Novel CSI-Free Symbol-Level Precoding for MU-MIMO Systems with MLD Receiver

Xiao Tong*, Lei Lei*†, Ang Li*†, and Christos Masouros‡
School of Information and Communications Engineering, Xi'an Jiaotong University, Xi'an, China*
The National Mobile Communications Research Laboratory, Southeast University, Nanjing, China†
The Department of Electronic and Electrical Engineering, University College London, London WC1E 7JE, U.K.‡
Email: xiao.tong.2023@stu.xjtu.edu.cn, {lei.lei, ang.li.2020}@xjtu.edu.cn, c.masouros@ucl.ac.uk

***Abstract*—In this work, we explore symbol-level precoding (SLP) and efficient decoding strategies for downlink transmission in multi-user multiple-input multiple-output (MU-MIMO) systems. We specifically study scenarios where the base station (BS) sends multiple multi-level modulated data streams to users for decoding. We formulate an optimization problem for joint symbol-level transmit precoding and receive combining. However, the receive combining matrix is dependent on the transmit symbols in the joint design scheme, thus, we employ maximum likelihood detection (MLD) method at the receiver side. we demonstrate that the smallest singular value of the precoding matrix significantly affects the MLD performance, while traditional SLP scheme returns a rank-one precoding matrix, which results in inferior error-rate performance to users. To overcome this challenge, we propose a novel channel state information (CSI)-Free SLP scheme that employs semidefinite programming (SDP) method to enable SLP technique in systems utilizing MLD decoding, where the design of the precoding matrix depends only on the modulated data symbols. Numerical simulations confirm that our proposed scheme substantially outperforms the traditional block diagonalization (BD) methods.**

***Index Terms*—MU-MIMO, channel state information (CSI)-Free, symbol-level precoding (SLP), maximum likelihood detection (MLD).**

I. Introduction

Multiple-input multiple-output (MIMO) communication is one of the promising technologies to achieve massive and low-latency communications by utilizing multiple antennas on both transmitter and receiver sides to simultaneously transmit multiple data streams to multiple users (MU) [1]. However, a primary limitation of MIMO systems is the interference arising from the simultaneous transmission of multiple signals using the same time and frequency resources. Therefore, it is crucial for the transmitter and receiver to exploit the spatial degrees of freedom offered by multiple antennas to mitigate the interference and improve decoding efficiency.

At the transmitter side, precoding has been extensively studied as an effective strategy to mitigate interference and enhance communication performance. Traditional precoding schemes utilize channel state information (CSI) to design the precoding matrix that maximizes some system performance while adhering to the power budget. Representative examples include zero-forcing (ZF) [2], regularized zero-forcing (RZF) [3], block diagonalization (BD) [4] as well as regularized block diagonalization (RBD) [5]. However, these schemes primarily focus on interference suppression, overlooking the potential benefits of interference, which can be further exploited at the symbol-level, a technique known as symbol-level precoding (SLP) [6].

SLP is an approach that leverages both CSI and modulated data symbols to design the precoding strategy on a symbol-by-symbol basis. In [7] and [8], the authors proposed closed-form SLP solutions for MU multiple-input single-output (MISO) downlink communication systems using phase shift keying (PSK) and quadrature amplitude modulation (QAM) modulation symbols, respectively. Furthermore, the bit error rate (BER) performance was improved for the symbol-by-symbol optimization and lower computational complexity algorithms were introduced. Additionally, SLP can be applied in large-scale mmWave communication systems to enhance overall system performance [9], [10]. In [9], a hybrid analog-digital precoding method was used to minimize the Euclidean distance between the optimal fully-digital and hybrid symbol-level precoders, where an additional switching network was used to achieve power savings by turning off some phase shifters. Further optimization of power consumption was explored in [10], where the transmitter and receiver were jointly designed for MIMO-OFDM system that use one-bit digital-to-analog converters (DACs) and analog-to-digital converters (ADCs).

Notably, the majority of existing SLP researches primarily focus on MU-MISO downlink systems, with only limited works addressing the challenges of MU-MIMO SLP problems [11], [12]. In [11], the closed-form transmit SLP matrix and receive combining matrix were derived for MU-MIMO systems employing PSK modulated symbols, where a novel regularized interference rejection combiner (RIRC) receiver was proposed for signal decoding. In [12], the authors considered a joint SLP and linear receive combining design problem to minimize the symbol error rate (SER) for MU-MIMO systems with QAM modulation. However, the dependency of the receive combining on the transmit signals was not fully addressed. Despite these efforts to improve MU-MIMO downlink communication performance with SLP, there is still need for more practical symbol-level schemes to be studied for MU-MIMO systems when using multi-level modulations such as QAM. At the receiver side, maximum likelihood detection (MLD) is a non-linear detection method that returns the optimal detection performance [13]. However, the disadvantage of this method is that its computational complexity increases exponentially as the modulation order or the number of data streams increases. Therefore, the QR-MLD method [14] and QRM-MLD method [15] were proposed to reduce the number of signal candidates that need to be searched via QR decomposition and *M-algorithm*.

Therefore in this paper, we aim to design practical symbol-level transmit precoding and receive decoding techniques for MU-MIMO communication systems with QAM modulation. The MLD method is utilized to address the dependence of the receive combining matrix on the transmit symbols. We propose a novel semidefinite programming (SDP)-based CSI-Free SLP scheme to maximize the smallest singular value of the SLP matrix, which not only facilitates efficient decoding at the receivers but also simplifies the optimization problem. Specifically, our novel SLP design relies only on the modulated data symbols and is independent of the CSI, which offers excellent communication performance while avoiding the performance losses from inaccurate channel estimation. Numerical results demonstrate that the proposed scheme significantly outperforms benchmark approaches, highlighting its effectiveness in enhancing system performance.

II. System Model And Problem Formulation

A. System Model

We study a downlink multi-stream MU-MIMO system, where the BS is equipped with N_T transmit antennas and simultaneously transmits L data streams to each user. Each user is equipped with N_R antennas, with the constraint $L \leq N_R$. The total number of transmit antennas satisfies $N_T \geq KL$, where K is the number of users served by the BS. We focus on downlink transmit precoding and receive combining design, where perfect CSI is assumed throughout the paper [7], [8]. The transmit symbol vector is assumed to be formed from a normalized QAM modulation constellation [8], and $\mathbf{s}_k \in \mathbb{C}^{L\times 1}$ represents the transmit signal for the k-th user. The transmit precoding matrix for the k-th user is denoted as $\mathbf{P}_k \in \mathbb{C}^{N_T\times L}$. Therefore, the transmit signal at the BS can be expressed as

$$\mathbf{x} = \mathbf{P}\mathbf{s} = \sum_{k=1}^{K} \mathbf{P}_k \mathbf{s}_k, \tag{1}$$

where $\mathbf{P} = [\mathbf{P}_1, \mathbf{P}_2, \cdots, \mathbf{P}_K] \in \mathbb{C}^{N_T\times KL}$ and $\mathbf{s} = \left[\mathbf{s}_1^T, \mathbf{s}_2^T, \cdots, \mathbf{s}_K^T\right]^T \in \mathbb{C}^{KL\times 1}$ represent the overall transmit precoding matrix and symbol vector, respectively. Then, the received signal at the k-th user is given by

$$\begin{aligned}\mathbf{y}_k &= \mathbf{H}_k\mathbf{x} + \mathbf{n}_k \\ &= \mathbf{H}_k\mathbf{P}_k\mathbf{s}_k + \mathbf{H}_k \sum_{i=1,i\neq k}^{K} \mathbf{P}_i\mathbf{s}_i + \mathbf{n}_k,\end{aligned} \tag{2}$$

where $\mathbf{H}_k \in \mathbb{C}^{N_R\times N_T}$ denotes the channel matrix between the k-th user and the BS, where each entry in $\mathbf{H}_k$ follows a standard complex Gaussian distribution, $\mathbf{n}_k \in \mathbb{C}^{N_R\times 1}$ represents the zero mean circularly symmetric complex Gaussian noise vector with $\mathbf{n}_k \sim \mathcal{CN}\left(\mathbf{0}, \sigma^2\mathbf{I}\right)$.

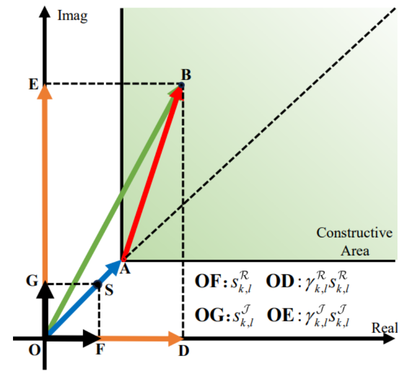

Fig. 1. An illustration for CI, 16QAM, ‘symbol-scaling’ metric.

To correctly decode the received signals, the k-th user decodes the received signal with a receive combining matrix $\mathbf{W}_k \in \mathbb{C}^{L\times N_R}$, then the decoded signal can be expressed as

$$\begin{aligned}\hat{\mathbf{s}}_k &= \mathbf{W}_k\left(\mathbf{H}_k \sum_{i=1}^{K} \mathbf{P}_i\mathbf{s}_i + \mathbf{n}_k\right) \\ &= \mathbf{W}_k\mathbf{H}_k\mathbf{P}_k\mathbf{s}_k + \mathbf{W}_k\mathbf{H}_k \sum_{i=1,i\neq k}^{K} \mathbf{P}_i\mathbf{s}_i + \mathbf{W}_k\mathbf{n}_k,\end{aligned} \tag{3}$$

B. Constructive Interference

For illustration, Fig. 1 depicts one-quarter of a nominal 16QAM constellation. Specifically, the constellation points and noiseless received signals can be mathematically decomposed into

$$\begin{aligned}&s_{k,l} = s^{\mathcal{R}}_{k,l} + s^{\mathcal{J}}_{k,l} \\ &\mathbf{w}_{k,l}\mathbf{H}_k\mathbf{P}\mathbf{s} = \gamma^{\mathcal{R}}_{k,l}s^{\mathcal{R}}_{k,l} + \gamma^{\mathcal{J}}_{k,l}s^{\mathcal{J}}_{k,l}, \forall k \in \mathcal{K}, \forall l \in \mathcal{L},\end{aligned} \tag{4}$$

where $s_{k,l}$ denotes the l-th data symbol for the k-th user, $s^{\mathcal{R}}_{k,l} = \Re\left\{s_{k,l}\right\}$ and $s^{\mathcal{J}}_{k,l} = j\cdot\Im\left\{s_{k,l}\right\}$ are the bases that are parallel to the detection thresholds for each constellation symbol. $\mathbf{w}_{k,l}$ is the l-th row of receiving combining matrix $\mathbf{W}_k$. $\gamma^{\mathcal{R}}_{k,l} \geq 0$ and $\gamma^{\mathcal{J}}_{k,l} \geq 0$ are the real-valued scaling coefficients. To concisely express the CI constraints, we define

$$\boldsymbol{\gamma}_{k,l} = \left[\gamma^{\mathcal{R}}_{k,l}, \gamma^{\mathcal{J}}_{k,l}\right]^T, \bar{\mathbf{s}}_{k,l} = \left[s^{\mathcal{R}}_{k,l}, s^{\mathcal{J}}_{k,l}\right]^T, \forall k \in \mathcal{K}, \forall l \in \mathcal{L}, \tag{5}$$

then $\mathbf{w}_{k,l}\mathbf{H}_k\mathbf{P}\mathbf{s}$ can be further simplified as

$$\mathbf{w}_{k,l}\mathbf{H}_k\mathbf{P}\mathbf{s} = \boldsymbol{\gamma}^T_{k,l}\bar{\mathbf{s}}_{k,l}, \forall k \in \mathcal{K}, \forall l \in \mathcal{L}. \tag{6}$$

For notational convenience, we introduce a set $\mathcal{O}$ that consists of the real or imaginary parts of the outer points that can be scaled, and a set $\mathcal{I}$ that consists of the real or imaginary parts of the inner points that can not be scaled [8]. Accordingly, the CI constraints for QAM modulations can be expressed as

$$\gamma^{\mathcal{O}}_{k,l} \geq t, \gamma^{\mathcal{I}}_{k,l} = t, \forall \gamma^{\mathcal{O}}_{k,l} \in \mathcal{O}, \forall \gamma^{\mathcal{I}}_{k,l} \in \mathcal{I}, \tag{7}$$

where t is the Euclidean distance between the CI region and decision boundary and more detailed description can be found in [6], [8].

C. Problem Formulation

Consistent with [8], we consider interference on the inner points as destructive and on the outer points as constructive. Following [11], we propose to jointly optimize the transmit precoding matrix and receive combining matrix to maximize the CI effect for the outer constellation points while maintaining performance for the inner constellation points. For the considered MU-MIMO system, the problem can be formulated as

$$\begin{aligned} \mathrm{P1}: \max_{\mathbf{W}_k,\mathbf{P},t,\mathbf{\Gamma}} \; & t \\ s.t. \quad & \mathbf{C1}: \mathbf{W}_k\mathbf{H}_k\mathbf{P}\mathbf{s} = \mathbf{\Gamma}_k\bar{\mathbf{s}}_k, \forall k \in \mathcal{K} \\ & \mathbf{C2}: t-\gamma_m^{\mathcal{O}} \le 0, \forall \gamma_m^{\mathcal{O}} \in \mathcal{O} \\ & \mathbf{C3}: t-\gamma_n^{\mathcal{I}} = 0, \forall \gamma_n^{\mathcal{I}} \in \mathcal{I} \\ & \mathbf{C4}: \|\mathbf{P}\mathbf{s}\|_2^2 \le p \\ & \mathbf{C5}: \|\mathbf{W}_k\|_2^2 \le 1, \forall k \in \mathcal{K}, \end{aligned} \tag{8}$$

where $\mathbf{\Gamma}_k = \left[\boldsymbol{\gamma}_{k,1}^T, \boldsymbol{\gamma}_{k,2}^T, \cdots, \boldsymbol{\gamma}_{k,L}^T\right]^T$ is the real-valued scaling vector and $\bar{\mathbf{s}}_k = \left[\bar{\mathbf{s}}_{k,1}^T, \bar{\mathbf{s}}_{k,2}^T, \cdots, \bar{\mathbf{s}}_{k,L}^T\right]^T$ is the transmit signal basis vector for the k-th user. The objective function of the formulated problem is the distance between the CI region and decision boundary. **C1-C3** represents the CI constraints for QAM modulation symbol. **C4** is the total transmit power budget and **C5** is the receive combining power budget to ensure the problem is bounded and feasible. We should note that the value of the power constraint at the right-hand side of **C5** will not affect the performance of the proposed scheme, because $\mathbf{W}_k$ is multiplied to the noise, too.

III. Symbol-Level Precoding with MLD Receiver

In P1, the variables of the transmit precoding matrix and the receive combining matrix are coupled with each other, which makes the problem difficult to directly solve. Furthermore, It can be observed that the receive combining matrix is related with the transmit symbols, which requires additional signaling exchange between the BS and users in practical wireless communication systems. Therefore, we consider the SLP design at the BS when users employ the MLD method to decode the received signals.

A. Performance Analysis of MLD Method

In a generic MU-MIMO system where users employ MLD, the estimation process can be expressed as

$$\begin{aligned} \hat{\mathbf{s}} &= \arg\min_{\tilde{\mathbf{s}}} \|\mathbf{y} - \mathbf{M}\tilde{\mathbf{s}}\|_2^2 \\ &= \arg\min_{\tilde{\mathbf{s}}} \|\mathbf{M}(\mathbf{s}-\tilde{\mathbf{s}}) + \mathbf{n}\|_2^2 \\ &\overset{(a)}{=} \arg\min_{\tilde{\mathbf{s}}} \left\{ \mathrm{Tr}\left[(\mathbf{s}-\tilde{\mathbf{s}})(\mathbf{s}-\tilde{\mathbf{s}})^H\mathbf{M}^H\mathbf{M}\right] + \sigma^2 \right\} \\ &= \arg\min_{\tilde{\mathbf{s}}} \left\{ \mathrm{Tr}\left[\tilde{\mathbf{S}}\tilde{\mathbf{M}}\right] + \sigma^2 \right\}, \end{aligned} \tag{9}$$

where the subscript k for the k-th user is omitted for clarity. Here, $\mathbf{M} = \mathbf{HP}$ denotes the equivalent channel matrix, $\tilde{\mathbf{s}}$ represents the candidate symbol vector selected from the constellation book. Additionally, we define $\tilde{\mathbf{S}} = (\mathbf{s}-\tilde{\mathbf{s}})(\mathbf{s}-\tilde{\mathbf{s}})^H$ and $\tilde{\mathbf{M}} = \mathbf{M}^H\mathbf{M}$. The step (a) can be achieved because the transmit symbols and received noise are independent. Furthermore, we introduce *Von Neumann's trace inequality* here for subsequent derivation.

Lemma 1 (*Von Neumann's trace inequality*): Let $\mathbf{A}$ and $\mathbf{B}$ be the N-dimensional Hermitian positive semi-definite matrices. Denote the eigenvalues of $\mathbf{A}$ and $\mathbf{B}$ as $\lambda_1(\mathbf{A}) \ge \lambda_2(\mathbf{A}) \ge \cdots \ge \lambda_n(\mathbf{A})$ and $\lambda_1(\mathbf{B}) \ge \lambda_2(\mathbf{B}) \ge \cdots \ge \lambda_n(\mathbf{B})$, respectively, arranged in non-decreasing order. Then we have

$$\mathrm{Tr}[\mathbf{AB}] \ge \sum_{n=1}^{N} \lambda_n(\mathbf{A})\lambda_{N-n+1}(\mathbf{B}). \tag{10}$$

Based on *Lemma 1*, we obtain:

$$\mathrm{Tr}\left[\tilde{\mathbf{S}}\tilde{\mathbf{M}}\right] \ge \sum_{l=1}^{L} \lambda_l\left(\tilde{\mathbf{S}}\right)\lambda_{L-l+1}\left(\tilde{\mathbf{M}}\right) \overset{(b)}{=} \lambda_1\left(\tilde{\mathbf{S}}\right)\lambda_L\left(\tilde{\mathbf{M}}\right), \tag{11}$$

where the step (b) follows from the fact that $rank(\tilde{\mathbf{S}}) = 1$. According to (9), under the same SNR conditions, a larger value of $\mathrm{Tr}(\tilde{\mathbf{S}}\tilde{\mathbf{M}})$ is more beneficial for correctly decoding the transmit symbols from the received signals that contain noise. This can be achieved by maximizing the lower bound of $\mathrm{Tr}(\tilde{\mathbf{S}}\tilde{\mathbf{M}})$. Moreover, $\lambda_1(\tilde{\mathbf{S}})$ remains constant once the candidate symbol vector is selected, and since $\tilde{\mathbf{M}}$ comprises the optimized transmit precoding matrix, the singular values of $\mathbf{P}$ will affect the MLD performance. $\lambda_L(\tilde{\mathbf{M}})$ can be expressed as

$$\lambda_L\left(\tilde{\mathbf{M}}\right) = \lambda_L\left(\mathbf{P}^H\mathbf{H}^H\mathbf{H}\mathbf{P}\right). \tag{12}$$

It can be inferred that $\mathbf{P}$ must be a full-rank matrix, with a larger smallest singular value being preferable. This is due to the fact that a larger smallest singular value of $\mathbf{P}$ will yield a correspondingly larger smallest singular value of $\tilde{\mathbf{M}}$, thereby enhancing MLD performance.

B. SLP Matrix Analysis

When the users apply the MLD method, the precoding matrix can be optimized by solving the following problem

$$\begin{aligned} \mathrm{P2}: \max_{\mathbf{P},t,\mathbf{\Gamma}} \; & t \\ s.t. \quad & \mathbf{C1}: \mathbf{GPs} = \mathbf{U}diag(\mathbf{\Gamma})\bar{\mathbf{s}} \\ & \mathbf{C2}: t-\gamma_m^{\mathcal{O}} \le 0, \forall \gamma_m^{\mathcal{O}} \in \mathcal{O} \\ & \mathbf{C3}: t-\gamma_n^{\mathcal{I}} = 0, \forall \gamma_n^{\mathcal{I}} \in \mathcal{I} \\ & \mathbf{C4}: \|\mathbf{P}\mathbf{s}\|_2^2 \le p, \end{aligned} \tag{13}$$

where $\mathbf{G}$ is a diagonal equivalent channel matrix, where each diagonal element $\mathbf{G}_k$ can be obtained by performing Singular Value Decomposition (SVD) on channel matrix $\mathbf{H}_k$. $\bar{\mathbf{s}} = \left[\bar{\mathbf{s}}_1^T, \bar{\mathbf{s}}_2^T, \cdots, \bar{\mathbf{s}}_K^T\right]^T$, $\mathbf{\Gamma} = \left[\mathbf{\Gamma}_1^T, \mathbf{\Gamma}_2^T, \cdots, \mathbf{\Gamma}_k^T\right]^T$ and $\mathbf{U} = \mathbf{I}_{KL} \otimes [1,1]$. P2 is a convex problem and can be solved by using the CVX tool.

Since the MLD method requires a larger smallest singular value of the transmit precoding matrix to enhance the decoding efficiency, the rank-one property of the SLP matrix makes the MLD method cannot be directly used to decode the received signals transmitted by the SLP scheme, as stated in *Corollary 1*.

Corollary 1: In P2, the optimized transmit SLP matrix $\mathbf{P}$ is rank-one.

Proof: We begin by transforming the power constraint in P2, where $\mathbf{Ps}$ can be decomposed as follows

$$\mathbf{Ps} = \sum_{i=1}^{KL} \mathbf{p}_i s_i, \forall i \in \mathcal{KL}. \tag{14}$$

$\mathbf{Ps}$ can be viewed as a single vector variable for P2, and the distribution of power among each $\mathbf{p}_i s_i$ does not impact the optimal solution. Thus, $\mathbf{p}_i s_i$ can be treated as identical, which results in

$$\|\mathbf{Ps}\|_2^2 = \|KL\mathbf{p}_i s_i\|_2^2 = K^2L^2 s_i^* \mathbf{p}_i^H \mathbf{p}_i s_i = KL\sum_{i=1}^{KL} s_i^* \mathbf{p}_i^H \mathbf{p}_i s_i, \tag{15}$$

then the power constraint is equivalent to

$$\sum_{i=1}^{KL} s_i^* \mathbf{p}_i^H \mathbf{p}_i s_i \leq \frac{p}{KL}. \tag{16}$$

By replacing **C4** in P2 with (16), P2 can be further transformed into

$$\begin{aligned} \text{P3}: \min_{\mathbf{P},t,\mathbf{\Gamma}} \quad & -t \\ s.t. \quad & \mathbf{C1}: \mathbf{g}_{k,l} \sum_{i=1}^{KL} \mathbf{p}_i s_i = \boldsymbol{\gamma}_{k,l}^T \bar{\mathbf{s}}_{k,l}, \forall k \in \mathcal{K}, \forall l \in \mathcal{L} \\ & \mathbf{C2}: t - \gamma_m^{\mathcal{O}} \leq 0, \forall \gamma_m^{\mathcal{O}} \in \mathcal{O} \\ & \mathbf{C3}: t - \gamma_n^{\mathcal{I}} = 0, \forall \gamma_n^{\mathcal{I}} \in \mathcal{I} \\ & \mathbf{C4}: \sum_{i=1}^{KL} s_i^* \mathbf{p}_i^H \mathbf{p}_i s_i \leq \frac{p}{KL}, \end{aligned} \tag{17}$$

where $\mathbf{g}_{k,l}$ is the l-th row of $\mathbf{G}_k$. The Lagrangian function of P3 can be expressed as

$$\begin{aligned} & L\left(\mathbf{p}_i, t, \alpha_{k,l}, \theta_m, \vartheta_n, \alpha_0\right) \\ & = -t + \sum_{k=1}^{K}\sum_{l=1}^{L} \alpha_{k,l}\left(\mathbf{g}_{k,l}\sum_{i=1}^{KL}\mathbf{p}_i s_i - \boldsymbol{\gamma}_{k,l}^T\bar{\mathbf{s}}_{k,l}\right) \\ & \quad + \sum_{m=1}^{card\{O\}} \theta_m\left(t-\gamma_m^O\right) + \sum_{n=1}^{card\{I\}} \vartheta_n\left(t-\gamma_n^I\right) \\ & \quad + \alpha_0\left(\sum_{i=1}^{KL} s_i^*\mathbf{p}_i^H\mathbf{p}_i s_i \leq \frac{p}{KL}\right), \end{aligned} \tag{18}$$

where $\alpha_{k,l}$, $\theta_m \geq 0$, ϑ_n and $\alpha_0 \geq 0$ represent the introduced dual variables, and each $\alpha_{k,l}$ and ϑ_n can be complex. Furthermore, the KKT conditions of (18) can be derived as

$$\frac{\partial L}{\partial t} = -1 + \sum_{m=1}^{card\{O\}} \theta_m + \sum_{n=1}^{card\{I\}} \vartheta_n = 0, \tag{19}$$

$$\theta_m\left(t-\gamma_m^O\right) = 0, \forall \gamma_m^{\mathcal{O}} \in \mathcal{O}, \tag{20}$$

$$t - \gamma_n^{\mathcal{I}} = 0, \forall \gamma_n^{\mathcal{I}} \in \mathcal{I}, \tag{21}$$

$$\frac{\partial L}{\partial \mathbf{p}_i} = \left(\sum_{k=1}^{K}\sum_{l=1}^{L}\alpha_{k,l}\mathbf{g}_{k,l}\right)s_i + \alpha_0 s_i s_i^* \mathbf{p}_i^H = \mathbf{0}, \forall i \in \mathcal{KL}, \tag{22}$$

$$\mathbf{g}_{k,l}\sum_{i=1}^{KL}\mathbf{p}_i s_i - \gamma_{k,l}^T\bar{\mathbf{s}}_{k,l} = 0, \forall k \in \mathcal{K}, \forall l \in \mathcal{L}, \tag{23}$$

$$\alpha_0\left(\sum_{i=1}^{KL} s_i^*\mathbf{p}_i^H\mathbf{p}_i s_i \leq \frac{p}{KL}\right) = 0, \tag{24}$$

It can be observed that $\alpha_0 \neq 0$ based on (22), and given $\alpha_0 \geq 0$, it follows that $\alpha_0 > 0$. This indicates the power constraint must be satisfied with equality when optimality is achieved. Therefore, $\mathbf{p}_i^H$ can be expressed as

$$\mathbf{p}_i^H = -\frac{1}{\alpha_0 s_i^*}\left(\sum_{k=1}^{K}\sum_{l=1}^{L}\alpha_{k,l}\mathbf{g}_{k,l}\right) = \frac{1}{s_i^*}\left(\sum_{k=1}^{K}\sum_{l=1}^{L}\chi_{k,l}\mathbf{g}_{k,l}\right), \tag{25}$$

where we define

$$\chi_{k,l} = -\frac{\alpha_{k,l}}{\alpha_0}, \forall k \in \mathcal{K}, \forall l \in \mathcal{L}. \tag{26}$$

Based on (25), $\mathbf{p}_i$ can be expressed as

$$\mathbf{p}_i = \left(\sum_{k=1}^{K}\sum_{l=1}^{L}\chi_{k,l}^*\mathbf{g}_{k,l}^H\right)\frac{1}{s_i}, \forall i \in \mathcal{KL}, \tag{27}$$

which further leads to

$$\mathbf{p}_i s_i = \sum_{k=1}^{K}\sum_{l=1}^{L}\chi_{k,l}^*\mathbf{g}_{k,l}^H, \forall i \in \mathcal{KL}. \tag{28}$$

It can be observed from (28) that $\mathbf{p}_i s_i$ is constant for any i. This indicates that different precoding vectors for different data symbols are correlated with each other, which subsequently result in $rank(\mathbf{P}) = 1$.

C. Proposed CSI-Free SLP-MLD Design

Based on the aforementioned analysis, we know that the smallest singular value of the SLP matrix should be maximized to enable MLD decoding in the users. However, in a typical MU-MIMO communication system, $\mathbf{P}$ is not a square matrix, which complicates the optimization of its smallest singular value. Therefore, we propose to decompose $\mathbf{P}$ into a Hermitian matrix and a normal matrix. The lower bound of the smallest singular value of $\mathbf{P}$ can be obtained by maximizing the smallest singular value of the corresponding Hermitian matrix. Consequently, a full-rank SLP matrix can be derived and the MLD method can be applied to decode the received signals.

Specifically, given that $\mathbf{P}$ is a $N_T \times KL$ dimensional matrix and $N_t \geq KL$ for MU-MIMO systems, $\mathbf{P}$ can be decomposed into two parts as

$$\mathbf{P} = \begin{bmatrix} \mathbf{P}_1 \\ \mathbf{P}_2 \end{bmatrix}, \tag{29}$$

where $\mathbf{P}_1 \in \mathbb{C}^{KL \times KL}$ is assumed to be a Hermitian matrix and $\mathbf{P}_2$ is a normal matrix of dimensions $(N_T - KL) \times KL$. $\mathbf{P}$ will be full-rank if $\mathbf{P}_1$ is optimized to be a full-rank matrix. Furthermore, according to the fundamental rank properties [16], it can be established that

$$\sigma_{\min}(\mathbf{P}) \geq \sigma_{\min}(\mathbf{P}_1) = \lambda_{\min}(\mathbf{P}_1), \tag{30}$$

where $\sigma_{\min}(\mathbf{P})$ represents the smallest singular value of $\mathbf{P}$ and it can be observed that the smallest eigenvalue of $\mathbf{P}_1$ is the lower bound of the smallest singular value of $\mathbf{P}$. Therefore, the task of maximizing the smallest singular value of $\mathbf{P}$ can be transformed into maximizing the smallest eigenvalue of $\mathbf{P}_1$.

Furthermore, it is important to emphasize that a larger smallest singular value of the transmit precoding matrix is more beneficial to the decoding efficiency of the users based on the preceding analysis in Section III-A. Therefore, the CI constraints **C1**-**C3** in (17), which aim to transform multi-user interference into CI are no longer necessary for optimizing the singular values, as users utilize the MLD method to decode received signals. Thus, the symbol-level smallest singular value maximization problem can be formulated as

$$\begin{aligned}
\text{P4}: \max_{\mathbf{P}_1, \mathbf{P}_2, z} \quad & z \\
s.t. \quad & \mathbf{C1}: \|\mathbf{P}_1\mathbf{s}\|_2^2 + \|\mathbf{P}_2\mathbf{s}\|_2^2 \leq p \\
& \mathbf{C2}: \mathbf{P}_1\mathbf{s} = K\mathbf{P}_{k,1}\mathbf{s}_k, \\
& \quad \mathbf{P}_2\mathbf{s} = K\mathbf{P}_{k,2}\mathbf{s}_k, \forall k \in \mathcal{K} \\
& \mathbf{C3}: \begin{bmatrix} \mathcal{R}(\mathbf{P}_1) & -\mathcal{I}(\mathbf{P}_1) \\ \mathcal{I}(\mathbf{P}_1) & \mathcal{R}(\mathbf{P}_1) \end{bmatrix} - z\mathbf{I} \succeq \mathbf{0},
\end{aligned} \tag{31}$$

where the CI constraints are omitted, $\mathbf{P}_{k,1}$ is a square matrix composed of the first L rows of $\mathbf{P}_k$, and $\mathbf{P}_{k,2}$ includes the last $N_T - L$ rows of $\mathbf{P}_k$. **C2** denotes the independent MLD decoding requirement and the objective function z represents the optimized smallest singular value of $\mathbf{P}_1$.

Notably, our goal is to maximize the smallest singular value of $\mathbf{P}_1$, which is uncorrelated with $\mathbf{P}_2$. Consequently, the optimal solution $\mathbf{P}_2^*$ in P4 is $\mathbf{0}$, as $\mathbf{P}_1$ and $\mathbf{P}_2$ are only coupled through the power budget. Thus, more power can be allocated to $\mathbf{P}_1$ to maximize the objective value. Therefore, P4 can be further simplified to

$$\begin{aligned}
\text{P5}: \max_{\mathbf{P}_1, z} \quad & z \\
s.t. \quad & \mathbf{C1}: \|\mathbf{P}_1\mathbf{s}\|_2^2 \leq p \\
& \mathbf{C2}: \mathbf{P}_1\mathbf{s} = K\mathbf{P}_{1,k}\mathbf{s}_k, \forall k \in \mathcal{K} \\
& \mathbf{C3}: \begin{bmatrix} \mathcal{R}(\mathbf{P}_1) & -\mathcal{I}(\mathbf{P}_1) \\ \mathcal{I}(\mathbf{P}_1) & \mathcal{R}(\mathbf{P}_1) \end{bmatrix} - z\mathbf{I} \succeq \mathbf{0}.
\end{aligned} \tag{32}$$

P5 is an SDP-based convex problem with fewer variables [16] , and can be solved by using the CVX tool. Moreover, there is no CSI involved in P5, only the transmit data symbols are utilized to design the SLP matrix. Consequently, this scheme can be termed an SDP-based CSI-Free SLP scheme, which not only effectively mitigates the negative impact of channel estimation errors on the precoding matrix but also simplifies the symbol-level optimization problem for MU-MIMO systems.

IV. Simulation Results

In this section, we provide a comparative analysis of the numerical results for the proposed SDP-based CSI-Free singular value optimization scheme, in contrast to the traditional transmit BD precoding paired with MLD estimation at the receivers. MonteCarlo simulations are utilized as the evaluation methodology. For each time slot, the transmit power budget is set to $p = 1\,\text{W}$, and the transmit SNR is defined as $\rho = 1/\sigma^2$. The elements of the channel matrix $\mathbf{H}$ are assumed to follow a standard complex Gaussian distribution, specifically $\mathbf{H}_{m,n} \sim \mathcal{CN}(0, 1)$. To ensure clarity, the following abbreviations will be consistently used throughout this section:

1) 'Traditional SLP': traditional optimization-based SLP scheme based on P2;
2) 'BD': traditional BD scheme as proposed in [4];
3) 'Joint Design': joint symbol-level transmit precoding and receive combining scheme proposed in [11];
4) 'SDP': proposed SDP-based CSI-Free symbol-level singular value optimization problem based on P5;
5) 'MLD' or 'QRM-MLD': the MLD or QRM-MLD method used at the user to decode the receive signals.

In Fig. 2, we compare the BER performance of the proposed scheme with the traditional BD scheme for 16QAM constellation with $N_T = 16, N_R = 8, L = 4$ and $K = 2$. It is observed that the 'SDP + MLD' scheme achieves the best BER performance, significantly outperforming the BD-based schemes. This is attributed to the utilization of symbol-by-symbol optimization and MLD method. Furthermore, the BER performance of the 'Traditional SLP' scheme is significantly degraded due to the rank-one property. In addition, the computational complexity of the MLD method increases exponentially with the number of data streams and the modulation order, thus, the QRM-MLD method is a more practical alternative. Therefore, we compare the BER performance of

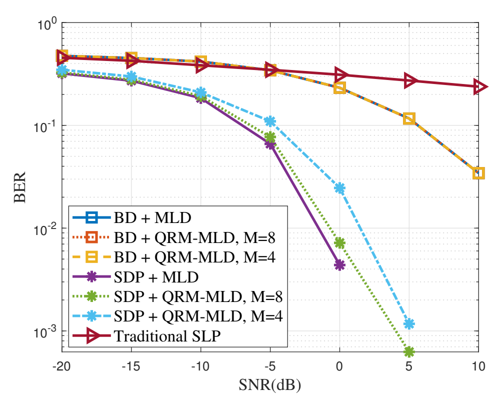

Fig. 2. BER v.s. SNR, 16QAM, $N_T = 16, N_R = 8, L = 4, K = 2, M = 8$ and $M = 4$.

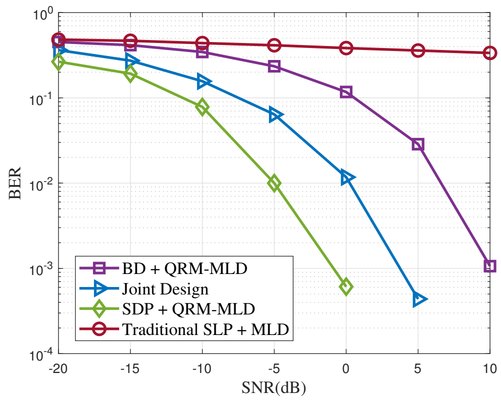

Fig. 3. BER v.s. SNR, 16QAM, $N_T = 32, N_R = 16, L = 4$ and $K = 2$.

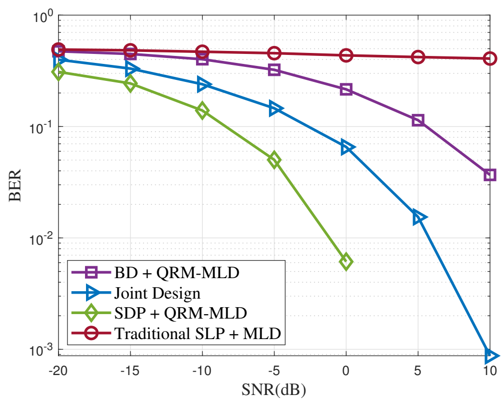

Fig. 4. BER v.s. SNR, 64QAM, $N_T = 32, N_R = 16, L = 4$ and $K = 2$.

the MLD method and the QRM-MLD method for clarity, setting $M = 8$ and $M = 4$. It can be observed that the MLD method and the QRM-MLD method achieve the same BER performance when using 'BD' precoding scheme, since the precoding matrix is forced to lie in the null space of the channel matrix. Furthermore, the QRM-MLD method exhibits a slight performance loss compared to the MLD method for 'SDP' scheme, this indicates a trade-off between the decoding performance and computational complexity.

In Fig. 3 and Fig. 4, we compare the BER performance of different schemes employing 16QAM and 64QAM constellation symbols. In both figures, $N_T = 32, N_R = 16, L = 4, K = 2$. It can be observed that the BER decreases with the increase of the SNR, and higher BER gains can be achieved in the high SNR regime. The proposed SLP-based schemes clearly outperform the 'BD' scheme due to their symbol-by-symbol optimization. However, the rank deficiency of the traditional SLP matrix results in the poorest performance for the 'Traditional SLP' scheme. Moreover, the performance improvement is more pronounced in the case of 'SDP' scheme compared to the 'Joint Design' scheme, this benefits from the use of the QRM-MLD method at the users. Additionally, it can be observed that under the same system parameters, 16QAM modulation achieves a lower BER compared to 64QAM modulation. This is due to the smaller distance between symbols and there are more inner points at higher modulation orders, which increases the likelihood of decoding errors.

V. Conclusion

In this work, we have investigated symbol-level transmit precoding for MU-MIMO communication systems utilizing QAM symbols. To address the dependence of the receive combining matrix on the transmit symbols, we have employed the MLD method to decode the received signals. Furthermore, we have proposed a novel CSI-Free SLP scheme focused on maximizing the smallest singular values to enable SLP method in systems utilizing MLD decoding. Numerical simulations have demonstrated that the proposed scheme significantly outperforms the traditional BD-based approach.

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