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Optimisation of Biomass-Energy-Water-Food Nexus under Uncertainty

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ABSTRACT

The three systems, water, energy and food, are intertwined since the effect of any of these systems can affect others. This study proposes a mathematical model incorporating uncertain parameters in the biomass energy-water-food nexus system. The novel aspects of this work include formulating and solving the problem as a mixed-integer linear program and addressing the presence of uncertain parameters through a two-stage stochastic mathematical programming approach. Taking maximising economic benefit as an objective function, this work compares the results of the deterministic model with the results computed by incorporating uncertainty in the model parameters. The results indicate that incorporation of uncertainty gives rise to reduced profitability, but increased greenhouse gas emission (GHG) as compared to the deterministic model. On the other hand, when minimisation of GHG emission is considered as an objective function, a significantly greater reduction in the profitability is observed for both, stochastic and deterministic, models. The model results are used for allocating optimal resources, reducing carbon footprint, increasing economic potential and managing resources sustainably in the whole system.

Keywords: biomass energy, optimisation, uncertain parameters

INTRODUCTION

The biomass energy-water-food nexus concept represents a complex interaction in managing limited resources in an innovative way [3, 4,11]. However, production of energy through fossil fuels is linked to global warming [1]. Achieving CO2 emission reduction target-2050 requires that the renewable energy share in electricity generation globally be increased from 29% in 2020 to over 60% in 2030 and to nearly 90% in 2050 [5]. Several studies postulate bioenergy as a cost effective and sustainable way to mitigate climate change [10,12]. Therefore, this study aims to create a mathematical model of biomass energy-water-food-nexus in the presence of uncertain parameter for optimal resource allocation. There are few studies reported on biomass-energy-water-food nexus incorporating uncertain parameters. Li *et al.* proposed a multi-objective optimisation model with a view to determining optimal policy options for the trade-off between financial benefit and environmental impact in the water-land-energy-livestock context with uncertain parameters in the nexus system [6]. Similarly, Lopez-Diaz *et al.* proposed an optimisation model to create efficient supply chain using material flow analysis among watershed, wastewater flown towards watershed, production and distribution of feedstocks, grains and biofuel [7]. Peña-Torres *et al.* proposed a multi-objective optimisation model with five different demands of resources with respect to water-food-energy nexus with a view to maximising economic benefits (EB) and reducing greenhouse gas emission (GHG) [9]. After analysing the literature, this study introduces novelties in the nexus which are described in the methods section.

METHODS

This study incorporates two mono-objective functions: maximising economic benefit (EB) and minimising greenhouse gas (GHG) emissions. Novel aspects of this research include the addition of effluent treatment plants (ETP), the integration of rainwater harvesting systems and the implementation of solar power generation systems across all sectors in the superstructure (figure 1). Moreover, this study has incorporated two-stage stochastic mathematical programming for managing uncertainty in the parameter. The superstructure of the system includes water subsystem comprised of water sources from power plants, aquifers, dams, rainwater and ETP. Moreover, fertiliser industry is included in the nexus for supplying fertiliser to agriculture sectors and external market.

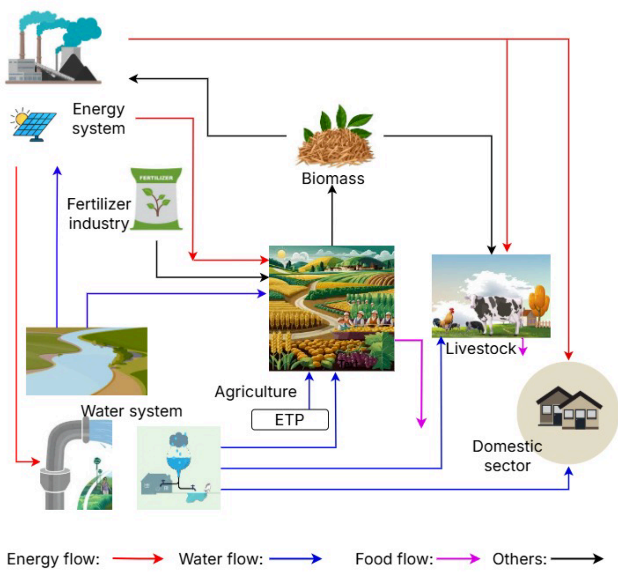

Figure 1: Superstructure of Biomass energy water food nexus

The energy subsystem mainly includes existing and new power plants, which utilise biomass, gasification gas and natural gas for producing electricity. On the other hand, electricity from the solar systems in each sector is used in the respective sector.

In this paper, we have generated results for each objective function separately instead of generating results for multi-objective function. The key features of the mathematical model are presented as follows:

Objective function 1: Maximising EB

$$Max\ EB = Revenue - TAC \tag{1}$$

Objective function 2: Minimising GHG

$$Min\ GHG = sum\ of\ GHG\ in\ all\ sectors \tag{2}$$

Water mass balance:

Amount of water remained in the sources within two time periods is equal to the sum of amount distributed and amount that already existed.

$$F_{o,t}^{Wsource} - F_{o,t-1}^{Wsource} = F_{o,t}^{existing} + \sum_A F_{o,A,t}^{Wsource-Actor} \tag{3}$$

Amount of water in actors is equal to the amount of water sent from water sources and the amount of water generated in rainwater harvesting systems.

$$F_{A,t}^{to-Actor} = \sum_o F_{o,A,t}^{WSource-Actor} + F_{A,t}^{Rain-Actor} \tag{4}$$

$$F_{A,t}^{Rain-Actor} = ARW_A . F_{A,t}^{precip} \tag{5}$$

Energy balance:

Amount of energy generation in unit is expressed in terms of amount of water utilised for this purpose in power plants.

$$GE_{g,t}^{Esource} = \epsilon^{fuel} . F_{g,t}^{Sea-Esource} \tag{6}$$

$$F_{g,t}^{Sea-Esource} = \omega^{water} . \left[NG_{g,t}^{Esource} + \sum_a AgWaste_{a,g,t}^{Esource} + \sum_h Gas_{h,g,t}^{Esource} \right] \tag{7}$$

Total energy in consuming bodies (actors) is equal to amount of energy flow from generation units to actor and the amount of solar energy generated in that sector.

$$E_{A,t}^{to-Actor} = \sum_g E_{g,A,t}^{Esource-Actor} + GE_{A,t}^{Solar-Actor} \tag{8}$$

$$GE_{A,t}^{Solar-Actor} = AsolarSector_A . SR_t \tag{9}$$

Domestic waste is used to produce gasification gas, which is sent to power plants for energy production and external market.

$$GGasDomWaste_{h,t} = \sum_g Gas_{h,g,t} + Gas_{h,t}^{ext} \tag{10}$$

Food mass balance in agriculture and livestock sectors:

Crops from agriculture sectors and meat from livestock sectors are sources of food which are distributed to domestic sectors and external markets.

$$AgL_{al,h,t}^{dom} = FoodDemand_{h,t}^{dom} \tag{11}$$

$$GRfood_{al,t}^{AgL} = AgL_{al,h,t}^{dom} + AgL_{al,t}^{ext} \tag{12}$$

Mathematical equations related to two-stage stochastic program:

Food demand in domestic sectors is considered as an uncertain parameter. The general formulation of two-stage stochastic program [2] is:

$$Max\ C^T x + \sum_k p_k q_k^T y_k \tag{13}$$

s.t. Ax=b

$$T_k x + W_k y_k = h_k \quad \forall k$$

$$x \geq 0, y_k \geq 0 \quad \forall k$$

Where p_k is probability of occurrence of uncertain parameters, x and y_k are first and second stage decision variables respectively.

The following equation (14) represents that raw food flow from agriculture and livestock sectors to domestic sector in time t is equal to uncertain food demand in domestic sector for scenario s.

$$AgL_{al,h,t}^{dom} = FoodDemand_{h,t,s}^{dom} \quad (14)$$

Similarly, total amount of food production in agriculture and livestock sectors is equal to amount sent to domestic sectors and external market for scenario s.

$$GRfood_{al,t}^{AgL} = FoodDemand_{h,t,s}^{dom} + AgL_{al,t,s}^{ext} \quad (15)$$

In this study, all decision variables, except variables related to uncertain parameter, are placed in the first stage. Ten scenarios of uncertain food demand in the domestic sectors are considered. The uncertainty is modelled through normal distribution with 20% standard deviation from the mean value. Distribution is done on twelve individual months of a year with 10 scenarios over three domestic sectors. The second stage decision variables are amount of food sent to domestic sectors and amount of food sent to the external market. The data for the base-case, deterministic model, is taken from Pena Torres *et al.* (2024) and Núñez-López, Rubio-Castro and Ponce-Ortega (2021). Both mathematical models are coded in GAMS software with CPLEX v. 22.1.0.0 solver for the MILP.

RESULTS

This section presents results of both deterministic and two-stage stochastic solutions. Optimal solutions are presented for four cases such as deterministic solution for maximising EB, stochastic solution for maximising EB, deterministic solution for minimising GHG emission and stochastic solution for minimising GHG emission. Running the model with taking objective function as maximising EB, the stochastic solution provides 11.14% less profit (figure 2) and 3.85% higher GHG emission (figure 3) than deterministic solution. The risk associated with uncertainty is the main cause of such differences. Similarly, for minimising GHG emission cases, the stochastic solution provides 33.46% less profit (figure 2) than deterministic solution but nearly same amount of GHG emission in both models (figures 3). In four types of models, the main profit contributing agent in the system is fertiliser industry. For model with objective function 1, the revenue from fertiliser industry for stochastic solution is 4.93% higher than deterministic solution while running the model with objective function 2, it generates same amount of revenue in the fertilizer industry for both deterministic and stochastic programs (figure 4, tables 1 and 2). In the following tables and figures, monetary values are expressed in million USD and mass quantities in million tons, except for crop distribution in stochastic solution, which is expressed in tons.

Table 1: Optimal solution for first objective function

Variables	Deterministic solution	Stochastic solution
EB	8.54E+3	7.59E+3
GHG	2.27E+6	2.36E+6
Crop supplied to domestic sectors	2.13E-1	second stage decision
Crop supplied to market	4.23	second stage decision
Revenue from fertilizer industry	5.33E+4	5.59E+4

Table 2: Optimal solution for second objective function

Variables	Deterministic solution	Stochastic solution
EB	1.49E+3	9.91E+2
GHG	1.65E+6	1.65E+6
Crop supplied to domestic sectors	1.11	second stage decision
Crop supplied to market	0.00	second stage decision
Revenue from fertilizer industry	4.93E+4	4.93E+4

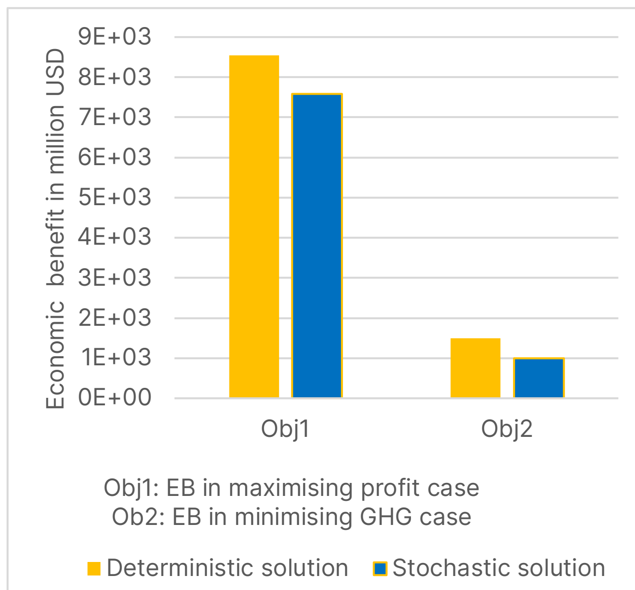

Figure 2. Economic benefit (EB) in two different mono-objective cases

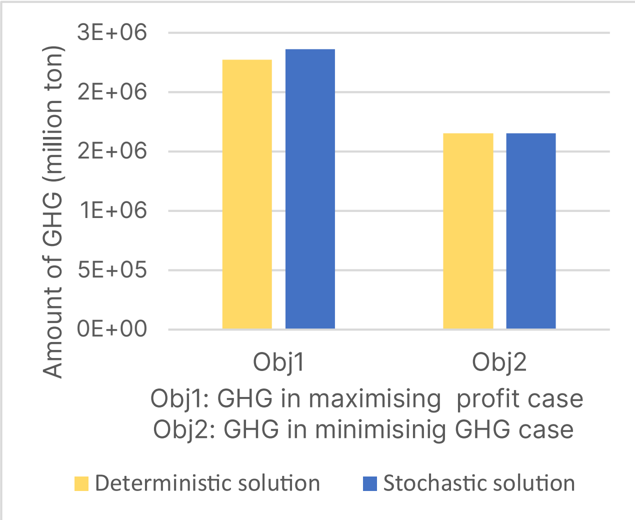

Figure 3. GHG emission in two different mono-objective cases

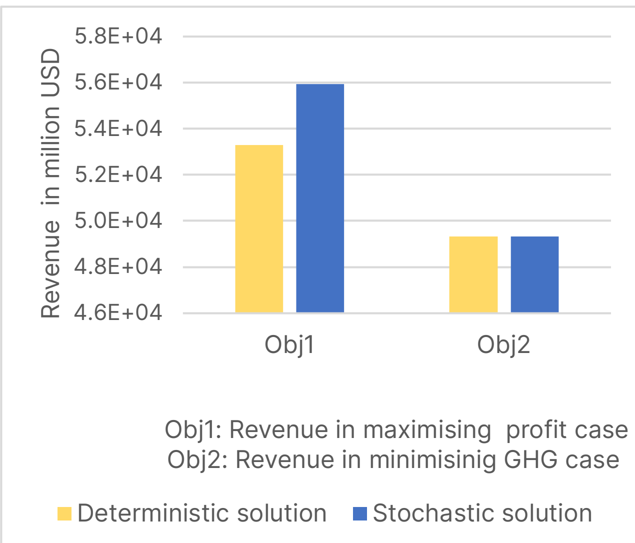

Figure 4. Revenue from fertilizer industry in two different mono-objective cases

Optimal number of installation decision has also been generated in this model for the four cases. The optimal number of installation decision of new power plant, gasification plant, rainwater harvesting system and solar panel for deterministic solution for maximising EB and minimising GHG emission cases are 0,0,1, 1 and 0,3,2,3 respectively; the increased number is due to focus on minimisation of GHG in the system. On the other hand, in the first stage of stochastic solution with maximising EB objective function case, the optimal number of installation decision of new power plant, gasification plant, rainwater harvesting system and solar panels are 0,0,1 and 3 respectively. However, the installation decision values become 0,3,2 and 3 respectively for minimising GHG objective function for stochastic solution. The second stage decision variables for stochastic solution are: amount of food, produced in agriculture and livestock sectors, sent to domestic sectors and amount of food sent to external market.

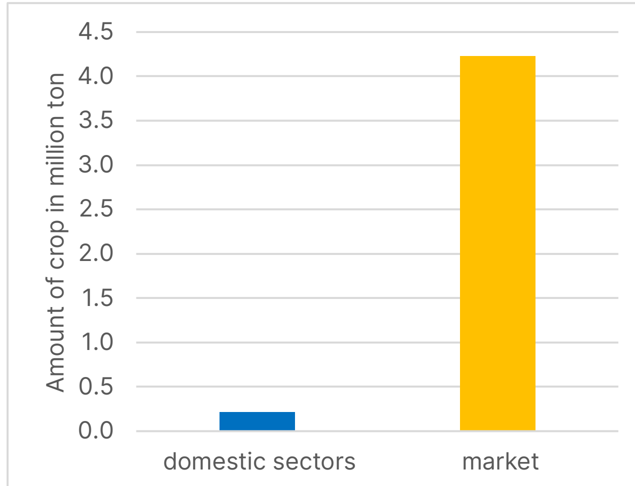

Figure 5. Deterministic solution: Crop supply from agriculture sectors to domestic sector and market (maximising EB case)

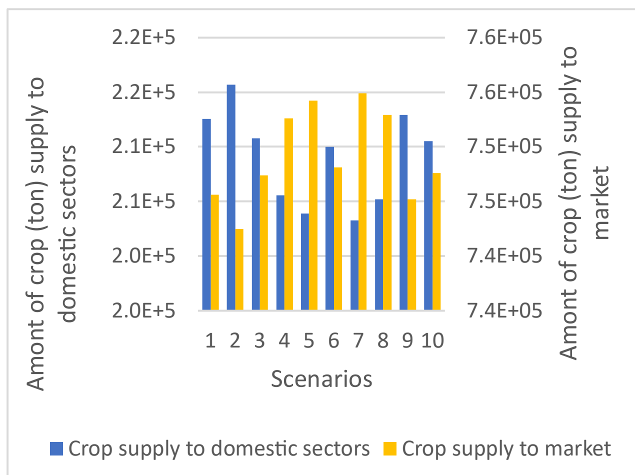

Figure 6. Stochastic solution: Crop supply from agriculture sectors (maximising EB case)

For all models, optimal solutions of meat production in livestock sectors are zero. For deterministic solution (scenario independent), it is found that the amount of food sent to external market is much higher than that sent to domestic sectors (figure 5). In the second stage, as soon as the demand scenario is realized, the distribution of food to domestic sectors and external market can be decided (figure 6 and 7). From the figures, it can be observed that the amount of food sent to market for the first mono objective function case is significantly higher than that amount of food sent to domestic sectors compared to second mono objective function case. Conversely, in first mono objective case, the amount of food sent to domestic sectors is significantly lower compared to second mono objective function case (figure 6, figure 7).

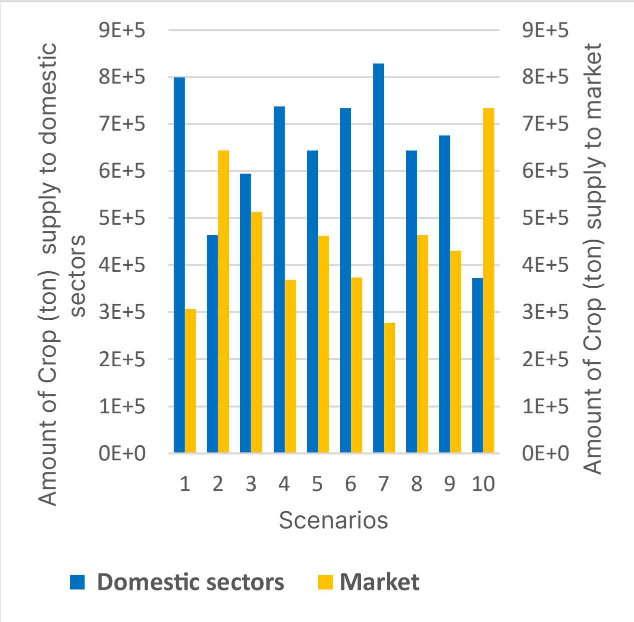

Figure 7. Stochastic solution: Crop supply from agriculture sectors (minimising GHG case)

CONCLUSIONS

First and second mono objective function of this study are maximisation of economic benefit and minimisation of greenhouse gas emission respectively. Both deterministic and stochastic model have been constructed based on the first and second objective functions in the context of optimisation of biomass energy water food nexus. Running the models with objective function 1 and 2 separately, optimal solutions have been generated for four types of cases which are 1) deterministic solution for maximising EB 2) deterministic solution for minimising GHG emission 3) stochastic solution for maximising EB and 4) stochastic solution for minimising GHG emission. The obtained results are then compared. For objective function 1, two-stage stochastic program generates 11.14% less profit than deterministic solution but greenhouse gas emission is increased by 3.85% as there are risks involved in the stochastic solution due to uncertain food demand parameter. Conversely, the model incorporating objective function 2 resulted in significantly lower profit accompanied by a substantial reduction in GHG emission across both deterministic and stochastic solutions. Moreover, optimal installation decisions are different in different cases. In stage two, amount of food distribution decision is determined as soon as the demand parameter is realized. In this study, the optimal solutions of the decision variables in second stage are generated and analysed for objective functions 1 and 2.

NOMENCLATURE

Index	Description
o	Sources of water
a	Agriculture sectors
c	Livestock sectors
m	Livestock types
v	Crop types
h	Domestic sectors
t	Time period in months
A	Actor (water and energy consuming bodies)
g	Energy generation units other than solar panels
sc	Sectors such as livestock, domestic, industry, agriculture
al	Agriculture and livestock sector

Acronym	Description
AgLI	Agriculture, livestock, fertiliser industry
ARW_A	Effective area for rainwater harvesting in the sectors
$AsolarSector_A$	Effective area for solar radiation in the sectors
$AgL_{al,h,t}^{dom}$	Raw food sent to domestic from agriculture and livestock sector
$AgL_{al,t}^{ext}$	Raw food sent to market from agriculture and livestock sector
$AgL_{al,t,s}^{ext}$	Raw food sent to market from agriculture and livestock sector in scenario s
EB	Economic benefit in USD
Esource	Electricity producing system such as power plants
ϵ^{fuel}	Fuel efficiency coefficient for energy production
$F_{A,t}^{Rain-Actor}$	Water harvested in all sectors such as domestic, livestock
$F_{A,t}^{precip}$	Rainwater precipitation flux in the sectors
$FoodDemand_{h,t}^{dom}$	Amount of food required in domestic sector
$FoodDemand_{h,t,s}^{dom}$	Amount of food required in domestic sector under scenario s
$F/E_{index}^{acronym}$	Input variable such as water/energy flow to a system
GHG	Greenhouse gas emission (carbon equivalent in ton)
$GRfood_{al,t}^{AgL}$	Production of raw food such as crop from agriculture and meat from livestock

$GGasDomWaste_{h,t}$	Amount of gasification gas generated in domestic sector h
$Gas_{h,g,t}$	Amount of gasification gas sent to power generation unit
$Gas^{ext}_{h,t}$	Amount of gasification gas sent to external market
$GE^{Esource}_{g,t}$	Energy generation in any unit
$GE^{Solar-Actor}_{A,t}$	Solar energy generation in the sectors
SR_t	Solar radiation flux in month t
TAC	Total annualised cost in USD
Wsource	Water source such as dam, deep well, aquifer, effluent treatment plant (ETP)
ω^{water}	Water efficiency coefficient for energy generation

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