# Assessment and Improvement of the Capabilities of a Window Correlator to Model GPS Multipath Phase Errors

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**The potential of output from a window correlator to mitigate GPS phase multipath is reviewed and assessed based on the analysis of data collected in controlled multipath environments under both static and kinematic conditions. Previous findings that the method is suboptimal for reflectors leading to additional path lengths of less than about 7 m are confirmed, and methods for combining this output with two other multipath indicators: time series of signal-to-noise ratios (SNRs) and estimates of code multipath from dual frequency code and phase combinations, are investigated. A new method to combine all three indicators has been found and its application is shown to improve the quality of GPS static phase data by between 10% and 20% depending on the length of the additional path travelled by the reflected signal. The method can be applied completely automatically as it uses just the three multipath indicators; no knowledge of the surrounding environment is required. The paper concludes with some suggested practical applications.**

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## I. INTRODUCTION

The dominant error source in the use of phase GPS for almost all kinematic applications, especially in civil engineering and robotics, is multipath. Multipath occurs when a direct signal from a GPS satellite is mixed with one that has been reflected from one or more surfaces and can, in theory, cause errors of up to a quarter of a carrier signal wavelength in a measured range (i.e., 47 mm on L1). It can occur at both a reference antenna and at a roving antenna, making studies of GPS phase multipath in both static and kinematic modes important. Typically, multipath induces errors of up to 10 mm and 20 mm, respectively, in horizontal and vertical kinematic positioning (although in some highly reflective environments these numbers can be larger, especially if the satellite geometry is poor) and it can seriously limit the use of GPS in some applications. For instance GPS is not sufficiently accurate to control pavement laying equipment and its accuracy as an attitude measuring system is still severely limited when short baselines are used. Driving down multipath errors is probably the single-most important objective of current research into the use of GPS for very high accuracy applications.

This paper is primarily concerned with the use of output from a window correlator to address the problem but it is well known that such output is not optimal when the additional path length of a reflected signal is smaller than about 7 m. In an attempt to address this problem the paper considers a number of ways of combining window correlator output with two other indicators of multipath; namely time series of signal-to-noise ratios (SNRs) and estimates of code multipath from dual frequency code and phase combinations. All three of these indicators have been used individually in the past as tools for multipath mitigation. In some cases they have been used to estimate the amounts of multipath error, leading to corrections that can be applied directly to the measurements, and in other cases they have been used simply to indicate the presence of multipath leading to the down-weighting of the affected measurements in the position solution. Here we attempt the former, i.e., to estimate the amount of phase multipath so that a correction can be applied.

The strengths and weaknesses of the methods currently associated with the three indicators are examined, especially their dependence on the additional length of the path travelled by the reflected signal. This is done both from a theoretical standpoint and from one based on data collected in controlled multipath environments (one very strong reflector placed at different distances from a GPS antenna) under both static and kinematic conditions. This aim of the investigation is to discover a way of combining the three indicators in an automatic way in order to estimate phase corrections and so improve solutions for position and attitude irrespective of the length of the additional path travelled by the reflected signal and without specific knowledge of the reflecting environment.

## II. PHASE MULTIPATH MITIGATION BY REFERENCE WAVEFORM CORRELATORS

### A. Review of the Technique

By the end of the 1990s, most GPS manufacturers had developed further the primary "narrow" code correlator [1] and implemented "reference waveform," also called "gated," correlators in their geodetic receivers [2–4].

A similar extension of the correlation techniques was introduced also for phase under the name of the "phase multipath mitigation window" (PMMW) by Leica Geosystems (LGS) in a world patent in 1996 [5]. At more or less the same time, Ashtech developed its "enhanced strobe" correlator [6]. Both parallel developments were based on very similar signal processing. This is summarized in the following three steps.

First, initialization and phase tracking with standard phase correlators take place. During this stage, the received GPS signal is multiplied by in-phase (I product) and in-quadrature (Q product) generated replicas in the different channels of the application specific integrated circuit (ASIC). The integration and dump of the I and Q outputs of the correlators are performed using all the signal samples. The sampling frequency is typically of about 40 MHz in the ASIC of geodetic receivers.

Second, the multipath phase tracking error is estimated with multipath mitigation (MM) phase correlators. To do this estimation, only very specific samples are integrated and dumped. These are located immediately after the direct signal code arrives, as shown in Fig. 1, whether there are code transitions or not. The polarity of the samples is that of the instantaneous code. So, in terms of polarity, the MM phase correlator is similar as the standard correlator. In terms of signal content, however, it uses only 1 sample out of 40 in the case of a 40 MHz sampling frequency. The multipath immunity of this correlator (for reflected signals that are sufficiently delayed) results from the fact that the samples are taken before the reflected signal code arrives.

Third, the outputs of the MM phase correlators are used to correct those of the standard correlators. The solution adopted in LGS's implementation, for example, is to keep the phase tracking process unchanged by integrating I and Q in the standard loop at the usual periods of, respectively, 20 ms and

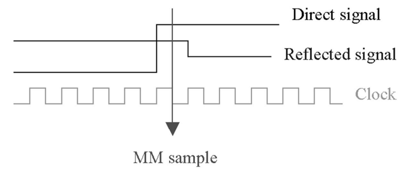


Fig. 1. Illustration of sampling strategy used in MM phase correlators.

5 ms. The MM phase correlator operates in parallel and outputs $I_{MM}$ and $Q_{MM}$ with its original sampling of 1 per 40 clock samples, but with a much larger integration period, typically 1 s, in order to improve the SNR. Of course this leads to time correlation in the output values of $I_{MM}$ and $Q_{MM}$, which might be noticeable in highly dynamic applications.

### B. Limitation of the Technique

The MM phase correlator works provided that the multipath signals are sufficiently delayed with respect to the direct signals, for the measurement samples to be taken before they arrive. This means that the sooner a sample of the received signal is taken after a code arrival, the better.

Hence, the precorrelation bandwidth of the received signal that enters the phase loop is of great importance. For most geodetic GPS receivers, this bandwidth is around 25 MHz, which means that a code transition will last for some 40 ns. Consequently, an equivalent delay must occur before taking a sample in the received signal. With a clock rate of 40 MHz, the first sample comes after a delay of 25 ns by which time the code transition is not quite complete. It is, however, considered to be just sufficient with regard to the bandwidth of the signal.

40 MHz (which corresponds to an additional path length of approximately 7.5 m) is therefore a good trade-off between the duration of the code transition and the capability of the correlator to mitigate multipath. Increasing this rate would enable multipath with shorter delays (i.e., from closer reflectors) to be mitigated but would lead to an increase in noise due to use of a less complete code transition.

In order to verify this theoretical limitation of the MM technique, a set of experiments has been designed in both static and kinematic modes. This is reported in the next section.

### C. Tests Program and Results

Experiments have been carried out within the framework of a cooperative program of research undertaken by Department of Geomatic Engineering at University College London, the Laboratoire Central des Ponts et Chaussées (LCPC) (with its positioning systems test bed: Sessyl) and Leica Geosystems.

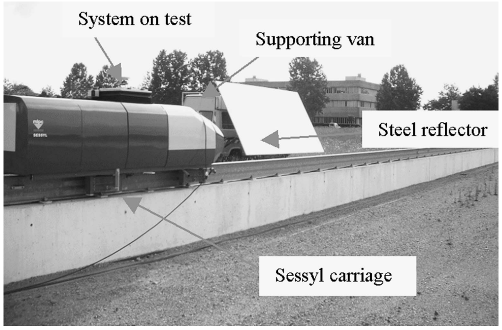

Fig. 2. SESSYL carriage near multipath panel.

During this campaign, static and kinematic tests were conducted. Leica System 500 receivers, logging at 1 Hz, with AT 502 lightweight antennas were used.

For the static tests, both base and rover antennas were set on tripods in the grass field surrounding the Sessyl test bed, for full 24 h periods. The baseline length was less than 100 m and had been surveyed previously so that it was known with an accuracy of the order of a millimetre.

The kinematic tests were carried out using the LCPC positioning systems test bed Sessyl. Sessyl consists of a closed-loop track (overall dimensions 81 m $\times$ 16 m), composed of a metal rail fixed upon a concrete wall, with a mobile carriage running automatically on it. The rover antenna was installed on the carriage. This provided a kinematic reference trajectory with an accuracy of the order of a millimetre. For these experiments, only the first straight of the track was used and runs were made every 20 min over an approximate 15 h period, at 0.05 m/s.

For the purpose of generating multipath in a controlled manner, an experimental set-up has been designed that includes a large metallic reflector (a 5 m by 2.5 m steel panel, visible in Fig. 2). This can be used to generate multipath with different characteristics depending on the distance that it is placed from an antenna. In the kinematic experiments, the time spent in front of the panel, which was placed along the track in the middle of the straight was around 100 s.

The reflector was placed during consecutive days at 1, 4, and 7 m from the antenna, on the antenna's north side (so that the reflector was facing south), in both the static and kinematic tests, and the reflector was tilted appropriately in order to avoid creation of multipath from low elevation satellites and diffraction at the horizon, while also avoiding the creation of multipath from satellites with an elevation greater than 70° (as these are used as differencing satellites in the double differencing process used for the MM technique assessment). An additional control day without the reflector was included in both the static and kinematic tests series.

TABLE I
L1 Phase DD Statistics in Static Mode (Rover on Tripod)

| Static Tests | (Size of Population) | $\sigma$ | $\sigma_{PMMW}$ | Gain |
|---|---|---|---|---|
| Reflector 1 m<br>0.7 m < addpath < 2.0 m | (~ 130000 data*) | 8.5 mm | 8.2 mm | 4% |
| No reflector | | 3.7 mm | 3.5 mm | 5% |
| Reflector 4 m<br>5.2 m < addpath < 7.3 m | (~ 68000 data) | 5.4 mm | 4.3 mm | 20% |
| No reflector | | 3.1 mm | 3.0 mm | 4% |
| Reflector 7 m<br>9.7 m < addpath < 11.9 m | (~ 22000 data) | 4.8 mm | 2.7 mm | 44% |
| No reflector | | 2.6 mm | 2.5 mm | 3% |

(*Elevation of satellites > 15°.)

TABLE II
L1 Phase DD Statistics in Kinematic Mode (Rover on Sessyl)

| Kinematic Tests | (Size of Population) | $\sigma$ | $\sigma_{PMMW}$ | Gain |
|---|---|---|---|---|
| Reflector 1 m<br>0.7 m < addpath < 2.0 m | (~ 5000 data*) | 9.2 mm | 8.7 mm | 6% |
| No reflector | | 5.1 mm | 4.9 mm | 3% |
| Reflector 4 m<br>3.0 m < addpath < 7.6 m | (~ 3500 data) | 5.9 mm | 5.0 mm | 14% |
| No reflector | | 3.0 mm | 2.8 mm | 7% |
| Reflector 7 m<br>5.1 m < addpath < 11.9 m | (~ 2000 data) | 4.7 mm | 3.3 mm | 29% |
| No reflector | | 2.5 mm | 2.4 mm | 4% |

(*Elevation of satellites > 15°.)

The results are summarised in Tables I and II, respectively for static and kinematic tests. Note that the accuracy gains in Tables I and II have been computed from $(\sigma - \sigma_{PMMW})/\sigma$, where $\sigma$ is the standard deviation of the observed-computed (O-C) double differences (DDs) of phase L1, referred to here as "*standard*" L1 phase; and $\sigma_{PMMW}$ is the standard deviation of the O-C DDs of the PMMW "*corrected*" L1 phase.

The O-C DDs of phase were computed using the known precise static baseline (for static tests) and the Sessyl reference (for kinematic tests). Example time series of O-C DDs of "*standard*" L1 phase for the static tests are shown in Fig. 4 for SV1.

Note that the size of the population (i.e., the number of (O-C) DDs) corresponds to the cumulated duration of every multipath time window for every satellite being tracked. The same time windows are used to compute statistics with and without the reflector. The reason for the level of the phase error at 1 m being significantly higher than that at 4 or 7 m is the presence of low-elevation satellites in the multipath time windows.

As predicted by the theory of the MM phase correlator, only multipath from reflected signals with

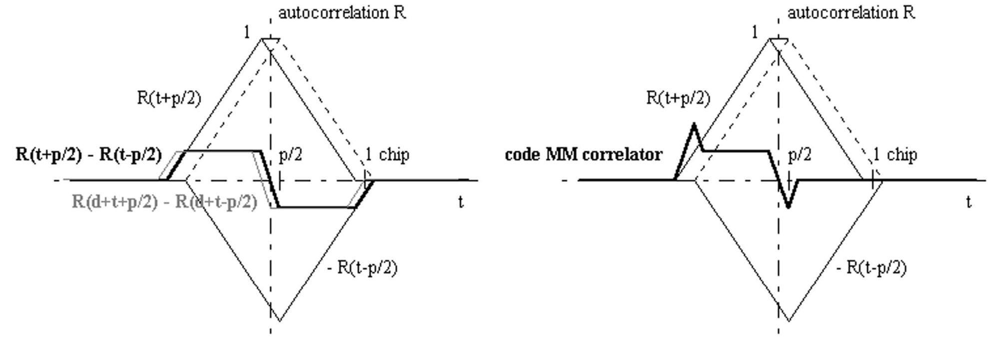


Fig. 3. Discrimination functions of commonly implemented narrow and MM code correlators.

an additional path length over 7.5 m are significantly mitigated in LGS's current implementation, while multipath from reflected signals with shorter path lengths are mitigated to a lesser extent (an improvement of 44% for a 7 m reflector compared with only 4% for a 1 m reflector).

Another important conclusion comes from the statistics when there is no reflector. It is noticeable that there is an improvement of a few % even in this case. This can be explained by the ability of the correlator to mitigate even weak multipath that exists in the general environment (i.e., even without the reflector).

Table II displays the results for the kinematic tests. The same trend as for static tests is observed, although the MM phase correlator is slightly less efficient, e.g. an improvement of 29% with the reflector at 7 m compared with 44% in the static case.

## III. MULTIPATH FUNCTIONAL THEORY

This section and specifically (1), (3), and (5) for phase, code, and SNR, makes the assumption that the reflections follow the law of the geometric optics, something which is classically accepted in the ray tracing model in multipath theory [7, 8, or 9].

### A. Calculation of the Phase Multipath Errors

The phase tracking loop that is implemented in the Leica receivers is still based on a standard correlator (the MM phase correlator operates in parallel).

In the presence of multipath, the phase error is given by (1) in the ray tracing model:

$$\Phi = a\tan((\Sigma_i \alpha_i \alpha'_i \sin\Theta_i)/(1+\Sigma_i \alpha_i \alpha'_i \cos\Theta_i)) \quad (1)$$

for a number $n$ of reflected signals (from $i = 1$ to $i = n$), where

$\alpha'_i = R(d_i + \tau)/R(\tau)$;

$R$ is the code autocorrelation function;

$d_i$ is the code delay of the $i$th reflected signal;

$\Theta_i$ is the phase delay of the $i$th reflected signal;

$\tau$ is the multipath code error;

$\alpha_i$ is the ratio of amplitude between $i$th reflected signal and the direct signal.

Equation (1) can be further simplified, given that, in most code tracking loops, the multipath code error $\tau$ is bounded, as are the code delays $d_i$ of the reflected signals that cause this code error. So, in MM code corrrelators $d_i$ and $\tau$ are small with respect to the chip length. Consequently, $\alpha'_i \sim 1$. Moreover, the hypothesis that $\alpha_i$ is small permits (1) to be further simplified to

$$\Phi \sim \Sigma_i \alpha_i \sin\Theta_i. \quad (2)$$

This last assumption is reasonable even with metallic reflectors since the gain patters of most antennas attenuate significantly reflected signals whose polarity is inversed.

### B. Calculation of the Code Multipath Errors

As far as the code measurements are concerned, the theory rests on the code tracking fundamental equation: DF = 0, where DF is the discrimination function implemented in the code correlator. The DFs generally used in receivers (including that used in most MM code correlators) have a similar central linear part (see Fig. 3), whatever the chip spacing $p$. This is given by (3):

$$\begin{aligned} \mathrm{DF} &= \cos\Phi[R(\tau+p/2) - R(\tau-p/2)] \\ &\quad + \Sigma_i \alpha_i \cos(\Phi-\Theta_i)[R(d_i+\tau+p/2) - R(d_i+\tau-p/2)] \\ &= 0. \end{aligned} \quad (3)$$

The hypotheses that the delays $d_i$ keep in the central linear part of the DF in a steady state of the receiver, and also that the ratio of amplitude is small, lead to a simplified equation for the multipath code error as follows:

$$\tau \sim \Sigma_i \alpha_i \cos\Theta_i * d_i. \quad (4)$$

### C. Impact of Multipath on Signal-to-Noise Ratio

Multipath also has effects on the SNR. Equation (5) describes the power of the received signal, composed of one direct component and one or several reflected components:

$$P = P_{\text{direct}} * R^2(\tau) * (1 + 2\Sigma_i \alpha_i \alpha_i' \cos\Theta_i) \qquad (5)$$

for a number $n$ of reflected signals (from $i = 1$ to $i = n$), where $P_{\text{direct}}$ is the power of the direct signal, which is usually given in dB: $P^{\text{dB}} = 10\log(P)$.

Considering again that $\alpha_i' \sim 1$ (which means that the multipath code error is small) and also making the hypothesis that $2\Sigma_i \alpha_i \cos\Theta_i \ll 1$, i.e., that the ratio of amplitude is small, leads to the approximation of the logarithm by its first order Taylor's expansion.

Practically the receiver outputs an estimation of the signal-to-noise ratio, denoted SNR (or C/N0, after normalizing by the bandwidth of the tracking loop). Hence, SNR (or C/N0) data do not address directly the power of the received signal. However, the above equation remains valid for the SNR, since $P^{\text{dB}}$ and the SNR differ only by an additive term corresponding to the power of the noise.

The variation of the power of the received signal (as well as that of SNR or C/N0) around its nominal (direct) value in dB is given by

$$\text{C/N0}_{\text{multipath}} \sim K\Sigma_i \alpha_i \cos\Theta_i \qquad (6)$$

where $K$ is a constant, independent of the index $i$ of the reflected signals. $K$ is known a priori and may differ from one receiver model to another, because the manufacturers do not exactly implement the same formula to output SNR (or C/N0).

For a GPS user, the practical derivation of $\text{C/N0}_{\text{multipath}}$ from C/N0 data requires that a calibration of the SNR be carried out in a multipath free environment, from which a template function (SNR versus satellite elevation) can be derived. Note that in order to simplify the notation, the next sections of this paper do not mention C/N0: only the term SNR is used.

## IV. FUNCTIONAL MODELLING AND ITS APPLICATION TO PHASE MULTIPATH MITIGATION

### A. Multipath Observables

There are a number of items of information that can be used to compute a correction for the multipath phase error. These are referred to here as multipath observables and three of these are summarised as follows:

1) the variation of the SNR, with respect to a satellite elevation template function;

2) the multipath code error, obtained by computing the variation around the average value of the ionospheric L1 and L2 combination [10]:

$$C1 - (1 + 2/((f1/f2)^2 - 1)\Phi1 + (2/((f1/f2)^2 - 1)\Phi2 \qquad (7)$$

where $\Phi1$ and $\Phi2$ are the phase measurements on L1 and L2 and f1 and f2 the nominal carrier frequencies;

3) the PMMW correction, a direct estimate of the phase error $\Phi$, that deteriorates when the multipath source is close.

Fig. 4 illustrates these observables. A snapshot for SV1 in each of the three consecutive days (panel distances of 1, 4, and 7 m) of the static tests is displayed. Signal-to-noise ratio (black, with 1 dB resolution), code error (clear grey, in m), PMMW (black, in cm) and additionally, the O-C DD of L1 phase (dark grey, in cm) are superimposed with their own units. The quadrature between the multipath phase error on the one hand, and the code error and SNR on the other, is clearly visible, except for the test corresponding to the reflector at only 1 m. These observables are here displayed for a specific satellite, but the patterns are similar for the others.

### B. Application of the Functional Modelling

The objective of the investigations reported in this section is to overcome the key limitation of the PMMW technique. The authors seek to obtain the same near 50% improvement, irrespective of the distance between the rover antenna and the reflector. In order to do this, they propose an algorithm that is based on multipath error modelling and reconstruction.

The SNR-based multipath phase error correction was first introduced by [8]. This method is based on the fact that the SNR varies harmonically (see (6)) around its nominal value in the presence of multipath. Several harmonics mix if multipath is due to several reflectors.

An identification of the amplitudes (denoted $\hat{A}_i$) and the arguments (denoted $h_i$) of multiple nonstationary sine waves embedded in a signal is possible by combining two classical algorithms of signal processing (first, an adaptive notch filter (ANF) for frequency and amplitude identification [11], and second, an adaptive least squares (ALS) for amplitude and argument identification [12]). The outputs ($\hat{A}_i$ and $h_i$) are such that:

$$S = \Sigma_i \hat{A}_{i\backslash S} \sinh_{i\backslash S} \qquad (8)$$

where $S$ is the input signal. Note that "$\backslash S$" denotes the dependency of the outputs ($\hat{A}_i$ and $h_i$) on the input signal.

This is applied to the variation of the SNR:

$$\text{SNR}_{\text{multipath}} = \Sigma_i \hat{A}_{i\backslash\text{SNR}} \sinh_{i\backslash\text{SNR}}. \qquad (9)$$

This identification provides a way of building a phase correction. Actually, the multipath phase error given in (2) shows the same amplitudes ($\alpha_i$) and

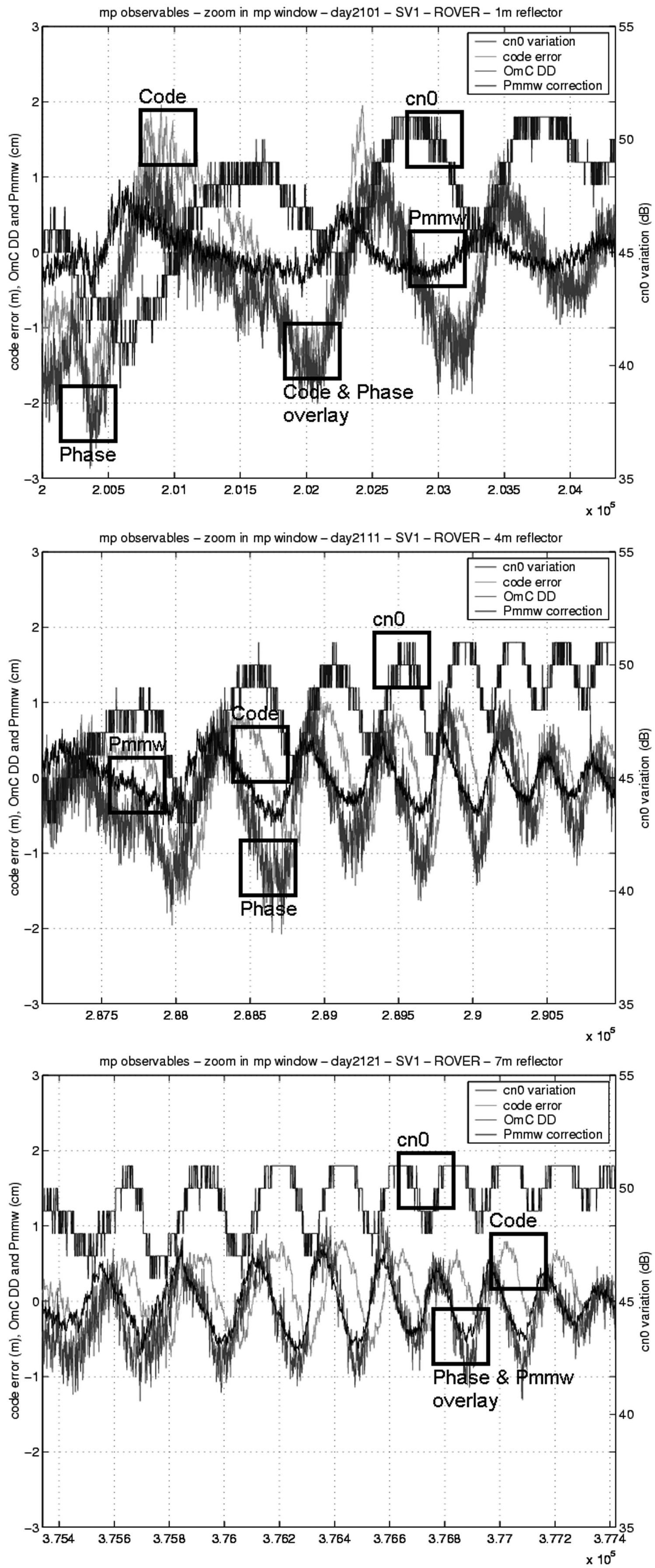


Fig. 4. Multipath observables corresponding to different distances of panel.

arguments ($\Theta_i$) as the multipath variation of the SNR in (6).

Hence, $\hat{A}_{i\backslash \mathrm{SNR}}$ and $h_{i\backslash \mathrm{SNR}}$ identified from the variation of the SNR can be introduced in (2) to compute the phase correction, changing sine into cosine (because of the quadrature between SNR and the multipath phase error), and dividing the amplitude by $K$:

$$\Phi = 1/K \Sigma_i \pm \hat{A}_{i\backslash \mathrm{SNR}} \cos(h_{i\backslash \mathrm{SNR}}). \tag{10}$$

However, the identification from the variation of the SNR does not give the sign of the arguments, since SNR only enables the recovery of the sine of these arguments and not their cosine. This sign ambiguity has a physical interpretation. The $i$th reflected signal travels an additional path length, to which the delay $d_i$ is proportional. While this additional path length varies, the phase shift $\Theta_i$ of the reflected signal (relative to the direct) rotates. The sense of the rotation depends on the additional path length increases or decreases.

A possible method for the estimation of the combination of the sign of the arguments, as suggested by [8], consists of estimating the set of signs that best fit the DD residuals. In real-time processing, such an estimation requires the analysis of residuals in a moving window, with a size that needs to be ascertained by tuning. This process induces a time shift that will impact on the ability of the method to be truly real-time. Also, if the environment changes rapidly, incorrect signs may result due to the fact that all residuals absorb part of the multipath error, especially if there are only a few satellites (and hence limited redundancy), which is often the case in typical multipath environments where visibility may be limited.

## V. MIXED MULTIPATH PHASE ERROR RECONSTRUCTION PROCESSES

The investigation reported in this section is based on the fact that the PMMW provides an estimate of the multipath phase error $\Phi$, including its sign. The principle of this new way to determine the sign of the phase correction was introduced in [13] and it is reproduced in Fig. 5.

The idea consists of mixing in a reconstruction process of the multipath phase error both its amplitude (given by the SNR) and its argument (given by the PMMW estimate of the phase error). Actually, the PMMW estimate is not completely null when the reflector is close by the antenna. It just gets attenuated, which still enables the identification of the argument.

Hence, the phase correction has, by nature, the same sign as the PMMW estimate, while its amplitude remains proportional to that of the SNR variation. This is mathematically given by (11):

**SNR-only based process**
ANF/ALS( SNR ) → amplitude & argument but ambiguous!

**SNR and PMMW based process**
ANF/ALS( SNR ) → amplitude (& argument unused)
ANF/ALS( PMMW ) → (amplitude) & argument <u>not</u> ambiguous!

amplitude (from SNR) ↘
mixed reconstruction process
argument (from PMMW) ↗

Fig. 5. SNR-only and SNR and PMMW based processes.

**SNR and multipath code error based process**
ANF/ALS( SNR ) → amplitude (& argument unused)
ANF/ALS( code error ) → (amplitude) & argument <u>not</u> ambiguous,
for very close reflector!

amplitude (from SNR) ↘
mixed reconstruction process
argument (from code error) ↗

Fig. 6. SNR and code error based processes.

$$\Phi = 1/K \Sigma_i \hat{A}_{i\backslash \mathrm{CN0}} \sinh_{i\backslash \mathrm{PMMW}} \tag{11}$$

where $h_{i\backslash \mathrm{PMMW}}$ comes from the ANF/ALS identification (see (12)) applied on the PMMW estimate:

$$\mathrm{PMMW} = \Sigma_i \hat{A}_{i\backslash \mathrm{PMMW}} \sinh_{i\backslash \mathrm{PMMW}}. \tag{12}$$

Another key observation concerns the multipath code error $\tau$. Whereas the use of the amplitude of this multipath observable makes the problem more intricate (because it is dependent not only on the multipath ratios of amplitude $\alpha_i$ but also on the code delays $d_i$), it seems that its argument shows a remarkable property that is unexpected from the theory: the argument of the multipath phase error progressively shifts as the additional path length (or code delay) of the reflected signal diminishes.

This is particularly true for the test at 1 m and it is visible on the upper Fig. 4. The code error, that was in phase with the SNR variation and in quadrature with the phase error for the tests at 4 and 7 m, shifts to become in quadrature with the SNR variation and consequently in phase with the phase error at around 1.5 m additional path. This is observed for all satellites. Further tests with different receivers might indicate whether or not this is due to details of LGS's implementation (e.g. sampling strategy or precorrelation bandwidth), and if it is only applicable to a certain class of multipath mitigation techniques.

There appears to be great potential in the use of this observation to compute multipath phase corrections for short additional path lengths. This is because code and phase errors are in phase and the drawback of the PMMW based identification at very short additional distances might be able to be solved using the observed code error directly.

Hence another algorithm can be designed (see Fig. 6).

The SNR and code error mixed process is mathematically given by (13):

$$\Phi = 1/K \Sigma_i \hat{A}_{i\backslash \mathrm{CN0}} \sinh_{i\backslash \mathrm{code\ error}} \tag{13}$$

where $h_{i\backslash \mathrm{code\ error}}$ comes from the ANF/ALS identification (see (14)) applied to the multipath code error:

$$\tau = \Sigma_i \hat{A}_{i\backslash \mathrm{code\ error}} \sinh_{i\backslash \mathrm{code\ error}}. \tag{14}$$

Both algorithms were first tested on the SV1 static data sets already used to illustrate the different multipath observables in Fig. 4. Fig. 7 displays the standard deviation of the corrected O-C phase DD versus the gain ($1/K$) applied in the phase error reconstruction processes. It is important to notice that a process can be deemed to be successful if the gain that corresponds to the minimum standard deviation does not depend on the distance to the reflector. The "black (+)," "clear grey (*)" and "dark grey (#)" lines refer, respectively, to the reconstruction processes with SNR only, with the PMMW estimate + SNR and lastly with the code error + SNR.

With the panel at 7 m, the best results are obtained by applying the PMMW correction directly (the corresponding standard deviation is figured by an horizontal black line). However, at 4 m the SNR and PMMW mixed process is more or less equivalent to the PMMW direct correction, even slightly better since the effectiveness of the PMMW starts to deteriorate. Moreover, the mixed process gives a more efficient or at least equal correction to the classical SNR based process, except in the case of the very close reflector (1 m) where the PMMW estimation has deteriorated. On the other hand, at 1 m the SNR based process remains potentially the most efficient (provided the sign ambiguity is solved).

None of the mixed reconstruction processes alone, however, appear to cope with a variable distance to the experimental panel. This contrasts with the SNR-only based process but that process remains ambiguous from a sign perspective. The next step therefore attempts to take advantage of both the PMMW and the code error in a new adaptive algorithm.

## VI. PROPOSITION OF A MULTIPATH ADAPTIVE ALGORITHM

Initial efforts at combining SNR with either the output from the PMMW correlator or the code data suggest that SNR is always useful to determine the amplitude of the multipath phase correction. However, the argument of the correction needs to be computed from either the PMMW estimation or the multipath code error, depending on the distance to the reflector. So, it appears that the two strategies are both potentially rather efficient, but for two different situations in terms of distance to the reflector. This section develops the fusion of these reconstruction processes.

An intuitive and basic idea of a possible fusion is presented here and represents a first step in designing an adaptive algorithm. The idea is that the reconstructed phase errors could be mixed with respect to the difference in phase of the multipath code error and the SNR variation as follows (note that the PMMW correction is not directly used here):

1) if they are in phase, use only the phase correction computed from the SNR variation;
2) if they are in quadrature, use only the phase correction computed from the code error;
3) between these two opposite situations, balance with a weighting of the two reconstructed phase errors in the final mixing fixed linearly with respect to the phase difference between code error and SNR variation. The following ratio is introduced

$$\gamma_i = |h_{i\backslash \mathrm{SNR}} - h_{i\backslash \mathrm{code\ error}}|/(\pi/2) \tag{15}$$

and the final reconstructed phase error results from the following combination:

$$\Phi = (1-\gamma_i)/K \Sigma_i \hat{A}_{i\backslash \mathrm{SNR}} \sinh_{i\backslash \mathrm{PMMW}} + \gamma_i/K \Sigma_i \hat{A}_{i\backslash \mathrm{SNR}} \sinh_{i\backslash \mathrm{code\ error}}. \tag{16}$$

Note that the PMMW, although it is unused in ratio $\gamma_i$, is still necessary to get the sign of the correction unambiguously when the reflector is sufficiently distant. This algorithm is tested on the data sets for SV1 previously used in this paper. The results of the gain tuning are given in Fig. 8.

From visual inspection on Fig. 8, it actually seems that this algorithm, at least on the selected satellite, gives significantly improved results when the reflector is at 1 m compared with those obtained by a PMMW direct correction. Moreover, the optimal gain tuning is stable (around unity in the case of receiver used). So, this algorithm has the advantage, contrary to the preceding ones, of being able to adapt to the close reflector situation, while remaining quite efficient when the reflector gets further away (4 and 7 m during the present tests). Note that at 4 m, the results are still the best with this algorithm, but at 7 m, the PMMW direct correction becomes the most efficient.

However, it should be noted that, despite the optimal gain tuning remains more or less stable, the sharpness of the parabolic curve reduces as the distance to the reflector increases, which is consistent with the main limitation of the modelling.

The practical application of this algorithm has been investigated by applying it to the entire data sets collected during the campaign of tests. In practice of course there is not usually an a priori way to

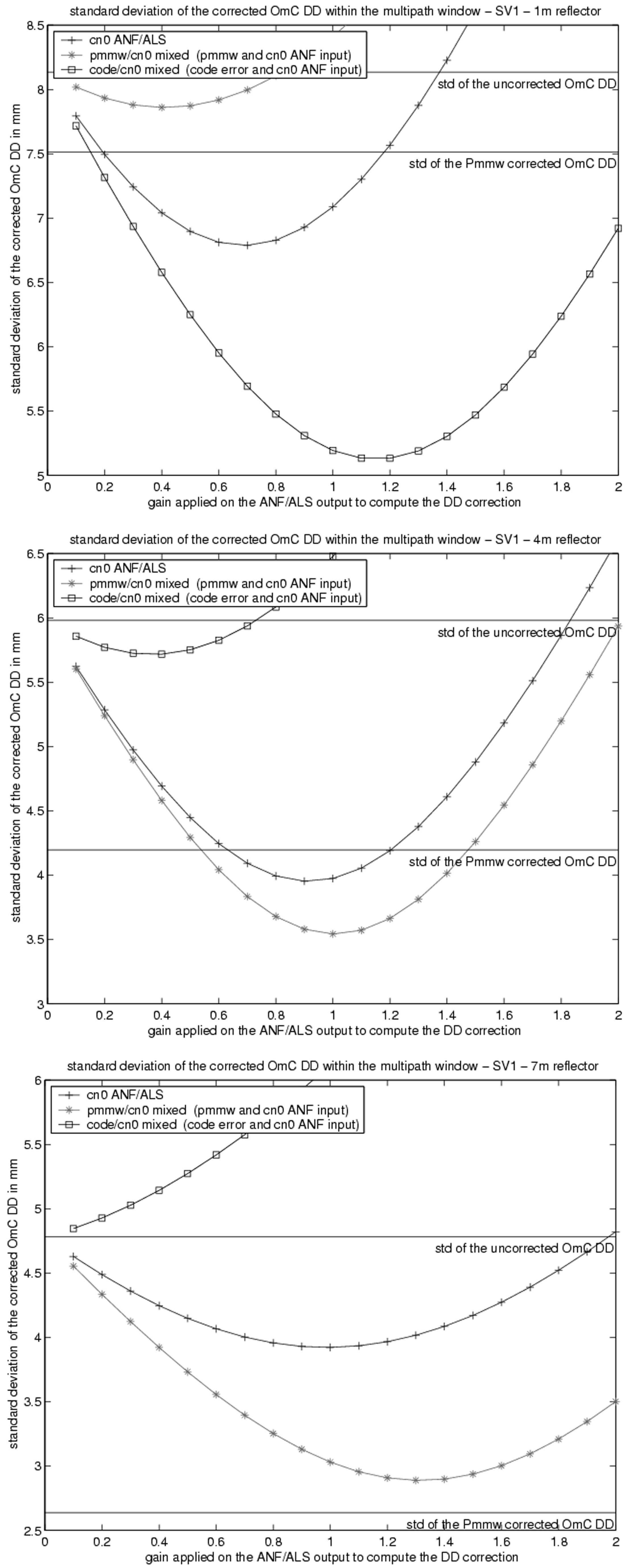


Fig. 7. Comparison of gain tuning of processes using SNR only, using SNR and PMMW, and using SNR and code data (panel at 1, 4, 7 m).

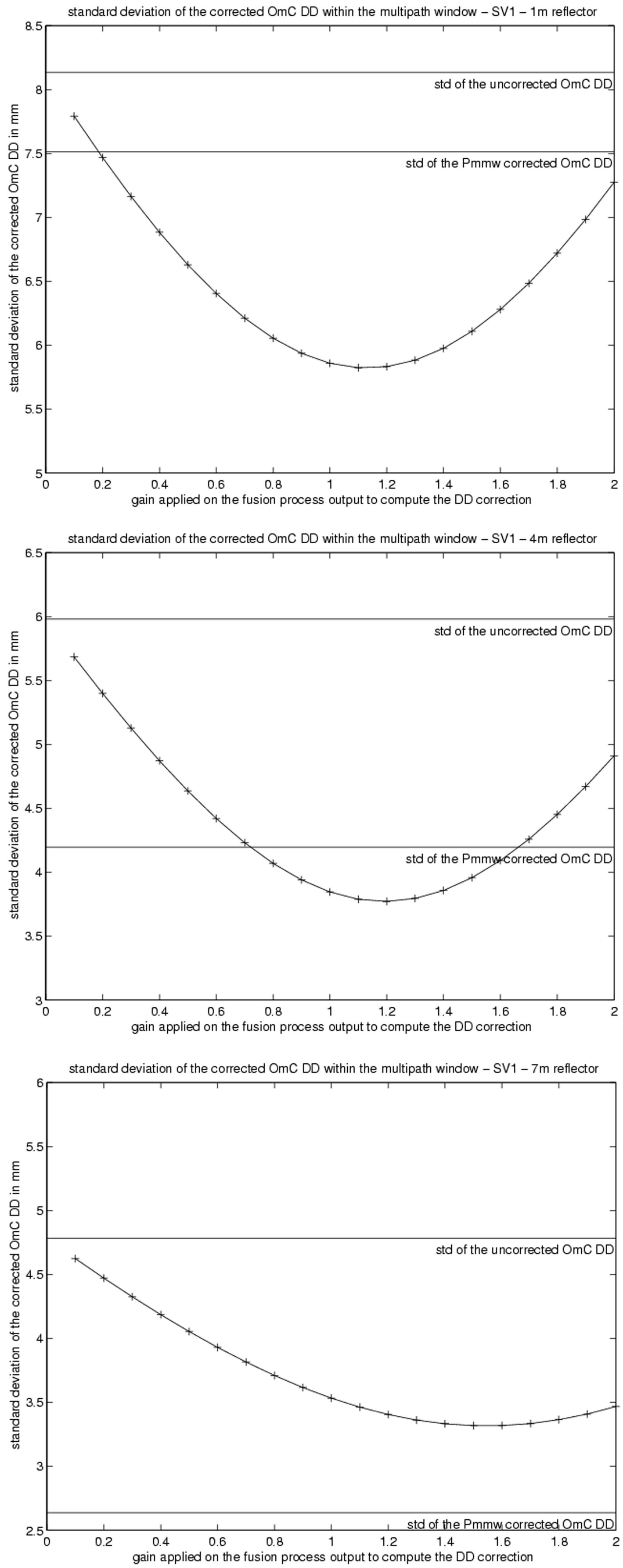


Fig. 8. Gain tuning of adaptive process using SNR, PMMW, and code data altogether (panel at 1, 4, 7 m).

TABLE TABLE III
L1 Phase DD Statistics in Static Mode (Rover on Tripod) After Applying Adaptive Algorithm

| Static Tests | (Size of Population) | $\sigma$ | $\sigma_n$ | Gain |
|---|---|---|---|---|
| Reflector 1 m<br>0.7 m < addpath < 2.0 m | (~130000 data*) | 8.5 mm | 7.8 mm | 9% |
| Reflector 4 m<br>5.2 m < addpath < 7.3 m | (~68000 data) | 5.4 mm | 4.3 mm | 20% |
| Reflector 7 m<br>9.7 m < addpath < 11.9 m | (~22000 data) | 4.8 mm | 3.8 mm | 22% |

(*Elevation of satellites > 15°.)

select geometrically the satellites that are affected by multipath as the environment is usually unknown. Multipath detection, including deciding whether reflections are from close or distant objects, must rely only on the code error observable, the SNR and the PMMW outputs (the three inputs to the algorithm), not on knowledge of the reflecting surfaces. Hence, any multipath mitigation algorithm has to be run continuously, for every satellite, for every epoch. There is also a problem in applying the algorithm in real-time due to the need to average the code error observable. This is treated by introducing a moving window, which of course entails some latency. The results given here were actually obtained by averaging the code error observable for every satellite over series of epochs that showed no loss of lock, something which would not strictly be possible in real-time.

Another issue arises from the use of the double-differencing process for the analysis. It is essential to keep in mind the fact that each DD involves a mixture of four reconstructed phase corrections and for these tests there is no a priori reason why the correction should not be applied to the differencing satellite, unlike in the earlier controlled experiments where the source of multipath was known. This modifies noticeably the conclusions that can be drawn due to the introduction of additional noise. Nevertheless the results summarised in Table III, show that the proposed method leads to significant levels of multipath mitigation, ranging from 9% in the presence of a very close reflector (significantly better than the PMMW) to over 20% for more distant reflectors. It is anticipated that even better results could be obtained if prior knowledge of positions and orientations of the reflectors was taken into account—so reducing the noise levels of the corrections. No improvements were seen after applying the method to the kinematic tests, probably due to the high noise levels introduced through the double differencing process. Indeed in some cases the application of the method appeared to worsen the results.

## VII. CONCLUSION

Several investigations have been carried out in order to overcome the key limitation of the multipath mitigation phase correlator (PMMW correlator for Leica System 500 receivers), i.e., the degradation of its efficiency as the additional path length travelled by the reflected signal decreases.

The main contribution of this work is a consideration of the combination of PMMW measurements with two other phase multipath observables: SNR and multipath code error. The following has been demonstrated.

1) The combination of PMMW and SNR is an efficient method for modelling multipath phase error resulting from reflectors more than a few metres away from the antenna. Essentially, the PMMW indicates the phase of the multipath error (without sign ambiguity) and the SNR its amplitude.

2) For close reflectors the multipath code error can be used instead of the PMMW to indicate the phase of the correction. In fact one of the outcomes of this work is the experimental demonstration that the multipath code error shifts in phase for additional distances of under a few metres. This property of the code might result from the specific implementation of the code correlator in the Leica System 500 and might be true only for that receiver. Further investigations are needed to understand this better.

An intelligent adaptive algorithm that can combine all three multipath observables (PMMW, SNR, and multipath code error) in order to estimate multipath phase errors, whatever the distance is to the reflector, has been designed.

In static mode, a real improvement of around 10% in the O-C DD of L1 phase is obtained in the case of a very close reflector versus only a few percent delivered by the PMMW phase correction. However, the algorithm only makes a 20% improvement when the reflector gets further away whereas application of the PMMW phase correction led to improvements reaching 50%. Research into a criterion on how to revert finally to the PMMW correction, with roughly 50% improvement beyond 7 m compared with 20% for the proposed combination would be useful.

In kinematic mode, the effectiveness of the algorithm could not be proved, and further investigations are needed. Moreover, for experiments that deal with so small residuals (mm) it would be particularly relevant, in order to come to a conclusion to see the performance of the technique in an even better controlled environment than the one we set-up (whose reference positions were known with only mm accuracy), such as using a GPS signal simulator.

Further investigations are also needed in the case of multiple reflections. Each reflection would require the determination of a corresponding amplitude in the SNR and a corresponding (signed) argument in the phase MM estimate. The robustness of the algorithm will probably decrease as the number of reflection to be combined increases but this supposition has not been evaluated quantitatively.

The results of this work are considered to be most relevant to applications such as

1) real-time kinematic GPS network base stations (particularly in situations where the environment around the antennas may change and prior calibration cannot be relied upon);

2) temporary base stations set-up for local static or kinematic GPS surveys on say a civil engineering construction site, when other kind of mitigation techniques based on long time series of data cannot be applied due to lack of sufficient data, and

3) vehicles whose motions lead to smooth changes in the relative geometry between the satellites, receiver and reflectors, such as slow moving road pavers and low Earth orbiting satellites.

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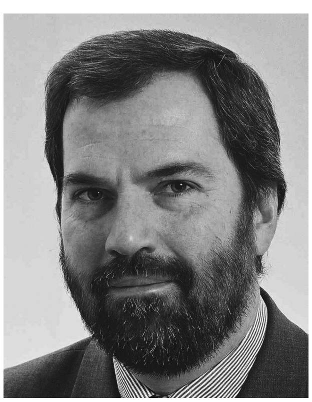

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