^+$, and (ii) $M+1$ qubits and $r = r_{W'}$. The main difference between these two protocols is the operation time $\tau^* = \pi/\omega$: it is shorter for the r_W^+ case since $\omega(r_W^+) > \omega(r_{W'})$. For a large number of outputs, this difference is negligible since $r_W \simeq r_{W'}$. Interestingly, the operation time decreases with the number of anticloning, implying that the protocols are robust in the presence of decoherence. We confirm this robustness in more detail below. In both cases r_W^+ and $r_{W'}$, the fidelity of the one-step anticloning procedure is comparable with that reported for cloning operations using a XY spin network [9] since it depends on the number of outputs M as $1/\sqrt{M}$. In the case of identical couplings, the fidelity of the target qubits is $\mathcal{F}^{\text{iden}} = \frac{1}{2}[1 + \frac{2}{M}]$ which is always less than \mathcal{F}^{sep} as well as being less than \mathcal{F}^{\pm} for $M > 4$. This behavior is comparable with that of a Heisenberg spin network since it depends on the number of outputs M as $1/M$ [9].

Decoherence will arise through two main channels: qubit dipole decay at rate Γ , and cavity decay with rate κ . A single trajectory in the quantum jump model [16] is well suited to evaluate the effects on the fidelity at the trapping time. We suppose that the photon decays are continuously monitored, and that the single trajectory is specified by the evolution of the system conditioned to no-photon detection. The conditional dynamics satisfies the dissipative Hamiltonian

$$\tilde{H} = H_I - i\Gamma \sum_{j=1}^M \sigma_j^+ \sigma_j^- - i\kappa a^\dagger a. \quad (14)$$

The (unnormalized) conditional state $|\Psi_{\text{cond}}(t)\rangle = \tilde{U}(t, 0)|\Psi(0)\rangle = \sum_{j=1}^{M+1} b_j(t)|\phi_j\rangle$ with $\tilde{U}(t, 0) = \exp[-i\tilde{H}t]$ and $\|P(0, t) = |\Psi_{\text{cond}}(t)\rangle\|^2$ being the probability of not detecting a photon in the interval $(0, t)$. The conditional state becomes

$$|\Psi_{\text{cond}}(t)\rangle = b_1(t)|\phi_1\rangle + b(t) \sum_{j=2}^M |\phi_j\rangle + b_{M+1}(t)|\phi_{M+1}\rangle \quad (15)$$

where

$$\begin{aligned} b_1(t) &= 1 + rb(t) \\ b(t) &= \alpha e^{-\Gamma t} [-1 + e^{(\Gamma-\kappa)t/2} (v + (\kappa - \Gamma)u/\Omega)] \\ b_{M+1}(t) &= -2i\omega\sqrt{r\alpha} e^{-(\Gamma+\kappa)t/2} u/\Omega \end{aligned} \quad (16)$$

with $\alpha = \gamma_1 \gamma / \omega^2$, $\Omega = \sqrt{4\omega^2 - (\kappa - \Gamma)^2}$, $u = \sin[\Omega t/2]$, and $v = \cos[\Omega t/2]$. An immediate and remark-

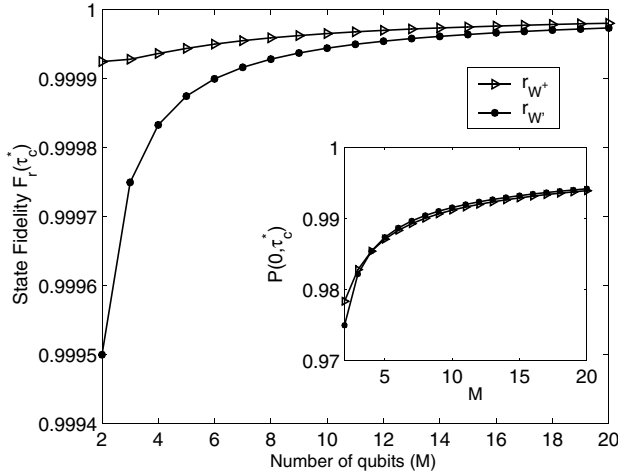


FIG. 2. Fidelity of the state obtained at $\tau_c^* = 2\pi/\Omega$ with respect to the pure state obtained at τ^* [$F_r(\tau_c^*)$] as a function of the number of qubits M . Two cases are shown: $r_{W'}^+$ (triangles) and $r_{W'}^-$ (circles). Inset: Probability of no-photon detection during interval $(0, \tau_c^*)$. Here $\kappa = 0.02\gamma$ and $\Gamma = 0.001\gamma$.

able conclusion from this calculation is that the vacuum trapping condition (i.e., $b_{j=M+1} = 0$ but $b_{j \neq M+1} \neq 0$) still arises. Moreover, it will arise for *any* number of qubits. This implies that the effects we have discussed are not just robust against decoherence: they are to a great extent *immune* to decoherence since $b_{j=M+1}$ is strictly zero at the renormalized trapping time $\tau_c^* = 2m\pi/\Omega$ with m odd, for any M . We now turn to the state fidelity F_r with respect to the pure system's state at $t = \tau^*$ [see Eq. (8)], i.e., $F_r = |\langle \Psi(t = \tau^*) | \tilde{\Psi}_{\text{cond}}(t = \tau_c^*) \rangle|$ where $|\tilde{\Psi}_{\text{cond}}(t = \tau_c^*)\rangle$ denotes the normalized conditional state. Several interesting features arise from the interplay between Γ and κ . For the situation in which $\Gamma = \kappa$, the fidelity F_r equals unity for any value of r and at any time. This is due to the fact that the non-Hermitian operator accounting for the dissipative interaction in \tilde{H} , is just the excitation number operator (i.e., $-i\Gamma\mathcal{N}$) hence the conditional state becomes $|\Psi_{\text{cond}}(t)\rangle = e^{-\Gamma t}|\Psi(t)\rangle$ and $P(0, t) = e^{-2\Gamma t}$. Therefore the decoherence sources can effectively be combined to produce a negligible net effect. This feature becomes more prominent as the number of qubits increases—see Fig. 2. The state fidelity F_r with $\Gamma \neq \kappa$, is shown in the two cases in which it is possible to either generate symmetric W entangled states or to obtain M anticloned with high fidelity: $r_{W'}^+$ and $r_{W'}^-$. In both situations the state fidelity moves closer to unity as the number of qubits increases, since the time interval required to achieve the desired state becomes shorter. Higher values of fidelity are obtained for the symmetric case $r_{W'}^+$ than in the $r_{W'}^-$ case. This effect can

be better appreciated for a small number of qubits. The probability of not detecting a photon in $(0, \tau_c^*)$ does not fall below 0.97 for $M = 2$ and becomes even closer to unity for higher numbers of qubits (see inset of Fig. 2). This again shows how efficient our protocols for entangling/anticloning are, and concludes the justification of the claims in this Letter.

The experimental control of the qubit-cavity couplings required by our scheme can be achieved by controlling the position of the qubits with respect to the field mode profile. For instance, optimal $1 \rightarrow 2$ anticloning with $M = 3$ requires the two target qubits to be located in equivalent positions while the initially excited qubit is placed elsewhere such that $r_{W'} = \sqrt{2}$. Such deterministic control of the variation of the qubit-field coupling strength has already been demonstrated in ion-cavity schemes [12]. In addition, controllable strong coupling between a single microwave photon and a superconducting qubit has recently been demonstrated [14]—this suggests a novel solid-state implementation of our scheme is also feasible.

In summary, we have shown how asymmetric cavity-qubit couplings can be exploited to perform robust, high-fidelity entangling and anticloning operations in a physically realizable multiqubit system.

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