Did Recent Inflation Reflect a Nonlinear Phillips Curve?

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To examine the potential role of a nonlinear Phillips curve in explaining recent US inflation, we combine evidence from both cross-city variation and aggregate time series data. A central challenge relates to how best to control for the potential confounding effect of inflation expectations. The paper explores several options. Our findings place into question the robustness that tight labour markets, due to a nonlinear Phillips curve, played a very important role in the recent inflation episode.

Prior to the Covid-19 pandemic in 2020, a substantial body of empirical research on inflation dynamics[1] had concluded that the Phillips curve was relatively flat. Under this interpretation, transitory episodes of labour market tightness were expected to exert only modest upward pressure on inflation. However, this view has been challenged by the inflationary surge observed between 2021 and 2023, a period characterized by elevated inflation, high job vacancy rates, and low unemployment.

In particular, Benigno and Eggertsson (2023) presents evidence in support of a strongly nonlinear Phillips curve, where inflation rises sharply once the vacancy-to-unemployment ratio becomes high.[2] This suggests that labour market tightness played a central role in the inflationary dynamics of late 2021 and 2022, and that the subsequent moderation in labour market conditions contributed significantly to the disinflation observed thereafter. Under this interpretation, inflationary pressures stemming from Covid-19-related supply shocks would have been considerably more muted had they not coincided with tight labour markets. Consequently, large interest rate hikes may not have been necessary to restore price stability in the absence of such tightness.

The central object of interest in this discussion hinges on the slope of the Phillips curve, namely the derivative of inflation with respect to labour market tightness, everything else (including inflation expectations) held constant. We consider a Phillips curve derived under the assumption[3] that firms are monop-

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[1]See, for example, Coibion and Gorodnichenko (2015), Blanchard (2016), Ball and Mazumder (2018), Hazell et al. (2022), Del Negro et al. (2020) and Beaudry, Hou and Portier (2024*a*).

[2]See also Harding, Lindé and Trabandt (2023), Gitti (2024), Cerrato and Gitti (2022), and Inoue, Rossi and Wang (2024) for further recent empirical evidence consistent with this nonlinearity.

[3]See Woodford (2003) and Galí (2015).

olistically competitive and can only reset price with exogenous probability or quadratic adjustment costs. Because price adjustment is constrained, firms set prices based on a forward-looking consideration of future marginal costs, rather than simply marking up over contemporaneous marginal cost. Since marginal cost can be expressed as a function of output under further assumptions[4], one obtains the canonical linearized New Keynesian Phillips curve:

$$\pi_t = \beta\pi^e_{t+1} + \kappa\, x_t + \varepsilon_t, \tag{1}$$

where π_t denotes current inflation, π^e_{t+1} is expected inflation in the subsequent period, ε_t captures supply-side (cost-push) shocks and x_t measures economic activity, either the output gap or a labour market tightness indicator such as cyclical unemployment or the vacancy to unemployment rate. In this work, we follow Benigno and Eggertsson (2023), Gitti (2024) and Barnichon and Shapiro (2024) in using the vacancy to unemployment ratio as a measure of economic activity. The parameter κ represents the slope of the Phillips curve. Equation (1) echoes the expectations-augmented Phillips curve first introduced by Phelps (1967) and Friedman (1968), but derived within a microfounded, forward-looking framework. It is crucial to note that the linear approximation leading to this specifications is taken around a low (near-zero) inflation steady state in which long-run expectations are assumed to be well anchored; a plausible assumption for the post-1990 U.S. economy. Thus, the expectations term π^e_{t+1} should be interpreted and measured by short-run inflation expectations.

To explore possible nonlinearity in the relation between labour market tightness and inflation, we maintain a linear specification with respect to expectations and supply shocks[5], while allowing for a nonlinearity in the partial relation between current inflation and economic activity x. This can be expressed as an extended Phillips curve of the form:

$$\pi_t = \beta\pi^e_{t+1} + f(x_t) + \varepsilon_t \tag{2}$$

where $f(\cdot)$ represents the potential nonlinear relation between activity and inflation. The empirical question we want to explore is whether a linear $f(\cdot)$ remains an acceptable representation when considering the inflationary episode associated with the Covid-19 and post-pandemic recovery.

It is worth noting that the Phillips curve represents only one component among the broader set of equilibrium conditions that determine inflation. Even if the Phillips curve slope was constant and small, that would not prevent labour market tightness (or the gap) to potentially cause substantial inflation. One mechanism which can generate such an outcome is if variations in labour market tightness

[4]These include that firms face isoelastic demand and produce with labor only.

[5]For alternative approaches incorporating time-varying parameters or fully nonlinear specifications, see repectively Inoue, Rossi and Wang (2024) and Blanco et al. (2024).

are highly persistent and expectations are rational.[6] In that case, the tightness will impact inflation directly via the slope of the Phillips curve and indirectly via an increase in expected inflation. However, in the current work, we aim to separate the direct and indirect channels by using proxies to control for the inflation expectations component, thereby allowing us to focus on shape of the Phillips curve as represented by $f(\cdot)$.

Knowing whether the Phillips curve is rather flat or highly nonlinear is potentially of first-order importance for the conduct of monetary policy. A strong prior that the Phillips curve is highly nonlinear − if unfounded − may lead to suboptimal policy decisions in the future. For instance, suppose we were to face a set of supply shocks similar in intensity to that observed during Covid-19, but that the resulting inflation does not arise in conjunction with tight labour markets. A mis-interpretation of recent experience could lead one to conclude that looking through such a shock may be warranted as inflation is likely to return to normal on its own and the cost of addressing such temporary inflation by restrictive policy may be viewed as excessive terms of output loss.[7] But if the recent evidence in favour of a nonlinear Phillips curve were in fact masking a de-anchoring of short run inflation expectations, in a future supply shock episode without tight labour markets one may under-appreciate the danger of inflation becoming intrenched in the absence of forceful monetary response. Looking though supply shocks may then become costly as it could produce very persistent inflation even if labour markets are rather loose.[8] Nonetheless, it should be noted that even if labour market tightness only has a weak direct effect of inflation, and that recent inflation was in large part driven by a short term de-anchoring of inflation expectations, this does not mean that a strong monetary response to tight labour markets is not in order. It could be that expectations react excessively to tight labour market and therefore a strong response to a burst in demand may be in order maintain anchored inflation expectations even in the absence of supply shocks.

Our objective is to highlight a set of data patterns which underscore both the potential relevance and fragility of the nonlinear Phillips curve hypothesis. To keep the presentation as transparent as possible, we focus attention on a set of

[6] Beaudry, Hou and Portier (2024*b*) argued that this mechanism is not strong enough to explain much of post Covid-19 inflation, and accordingly proposed a mechanism that departs from full information rational expectations. A rational expectations view may nonetheless explain some of the inflation surge, as shown by Hazell and Hobler (2024). In particular, the later use a high frequency narrative approach to show that the 2021 deficits may have caused tight labour markets and therefore accounted for a significant share of the 2021-22 inflation.

[7] A central tenant of flexible inflation targeting – versus strict inflation targeting– is that temporary deviations of inflation from target are view as acceptable as long as inflation is expected to return to target over a short horizon. A flexible inflation targeting framework helps explain why many monetary authorities that viewed inflation expectations as well anchored decided to look through supply shocks during 2021.

[8] This danger arises if short run expectations react excessively– relative to a rational expectation benchmark – to supply shocks. If instead, short run inflation expectations react excessive to labour market tightness but not to supply shocks, then in the absence of tight labour markets, monetary authorities may not need to worry about de-anchoring.

simple figures. Section B in the Supplemental Appendix to the paper provides the regressions underlying the figures.[9] In summary, our figures show that *(i)* evidence of nonlinearity in the aggregate time series disappears when controlling for household or firms inflation expectations, but not professional inflation expectations, *(ii)* evidence of nonlinearity in the cross-section disappears when controlling for expectations using either local households expectations or by including a full set of time fixed effects. We further link these two perspectives together by comparing the behaviour of the time-fixed effects drawn from the cross section data with the different aggregate measures of inflation expectations.

I. A Visual Exploration of Inflation Patterns

In Panels A and B of Figure 1, we plot quarterly observations of core CPI inflation against the vacancy to unemployment ratio over the period 2000-2023. Panel A presents aggregate observations for the US, while Panel B presents US city level observations. The city level observations are for 19 major Metropolitan statistical areas (MSA).[10] In both cases, we superimpose an estimated cubic relationship between the two variables to express nonlinearity.

In both Panels A and B of Figure 1, inflation seems almost unrelated to labour market tightness at low levels of tightness – as would be suggested by a flat Phillips curve view- but a strong positive relationship is apparent at high levels of tightness. In particular, with labour market tightness measured by the vacancy to unemployment ratio, a change in relationship between inflation and tightness appears to arise when the vacancy-to-unemployment ratio is above one. This is clear in both the aggregate data and city level data. In both figures, we have marked in dark data points for which θ, as measured by the vacancy-to-unemployment rate, is greater than 1. As can be seen on Panel C, most of these dark dots arise in the post-2021 period. Therefore, the apparent strong positive relationship between inflation and labour market tightness is mostly driven by observations post Covid-19. This intriguing pattern has been interpreted by many, most notably Benigno and Eggertsson (2023) and Gitti (2024), as suggesting a strongly nonlinear Phillips curve, with the effects of labour market tightness on inflation being much stronger when the vacancy to unemployment ratio rises above one.

When looking at Panels A and B of Figure 1, the nonlinear relation between inflation and the vacancy-to-unemployment ratio is very salient, but obviously that does not imply causality. In particular, from a Phillips curve perspective, the high level of inflation post Covid-19 could be due to a combination of factors such as cost shocks, inflation expectations, and labour market tightness. Untangling the respective role of these different forces in driving inflation can be difficult because they tended to move together over the post Covid-19 period. To visualize the potential simultaneity problem, in Panels C and D of Figure 1 we plot the time

[9] Results from a more exhaustive empirical exploration are available from the authors upon request.
[10] The details about their construction are presented in Section A of the Supplemental Appendix.

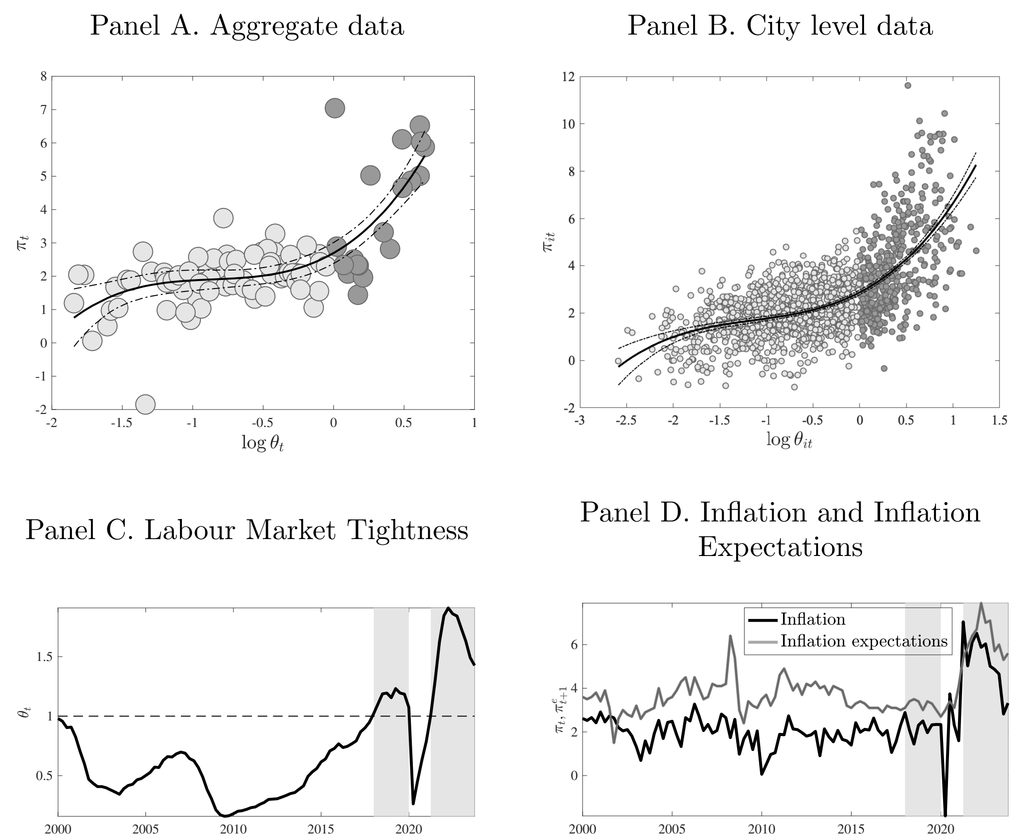

Figure 1. Labour Market Tightness, Inflation and Inflation Expectations, Raw Data

Note: Each dot represents a quarter (Panel A) or a quarter-city (Panel B). Labour market tightness is measured as $\log \theta$, where θ is the ratio of job vacancies to unemployment (V/U). Inflation is quarter-to-quarter (Panels A and D) or year-to-year (Panel B) CPI core inflation (annualized). For Panels A and B, light gray dots correspond to $\log \theta \leq 0$, dark gray dots correspond to $\log \theta > 0$ and the black line is the fitted cubic relation between π and $\log \theta$, dotted lines delimit the 95% confidence interval. Expectations are mean one-year-ahead and obtained from the Michigan Survey of Consumers. Grey areas represent quarters with $\theta > 1$. Sample is 2000Q1-2023Q4 for Panels A, C and D and 2001Q4-2024Q3 for Panel B.

path of the aggregate vacancy-to-unemployment ratio in Panel C and inflation (actual and expected inflation using the Michigan Survey of Consumer Expectations) in Panel D. As can be seen, expected inflation tended to be high precisely when the vacancy-to-unemployment ratio was high, which suggests that the high inflation over this period could potentially reflect– at least in part – a short-run de-anchoring of inflation expectations potentially induced by a series of supply shocks.[11]

In order to get a better sense of whether the apparent nonlinearity in the inflation-labour-market tightness relation seen in Figure 1 may be causal, the cross city data has important advantages relative to the time series data. This was the point emphasized in pre Covid-19 study of the Phillips curve by Fitzgerald and Nicolini (2014), McLeay and Tenreyro (2020) and Hazell et al. (2022), and post Covid-19 by Gitti (2024). In particular, the cross sectional data allows one to control for common cost shocks and common aggregate inflation expectations which could be affecting inflation at the same time as a tight labour market. We pursue this in Figure 2, where we plot the residuals of a two-way linear fixed effects regression. In Panel A we plot city level inflation against city level unemployment-to-vacancy ratio controlling for city fixed effects, while in Panel B we control for both city and time fixed effects. Again, we superimpose an estimated cubic relationship between the two variables. As can be seen, the removal of city level fixed effects has very little effect in comparison to Figure 1, and we continue to see a marked nonlinear relationship. In contrast, when removing time fixed effects, as done in Panel B, we no longer see any evidence of nonlinearity. [12] This observation regarding the disappearance of nonlinearity when controlling for time fixed effects is robust to using monthly frequency observations instead of quarterly observations, and to controlling in that case for either a full set of monthly fixed effects or only controlling for year-quarter fixed effects. It is worth noting that Gitti (2024) reports some evidence of nonlinearity using city level date at the monthly frequency. As shown in Section F of the Supplemental Appendix, the difference in findings comes from the inclusion in Gitti (2024) of city-specific-year-quarterly fixed effects in a monthly regression instead of allowing for the more easily interpretable year-monthly fixed effects or year-quarterly fixed effects.

This absence of nonlinearity arises when controlling for time effects despite the fact that there remains considerable cross-section variation in the local tightness even after removing city and time fixed effects.[13] This places into question the causal interpretation of the nonlinear relationship observed in these data in Figure

[11]Beaudry, Hou and Portier (2024*b*) develop in more depth such an explanation.

[12]Much of our analysis focuses on quarter-to-quarter inflation measures since this maps more easily to the theory when the data is at the quarterly frequency. However, for the city level observations, we have chosen to begin by presenting the year-to-year inflation observations since the nonlinearity is more striking and it nevertheless disappears when simply controlling for time fixed effects (see Section B in the Supplemental Appendix).

[13]This result is similar to Figure 7 in Barnichon and Shapiro (2024), where nonlinearity in the MSA-level Phillips curve vanishes when a Beveridge curve shift variable is introduced. It is also in line with Beschin et al. (2025) analysis of the Euro area inflation using subnational regional data.

1, and points instead to the possibility that this observed nonlinearity reflects underlying common factors distinct from labour market tightness that arrived post Covid-19.

According to the theory behind the New Keynesian Phillips curve, the disappearance of the nonlinearity observed in Panel B of Figure 2 relative to Panel B of Figure 1 is due to controlling for common forces such as cost shocks and movements in inflation expectations through the inclusion of time dummies. To get a sense for whether inflation expectations may be playing an especially important role in this transformation, in Panel C of Figure 2 we continue to use the cross city data but aim to control directly for inflation expectations instead of including time fixed effects. In the New Keynesian Phillips curve, inflation expectations should enter with a coefficient β equal to agents' discount factor. This suggests regressing quarter-to-quarter inflation minus β times one-quarter-ahead expected inflation[14] on labour market tightness and city fixed-effects and plotting residualized inflation against $\log \theta$. Using a discount factor $\beta = .99$, Panel C of Figure 2 shows that controlling for inflation expectations this way, we again find no sign of nonlinearity.[15]

Panels B and C of Figure 2 suggest that the cross-city data provide very little evidence in support of a nonlinear Phillips curve and hint to the possibility that the initially apparent nonlinearity could reflect the confounding effect of increased inflation expectations. The need to properly control for expectations, as expressed by an "Expectations-Augmented" Phillips curve first advanced by Phelps (1967) and Friedman (1968), was a key lesson learnt during the inflation episode of the 1970. In parallel to Panel C of Figure 2, we can do a similar exercise with the aggregate data. This is presented in Panel D of Figure 2, where we plot inflation minus .99 times expected inflation against the vacancy to unemployment ratio. In this figure, expected inflation is measured by median expected inflation of households in the Michigan Survey of Consumers. The nonlinearity –which was quite striking in the raw data in Panel A of Figure 1– also now disappears when controlling for inflation expectations this way. There is no longer any significant evidence that the Phillips curve becomes steeper for $\log \theta > 0$ and if anything, the slope seems to become smaller.

While the cross-city data allowed us to implicitly control jointly for common supply shocks and inflation expectations using time dummies– without needing to take a stance on the proper measure for inflation expectations– our plot in Panel D of Figure 2 is based on a specific measure for inflation expectations, that is, one drawn from the Michigan Survey of Consumers. Given that measures of inflation expectations can vary across different sources, it is important to verify which aggregate inflation patterns are robust when controlling for different measure

[14]See Section C in the Supplemental Appendix for how we extract quarter-to-quarter inflation expectations from year-to-year ones.

[15]Here we use the inflation expectations at the MSA level, as constructed by Binder, Kamdar and Ryngaert (2024). Results are very similar if we use β anywhere in the range $[.7, 1]$, as shown in Section D in the Supplemental Appendix.

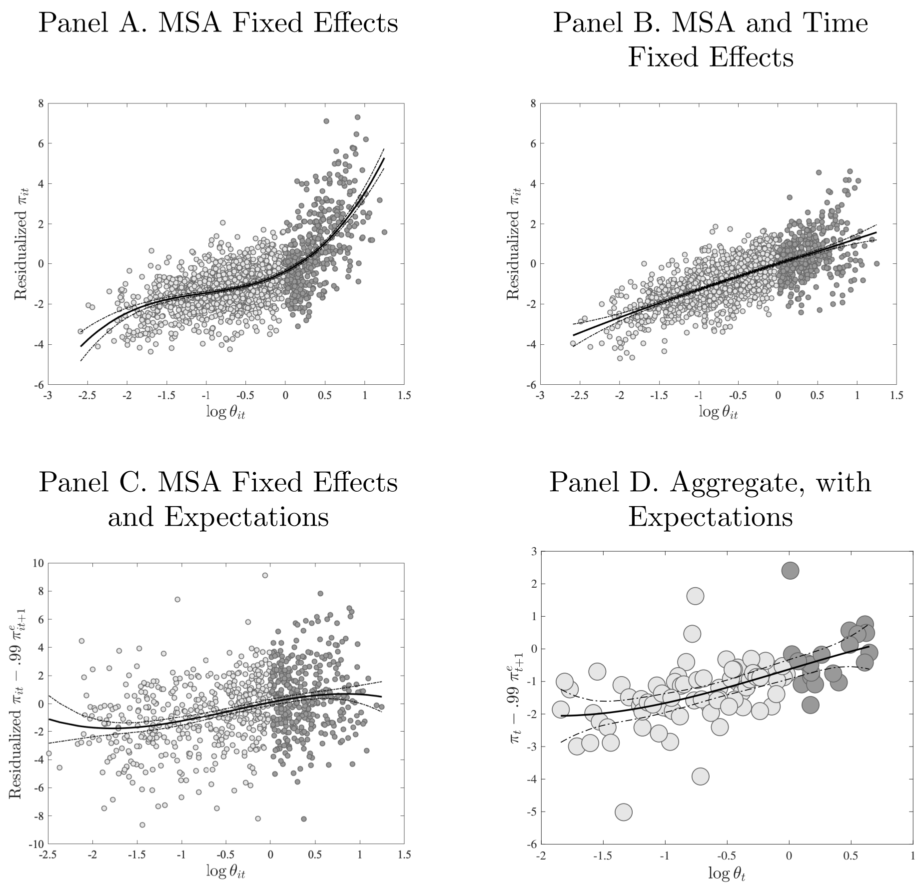

FIGURE 2. INFLATION AND LABOUR MARKET TIGHTNESS, CITY LEVEL DATA AND AGGREGATE DATA

Note: Each dot represents a quarter-city in Panels A, B and C, and a quarter in Panel D. Dark dots indicate observations with $\log\theta > 0$ and light dots observations with $\log\theta \leq 0$. Inflation is quarter-to-quarter (Panel C and D) or year-to-year (Panels A and B) CPI core inflation (annualized). In Panel A, residualized inflation is obtained from the two-way linear fixed effects regression $\pi_{it} = \alpha_i + \kappa \log\theta_{it} + \varepsilon_{it}$ and computed as $\pi_{it} - \alpha_i$. In Panel B, residualized inflation is obtained from the two-way linear fixed effects regression $\pi_{it} = \alpha_i + \gamma_t + \kappa \log\theta_{it} + \varepsilon_{it}$ and is computed as $\pi_{it} - \alpha_i - \gamma_t$. In Panel C, residualized inflation is obtained from the two-way linear fixed effects regression $\pi_{it} - .99\ \pi^e_{it+1} = \alpha_i + \kappa \log\theta_{it} + \varepsilon_{it}$ and computed as $\pi_{it} - .99\pi^e_{it+1} - \alpha_i$. The measure of π^e_{it+1} is obtained from Binder, Kamdar and Ryngaert (2024). In Panel D, expectations are median one-year-ahead and obtained from the Michigan Survey of Consumers. Both expectations are adjusted to proxy one-quarter-ahead expectations. In all panels, the black line is the fitted cubic relation between (residualized) π and $\log\theta$, dotted lines delimit the 95% confidence interval. Sample is 2001Q4-2024Q3 for Panels A, B and C and 2000Q1-2023Q4 for Panel D.

of inflation expectations. Panel A of Figure 3 plots time series for two types of expectations. We report three "experts" expectations sources (the Cleveland Fed., the Survey of Professional Forecasters and the Livingston survey) on the one side, and complement this with a measure of expectations drawn from firms (Survey of Firms Expectations) and two measures drawn from households (Survey of Consumer Expectations and the Michigan Survey of Consumers[16]) on the other side. All series are normalized to zero in 2020Q1. We observe that firms and households measures are pretty similar, and all increase strongly in the Covid-19/post-Covid-19 period. Experts measures also move together but are much less reactive than that of households or firms during the Covid-19 period.

With a Phillips curve like Equation (2), it is quite evident that the more responsive expectations will be during the Covid-19 period, the less nonlinearity in the Phillips curve will be needed to fit the data. This is shown in Panels B to G of Figure 3. On these panels, we are controlling for inflation expectations by plotting $\pi_t - .99\ \pi^e_{t+1}$ against log labour market tightness for different measures of inflation expectations. There is a clear distinction in the results: the nonlinear relationship between inflation and the vacancy-to-unemployment ratio appears robust to controlling for inflation expectations in the cases where we use experts measures. In contrast, when controlling for expectations using firms or households measures of inflation expectations there is again no longer any significant evidence that the Phillips curve becomes steeper for $\log \theta > 0$.[17] This distinction helps explain the results obtained by Benigno and Eggertsson (2023) as they use expert based expectations when presenting evidence in support of a highly nonlinearly Phillips curve. The possibility that firm or household expectations may better explain inflation in turbulent times than expert expectations is suggested in the work of Coibion and Gorodnichenko (2015) which focuses on the missing deflation during the Great Recession. In that work, they showed that the perception of missing disinflation after the Great Recession disappears when using household measures of inflation expectations. Conceptually, firm expectations are likely the more relevant measure for inflation expectations in the Phillips curves as it is firms that set prices. While we have a comprehensive measure of firm expectations in SoFIE for only a short period, it is interesting to note that this measure of inflation expectations track closely those of households post 2020.[18]

It certainly remains debatable which measure of inflation expectations is best to use when wanting to examine the slope of the Phillips curve. Experts expectations reflect well-informed agents with access to a broad set of data and models, but may differ from the expectations of economic agents (households/firms) who are more directly involved in price and wage setting. Household inflation expecta-

[16] In Panel A and B of Figure 3, we use the Michigan Survey of Consumers mean expectation to show robustness with respect to the median measure we used in Panel D of Figure 2.

[17] This result can also be found in Chodorow-Reich (2024).

[18] Another survey of firms expectations is the Business Inflation Expectations (BIE) conducted by the Atlanta Fed. The question on price inflation expectations is unfortunately only asked since December 2020 and not every quarters. That series is very close to the SoFIE one.

tions reflect perceptions of the general public, which are relevant for consumption and potentially wage-setting behavior, but tend to be noisier and biased. Firms inflation expectations may reflect more direct sensitivity to costs and demand pressures, but are still not available on long samples. Reis (2023) discusses how to best navigate with these various measures of inflation expectations. One observation we can make is that expert measures did a poorer job at predicting inflation over the post 2020 period. Over the sample 2020Q2-2023Q4 that incorporates the inflation surge, the root mean square error between expectations and headline inflation was lower for firms and households measures (Michigan Survey of Consumers: 2.9, Survey of Consumers Expectations: 2.7, Survey of Firms Expectations: 3.3) than for experts ones (Cleveland Fed.: 4, Livingston survey: 3.5, Survey of Professional Forecasters: 3.8).[19] We will return to such a comparison when discuss the inflation experience of the seventies.

One way to assess which measure of inflation expectations is more relevant for understanding inflation surges is to compare the time fixed effects from city-level panel data with various aggregate inflation expectation measures. To this end, in Panel D of Figure 4, we plot the time fixed effect implicit in Panel B of Figure 2.[20]. We super-impose four measures of inflation expectations. In order to fully understand this plot, it is useful to start with the time fixed effects estimated with quarter-to-quarter headline inflation, which corresponds to Panel A of Figure 4. In principle, the time fixed effects extracted from the city level data should incorporate supply shocks to the aggregate Phillips curve and the relevant concept of inflation expectations driving the common component of inflation across cities. These supply shocks are more likely to show up in headline inflation than in the core one. Indeed, Panel A shows that time fixed effects are above the expectation series before mid-2022, and then below as supply shocks reverted. When choosing core inflation (Panel B), supply shocks are likely to be partially eliminated. This is indeed what we observe, and 2020Q3 and 2021Q2 appear to stand out as periods in which supply shocks had a direct impact on core inflation. The rest of the time, time fixed effects are close to the firms and households measure of expectations. If we consider professional expectations, one would need to assume that core inflation still incorporates large and persistent supply shocks before and after 2022. Panels C and D use year-to-year inflation to estimate time fixed effects, and the time fixed effects are therefore smoothed versions of the quarter-to-quarter estimates. As can be seen on Panel D (with core year-to-year inflation), the time fixed effects map very closely the household and the firm level inflation expectations series over the Covid-19 period, while expert measures are substantially below the estimated time effects.

Our take away from this exploration of data is that a nonlinear Phillips curve

[19]The RMSE is computed as $\sqrt{\left(\sum(\pi_t - \pi^e_{t-1,t})^2\right)}$. In periods where inflation is quite stable, experts tend to do better at forecasting inflation than households.

[20]These are the quarter-year fixed effects obtained by regressing city level year-to-year core inflation on city dummies, a cubic in tightness and the full set of time dummies

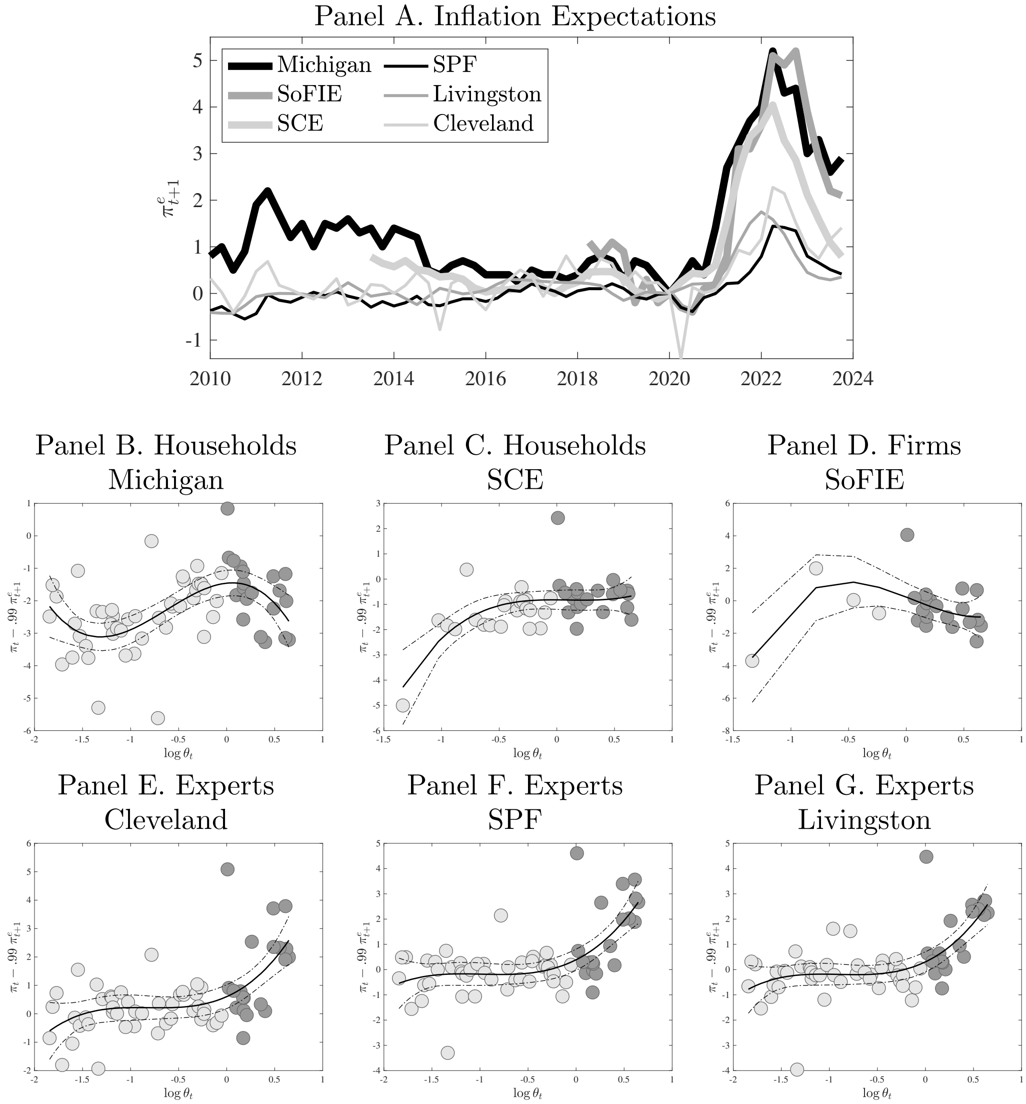

Figure 3. Inflation and Labour Market Tightness, Controlling for Various Expectations

Note: On Panel A, all series are adjusted to take value 0 in 2020Q1. On Panels B to G, each dot represents a quarter. Dark dots indicate observations with $\log\theta \geq 0$ and light dots observations with $\log\theta < 0$. The sample varies with the availability of the measure of expectations we use. Inflation is quarter-to-quarter CPI core inflation (annualized). All inflation expectations are one-year-ahead and adjusted to proxy one quarter-ahead expectations. "Cleveland " is the inflation expectations series published by the Federal Reserve Bank of Cleveland, "SPF" is the Survey of Professional Forecasters published by the Federal Reserve Bank of Philadelphia, "Livingston" is the Livingston Survey published by the Federal Reserve Bank of Philadelphia, "SCE" is the Survey of Consumer Expectations series published by the Federal Reserve Bank of New York, "SoFIE" is the Survey of Firms' Inflation Expectations series published by the Federal Reserve Bank of Cleveland, "Michigan" is the mean inflation expectations series of the Surveys of Consumers published by the University of Michigan. Sample always ends in 2023Q4. It starts in 2008Q1 for Michigan, Cleveland and SPF, 2013Q3 for SCE and 2018Q2 for SoFIE. The black line is the fitted cubic relation between the y-axis variable and $\log\theta$, dotted lines delimit the 95% confidence interval.

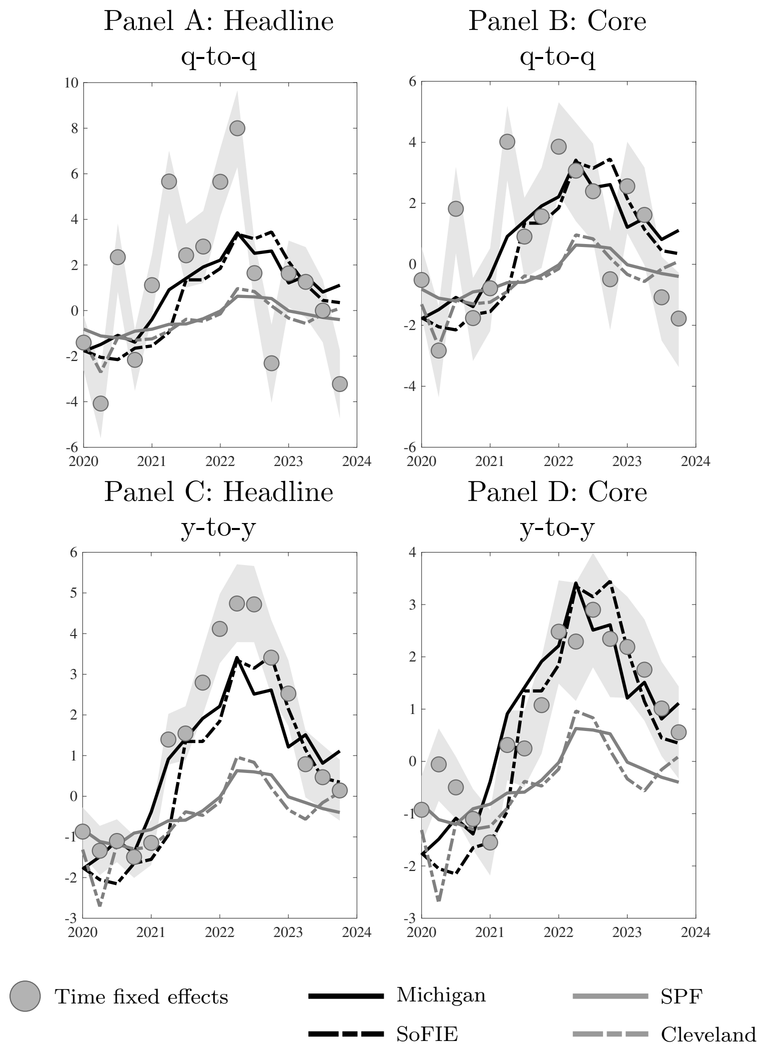

Figure 4. Time Fixed Effects and Inflation Expectations

Note: Each dot represents a quarter time fixed effect γ_t, as obtained from the two-way fixed effects regression $\pi_{i,t} = \kappa \log \theta_{i,t} + \kappa_2 (\log \theta_{i,t})^2 + \kappa_3 (\log \theta_{i,t})^3 + \gamma_i + \gamma_t + \varepsilon_{i,t}$, with gray area representing ± 2 standard deviations. The regression is done using quarter-to-quarter or year-to-year, headline or core inflation measures at the MSA level. Expectations series are one-year-ahead and demeaned over their maximum respective sample. "Cleveland " is the inflation expectations series published by the Federal Reserve Bank of Cleveland, "SPF" is the Survey of Professional Forecasters published by the Federal Reserve Bank of Philadelphia, "SoFIE" is the Survey of Firms' Inflation Expectations series published by the Federal Reserve Bank of Cleveland, "Michigan" is the mean inflation expectations series of the Surveys of Consumers published by the University of Michigan. On the plots, sample always ends in 2023Q4. It starts in 2002Q1 for Michigan, Cleveland and SPF and 2018Q2 for SoFIE.

interpretation of recent inflation outcomes is questionable. There seems to be little evidence in its support in the cross-section data once one includes inflation expectations or time fixed effects. In the time series data, controlling for firms or households inflation expectations is sufficient to explain the apparent non-linearity. The only case for which we do find evidence in support of a nonlinear Phillips curve interpretation of recent inflation is when focusing on aggregate data and controlling for inflation expectations using expert measures. However, before forming any conclusion, it is important to point out several caveats and weaknesses of our approach which highlights why more work is likely needed before getting a definitive answer.

II. Caveats, Challenges and Longer Run Evidence

In this section, we first discuss some of the caveats and challenges associated with our analysis, before examining evidence over longer samples.

A. Caveats and Challenges

Although the data patterns we presented call into question the robustness of the view that a nonlinear Phillips curve has played a significant role in recent inflation, several considerations warrant caution against over-interpreting these findings. All the results conveyed in the figures are based on partial correlations which may not map directly to the structural parameters of interest. In particular, it is well known that supply shocks can create confounding effects when estimating the slope of the Phillips curve, even after controlling for inflation expectations, if these shocks are correlated with labour market tightness. For example, this could cause a negative bias if driven by countercyclical policy as emphasized in McLeay and Tenreyro (2020), or it could cause a positive bias during a period like Covid-19 where supply shocks may have been positively correlated with tight labour markets.[21] In the time series, reaching identification of the structural parameters of the Phillips curve is notoriously challenging as noted in Mavroeidis, Plagborg-Møller and Stock (2014), and we do not claim to have an ideal solution. In Section E of the Supplemental Appendix, we report several instrumental variable exercises to counter such biases. These exercises confirm our results, but none of them are above all criticism.

The cross section specification alleviates some of these endogeneity issues as the time fixed effects potentially control for movements in aggregate inflation expectations, aggregate supply shocks, and counter-cyclical monetary policy. But city-specific supply shocks can still be a threat to causal identification. Unfortunately, instruments such as the predicted change in tradeable employment used in

[21] If the strong correlation between inflation and tight labour markets observed during the Covid period was mainly due to supply shocks being positively correlated with tight labour markets, then the pattern should remain even after controlling for inflation expectations.

Hazell et al. (2022) have low power in the post-COVID 19 period when including city and time effects.[22]

It is also possible that including time fixed effects in the cross-section analysis is actually over-controlling as the source of nonlinearity may be being picked up by the fixed effect. For example, this could arise in a menu cost setup with national pricing where the frequency of prices changes affects the slope of the Phillips curve when aggregate inflation increases.[23] In such a case, the source of the nonlinearity would not appear in cross-section when controlling fully for time effects. However, this is a very different type of non-linearity that upon which we focus here since the Phillips curve may be nonlinear in manners not captured by a focus on labour market tightness. This could arise for example due to capacity constraints, attention thresholds or quasi-kinked demand curves. None of these cases are directly addressed in our analysis.

B. Longer Samples

Up to now, we have mainly focused on a sample starting in 2000, a period where long run inflation expectations are considered reasonably anchored and for which we have city level observations. For the aggregate data, results we presented are almost unchanged if extending the sample to the full post Volcker period and adding controls such as a proxy for supply shocks and lagged inflation, as shown in Section B.3 of the Supplemental Appendix. This should not be surprising as the vacancy to unemployment ratio never exceeds one in the 80s and 90s, and therefore this longer period provide little scope to learn about whether the Phillips curve becomes very steep when labour markets are very tight.

Over a much longer sample – from 1960Q1 to 2023Q3 – the evidence in favour of nonlinear relationship also disappears once we control for short run household inflation expectations in same manner we have done for Panel D of Figure 2. In other words, even when considering a much longer sample, evidence for a nonlinear Phillips curve seems rather fragile. Nonetheless it is worth noting that, if we focus exclusively on the 1960Q1-1969Q4 period, then we do find some support for what may be a nonlinear Phillips curve, as shown in Figure 5. This is consistent with the discussion in Benigno and Eggertsson (2023) which is motivated in large part by nonlinear Phillips curve type patterns observed in the 1960s.[24] It should

[22]We have used shift-share instrument constructed in the line of Hazell et al. (2022). In the first stage estimation, after partialling out time and MSA fixed effects and clustering at year × MSA level, the instruments appears to be strong enough for full sample (F-stat=15.2) and pre-COVID sample (F-stat=18.4), but quite weak for post-COVID sample (F-stat=0.7).

[23]See Cavallo, Lippi and Miyahara (2024), Blanco et al. (2024) and Karadi et al. (2024) for such models of the recent inflation surge. More generally, Auclert et al. (2024) have shown that in a broad class of menu cost models, there is a limited quantitative role for Phillips curve aggregate nonlinearity and state dependence given shocks that generate inflation of up to 5%.

[24]Benigno and Eggertsson (2023) also examine inflation periods prior to 1960. However, data on inflation expectations pre-1960 are not readily available, making it very difficult to disentangle different forces affecting inflation. In particular, since many of the pre-1960 inflation surges in the US are associated with the begin or ends of wars, this bring in a whole new set of issues including the effects of price controls and potential periods of fiscal dominance.

nevertheless be noted that controlling for inflation expectations does substantially decrease the size of the inflation that arose in the late 60s. There may also be some indication that inflation, as suggested in Rudd (2022), was more U-shaped than L-shaped over this period. According to Rudd, the main inflation puzzle in the 1960s may be the low inflation during the early half of the decade as opposed to the higher inflation in the second half.

It is also interesting to focus attention on the large inflation surge of the seventies. In particular, we can use this period to examine whether the relative performance of households versus experts in predicting inflation appears similar to that observed during Covid-19. Recall that during Covid-19, according to our measures, households and firms predicted inflation better than experts. Such a pattern is potentially consistent with the short run de-anchoring of household/firm expectations (following a set of supply shocks) having an important direct role in driving inflation in that episode. Although we only have two measures for inflation expectations during the inflation period of 1973Q1 to 1981Q4; a household measure from the Michigan Survey of Consumers and the expert measure from the Livingstone survey, we see in a similar pattern in the 70s episode to that during Covid-19 (Figure 5). Households predicted headline inflation over the period 1973Q1-1981Q4 with an average root-mean-square forecast error of 2.1 while professionals in the Livingston survey had an average forecast error of 2.8. This offers further support to the possibility that households expectations may play an especially important role in driving inflation after a set of supply shocks.

III. Conclusion

The main message of this paper is that there are two very different interpretations of the recent inflation experience and they are hard to disentangle. Being cautious, one should dismiss neither readily until better data or methods can more definitely resolve this issue and, most importantly, one should embrace this ambiguity if one wants to avoid policy errors in the future. On the one hand, there is the view that– in addition to supply shocks– labour market tightness played a very important direct role in generating high inflation post 2020 because the Phillips curve is highly nonlinear and the labour market was very tight. On the other hand, there is the view that the Phillips curve is likely quite flat and that a de-anchoring of short run inflation expectations likely played a central role in realized inflation dynamics.[25] The main difficulty is differentiating between these two views, and weighting their respective merits, relates to the difficulty of knowing how to properly control for inflation expectations. The main reason it is so important to differentiate between these two views is that they could lead

[25] Such a de-anchoring of inflation expectations, especially when viewed from a behavioural perspective, could be ignited different forces. For example, it could be that board based supply shocks create confusion that directly transmits into persistent inflation expectations as discussed in Beaudry, Hou and Portier (2024*a*). Alternatively, it could be that inflation expectations react strongly to tight labour markets even if the latter has very little direct effect on inflation.

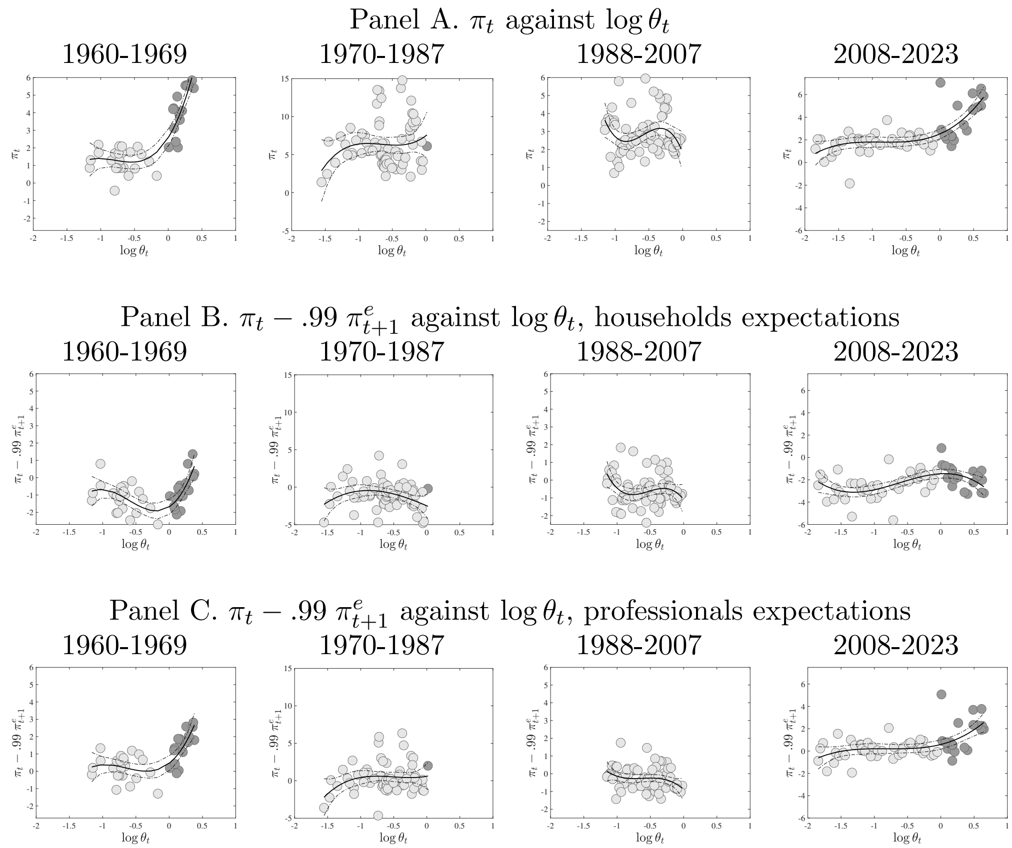

Figure 5. Inflation and Labour Market Tightness, Various Sub-periods

Note: Each dot represents a quarter. Dark dots indicate observations with $\log \theta \geq 0$ and light dots observations with $\log \theta < 0$. Inflation is quarter-to-quarter CPI core inflation (annualized). For households expectations, we use mean inflation expectations from the Michigan Survey of Consumers for the subperiods 1960-1969 and 1970-1987 as median ones are not available before 1978. For professional expectations, we use one-year ahead inflation expectations of the Federal Reserve Bank of Cleveland, which are quarterly and available since 1982Q2. The series is patched backward to 1960Q1 with the 12-month inflation expectations from the Livingston survey. Since the latter is twice yearly, missing observations are interpolated though a spline curve-preserving function. All expectations are adjusted to proxy one quarter-ahead expectations (see Supplemental Appendix C). The black line is the fitted cubic relation between the y-axis variable and $\log \theta$, dotted lines delimit the 95% confidence interval.

to different policy responses when faced with stagflation.[26]

To address this issue, we began by looking at cross-city data. One of the advantage of using cross-city data is that common inflation expectations can be controlled by the use of time dummies, without a need to take a firm stand on how to measure inflation expectations. From the city level data, we found no evidence in support of a nonlinear Phillips curve once expectations of a complete set of time dummies were allowed in the regression. However, as we have emphasized, it could still be possible that the nonlinearity of the Phillips curve is only an aggregate phenomena for which cross city data is not informative.

When looking at the aggregate level evidence, we showed how the evidence for or against a nonlinear Phillips curve was highly sensitive to which measure of inflation expectations one considers most relevant in determining of inflation. If one focuses on aggregate evidence and has a strong prior that the inflation expectations of professional forecasters are the most relevant for thinking about inflation, one can come to the conclusion that the Phillips is highly nonlinear, labour market tightness was a very important driver of inflation post Covid-19 and that inflation expectations played a minor role. In contrast, if one thinks that firm or household expectations are more relevant for thinking about inflation (and implicitly wage) determination, then one comes to the opposite conclusion: the Phillips curve appears linear, quite flat and that short run inflation expectations played an important role in recent inflation dynamics because they can de-anchor easily following supply shocks. We believe that our results tilt in favour of a linear view of the Phillips curve and the potential de-anchoring of short run expectations, but do not claim that the nonlinear view can be rejected. More work is needed.

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[26]Differentiating between these two is could be less important if the economy if subject to a large increase in demand, as in such a case it is generally desirable to tighten monetary policy to keep the economy close to its natural level.

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Did Recent Inflation Reflect a Nonlinear Phillips Curve?

Supplemental Appendix

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November 5, 2025

A. Data Appendix

1. Aggregate Data

Labour Market tightness θ: Computed in Michaillat & Saez [2024],[1] obtained from Pascal Michaillat webpage (link).

Core inflation π: "CPILFES", obtained from from the FRED database (link).

Inflation expectations: We use one-year-ahead inflation expectations. "Michigan" is the inflation expectations series of the Surveys of Consumers published by the University of Michigan (link). "Cleveland " is the inflation expectations series published by the Federal Reserve Bank of Cleveland (link), "SPF" is the Survey of Professional Forecasters published by the Federal Reserve Bank of Philadelphia (link), "Livingston" is the Livingston Survey published by the Federal Reserve Bank of Philadelphia (link), "SCE" is the Survey of Consumer Expectations series published by the Federal Reserve Bank of New York (link), "SoFIE" is the Survey of Firms' Inflation Expectations series published by the Federal Reserve Bank of Cleveland (link), "BIE" is the Business Inflation Expectations survey published by the Federal Reserve Bank of Atlanta (link).

Supply shocks v: As in Benigno and Eggertsson (2023), we proxy supply shocks v constructed as the four-quarter average of the principal component of the following three series: headline shocks, both to CPI and PCE, and import shock. The CPI or PCE headline shock is the difference between the annualized quarterly inflation rate computed using the CPI or PCE price index and that computed using the CPI or PCE price index excluding energy and food. The import shock is the difference between the annualized quarterly inflation rate computed using the import-price deflator and that computed using the GDP deflator. All series are obtained from the FRED database (link).

2. MSA Data

Labour Market Tightness: unemployment numbers at the MSA level and vacancy numbers at the State level are obtained from Job Openings and Labor

[1] Michaillat, P. and Saez, E. (2024), "$u^\star = \sqrt{u\,v}$: The Full-Employment Rate of Unemployment in the United States", Brookings Papers on Economic Activity, Vol. 55.

Turnover Survey (JOLTS) of U.S. Bureau of Labor Statistics (BLS) (link). The weights used to compute the MSA level vacancies are from U.S. Census 2020 available through IPUMS (link).

Inflation: Headline CPI is all items for all urban consumers and Core CPI is all items less food and energy for all urban consumers at MSA levels. Both are available from the BLS (link).

Inflation expectations: We use one year ahead inflation expectations reconstructed from Michigan Survey of Consumers, obtained from Binder, Kamdar and Ryngaert (2024).

B. Regression Tables

1. *Baseline Quarterly Data, City-Level Data*

In the following tables, we display the OLS regression results for the regression lines we presented in Figure 2 of the main text. Furthermore, we explore two different specifications of nonlinearity: we either allow for a cubic relationship between inflation and labor market tightness as we plotted in the figures:

$$\pi_{i,t} = \kappa \log \theta_{i,t} + \kappa_2 (\log \theta_{i,t})^2 + \kappa_3 (\log \theta_{i,t})^3 + \gamma_i + \gamma_t + \varepsilon_{i,t} \tag{B.1}$$

or we consider a piece-wise linear form where the slope of the Phillips Curve changes when $\log \theta_{i,t} > 0$:

$$\pi_{i,t} = \kappa \log \theta_{i,t} + \kappa_{\theta \times D} (D_{i,t} \times \log \theta_{i,t}) + \gamma_i + \gamma_t + \varepsilon_{i,t} \tag{B.2}$$

where $D_{i,t} = \mathbb{1}(\log \theta_{i,t} > 1)$.

Columns (3) and (4) in Table B.1 report the estimates from Equation (B.1), which correspond to the cubic regression lines shown in Panels A and B of Figure 2 in the main text. Columns (1) and (2) show results from the piecewise linear Equation (B.2). Both sets of estimates indicate that the Phillips Curve looks strongly nonlinear, with a steeper slope when the labor market is overly tight, but when we control for MSA fixed effects only. Once we also include time fixed effects, this nonlinearity largely disappears.

We next show that the same pattern holds when we estimate the Phillips Curve using the theoretically preferred quarter-to-quarter Core CPI. Table B.2 presents these results: once again, we find no evidence of a steeper Phillips Curve when time fixed effects are included.

Finally, we estimate the Phillips Curve using MSA-level one-year-ahead inflation expectations constructed by Binder, Kamdar and Ryngaert (2024), with quarter-to-quarter Core CPI. In these regressions, we either include expectations directly as a regressor, or use the transformed term $\pi_{i,t} - \beta \pi^e_{i,t+1}$ with $\beta = 0.99$

Table B.1—City Level Estimation of Phillips Curve, y-to-y Core CPI

	Piecewise Linear			Cubic	
	(1)	(2)		(3)	(4)
κ	0.77***	1.35***	κ	2.17***	1.17***
	(0.09)	(0.21)		(0.18)	(0.30)
$\kappa_{\theta\times D}$	3.51***	-0.23	κ_2	1.59***	-0.07
	(0.48)	(0.57)		(0.27)	(0.28)
			κ_3	0.51***	0.02
				(0.12)	(0.10)
# Obs.	1492	1492		1492	1492
Adjusted R^2	0.54	0.69		0.53	0.69
MSA F.E.	Y	Y		Y	Y
Time F.E.	N	Y		N	Y

Note: The dependant variable π_{it} is core CPI inflation at the MSA level. We iterate expected inflation forward and assume common across MSAs long-run expectations anchored by monetary policy as in Hazell et al. (2022). Columns (1) and (2) are estimates of equation (B.2) and columns (3) and (4) are estimates of equation (B.1). Stardard Errors are clustered at year×MSA level. Sample is 2001Q4-2024Q3.

Table B.2—City Level Estimation of Phillips Curve, q-to-q Core CPI

	Piecewise Linear			Cubic	
	(1)	(2)		(3)	(4)
κ	1.07***	1.72***	κ	2.08***	1.35***
	(0.11)	(0.26)		(0.21)	(0.37)
$\kappa_{\theta\times D}$	2.52***	-0.74	κ_2	1.17***	-0.27
	(0.57)	(0.69)		(0.32)	(0.34)
			κ_3	0.38***	-0.04
				(0.13)	(0.13)
Observations	1555	1555		1555	1555
Adjusted R^2	0.20	0.41		0.20	0.41
MSA F.E.	Y	Y		Y	Y
Time F.E.	N	Y		N	Y

Note: The dependant variable π_{it} is core CPI inflation at the MSA level. We iterate expected inflation forward and assume common across MSAs long-run expectations anchored by monetary policy as in Hazell et al. (2022). Columns (1) and (2) are estimates of equation (B.2) and columns (3) and (4) are estimates of equation (B.1). Stardard Errors are clustered at year×MSA level. Sample is 2001Q4-2024Q3.

to help address the endogeneity of expectations.[2] All regressions include time fixed effects. Column (3) of Table B.3 corresponds to the regression line shown in Panel C of Figure 2 in the main text. Across all specifications that control for local expectations, we find no evidence of nonlinearity in the Phillips Curve.

Table B.3—City Level Estimation of Phillips Curve with Local Expectations, q-to-q Core CPI

	Piecewise Linear			Cubic	
	(1)	(2)		(3)	(4)
β	0.79***	.99†	β	0.81***	.99†
	(0.09)	–		(0.09)	–
κ	1.15***	1.14***	κ	1.53***	1.37***
	(0.15)	(0.15)		(0.23)	(0.21)
$\kappa_{\theta\times D}$	0.58	0.06	κ_2	0.05	-0.18
	(0.61)	(0.52)		(0.32)	(0.28)
			κ_3	-0.10	-0.19
				(0.16)	(0.14)
Observations	860	860		860	860
Adjusted R^2	0.33	0.13		0.33	0.13
MSA F.E.	Y	Y		Y	Y
Time F.E.	N	N		N	N

Note: The dependant variable π_{it} is core CPI inflation at the MSA level. A † stands for a non-estimated coefficient. Columns (1) and (2) are estimates of equation (B.2) and columns (3) and (4) are estimates of equation (B.1). Standard Errors are clustered at year×MSA level. Sample is 2001Q4-2024Q3.

2. *Aggregate Data*

We report in Table B.4 parameters estimates corresponding to Panel A of Figure 1, Panel D of Figure 2 and Panels B to G of Figure 3. With the same data, we also estimate a piecewise-linear specification (Table B.5), where the slope can change value for $\theta > 1$, with similar conclusion regarding the nonlinearity of the Phillips curve. The estimated equations are (omitting constants):

$$\pi_t = \beta\pi^e_{t+1} + \kappa\log\theta_t + \kappa_2(\log\theta_t)^2 + \kappa_3(\log\theta_t)^3 + \varepsilon_t \tag{B.3}$$

$$\pi_t = \beta\pi^e_{t+1} + \kappa\log\theta_t + \kappa_{\theta\times D}(D_t \times \log\theta_t) + \varepsilon_t \tag{B.4}$$

where π_t is core quarter-to-quarter inflation, π^e_{t+1} is a measure of next quarter inflation expectation, $\theta_t = \frac{V_t}{U_t}$, v_t is a measure of supply shocks and D_t an indicator variable for $\theta_t > 1$.

3. *Aggregate Data Adding Controls*

Here we check that our results survive when we estimate β and add a measure of supply shocks and lagged inflation as controls, in order to estimate a relation

[2]Section D in the Supplemental Appendix evaluates the sensitivity of our results to the choice of β.

TABLE B.4—ESTIMATION OF THE AGGREGATE PHILLIPS CURVE (B.3) OF FIGURES 1, 2 AND 3

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
β	$0^{\dagger}$	$.99^{\dagger}$	$.99^{\dagger}$	$.99^{\dagger}$	$.99^{\dagger}$	$.99^{\dagger}$	$.99^{\dagger}$	$.99^{\dagger}$
κ	2.41***	1.14***	0.30	-0.03	-2.79**	1.51***	1.91***	1.82***
	(0.28)	(0.26)	(0.33)	(0.44)	(1.28)	(0.35)	(0.34)	(0.33)
κ_2	2.60***	-0.01	-2.43***	-0.36	-0.43	1.90***	2.02***	2.11***
	(0.57)	(0.54)	(0.67)	(0.80)	(1.65)	(0.70)	(0.70)	(0.67)
κ_3	1.01***	-0.11	-1.29***	1.20	2.81	0.78**	0.68*	0.79**
	(0.31)	(0.29)	(0.36)	(0.82)	(1.80)	(0.39)	(0.38)	(0.37)
N	96	96	64	42	23	64	64	64
adj. R^2	0.53	0.31	0.26	0.35	0.25	0.33	0.39	0.41

Note: The dependant variable π_t is core CPI inflation. A † stands for a non-estimated coefficient. Constants are omitted. Column (1) is for raw data and corresponds to Panel A of Figure 1. Column (2) adjusts for inflation expectations (median Michigan expectations) and corresponds to Panel D of Figure 2. Columns (3) to (8) correspond to Panels B to G of Figure 3. The first three of these columns use households and firms expectations: (3) uses mean Michigan expectations, (4) the Survey of Consumer Expectations, (5) the Survey of Firms' Inflation Expectations. The last three of these columns use professional expectations: (6) uses then Cleveland Fed. measure, (7) the Survey of Professional Forecasters and (8) the Livingston survey. All these expectations are one year-ahead and adjusted to proxy one quarter-ahead expectations (see Supplemental Appendix C). Sample always ends in 2023Q4. It starts in 2000Q1 for Michigan, Cleveland and SPF, 2013Q3 for SCE and 2018Q2 for SoFIE.

TABLE B.5—ESTIMATION OF THE PIECEWISE-LINEAR AGGREGATE PHILLIPS CURVE (B.4)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
β	$0^{\dagger}$	$.99^{\dagger}$	$.99^{\dagger}$	$.99^{\dagger}$	$.99^{\dagger}$	$.99^{\dagger}$	$.99^{\dagger}$	$.99^{\dagger}$
κ	0.67***	0.87***	1.02***	1.86***	1.98*	0.39	0.30	0.33
	(0.20)	(0.19)	(0.24)	(0.45)	(1.06)	(0.24)	(0.24)	(0.23)
$\kappa_{\theta \times D}$	4.02***	0.43	-2.48***	-2.54**	-5.08**	2.55***	3.42***	3.30***
	(0.76)	(0.72)	(0.92)	(1.03)	(2.18)	(0.95)	(0.94)	(0.89)
N	96	96	64	42	23	64	64	64
adj. R^2	0.54	0.32	0.21	0.29	0.13	0.32	0.39	0.41

Note: The dependant variable π_t is core CPI inflation. A † stands for a non-estimated coefficient. Constants are omitted. Column (1) is for raw data and corresponds to Panel A of Figure 1. Column (2) adjusts for inflation expectations (median Michigan expectations) and corresponds to Panel D of Figure 2. Columns (3) to (8) correspond to Panels B to G of Figure 3. The first three of these columns use households and firms expectations: (3) uses mean Michigan expectations, (4) the Survey of Consumer Expectations, (5) the Survey of Firms' Inflation Expectations. The last three of these columns use professional expectations: (6) uses then Cleveland Fed. measure, (7) the Survey of Professional Forecasters and (8) the Livingston survey. All these expectations are one year-ahead and adjusted to proxy one quarter-ahead expectations (see Supplemental Appendix C). Sample always ends in 2023Q4. It starts in 2000Q1 for Michigan, Cleveland and SPF, 2013Q3 for SCE and 2018Q2 for SoFIE.

comparable to Benigno and Eggertsson (2023). The equation we estimate are

$$\text{(B.5)} \quad \pi_t = \beta\pi^e_{t+1} + \kappa \log\theta_t + \kappa_2(\log\theta_t)^2 + \kappa_3(\log\theta_t)^3 + \gamma_v v_t + \gamma_{\pi_{-1}}\pi_{t-1} + \varepsilon_t$$

$$\text{(B.6)} \quad \pi_t = \beta\pi^e_{t+1} + \kappa \log\theta_t + \kappa_{\theta\times D}(D_t \times \log\theta_t) + \gamma_v v_t + \gamma_{D\times v}(D_t \times v_t) + \gamma_{\pi_{-1}}\pi_{t-1} + \varepsilon_t$$

where π_t is core quarter-to-quarter inflation, π^e_{t+1} is a measure of next quarter inflation expectation, $\theta_t = \frac{V_t}{U_t}$, v_t is a measure of supply shocks and D_t an indicator variable for $\theta_t > 1$. We restrict estimation to the post-Volcker period when long run expectations can reasonably be thought as stable. This is illustrated in Tables B.6 and B.7. We see that for both the cubic and piecewise-linear specifications, nonlinearity shows up with the experts measure of inflation expectations (Cleveland Fed.), but disappear when using a household measure (Michigan Survey of Consumers, mean or median). We also consider the post-2008 period and a longer sample that starts in 1960 and obtain similar results. The only case in which we do find a marginally significant (at 8%) steepening of the Phillips curve for high levels of θ with households expectations is for the post-2008 period when we use the piecewise-linear specification, when we choose median expectations from the Michigan Survey of Consumer and when we include lagged inflation (which enters then with a counterintuitive negative sign) (see Column (6) of Table B.8).

Table B.6—Estimation of Phillips Curves, Piecewise Linear Specification (B.6)

	Cleveland		Michigan (mean)		Michigan (median)	
	(1)	(2)	(3)	(4)	(5)	(6)
β	0.84***	0.64***	0.80***	0.54***	1.08***	0.69***
	(0.08)	(0.10)	(0.09)	(0.09)	(0.14)	(0.14)
κ	0.12	0.16	0.77***	0.58***	0.77***	0.55***
	(0.17)	(0.16)	(0.17)	(0.16)	(0.18)	(0.16)
$\kappa_{\theta\times D}$	4.00***	2.79***	-1.42*	-1.53**	-0.25	-0.86
	(0.64)	(0.76)	(0.83)	(0.76)	(0.83)	(0.75)
γ_v	-0.01	-0.01	-0.05***	-0.03**	-0.05***	-0.03**
	(0.01)	(0.01)	(0.02)	(0.01)	(0.02)	(0.02)
$\gamma_{v\times D}$	0.14***	0.12**	0.14**	0.10**	0.10*	0.07
	(0.05)	(0.05)	(0.05)	(0.05)	(0.06)	(0.05)
$\gamma_{\pi_{-1}}$		0.23***		0.37***		0.44***
		(0.08)		(0.07)		(0.07)
N	144	144	144	144	144	144
adj. R^2	0.62	0.64	0.56	0.63	0.51	0.61

Note: The dependant variable π_t is core CPI inflation. v is a measure of supply shocks. All expectations measures are one year-ahead and adjusted to proxy one quarter-ahead expectations (see Supplemental Appendix C). Sample is 1988Q1-2023Q4.

Table B.7—Estimation of Phillips Curves, Cubic Specification (B.5)

	Cleveland		Michigan (mean)		Michigan (median)	
	(1)	(2)	(3)	(4)	(5)	(6)
β	0.86***	0.64***	0.78***	0.51***	1.09***	0.69***
	(0.08)	(0.11)	(0.08)	(0.09)	(0.13)	(0.14)
κ	1.83***	1.31***	0.11	-0.06	0.49	0.14
	(0.24)	(0.29)	(0.31)	(0.29)	(0.31)	(0.28)
κ_2	2.76***	1.90***	-0.14	-0.42	0.59	-0.02
	(0.48)	(0.54)	(0.59)	(0.55)	(0.59)	(0.54)
κ_3	1.12***	0.77***	0.16	-0.04	0.50	0.14
	(0.27)	(0.28)	(0.31)	(0.29)	(0.31)	(0.29)
γ_v	-0.01	-0.01	-0.04**	-0.03*	-0.05***	-0.03**
	(0.01)	(0.01)	(0.02)	(0.01)	(0.02)	(0.02)
$\gamma_{\pi_{-1}}$		0.26***		0.38***		0.42***
		(0.08)		(0.07)		(0.07)
N	144	144	144	144	144	144
adj. R^2	0.61	0.63	0.54	0.62	0.51	0.61

Note: The dependant variable π_t is core CPI inflation. v is a measure of supply shocks. All expectations measures are one year-ahead and adjusted to proxy one quarter-ahead expectations (see Supplemental Appendix C). Sample is 1988Q1-2023Q4.

Table B.8—Estimation of Piecewise Linear Phillips Curve (B.3), Post-2008 Sample

	Cleveland		Michigan (mean)		Michigan (median)	
	(1)	(2)	(3)	(4)	(5)	(6)
β	0.58**	0.59**	0.58***	0.63***	0.88***	0.96***
	(0.26)	(0.26)	(0.13)	(0.14)	(0.20)	(0.20)
κ	0.50**	0.51**	0.83***	0.91***	0.84***	0.92***
	(0.24)	(0.24)	(0.22)	(0.22)	(0.22)	(0.22)
$\kappa_{\theta \times D}$	3.26***	3.68***	0.49	1.38	1.20	2.18*
	(1.04)	(1.34)	(1.22)	(1.33)	(1.10)	(1.23)
γ_v	-0.01	-0.01	-0.03*	-0.04*	-0.04**	-0.05**
	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)
$\gamma_{v \times D}$	0.17***	0.17***	0.14**	0.16***	0.11*	0.13**
	(0.06)	(0.06)	(0.05)	(0.05)	(0.06)	(0.06)
$\gamma_{\pi_{-1}}$		-0.07		-0.21		-0.22*
		(0.14)		(0.13)		(0.13)
N	64	64	64	64	64	64
adj. R^2	0.63	0.62	0.69	0.70	0.69	0.70

Note: The generic Phillips curve we estimate is $\pi_t = \beta\pi^e_{t+1} + \kappa \log\theta_t + \kappa_{\theta\times D}(D_t \times \log\theta_t) + \gamma_v v_t + \gamma_{D\times v}(D_t \times v_t) + \gamma_{\pi_{-1}}\pi_{t-1} + \varepsilon_t$. Constants are omitted. Inflation is quarter-to-quarter CPI core inflation (annualized). D is a dummy for $\log\theta_t > 0$ and v is a measure of supply shocks. Expectation measures are one year-ahead and adjusted to proxy one quarter-ahead expectations (see Supplemental Appendix C). Sample is 2008Q1-2023Q4.

Table B.9—Estimation of Cubic Phillips Curve (B.4), Post-2008 Sample

	Cleveland		Michigan (mean)		Michigan (median)	
	(1)	(2)	(3)	(4)	(5)	(6)
β	0.66**	0.66**	0.64***	0.69***	0.99***	1.10***
	(0.26)	(0.27)	(0.14)	(0.15)	(0.20)	(0.21)
κ	1.86***	1.79***	1.10**	1.38**	1.42***	1.83***
	(0.45)	(0.59)	(0.45)	(0.53)	(0.38)	(0.47)
κ_2	2.44***	2.36**	-0.26	-0.01	0.25	0.63
	(0.82)	(0.96)	(1.02)	(1.05)	(0.88)	(0.91)
κ_3	1.01**	0.98**	-0.28	-0.20	-0.07	0.04
	(0.45)	(0.49)	(0.51)	(0.52)	(0.46)	(0.46)
γ_v	-0.01	-0.01	-0.01	-0.02	-0.03	-0.04*
	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)
$\gamma_{\pi_{-1}}$		0.02		-0.14		-0.18
		(0.14)		(0.13)		(0.13)
N	64	64	64	64	64	64
adj. R^2	0.59	0.58	0.66	0.66	0.68	0.69

Note: The generic Phillips curve we estimate is $\pi_t = \beta\pi^e_{t+1} + \kappa \log\theta_t + \kappa_2(\log\theta_t)^2 + \kappa_3(\log\theta_t)^3 + \gamma_v v_t + \gamma_{\pi_{-1}}\pi_{t-1} + \varepsilon_t$, where v is a measure of supply shocks. Constants are omitted. Inflation is quarter-to-quarter CPI core inflation (annualized). Columns (1) and (2) use the Federal Reserve Bank of Cleveland measure of inflation expectations, Columns (3) and (4) use the Michigan Survey of Consumers measure of mean inflation expectations. Both measures are one year-ahead and adjusted to proxy one quarter-ahead expectations (see Supplemental Appendix C). Sample is 2008Q1-2023Q4.

Table B.10—Estimation of Piecewise Linear Phillips Curve (B.3), Long Sample

	Cleveland		Michigan (mean)		Michigan (median)	
	(1)	(2)	(3)	(4)	(5)	(6)
β	1.03***	0.73***	0.97***	0.57***	1.25***	0.85***
	(0.04)	(0.07)	(0.04)	(0.06)	(0.05)	(0.09)
κ	0.10	0.25	0.38*	0.48***	0.68***	0.62***
	(0.19)	(0.18)	(0.21)	(0.18)	(0.22)	(0.20)
$\kappa_{\theta\times D}$	4.33***	2.54***	-1.01	-1.44*	-0.94	-1.18
	(0.80)	(0.85)	(0.90)	(0.78)	(0.90)	(0.85)
γ_v	0.05***	0.04***	0.00	-0.00	-0.07***	-0.05***
	(0.01)	(0.01)	(0.01)	(0.01)	(0.02)	(0.02)
$\gamma_{v\times D}$	0.01	0.02	-0.01	0.00	0.10	0.08
	(0.06)	(0.06)	(0.07)	(0.06)	(0.07)	(0.07)
$\gamma_{\pi_{-1}}$		0.29***		0.44***		0.33***
		(0.06)		(0.05)		(0.06)
N	256	255	256	255	184	184
adj. R^2	0.80	0.82	0.75	0.82	0.77	0.80

Note: The generic Phillips curve we estimate is $\pi_t = \beta\pi^e_{t+1} + \kappa\log\theta_t + \kappa_{\theta\times D}(D_t \times \log\theta_t) + \gamma_v v_t + \gamma_{D\times v}(D_t \times v_t) + \gamma_{\pi_{-1}}\pi_{t-1} + \varepsilon_t$. Constants are omitted. Inflation is quarter-to-quarter CPI core inflation (annualized). D is a dummy for $\log\theta_t > 0$ and v is a measure of supply shocks. The Federal Reserve Bank of Cleveland measure of inflation expectations is patched with the Livingston survey measure before 1982Q2). Note that in Columns (5) and (6) (median inflation expectations from the Michigan Survey of Consumers), we start in 1978Q1 (no median data before). Expectations measures are one year-ahead and adjusted to proxy one quarter-ahead expectations (see Supplemental Appendix C). Sample is 1960Q1-2023Q4 for Columns (1) to (4), 1978Q1-2023Q4 for Columns (5) and (6).

Table B.11—Estimation of Cubic Phillips Curve (B.4), Long Sample

	Cleveland		Michigan (mean)		Michigan (median)	
	(1)	(2)	(3)	(4)	(5)	(6)
β	1.03***	0.72***	0.97***	0.58***	1.26***	0.89***
	(0.04)	(0.07)	(0.04)	(0.06)	(0.05)	(0.09)
κ	1.88***	1.31***	-0.33	-0.23	-0.08	-0.10
	(0.29)	(0.30)	(0.32)	(0.28)	(0.34)	(0.32)
κ_2	2.56***	1.40**	0.00	-0.56	0.64	0.16
	(0.58)	(0.59)	(0.64)	(0.56)	(0.63)	(0.61)
κ_3	1.01***	0.51	0.36	-0.06	0.76**	0.41
	(0.32)	(0.31)	(0.35)	(0.31)	(0.35)	(0.34)
γ_v	0.05***	0.03***	-0.00	-0.00	-0.07***	-0.05***
	(0.01)	(0.01)	(0.01)	(0.01)	(0.02)	(0.02)
$\gamma_{\pi_{-1}}$		0.31***		0.43***		0.30***
		(0.06)		(0.05)		(0.06)
N	256	255	256	255	184	184
adj. R^2	0.80	0.82	0.76	0.82	0.78	0.80

Note: The generic Phillips curve we estimate is $\pi_t = \beta\pi^e_{t+1} + \kappa \log\theta_t + \kappa_2(\log\theta_t)^2 + \kappa_3(\log\theta_t)^3 + \gamma_v v_t + \gamma_{\pi_{-1}}\pi_{t-1} + \varepsilon_t$, where v is a measure of supply shocks. Constants are omitted. Inflation is quarter-to-quarter CPI core inflation (annualized). The Federal Reserve Bank of Cleveland measure of inflation expectations is patched with the Livingston survey measure before 1982Q2). Note that in Columns (5) and (6) (median inflation expectations from the Michigan Survey of Consumers), we start in 1978Q1 (no median data before). Expectations measures are one year-ahead and adjusted to proxy one quarter-ahead expectations (see Supplemental Appendix C). Sample is 1960Q1-2023Q4 for Columns (1) to (4), 1978Q1-2023Q4 for Columns (5) and (6).

C. Transforming One-Year-Ahead Inflation Expectations into One-Quarter-Ahead Inflation Expectations

The various inflation expectations we use are one-year-ahead. For example, in the Michigan Survey of Consumers, every month a representative sample of consumers are asked the following question: "*By about what percent do you expect prices to go (up/down) on the average, during the next 12 months?*" The answer to this question is then the one-year-ahead inflation expectations $E_t\pi_{t+4,t}$. To keep consistency with the quarter-to-quarter inflation when estimating a New-Keynesian Phillips curve, we rescale the one-year-ahead expected inflation in the following way.[3] Note that this rescaling only affects the estimate of β.

We first assume that realized quarter-to-quarter inflation follows an AR(1) process with persistence ρ_π:

$$\pi_{t+1,t} = \rho_\pi \pi_{t,t-1} + \epsilon_t \tag{C.1}$$

Consumers may or may not have the correct belief on ρ_π. We assume they believe that persistence is $\widetilde{\rho}$, so that the perceived law of motion of inflation is

$$\pi_{t+1,t} = \widetilde{\rho}\pi_{t,t-1} + \epsilon_t \tag{C.2}$$

Consumers observe a noisy signal on inflation:

$$s_t = \pi_{t,t-1} + \eta_t \tag{C.3}$$

where s_t is a noisy signal on inflation and η_t is the noise. We assume the noise is mean zero, i.i.d., orthogonal to ϵ_t and independent across time following the standard literature of noisy information models. Consumers will form quarter-to-quarter inflation expectation, denoted by $E_t\pi_{t+1,t}$, using a Kalman filter:

$$E_t\pi_{t+1,t} = \widetilde{\rho}E_t\pi_{t,t-1} = \widetilde{\rho}(1-K)E_{t-1}\pi_{t,t-1} + \widetilde{\rho}K\pi_{t,t-1} + \widetilde{\rho}K\eta_t \tag{C.4}$$

where K is the Kalman gain.

We do observe one-year-ahead expected inflation:

$$E_t\pi_{t+4,t} \equiv E_t(\pi_{t+4,t+3} + \pi_{t+3,t+2} + \pi_{t+2,t+1} + \pi_{t+1,t})$$

Using the perceived law of motion (C.2):

$$\begin{aligned} E_t\pi_{t+4,t} &= (1+\widetilde{\rho}+\widetilde{\rho}^2+\widetilde{\rho}^3)E_t\pi_{t+1,t} \\ &= (1+\widetilde{\rho}+\widetilde{\rho}^2+\widetilde{\rho}^3)\big(\widetilde{\rho}(1-K)E_{t-1}\pi_{t,t-1} + \widetilde{\rho}K\pi_{t,t-1} + \widetilde{\rho}K\eta_t\big) \end{aligned} \tag{C.5}$$

[3] For details of this approach extended to multi-variable joint learning environment, see Hou [2020], 'Uncovering Subjective Models from Survey Expectations', Simon Fraser University Working paper, available at SSRN 3728884.

We use the $t-1$ version of (C.5) and plug it in the above equation to obtain:

$$E_t\pi_{t+4,t} = \underbrace{\widetilde{\rho}(1-K)}_{\psi_1} E_{t-1}\pi_{t+3,t-1} + \underbrace{(1+\widetilde{\rho}+\widetilde{\rho}^2+\widetilde{\rho}^3)\widetilde{\rho}K}_{\psi_2}\pi_{t,t-1} + (1+\widetilde{\rho}+\widetilde{\rho}^2+\widetilde{\rho}^3)\widetilde{\rho}K\eta_t \tag{C.6}$$

We can estimate equation (C.6) with OLS because η_t is the i.i.d noise orthogonal to inflation.[4] We need to use quarter-to-quarter (not annualized) inflation for $\pi_{t,t-1}$ and year-ahead expected inflation and its lag from the Michigan Survey of Consumers. We use Headline CPI as proxy for $\pi_{t,t-1}$.

Given the estimate on the perceived persistence of inflation, the quarter-to-quarter expected inflation is implied by equation (C.5):

$$E_t\pi_{t+1,t} = \frac{1}{1+\widetilde{\rho}+\widetilde{\rho}^2+\widetilde{\rho}^3}E_t\pi_{t+4,t} \tag{C.7}$$

D. Sensitivity Analysis with Different Values of β

Here we estimate the Phillips Curve with MSA-level one-year-ahead inflation expectations constructed by (Binder, Kamdar and Ryngaert 2024):

$$\pi_{i,t} - \beta\pi^e_{i,t+1} = \kappa\log\theta_{i,t} + \kappa_{\theta\times D}(D_{i,t}\times\log\theta_{i,t}) + \gamma_i + \gamma_t + \varepsilon_{i,t} \tag{D.1}$$

We present the results with a range of β values from 0.99 to 0.59 in Table D.1. It can be seen that the estimate of κ is quite stable across different values of β, whereas the estimate on the nonlinear part, $\kappa_{\theta\times D}$, is decreasing in β. To obtain significant steepening one needs β to be as low as 0.64.

Table D.1—Estimation of the Piecewise-Linear Phillips Curve with Various Values of β

	Change forcing β								
β	0.99	0.94	0.89	0.84	0.79	0.74	0.69	0.64	0.59
κ	1.14***	1.14***	1.14***	1.15***	1.15***	1.15***	1.15***	1.15***	1.15***
	(0.16)	(0.16)	(0.15)	(0.15)	(0.15)	(0.15)	(0.15)	(0.15)	(0.15)
$\kappa_{\theta\times D}$	0.06	0.19	0.33	0.46	0.59	0.72	0.85	0.98*	1.11**
	(0.51)	(0.52)	(0.52)	(0.52)	(0.53)	(0.53)	(0.53)	(0.54)	(0.54)
Observations	860	860	860	860	860	860	860	860	860
Adjusted R^2	0.13	0.13	0.14	0.15	0.15	0.16	0.17	0.17	0.18
MSA F.E.	Y	Y	Y	Y	Y	Y	Y	Y	Y
Time F.E.	N	N	N	N	N	N	N	N	N

Note: We estimate $\pi_{i,t} - \beta\pi^e_{i,t+1} = \kappa\log\theta_{i,t} + \kappa_{\theta\times D}(D_{i,t}\times\log\theta_{i,t}) + \gamma_i + \gamma_t + \varepsilon_{i,t}$ with different values of β. The measure of inflation is quarter-to-quarter (annualized) Core CPI. Sample is 2001Q4-2024Q3.

[4] If η_t is serially correlated, $\widetilde{\rho}$ will be overestimated, resulting in lower-than-actual $E_t\pi_{t+1,t}$. Using such $E_t\pi_{t+1,t}$ in estimation will lead to over-estimated β. To alleviate this concern, we show in Table D.1 that our results are robust to different values of β.

E. Aggregate Estimation Results with IV Estimation

Here we repeat the nonlinear Phillips Curve estimations when instrumenting tightness and lagged inflation by their first and second lag.[5] We also do an exercise (post 2008) using monetary policy shocks to instrument tightness, expectations and lagged inflation (as suggested by Barnichon & Mesters [2020] and done in Beaudry, Hou and Portier (2024*a*)).[6] In both cases, we confirm the OLS results but the exercise must be taken with a grain of salt.

Table E.1—Estimation of Piecewise Linear Phillips Curve (B.3), IV Using Lags as Instruments, Post Volcker

	(1)	(2)	(3)	(4)
κ	0.03	-0.08	0.70***	0.19
	(0.25)	(0.17)	(0.23)	(0.12)
$\kappa_{\theta\times D}$	4.14***	1.33	-0.53	-0.98*
	(0.60)	(1.55)	(1.11)	(0.55)
β	0.83***	0.31	0.68***	0.16*
	(0.11)	(0.29)	(0.15)	(0.09)
γ_v	-0.01	-0.01	-0.04***	-0.01*
	(0.01)	(0.01)	(0.01)	(0.01)
$\gamma_{v\times D}$	0.15***	0.06***	0.16***	0.07***
	(0.05)	(0.02)	(0.03)	(0.02)
$\gamma_{\pi_{-1}}$		0.66**		0.85***
		(0.28)		(0.08)
N	144	144	144	144

Note: The generic Phillips curve we estimate is $\pi_t = \beta\pi^e_{t+1} + \kappa\log\theta_t + \kappa_{\theta\times D}(D_t \times \log\theta_t) + \gamma_v v_t + \gamma_{D\times v}(D_t \times v_t) + \gamma_{\pi_{-1}}\pi_{t-1} + \varepsilon_t$. Constants are omitted. Inflation is quarter-to-quarter CPI core inflation (annualized). D is a dummy for $\log\theta_t > 0$ and v is a measure of supply shocks. Columns (1) and (2) use the Federal Reserve Bank of Cleveland measure of inflation expectations (patched with the Livingston survey for the first part of the sample), Columns (3) and (4) use the Michigan Survey of Consumers measure of mean inflation expectations. Both measures are one year-ahead and adjusted to proxy one quarter-ahead expectations (see Supplemental Appendix C). $\log\theta_t$, $D_t \times \log\theta_t$ and π_{t-1} are instrumented by their two first lags. All results are using IV-GMM procedure, Newey-West HAC standard errors with six lags are reported in parentheses. Sample is 1988Q1-2023Q4.

[5] Mavroeidis, Plagborg-Møller and Stock (2014) have discussed the relative fragility of using such instruments.

[6] We have there 12 instruments and 114 data points, which most likely raises a many instruments problem.

TABLE E.2—ESTIMATION OF PIECEWISE LINEAR PHILLIPS CURVE (B.3), IV USING MONETARY SHOCK AS INSTRUMENTS, 2008Q1-2023Q4

	(1)	(2)	(3)	(4)
β	0.85***	0.94***	0.68***	0.51
	(0.23)	(0.29)	(0.25)	(0.32)
κ	0.17	0.22	0.82***	0.57**
	(0.22)	(0.31)	(0.13)	(0.29)
$\kappa_{\theta\times D}$	9.63***	10.68***	0.81	0.66
	(3.00)	(3.70)	(2.42)	(2.60)
γ_v	-0.01	-0.02	-0.04***	-0.02
	(0.01)	(0.02)	(0.01)	(0.02)
$\gamma_{v\times D}$	0.12	0.15*	0.11***	0.12***
	(0.08)	(0.09)	(0.04)	(0.04)
$\gamma_{\pi_{-1}}$		-0.19		0.13
		(0.23)		(0.19)
N	114	114	114	114

Note: The generic Phillips curve we estimate is $\pi_t = \beta\pi^e_{t+1} + \kappa \log\theta_t + \kappa_{\theta\times D}(D_t \times \log\theta_t) + \gamma_v v_t + \gamma_{D\times v}(D_t \times v_t) + \gamma_{\pi_{-1}}\pi_{t-1} + \varepsilon_t$. Constants are omitted. Inflation is quarter-to-quarter CPI core inflation (annualized). D is a dummy for $\log\theta_t > 0$ and v is a measure of supply shocks. Columns (1) and (2) use the Federal Reserve Bank of Cleveland measure of inflation expectations, Columns (3) and (4) use the Michigan Survey of Consumers measure of mean inflation expectations. Both measures are one year-ahead and adjusted to proxy one quarter-ahead expectations (see Supplemental Appendix C). π^e_{t+1}, $\log\theta_t$, $D_t \times \log\theta_t$ and π_{t-1} are instrumented by six lags of a measure of monetary shocks and six lags of their square. The measure of monetary shocks is the updated series of Bu et al. [2021], as kindly provided by the authors. All results are using IV-GMM procedure, Newey-West HAC standard errors with six lags are reported in parentheses. Sample is 1988Q1-2023Q4 .

TABLE E.3—ESTIMATION OF CUBIC PHILLIPS CURVE (B.4), IV USING LAGS AS INSTRUMENTS, 2008Q1-2023Q4

	(1)	(2)	(3)	(4)
κ	1.62***	0.47	0.40	-0.15
	(0.21)	(0.53)	(0.42)	(0.20)
κ_2	2.90***	1.01	0.30	-0.44
	(0.39)	(1.23)	(0.84)	(0.47)
κ_3	1.24***	0.44	0.33	-0.18
	(0.22)	(0.57)	(0.43)	(0.27)
β	0.90***	0.38	0.66***	0.14*
	(0.09)	(0.32)	(0.14)	(0.08)
γ_v	-0.01	-0.01	-0.04**	-0.01
	(0.02)	(0.01)	(0.02)	(0.01)
$\gamma_{\pi_{-1}}$		0.61*		0.86***
		(0.32)		(0.09)
N	144	144	144	144

Note: The generic Phillips curve we estimate is $\pi_t = \beta\pi^e_{t+1} + \kappa\log\theta_t + \kappa_2(\log\theta_t)^2 + \kappa_3(\log\theta_t)^3 + \gamma_v v_t + \gamma_{\pi_{-1}}\pi_{t-1} + \varepsilon_t$, where v is a measure of supply shocks. Constants are omitted. Inflation is quarter-to-quarter CPI core inflation (annualized). Columns (1) and (2) use the Federal Reserve Bank of Cleveland measure of inflation expectations, Columns (3) and (4) use the Michigan Survey of Consumers measure of mean inflation expectations. Both measures are one year-ahead and adjusted to proxy one quarter-ahead expectations (see Supplemental Appendix C). $\log\theta_t$, $(\log\theta_t)^2$, $(\log\theta_t)^3$ and π_{t-1} are instrumented by their two first lags. Sample is 1988Q1-2023Q4.

Table E.4—Estimation of Cubic Phillips Curve (B.4), IV Using Monetary Shock as Instruments, 2008Q1-2023Q4

	(1)	(2)	(3)	(4)
κ	2.59***	2.09**	0.78	0.43
	(0.70)	(0.86)	(1.04)	(0.98)
κ_2	3.37***	2.87**	1.00	1.04
	(1.15)	(1.24)	(1.07)	(1.07)
κ_3	1.28**	1.14*	0.61	0.71
	(0.61)	(0.62)	(0.53)	(0.55)
β	0.43***	0.36**	0.51**	0.39
	(0.13)	(0.15)	(0.24)	(0.26)
γ_v	-0.00	-0.00	-0.03	-0.02
	(0.02)	(0.02)	(0.02)	(0.02)
$\gamma_{\pi_{-1}}$		0.16		0.23
		(0.17)		(0.16)
N	114	114	114	114

Note: The generic Phillips curve we estimate is $\pi_t = \beta\pi^e_{t+1} + \kappa \log\theta_t + \kappa_2(\log\theta_t)^2 + \kappa_3(\log\theta_t)^3 + \gamma_v v_t + \gamma_{\pi_{-1}}\pi_{t-1} + \varepsilon_t$, where v is a measure of supply shocks. Constants are omitted. Inflation is quarter-to-quarter CPI core inflation (annualized). Columns (1) and (2) use the Federal Reserve Bank of Cleveland measure of inflation expectations, Columns (3) and (4) use the Michigan Survey of Consumers measure of mean inflation expectations. Both measures are one year-ahead and adjusted to proxy one quarter-ahead expectations (see Supplemental Appendix C). π^e_{t+1}, $\log\theta_t$, $(\log\theta_t)^2$, $(\log\theta_t)^3$ and π_{t-1} are instrumented by six lags of a measure of monetary shocks and six lags of their square. The measure of monetary shocks is the updated series of Bu et al. [2021], as kindly provided by the authors. Sample is 1988Q1-2023Q4.

F. Understanding the Differences with Gitti (2024)

We first redo our analysis with monthly data at city level by estimating our baseline specifications (B.2) and (B.1). In particular, we compare the results with either monthly, quarterly, or quarter-MSA fixed effects as in Gitti (2024).[7]

The results in Table F.1 show no significant evidence for nonlinearity in Phillips Curve when controling for either quarter-year or month-year fixed effects. However, as in Gitti (2024) we find a significant nonlinear relationship when we estimate the piecewise linear specification and control for MSA and quarter-year × MSA fixed effects.

It is worth pointing out that our baseline specifications are slightly different from that in Gitti (2024). To most clearly see the differences, we have also directly estimated the specification from Gitti (2024):

$$\begin{aligned}\pi_{it} =& c + \alpha_i + \gamma_{t_q} + \delta_{it_q} + \psi^1_\theta \log\theta_{it} + \psi^2_\theta \log\theta_{it} \times I_{\{\theta_{it}>1\}} \\ &+ \psi_I I_{\{\theta_{it}>1\}} + \psi_p p^x_{it} + \varepsilon_{it}\end{aligned} \tag{F.1}$$

where α_i is MSA fixed effect, γ_{t_q} is year-quarter fixed effect, δ_{it_q} is the interaction between year-quarter and MSA fixed effects. p^x_{it} is local relative price of intermediate inputs in MSA i and year-month t, measured as the log ratio of national

[7]We use monthly year-to-year Core CPI interpolated in the same way as in Gitti (2024).

Table F.1—City Level Estimation of Phillips Curve at Monthly Frequency with Different Dummies

	Piecewise Linear (B.2)				Cubic (B.1)		
	(1)	(2)	(3)		(4)	(5)	(6)
κ	1.26***	1.18***	0.24***	κ	1.11***	1.06***	0.49***
	(0.19)	(0.17)	(0.08)		(0.28)	(0.24)	(0.09)
$\kappa_{\theta\times D}$	-0.20	-0.16	0.45**	κ_2	-0.09	-0.08	0.12
	(0.54)	(0.49)	(0.18)		(0.25)	(0.23)	(0.09)
				κ_3	-0.00	-0.00	0.00
					(0.09)	(0.08)	(0.03)
Observations	4420	4420	4401		4420	4420	4401
Adjusted R^2	0.70	0.70	0.97		0.70	0.70	0.97
MSA F.E.	Y	Y	Y		Y	Y	Y
Year-Quarter F.E.	N	Y	Y		N	Y	Y
Year-Month F.E.	Y	N	N		Y	N	N
Year-Quarter × MSA F.E.	N	N	Y		N	N	Y

Note: The dependant variable π_{it} is core CPI inflation at the MSA level. We iterate expected inflation forward and assume common across MSAs long-run expectations anchored by monetary policy as in Hazell et al. (2022). Columns (1)-(3) are estimates of equation (B.2) and columns (4)-(6) are estimates of equation (B.1). All results controlling for MSA fixed effects. Columns (1) and (4) control for full set of month-year fixed effects, columns (2) and (5) control for quarter-year fixed effects, column (3) and (6) control for quarter-year × MSA fixed effects as in Gitti (2024). Standard Errors are clustered at year×MSA level. Sample is 2001M12-2024M8.

manufacturing PPI in year-month t and all-items CPI in MSA i and year-month t. For inflation π_{it}, we follow Gitti (2024) and use year-to-year inflation at monthly frequency. One thing to point out is the specification (F.1) is different from the one we have estimated (see Supplemental Appendix B.1), as the controls p^x_{it} are specific to the vertical supply chain structure in Gitti (2024). However, the results remain qualitatively in line with those we show in Supplemental Appendix B.1.

In Table F.2, we are able to replicate the evidence of "steepening" Phillips Curve in column (2) once we control for year-quarter × MSA fixed effects. However, there is no sign of steepening if either quarter-year or month-year fixed effects are controled. To this end, we think what is driving the difference between our main results with Gitti (2024) is the introduction of city-specific-year-quarter fixed effects in the monthly regression, which means that the slope of the Phillips curve is being identified by the within city variation across three month within a quarter. With this specification, as we have discussed in the main text, there remains some signs of nonlinearity. However, we want to point out that the residual nonlinearity found with this specification provides very little support for a steep Phillips curve during the Covid-19 period. In fact, when including city-specific-year-quarter fixed effects, one estimates an extremely flat Phillips curve – albeit, slightly steeper when the vacancy-to-unemployment v/u ratio is above 1– with the slope when $v/u > 1$ being in fact smaller than the slope estimated by Benigno and Eggertsson (2023) when $v/u < 1$. So this type of specification suggest that the Phillips curve may be even flatter that pre-Covid-19 estimates

Table F.2—Estimation of Equation (F.1) with Different Fixed Effects

	(1)	(2)	(3)	(4)
ψ_θ^1	0.90***	0.23***	1.36***	1.46***
	(0.098)	(0.084)	(0.180)	(0.200)
ψ_θ^2	3.11***	0.44**	-0.32	-0.39
	(0.537)	(0.184)	(0.471)	(0.518)
ψ_I	-0.20	0.04	-0.03	-0.04
	(0.176)	(0.043)	(0.124)	(0.130)
ψ_p	3.47***	-0.29	-8.52***	-9.57***
	(0.967)	(0.951)	(1.890)	(2.094)
Observations	4420	4401	4420	4420
MSA F.E.	Y	Y	Y	Y
Year-Quarter F.E.	N	Y	Y	N
Year-Month F.E.	N	N	N	Y
Year-Quarter × MSA F.E.	N	Y	N	N

Note: All columns controlling for MSA fixed effects. Column (2) controls for quarter-year fixed effects and interactions between quarter-year and MSA fixed effects as in Gitti (2024). Column (3) controls for quarter-year fixed effects. Column (4) controls for month-year fixed effects. Standard Errors are clustered at year×MSA level. Sample is 2001M12-2024M9.

may have suggested, with labour market have very little effect on inflation both when v/u is greater or smaller than zero.