The constraints of packing rooms together, and the flexibility of dimensioning allowed by rectangular arrangements, explain the predominance of the right angle in architectural plans.

Why are most buildings rectangular?

Philip Steadman

The paper offers evidence to show that the geometry of the majority of buildings is predominantly rectangular, and asks why this should be. Various hypotheses are reviewed. It is clear that, in the vertical direction, rectangularity is at least in part to do with the force of gravity. In the horizontal plane, departures from rectangularity tend to be found in buildings consisting of single spaces, and around the peripheries of plans made up of many spaces. This suggests strongly that the causes of rectangularity in multi-room plans lie in the constraints of packing those rooms closely together. A geometrical demonstration comparing room shapes and room arrangements on square, triangular and hexagonal grids indicates that it is the *superior flexibility of dimensioning* allowed by rectangular packings that leads to their predominance.

Introduction: is it the case?
Why are most buildings rectangular? This is a fundamental question that is rarely asked. Perhaps visiting Martians – assuming their interests tended to the geometrical – might want to raise the issue. Certainly from the evidence of science fiction films the dwellings of aliens seem to be non-rectangular – presumably to signal their exoticism and unEarthliness. The question is worth pursuing all the same, I believe, because of its implications for a theory of built form. By 'buildings' I do not just mean considered and prestigious works of architecture, which possibly tend more often than others to the non-orthogonal and curvilinear, for reasons that may become clear. I mean the totality of buildings of all types, the whole of the stock, both architect-designed and 'without architects', present and past. And to be slightly more specific about the question itself: I mean to ask 'Why is the geometry of the majority of buildings *predominantly* rectangular?' There will, certainly, be many small departures from rectangularity in otherwise rectangular buildings, and it would be rare indeed to find a building in which every single space and component was perfectly rectangular. We are talking about general tendencies here.

Is it actually true? Everyday experience would

indicate that it is: but we do not have to rely just on subjective impressions. At least two surveys have been made, to my knowledge, of the extent of rectangularity in the building stock. The first was a survey of houses carried out in the 1930s by the American architect Albert Farwell Bemis, reported in his book *The Evolving House*.[1] Bemis was interested in the potential for prefabrication in house building. Taking for granted that components making up any prefabricated system would have to be rectangular, he measured a sample of 217 conventionally constructed houses and apartments in Boston, to determine what percentage of their total volume – conceiving the building as a 'solid block' in each case – was organised according to a rectangular geometry. He found that proportion to be 83%, the remaining 17% being largely attributed to pitched roofs.[2] This measurement gave Bemis an indication of the extent to which current house designs could be replicated with standardised rectangular components.

The second survey was made by M. J. T. Krüger in the 1970s, in the course of a study of urban morphology and the connectivities between streets, plots and buildings.[3] Krüger took as his sample the entire city of Reading (Berkshire), and included buildings of all types. He inspected just the outlines of their perimeters in plan, working from Ordnance Survey maps. (Of course these outlines would have been somewhat simplified in the maps.) He distinguished plan shapes that were completely rectangular in their geometry (all external walls were straight line segments, and the angles between them right angles) from plans with curved walls, or with straight walls but set at acute or obtuse angles. He found that 98% of the plan shapes were rectangular on this definition. On the evidence of these two studies then – and acknowledging their limited historical and geographical scope – it *does* seem fair to say that the majority of buildings are predominantly rectangular.

Hypotheses
At least part of the answer to our question in relation to the vertical direction is no great mystery: rectangularity in buildings has much to do with the

force of gravity. Floors are flat so that we, and pieces of furniture, can stand easily on them. Walls and piers and columns are made vertical so that they are structurally stable and the loads they carry are transferred directly to the ground – although there are obviously many exceptions. Larger buildings as a consequence tend to be made up, as geometrical objects, from the horizontal layers of successive floors. In trying to understand general rules or tendencies it is often informative to look at exceptions, at 'pathological cases'. When are floors *not* flat? The most obvious examples are vehicle and trolley ramps – but we humans seem to prefer to rise or descend, ourselves, on the flat treads of stairs or escalators. Theatres and lecture rooms have raked floors; but these, like staircases, are not locally sloped, and consist of the shallow steps on which the rows of seats are placed. Truly sloping floors on which the occupants of buildings are expected to walk are rare; and where they do occur – as in the helical galleries of Frank Lloyd Wright's Guggenheim Museum in New York – can be a little disturbing and uncomfortable.

The rectangularity of buildings in the horizontal plane is more mysterious. I have solicited explanations from a number of colleagues who combine an interest in architecture with a mathematical turn of mind, and they have offered different ideas.

1. The cause, they suggest, lies in the use by architects of instruments – specifically drawing boards with T-squares and set-squares – that make it easier to draw rectangles than other shapes. (Techniques for surveying and laying out plans on the ground might also favour right angles.) We know that drawing apparatus of this general kind is very ancient. The earliest known examples are from Babylonia and are dated to around 2130 BC *[1]*. However, we also know that very many, in fact almost certainly the majority of buildings in history, were built without drawings of any kind; and that many of these were nevertheless rectangular. So this explanation is clearly inadequate.

2. The cause is to be found deeper in our culture and intellectual make-up, and has to do with Western mathematical conceptions of three-dimensional space – with the geometry of Euclid, and with the superimposition on to mental space of the orthogonal coordinate systems of Descartes. (Architects' drawing equipment would then be just one symptom of this wider conceptualisation.) However this argument is subject to the same objections as the first. What about all those rectangular buildings produced in non-Western cultures, or in the West but before Greek geometry, or erected by builders who had absolutely no knowledge of Western geometrical theory?

3. The cause is to be found yet deeper still in our psychology, and has to do with the way in which we conceptualise space in relation to the layout, mental image and functioning of our own bodies. Our awareness of gravity and the earth's surface creates the basic distinction between 'up' and 'down'. The design of the body for locomotion, and

1   Carved representation of a drawing board with scribing instrument and scale, from a statue of Gudea of Ur, c. 2130 BC (Musée du Louvre)

the placing of the eyes relative to the direction of this movement, creates the distinction – in this argument – between 'forward' and 'backward'. We now have two axes at right angles. A third orthogonal axis, distinguishing 'left' from 'right', reflects the bilateral symmetry, in this direction, of arms, legs, eyes and ears. When we walk, we steer and turn by reference to this sense of left and right. We organise our buildings accordingly.[4]
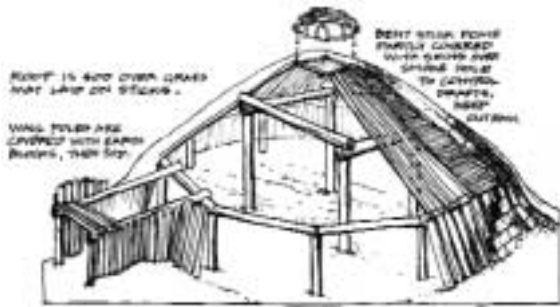
This third argument for a general rectangularity in buildings is subtler if even more speculative than the previous two. If true, it would clearly apply to humans and their buildings in all times and places. Such a mental and bodily disposition to see and move in the world relative to an orthogonal system of 'body coordinates' might conceivably – although this is *very* hypothetical – account for the extent to which we humans are comfortable in buildings whose geometry is itself rectangular. But this is *not* in my view the explanation – or at least not the sole explanation – for the occurrence of that rectangularity in the first place.

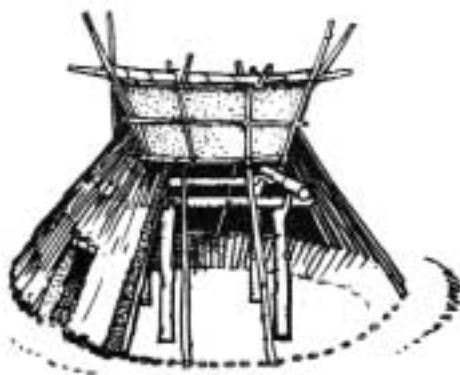### Where do departures from rectangularity occur in plans?

Once again it is instructive to examine some counter-examples. In what circumstances do we tend to find buildings, or parts of buildings, which are not rectangular in plan? Many buildings that comprise just one room – or one large room plus a few much smaller attached spaces, such as porches or lobbies – have plan perimeters whose shapes are circles, ellipses, hexagons or octagons. Primitive and vernacular houses provide many familiar examples of circular one-room plans: igloos, Mongolian yurts, tepees, Dogon and Tallensi huts *[2]*. Temples, chapels and other small places of worship are often single spaces, and again their plan shapes are frequently non-rectangular. You could cite circular temple plans from many cultures and periods. Figure 3 reproduces a detail of a plate from J.-B. Séroux d'Agincourt's *Histoire de l'Art, Architecture* showing circular religious

2a

2b

2c

buildings dating from the fourth to the sixteenth century [3]. The space of the auditorium is dominant in the forms of some theatres, and here too we find that where the plan of the auditorium is a semi-circle, a horseshoe, or a trapezium, then it can give this shape to the perimeter of the building [4].
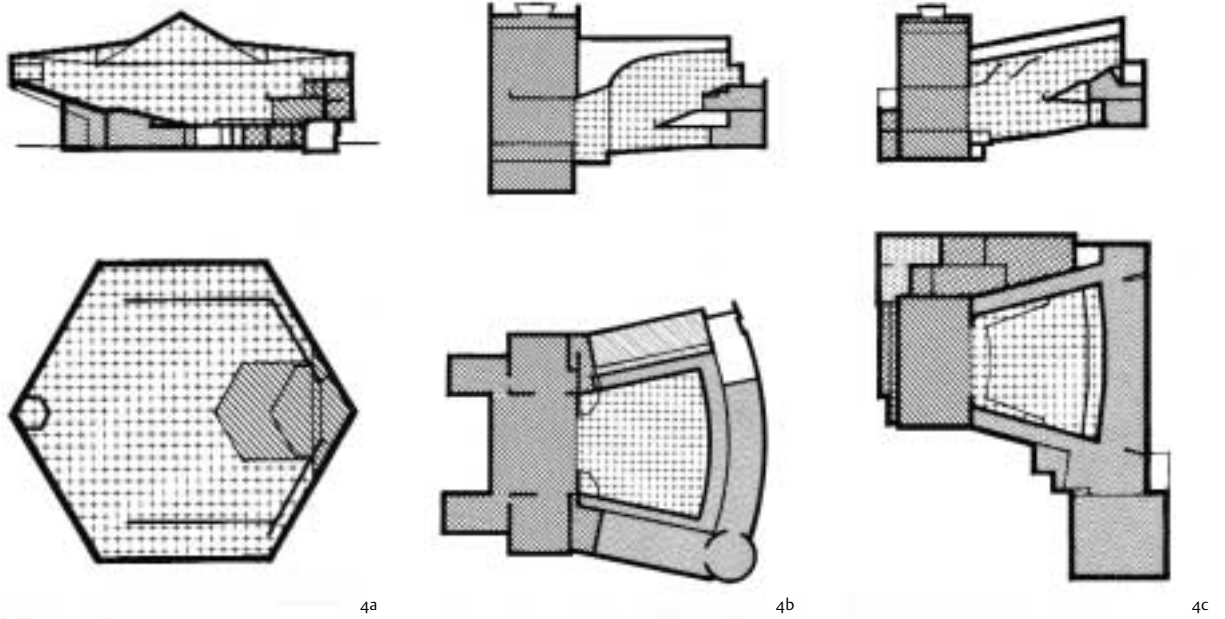
These are buildings whose plans consist of, or are dominated by, one single space. A second type of situation in which curvilinear or other non-orthogonal elements are often found – in the plans of buildings that may consist of many rooms – is around the building's outer edges. Many of the otherwise rectangular churches in Figure 3 have semi-circular apses. In simple rectangular modern houses we find semi-circular or angled bay windows. These provided one of the more frequent departures from rectangularity in Bemis's survey (the most obvious non-orthogonality in many buildings in the vertical direction – as confirmed by Bemis – is in their pitched roofs; again on the outer surfaces of the built forms, by definition).

Figure 5 illustrates the plans of an apartment building at Sausset-les-Pins in France, designed by André Bruyère and completed in 1964. From the exterior the block gives the impression of being designed according to an entirely 'free-form' curvilinear geometry. Closer inspection of the interior, however, shows the majority of the rooms in the flats to be simple rectangles, or near approximations to rectangles. The curved profile of the exterior is created by curving some of the exterior-facing walls of the living rooms, and by adding balconies with curved outlines [5]. Many late twentieth-century office buildings that have bulging facades are similarly curved only on these exterior surfaces, with conventional rectangular layouts concealed behind.

Naval architecture offers some interesting parallels here. The hulls of ships are doubly curved, for obvious hydrodynamic reasons. In smaller boats with undivided interiors the plan shape of the single cabin follows directly the internal lines of the hull. But in ocean liners with many spaces, the interior layout tends to consist mostly of rectangular rooms, with only the curved walls of those cabins that lie on the two outer sides of the ship taking up the curvature of the hull [6].
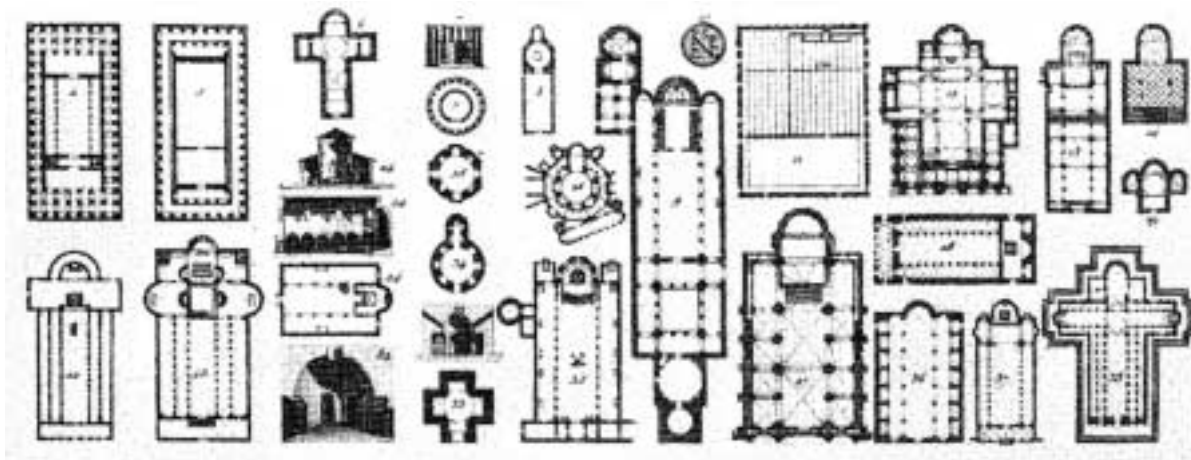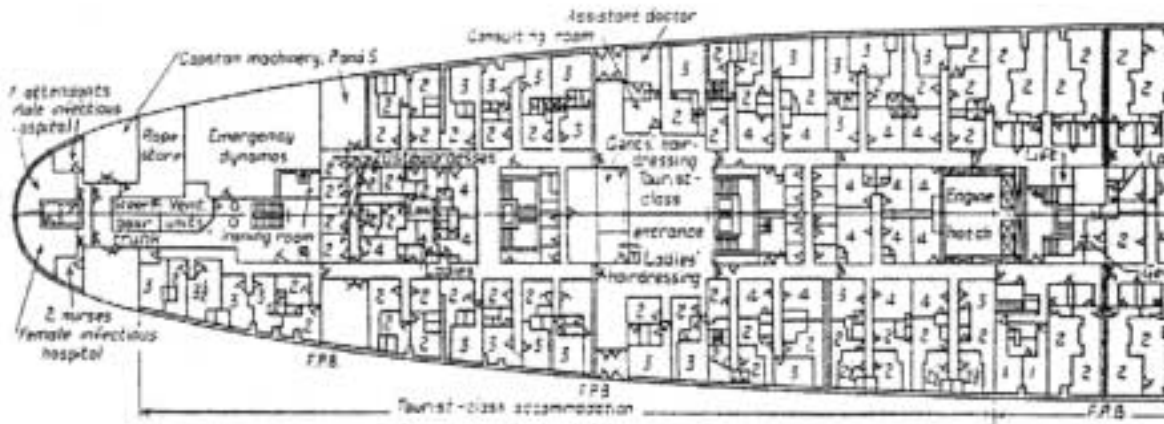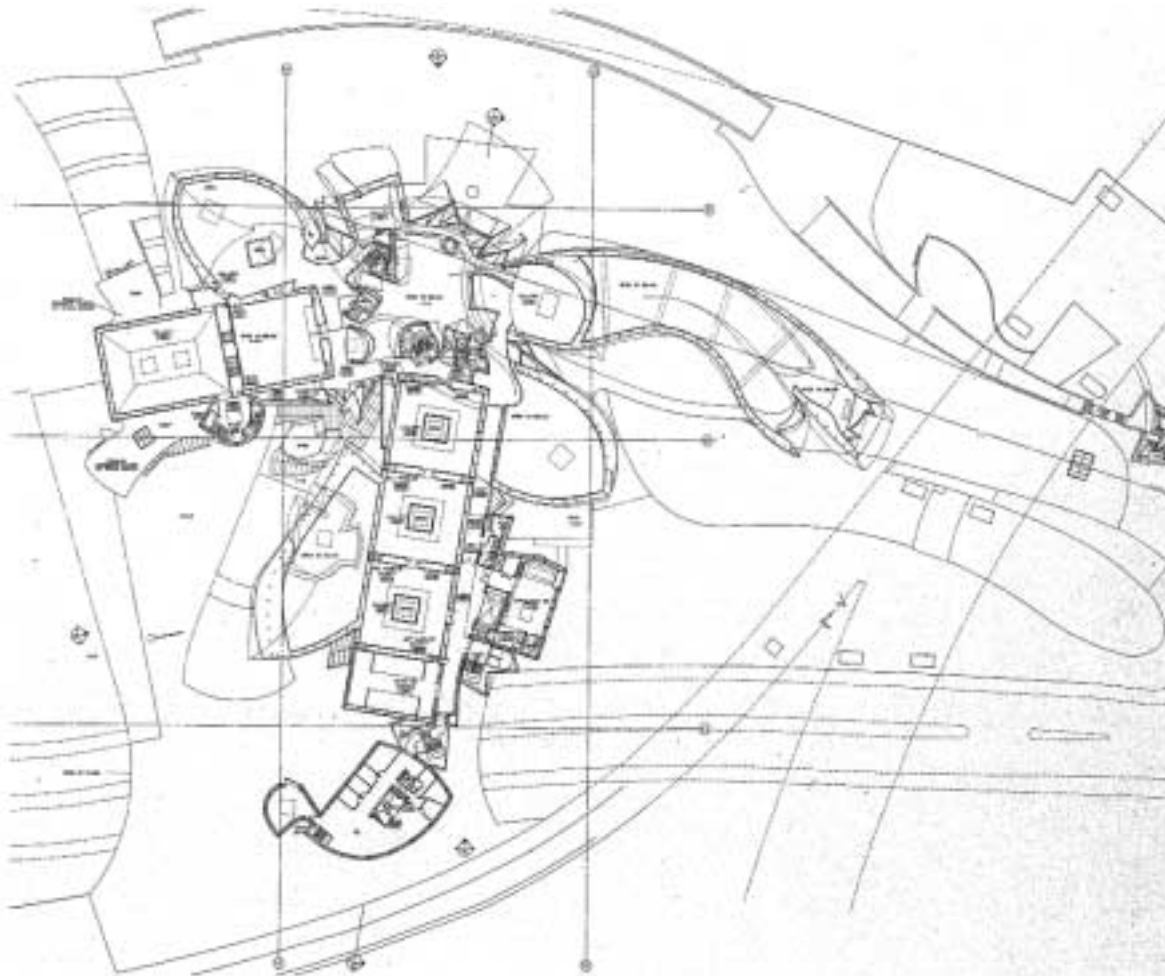
2 Traditional and vernacular houses with circular plans:
a Mongolian yurt
b Mandan earth lodge
c Neolithic northern Japanese shelter (reprinted with permission from *Shelter* © 1973, Shelter Publications, California, p. 16, p. 18 and p. 21)

3 Detail of plate 'Summary and General Catalogue of the Buildings That Constitute the History of the Decadence of Architecture' showing religious buildings from the fourth to the sixteenth century

with circular plans, as well as some rectangular buildings with semi-circular apses, from J.-B. Séroux d'Agincourt, *Histoire de l'Art par les Monuments Depuis sa Décadence au IVe Siècle jusqu'à son Renouvellement au XVIe* (Paris, 1811–23)

3

4a   4b   4c

5

6

7

6 Part plan of B Deck of
  the 'Queen Mary'

7 Third floor plan of the
  Guggenheim
  Museum, Bilbao by
  Frank Gehry, 1997
  (from C. van
  Bruggen,
  *Guggenheim
  Museum Bilbao*
  (1998), p. 151). Notice

the rectangular
planning of the
smaller galleries and
the office wing

8 Plan of Altes
  Museum, Berlin by
  K. F. Schinkel, 1823–30,
  to show the central
  circular hall created
  within an otherwise
  rectangular plan
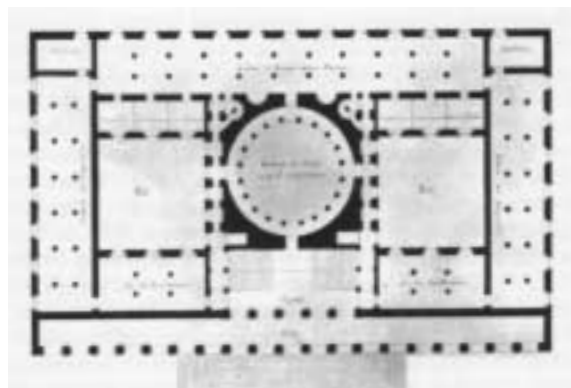
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If one were to ask the general public to name one contemporary building, above all others, whose form is definitively 'free' and non-rectangular, the most frequent answer might well be Frank Gehry's Guggenheim Museum in Bilbao. Without question the external titanium shell is geometrically complex. But if one looks deeper into the building one finds that in some places this shell is used to enclose large 'free-form' galleries, practically detached from the remainder of the structure and so very nearly 'single-room buildings' in themselves [7]. Meanwhile, in those parts of the Museum where many rooms of comparable dimensions are located together, such as the multiple smaller galleries, and the administrative offices, the planning reverts to an orthogonal geometry.

It is true that in certain classically planned buildings with many rectangular rooms, as for example villas or country houses, there can be spaces deep in the interior, such as central halls, whose plans are circular, polygonal or elliptical [8]. However, these are produced by filling out the corners of rectangular spaces – in Beaux Arts terms the *poché* – with solid masonry, cupboards, lobbies or spiral stairs; and the overall planning discipline remains rectangular.

What all this evidence indicates, I would suggest, is that rectangularity in buildings in the horizontal plane has to do, crucially, with *the packing together of rooms in plan.* When many rooms of similar or varying sizes are fitted together so as to pack without interstices, it is *there* that rectangularity is found. Non-rectangular shapes occur on the edges of plans, or in single-room buildings, since in both cases the exigencies of close packing do not apply.

Packings of squares, triangles and hexagons

One reasonable objection to this line of argument would seem to be that it is possible to pack other two-dimensional shapes besides rectangles to fill the plane. Among regular figures (equal length sides, equal angles), there are just three shapes that tessellate in this way: squares, equilateral triangles and hexagons. Some architects have laid out plans very successfully on regular grids of triangles or hexagons. Figure 9 shows Frank Lloyd Wright's Sundt House project of 1941, and Figure 10 shows Bruce Goff's house for Joe Price of 1956. Use of a triangular organising grid does not force the designer into making all rooms simple triangles. The triangular units can be aggregated into parallelograms, trapezia and other shapes (including hexagons), as evidenced in the Wright and Goff plans [9 & 10]. The triangular grid, that is, offers a certain *flexibility of room shape* to the architect. Similarly, unit hexagons in a hexagonal grid can be joined together to make other more complex shapes.

When I started to think about this particular issue of flexibility in planning, I imagined that a regular square grid perhaps offered *more* possibilities for room shapes, and *more* possibilities for arranging those shapes, than did a triangular or a hexagonal grid – and that here might lie part of an explanation for the prevalence of rectangularity in buildings. The
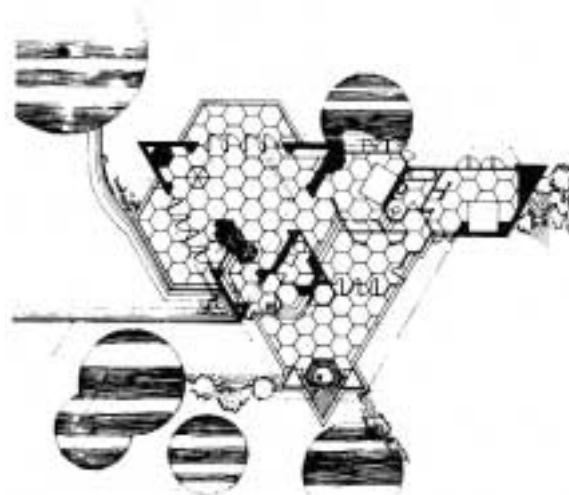

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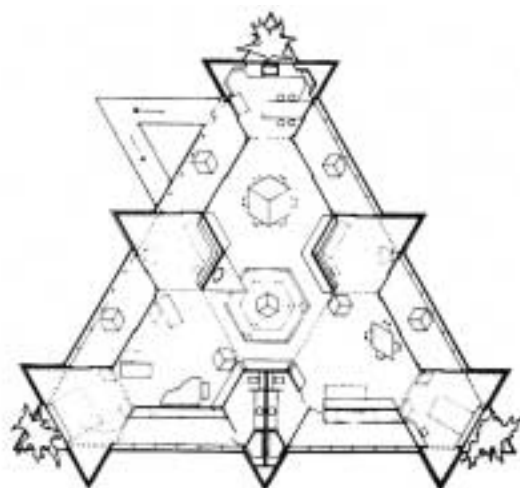
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following demonstration, though very far from providing any kind of mathematical proof, serves however to indicate that my intuition was wrong.

Consider three fragmentary square, triangular and hexagonal grids, each comprising nine grid cells [11]. We can imagine that the plans of simple buildings – perhaps small houses – are to be laid out on the grids with their walls following selected grid lines. How many distinct shapes for rooms can be made by joining adjacent grid cells together, in each case? Let us confine our attention, since we are thinking about rooms in buildings, to *convex* shapes, convexity being a characteristic of most small architectural spaces. Let us also count shapes that are geometrically similar, but are of different sizes (made up of different numbers of grid cells) as being distinct. Thus on the square grid we can make three different square shapes, with one cell, four cells or nine cells. Our criterion of convexity means that there is only *one* shape that can be made on the hexagonal grid: the unit hexagonal cell itself. All shapes made by aggregations of hexagonal cells are non-convex. On the square grid by contrast it is possible to make six distinct shapes [12]; but on the triangular grid it is possible to make 10 distinct shapes [13]. The triangular grid, contrary to what I had expected, offers a *greater* range of shapes for rooms than does the square grid.

11
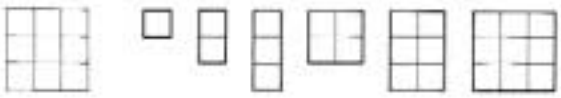
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9   Plan of Sundt house
    project by Frank
    Lloyd Wright, 1941,
    laid out on a
    triangular grid

10 Plan of house for Joe
    Price by Bruce Goff,
    Bartlesville,
   Oklahoma, 1956-,
    also laid out on a
    triangular grid

11 Fragments of grids,
    each comprised of
    nine cells whose

shapes are squares,
equilateral triangles,
and regular
hexagons

12 The six possible
    convex shapes made
    by aggregating unit
    cells in the square
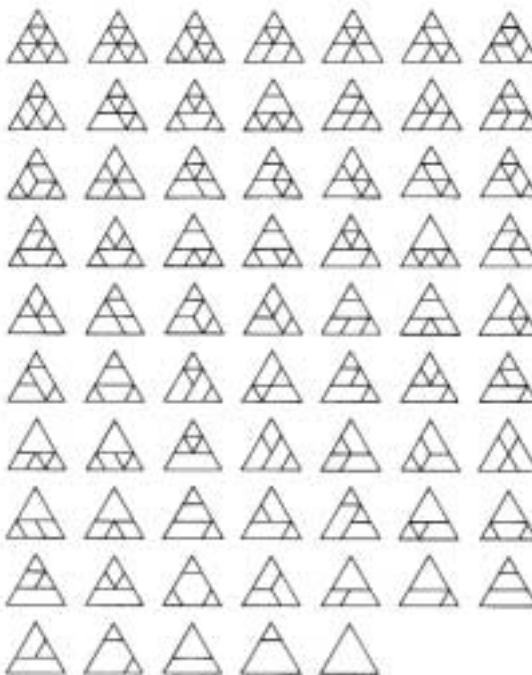   grid of Figure 11

13 The 10 possible
    convex shapes made
    by aggregating unit
    cells in the triangular
    grid of Figure 11

14

Perhaps these shapes made by aggregating triangular units, although more numerous, cannot be packed together without gaps in as many distinct *arrangements* as can the shapes made from unit squares? For square grids, this question of possible arrangements was intensively studied, in effect, by several authors, during the 1970s and '80s. The purpose of that work was more general, as I shall explain. But one incidental result was to show for a 3 x 3 square grid, how many different arrangements are possible, of rectangular and square shapes made up from different numbers of grid cells, packed to fill the grid completely. These possibilities, of which there are 53, are all illustrated *[14]*. (The enumeration is derived, with modifications and additions, from Bloch.)[5] No account is taken, in arriving at this total, of the particular orientation in which an arrangement is set on the page. That is to say, the same arrangement, simply rotated through 90° or 180° or 270°, is not regarded as 'different'. Certain arrangements can exist in distinct left-handed and right-handed versions. Just one representative is taken in each case. The count includes the two extreme cases in which the packing consists of nine unit squares, and of one single 3 x 3 square.

The enumeration of these square grid packings was made by computer. Essentially the same results were achieved by Mitchell, Steadman, Bloch,
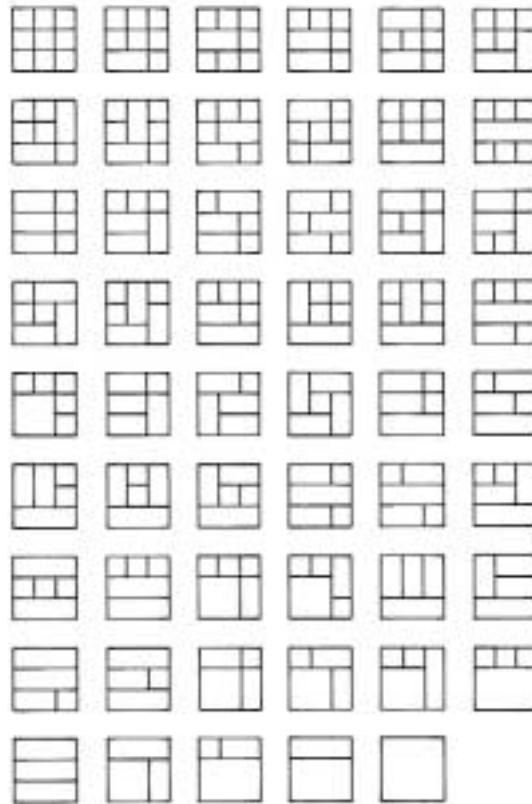
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14 All 53 possible
    arrangements in
    which combinations
    of the rectangular
    shapes shown in
    Figure 12 can be
    packed, without
    interstices, into the
    square grid of Figure
    11. Differences solely
   by rotation and
   reflection are ignored

15 All 68 possible
    arrangements in
    which combinations
    of the shapes shown
    in Figure 13 can be
    packed, without
    interstices, into the
    triangular grid of
    Figure 11. Differences
    solely by rotation
    and reflection are
    ignored

Krishnamurti, Earl and Flemming using several different algorithms.[6] I have carried out a similar exercise by hand for the grid of nine triangular cells, to count the number of arrangements in which combinations of the shapes illustrated in Figure 13 can be packed to fill this grid. Once again, differences by rotation and reflection are ignored. My results are illustrated [15]. The number of possibilities is 68. Here once again, and contrary to my expectation, the triangular grid offers rather *more* possible arrangements of shapes than does the square grid.

16 *a* A packing of rectangles within the square grid of Figure 11, set on an *x, y* coordinate system. The spacing of the grid lines is given by the dimensions $x_1, x_2, x_3, y_1, y_2, y_3$
b The same packing enlarged, by a similarity transformation
c The same packing stretched, by a shear transformation

d The same packing transformed by changing the spacing of the grid lines in different ratios

17 *a* A packing of shapes (compare Figure 13) within the triangular grid of Figure 11
b The same packing enlarged, by a similarity transformation
c The same packing stretched, by a shear transformation

d The same packing transformed by changing the spacing of the grid lines in different ratios
d The same packing transformed by changing the spacing of the grid lines in different ratios

18 Plan of the Jones House by Bruce Goff, Bartlesville, Oklahoma 1958, laid out as a semi-regular tessellation of octagons and squares

16a

16b

16c

16d

17a

17b

17c

17d

18
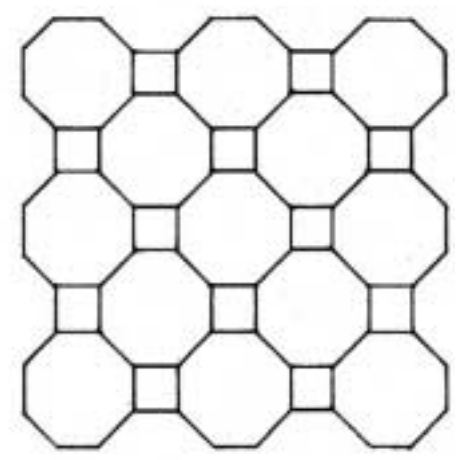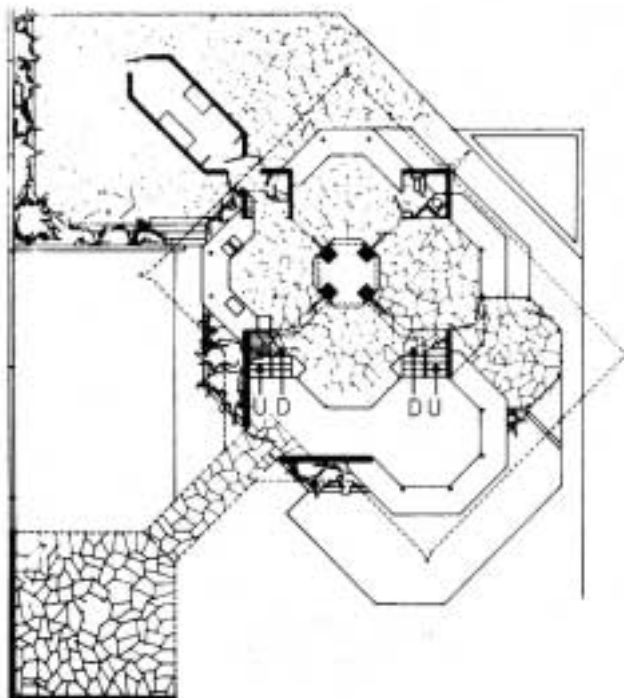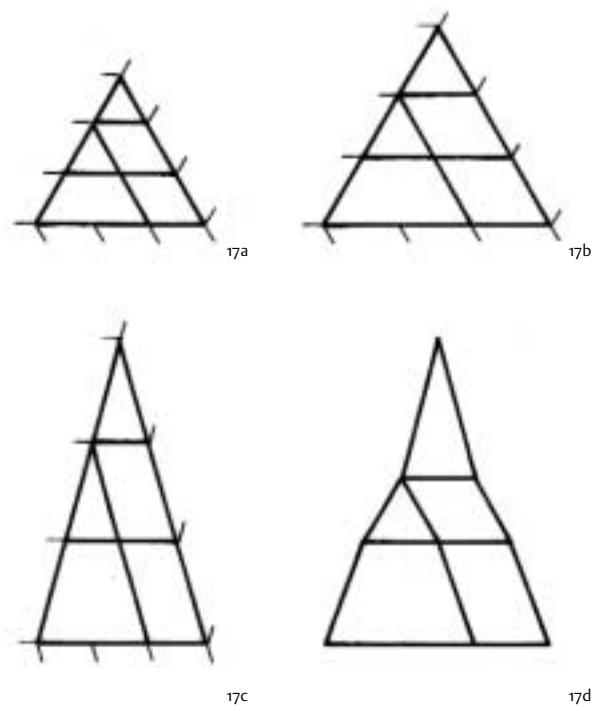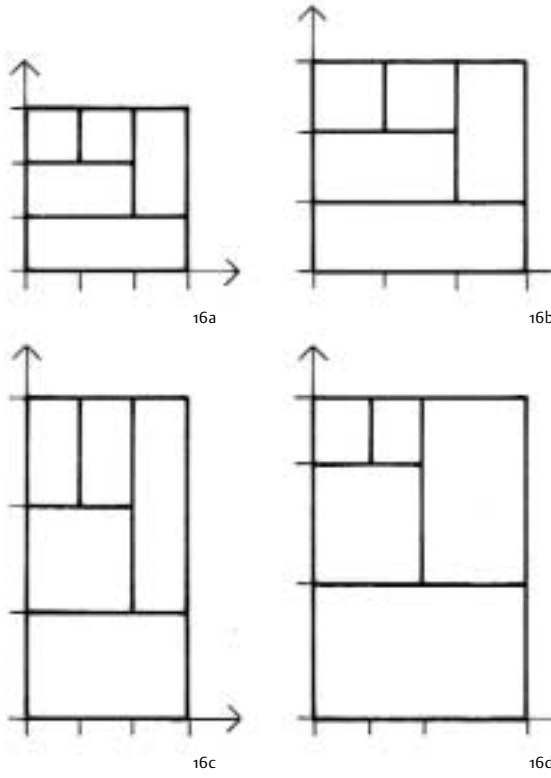
The flexibility of dimensioning offered by rectangular packings

This demonstration however ignores one very important aspect of flexibility in the possible packings of shapes to fill the plane. The wider aim of the computer work mentioned above was to enumerate possibilities for packing rectangles of *any dimensions whatsoever* – not just rectangular shapes made by aggregating square cells of some given unit size.

Consider the packing of shapes on the square grid in Figure 16a. Here the arrangement is set on a system of x, y coordinates, as shown. The spacing of the grid lines in x and y is given by a series of dimensions $x_1, x_2, x_3, y_1, y_2, y_3$. In the packing of unit squares as shown in the figure these dimensions are of course all equal [16a]. Suppose, however, that the x and y values are changed. If they are all multiplied, or divided, in the same ratio, then the packing as a whole will simply be enlarged or shrunk in size, but will remain otherwise unchanged [16b]. (In mathematical terms this is a *similarity transformation*.) If the x values are all multiplied in the same ratio, while the y values are all multiplied in a different ratio, the arrangement will be stretched or shrunk in one direction or the other [16c]. (This is a *shear transformation*.) Finally, individual x and y values can be altered in different ratios, so as to cause local shrinking or stretching of different component rectangles [16d]. All the shapes in the packing nevertheless remain rectangular throughout.

In this way any *dimensioned* version whatever of the basic arrangement of Figure 16a can in principle be generated. These versions are infinitely numerous, since any of the x and y grid dimensions can be altered by increments that can be as small as we wish. The same process of assigning dimensions can be carried out for all the different arrangements on 3 x 3 square grids in Figure 14; and indeed for all arrangements on 1 x 3, or 2 x 2, or 2 x 3, or 3 x 4 grids and so on. Such an approach makes it possible to generate absolutely any packing of rectangles, of whatever dimensions, within a simple rectangular boundary. This was the goal of the computer research referred to earlier (see note 6).

Now imagine a similar process of dimensioning being applied to the grid of nine equilateral triangles [17a] (or to the grid of nine unit hexagons). The entire pattern can be subjected to a similarity transformation, enlarging or reducing it in size, without difficulty [17b]. But any overall shear transformation results in changes in the internal angles of all the triangles [17c]. And any change in the spacing of some of the grid lines results in changes in the internal angles of the triangles between those lines [17d] – indeed some 'grid lines' now become bent, and the very idea of a grid ceases to apply. In giving different dimensions to the grid spacing in square grids, we generate shapes that are always rectangles. In giving different dimensions to the spacings of lines in grids of equilateral triangles, by contrast, we generate shapes that are always triangles, certainly, but they are no longer equilateral triangles. A similar argument applies to the hexagonal grid.

Here, I would propose, we are approaching the heart of the issue, the key reason for the superior flexibility of rectangular packing over other shapes that fill the plane. That flexibility lies in part in the variety of possibilities for *configurations* of rectangles, irrespective of their sizes; but much more, in the flexibility of assigning different *dimensions* to those configurations, *while preserving their rectangularity. Any rectangular packing can be dimensioned as desired in the general way illustrated in Figure 16* – in principle in an infinity of ways – and the component shapes will all still be rectangles.

Looking at this flexibility from another point of view: it is always possible to divide any rectangle within a packing into two rectangles, and to divide each of these rectangles into two further rectangles, and so on. In the context of plan layout, the designer can decide to turn one proposed rectangular room into two. More generally, he or she can squeeze groups of rectangular spaces together, or can pull them apart and slide others in between. In buildings once constructed, a new partition wall can divide any rectangular room into two rectangular parts. (It is equally possible to divide any triangle into two triangles – but those component triangles will have different internal angles from the first.)

I have confined discussion so far to packings of regular figures – squares, equilateral triangles, hexagons – and how these may be dimensioned. Beyond these, there are packings of other shapes, notably the semi-regular or Archimedean tessellations, which are made up of combinations of different regular figures. Some of these have served on occasion as the basis for architectural plans, as for example Bruce Goff's Jones House of 1958, whose rooms are alternate squares and octagons [18]. And there is an infinite variety of irregular shapes that can fill the plane, examples of which form the basis of many of M. C. Escher's irritating puzzle pictures. But the argument about inflexibility of dimensioning applies, I would suggest, with even greater force in all these cases. Goff's planning of the Jones House is ingenious. But he picked a geometrical straitjacket for himself, and it is not surprising that few others have followed his lead.

Close packing of components in buildings and other artefacts

Up to this point we have looked only at the problems of arranging rooms in architectural plans. But the solid structure of any building – unless it is wholly of mud or concrete – itself consists of packings of three-dimensional components at a smaller scale: bricks, beams, door and window frames, floorboards, floor tiles. Here too, rectangularity generally prevails, for the same geometrical reasons, I would suggest, as already outlined. (And even concrete needs formwork, often assembled from rectangular members.) Rectangular rooms can readily be formed out of rectangular components of construction. This rectangularity of building components was Bemis's concern: to what extent would it be possible to construct houses from rectangular pre-made parts that would be larger than standard bricks or

timbers, but could still be fitted together in many ways? In traditional brickwork, it is where walls meet at angles other than 90º that there is a need for differently shaped, hence more expensive 'specials'. Otherwise, so long as brick-based modular dimensions are generally adhered to, the standard brick will serve throughout. Pieces of timber in the form of parallelepipeds can always be cut into smaller paralellepipeds, and rectangular sheets of board into smaller rectangles. Building components must pack vertically as well as horizontally, so here is another reason besides gravity for the appearance of rectangularity in the vertical direction.

We find rectangularity in other types of artefact, for essentially the same reasons, I believe, of flexibility in the assembly or subdivision of parts. Woven cloth, with its weft and warp, is produced in rectangles because of the basic technology of the loom. Many types of traditional garment – shirts, ponchos, trousers, coats, kimonos – are then sewn from square or rectangular pieces cut from these larger sheets, so as not to waste any of the valuable and laboriously made cloth.[7] Such garments, when old, are picked apart and the pieces reused for other purposes. Figure 19 shows a nineteenth-century agricultural labourer's smock from Sussex, with the pattern of rectangular pieces from which it is assembled [19].

Paper too is manufactured in rectangles that can be cut in many ways without waste to create smaller sheets of different sizes. The rectangularity of the paper fits with the rectangularity of pages of printed type. In traditional printing methods each letter and space was represented by a separate rectangular slug of metal, the whole assemblage of letters – possibly of many different sizes – clamped together in a rectangular frame or 'chase' [20]. The art of newspaper and magazine layout lies in the fitting together of differently sized pictures, headlines and blocks of type, to fill each page. Much furniture is built of course from essentially rectangular wooden components, and the furniture's rectangularity allows it to fit in the corners of rectangular rooms. In the denser parts of cities, complete rectangular buildings are packed close together on sites that are themselves rectangular; and these sites pack to fill the complete area of city blocks. This was part of the reason for Krüger's interest in plan shapes.

A transition from round to rectangular in vernacular houses?

We have noticed how circularity in plan is often a characteristic of freestanding, widely spaced, single-room houses in pre-industrial societies. (The circular plan may derive in part from a system of construction where the roof is supported on a central pole, or forms a self-supporting cone or dome.) One might imagine that with increasing wealth there could be a change, at some point in time, from single-room to multi-room houses. Another possibility is that in circumstances where land was in short supply – as for example where it was necessary to confine many houses within a defensive perimeter – then those houses might have needed to be packed tightly together. In both cases the theory of rectangular packing proposed here might lead you to expect a transition in traditional houses from circular to rectangular plan shapes. Can you find actual evidence of such changes?

The American archaeologist Kent Flannery made a comparative analysis of the forms of villages and
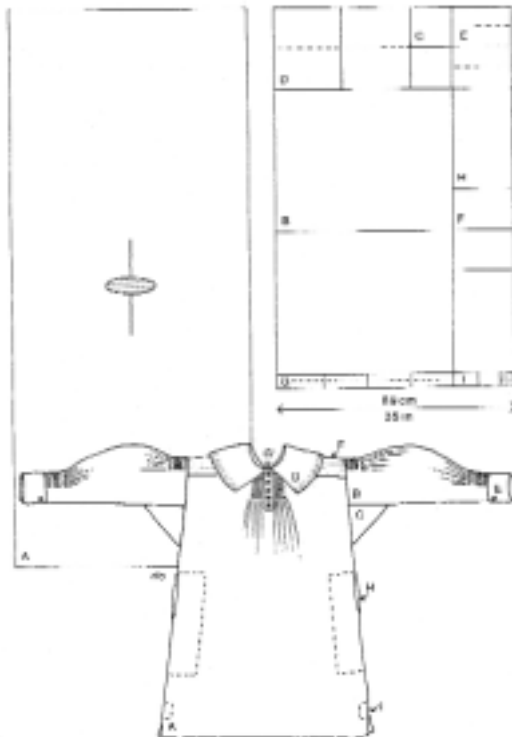
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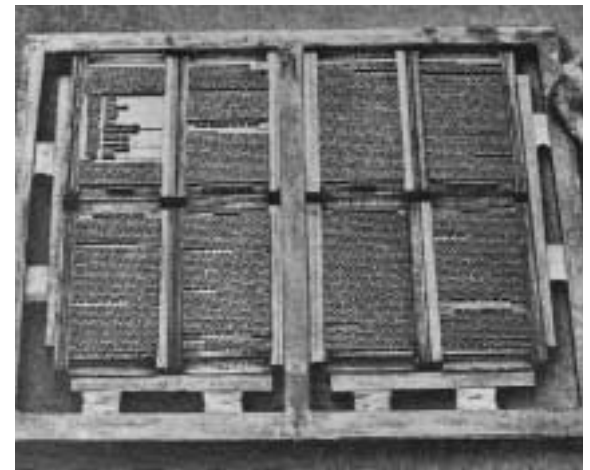
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19 Man's smock, probably from Sussex, 1860-80; and the pattern from which it was made. One rectangular piece of cloth serves for the body of the garment. All other pieces are also rectangular and are cut from a second rectangle of cloth without waste. (From D. K. Burnham, *Cut My Cote* (1973), Figure 12, p. 18)

20 Rectangular metal slugs carrying letters or acting as spacers, clamped together into a rectangular frame or 'chase', as used in traditional printing methods. (From P. Gaskell, *A New Introduction to Bibliography* (1974), Figure 43, reprinted by permission of Oxford University Press)

village houses in the period when permanent settlements first appeared after the end of the Pleistocene.[8] He looked at examples in the Near East, Africa, the Andes and Mesoamerica. He found two broad types of settlement: compounds consisting of small circular huts, and 'true villages' with larger rectangular houses. The round hut was characteristic of nomadic or semi-nomadic communities, and consisted of a single space, housing one or at most two people. The rectangular house typically had several rooms, accommodated an entire family, tended to be extended over time, and was found in fully sedentary communities. In some cases these rectangular houses were indeed concentrated together for the purposes of defence. In Flannery's own words:

> 'Rectangular structures replace circular ones through time in many archaeological areas around the world (although reversals of this trend occur).'[9]

Figure 21 shows this process in action. The photograph is reproduced from Bernard Rudofsky's *Architecture Without Architects*, and shows an aerial view of Logone-Birni in the Cameroun.[10] There are freestanding circular huts with roofs both inside and outside the walled compounds. But the contiguous roofed structures within these compounds – together with some roofless enclosures that are packed closely with those buildings – are for the most part rectangular [21].

Other examples can be seen in the vernacular architecture of Europe. The typical hut or *borie* of the Vaucluse region of France is a stone-built cylindrical one-room structure with a corbelled domical stone roof.[11] There are a few existing multi-room rectangular *bories*. In the example of Figure 22, the room layout seems to be in some sort of

transitional stage between a squashing together of circles, and an emerging rectangularity [22].[12] The *trullo* of Apulia has a stone structure similar to the *borie*. According to Fauzia Farneti the *trullo* was originally a temporary dwelling with a circular plan found in rural areas, and later evolved to have a rectangular exterior and a circular interior.[13] These rectangular *trulli*, in their final form, became the repeated structural units of multi-room houses.

A parting shot
Finally, why might we expect to find more departures from rectangularity in the work of 'high architects' than in the general run of more everyday buildings? Many contemporary architects, it seems to me, find the rectangular discipline imposed by the necessary constraints of the close packing of rooms – paradoxically, and despite its flexibility – to be an irksome prison; and they try to escape from it. They gravitate towards building types such as art museums and theatres, not just because these are prestigious and well-funded cultural projects with imaginative clients, but also because they can provide opportunities for spaces that are close to being 'single-room structures', which can be treated sculpturally. It is possible that architects might choose to adopt a non-orthogonal geometry in order, precisely, to set their work apart from the majority of the building stock. Rectangularity can be avoided on the external surfaces of buildings as we have seen: so there is much free play here for architectural articulation and elaboration of a non-orthogonal character. But this treatment comes at a cost, and in more utilitarian buildings it may be dispensed with.


21

21 Aerial view of Logone-Birni in the Cameroun (from B. Rudofsky, *Architecture Without Architects* (1964), plate 132, p. 131). Notice the freestanding circular huts, and the rectangular huts and rectangular unroofed enclosures packed within the compounds

22 Plan of a stone *borie* at Lacoste in the Vaucluse region of France, with seven spaces (from *Bories* (1994), p. 177). The plan appears to represent a transitional stage between the circular single-room *borie*, and a packing of several rectangular spaces
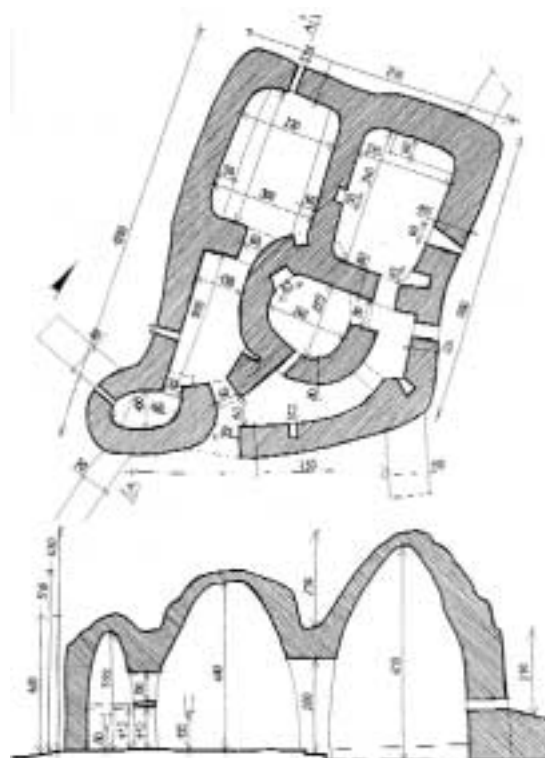
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Notes

1. Albert Farwell Bemis and John Burchard, *The Evolving House*, 3 vols., (Cambridge, MA: MIT, Technology Press, 1933-36).
2. *Ibid.*, vol. 3, Appendix A, pp. 303-316. Bemis extrapolated these data to the entire housing stock of the United States, based on the frequency of different house types, to give a figure of 88.5% rectangularity. He also measured the rectangularity of the structural components of a sample detached house with hipped roof, bow windows and dormers. He found that 97% of those components by number were rectangular.
3. Mario J. T. Krüger, 'An approach to built-form connectivity at an urban scale: system description and its representation', *Environment and Planning B*, vol. 6 (1979), 67-88.
4. This proposal comes from Joe Rooney (personal communication).
5. Cecil J. Bloch, *A Formal Catalogue of Small Rectangular Plans: Generation, Enumeration and Classification* (PhD thesis, University of Cambridge School of Architecture, 1979). Bloch enumerated packings of rectangles on 'minimal gratings' – that is to say where all grid lines coincide with the edges of rectangles. The number of distinct packings on 3 x 3 minimal gratings given by Bloch is 33. To these we must add – for the present enumeration – packings that correspond to 2 x 2 and 2 x 3 minimal gratings, suitably dimensioned to fill the 3 x 3 grid. Bloch did not count packings in which the number of rectangles equals the number of cells in the grating. These (on 1 x 1, 1 x 2, 1 x 3, 2 x 3 and 3 x 3 gratings) are also included here, appropriately dimensioned to the 3 x 3 grid in each case.
6. William J. Mitchell, J. Philip Steadman and Robin S. Liggett, 'Synthesis and optimisation of small rectangular floor plans', *Environment and Planning B*, vol. 3 (1976), 37-70; Christopher F. Earl, 'A note on the generation of rectangular dissections', in *Environment and Planning B*, vol. 4 (1977), 241-246; Ramesh Krishnamurti and Peter H. O'N. Roe, 'Algorithmic aspects of plan generation and enumeration', *Environment and Planning B*, vol. 5 (1978), 157-177; Cecil J. Bloch and Ramesh Krishnamurti, 'The counting of rectangular dissections', *Environment and Planning B*, vol. 5 (1978), 207-214; Ulrich Flemming, 'Wall representations of rectangular dissections and their use in automated space allocation', *Environment and Planning B*, vol. 5 (1978), 215-232; Cecil J. Bloch, 'An algorithm for the exhaustive enumeration of rectangular dissections', *Transactions of the Martin Centre for Architectural and Urban Studies*, vol. 3 (1978), 5-34. See also the general discussion in J. Philip Steadman, *Architectural Morphology* (London: Pion, 1979).
7. Dorothy K. Burnham, *Cut My Cote* (Toronto, Canada: Textile Department, Royal Ontario Museum, 1973).
8. Kent V. Flannery, 'The origins of the village as a settlement type in Mesoamerica and the Near East: a comparative study', in *Man, Settlement and Urbanism*, ed. by P. J. Ucko, R. Tringham and G. W. Dimbleby (London: Duckworth, 1972), pp. 23-53.
9. *Ibid.*, pp. 29-30.
10. Bernard Rudofsky, *Architecture Without Architects* (New York: Museum of Modern Art, 1964. Reprinted, London: Academy Editions, 1973).
11. Pierre Desaulle, *Les Bories de Vaucluse: Région de Bonnieux* (Paris: A. et J. Picard, 1965).
12. I am grateful to Bill Hillier for drawing my attention to *bories*, and for taking me to see the very building of Figure 22. It should be said however that a minority of freestanding single-room *bories* have rectangular plans; so this interpretation could be open to challenge.
13. *Encyclopedia of Vernacular Architecture of the World*, ed. by Paul Oliver (Cambridge: Cambridge University Press, 3 vols., 1997), p. 1568.

Illustration credits

Acknowledgements

Biography

Philip Steadman is Professor of Urban and Built Form Studies at the Bartlett School, University College London. He trained as an architect at Cambridge University and taught previously at Cambridge and the Open University. He has published two books on the forms of buildings: *The Geometry of Environment* (with Lionel March, 1971) and *Architectural Morphology* (1983). He is working on a new book on *Building Types and Built Forms* (2020 approx).

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