

## Measurement of the $\tau$ Lepton Mass and an Upper Limit on the Mass Difference between $\tau^+$ and $\tau^-$

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The mass of the  $\tau$  lepton has been measured in the decay mode  $\tau \rightarrow 3\pi\nu_\tau$  using a pseudomass technique. The result obtained from  $414 \text{ fb}^{-1}$  of data collected with the Belle detector is  $M_\tau = [1776.61 \pm 0.13(\text{stat}) \pm 0.35(\text{sys})] \text{ MeV}/c^2$ . The upper limit on the relative mass difference between positive and negative  $\tau$  leptons is  $|M_{\tau^+} - M_{\tau^-}|/M_\tau < 2.8 \times 10^{-4}$  at 90% confidence level.

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Masses of quarks and leptons are fundamental parameters of the Standard Model (SM). High precision measurements of the mass, lifetime, and the leptonic branching fractions of the  $\tau$  lepton can be used to test the lepton universality hypothesis embedded in the SM. The present PDG value of the  $\tau$  mass [1] is dominated by the result of the BES Collaboration [2] and has an accuracy of about  $0.3 \text{ MeV}/c^2$ . The same level of accuracy in  $\tau$  mass measurement was recently reported by the KEDR Collaboration [3]. The data collected by the Belle experiment allow a measurement with similar accuracy to the BES and KEDR experiments but with different systematic uncertainties; the latter experiments analyze the cross section for  $\tau$  pair production near threshold while Belle measures the four-momenta of the visible  $\tau$  decay products at a center-of-mass (c.m.) energy of  $\sqrt{s} = 10.58 \text{ GeV}$ . Eventually, by combining these high precision measurements, we will significantly improve the accuracy of the  $\tau$  mass determination.

Separate measurements of the masses of the  $\tau^+$  and  $\tau^-$  leptons in Belle allow us to test the CPT theorem, which demands their equality. As CPT is a fundamental assumption of quantum field theory, which is used to calculate all fundamental processes, a discovery of a violation in CPT would be revolutionary, and therefore all tests of it are extremely important. A similar test was previously performed by OPAL at LEP [4] with the result  $(M_{\tau^+} - M_{\tau^-})/M_\tau < 3.0 \times 10^{-3}$  at 90% C.L.

To measure the  $\tau$  mass, we use a pseudomass technique that was first employed by the ARGUS collaboration [5]. This technique relies on the reconstruction of the invariant

mass and energy of the hadronic system in hadronic  $\tau$  decays. We use the  $e^+e^- \rightarrow \tau^+\tau^-$  event candidates with following  $\tau \rightarrow 3\pi\nu$  decay. The analyzed variable is

$$M_{\min} = \sqrt{M_X^2 + 2(E_{\text{beam}} - E_X)(E_X - P_X)}, \quad (1)$$

which is less than or equal to the  $\tau$  lepton mass. Here,  $M_X$ ,  $E_X$ , and  $P_X$  are the invariant mass, energy, and absolute value of the momentum, respectively, of the hadronic system in the  $e^+e^-$  c.m. frame, and  $E_{\text{beam}}$  is the energy of the electron (or positron) in this frame. In the absence of initial and final state radiation and assuming a perfect measurement of the four-momentum of the hadronic system, the distribution of  $M_{\min}$  extends up to and has a sharp edge at  $M_\tau$ . Initial (ISR) and final (FSR) state radiation as well as the finite momentum resolution of the detector smear this edge. We can use the edge position from a fit to the experimental  $M_{\min}$  distribution as an estimator of the  $\tau$  mass, since the background processes in the selected  $\tau^+\tau^-$  sample have a featureless  $M_{\min}$  distribution near  $M_\tau$ .

The analysis presented here is based on  $414 \text{ fb}^{-1}$  of data taken at the  $Y(4S)$  resonance ( $\sqrt{s} = 10.58 \text{ GeV}$ ) with the Belle detector at the KEKB asymmetric-energy  $e^+e^-$  collider [6]. A detailed description of the Belle detector is given elsewhere [7]. We mention here only the detector components essential for the present analysis.

Charged tracks are reconstructed from hit information in a central drift chamber (CDC) located in a 1.5 T solenoidal magnetic field. The  $z$  axis of the detector and the direction of the magnetic field are aligned antiparallel to the positron beam. Track trajectory coordinates of the charged particles

near the collision point are provided by a silicon vertex detector. Photon detection and energy measurement are performed with a CsI(Tl) electromagnetic calorimeter (ECL). Identification of charged particles is based on the information from the time-of-flight counters (TOF) and silica aerogel Cherenkov counters (ACC). The ACC provides good separation between kaons and pions or muons at momenta above 1.2 GeV/c. The TOF system consists of a barrel of 128 plastic scintillation counters, and is effective in  $K/\pi$  separation mainly for tracks with momentum below 1.2 GeV/c. The lower energy tracks are also identified using specific ionization ( $dE/dx$ ) measurements in the CDC. Electrons are identified by combining information from the ECL, ACC, TOF, and CDC [8]. The magnet return yoke is instrumented to form the  $K_L$  and muon detector, which detects muon tracks [9] and provides trigger signals. The responses from these detectors determine the likelihood  $L_i$  of particle type  $i \in \{e, \mu, \pi, K, p\}$ . A charged particle is identified as an electron if the corresponding likelihood ratio [8],  $P_e > 0.9$  or if the electron mass hypothesis has the highest probability. The electron efficiency for  $P_e > 0.9$  is approximately 90% for a single electron embedded into a hadronic event. Charged particles are identified as muons if the corresponding muon likelihood ratio [9]  $P_\mu > 0.8$ . The muon detection efficiency for this measurement is approximately 91%. The corresponding likelihood ratio cut for kaons and protons is 0.8. The kaon and proton identification efficiencies are about 80%. All charged tracks that are not identified as an electron, muon, kaon, or proton are treated as pions. The  $K_S^0$  candidates are formed from pairs of charged tracks intersecting in a secondary vertex more than 0.3 cm from the beam spot in the plane transverse to the beam axis; the  $\chi^2/Ndf$  of the vertex fit is required to be less than 11/1, and the  $\pi^+\pi^-$  invariant mass of the candidate is required to be  $0.48 \text{ GeV}/c^2 < M_{\pi^+\pi^-} < 0.52 \text{ GeV}/c^2$ . The  $\pi^0$  candidates are formed from pairs of photons, each with energy greater than 0.1 GeV, that satisfy the condition  $0.115 \text{ GeV}/c^2 < M_{\gamma\gamma} < 0.152 \text{ GeV}/c^2$ . The chosen invariant mass cuts correspond to about 95% of the signal acceptance.

We select events that have one  $\tau$  lepton decaying leptonically into  $l\bar{\nu}_l\nu_\tau$  and the other into three charged pions and a neutrino. For the entire event, we require three charged pions and one lepton (either muon or electron) with net charge equal to zero. The number of charged kaons, protons,  $K_S^0$  mesons, and  $\pi^0$  mesons should be equal to zero. After applying all selection criteria,  $5.8 \times 10^6$  events remain.

The  $M_{\min}$  distribution for the  $\tau \rightarrow 3\pi\nu_\tau$  data is shown in Fig. 1. A fit was performed to these data with the empirical edge function

$$F(x) = (P_3 + P_4x) \arctan[(x - P_1)/P_2] + P_5 + P_6x, \quad (2)$$

where  $P_i$  are parameters of the fit. The fit range 1.72–1.80 GeV/ $c^2$  is chosen. The value of the uncorrected  $\tau$  mass estimator,  $P_1$ , obtained from the fit is  $P_1 = 1777.77 \pm 0.13 \text{ MeV}/c^2$ .

To obtain the value of the  $\tau$  mass from the  $\tau$  mass estimator  $P_1$ , we use three Monte Carlo samples of  $\tau^+\tau^-$  events where one  $\tau$  decays leptonically and the other one decays into three charged pions and neutrino in a complete and fully modeled response of the Belle detector. The KORALB generator [10] is used for the Monte Carlo  $e^+e^- \rightarrow \tau^+\tau^-$  event production. The input  $\tau$  masses of the above Monte Carlo samples are equal to 1777.0 MeV/ $c^2$ , 1776.0 MeV/ $c^2$  and 1776.8 MeV/ $c^2$ . The statistics of each sample is approximately equal to that of the data. The differences between the fitted estimator  $P_1$  and the input  $\tau$  mass for these samples are  $\Delta_1 = (1.27 \pm 0.12) \text{ MeV}/c^2$ ,  $\Delta_2 = (1.29 \pm 0.05) \text{ MeV}/c^2$ , and  $\Delta_3 = (1.06 \pm 0.04) \text{ MeV}/c^2$  for the first, second, and third sample, respectively. To convert the  $\tau$  mass estimator  $P_1$  to  $M_\tau$ , we use the weighted mean and dispersion of  $\Delta_1$ ,  $\Delta_2$ , and  $\Delta_3$  to obtain the estimator correction  $\bar{\Delta} = (1.16 \pm 0.14) \text{ MeV}/c^2$ .

The subtraction of this value from the edge position parameter  $P_1$  in data gives  $M_\tau = 1776.61 \pm 0.13(\text{stat}) \pm 0.14(\text{MC}) \text{ MeV}/c^2$ , where MC means the error due to limited Monte Carlo statistics.

To study the systematic uncertainty due to the choice of the edge parameterization, we use the following alternate functions:

$$F_1(x) = (P_3 + P_4x) \frac{x - P_1}{\sqrt{P_2 + (x - P_1)^2}} + P_5 + P_6x, \quad (3)$$

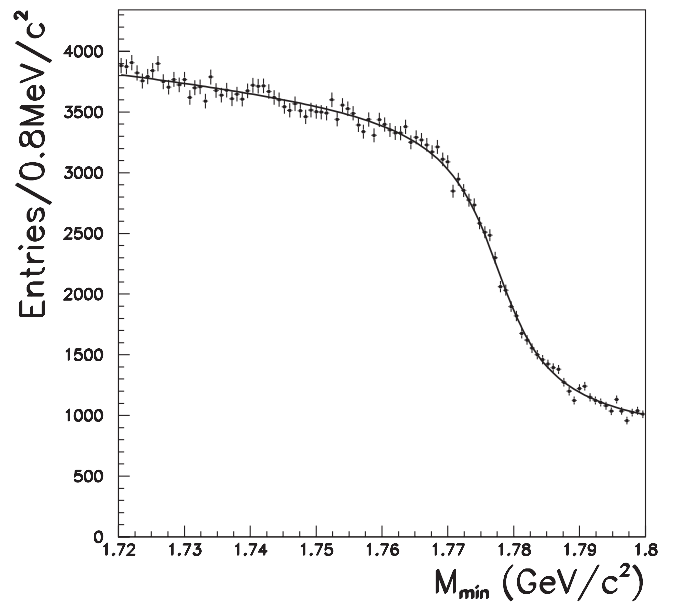


FIG. 1. The pseudomass distribution  $M_{\min}$  for the  $\tau^\pm \rightarrow 3\pi^\pm\nu_\tau$  candidates. The points with error bars are data and the solid line is the result of the fit with the function (2).

$$F_2(x) = (P_3 + P_4x) \frac{-1}{1 + \exp[(x - P_1)/P_2]} + P_5 + P_6x \quad (4)$$

for the fit to the  $M_{\min}$  distribution. Here,  $P_i$  are the parameters of the fit.

The above procedure for  $\tau$  mass extraction is repeated successively for the data and MC samples with each of these functions. The extracted values for the  $\tau$  mass obtained with the functions (3) and (4) are  $[1776.85 \pm 0.13(\text{stat}) \pm 0.12(\text{MC})] \text{ MeV}/c^2$  and  $[1776.52 \pm 0.12(\text{stat}) \pm 0.10(\text{MC})] \text{ MeV}/c^2$ , respectively.

We take for the measured value of the  $\tau$  mass the one obtained using function (2):

$$M_\tau = M_1 = 1776.61 \pm 0.13(\text{stat}) \text{ MeV}/c^2. \quad (5)$$

The square root of the variance of the obtained  $\tau$  masses,  $0.18 \text{ MeV}/c^2$  is taken as systematic uncertainty due to the choice of the edge parameterization. This value exceeds the error of  $0.14 \text{ MeV}/c^2$ , which is assigned as a systematic uncertainty due to limited Monte Carlo statistics. The use of a different fit range gives a much smaller shift in  $\tau$  mass of  $0.04 \text{ MeV}/c^2$ , which we include in the systematic uncertainty.

In this analysis, we use the beam energy calibrated using the beam-energy constrained mass of fully reconstructed  $B$  decays on a run-by-run basis. We estimate the uncertainty of the uncorrected beam energy to be less than  $1.5 \text{ MeV}$ , which includes the uncertainty of  $B$  mass, tracking system calibration, and the effect of the  $Y(4S)$  width. Using the Monte Carlo samples, we find that this uncertainty propagates to a systematic uncertainty on the  $\tau$  mass of  $0.26 \text{ MeV}/c^2$  under the assumption that the value  $1.5 \text{ MeV}$  is fully due to uncertainty of the beam-energy calibration.

As a crosscheck of the result obtained from the fully reconstructed  $B$  decays, we analyze the distribution of the variable  $\Delta_{\text{ME}} = [M(\mu^+\mu^-) - 2E_{\text{beam}}]^\circ\text{C}$  for  $e^+e^- \rightarrow \mu^+\mu^-$  data events, where  $E_{\text{beam}}$  is provided by KEKB without  $B$  mass correction. If a systematic shift exists in the beam energy or tracking system calibration, we would expect some shift of the maximum of this distribution from zero. We fit the  $\Delta_{\text{ME}}$  distribution to a sum of two Gaussians with the same central value multiplied by a cubic polynomial to take into account the peak asymmetry due to ISR. To check the consistency of this fitting procedure, we apply it to Monte Carlo  $e^+e^- \rightarrow \mu^+\mu^-$  events with ISR and FSR (KKMC generator) [11] that pass through the full Belle simulation and reconstruction procedures. The  $\Delta_{\text{ME}}$  distributions for data and Monte Carlo calculations are shown in Fig. 2 together with results of the fit. The reduced goodness-of-fit values  $\chi^2/Ndf$  are 0.9 and 1.06 for the data and Monte Carlo, respectively, where  $Ndf = 51$  is the number of degrees of freedom.

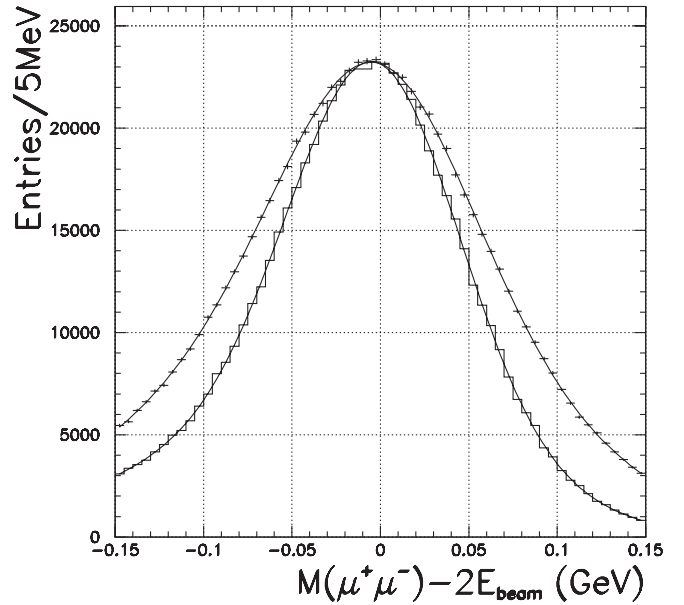


FIG. 2. The  $[M(\mu^+\mu^-) - 2E_{\text{beam}}]$  distributions for the data (points with errors) and Monte Carlo calculations (histogram without errors). The curves show the results of the fit to the data and Monte Carlo calculations by the sum of two Gaussians multiplied by a cubic polynomial.

While the resolution of the  $\Delta_{\text{ME}}$  variable is not well described by Monte Carlo simulations, the peak position coincides for data and simulation. To estimate the systematics due to the difference in momentum resolution between data and Monte Carlo calculations, we included additional smearing of the track momenta in the Monte Carlo samples by a Gaussian with  $\sigma = 1.02 \times 10^{-3} p^2$  (the units of  $\sigma$  and  $p$  are  $\text{GeV}/c$ ). The consistency in  $\Delta_{\text{ME}}$  between data and Monte Carlo becomes much better after this procedure. The shift in the edge position parameter  $P_1$  is negligible (less than  $0.02 \text{ MeV}/c^2$ ) and is included in total systematics.

The difference between data and Monte Carlo peak positions obtained from the fit is  $\delta\Delta_{\text{ME}} = 3 \pm 2 \text{ MeV}$ . This difference comes from the imperfect calibration of both the beam energy and tracking system. We analyze two extreme cases when the shift  $\delta\Delta_{\text{ME}}$  is due to the imperfect calibration of either (1) the beam energy or (2) the tracking system.

For the first case, we have  $\Delta E_{\text{beam}} = \delta\Delta_{\text{ME}}/2 = 1.5 \text{ MeV}$ , which is consistent with accuracy of the beam-energy calibration obtained from the reconstruction of the exclusive  $B$  decays. To estimate the shift of the  $\tau$  mass for the second case, we construct the  $M_{\min}$  Monte Carlo distributions for an input  $\tau$  mass equal to  $1777.0 \text{ MeV}/c^2$  for unmodified pion momenta and for momenta shifted by  $\Delta p/p = \pm 3/10580 = 2.8 \times 10^{-4}$ . We obtain a mass shift in the range  $0.10\text{--}0.15 \text{ MeV}/c^2$ , which is smaller than the shift observed when  $\delta\Delta_{\text{ME}}$  includes the full

beam-energy uncertainty ( $0.26 \text{ MeV}/c^2$ ). We take this conservative assumption and assume a systematic uncertainty due to the combined imperfections of the beam energy and tracking system calibration of  $0.26 \text{ MeV}/c^2$ .

To estimate the systematic uncertainty due to the model dependence of the spectrum of the  $3\pi$  system in the  $\tau$  decay, we vary the mass and width of the  $a_1(1260)$  meson in the range  $\pm 300 \text{ MeV}/c^2$  from the nominal PDG values. We find the shift in the edge position due to this variation to be negligible (less than  $0.02 \text{ MeV}/c^2$ ).

Systematic uncertainties from misidentified  $\tau$  decay products and from non- $\tau^+\tau^-$  events are negligible (less than  $0.01 \text{ MeV}/c^2$ ), since their  $M_{\min}$  distributions show no significant structure in the region of the  $\tau$  mass. We got this estimation by fitting Monte Carlo  $M_{\min}$  distribution with different background contaminations. We also checked that at the generator level, KKMC and KORALB give the same edge shift of the  $M_{\min}$  distribution due to ISR and FSR. The difference in the shifts is smaller than  $0.01 \text{ MeV}$ .

The list of the analyzed sources of systematics is given in Table I. The final result is  $M_\tau = [1776.61 \pm 0.13(\text{stat}) \pm 0.35(\text{syst})] \text{ MeV}/c^2$ . In the analysis, we assume that the neutrino mass is equal to zero. According to MC calculations, a change in the neutrino mass from zero to  $10 \text{ MeV}/c^2$  leads to a shift in the edge position of the pseudomass distribution by  $-0.1 \text{ MeV}/c^2$ .

The pseudomass method allows a separate measurement of the masses of the positively and negatively charged  $\tau$  leptons. A mass difference between positive and negative  $\tau$  leptons would result in a difference in the energy between the  $\tau$ 's produced in the  $e^+e^-$  collision. This in principle makes the assumption  $E_\tau = E_{\text{beam}}$  invalid. The  $M_{\min}$  distributions for positive and negative  $\tau$ 's decaying into  $3\pi\nu_\tau$  are shown in Fig. 3 together with the results of the fit. Good agreement is found between the distributions for  $\tau^+$  and  $\tau^-$ . The mass difference obtained from independent fits to these distributions is  $M_{\tau^+} - M_{\tau^-} = (0.05 \pm 0.23) \text{ MeV}/c^2$ .

Most sources of systematic uncertainty on  $\tau$  mass affect positive and negative  $\tau$  leptons equally so that their contributions to the mass difference (and its uncertainty) cancel. One exception is different interactions of particles and antiparticles in the detector material. For example, the

numbers of positive and negative triplets for the selected pions are not equal to each other. However, the description of this difference by the MC calculations is reasonably accurate. In the data, the ratio of the number of negative to the number of positive triplets is 1.034 while in MC calculations, this ratio is equal to 1.031. To estimate a systematic shift in the mass difference between  $\tau^+$  and  $\tau^-$ , we compare the peak positions of  $\Lambda_c \rightarrow pK^-\pi^+$  and  $\bar{\Lambda}_c \rightarrow \bar{p}K^+\pi^-$ ,  $D^+ \rightarrow \phi(1020)\pi^+$  and  $D^- \rightarrow \phi(1020)\pi^-$ ,  $D_s^+ \rightarrow \phi(1020)\pi^+$ , and  $D_s^- \rightarrow \phi(1020)\pi^-$ . The average relative mass shift from the decay modes listed above is approximately  $0.8 \times 10^{-4}$ . Assuming the CPT is valid, this value is used as the systematic uncertainty in the relative mass difference between  $\tau^+$  and  $\tau^-$  and corresponds to a systematic uncertainty in the mass difference of  $0.14 \text{ MeV}/c^2$ .

Adding the statistical and systematic errors in quadrature, we obtain  $M_{\tau^+} - M_{\tau^-} = (0.05 \pm 0.27) \text{ MeV}/c^2$ . This result can be expressed as an upper limit on the relative mass difference [12]  $|M_{\tau^+} - M_{\tau^-}|/M_\tau < 2.8 \times 10^{-4}$  at 90% C.L. Good agreement of the  $M_{\min}$  distributions for positive and negative  $\tau \rightarrow 3\pi\nu_\tau$  decays shows that CPT invariance is respected at the present level of experimental accuracy.

To summarize, we have measured the mass of the  $\tau$  lepton from the pseudomass distribution of  $\tau$  decays into three charged pions and a neutrino. The result is

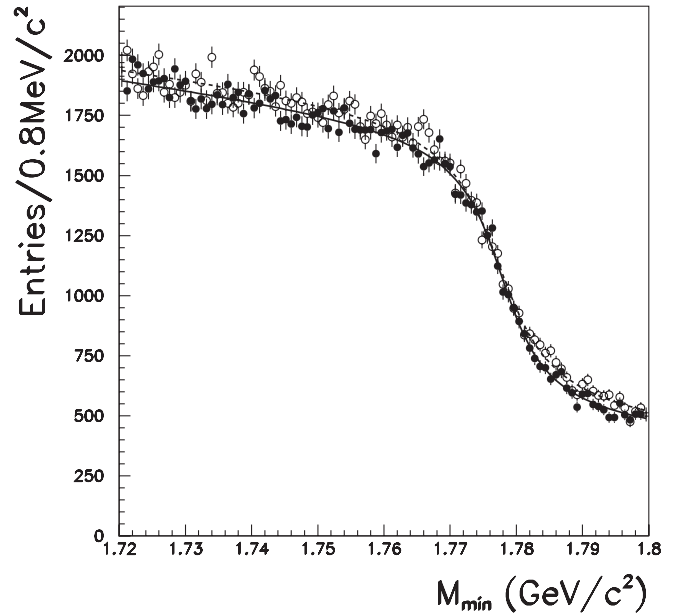


FIG. 3. The distribution of the pseudomass  $M_{\min}$  for the decay  $\tau \rightarrow 3\pi^\pm\nu_\tau$ , shown separately for positively and negatively charged  $\tau$  decays. The solid points with error bars correspond to  $\tau^+$  decays, while the open points with error bars are  $\tau^-$  decays. The solid curve is the result of the fit to the pseudomass distribution of  $\tau^+$  with function (2) while the dashed one is for the  $\tau^-$ .

TABLE I. Summary of systematic uncertainties.

| Source of systematics                    | $\sigma$ , $\text{MeV}/c^2$ |
|--|-----------------------------|
| Beam energy and tracking system          | 0.26                        |
| Edge parameterization                    | 0.18                        |
| Limited MC statistics                    | 0.14                        |
| Fit range                                | 0.04                        |
| Momentum resolution                      | 0.02                        |
| Model of $\tau \rightarrow 3\pi\nu_\tau$ | 0.02                        |
| Background                               | 0.01                        |
| Total                                    | 0.35                        |

$$M_{\tau} = [1776.61 \pm 0.13(\text{stat}) \pm 0.35(\text{sys})] \text{ MeV}/c^2,$$

in good agreement with the current world average and of comparable accuracy. Independent measurements of the positive and negative  $\tau$  mass are obtained to test CPT symmetry. The measured values are consistent and an upper limit on the relative mass difference is

$$|M_{\tau^+} - M_{\tau^-}|/M_{\tau} < 2.8 \times 10^{-4} \quad \text{at 90\%C.L.},$$

1 order of magnitude better than the previous limit from OPAL.

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