Performance Enhancement of Multiuser MIMO Wireless Communication Systems

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***Abstract*—This paper describes a new approach to the problem of enhancing the performance of a multiuser multiple-input–multiple-output (MIMO) system for communication from one base station to many mobile stations in both frequency-flat and frequency-selective fading channels. This problem arises in space-division multiplexing systems with multiple users where many independent signal streams can be transmitted in the same frequency and time slot through the exploitation of multiple antennas at both the base and mobile stations. Our new approach is based on maximizing a lower bound for the product of signal-to-interference plus noise ratio (SINR) of a multiuser MIMO system. This provides a *closed-form* (noniterative) solution for the antenna weights for all the users, under the constraint of fixed transmit power. Our solution is shown by simulation to have better performance than previously proposed iterative or noniterative solutions. In addition, our solution requires significantly reduced complexity over a gradient search-based method that directly optimizes the product SINRs while still maintaining similar performance. Our solution assumes channel state information is present at the base station or transmitter.**

***Index Terms*—Cochannel interference, flat and frequency-selective fading channels, intersymbol interference, multiple-input–multiple-output (MIMO) capacity, smart antennas, wireless communication systems.**

I. Introduction

PROVIDING high data rate transmission on the order of several megabits per second (Mbits/s) is important to future wireless communications [1], [2]. In recent years, antenna systems which employ multiple antennas at both the base station (BS) and mobile station (MS), operating in space–time, have been proposed and demonstrated to significantly increase system performance as well as capacity [3]–[7]. The merit of using multiple antennas or space diversity is that no bandwidth expansion or increase in transmitted power is required for capacity and performance improvements.

In this paper, we suggest a new approach to the problem of enhancing the performance (in terms of average user error probability) for multiuser multiple-input–multiple-output (MIMO) systems for transmission from one BS to many MS (or point-to-multipoint) in both frequency flat and selective fading channels. Our approach is based on deriving a lower bound of the product signal-to-interference plus noise ratio (SINR) of the multiuser system and then maximizing it. The major advantage of this approach is that the relationship between the transmit antenna weights is made independent. In addition, the lower bound approximately describes the effect of cochannel interference (CCI) on the performance of the multiuser MIMO system. From the lower bound, analytic expressions for finding the joint optimal antenna weights which maximize the lower bound of the product SINR can be found.

In related work [3]–[5], space–time or frequency codes that allow space-division multiplexing (SDM) are investigated for increasing the capacity of a point-to-point transmission. It was also demonstrated that good codes exist for achieving extraordinary capacity with or without channel state information (CSI) at the transmitter. In [6], Wong *et al.* studied the optimization of transmit and receive antenna weights, operating jointly, in the sense of maximizing receive SINR, for intersymbol interference (ISI) mitigation and CCI suppression in point-to-point transmission. The problem considered is a single-user MIMO system corrupted by fixed uncontrollable CCI. In addition to exploiting multiple signaling spatial dimension, space diversity can also be employed for support of multiple users, transmitting in the same frequency band and time slot [7]–[11]. In [7], the flat fading weights solution proposed in [6] is used for a multicarrier MIMO system. However, a complicated iteration process is required, and the computational complexity could grow exponentially as the number of users increases.

Our work is different in that we obtain an analytic expression for the antenna weights in a multiuser MIMO system, where several users occupy the same frequency band, time slot, and multiple spatial dimensions. This solution is shown by simulation to have better performance than previously proposed iterative or noniterative solutions. In addition, our solution requires significantly reduced complexity over a gradient search-based method that directly optimizes the product of SINRs while still maintaining similar performance. It is assumed in our solution that CSI is known at both the transmitter and receivers and that multipath characteristics remain approximately constant over a block of bits.

This paper is organized as follows. In Section II, we introduce some necessary notation for a multiuser MIMO system that is used throughout the paper. Section III derives a lower bound of the system product SINR and obtains a closed-form solution for the joint optimization of transmit and receive antennas weights for a multiuser MIMO system in multipath fading environments. In Section IV, simulation setup and results are presented. Finally, we conclude the paper in Section V.

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II. Multiuser MIMO System Model

The system configuration of the multiuser MIMO antenna system is shown in Fig. 1 where one BS is transmitting to M MS. For the MIMO system, n_T antennas are located at the BS and n_{R_m} antennas are located at the MS. We first consider the link between the BS and a single user m. Data is transmitted in blocks of symbols of length N, and the number of spatial subchannels (or spatial dimensions) per user is denoted by K_m. Therefore, the total number of symbols sent per user is NK_m, and this is written in packet format $\mathbf{z}_m = [z_{1,1}^{(m)} z_{2,1}^{(m)} \cdots z_{N,1}^{(m)} z_{1,2}^{(m)} \cdots z_{N,2}^{(m)} \cdots z_{N,K_m}^{(m)}]^T$ where $z_{n,k}^{(m)}$ is the kth dimension of the nth symbol transmitted by the mth user, and the superscript T denotes the transpose operation. The packet is multiplied by a $N \times NK_m$ transmission matrix

$$\mathbf{T}_k^{(m)} = \begin{bmatrix} t_{1,1}^{(m,k)} & t_{1,2}^{(m,k)} & \cdots & t_{1,NK_m}^{(m,k)} \\ t_{2,1}^{(m,k)} & t_{2,2}^{(m,k)} & & \vdots \\ \vdots & & \ddots & \\ t_{N,1}^{(m,k)} & \cdots & & t_{N,NK_m}^{(m,k)} \end{bmatrix} \tag{1}$$

to produce a packet $\mathbf{x}_{m,k} = \mathbf{T}_k^{(m)} \mathbf{z}_m$, which is transmitted by the kth BS antenna to the mth mobile in a block of length N (see Fig. 2).

At the MS, n_{R_m} antennas are used for reception and the channel between the kth BS antenna, and ℓth MS antenna is assumed quasi-stationary and can be considered as time invariant over a packet, so that it can be characterized by a Toeplitz matrix

$$\mathbf{H}_{\ell,k}^{(m)} = \begin{bmatrix} h_0^{(m,\ell,k)} & 0 & 0 & \cdots & & 0 \\ h_1^{(m,\ell,k)} & h_0^{(m,\ell,k)} & 0 & \cdots & & \vdots \\ \vdots & \ddots & \ddots & \ddots & & \\ h_\nu^{(m,\ell,k)} & \cdots & h_0^{(m,\ell,k)} & & 0 & \cdots \\ 0 & \ddots & & & \ddots & 0 \\ \vdots & & & & \ddots & h_0^{(m,\ell,k)} \\ 0 & & & & & h_1^{(m,\ell,k)} \\ \vdots & & & \ddots & \ddots & \vdots \\ 0 & \cdots & & & 0 & h_\nu^{(m,\ell,k)} \end{bmatrix} \tag{2}$$

where the received packet is given by

$$\mathbf{y}_{m,\ell} = \sum_{k=1}^{n_T} \mathbf{H}_{\ell,k}^{(m)} \mathbf{x}_{m,k}.$$

The maximum delay is assumed to last for ν samples and the discrete-time channel gains are defined by a multiray model as in [6] and [12], so the dimensions of $\mathbf{H}_{\ell,k}^{(m)}$ are $(N+\nu) \times N$.

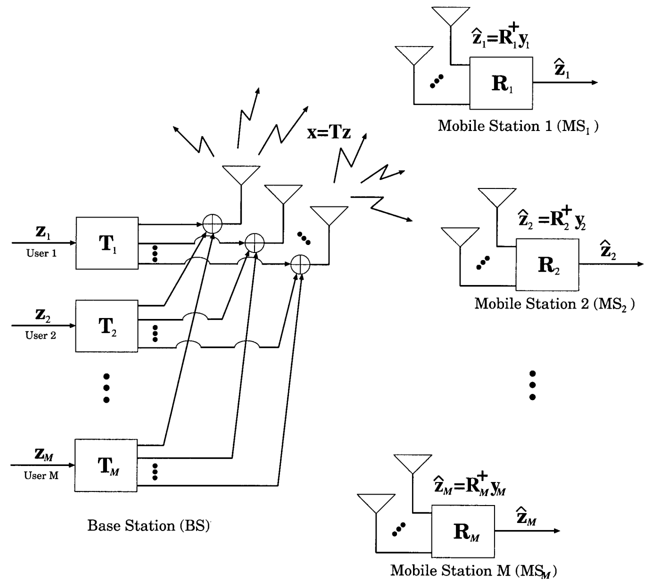

Fig. 1. System configuration of a multiuser MIMO antenna system.

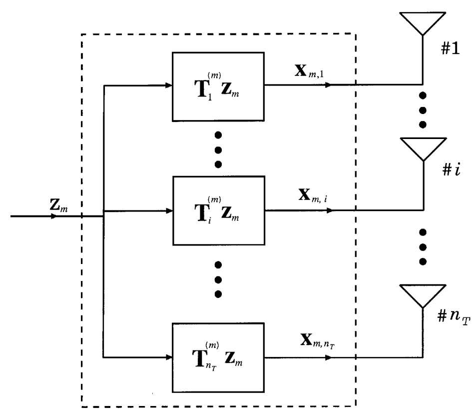

Fig. 2. System configuration of space–time preprocessing network for the mth user.

The received packet, $\mathbf{y}_{m,\ell}$, is then weighted in space and time by a matrix $\mathbf{R}_\ell^{(m)\dagger}$, where

$$\mathbf{R}_\ell^{(m)} = \begin{bmatrix} r_{1,1}^{(m,\ell)} & r_{1,2}^{(m,\ell)} & \cdots & r_{1,NK_m}^{(m,\ell)} \\ r_{2,1}^{(m,\ell)} & r_{2,2}^{(m,\ell)} & & \vdots \\ \vdots & & \ddots & \\ r_{N+\nu,1}^{(m,\ell)} & \cdots & & r_{N+\nu,NK_m}^{(m,\ell)} \end{bmatrix} \tag{3}$$

and the superscript $\dagger$ denotes the conjugate transpose operation, to produce an estimate $\hat{\mathbf{z}}_m$ of the original packets. Writing the packet transmitted from all n_T antennas as $\mathbf{x}_m = [\mathbf{x}_{m,1}; \mathbf{x}_{m,2}; \ldots; \mathbf{x}_{m,n_T}]^T$ and the received data by all n_{R_m} antennas as $\mathbf{y}_m = [\mathbf{y}_{m,1}; \mathbf{y}_{m,2}; \ldots; \mathbf{y}_{m,n_{R_m}}]^T$, we can write the received signal of the entire MIMO system as

$$\mathbf{y}_m = \mathbf{H}_m \mathbf{x}_m + \mathbf{n}_m \tag{4}$$

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where $\mathbf{n}_m$ is the noise vector that is assumed to be additive white Gaussian noise (AWGN) with power σ_n^2. Likewise, $\mathbf{H}_m$ is given by

$$\mathbf{H}_m = \begin{bmatrix} \mathbf{H}_{1,1}^{(m)} & \mathbf{H}_{1,2}^{(m)} & \cdots & \mathbf{H}_{1,n_T}^{(m)} \\ \mathbf{H}_{2,1}^{(m)} & \mathbf{H}_{2,2}^{(m)} & \cdots & \mathbf{H}_{2,n_T}^{(m)} \\ \vdots & & \ddots & \vdots \\ \mathbf{H}_{n_{R_m},1}^{(m)} & \cdots & & \mathbf{H}_{n_{R_m},n_T}^{(m)} \end{bmatrix} \tag{5}$$

where $\mathbf{H}_{\ell,k}^{(m)}$ is defined in (2). The estimate, $\hat{\mathbf{z}}_m$, can then be written as

$$\hat{\mathbf{z}}_m = \mathbf{R}_m^\dagger \mathbf{H}_m \mathbf{T}_m \mathbf{z}_m + \mathbf{R}_m^\dagger \mathbf{n}_m \tag{6}$$

where $\mathbf{R}_m \triangleq [\mathbf{R}_1^{(m)}; \mathbf{R}_2^{(m)}; \ldots; \mathbf{R}_{n_{R_m}}^{(m)}] \in \mathbb{C}^{(N+\nu)n_{R_m} \times NK_m}$ denotes the space–time weights operating on the received signals, and $\mathbf{T}_m \triangleq [\mathbf{T}_1^{(m)}; \mathbf{T}_2^{(m)}; \ldots; \mathbf{T}_{n_T}^{(m)}] \in \mathbb{C}^{Nn_T \times NK_m}$ denotes the space–time weights operating on the transmitted packet.

By considering all M users, we obtain an M-user MIMO system, where n_T antennas are located at the BS and n_{R_m} antennas are located at the mth MS, as

$$\hat{\mathbf{z}}_m = \mathbf{R}_m^\dagger \left(\sum_{m'=1}^{M} \mathbf{H}_m \mathbf{T}_{m'} \mathbf{z}_{m'} + \mathbf{n}_m \right) \qquad \forall m \tag{7}$$

where $\mathbf{z}_{m'} \in \mathbb{C}^{NK_{m'}}$ denotes the symbols transmitted from the m'th user.

Note that there are NK_m symbols transmitted in N symbol durations by the mth user, and it is known [5] that the number of spatial dimensions should be bounded by

$$K_m \le \frac{\mathrm{rank}(\mathbf{H}_m)}{N} \le \min\left\{ \left(1+\frac{\nu}{N}\right) n_{R_m}, n_T \right\} \qquad \forall m. \tag{8}$$

In addition, the above formulation assumes that $\mathbf{H}_m$ are uncorrelated with themselves and $\mathbf{z}_m$.

We define the SINR of the kth dimension of the nth symbol from the mth user, $\gamma_{n,k}^{(m)}$, as

$$\gamma_{n,k}^{(m)} \triangleq \frac{\mathrm{E}\left[\left|z_{n,k}^{(m)}\right|^2\right] \left|\mathbf{r}_{n,k}^{(m)\dagger} \mathbf{H}_m \mathbf{t}_{n,k}^{(m)}\right|^2}{\mathrm{E}\left[\left|\mathbf{r}_{n,k}^{(m)\dagger} \left(\sum_{\tilde{m}=1}^{M} \sum_{\tilde{k}=1}^{K_{\tilde{m}}} \sum_{\substack{\tilde{n}=1 \\ (\tilde{m},\tilde{k},\tilde{n}) \neq (m,k,n)}}^{N} \mathbf{H}_m \mathbf{t}_{\tilde{n},\tilde{k}}^{(\tilde{m})} z_{\tilde{n},\tilde{k}}^{(\tilde{m})} + \mathbf{n}_m \right)\right|^2\right]} \tag{9}$$

where $\mathbf{t}_{n,k}^{(m)}$ and $\mathbf{r}_{n,k}^{(m)}$ are, respectively, the transmit and receive weight vectors, such that $\mathbf{T}_m = [\mathbf{t}_{1,1}^{(m)} \mathbf{t}_{2,1}^{(m)} \cdots \mathbf{t}_{N,K_m}^{(m)}]$ and $\mathbf{R}_m = [\mathbf{r}_{1,1}^{(m)} \mathbf{r}_{2,1}^{(m)} \cdots \mathbf{r}_{N,K_m}^{(m)}]$, and the notation $(\tilde{m}, \tilde{k}, \tilde{n}) \neq (m, k, n)$ is used to indicate that the term $\tilde{m} = m, \tilde{k} = k, \tilde{n} = n$ is disallowed. The numerator in (9) denotes the received signal power of the symbol, $z_{n,k}^{(m)}$, and the denominator denotes the received powers of CCI and noise. In addition, $|\mathbf{r}_{n,k}^{(m)\dagger} \mathbf{H}_m \mathbf{t}_{\tilde{n},\tilde{k}}^{(\tilde{m})}|^2$ is regarded as the effective channel power (channel power combined by the antennas) for $z_{\tilde{n},\tilde{k}}^{(\tilde{m})}$. To make the analysis more succinct and without loss of generality, we shall assume that $\mathrm{E}[|z_{n,k}^{(m)}|^2] = 1$ throughout the paper.

III. Performance Enhancement of Multiuser MIMO Systems

Our objective is to optimize a performance measure, C, of the multiuser MIMO system so that we can find the antenna weights at the BS and MS that enable multiuser SDM, subject to some constraints. This can be written as

$$(\mathbf{T}_m, \mathbf{R}_m)_{\mathrm{opt}} = \arg \max_{\substack{\mathbf{T}_m, \mathbf{R}_m \\ \|\mathbf{t}_{n,k}^{(m)}\|^2 = P_T \\ \mathrm{Rank}(\mathbf{T}_m) = NK_m}} C \qquad \forall m \tag{10}$$

where the power constraint, $\|\mathbf{t}_{n,k}^{(m)}\|^2 = P_T$, ensures a fixed transmit power at the BS while the rank requirement, $\mathrm{Rank}(\mathbf{T}_m) = NK_m$, ensures that all the transmitted symbols are received.

The difficulty with this approach is defining the performance measure, and here, we use the product of SINR. That is, we have

$$(\mathbf{T}_m, \mathbf{R}_m)_{\mathrm{opt}} = \arg \max_{\substack{\mathbf{T}_m, \mathbf{R}_m \\ \|\mathbf{t}_{n,k}^{(m)}\|^2 = P_T \\ \mathrm{Rank}(\mathbf{T}_m) = NK_m}} \prod_{m=1}^{M} \prod_{k=1}^{K_m} \prod_{n=1}^{N} \gamma_{n,k}^{(m)}. \tag{11}$$

This measure has been motivated from consideration of the capacity for MIMO systems with parallel uncoupled channels [5], which can be written as

$$C = \frac{1}{NK_m} \sum_{i=1}^{NK_m} \log_2 (1 + \mathrm{SNR}_i) \tag{12}$$

where NK_m is the number of parallel uncoupled channels, and SNR_i denotes the signal-to-noise ratio (SNR) of the ith channel. The capacity in (12) can be lower bounded so that

$$C > \frac{1}{NK_m} \log_2 \prod_{i=1}^{NK_m} \mathrm{SNR}_i \tag{13}$$

where similarity to (11) can be seen. In the absence of CCI, the product SNR is related to the capacity of parallel uncoupled channels. Likewise, when there are many interferers present ($M \gg 2$ and $K_m > 1$), the CCI will be approximately Gaussian distributed (by the central limit theorem) and we can, therefore, replace the SNR terms in (13) with SINR, giving us (11). When the number of interferers is not large, our expression will only be approximate, but hopefully, reasonably accurate. More pragmatically, we can also note that when maximizing the product SINR, there is a tendency to jointly maximize the SINR of all the channel symbols, and this leads to a reduction of average bit error probability. For example, using the sum of SINR tends to underrepresent channels which perform poorly. Of all the various methods we have studied (see Section IV), the use of the product of SINR as the performance measure is shown to be effective and produces the best bit error rate (BER) results for the multiuser situation as compared to other methods.

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Returning to the optimization of (11), we first note that the receive weight vectors, $\mathbf{r}_{n,k}^{(m)}$, for a particular user signal, do not affect the SINR of other user signals. Therefore, the optimum receive weights [in the sense of maximizing (11)] can be found from the traditional smart antenna algorithm as [13]

$$\mathbf{r}_{n,k}^{(m)} = \mu \mathbf{\Phi}_{m,n,k}^{-1} \mathbf{H}_m \mathbf{t}_{n,k}^{(m)} \tag{14}$$

where

$$\mathbf{\Phi}_{m,n,k} = \sum_{\substack{\tilde{m}=1 \\ (\tilde{m},\tilde{k},\tilde{n}) \neq (m,k,n)}}^{M} \sum_{\tilde{k}=1}^{K_{\tilde{m}}} \sum_{\tilde{n}=1}^{N} \mathbf{H}_m \mathbf{t}_{\tilde{n},\tilde{k}}^{(\tilde{m})} \mathbf{t}_{\tilde{n},\tilde{k}}^{(\tilde{m})\dagger} \mathbf{H}_m^{\dagger} + \sigma_n^2 \mathbf{I} \tag{15}$$

and μ is an arbitrary real constant.

Using the receive weights in (14), the SINR, $\gamma_{n,k}^{(m)}$, in (9) can be expressed as

$$\gamma_{n,k}^{(m)} = \mathbf{t}_{n,k}^{(m)\dagger} \mathbf{H}_m^{\dagger} \mathbf{\Phi}_{m,n,k}^{-1} \mathbf{H}_m \mathbf{t}_{n,k}^{(m)}. \tag{16}$$

It can then be shown (see Appendix I) that

$$\prod_{m=1}^{M} \prod_{k=1}^{K_m} \prod_{n=1}^{N} \gamma_{n,k}^{(m)} \geq \rho \prod_{m=1}^{M} \prod_{k=1}^{K_m} \prod_{n=1}^{N} \times \frac{\mathbf{t}_{n,k}^{(m)\dagger} \mathbf{H}_m^{\dagger} \mathbf{H}_m \mathbf{t}_{n,k}^{(m)}}{\mathbf{t}_{n,k}^{(m)\dagger} \left[\sum_{\tilde{m}=1}^{M} Q_{\tilde{m}} \left(\mathbf{H}_{\tilde{m}}^{\dagger} \mathbf{H}_{\tilde{m}} + \frac{\sigma_n^2}{P_T} \mathbf{I} \right) \right] \mathbf{t}_{n,k}^{(m)}} \tag{17}$$

where ρ is a real constant, and

$$Q_{\tilde{m}} = \begin{cases} K_{\tilde{m}} N, & \tilde{m} \neq m \\ K_m N - 1, & \tilde{m} = m. \end{cases} \tag{18}$$

A tighter lower bound can be obtained, but the lower bound used here has certain features that facilitate a *closed-form* solution for the antenna weights. Specifically, the lower bound (17) is a product of factors in which each factor depends only upon a particular transmit weight vector. In other words, the interdependence of multiuser transmit weights is removed, so that the maximization of the lower bound can be achieved by performing maximization of each individual factor, while ensuring a low level of CCI. Consequently, iterations for adapting multiuser antenna weights are avoided. Accordingly, our objective is now to

$$\max_{\mathbf{t}_{n,k}^{(m)}} \frac{\mathbf{t}_{n,k}^{(m)\dagger} \mathbf{H}_m^{\dagger} \mathbf{H}_m \mathbf{t}_{n,k}^{(m)}}{\mathbf{t}_{n,k}^{(m)\dagger} \left[\sum_{\tilde{m}=1}^{M} Q_{\tilde{m}} \left(\mathbf{H}_{\tilde{m}}^{\dagger} \mathbf{H}_{\tilde{m}} + \frac{\sigma_n^2}{P_T} \mathbf{I} \right) \right] \mathbf{t}_{n,k}^{(m)}} \quad \forall m, n, k \tag{19}$$

subject to the rank and power constraints in (10).

To do so, we first let $\mathbf{t}_{n,k}^{(m)} = d_{n,k}^{(m)} \mathbf{W}_m \mathbf{s}_{n,k}^{(m)}$ such that

$$\mathbf{W}_m^{\dagger} \left[\sum_{\tilde{m}=1}^{M} Q_{\tilde{m}} \left(\mathbf{H}_{\tilde{m}}^{\dagger} \mathbf{H}_{\tilde{m}} + \frac{\sigma_n^2}{P_T} \mathbf{I} \right) \right] \mathbf{W}_m = \mathbf{I} \tag{20}$$

where $\mathbf{s}_{n,k}^{(m)}$ is arbitrary, and $d_{n,k}^{(m)}$ is a real constant to ensure that $\|\mathbf{t}_{n,k}^{(m)}\| = \sqrt{P_T}$. Then, the cost function to be maximized becomes

$$\max_{\mathbf{s}_{n,k}^{(m)}} \mathbf{s}_{n,k}^{(m)\dagger} \mathbf{W}_m^{\dagger} \mathbf{H}_m^{\dagger} \mathbf{H}_m \mathbf{W}_m \mathbf{s}_{n,k}^{(m)} \tag{21}$$

subject to the unity norm condition and $\mathrm{Rank}(\mathbf{S}_m) = NK_m$, where $\mathbf{S}_m$ is defined in a similar fashion to $\mathbf{T}_m$. As we can see, the cost functions for a particular m are all identical. Therefore, together with the rank requirement, the optimum weights $\mathbf{s}_{n,k}^{(m)}$ can be found using an eigenvalue decomposition (EVD) of $\mathbf{W}_m^{\dagger} \mathbf{H}_m^{\dagger} \mathbf{H}_m \mathbf{W}_m$. If we define $\mathbf{E}_m$ as the matrix whose columns correspond to the NK_m largest eigenvalues of $\mathbf{W}_m^{\dagger} \mathbf{H}_m^{\dagger} \mathbf{H}_m \mathbf{W}_m$, the corresponding optimum antenna weights are then given by

$$\mathbf{T}_m = \mathbf{W}_m \mathbf{E}_m \mathbf{D}_m \tag{22}$$

where $\mathbf{W}_m$ satisfies (20), and

$$\mathbf{D}_m = \mathrm{diag}\left(d_{1,1}^{(m)}, \ldots, d_{N,1}^{(m)}, d_{1,2}^{(m)}, \ldots, d_{N,K_m}^{(m)} \right). \tag{23}$$

Intuition about this scheme can be obtained by considering a special case of a two-user system ($M = 2$), with two antennas at the BS ($n_T = 2$), and both MS have two antennas ($n_{R_1} = n_{R_2} = 2$) and transmits in only one dimension (i.e., $K_1 = K_2 = 1$) in flat fading channels. In a flat fading radio environment, ISI is negligible, so that the same set of weights can be used for the entire packet (i.e., $N = 1$). In fact, the block transmission is only necessary in frequency-selective fading channels where there is a significant delay spread.

Using (16) and (48) in Appendix II, the SINR for the first user, $\gamma^{(1)}$, and second user, $\gamma^{(2)}$, are, respectively, given by

$$\gamma^{(1)} = \frac{1}{\sigma_n^2} \left[\mathbf{t}_1^{\dagger} \mathbf{H}_1^{\dagger} \mathbf{H}_1 \mathbf{t}_1 - \frac{\left| \mathbf{t}_1^{\dagger} \mathbf{H}_1^{\dagger} \mathbf{H}_1 \mathbf{t}_2 \right|^2}{\mathbf{t}_2^{\dagger} \mathbf{H}_1^{\dagger} \mathbf{H}_1 \mathbf{t}_2 + \sigma_n^2} \right] \tag{24}$$

and

$$\gamma^{(2)} = \frac{1}{\sigma_n^2} \left[\mathbf{t}_2^{\dagger} \mathbf{H}_2^{\dagger} \mathbf{H}_2 \mathbf{t}_2 - \frac{\left| \mathbf{t}_2^{\dagger} \mathbf{H}_2^{\dagger} \mathbf{H}_2 \mathbf{t}_1 \right|^2}{\mathbf{t}_1^{\dagger} \mathbf{H}_2^{\dagger} \mathbf{H}_2 \mathbf{t}_1 + \sigma_n^2} \right] \tag{25}$$

where $\mathbf{t}_1$ and $\mathbf{t}_2$ denote the transmit weight vectors for the first and second users, respectively. To maximize the product SINR of the whole system, we need to maximize the positive terms and minimize the negative terms in (24) and (25) jointly. These negative terms involve the correlation between $\mathbf{t}_1$ and $\mathbf{t}_2$, thereby making the optimization very complicated.

Using our lower bound, (17) can be expressed as

$$\gamma^{(1)} \gamma^{(2)} \geq \left[\frac{\mathbf{t}_1^{\dagger} \mathbf{H}_1^{\dagger} \mathbf{H}_1 \mathbf{t}_1}{\mathbf{t}_2^{\dagger} \left(\mathbf{H}_1^{\dagger} \mathbf{H}_1 + \frac{\sigma_n^2}{P_T} \mathbf{I} \right) \mathbf{t}_2} \right] \left[\frac{\mathbf{t}_2^{\dagger} \mathbf{H}_2^{\dagger} \mathbf{H}_2 \mathbf{t}_2}{\mathbf{t}_1^{\dagger} \left(\mathbf{H}_2^{\dagger} \mathbf{H}_2 + \frac{\sigma_n^2}{P_T} \mathbf{I} \right) \mathbf{t}_1} \right]. \tag{26}$$

Note that the lower bound of the product SINR considers the interference denominators in (26) as the sum of the total received channel power (combined by the transmit antennas) from the

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other users and noise, where, in fact, the total undesired power would be less as the receive weights suppress CCI. Hence, the cost function we use in this approach attempts to suppress the received "transmit channel" and noise powers. Accordingly, the cost function (19) can be thought of maximizing the transmit SINR, and we shall refer to the system as *Maximum Transmit SINR* (MTxSINR). Maximization of (26) by (22) then gives the optimum transmit weights as

$$\mathbf{t}_\ell = d_\ell \mathbf{W}_\ell \mathbf{e}_\ell \qquad \text{for } \ell = 1, 2 \tag{27}$$

where d_ℓ are real constants, $\mathbf{W}_\ell$ are the whitening matrices defined in (20), and $\mathbf{e}_\ell$ are the eigenvectors which correspond to the largest eigenvalue of $\mathbf{W}_\ell^\dagger \mathbf{H}_\ell^\dagger \mathbf{H}_\ell \mathbf{W}_\ell$.

IV. Simulation Results

A. Configuration

The proposed MTxSINR system is investigated for a time-division multiple-access (TDMA)-based wireless communication system in fading channels under the assumption of knowledge of perfect CSI. Quaternary phase-shift keying (QPSK) modulation is used for transmission. Average product SINR and average user bit error probability are provided for various AWGN (SNR). We assume the channel fading is quasi-stationary so that the channels within a packet are invariant, but the channels from packet to packet vary independently. For each simulation, data blocks consisting of 50 data symbols (i.e., $N = 50$) are transmitted with more than 10 000 independent channel realizations (for each channel realization, it includes the channel matrices of all users at a particular time instant). Only the results on average are shown for both product SINR and BER.

Results are compared with various alternative solutions, including a direct transmission (DTx) system, singular value decomposition and minimum mean-square error (SVD-MMSE) system, and joint approximate diagonalization of eigenmatrices (JADE) system. A comparison with a very loose performance bound is also provided. The details of these systems are discussed in the following subsections.

1) Performance Bound: The performance bound we use assumes no CCI is present and that singular value decomposition (SVD) is performed for every user link (which is optimal in the absence of CCI). As a consequence, the transmit weight matrix for the mth user is

$$\mathbf{T}_m = \sqrt{P_T}\, \mathbf{V}_m \tag{28}$$

and

$$\mathbf{R}_m = \mathbf{U}_m \tag{29}$$

where $\mathbf{V}_m(\mathbf{U}_m)$ is the matrix whose columns are the right (left) singular vectors which correspond to the NK_m largest singular values of $\mathbf{H}_m$, and the norm of each column vector of $\mathbf{T}_m$ is $\sqrt{P_T}$ (satisfying the power constraint). Also note that the bound does not involve CCI or assumes that CCI is completely eliminated. Hence, it is the upper bound for the true system capacity. As a result, the corresponding capacity of the system is found as

$$C_{\text{upper}} = \sum_{m=1}^{M} \sum_{k=1}^{K_m} \log_2 \left(1 + \frac{\lambda_k^{(m)}}{\sigma_n^2} \right) \tag{30}$$

where $\lambda_k^{(m)}$ is the kth largest eigenvalue of $\mathbf{H}_m^\dagger \mathbf{H}_m$.

2) DTx: A straightforward approach, which we refer to as a DTx system, is considered. This approach partitions the transmit antennas and directly assigns different antennas to different users. As such, the transmit weights are

$$[\mathbf{T}_1 \mathbf{T}_2 \cdots \mathbf{T}_M] = \sqrt{P_T}\, \mathbf{I}_K \tag{31}$$

where $\mathbf{I}_K$ is an $K \times K$ identity matrix, and $K = \sum_{m=1}^{M} K_m$. The corresponding optimal receive weights that maximize the SINR (and, hence, the capacity) for a given set of transmit weights are then found from (14).

3) SVD-MMSE: For a single-user MIMO system, it is found [5] that the best way to transmit data into multiple spatial dimensions is through the use of SDM, by SVD of the channel matrix. In a multiuser system, we can use the method to distribute the data across space for increasing the capacity or spectral efficiency of the system. As a result, the transmit weights are found from (28). In addition, because of CCI, the receive weights need to be found from (14) for CCI suppression. This system uses SVD for transmitting and MMSE for reception. Thus, we refer to this system as a SVD-MMSE system.

4) JADE: In [14], Cardoso *et al.* proposed an iterative approach for JADE. It suggests that for a set of M complex Hermitian matrices, $\mathbf{G}_m$, it is possible to find a unitary matrix $\mathbf{U}$ that minimizes

$$\sum_{m=1}^{M} \text{off}\left(\mathbf{U} \mathbf{G}_m \mathbf{U}^\dagger \right) \tag{32}$$

where

$$\text{off}(\mathbf{A}) \triangleq \sum_{1 \le k \ne \ell \le N} |a_{k,\ell}|^2 . \tag{33}$$

The idea is to maximize the desired or diagonal signals and minimize the undesired or off-diagonal signals. However, it should be noted that not all diagonal elements are used. Therefore, there will be some loss in the degree of freedom for adapting the weights. Using JADE on the matrices $\mathbf{H}_m^\dagger \mathbf{H}_m$, we obtain

$$[\mathbf{T}_1 \mathbf{T}_2 \cdots \mathbf{T}_M] = \sqrt{P_T}\, \mathbf{U} \tag{34}$$

for the weights operating on the transmitted packets, while (14) is used as the weights operating on the received signals in minimizing the interuser and interchannel interference.

B. Multiray Channel Model

The antenna elements transmit or receive information through a wireless communication channel, which is here characterized by a multipath fading model. For a particular channel, the multipath model is represented by its channel impulse response using a multiray model defined as [12]

$$c_{\ell,k}(t) = \sum_{i=0}^{I-1} \beta_{\ell,k}^i \delta(t - \tau_{\ell,k}^i) \tag{35}$$

where the subscripts ℓ, k refer to the channel between the ℓth and kth antenna at the BS and MS, respectively. Likewise, $\beta_{\ell,k}^i$ and $\tau_{k,\ell}^i$ are, respectively, the complex gain and time delay for

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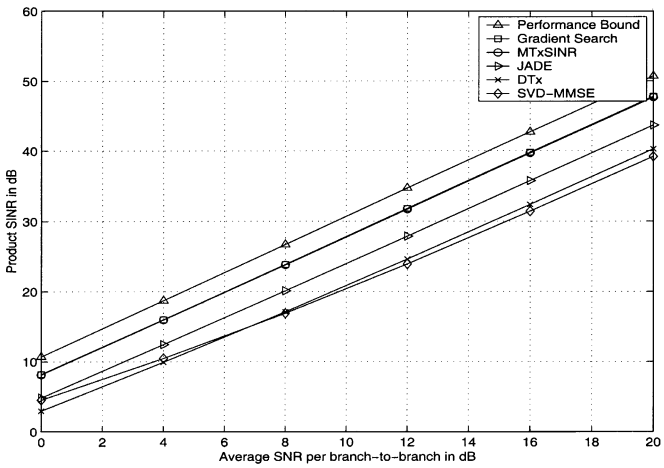

Fig. 3. Simulation results for a two-user MIMO system with two BS antennas, two antennas/MS, and $(K_1, K_2) = (1, 1)$.

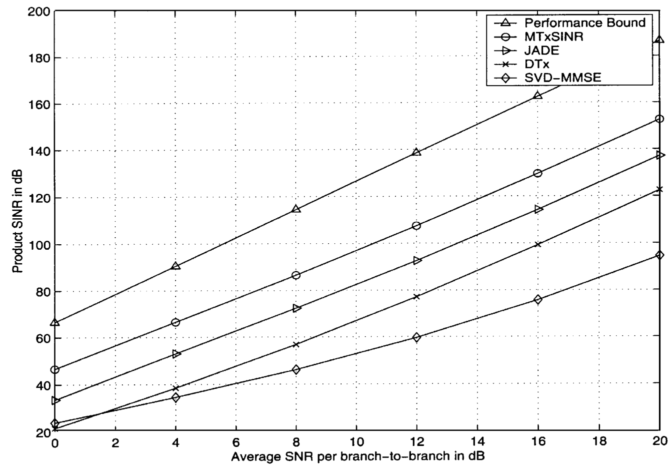

Fig. 5. Simulation results for a three-user MIMO system with six BS antennas, six antennas/MS, and $(K_1, K_2, K_3) = (2, 2, 2)$.

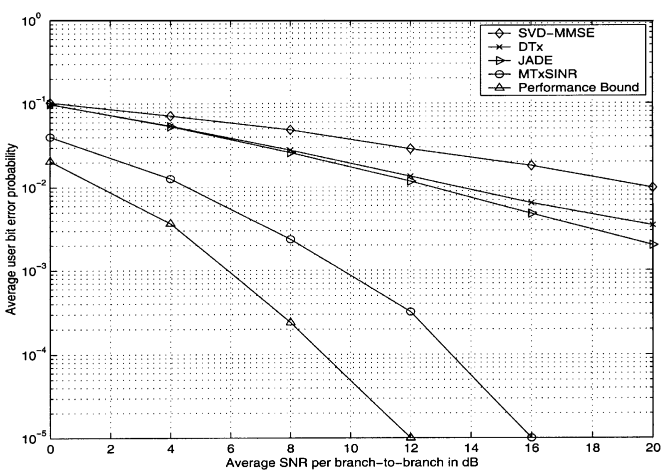

Fig. 4. Average error probabilities for a two-user MIMO system with two BS antennas, two antennas/MS, and $(K_1, K_2) = (1, 1)$.

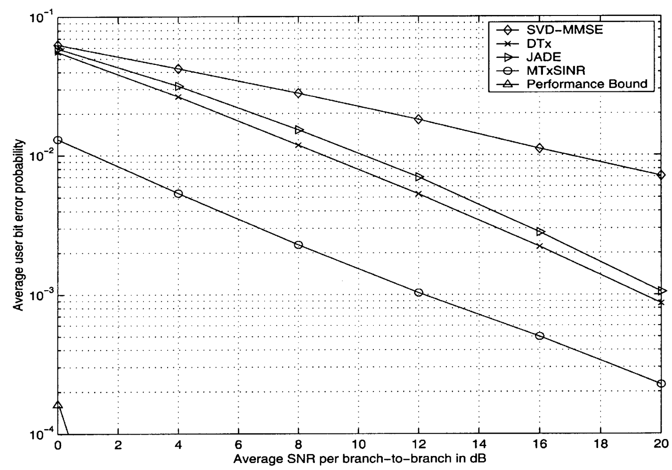

Fig. 6. Average error probabilities for a three-user MIMO system with six BS antennas, six antennas/MS, and $(K_1, K_2, K_3) = (2, 2, 2)$.

the ith path of the diversity channel. In the simulation, for frequency-selective fading channels, the channel of each link contains three random multipath components, while only one path is assumed for flat fading channels.

To determine $\beta^i_{\ell,k}$, in this paper, a statistical approach is used to allow easier control of channel parameters such as delay spread. Assume that paths with different delays are uncorrelated (i.e., uncorrelated scattering) and that the paths are uncorrelated for each antenna branch so as to provide perfect spatial diversity. As a result, path gains $\beta^{i_1}_{\ell_1,k_1}$ and $\beta^{i_2}_{\ell_2,k_2}$ are uncorrelated if $\ell_1 \neq \ell_2$, or $k_1 \neq k_2$, or $i_1 \neq i_2$. This will be realistic if antenna spacings at the MS and BS are, respectively, greater than 0.4 and 20–40 wavelengths. We model $\beta^i_{\ell,k}$ statistically by zero mean, complex Gaussian random variables, with their power following the exponential delay profile given by

$$\mathrm{E}\left[|c_{\ell,k}(t)|^2\right] = \begin{cases} \dfrac{1}{D}\exp\left(-\dfrac{t}{D}\right), & \text{for } t \geq 0 \\ 0, & \text{elsewhere.} \end{cases} \tag{36}$$

Hence, $h_j^{(m,\ell,k)}$ in (2) is equal to $(p \otimes c_{\ell,k} \otimes p)(t)|_{t=jT}$, where $\otimes$ denotes the convolution operator between two continuous time signals, and $p(t)$ is the pulse-shaping filter at the transmitter and receiver. The pulse-shaping filter used in the simulation is root raised cosine pulse with a rolloff factor of 0.3. For simplicity, we consider only the paths with delays less than five normalized root mean square (rms) delay spread, which is defined as $D \triangleq \tau_{\mathrm{rms}}/T$, where τ_{rms} and T are the rms delay spread and symbol period, respectively. In addition, we specifically assume the path delay as $\tau^i_{\ell,k} = (5D/(I-1))i$. In the simulation, $D = 0.5$ is simulated for frequency-selective fading.

C. Results

In Figs. 3–6, results are provided for flat Rayleigh fading channels for different multiuser MIMO systems when the BS and each MS have the same number of antennas, $n_T = n_{R_m}\ \forall m$. The results for MTxSINR, JADE, DTx, SVD-MMSE, and the performance bound are shown.

In Fig. 3, results are provided for the configuration of a two-user MIMO system ($M = 2$) that has two BS antennas ($n_T = 2$), and the number of antennas per MS equals two ($n_{R_1} = n_{R_2} = 2$). A close observation of this figure indicates that the performance of MTxSINR is better than all the other methods. For this example, we also compare our results with

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those based on a gradient search method which searches for the weights that maximize (11). The gradient search method is based on a steepest descent algorithm or Lagrange multipliers method. This method solves constrained optimization problems using a sequential quadratic programming algorithm and quasi-Newton gradient search techniques [15]. Results for the gradient search for systems with more users are not included, due to the high dimensionality of the problem. Results show that the performance difference between MTxSINR and the gradient search is less than 0.1 dB. Results also demonstrate that the upper bound has about 1.5 dB gain compared with the search or MTxSINR. However, we do not know how tight the bound is. In addition, note that SVD-MMSE has the worst performance for SNR $>$ 7 dB. This is because SVD-MMSE performs optimization only for the desired user link in the absence of CCI, and it is likely to cause severe CCI to other MS, thereby decreasing the SINR. Results also show that the performance of DTx (which does not make use of CSI at the BS) is even better than that of SVD-MMSE for SNR $>$ 7 dB. Moreover, JADE has only a 2 dB gain in average SNR compared with DTx. As such, the adaptation performed by JADE does not seem to give any performance gain. However, by using MTxSINR, an increase of more than 4 dB in average SNR is possible compared with DTx. Results for average user bit error probability are also provided for evaluation. In Fig. 4, results demonstrate that MTxSINR has much lower error probability for any given average SNR compared with other existing systems.

Similar results are provided in Fig. 5, but for the configurations of three-user ($M = 3$), six BS antennas ($n_T = 6$) and six antennas per MS ($n_{R_1} = n_{R_2} = n_{R_3} = 6$) are considered. Results illustrate consistent results similar to those listed in Fig. 3. However, note that the performance difference between the bound and MTxSINR becomes larger. This is due to the fact that as the number of users increases, consideration of CCI in the bound derivation is not taken into account and becomes even more and more unrealistic. Results also demonstrate that more than 5.5 dB gain in average SNR can be achieved by using MTxSINR compared with DTx.

In addition to the results for the average product SINR, Fig. 6 provides the average user bit error probability for the same configuration as that of Fig. 5. As can be seen, higher product SINR does not imply lower average user bit error probability. This can be explained by recognizing that the average error probability is usually dominated by the error probability of the channel with the worst condition. Results illustrate that, in terms of average user bit error probability, DTx has a slightly better performance (about 1 dB gain in average SNR) than JADE. Moreover, results demonstrate that MTxSINR has now about 8 dB gain in average SNR compared with DTx.

In practice, MS may only be able to contain two antennas. Thus, in the following, we specifically provide results for the configurations of a small number of MS antennas to evaluate the performance of our proposed system.

In Figs. 7 and 8, results are provided for a three-user ($M = 3$), six BS antennas ($n_T = 6$), and only two antennas per MS ($n_{R_1} = n_{R_2} = n_{R_3} = 2$) MIMO system. Notice that the number of antennas per MS is equal to the number of spatial

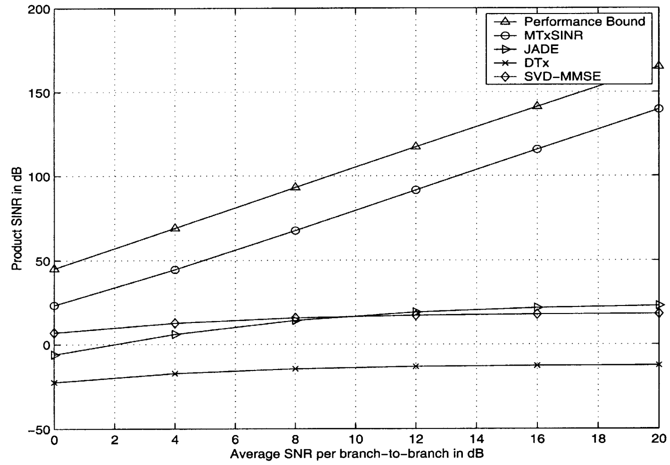

Fig. 7. Simulation results for a three-user MIMO system with six BS antennas, two antennas/MS, and $(K_1, K_2, K_3) = (2, 2, 2)$.

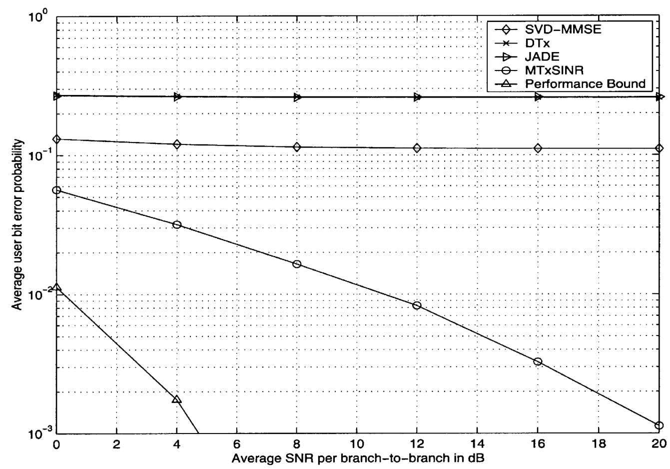

Fig. 8. Average error probabilities for a three-user MIMO system with six BS antennas, two antennas/MS, and $(K_1, K_2, K_3) = (2, 2, 2)$.

dimensions for each user (i.e., $n_{R_m} = K_m = 2\ \forall m$). Thus, interference cancellation relies greatly on the adaptation of BS weights. Results in Fig. 7 show that the performance of SVD-MMSE, DTx, and JADE converge and are interference limited. In contrast, the product SINR of MTxSINR can grow linearly with SNR. Therefore, the capacity of the system is expected to be able to grow linearly with SNR without causing severe CCI, with only two antennas at each MS. Results in Fig. 8 reveal that the CCI causes irreducible error floors for SVD-MMSE, DTx, and JADE. However, the average user bit error probability drops linearly with SNR. Roughly speaking, by adding two more BS antennas, the system can support one more MS that is able to transmit in two spatial dimensions, at the same frequency band and time slot.

In Figs. 9–12, results for various numbers of BS or MS antennas of a three-user MTxSINR system are provided and compared. Figs. 9 and 10 show the results for a three-user ($M = 3$), six BS antennas ($n_T = 6$) MTxSINR system with two, three, four, five, or six antennas per MS ($n_{R_m} = 2, 3, 4, 5, 6\ \forall m$). Results in Fig. 9 reveal that in general, as the number of antennas per MS increases, higher product SINR can be achieved. However, the performance of MTxSINR with three, four, or five

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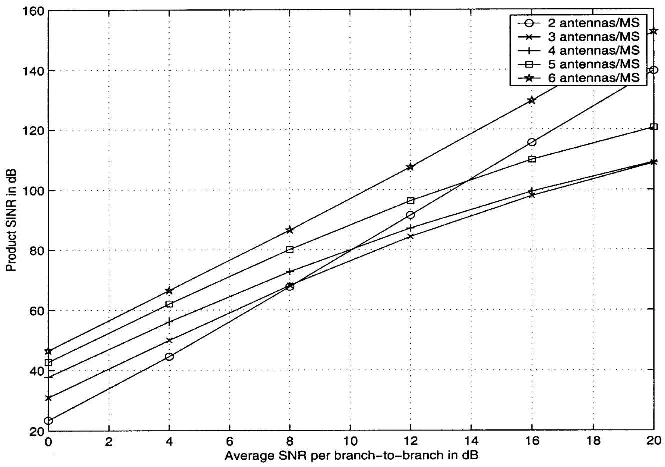

Fig. 9. Simulation results for a three-user MTxSINR system with six BS antennas, $(K_1,\, K_2,\, K_3) = (2,\, 2,\, 2)$, and various number of antennas/MS.

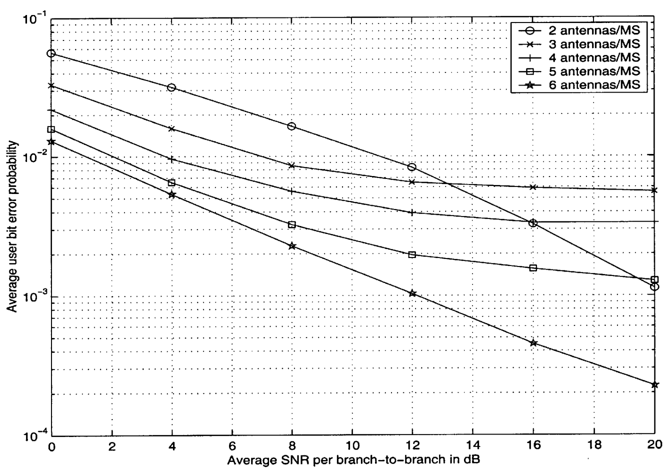

Fig. 10. Average error probabilities for a three-user MTxSINR system with six BS antennas, $(K_1,\, K_2,\, K_3) = (2,\, 2,\, 2)$, and various number of antennas/MS.

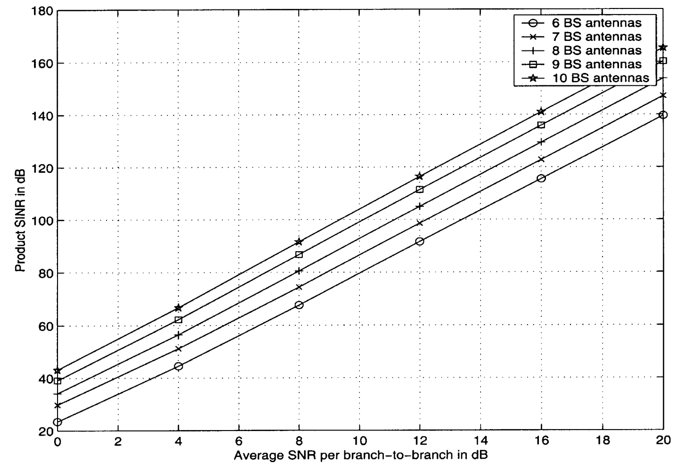

Fig. 11. Simulation results for a three-user MTxSINR system with two antennas/MS, $(K_1,\, K_2,\, K_3) = (2,\, 2,\, 2)$, and various number of BS antennas.

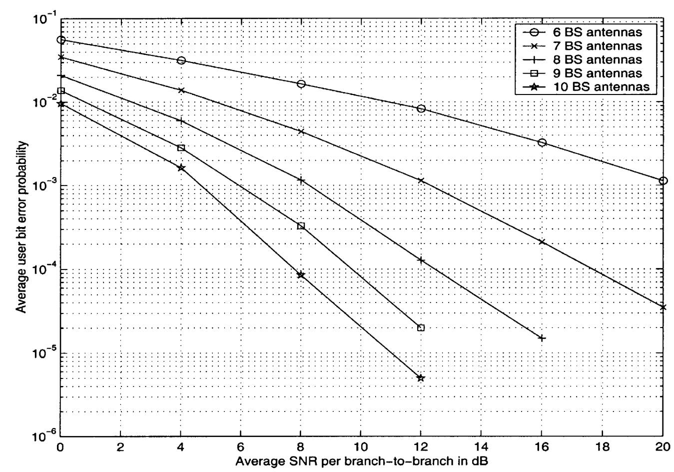

Fig. 12. Average error probabilities for a three-user MTxSINR system with two antennas/MS, $(K_1,\, K_2,\, K_3) = (2,\, 2,\, 2)$, and various number of BS antennas.

antennas per MS for high SNR values diminish and could have much worse performance than that for two antennas per MS. This can be explained by recognizing that our MTxSINR solution is suboptimal, which provides a closed-form solution by neglecting the interdependence of different users. This solution relies greatly on the transmit antennas for controlling the interference at the mobile receivers, which trades off between the complexity of the solution and the interference rejection capability. To be more specific, the reason for the results in Figs. 9 and 10 can be explained by the following. When the number of MS antennas increases, it is found that the transmitting combining gets worse. The reason for this can be seen by an example. When there are six transmit antennas and two antennas for each of the three MS, there are enough degrees of freedom at the transmitter to independently handle all six receive antennas (i.e., two antennas times three MS). However, once the total number of receive antennas goes beyond six, the transmit performance degrades, since there is now fewer transmit antennas than receive antennas and no degrees of freedom left at the transmitter. This explains why the example of BER in Fig. 10 for two antennas per MS has no error floor while those with three to five antennas has an error floor. Complementing the effect of the reduced transmitter performance is the improved receiver combining as the number of receive antennas increases. In particular, as the number of receive antennas per MS becomes greater than the number of transmit antennas, there are enough degrees of freedom at the receiver to again retrieve all the transmitter signals. Therefore, the error floor will again disappear when the number of MS antennas becomes equal to or greater than six, as shown in Fig. 10.

Results in Figs. 11 and 12 are provided for a three-user ($M = 3$) MTxSINR system for the configurations of two antennas per MS ($n_{R_m} = 2\ \forall m$), and various number BS antennas. Results indicate that an increase in the product SINR as well as reduction in average user error probability can be achieved by increasing the number of BS antennas. As low as 10^{-4} average error probability is possible for three cochannel users at SNR $=$8, 10, and 13 dB when ten, nine, and eight antennas are located at the BS, respectively, and two antennas are located at each MS. Results also demonstrate that as the number of BS antennas increases, the performance can grow without bound as SNR is increased.

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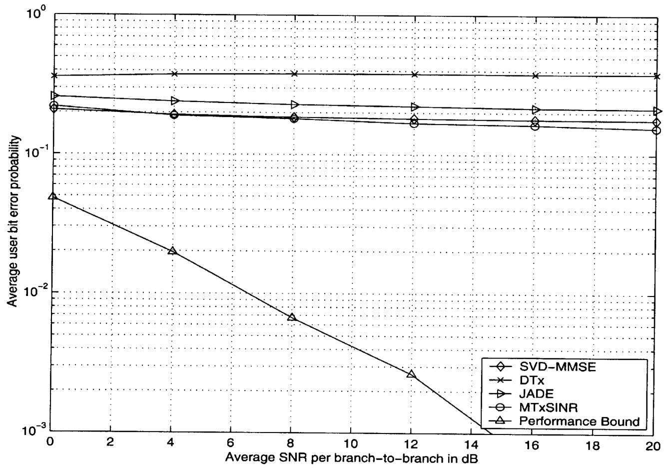

Fig. 13. Average error probabilities for a three-user MTxSINR system with one antenna/MS, $(K_1, K_2, K_3) = (1, 1, 1)$ in frequency-selective fading channels.

In Fig. 13, results are provided for a three-user ($M = 3$) system for the configuration of three BS antennas ($n_T = 3$) and one antenna per MS ($n_{R_m} = 1 \, \forall m$) in frequency-selective fading channels with delay spread $D = 0.5$. As results indicate, the performances of all the approaches including MTxSINR are not satisfying, although MTxSINR has comparatively better performance. The reason is that despite the block transmission approach could somehow handle the multipath, it seems that MTxSINR is unable to manage all the potential interference due to the substantial increase of CCI and ISI. This is also due to the lower bound of the product SINR being far more inaccurate in the case of frequency-selective fading channels. Therefore, future work has to be done in order to come up with a more accurate objective function for a multiuser multipath scenario.

Before finishing this section, we present some simple rules of thumb to provide BER results that do not have an error floor. These rules of thumb are only useful for flat fading channels and are given as follows.

Condition 1—The Number of Transmit (BS) Antennas: The number of antennas at the transmitter (or BS) should be equal to or greater than the total number of cochannel signals within the system (i.e., $n_T \geq K_1 + K_2 + \cdots + K_M$).

Condition 2—The Number of Receive (MS) Antennas: The minimum number of receive antennas at the mth MS should be equal to the number of spatial dimensions of that mobile (i.e., $n_{R_m} = K_m$) or equal to or greater than the total number of cochannel signals within the system (i.e., $n_{R_m} \geq K_1 + K_2 + \cdots + K_M$).

Reference to Fig. 10 will reveal the usefulness of these rules of thumb.

V. Conclusion

In this paper, we have considered the performance of a multiuser MIMO system. By exploiting space diversity, SDM can be implemented to enhance capacity for multiuser wireless communications. We have developed a *closed-form* solution for the antenna weights which are based on optimizing a lower bound of product SINR. When maximizing the product SINR, there is a tendency to jointly maximize the SINR of all the channel symbols, and this leads to a reduction of average bit error probability. Simulation results reveal that proposed MTxSINR system performs nearly as well as a gradient search-based optimization approach for a two-user MIMO system. In addition, results have shown that by using MTxSINR, only a few MS antennas are required, at the expense of BS antennas, for obtaining both high product SINR and low average user error probability. Finally, it has been demonstrated that MTxSINR is a promising technique for implementing SDM for high rate and reliable multiuser wireless communications.

Appendix I
Derivation of the Lower Bound of Product SINR

Using the inequality (47) derived in Appendix II, we can obtain a lower bound of the symbol SINR for the kth spatial dimension of the mth user as in (16)

$$\gamma_{n,k}^{(m)} \geq \frac{\mathbf{t}_{n,k}^{(m)\dagger}\mathbf{H}_m^{\dagger}\mathbf{H}_m\mathbf{t}_{n,k}^{(m)}}{\sigma_n^2} \prod_{\substack{\tilde{m}=1 \\ (\tilde{m},\tilde{k},\tilde{n})\neq(m,k,n)}}^{M} \prod_{\tilde{k}=1}^{K_{\tilde{m}}} \prod_{\tilde{n}=1}^{N} \times \frac{\sigma_n^2}{\mathbf{t}_{\tilde{n},\tilde{k}}^{(\tilde{m})\dagger}\mathbf{H}_m^{\dagger}\mathbf{H}_m\mathbf{t}_{\tilde{n},\tilde{k}}^{(\tilde{m})} + \sigma_n^2}. \quad (37)$$

Together with the power constraint $\|\mathbf{t}_{n,k}^{(m)}\| = \sqrt{P_T}$, we can have the lower bound

$$\prod_{m=1}^{M}\prod_{k=1}^{K_m}\prod_{n=1}^{N}\gamma_{n,k}^{(m)} \geq \rho_1 \prod_{m=1}^{M}\prod_{k=1}^{K_m}\prod_{n=1}^{N} \times \left[\frac{\mathbf{t}_{n,k}^{(m)\dagger}\mathbf{H}_m^{\dagger}\mathbf{H}_m\mathbf{t}_{n,k}^{(m)}}{\prod_{\substack{\tilde{m}=1 \\ (\tilde{m},\tilde{k},\tilde{n})\neq(m,k,n)}}^{M}\prod_{\tilde{k}=1}^{K_{\tilde{m}}}\prod_{\tilde{n}=1}^{N} \mathbf{t}_{\tilde{n},\tilde{k}}^{(\tilde{m})\dagger}\left(\mathbf{H}_m^{\dagger}\mathbf{H}_m + \frac{\sigma_n^2}{P_T}\right)\mathbf{t}_{\tilde{n},\tilde{k}}^{(\tilde{m})}}\right] \quad (38)$$

where $\rho_1 \triangleq (\sigma_n^2)^{(N\sum_m K_m)(N\sum_m K_m - 2)}$. Furthermore, by rearranging the factors of the lower bound, it can be easily shown that the lower bound can be rewritten as a product of another factor as

$$\prod_{m=1}^{M}\prod_{k=1}^{K_m}\prod_{n=1}^{N}\alpha_{n,k}^{(m)} \quad (39)$$

with $\alpha_{n,k}^{(m)}$ defined by (40), shown at the top of the next page. Note that $\alpha_{n,k}^{(m)}$ depends only on $\mathbf{t}_{n,k}^{(m)}$. As such, we have now obtained a lower bound which decorrelates the interdependence of multiuser antenna weights.

The obtained lower bound involves the product of the weights, so it is expected that the maximization of the lower bound under a closed-form solution is not possible. Therefore, another lower bound which removes nonlinearity in the optimization needs to be found. To do so, we derive a further

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$$\alpha_{n,k}^{(m)} \triangleq \frac{\mathbf{t}_{n,k}^{(m)\dagger}\mathbf{H}_m^\dagger\mathbf{H}_m\mathbf{t}_{n,k}^{(m)}}{\left[\mathbf{t}_{n,k}^{(m)\dagger}\left(\mathbf{H}_m^\dagger\mathbf{H}_m+\frac{\sigma_n^2}{P_T}\mathbf{I}\right)\mathbf{t}_{n,k}^{(m)}\right]^{NK_m-1}\prod_{\substack{\tilde{m}=1\\ \tilde{m}\neq m}}^{M}\left[\mathbf{t}_{n,k}^{(m)\dagger}\left(\mathbf{H}_{\tilde{m}}^\dagger\mathbf{H}_{\tilde{m}}+\frac{\sigma_n^2}{P_T}\mathbf{I}\right)\mathbf{t}_{n,k}^{(m)}\right]^{NK_{\tilde{m}}}} \tag{40}$$

lower bound by applying a single mathematical inequality $((1/L)\sum_{\ell=1}^{L}a_\ell \geq \sqrt[L]{\prod_{\ell=1}^{L}a_\ell})$. Then, we can get

$$\alpha_{n,k}^{(m)} \geq \frac{(N_s)^{N_s}\mathbf{t}_{n,k}^{(m)\dagger}\mathbf{H}_m^\dagger\mathbf{H}_m\mathbf{t}_{n,k}^{(m)}}{\left\{\mathbf{t}_{n,k}^{(m)\dagger}\left[\sum_{\tilde{m}=1}^{M}Q_{\tilde{m}}\left(\mathbf{H}_{\tilde{m}}^\dagger\mathbf{H}_{\tilde{m}}+\frac{\sigma_n^2}{P_T}\mathbf{I}\right)\right]\mathbf{t}_{n,k}^{(m)}\right\}^{N_s}} \tag{41}$$

where $N_s = N\sum_{m=1}^{M}K_m - 1$, and

$$Q_{\tilde{m}} \triangleq \begin{cases} K_{\tilde{m}}N, & \tilde{m}\neq m\\ K_mN-1, & \tilde{m}=m.\end{cases} \tag{42}$$

By noting that each factor in the denominator must be less than the largest eigenvalue of $\sum_{\tilde{m}=1}^{M}Q_{\tilde{m}}\,(\mathbf{H}_{\tilde{m}}^\dagger\mathbf{H}_{\tilde{m}}+(\sigma_n^2/P_T)\mathbf{I})$, say λ_m, we have also

$$\alpha_{n,k}^{(m)} \geq \left(\frac{N_s}{\lambda_m}\right)^{N_s}\times\frac{\mathbf{t}_{n,k}^{(m)\dagger}\mathbf{H}_m^\dagger\mathbf{H}_m\mathbf{t}_{n,k}^{(m)}}{\mathbf{t}_{n,k}^{(m)\dagger}\left[\sum_{\tilde{m}=1}^{M}Q_{\tilde{m}}\left(\mathbf{H}_{\tilde{m}}^\dagger\mathbf{H}_{\tilde{m}}+\frac{\sigma_n^2}{P_T}\mathbf{I}\right)\right]\mathbf{t}_{n,k}^{(m)}}. \tag{43}$$

Finally, we obtain the inequality

$$\prod_{m=1}^{M}\prod_{k=1}^{K_m}\prod_{n=1}^{N}\gamma_{n,k}^{(m)} \geq \rho\prod_{m=1}^{M}\prod_{k=1}^{K_m}\prod_{n=1}^{N}\times\frac{\mathbf{t}_{n,k}^{(m)\dagger}\mathbf{H}_m^\dagger\mathbf{H}_m\mathbf{t}_{n,k}^{(m)}}{\mathbf{t}_{n,k}^{(m)\dagger}\left[\sum_{\tilde{m}=1}^{M}Q_{\tilde{m}}\left(\mathbf{H}_{\tilde{m}}^\dagger\mathbf{H}_{\tilde{m}}+\frac{\sigma_n^2}{P_T}\mathbf{I}\right)\right]\mathbf{t}_{n,k}^{(m)}} \tag{44}$$

where

$$\rho = \rho_1\prod_{m=1}^{M}\prod_{k=1}^{K_m}\prod_{n=1}^{N}\left(\frac{N_s}{\lambda_m}\right)^{N_s}. \tag{45}$$

APPENDIX II

Here it is shown that for any vector $\mathbf{v}_K$ and a nonsingular Hermitian matrix $\mathbf{\Theta}_K$, given by

$$\mathbf{\Theta}_K = \sum_{\ell=1}^{K-1}\mathbf{v}_\ell\mathbf{v}_\ell^\dagger + \sigma^2\mathbf{I} \tag{46}$$

we have

$$\mathbf{v}_K^\dagger\mathbf{\Theta}_K^{-1}\mathbf{v}_K \geq \frac{\mathbf{v}_K^\dagger\mathbf{v}_K}{\sigma^2}\prod_{\ell=1}^{K-1}\frac{\sigma^2}{\mathbf{v}_\ell\mathbf{v}_\ell^\dagger+\sigma^2}. \tag{47}$$

To show this, first recall that for a matrix $\mathbf{Q} = \mathbf{a}\mathbf{a}^\dagger + \mathbf{S}$, where $\mathbf{Q}$ and $\mathbf{S}$ are square matrices of the same size, and $\mathbf{a}$ is a column vector, its inverse can be expressed as [13]

$$\mathbf{Q}^{-1} = \mathbf{S}^{-1} - \frac{\mathbf{S}^{-1}\mathbf{a}\mathbf{a}^\dagger\mathbf{S}^{-1}}{\mathbf{a}^\dagger\mathbf{S}^{-1}\mathbf{a}+1}. \tag{48}$$

Accordingly, we have

$$\mathbf{\Theta}_k^{-1} = \mathbf{\Theta}_{k-1}^{-1} - \frac{\left|\mathbf{v}_{k-1}^\dagger\mathbf{\Theta}_{k-1}^{-1}\mathbf{v}_{k-1}\right|^2}{\mathbf{v}_{k-1}^\dagger\mathbf{\Theta}_{k-1}^{-1}\mathbf{v}_{k-1}+1} \tag{49}$$

and therefore

$$\mathbf{v}_\ell^\dagger\mathbf{\Theta}_I^{-1}\mathbf{v}_\ell \leq \mathbf{v}_\ell^\dagger\mathbf{\Theta}_J^{-1}\mathbf{v}_\ell \qquad \text{for } I > J. \tag{50}$$

Now, using (49), we can write

$$\mathbf{v}_K^\dagger\mathbf{\Theta}_K^{-1}\mathbf{v}_K = \mathbf{v}_K^\dagger\mathbf{\Theta}_{K-1}^{-1}\mathbf{v}_K - \frac{\left|\mathbf{v}_K^\dagger\mathbf{\Theta}_{K-1}^{-1}\mathbf{v}_{K-1}\right|^2}{\mathbf{v}_{K-1}^\dagger\mathbf{\Theta}_{K-1}^{-1}\mathbf{v}_{K-1}+1}. \tag{51}$$

Also, by the Schwartz inequality (i.e., $|\mathbf{a}^\dagger\mathbf{b}|^2 \leq |\mathbf{a}|^2|\mathbf{b}|^2$), it is easy to see that

$$\mathbf{v}_K^\dagger\mathbf{\Theta}_K^{-1}\mathbf{v}_K \geq \frac{\mathbf{v}_K^\dagger\mathbf{\Theta}_{K-1}^{-1}\mathbf{v}_K}{\mathbf{v}_{K-1}^\dagger\mathbf{\Theta}_{K-1}^{-1}\mathbf{v}_{K-1}+1}. \tag{52}$$

In addition, using (50) in the denominator of the right-hand side, we can have a further lower bound. That is, we have

$$\begin{aligned}\mathbf{v}_K^\dagger\mathbf{\Theta}_K^{-1}\mathbf{v}_K &\geq \frac{\mathbf{v}_K^\dagger\mathbf{\Theta}_{K-1}^{-1}\mathbf{v}_K}{\frac{\mathbf{v}_{K-1}^\dagger\mathbf{v}_{K-1}}{\sigma^2}+1}\\ &= \mathbf{v}_K^\dagger\mathbf{\Theta}_{K-1}^{-1}\mathbf{v}_K\left(\frac{\sigma^2}{\mathbf{v}_{K-1}^\dagger\mathbf{v}_{K-1}+\sigma^2}\right).\end{aligned} \tag{53}$$

As a result, repeatedly using (53), we have (47).

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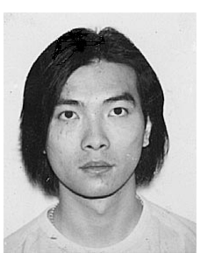

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