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Inconsistency of the Capital Asset Pricing Model in a Multi-Currency Environment

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ABSTRACT

The capital asset pricing model (CAPM) is a widely adopted model in asset pricing theory and portfolio construction because of its intuitive nature. One of its main conclusions is that there exists a global market portfolio that each rational investor should hold in proportion to the risk-free asset. In this paper, we demonstrate theoretically and through an example that the CAPM cannot hold in a multi-currency environment. This is because it produces different market risk premia depending on the investor's base currency unless each exchange rate is uncorrelated with the asset prices in the portfolio.

1 | Introduction

The capital asset pricing model (CAPM) (Sharpe 1964; Lintner 1965; Mossin 1966) is an easily understandable and straightforward model that explains the relationship between risk and expected return in an efficient market and is widely regarded as the initial and most commonly used asset pricing model. Together with the Markowitz portfolio selection model (Markowitz 1952), it is at the foundation of modern financial theory. Despite numerous proposed advancements over the past five decades, the original capital asset pricing and Markowitz models remain the primary tools used by scholars and investors for asset pricing and allocation (Rubinstein 2002).

The Markowitz (1952) portfolio selection model proposes that constructing portfolios with minimum variance given an expected return constraint can generate an efficient frontier, where each portfolio on the frontier provides either the highest expected return for a given level of risk or the lowest risk for a target return. The CAPM utilises this result by demonstrating that, under specific assumptions, in equilibrium, the market portfolio, calculated by dividing each asset's market capitalization by the total market capitalization of all assets, lies on the efficient frontier. Consequently, a linear relationship between the risk premia of the asset and of the market portfolio can be established. This relationship is captured by the asset's beta, which is the covariance of the asset return with the market portfolio return, standardised by the variance of the market portfolio.

The international capital asset pricing model (ICAPM), sometimes also referred to as the global capital asset pricing model (GCAPM), has been proposed as an extension of the traditional CAPM to address the challenge of a multi-currency environment (Solnik 1974; Stulz 1981; Adler and Dumas 1983; Serita 1991; Wilkie 1997; Thomson et al. 2016). The ICAPM recognizes that investors are concerned with consumption in their respective local currency and therefore evaluate portfolio risk, which includes both market and currency risks, differently based on their base currency. While the literature on the ICAPM focuses on taking into consideration currencies as another factor for the estimation of risk premia, our paper shows that including currencies that are correlated with the assets produces different risk premia depending on the base currency.

The aim of the present study is to contribute to the theoretical discussion surrounding the CAPM by examining its applicability in a multi-currency environment. The question posed here is: if an investor calculates their implied returns in a base currency, for example, US Dollar (USD), and converts these returns to another currency, for example, Euro (EUR), will they retrieve the same implied returns if they convert the asset's price in EUR and calculate their implied risk premia? In other words, are the implied returns in USD converted to EUR the same as the implied returns calculated in EUR? As we shall see, these two values are not equal if asset returns are correlated with the currency rate, which makes the CAPM inconsistent in a multi-currency environment: investors operating with different base currencies may imply different equilibrium risk premia. We prove this inconsistency and illustrate it with a simple example.

Our findings have significant empirical and practical implications for the asset-management industry. Empirically, the traditional CAPM may be untestable in an international context, as the risk premia obtained differ depending on the investor's base currency. From a practical perspective, the Black and Litterman (1992) model, which extends the CAPM to estimate implied expected returns and incorporate investor views, faces challenges in this setting. Specifically, our results suggest that the CAPM used as a neutral starting point in the Black–Litterman model cannot consistently estimate expected risk premia for investors with different base currencies.

In their seminal work, Black and Litterman (1992) argued that incorporating the global CAPM equilibrium improves investment models, stating: “Consideration of the global CAPM equilibrium can significantly improve the usefulness of these models. In particular, equilibrium returns for equities, bonds, and currencies provide a neutral starting point for estimating the set of expected excess returns needed to drive the portfolio optimization process. This set of neutral weights can then be tilted in accordance with the investor's view.” While their results are presented from a USD perspective, they also suggest that similar findings would hold for other currencies (Black and Litterman 1992, 30). However, our analysis indicates that market risk premia in the Black–Litterman model cannot be uniquely determined across base currencies. Moreover, the process of deriving market-implied views must be reconsidered when asset and currency correlations are non-zero. Consequently, the neutral starting point for asset allocation in the Black–Litterman model is inherently dependent on the investor's base currency, warranting further investigation into its application in an international context.

Our results emphasise the importance of properly accounting for currency risk in international markets. If market participants universally hedged currency risk, the correlations between asset returns and currencies would vanish, and the CAPM could hold in its traditional form. However, this is not the case. As Black (1989) demonstrated, the optimal level of exchange rate hedging is always less than 100%, and hedging entirely in forward markets is costly. Additionally, full hedging eliminates potential gains from currency returns (Glen and Jorion 1993) and the natural hedging benefits that currency exposure provides against underlying asset risks (Campbell et al. 2010).

These considerations underscore that currency risk is priced in financial markets. For instance, Karolyi and Wu (2022) showed that the magnitude and significance of currency risk premia are influenced by firm characteristics, industry sectors, and the degree of internationalisation of the firms, highlighting the complexity of currency risk pricing and its relationship to economic fundamentals.

While the CAPM remains a valuable framework for asset valuation, our findings highlight the need for careful consideration when applying it in an international context. Asset risk premia can vary significantly and even exhibit opposite signs, depending on the chosen reference currency. This variability underscores the necessity of incorporating currency risk into asset pricing and portfolio optimization models. The more recent literature has suggested additional risk factors (Brusa et al. 2014; Opie and Riddiough 2020; Nucera et al. 2023).

The remainder of this paper is organised as follows. Section 2 introduces the international CAPM and shows that the CAPM cannot hold in a multi-currency environment unless the correlations of the assets and exchange rates are zero. Section 3 provides an example showing that the CAPM does not hold in a multi-currency environment. Finally, Section 4 concludes the paper and discusses opportunities for future research.

2 | Inconsistency of the CAPM in a Multi-Currency Environment

To illustrate the inconsistency of the CAPM in a multi-currency framework, let us consider an investor with base currency k, for example $k = \$, €, \ldots$, and let us assume that there is no segmentation of the international capital market, that is, national capital markets are perfectly integrated. When markets are segmented, expected returns are shaped by country-specific risks and domestic market conditions. Segmentation leads to differences in asset pricing relationships across countries, with local risk factors playing a dominant role (Karolyi and Stulz 2003). In such cases, investors are exposed to both global and local sources of risk, and expected returns reflect a combination of the two. The key theoretical implication is that if a group of investors does not participate in international markets, the world market portfolio becomes inefficient. As a result, the traditional ICAPM must be extended to include an additional factor that captures the portion of domestic risk which cannot be diversified internationally due to segmentation. The degree of market integration determines the importance of this factor: as integration increases, the premium required for this undiversifiable domestic risk declines, thereby reducing the international cost of capital. Quantifying this trade-off explicitly would require a theoretical framework that models the underlying economic forces driving the transition from segmentation to integration. A possible approach is presented in Arouri et al. (2012). However, as shown in the present paper, the issue of the inconsistency in the CAPM equation still remains and cannot be resolved introducing multiple factors.

The single-factor ICAPM states that, if markets are in equilibrium, then the risk premia in currency k, that is, the additional return that investors demand for taking the risk of holding assets denominated in a foreign currency, are

$$\mathbf{r}_k = \frac{R_k}{\sigma_k}\Sigma_k\mathbf{w}. \tag{1}$$

Here, $\mathbf{r}_k$ is the vector of risk premia in currency k, whose element $r_{i/k}$ is the risk premium of asset i, $i = 1, \ldots, n$, in currency k; Σ_k is the covariance matrix in currency k, whose element $\sigma_{i/k,j/k}$ is the covariance between the prices of assets i and j in currency k; σ_k is the volatility of the world market portfolio priced in currency k, and $\mathbf{w}$ is the vector of the asset weights of the world market portfolio, whose element w_j is the weight of asset j, $j = 1, \ldots, n$. The Sharpe ratio in currency k,

$$R_k = \frac{r_k}{\sigma_k}, \tag{2}$$

is the ratio of the risk premium r_k of the world portfolio in currency k and its volatility σ_k. The risk premium is the difference $r_k = \mu_k - r_{\mathrm{f},k}$ between the expected return μ_k and the risk-free interest rate $r_{\mathrm{f},k}$, both in currency k. The Sharpe ratio divided by the volatility of the market portfolio, R_k / σ_k, is commonly interpreted as the market price of risk.

Equation (1) implies that to calculate the vector of the implied risk premia, an assumption should be made about the market portfolio's Sharpe ratio, and the CAPM assumption that all investors have the same market portfolio regardless of their base currency should hold. However, as we shall see, this assumption does not hold unless the returns of the assets in the portfolio and the exchange rates are uncorrelated.

If an investor wants to calculate the implied risk premium of asset i in the base currency k, they would use Equation (1) and get

$$r_{i/k} = \frac{R_k}{\sigma_k}\sum_{j=1}^{n}\sigma_{i/k,j/k}w_j = \beta_{i/k}r_k, \tag{3}$$

where $\beta_{i/k}$ is the beta of asset i in currency k,

$$\beta_{i/k} = \frac{1}{\sigma_k^2}\sum_{j=1}^{n}\sigma_{i/k,j/k}w_j. \tag{4}$$

For instance, an investor with USD as their base currency would use

$$r_{i/\$} = \frac{R_\$}{\sigma_\$}\sum_{j=1}^{n}\sigma_{i/\$,j/\$}w_j = \beta_{i/\$}r_\$. \tag{5}$$

The price $S_{i/€}(t)$ of an asset i in EUR at time t is obtained from its price $S_{i/\$}(t)$ in USD multiplying the latter by the exchange rate of 1 USD to $X_{\$/€}(t)$ EUR,

$$S_{i/€}(t) = S_{i/\$}(t)X_{\$/€}(t). \tag{6}$$

Assuming that both assets and the FX rate follow a log-normal distribution, the asset price in currency k after a time interval $\Delta t = t_1 - t_0$ (assumed to be the time horizon of market participants, or the duration between portfolio readjustments) is

$$S_{i/k}(t_1) = S_{i/k}(t_0)\exp\left[\left(r_{i/k} + r_{\mathrm{f},k} - q_{i/k} - \frac{1}{2}\sigma_{i/k}^2\right)\Delta t + \sigma_{i/k}\varepsilon_{i/k}\sqrt{\Delta t}\right], \quad k = \$, €, \tag{7}$$

where $q_{i/k}$ is the convenience (e.g., dividend) yield of asset i in currency k that for simplicity in the following we will set to 0, $\sigma_{i/k}$ is the volatility of asset i in currency k, and $\varepsilon_{i/k}$ is a standard normal random variable. In the same manner, the FX rate after a time interval $\Delta t = t_1 - t_0$ is

$$X_{\$/€}(t_1) = X_{\$/€}(t_0)\exp\left[\left(r_{\$/€} + r_{\mathrm{f}/€} - r_{\mathrm{f}/\$} - \frac{1}{2}\sigma_{\$/€}^2\right)\Delta t + \sigma_{\$€/}\varepsilon_{\$/€}\sqrt{\Delta t}\right], \tag{8}$$

where $r_{\$/€}$ is the risk premium of the FX rate, $\sigma_{\$/€}$ is the volatility of the FX rate, and $\varepsilon_{\$/€}$ is a standard normal random variable. From Equations (6–8) we get

$$S_{i/\$}(t_1)X_{\$/€}(t_1) = S_{i/\$}(t_0)X_{\$/€}(t_0) \times\exp\left[\left(r_{i/\$} + r_{\mathrm{f}/€} - \frac{1}{2}\sigma_{i/\$}^2 + r_{\$/€} - \frac{1}{2}\sigma_{\$/€}^2\right)\Delta t + \left(\sigma_{i/\$}\varepsilon_{i/\$} + \sigma_{\$/€}\varepsilon_{\$/€}\right)\sqrt{\Delta t}\right], \tag{9}$$

which must be equal to

$$S_{i/€}(t_1) = S_{i/€}(t_0)\exp\left[\left(r_{i/€} + r_{\mathrm{f}/€} - \frac{1}{2}\sigma_{i/€}^2\right)\Delta t + \sigma_{i/€}\varepsilon_{i/€}\sqrt{\Delta t}\right]. \tag{10}$$

In the following, we set without loss of generality $\Delta t = 1$ units of time. Therefore, we must have

$$r_{i/€} - \frac{1}{2}\sigma_{i/€}^2 = r_{i/\$} - \frac{1}{2}\sigma_{i/\$}^2 + r_{\$/€} - \frac{1}{2}\sigma_{\$/€}^2 \tag{11}$$

and

$$\sigma_{i/€}\varepsilon_{i/€} = \sigma_{i/\$}\varepsilon_{i/\$} + \sigma_{\$/€}\varepsilon_{\$/€}. \tag{12}$$

This is possible if and only if

$$\sigma_{i/€}^2 = \sigma_{i/\$}^2 + \sigma_{\$/€}^2 + 2\rho_{i/\$,\$/€}\sigma_{i/\$}\sigma_{\$/€}, \tag{13}$$

where $\rho_{i/\$,\$/€}$ is the correlation of the price of asset i in USD and the FX rate $X_{\$/€}$. Replacing this condition in Equation (11), we have

$$r_{i/€} = r_{i/\$} + r_{\$/€} + \rho_{i/\$,\$/€}\sigma_{i/\$}\sigma_{\$/€}. \tag{14}$$

All quantities in Equation (14) are known, except the FX risk premium $r_{\$/€}$. Although we know that $r_{\$/€}$ should be the same across all assets i for a given currency pair, we now show that this condition can only hold if $\rho_{i/\$,\$/€} = 0$ for all i.

Equation (12) involves, in the RHS, the innovation $\varepsilon_{i/€}$ affecting the return of asset i in €, and $\varepsilon_{i/\$}$ and $\varepsilon_{\$/€}$, that is, the innovations affecting the return of the same asset expressed in \$ and the one entering the currency rate $\$/€$. For the equality to be valid in law, we need to assume that the innovations distribution belongs to the elliptical class, the well-known sufficient condition that guarantees the validity of the domestic CAPM (Owen and Rabinovitch 1983). If innovations are not elliptical, there is no guarantee that the distribution of the sum of the random variables in the LHS belong to the same class as the distribution of the random variables in the RHS. Here, we have made the much simpler assumption of normality in order to reduce the complexity of the calculations and of the illustrative example.

The estimation is conducted on the log of the exchange rate, whose increments are assumed to be i.i.d., thereby satisfying the stationarity requirement for the statistical properties (mean, variance, and covariances) used in our analysis. We acknowledge that assuming a non-stationary foreign exchange rate process may not be the best choice. However, this assumption is quite common for pricing currency options, where exchange rates are typically modelled as lognormal non-stationary processes, starting from the classic Garman and Kohlhagen (1983) paper and continuing with the Heston (1993) model. If the investment horizon is relatively short, from a month to a year, which is standard in empirical CAPM tests, the implications of non-stationarity in the currency rate are limited and do not pose a significant concern.

The determination of the implied risk premia in EUR requires the corresponding covariance matrix in Euro. This can be obtained by using Equation (9) for assets i and j and then by computing the covariance between the log-returns of the two assets. It turns out that (Fusai et al. 2024)

$$\sigma_{i/€,j/€} = \sigma_{i/\$,j/\$} + \sigma_{i/\$,\$/€} + \sigma_{j/\$,\$/€} + \sigma^2_{\$/€}, \tag{15}$$

where $\sigma_{i/\$,\$/€}$ is the covariance of asset i in USD with the FX currency rate. In Appendix A we also illustrate how to implement the transformation of the covariance from one currency to another via a simple matrix multiplication as illustrated in Fusai et al. (2024).

If we knew the value of the FX risk premium $r_{\$/€}$, then, using Equation (14), we could calculate the Sharpe ratio of the EUR portfolio,

$$R_€ = \frac{1}{\sigma_€}\left(r_{\$/€} + \sum_{j=1}^{n} w_j r_{j/\$} + \sum_{j=1}^{n} w_j \sigma_{j/\$,\$/€} \right), \tag{16}$$

where $\sum_{j=1}^{n} w_j \sigma_{j/\$,\$/€}$ is the covariance of the portfolio in USD and the FX return. Equation (16) is valid assuming that the portfolio is continuously rebalanced (Merton 1990, 127), that is, if the time interval Δt shrinks to zero. Substituting Equation (16) into Equation (1) with $k = €$ gives the risk premium in EUR of asset i,

$$r_{i/€} = \beta_{i/€}\left(r_{\$/€} + \sum_{j=1}^{n} w_j r_{j/\$} + \sum_{j=1}^{n} w_j \sigma_{j/\$,\$/€} \right). \tag{17}$$

Combining Equations (14) and (17), we solve for the FX risk premium, and we obtain

$$r_{\$/€} = \frac{1}{\beta_{i/€} - 1}\left[r_{i/\$} + \rho_{i/\$,\$/€}\sigma_{i/\$}\sigma_{\$/€} - \beta_{i/€}\left(r_\$ + \sum_{j=1}^{n} w_j \sigma_{j/\$,\$/€} \right) \right]. \tag{18}$$

We prove now that the CAPM holds only if $\rho_{i/\$,\$/€} = 0, \ i = 1, \ldots, n$. Indeed, with this assumption Equation (18) becomes

$$r_{\$/€} = \frac{r_{i/\$} - r_\$ \beta_{i/€}}{\beta_{i/€} - 1}. \tag{19}$$

Moreover, using the zero-correlation assumption and converting the covariances from EUR to USD using Equation (15), we have

$$\beta_{i/€} = \frac{1}{\sigma^2_€}\sum_{j=1}^{n} w_j \sigma_{i/€,j/€} = \frac{1}{\sigma^2_€}\sum_{j=1}^{n} w_j \sigma_{i/\$,j/\$} + \frac{\sigma^2_{\$/€}}{\sigma^2_€} = \beta_{i/\$}\frac{\sigma^2_\$}{\sigma^2_€} + \frac{\sigma^2_{\$/€}}{\sigma^2_€}. \tag{20}$$

Under the assumption of zero correlation, using (13), we have $\sigma^2_{i/€} = \sigma^2_{i/\$} + \sigma^2_{\$/€}$, and then at portfolio level it must hold also $\sigma^2_€ = \sigma^2_\$ + \sigma^2_{\$/€}$. In this way, we rewrite Equation (20) as

$$\beta_{i/€} - 1 = \left(\beta_{i/\$} - 1\right)\frac{\sigma^2_\$}{\sigma^2_€}, \tag{21}$$

and a simple formula to transform the asset beta from one currency to another, which is valid only assuming a zero correlation with the exchange rates. Inserting Equations (3) and (21) into Equation (19), we get

$$r_{\$/€} = r_\$ \frac{\beta_{i/\$} - \beta_{i/€}}{\left(\beta_{i/\$} - 1\right)\frac{\sigma^2_\$}{\sigma^2_€}} \tag{22}$$

and finally

$$r_{\$/€} = r_\$\left(\frac{\sigma^2_€}{\sigma^2_\$} - 1 \right), \tag{23}$$

that is, when the correlation between the price of asset i and an exchange rate is zero, the FX risk premium is the same for all assets. However, if the zero correlation assumption is not satisfied this is not guaranteed. Indeed, we illustrate in the next section with a numerical example that distinct FX equilibrium risk premia exist for each asset. This finding renders the CAPM inconsistent. An additional remark can be made regarding Equation (23): even if the assets are uncorrelated with the currency rates, the FX risk premium can be either positive or negative, depending on the ratio of the variances of the market portfolio expressed in the two currencies. This implies that the forward currency rate is a biased predictor of the future spot currency rate, providing a theoretical foundation for the empirical findings of Sarno et al. (2012).

This observation is related to the Siegel (1972) paradox, which involves the expected value of the reciprocal of the exchange rate. By applying Jensen's inequality, Siegel showed that the forward exchange rate is a biased predictor of the future spot rate.

However, it is important to emphasize that our result pertains to asset risk premia rather than currency risk premia. Moreover, the expression in Equation (14) involves the expected value of the product of two random variables: the asset price in one currency and the corresponding exchange rate. Unlike the Siegel paradox, where the bias arises from the convexity of the reciprocal function, our setting considers a product, whose expectation can be either greater or smaller than the product of t.

Our conclusion immediately extends to multi-factor models, for example, the Fama and French (1992) three-factor model. Using the latter, the risk premium of asset i in dollars is

$$r_{i/\$} = \beta_{i1,\$} f_{1,\$} + \beta_{i2,\$} f_{2,\$} + \beta_{i3,\$} f_{3,\$}, \tag{24}$$

where $\beta_{ij,\$}$ are the factor loadings of asset i in dollars with respect to factor $f_{j,\$}$. For the multi-factor model to hold whatever the currency, each asset must be uncorrelated with the exchange rate. The argument is as follows.

Let us now consider so-called *mimicking portfolios*, that is, portfolios constructed to have exposure to a single factor while having zero exposure to the remaining factors. In this way, the risk premia of these portfolios in a given currency can be expressed in terms of a single factor, as in the univariate CAPM.

This allows us to repeat the same argument presented in Section 2 and show that the risk premia of these mimicking portfolios can vary across currencies and may even change sign unless the correlation between the return of the mimicking portfolio and the exchange rate is zero.

Note that for the CAPM to hold, it is not sufficient for the market portfolio to be uncorrelated with the exchange rate. In fact, the covariance between an individual asset and the exchange rate will be zero only if both the covariance between the market portfolio and the exchange rate, and the covariance between the asset's idiosyncratic (residual) risk and the exchange rate are zero.

3 | A Simple Example Illustrating the Inconsistency of the International CAPM

To demonstrate the inadequacy of the CAPM in a multi-currency environment, we present an example that highlights the divergence in equilibrium risk premia obtained when moving from one currency to another. Without loss of generality, we assume that the risk-free rates in the three currencies are identical in this example.

Let us consider a scenario where an investor holds a portfolio consisting of three assets: Apple (AAPL), Volkswagen (VOW), and Unilever (ULVR), each denominated in a different currency (USD, EUR, and British Pound [GBP]). To analyse this portfolio, we use the monthly time series in USD of the three assets, spanning from 29 January 2010 to 30 September 2022. We also incorporate the time series for the EUR/USD and GBP/USD exchange rates over the same period to obtain the covariance matrix presented in Table 1. For instance, the covariance between the log-returns of AAPL and VOW is 2.31. We use this covariance matrix $\Sigma_\$$ as a starting point to derive the implied equilibrium risk premia in EUR.

TABLE 1 | Covariance matrix in USD.

	AAPL/USD	VOW/USD	ULVR/USD	USD/EUR
AAPL/USD	6.04	2.31	0.98	−0.25
VOW/USD	2.31	10.39	1.34	−0.97
ULVR/USD	0.98	1.34	2.37	−0.48
EUR/USD	−0.25	−0.97	−0.48	0.62

Note: The covariance matrix of assets in USD and USD/EUR exchange rate with values multiplied by 1000.

With $\Sigma_\$$ as a starting point, we assume a Sharpe ratio of $R_\$ = 0.5$ for the US market portfolio to derive the equilibrium risk premia in USD, as shown in Equation (5). This choice of the Sharpe ratio is somewhat arbitrary, but it does not materially affect the paper's results. The value is reasonable, as it reflects moderate risk-adjusted performance—consistency with traditional portfolios (e.g., a 60/40 equity-bond mix). For instance, it can be justified by assuming an annualised volatility of 20% and an excess market return of 10%, both of which are empirically plausible figures. We also assume that the world market portfolio is equally weighted, as indicated in column 2 of Table 2. The volatility of the market portfolio in USD is $\sigma_\$ = \sqrt{\mathbf{w}^\top \Sigma_\$ \mathbf{w}} = 5.585\%$. The covariances of each asset with the market portfolio are $\Sigma_\$ \mathbf{w} = 3.11$ (AAPL), 4.68 (VOW) and 1.56 (ULVR). The resulting USD risk premia are calculated by applying Equation (1) and are presented in column 3 of Table 2. For instance, the risk premium in USD for AAPL is 2.79%.

To convert the implied equilibrium risk premia from USD to EUR, we first need to determine the risk premium of the USD/EUR exchange rate. Assuming that the covariances between the assets denominated in USD and the USD/EUR exchange rate are as shown in Table 1, we use Equation (18) to determine the USD/EUR risk premium required to make the converted implied returns from USD to EUR equivalent to the risk premia calculated directly in EUR. For Apple, this risk premium is found to be -2.52%. Using this value and Equation (14), we obtain the fourth column of Table 2, which presents the risk premia in EUR for VOW and ULVR. For instance, the risk premia in Euros for VOW and ULVR are calculated to be 1.573% and −1.170%, respectively. We can now compute the risk premium in EUR of the market portfolio using the average of the EUR risk premia in column 4 of Table 2, which results in a value of 0.214%. We also need to determine the covariances in EUR of each asset with the market portfolio. To accomplish this, we convert the USD covariance matrix to EUR using Equation (15).

The resulting EUR covariance matrix $\Sigma_€$ is given in Table A1 in Appendix A. This matrix is used to calculate the betas in EUR for each asset, which are presented in column 5 of Table 2. Using Equation (1) with $k = €$, we compute the implied risk premia in EUR and report them in column 6 of the same table.

However, our calculations reveal that the implied risk premia in EUR differ from the equilibrium risk premia in USD converted

TABLE 2 | Calculation of risk premia in EUR when the correlation is not zero.

Asset	Weights w_i	USD implied risk premia $r_{i/\$}$	EUR risk premia, Equation (17)	EUR beta $\beta_{i/€}$	EUR implied risk premia $r_{i/€}$ from Equation (1)
AAPL	0.333	2.79%	0.239%	1.118	0.239%
VOW	0.333	4.19%	1.573%	1.444	0.309%
ULVR	0.333	1.40%	− 1.170%	0.439	0.094%

Note: The calculation of EUR asset risk premia assuming non-zero correlation among assets and currency rates. The table allows the comparison between the EUR risk premia (column 4) derived from the implied USD risk premia (column 3) and the implied EUR risk premia (column 6). To illustrate, the Apple EUR risk premium $r_{AAPL/€}$ of 0.239% in column 4 is calculated using Equation (14): $r_{AAPL/€} = r_{AAPL/\$} + r_{\$,€} + \sigma_{AAPL/\$,\$/€} = 2.79\% - 2.52\% - 0.025\% = 0.239\%$. Similarly, for VOW, we obtain $r_{VOW/€} = 4.19\% - 2.52\% - 0.097\% = 1.573\%$. The beta in EUR of each asset (column 5) is obtained by converting the USD covariance matrix $\Sigma_\$$ to EUR ($\Sigma_€$), as shown in Table A1 in Appendix A, and then computing $\Sigma_€ \mathbf{w} / \left(\mathbf{w}^\top \Sigma_€ \mathbf{w}\right)$. With these betas and the portfolio risk premium in EUR $(0.239\% + 1.573\% - 1.170\%)/3 = 0.214\%$, we use Equation (1) to derive the implied risk premia of each asset that are reported in column 6.

TABLE 3 | Calculation of risk premia in EUR when the correlation is zero.

Asset	USD implied risk premia $r_{i/\$}$	Currency risk premium $r_{\$,€}$	$r_{i/€}$ from Equation (17)	EUR beta $\beta_{i/€}$	EUR implied risk premia $r_{i/€}$ from Equation (1)
AAPL	2.79%	0.56%	3.343%	0.998	3.343%
VOW	4.19%	0.56%	4.748%	1.417	4.748%
ULVR	1.40%	0.56%	1.956%	0.584	1.956%

Note: EUR asset risk premia assuming zero correlation among assets and currency rates. Column 2 contains the USD risk premia $r_{i/\$}$ from column 3 in Table 2; column 3 gives the FX risk premium $r_{\$,€}$ computed according to Equation (18) where we set $\rho_{i/\$,\$/€} = \sigma_{j/\$,\$/€} = 0$, or equivalently Equation (23); column 4 gives the EUR risk premia computed from USD risk premia applying Equation (14); column 5 gives the beta of asset i in EUR $\beta_{i/€}$ computed by converting the USD covariance matrix to EUR in Table A3 in Appendix A and then computing $\Sigma_€ \mathbf{w} / \left(\mathbf{w}^\top \Sigma_€ \mathbf{w}\right)$; the last column reports the implied EUR risk premia from Equation (1), using the betas in EUR and the EUR portfolio risk premium of $3.349\% = (3.343\% + 4.748\% + 1.956\%)/3$.

to EUR using Equation (14). These two sets of risk premia are only equivalent, by construction, for AAPL. For the CAPM to be valid, they should be equivalent for all assets. In fact, as we proved, the risk premia presented in columns 4 and 6 of Table 2 are only consistent when the covariances between the asset prices and the EUR/USD exchange rate are zero.

Let us assume a zero-covariance between the assets and the currency rates. We then use Equation (18) to determine the equilibrium FX risk premium, which is found to be $r_{\$/€} = 0.56\%$ for all assets. We can now apply Equation (14) to convert the USD risk premia to EUR risk premia, which are reported in column 4 of Table 3 and give a risk premium of the EUR portfolio equal to 3.349%. We can also convert the USD covariance matrix $\Sigma_\$$ of Table A2 to EUR. The EUR covariance matrix $\Sigma_€$ is given in Table A3 in Appendix A and can be used to calculate the betas in EUR for each asset. They are reported in column 5 of Table 3. With the EUR portfolio risk premium and the EUR betas, we use Equation (1) to calculate the implied risk premium in EUR for each asset. The final result is presented in the last column of Table 3, and we obtain the same risk premia as in column 4 of the same table. This confirms that the implied EUR risk premia are equal only when the correlation between the assets and the USD/EUR exchange rate is zero.

4 | Conclusion

The CAPM is an intuitive model and a useful starting point in asset allocation and portfolio construction. However, as we have shown in this study, it not only fails to hold empirically in a single-currency world, but it also provides inconsistent results in a multi-currency world. We expect that in any portfolio construction exercise, investors will find that all currencies will be correlated with asset classes in one way or another. Thus, each base currency will imply different asset risk premia and therefore lead to different optimal allocations. This is inconsistent with the traditional ICAPM result that the asset risk premia are equal regardless of the base currency.

The same inconsistency also applies to multi-factor asset pricing models in an international context, independently of the number of factors. This has a significant implication for investors and asset managers, as they rely heavily on the Black and Litterman (1992) model for asset allocation decisions. The model uses a global market portfolio as a starting point, where it is assumed that all investors should hold the same portfolio regardless of their base currency. We have shown that this is not the case, as investors with different base currencies will estimate different risk premia.

Our findings raise concerns about the meaningfulness of empirical tests of the CAPM in an international setting. According to our results, the asset risk premia can vary significantly depending on the sign of the covariance between the asset return and the exchange rate, as well as on the choice of reference currency. As a consequence, an estimated asset risk premium cannot be assigned a clear or consistent interpretation across different currency perspectives. Karolyi and Wu (2022) pointed to the limitations of models with internationally perfect financial markets in explaining portfolio holdings and their time variations. Their results could potentially be due to the inconsistency illustrated in the present paper.

The raised inconsistency of the ICAPM opens the door for future research on asset pricing and allocation, specifically on how to estimate the risk premia in a multi-currency portfolio and how to use them in portfolio construction (Lustig et al. 2011; Corte et al. 2016).

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Conflicts of Interest

The authors declare no conflicts of interest.

Data Availability Statement

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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Appendix A

Converting a Covariance Matrix Among Currencies

In order to convert a covariance matrix for assets whose prices are all in the same currency, for example, USD, to another currency, for example, EUR, we perform the calculation (Fusai et al. 2024)

$$\Sigma_{€} = \mathbf{B}^{\top}\Sigma_{\$}\mathbf{B}. \tag{A1}$$

In our example, involving three assets and the currency rate EUR/USD, the matrix $\mathbf{B}$ is

$$\mathbf{B} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix} \tag{A2}$$

while $\Sigma_{\$}$ is in Table 1. By performing the above product, we obtain the covariance matrix in EUR $\Sigma_{€}$ in Table A1. This matrix is then used to compute the beta of each asset in EUR by computing $\Sigma_{€}\mathbf{w} / \left(\mathbf{w}^{\top}\Sigma_{€}\mathbf{w}\right)$. The betas expressed in EUR for each asset are reported in the first column of Table 3. Let us now assume that the covariance between the different assets in USD and the currency rate USD/EUR is zero: to do so, we modify the last column and the last row of the covariance matrix in Table 1 as in Table A2.

The resulting covariance matrix $\Sigma_{€}$ in EUR is finally presented in Table A3.

TABLE A1 | EUR covariance matrix converted from the USD covariance matrix in Table 1.

	AAPL/EUR	VOW/EUR	ULVR/EUR
AAPL/EUR	6.17	1.72	0.88
VOW/EUR	1.72	9.08	0.52
ULVR/EUR	0.88	0.52	2.04

TABLE A2 | USD covariance matrix assuming zero covariance between asset and FX returns.

	AAPL/USD	VOW/USD	ULVR/USD	USD/EUR
AAPL/USD	6.04	2.31	0.98	0.00
VOW/USD	2.31	10.39	1.34	0.00
ULVR/USD	0.98	1.34	2.37	0.00
USD/EUR	0.00	0.00	0.00	0.62

TABLE A3 | Covariance matrix in EUR converted from the covariance matrix in USD assuming zero correlation among assets and FX returns.

	AAPL/EUR	VOW/EUR	ULVR/EUR
AAPL/EUR	6.66	2.93	1.61
VOW/EUR	2.93	11.01	1.96
ULVR/EUR	1.61	1.96	2.99