Skyrmion Liquid in SU(2)-invariant Quantum Hall Systems

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Abstract

We report on a theory of the Skyrmion states which occur in quantum Hall regime near certain filling fractions. It is shown that in the limit of zero Zeeman coupling in a realistic temperature range the Skyrmion plasma is a liquid described by the effective model of massive two-dimensional Dirac fermions.

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There are several physically different realizations of Quantum Hall systems where electrons carry additional SU(2) degrees of freedom. In the simplest case such degree of freedom is electron spin. In free space electrons have the Landee factor $g_L = 2$ and the Zeeman splitting $g_L\mu_B B$ is precisely equal to the distance beetween the Landau levels ω_c. However, in real Quantum Hall devices there are several reasons for the Zeeman energy to be much smaller than the Landau splitting. The first reason is the small effective mass ($m^* \sim 0.068 m_e$ in GaAs) which leads to the increase in the cyclotron frequency ω_c, the second is the spin orbit coupling which reduces g_L by roughly a factor of 4. Thus the ratio $g_L\mu_B B/\omega_c$ in GaAs is about 0.02 which makes spin fluctuations an important degree of freedom to be taken into account. The Zeeman coupling may be further reduced by the effects of pressure, perhaps even to zero.

A second example of a multi-component system is silicon where the conduction band minimum occurs at six symmetry equivalent points lying near the zone boundary. In the presence of the oxide barrier in a Si MOSFET device and a very large confining electric field perpendicular to the barrier the cubic symmetry is broken and only two valleys contribute to the low energy properties. The remaining effective Hamiltonian has the SU(2) symmetry just like a spin-1/2 system [1].

Recently it has been proposed that multi-component Quantum Hall systems may develop sophisticated spin (or pseudospin) textures when the Landau level filling factor slightly deviates from $\nu_0 = 1/(2m+1)$[2]. The recent Knight shift measurements give an experimental support to this prediction [3]. The incompressible ground state of a two-dimensional electron gas at these filling factors is ferromagnetic. The origin of ferromagnetism is due to exchange energy; when Zeeman energy is weak or even absent (as in Si devices) the direction of the ferromagnetic order parameter described by the unit vector field $\mathbf{n}(\tau, x)$ can deviate significantly from the direction of the external magnetic field. According to Refs. [2],[4], [5], in incompressible Quantum Hall states the density of topological charge of the vector field $\mathbf{n}$ is directly related to the density of the extra electric charge:

$$q = -\frac{\nu}{8\pi}\epsilon_{\mu\nu}\left(\mathbf{n}.[\partial_\mu \mathbf{n} \times \partial_\nu \mathbf{n}]\right) \tag{1}$$

The same authors suggest the following Lagrangian to describe spin textures:

$$L = \int d^2x[\mathcal{L}_1 + \mathcal{L}_2], \tag{2}$$

$$\mathcal{L}_1 = \frac{\nu}{4\pi l_B^2}\mathbf{A}\left[\mathbf{n}(\mathbf{r})\right]\partial_t \mathbf{n}(\mathbf{r}) + \frac{\rho_s}{2}|\partial_\mu \mathbf{n}|^2 + \frac{2\mu q}{\nu}, \tag{3}$$

$$\mathcal{L}_2 = \frac{1}{2}\int d^2x' q(r)V(r-r')\frac{e^2}{\epsilon|r-r'|}q(r') - g_L B n^z, \tag{4}$$

where $\mathbf{A}\left[\mathbf{n}(\mathbf{r})\right]$ is the vector potential of a unit monopole in spin space, i.e., $\nabla_{\mathbf{n}} \times \mathbf{A} = \mathbf{n}$. At large distances $|r| >> l_B$ the interaction is via the ordinary Coulomb potential: $V(r-r') = \frac{e^2}{\epsilon|r-r'|}$. $\mathcal{L}_1$ is the Lagrangian density of a ferromagnet with a non-zero topological charge stabilized by the chemical potential μ. The quantity ρ_s is the stiffness; its physical origin is the loss of exchange and correlation energy when the spin orientation varies in space. The estimates made for the case when the exchange energy is of the Coulomb origin give [5]

$$\begin{aligned} \rho_s &= a\frac{e^2}{\epsilon l_B} \\ a &= 2.49\times 10^{-2}(\nu=1),\ 9.23\times 10^{-4}(\nu=1/3),\ 2.34\times 10^{-4}(\nu=1/5) \end{aligned} \tag{5}$$

where l_B is the magnetic length. For $\nu = 1$ the stiffness $\rho_s \approx 4K$ for B =10T.

Let us start from the case when the Coulomb interaction and the Zeeman energy are absent (the former one will be reintroduced later). We choose the following convinient parametrization of the vector field, **n**:

$$w = \frac{n_x + \mathrm{i}n_y}{1-n^z}, \; n_x + \mathrm{i}n_y = \frac{2w}{1+|w|^2}, \; n^z = \frac{|w|^2-1}{1+|w|^2} \tag{6}$$

In these notations the static part of the Lagrangian (3) is,

$$L_1 = \int \frac{4d^2x}{(1+|w|^2)^2}\left[\rho_s(|\partial w|^2 + |\bar\partial w|^2) - \frac{\mu}{4\pi}(|\partial w|^2 - |\bar\partial w|^2)\right] \tag{7}$$

where $z = x + iy$, $\partial = \partial/\partial z$ and $\bar\partial = \partial/\partial\bar z$.

The energy is minimised on configurations called instantons or Skyrmions [6]:

$$w(z) = h\prod_{i=1}^{N}\left(\frac{z-a_i}{z-b_i}\right) \tag{8}$$

where a, b, h are parameters, a_i and b_i being called coordinates of the instanton. According to ref.[6] the energy of such configuration is proportional to its topological charge and does not depend on $\{h, a_i, b_i\}$: $E = (4\pi\rho_s - \mu)N$.

To find the partition function

$$Z = \int D\mathbf{n}\delta(\mathbf{n}^2(x) - 1)\exp[-\frac{1}{T}(E - \mu N)] \tag{9}$$

in a semiclassical approximation it is necessary to expand the energy functional (7) near the classical solutions (8) and after calculating the corresponding Gaussian integral over the fluctuations sum the contributions with different values of parameters h, a_i, b_i. Such a program was carried out by Fateev *et al.* [7] (see also [8]) who obtained the following result:

$$Z = \sum_N \frac{1}{(N!)^2}\left(\frac{m}{2\pi}\right)^{2N}\int\frac{d^2h}{(1+|h|^2)^2}\prod_{i=1}^{N}d^2a_i d^2b_i\frac{\prod_{i<j}|a_i-a_j|^2|b_i-b_j|^2}{\prod_{i,j}|a_i-b_j|^2} \tag{10}$$

where

$$m \approx l_B^{-1}\left(\frac{T}{\rho_s - \mu/4\pi}\right)\exp[-\frac{2\pi(\rho_s-\mu/4\pi)}{T}] \tag{11}$$

This equation shows that the grand partition function of Skyrmions coincides with the partition function for the classical two-dimensional Coulomb plasma at temperature $t = 1/4\pi$ with a_i and b_i being coordinates of positive and negative charges. Such partition function can also be written as the partition function of the free Dirac fermions:

$$Z = \int D\psi D\bar\psi\exp[\int d^2x\bar\psi(\mathrm{i}\gamma_\alpha\partial_\alpha - m)\psi] \tag{12}$$

In order to express m in terms of the extra charge density of the Quantum Hall state, we have to differentiate the free energy with respect to μ:

$$q \equiv \frac{1}{2\pi l_B^2}\left|\frac{\nu}{\nu_0}-1\right| = -\frac{\partial\Omega}{\partial\mu} = T\frac{\partial m^2}{\partial\mu}\int\frac{d^2k}{(2\pi)^2}\frac{1}{k^2+m^2} = \frac{m^2}{\pi}\ln\left(1/m\lambda\right) \tag{13}$$

where $\lambda \sim \frac{e^2}{\epsilon T}$ is the minimal distance between the Skyrmions determined by the Coulomb repulsion (renormalization of the minimal distance cut-off is the only plausable effect of the Coulomb interaction one may think of). Solving Eq.(13) with respect to m we obtain with the logarithmic accuracy

$$m^2 \approx -2\pi q/\ln(\pi\lambda^2 q) \tag{14}$$

Let us discuss conditions which must be fulfilled for this result to be self-consistent. Firstly we have considered only static configurations of the field. This is equivalent to projecting out the zeroth Matsubara frequency and so we introduce an energy cut off at the first Matsubara frequency, $\omega = 2\pi T$. The spectrum of magnons is given by [5] $\omega = \frac{4\pi\rho_s}{\nu}(kl_B)^2$ which implies a momentum cut off $\Lambda = \frac{1}{l_B}\left(\frac{T}{2\rho_s}\right)^{\frac{1}{2}}$. In the absence of s Skyrmions the static O(3) non-linear sigma model decays exponentially at distances larger than,

$$\xi \sim \Lambda^{-1}\frac{T}{\rho_s}\exp\left(2\pi\frac{\rho_s}{T}\right) \sim l_B\left(\frac{T}{\rho_s}\right)^{\frac{1}{2}}\exp\left(2\pi\frac{\rho_s}{T}\right) \tag{15}$$

If $q\xi^2 < 1$ the instantons are screened by fluctuations around the classical minima, interact weakly and their influence is small. Thus we need $q\xi^2 >> 1$, that is,

$$T << 2\pi\rho_s \tag{16}$$

Secondly, we have neglected quantum fluctuations. The magnon spectrum means that these fluctuations may be neglected if

$$2\pi T >> \frac{4\pi\rho_s}{\nu}(ml_B)^2 \tag{17}$$

which, using equations (13) and (14), leads to the condition

$$T >> T_c \equiv \frac{2\rho_s}{\nu}\left|\frac{\nu}{\nu_0}-1\right|\left\{\ln\left(\frac{2l_B^2}{\lambda^2|\nu/\nu_0-1|}\right)\right\}^{-1} \tag{18}$$

Finally, the Coulomb repulsion means that we must have $k\lambda^2 << 1$ which gives the further constraint

$$T >> m^{\frac{1}{2}}\frac{e^2}{\epsilon} \sim \left|\frac{\nu}{\nu_0}-1\right|^{\frac{1}{4}}\frac{e^2}{\epsilon l_B^2} \tag{19}$$

Note that the length m^{-1} is much smaller than ξ and that it has a much slower temperature dependence; logarithmic compared with exponential.

Both conditions (16) and (18) are entirely realistic. In this temperature range the system is in a liquid phase with a finite correlation length $\sim m^{-1}$ with m given by Eq.(14).

As we have remarked above, in the absence of the Zeeman energy the Coulomb interaction just prevents Skyrmions from approaching each other too closely. If $|\nu/\nu_0 - 1|\lambda^2 << 1$ the Coulomb interaction just renormalizes the ultraviolet cut-off. Therefore we have solved the problem for systems with zero Landee factor. In the realistic temperature range these systems are in a liquid phase. The case of finite Zeeman coupling will be presented elsewhere.

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