# A E-Companion for "Incentivizing flexible workers in the gig economy: The case of ride-hailing"

## A.1 Summary of Notations

- $n$ represents the number of locations.
- $T = (a_{i,j})_{n\times n}$ represents the transition matrix where $a_{i,j} > 0$ denotes the probability that a person at location $i$ wishes to travel to location $j$.
- $\theta$ represents the total number of drivers. The number of passengers is normalized to 1.
- $\delta_{i,j}$ is the physical distance between locations $i$ and $j$.
- $p_i$ is the per-mile price for a ride originating from location $i$.
- $c_i$ is the commission rate (percentage of the revenue that the driver takes home) for a ride originating from location $i$.
- $x_i$ and $y_i$ denote the number of cars and people at location $i$ with $\sum_{i=1}^{n} x_i = \theta$ and $\sum_{i=1}^{n} y_i = 1$.
- $r_i = y_i(1 - p_i)$ denotes the number of people willing to hire a car at location $i$.
- $m_i = \min\{r_i, x_i\}$ is the number of matches generated at location $i$.
- $\eta_i$ is the probability that a driver who is searching at location $i$ finds a passenger.
- $w$ refers to the wage that drivers could earn in the labor market.
- $\varphi_h$ is the probability that a driver matched with a passenger continues to offer driving services in the next period.
- $\varphi_l$ is the probability that a driver without a match continues to remain in service in the next period. We assume that $\varphi_h > \varphi_l$, i.e. being matched increases the likelihood of remaining in the service while being unmatched increases the likelihood of dropping out.
- $\sigma = (\sigma_1, ..., \sigma_n)$ is the unique steady-state vector of the transition matrix $T$.
- $M = \sum_{j=1}^{n} m_j$ denotes the total matches generated.
- $d_i = \sum_{j=1}^{n} a_{i,j}\delta_{i,j}$ is the average trip length of a ride originating from $i$. We label the locations from 1 to $n$ in such a way that $d_1 < d_2 < ... < d_n$.
- $\pi$ is the platform's per-period earnings.
- $\mathbb{E}_\sigma(d) = \sum_{i=1}^{n} \sigma_i d_i$ is a weighted sum of distances $d_1, ..., d_n$ and can be interpreted as the average trip length in the city. $\mathbb{E}_\sigma(\sqrt{d}) = \sum_{i=1}^{n} \sigma_i \sqrt{d_i}$ is similar.
- $\bar{\theta}_i$ is the threshold (minimum number of cars) necessary to maintain the interior equilibrium under model $i = 1, ..., 4$.
- $h(\mathbf{p}) = \sum_{i=1}^{n} \frac{\sigma_i}{1-p_i}$ and $g(\mathbf{p}) = \sum_{i=1}^{n} \sigma_i p_i d_i$.

- $\theta = \mu w$ represents the labor supply function, where $w$, with some abuse of notation, is expected earnings and $\mu$ is the labor market sensitivity parameter.
- $\theta_i^*$ is the optimal entry under model $i$.
- $cs_i$ is the consumer surplus at location $i$, while $CS$ is the total consumer surplus.
- $\alpha \in (0, 1)$ is the proportion of (behavioral) customers who turn off their app and exit the platform when they realize that it is using location-specific pricing and not uniform pricing.
- $\epsilon(p_i, \mu)$ is the elasticity of the price $p_i$ with respect to $\mu$. Similarly, $\epsilon(c_i, \mu)$ is the elasticity of commission rate $c_i$ with respect to $\mu$.

## A.2 Driver Entry: An Alternative Approach

In the main text, driver entry is modeled by the relationship $\theta = \mu w$, where $\mu$ captures the labor market's sensitivity to earning opportunities. Here, instead of relying on a single parameter, we offer a more granular approach to modeling driver entry and examine the robustness of our earlier results.

To start, suppose that the potential driver force consists of $I$ distinct groups, each with a different likelihood of joining the platform. Specifically, each individual in group $i = 1, 2, ..., I$ has a reservation wage to participate, and these reservation wages are distributed according to a cumulative distribution function $F_i$. An individual participates only if his reservation wage is less than or equal to the expected earnings in the market, $w$. Consequently, the total number of participants from group $i$ is equal to

$$\psi_i \cdot F_i(w),$$

where $\psi_i$ is the measure of individuals in that group. Summing over all groups, the total number of drivers entering the platform is equal to

$$\theta = \sum_{i=1}^{I} \psi_i \cdot F_i(w). \tag{18}$$

With this new approach, we depart from a single sensitivity parameter, and instead model driver entry as a composite expression. It depends on the size of each group, $\psi_i$, their likelihood of participation, $F_i$, and the expected earnings, $w$. We now proceed to derive the optimal entry under Model 4. While the main text is based on four operational models, for brevity, here we focus on Model 4, omitting model-specific indices when understood.

The profit is given by (13). Substituting for prices, given by (32), the expression becomes

$$\pi = \mathbb{E}_\sigma\left(d\right)\theta - \mathbb{E}_\sigma^2(\sqrt{d})\theta^2 - w\theta\left(1 - \varphi_h\right).$$

The platform solves $\max_\theta \pi(p)$ subject to (18). The first-order condition yields[25]

$$\frac{d\pi}{d\theta} = \mathbb{E}_\sigma\left(d\right) - 2\theta\mathbb{E}_\sigma^2(\sqrt{d}) - w\left(1 - \varphi_h\right) - \theta\left(1 - \varphi_h\right)\frac{dw}{d\theta} = 0.$$

Applying the Implicit Function Theorem to (18), we have

$$\frac{dw}{d\theta} = \frac{1}{\sum_{i=1}^{I} \psi_i \cdot f_i(w)}.$$

Thus, after substituting for (18), the first-order condition becomes

$$\mathbb{E}_\sigma\left(d\right) = 2\mathbb{E}_\sigma^2(\sqrt{d})\sum_{i=1}^{I} \psi_i \cdot F_i(w) + w\left(1 - \varphi_h\right) + \left(1 - \varphi_h\right)\frac{\sum_{i=1}^{I} \psi_i \cdot F_i(w)}{\sum_{i=1}^{I} \psi_i \cdot f_i(w)}. \tag{19}$$

[25] It is straightforward to verify that the second order condition holds under mild conditions. Indeed, note that

$$\frac{d^2\pi}{d\theta^2} = -2\mathbb{E}_\sigma^2(\sqrt{d}) - 2\frac{dw}{d\theta}\left(1 - \varphi_h\right) - \theta\left(1 - \varphi_h\right)\frac{d^2 w}{d\theta^2}.$$

The first two terms are negative, whereas the last term can be positive or negative. Unless $\varphi_h$ is too small and $F_i$s are extremely convex, the total sum remains negative, satisfying the second order condition.

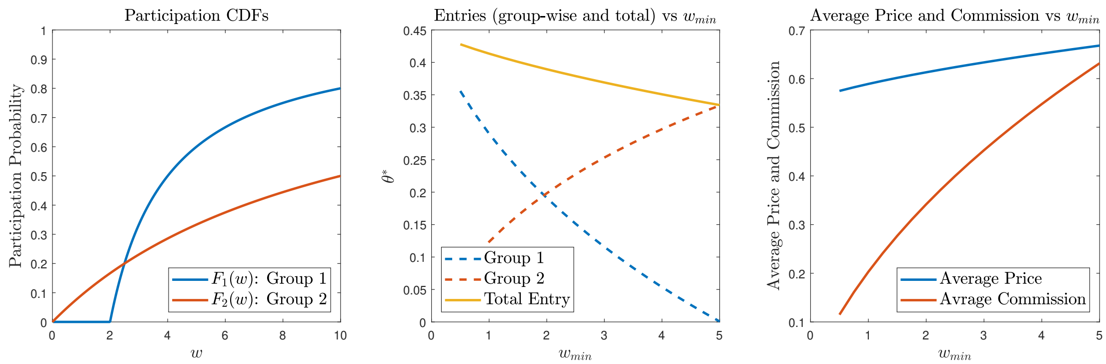


Figure 9: CDFs, Entry, Average Price and Commission

Drivers can be categorized in many ways, such as by gender, age, whether they use the platform for primary or supplemental income, whether they are exclusive drivers or multi-homing drivers, and so on. After compiling the necessary information—including the number of different groups, their density functions, and population measures—the platform can determine the optimal compensation level $w$ using (19). The corresponding level of entry $\theta$ can then be recovered via (18). Once $\theta$ is pinned down, the remaining objects—prices, commissions, profits—can be determined as was done previously.

As an example, suppose there are two distinct groups with equal population size, i.e. let $I = 2$ with $\psi_1 = \psi_2 = 1$. Suppose that the CDF of Group 1 is given by

$$F_1(w) = \begin{cases} 0 & \text{if } w < w_{min} \\ \frac{w - w_{min}}{w} & \text{if } w \geq w_{min}, \end{cases}$$

where $w_{min}$ is the minimum earnings threshold below which Group 1 drivers do not participate. Once this threshold is met, though, they are not too selective about additional compensation. (Fig. 9 illustrates $F_1$ for $w_{min} = 2$.) It is sensible to think that this behavior reflects drivers who rely on the platform for their primary income and need a minimum level of earnings to justify their participation.

For Group 2, the CDF is given by

$$F_2(w) = \frac{w}{w + s},$$

where a higher value of $s$ shifts the CDF downward, indicating a lower likelihood of participation. As an illustration, Fig. 9 plots $F_2$ for $s = 10$. Group 2 drivers are more flexible and, unlike Group 1 drivers, respond to earning opportunities even at low levels of $w$. This behavior is consistent with individuals who drive to earn supplementary income.

Given $F_1$ and $F_2$, we simulate key equilibrium outcomes against $w_{min}$, the earning threshold below which no one from Group 1 participates. (As in earlier examples, we consider a city with five locations, using the layout and transition matrix shown in Fig. 1, right panel.) A higher $w_{min}$ shifts $F_1$ to the right, making the drivers in this group less responsive to earning opportunities, and therefore, less likely to participate. In the simulation we vary $w_{min}$ between 0.5 to 5. As $w_{min}$ increases, Group 1 drivers become less responsive to driving opportunities, and consequently, entry from Group 1 drops significantly. In contrast, entry by Group 2 rises, but total entry still decreases

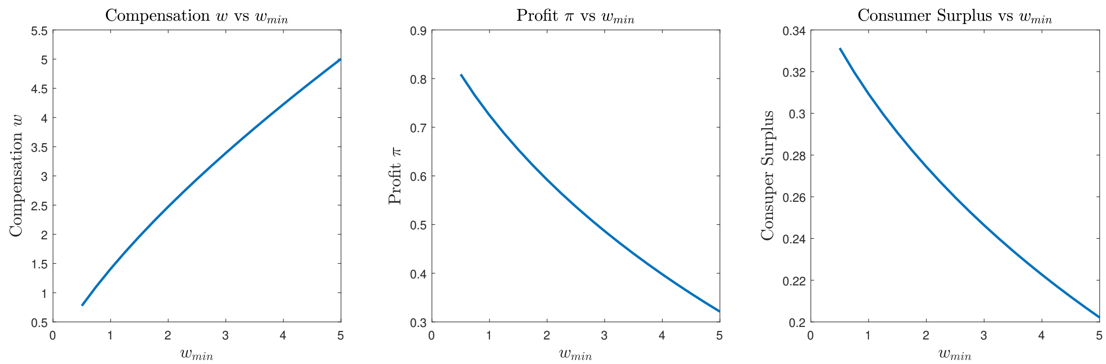


Figure 10: Driver compensation, Profits and Consumer Surplus

(Fig. 9, middle panel).

To see why, note that the platform raises both prices and commission rates across all locations to address the reduced responsiveness of Group 1 drivers. These adjustments result in an overall rise in driver compensation $w$. Since $F_2$ remains unchanged, the higher compensation attracts more drivers from Group 2, sharply increasing their presence in the driver fleet. However, the shifting $F_1$ means that, despite the rise in compensation, fewer drivers from Group 1 choose to join the platform, leading to a decline in total entry. Finally, the reduced total entry, combined with higher prices and increased driver compensation, results in lower profits and consumer surplus. (All these claims can be verified via Fig. 9 and 10.) This richer model of driver entry allows the platform to better anticipate participation behavior by each group, predict the composition of the driver fleet, and assess how different types of drivers respond to changes in commissions and prices.

These findings are consistent with the results presented in the previous section. Indeed, a rise in $w_{min}$ in this alternative version of driver entry is akin to a fall in $\mu$ in the benchmark model.[26] Proposition 7 shows that when $\mu$ declines, prices and commissions rise, $w$ increases, entry falls, and both profits and consumer surplus fall. The simulations in here confirm these insights, reinforcing the robustness of our earlier results.

Further note that as $w_{min}$ rises, the platform responds by increasing commission rates much more sharply than prices (Fig. 9, right panel). A modest price increase is still required to balance the demand with the reduced overall driver supply, but it remains significantly smaller in magnitude than the corresponding adjustment in commissions. This observation is in line with Remark 2, which argues that commissions are a more effective tool than prices for managing labor market fluctuations. The intuition is the same: higher commissions directly incentivize drivers without reducing customer demand. In contrast, raising prices suppresses demand, making it a less effective way to incentivize drivers.

To conclude, instead of summarizing labor market sensitivity with a single parameter, here we explicitly account for differences across driver groups, including their population sizes and participation behavior. Despite these added details, the main insights remain unchanged, which reinforces the validity of our original approach and confirms the robustness of our earlier results.

[26] As $w_{min}$ increases from 0.5 to 5, the resulting $w$ rises from 0.78 to 5, while $\theta$ drops from 0.43 to 0.33. In the benchmark model the relationship between $w$ and $\theta$ is given by $\theta = \mu w$, which means that the implied $\mu$ falls from 0.55 to 0.07.

### A.3 Proofs

**Proof of Lemma 1.** A Markov chain with a finite state space is said to be *regular* if a power of its transition matrix has only positive entries. In our model $n$ is finite and since $a_{i,j} > 0$ it is easy to verify that $T^2$ has only positive entries. It follows that the Markov chain associated with $T$ is regular and thus ergodic. For an ergodic Markov chain, there is a unique steady state vector

$$\sigma = (\sigma_1, .., \sigma_n) \text{ with } \sigma_i > 0 \text{ and } \sum_{i=1}^{n} \sigma_i = 1$$

satisfying $\sigma = \sigma T$. Furthermore, any vector $\mathbf{v} > \mathbf{0}$ such that $\mathbf{v} = \mathbf{v}T$ must be a multiple of $\sigma$ (see Grinstead and Snell (1998), Theorem 11.10). In the steady state $\mathbf{m} = \mathbf{m}T$, which implies that $m_i = \zeta\sigma_i$, where $\zeta$ is a positive scalar. Since $\sum_{i=1}^{n} \sigma_i = 1$ we have $\zeta := \sum_{i=1}^{n} m_i \equiv M$. ■

**Proof of Lemma 2.** By contradiction, suppose that at location 1 we have $x_1 < r_1$. Since $\eta_i < 1$ for $i \geq 2$ we have $x_i > r_i$ for $i \geq 2$. It follows that $m_1 = x_1$ and $m_i = r_i$; thus

$$M = \sum_{i=1}^{n} m_i = x_1 + \sum_{i=2}^{n} r_i.$$

Per (2), $m_i = \sigma_i M$; hence $x_1 = \sigma_1 M$ and $r_i = \sigma_i M$ and therefore $r_i = \sigma_i x_1/\sigma_1$ for $i \geq 2$. At location 1 we have $\eta_1 = 1$, thus (4) implies

$$pcd_1 = w\left(1 - \varphi_h\right).$$

Substituting this relationship into (6) yields

$$x_1 = \frac{\sigma_1\left(1 - \varphi_l\right) d_1 \theta}{\left(1 - \varphi_h\right) \mathbb{E}_\sigma\left(d\right) + d_1\left(\varphi_h - \varphi_l\right)}.$$

Noting (i) $r_i = (1-p)\, y_i$, (ii) $\sum_{i=1}^{n} y_i = 1$, (iii) $\sum_{i=1}^{n} \sigma_i = 1$ and (iv) $r_i = \sigma_i x_1/\sigma_1$ for $i \geq 2$, we have

$$x_1 < r_1 \quad \Leftrightarrow \quad p < \bar{p} \equiv 1 - \frac{x_1}{\sigma_1}.$$

Furthermore, using these equalities, the platform's profit in (8) can be written as

$$\pi = p\mathbb{E}_\sigma\left(d\right)\frac{x_1}{\sigma_1} - w\left[\left(1 - \varphi_l\right)\theta - \left(\varphi_h - \varphi_l\right)\frac{x_1}{\sigma_1}\right].$$

The profit function $\pi$ rises in $p$, which means that setting $p < \bar{p}$ is suboptimal to $p = \bar{p}$. Thus, an outcome with $x_1 < r_1$ cannot be an equilibrium. ■

**Proof of Proposition 1.** Recall that $r_i \leq x_i$ for all $i$; thus $m_i = r_i$ and $\eta_i = r_i/x_i$ Furthermore, since (i) $m_i = \sigma_i M$, (ii) $r_i = y_i\,(1-p)$ and (iii) $\sum_{i=1}^{n} y_i = 1$ we have

$$M = 1 - p \quad \text{and} \quad r_i = \sigma_i\left(1 - p\right),$$

and therefore

$$\pi = p\left(1-p\right)\mathbb{E}_\sigma\left(d\right) - w\left[\left(1 - \varphi_l\right)\theta - \left(\varphi_h - \varphi_l\right)\left(1 - p\right)\right].$$

The platform's problem is

$$\max_p \pi \text{ s.t. } r_i \leq x_i \text{ for all } i,$$

where the constraints obtain per Lemma 2. The objective function is concave in $p$; thus, ignoring the constraints, the first-order condition yields the global maximum

$$p^{int} = \frac{1}{2} - \frac{(\varphi_h - \varphi_l)\, w}{2\mathbb{E}_\sigma (d)}.$$

Now focus on the constraints. First note that $r_i \leq x_i \Leftrightarrow \eta_i \leq 1$. Since $d_1 < d_2 < ... < d_n$, only $\eta_1$ can be equal to 1, while all other $\eta_i$s must be strictly less than 1. The relevant constraint, therefore, is the first one $\eta_1 \leq 1$. Equation (4) implies.

$$\eta_1 \leq 1 \Leftrightarrow w(1 - \varphi h) \leq pcd_1.$$

Furthermore, using (7) we have

$$c = \frac{w(1 - \varphi_l)\,\theta}{(1 - p)\, p\mathbb{E}_\sigma (d)} - \frac{(\varphi_h - \varphi_l)\, w}{p\mathbb{E}_\sigma (d)}. \tag{20}$$

Substituting $p^{int}$ and $c$ into the inequality above yields $\eta_1 \leq 1 \quad \Leftrightarrow \quad \theta \geq \bar{\theta}_1$, where

$$\bar{\theta}_1 := \left[\frac{(1 - \varphi_h)\,\mathbb{E}_\sigma (d)}{d_1 (1 - \varphi_l)} + \frac{\varphi_h - \varphi_l}{1 - \varphi_l}\right] \left(\frac{1}{2} + \frac{(\varphi_h - \varphi_l)\, w}{2\mathbb{E}_\sigma (d)}\right),$$

i.e. if there are sufficiently many cabs in the city, then the constraint is slack and $p^{int}$ is feasible. We refer to this outcome as the interior equilibrium. If however, $\theta < \bar{\theta}_1$ then the constraint $\eta_1 \leq 1$ binds, and therefore per equation (4) we have

$$pcd_1 = w(1 - \varphi_h).$$

Substituting for $c$ yields the corner equilibrium price

$$p^{cor} = 1 - \frac{d_1 (1 - \varphi_l)\,\theta}{\mathbb{E}_\sigma (d)(1 - \varphi_h) + d_1 (\varphi_h - \varphi_l)}. \tag{21}$$

Commission rates $c^{int}$ and $c^{cor}$ can be obtained by substituting $p^{int}$ and $p^{cor}$ into (20). ■

**Proof of Lemma 3.** Fix some $\mathbf{p}$ and let $\mathcal{S}_0$ denote the set of locations in which demand is greater than or equal to supply, with at least one location exhibiting excess demand, and $\mathcal{S}_1$ the set of locations with excess supply, i.e. $\mathcal{S}_0 = \{i \in \mathbb{N} : r_i \geq x_i\}$ with at least one inequality strict and $\mathcal{S}_1 = \{i \in \mathbb{N} : r_i < x_i\}$.

*Case 1 -* $\mathcal{S}_1 \neq \emptyset$ : Since $x_i \leq r_i$ we have $\eta_i = 1$ for all $i \in \mathcal{S}_0$. Similarly $x_i > r_i \Leftrightarrow \eta_i < 1$ for all $i \in \mathcal{S}_1$. The indifference condition (4) implies

$$p_i d_i = p_j d_j, \text{ for all } i, j \in \mathcal{S}_0 \qquad \text{and} \qquad p_i d_i < p_j d_j, \text{ for all } i \in \mathcal{S}_0 \text{ and } j \in \mathcal{S}_1.$$

Suppose that the platform leaves prices in $\mathcal{S}_1$ intact but increases prices in $\mathcal{S}_0$ to $p_i' = p_i + \varepsilon_i$, where the vector $\varepsilon$ is positive but infinitesimally small, satisfying

$$\varepsilon_i d_i = \varepsilon_j d_j, \text{ for all } i, j \in \mathcal{S}_0.$$

Note that $p'_i d_i = p'_j d_j$, which means $\eta'_i = \eta'_j$ for all $i, j \in \mathcal{S}_0$. It follows that either $\eta'_i = 1$ or $\eta'_i < 1$ for all $i \in \mathcal{S}_0$. Since $\varepsilon$ can be arbitrarily small, it can be chosen to ensure that

$$\eta'_i = 1, \text{ for all } i \in \mathcal{S}_0 \quad \text{ while } \quad p'_i d_i < p_j d_j, \text{ for all } i \in \mathcal{S}_0 \text{ and } j \in \mathcal{S}_1,$$

Locations in $\mathcal{S}_0$: Prices are higher after the intervention. As for the number of rides, before the intervention $m_i = x_i$. The fact that $\eta'_i = 1$ implies that after the intervention we have $m'_i = x'_i$; however, note that $x'_i > x_i$ because now more drivers search in $\mathcal{S}_0$. Since both the prices and the number of rides go up, the platform earns more in $\mathcal{S}_0$ than it did before.
Locations in $\mathcal{S}_1$: Prices remain intact. The number of rides also remains the same. To see why, note that after the intervention we have $p'_i d_i < p_j d_j$ for all $i \in \mathcal{S}_0$ and $j \in \mathcal{S}_1$, which means that $\eta'_j < 1$ for all $j \in \mathcal{S}_1$. This, in turn, implies that the number of matches at each location in $\mathcal{S}_1$ remains the same. It follows that the platform earns the same in $\mathcal{S}_1$ as it did before. The intervention allows the platform to move some idle drivers in $\mathcal{S}_1$ to $\mathcal{S}_0$ and earn more; thus, the initially conjectured outcome cannot be an equilibrium.

*Case 2 - $\mathcal{S}_1 = \emptyset$:* Along this outcome $x_i \leq r_i$ for all $i = 1, ..., n$ with at least one inequality strict; thus $\eta_i = 1$ for all $i = 1, .., n$. Pick location $j$ as a reference point, and note that since $\eta_i = 1$ the indifference condition (4) becomes $p_i d_i = p_j d_j$, for all $i$. Substituting this relationship into (6) we obtain $x_i = \sigma_i \theta$ for all $i$. Recall that $r_i = y_i (1 - p_i)$; thus $x_i \leq r_i \Leftrightarrow y_i \geq \sigma_i \theta / (1 - p_i)$, with at least one inequality strict. It follows that

$$\sum_{i=1}^{n} y_i > \sum_{i=1}^{n} \frac{\sigma_i \theta}{1 - p_i} \quad \Leftrightarrow \quad \Delta(p_j) := \theta - \left[ \sum_{i=1}^{n} \frac{\sigma_i d_i}{d_i - p_j d_j} \right]^{-1} < 0.$$

The second step obtains because $\sum_{i=1}^{n} y_i = 1$ and $p_i d_i = p_j d_j$. The inequality $\Delta(p_j) < 0$ is strict because at least one location has $x_i < r_i$. Note that $\Delta$ increases in $p_j$ and $\Delta(1) > 0$. Since $\Delta(p_j) < 0$, there exists some $p'_j \in (p_j, 1)$ satisfying $\Delta(p'_j) = 0$. So, if the platform increases $p_j$ to $p'_j$ at location $j$, while also ensuring that $p'_i d_i = p'_j d_j$ at other locations, then $x'_i = r'_i$ for all $i$, i.e. no location exhibits excess demand. Prior to the intervention we had $x_i \leq r_i$, with at least one strict inequality; thus the number of rides was equal to $m_i = x_i = \sigma_i \theta$ for all $i$. After the intervention, we have $x'_i = r'_i$; thus, the number of rides is still equal to $m'_i = x'_i = \sigma_i \theta$ for all $i$. Prices, on the other hand, are now higher, which means that the platform earns more than before. It follows that the initially conjectured outcome cannot be an equilibrium. ■

**Proof of Proposition 2.** Ignoring the constraints, the platform solves

$$\max_{\mathbf{p}} \pi(\mathbf{p}) = \max_{\mathbf{p}} \; [g(\mathbf{p}) + w(\varphi_h - \varphi_l)] \, h(\mathbf{p})^{-1} - w(1 - \varphi_l)\,\theta.$$

The first order condition with respect to $p_i$ implies

$$(1 - p_i)^2 d_i = [g(\mathbf{p}) + w(\varphi_h - \varphi_l)] \, h(\mathbf{p})^{-1} \text{ for all } i = 1, ..., n.$$

It follows that $p_j = 1 - (1 - p_i)\sqrt{d_i/d_j}$; hence

$$p_i^{int} = 1 - \frac{\mathbb{E}_\sigma(d) + w(\varphi_h - \varphi_l)}{2\sqrt{d_i}\mathbb{E}_\sigma(\sqrt{d})}. \tag{22}$$

Substituting $p_i$ into (7) yields the equilibrium commission rate

$$c^{int} = \frac{4w(1-\varphi_l)\theta\mathbb{E}_\sigma\left(\sqrt{d}\right)^2}{\mathbb{E}_\sigma(d)^2 - w^2(\varphi_h - \varphi_l)^2} - \frac{2w(\varphi_h - \varphi_l)}{\mathbb{E}_\sigma(d) - w(\varphi_h - \varphi_l)}.$$

**Claim 1** *The objective function* $\pi(\mathbf{p})$ *is strictly concave in* $\mathbf{p}$.

We start by showing that $h^{-1}(\mathbf{p})$is strictly concave. The strategy is to establish that $h^{-1}(\mathbf{p})$ lies underneath its linearization at some $\mathbf{p}^0$, which is given by

$$\hat{h}^{-1}(\mathbf{p}) = h^{-1}(\mathbf{p}^0) + \nabla h^{-1}\left(\mathbf{p}^0\right)\left(\mathbf{p} - \mathbf{p}^0\right) = \sum_i \frac{\sigma_i(1-p_i)}{(1-p_i^0)^2} \cdot \left[\sum_{i=1} \frac{\sigma_i}{1-p_i^0}\right]^{-2}.$$

The function is concave if $h^{-1}(\mathbf{p}) < \hat{h}^{-1}(\mathbf{p})$, i.e. if

$$\left[\sum_{i=1}^n \frac{\sigma_i}{1-p_i^0}\right]^2 < \sum_{i=1}^n \frac{\sigma_i(1-p_i)}{(1-p_i^0)^2} \sum_{i=1}^n \frac{\sigma_i}{1-p_i}.$$

Letting $t_i \equiv \sqrt{\frac{\sigma_i(1-p_i)}{(1-p_i^0)^2}}$ and $s_i \equiv \sqrt{\frac{\sigma_i}{1-p_i}}$, the inequality becomes

$$\left[\sum_{i=1}^n t_i s_i\right]^2 < \sum_{i=1}^n t_i^2 \sum_{i=1}^n s_i^2.$$

The result follows from Cauchy-Scwharz. Note that the inequality is strict; thus $h^{-1}(\mathbf{p})$is strictly concave. Observe that $\pi(\mathbf{p}) = [g(\mathbf{p}) + w(\varphi_h - \varphi_l)]\, h^{-1}(\mathbf{p})$ minus a constant, where $g$ is linear and increasing; whereas $h^{-1}$ is strictly concave and decreasing in $p$. Thus $\pi$ is strictly concave (Boyd et al. (2004), pg. 119).

For this (interior) equilibrium to emerge we need $r_1 \leq x_1 \Leftrightarrow \eta_1 \leq 1$ which is equivalent to

$$w(1-\varphi_h) \leq p_1 d_1 c,$$

i.e. the constraint at location 1 ought to be slack. After substituting for $c$, the condition is equivalent to $\theta \geq \bar{\theta}_{2,0}$, where

$$\bar{\theta}_{2,0} := \left\{\frac{\mathbb{E}_\sigma(d) + w(\varphi_h - \varphi_l)}{2(1-\varphi_l)\mathbb{E}_\sigma(\sqrt{d})^2}\right\}\left\{\frac{(1-\varphi_h)\{\mathbb{E}_\sigma(d) - w(\varphi_h - \varphi_l)\}}{2p_1 d_1} + \varphi_h - \varphi_l\right\}. \tag{23}$$

If the constraint is slack at location 1 then it is slack at every other location (Lemma 4); thus $\theta > \bar{\theta}_{2,0}$ is sufficient. Finally, the inequality $p_i^{int} < p_{i+1}^{int}$ follows from the fact that $d_i < d_{i+1}$. ■

**Proof of Lemma 4.** If $\lambda_k = 0$ then the constraint is slack at location $k$, thus

$$r_i < x_i \quad \Rightarrow \quad \left[g(\mathbf{p}) + (\varphi_h - \varphi_l)\frac{w}{c}\right] h(\mathbf{p})^{-1} - \theta(\varphi_h - \varphi_l)\frac{w}{c} < p_k d_k \theta \quad \text{(i)}$$

Furthermore, the first-order condition implies

$$(1 - p_k)^2 d_k = [g(\mathbf{p}) + w(\varphi_h - \varphi_l)] h(\mathbf{p})^{-1} \quad \text{(ii)}$$

Now by contradiction suppose $\lambda_{k+1} > 0$. Since the constraint is assumed to bind at location $k+1$ we have

$$p_{k+1} d_{k+1} \theta = \left[g(\mathbf{p}) + (\varphi_h - \varphi_l)\frac{w}{c}\right] h(\mathbf{p})^{-1} - \theta(\varphi_h - \varphi_l)\frac{w}{c} \quad \text{(iii)}$$

The profit function $\pi$ is strictly concave. The constraint is assumed to bind at location $k+1$. This implies

$$d_{k+1}(1 - p_{k+1})^2 < [g(\mathbf{p}) + w(\varphi_h - \varphi_l)] h(\mathbf{p})^{-1} \quad \text{(iv)}$$

Since $d_{k+1} > d_k$, equations (ii) and (iv) together imply that

$$d_k(1 - p_k)^2 > d_{k+1}(1 - p_{k+1})^2 \Rightarrow p_{k+1} > p_k.$$

Notice, however, (i) and (iii) together imply that $p_k > p_{k+1}$; a contradiction. Thus $\lambda_{k+1}$ must be zero. The second part of the Lemma is proved similarly. ■

**Proof of Proposition 3**. Equation (10) implies that

$$p_{i,k} d_i = p_{1,k} d_1 \text{ for } \; i = 1, ..., k.$$

The inequality $p_{1,k} > ... > p_{k,k}$ follows from the fact that $d_1 < ... < d_n$. Similarly, equation (11) implies that

$$(1 - p_{i,k})\sqrt{d_i} = (1 - p_{n,k})\sqrt{d_n} \quad \text{for } \; i = k+1, ..., n.$$

Again, the inequality $p_{n,k} > ... > p_{k+1,k}$ follows from $d_1 < ... < d_n$. To compute the commission rate, note $\eta_i = 1$ for $i = 1, .., k$, thus equation (4) becomes

$$c_k p_{i,k} d_i = w(1 - \varphi_h), \text{ for } i = 1, .., k.$$

Noting that $p_{i,k} d_i = p_{1,k} d_1$, this relationship implies

$$c_k = w(1 - \varphi_h) / p_{1,k} d_1. \tag{24}$$

Therefore, if a feasible $p_{1,k}$ exists then $c_k$ can be computed using the relationship above. In what follows, we will show the existence of such a $p_{1,k}$. With no loss in generality let $\varphi_h = \varphi_l$, thus equations (10) and (11) can be rewritten as

$$\Omega := g(\mathbf{p}) / h(\mathbf{p}) = p_i d_i \theta \text{ for } \; i = 1, ..., k \quad \text{and} \quad (1 - p_i)^2 d_i = \Omega \;\text{ for }\; i = k+1, ..., n.$$

The prices in regime$-k$ can be written in terms of $p_{1,k}$ as follows:

$$p_{i,k} = p_{1,k} d_1 / d_i \text{ for } i = 1, .., k \;\text{ and }\; p_{i,k} = 1 - \sqrt{p_{1,k} d_1 \theta / d_i} \text{ for } i = k+1, .., n. \tag{25}$$

Substituting these relationships into the equality $p_{1,k}d_1\theta = \Omega$ yields

$$\Delta_k\left(p_{1,k}\right) := \frac{p_{1,k}d_1\sum_{i=1}^{k}\sigma_i + \sum_{i=k+1}^{n}\sigma_i d_i - \sqrt{p_{1,k}d_1\theta}\sum_{i=k+1}^{n}\sigma_i\sqrt{d_i}}{\sum_{i=1}^{k}\frac{\sigma_i d_i}{d_i - p_{1,k}d_1} + \frac{1}{\sqrt{p_{1,k}d_1\theta}}\sum_{i=k+1}^{n}\sigma_i\sqrt{d_i}} - p_{1,k}d_1\theta = 0. \tag{26}$$

The rest of the proof is by induction. The first step is to show that when $k = 1$ there exits some $p_{1,1} \in (0,1)$ satisfying $\Delta_1 = 0$. First, note that if $p_{1,1} = 1$ then $\Delta_1(1) < 0$. Indeed when $p_{1,1} = 1$ the expression $\frac{\sigma_1 d_1}{d_1 - p_{1,k}d_1}$ in the denominator tends to infinity; rendering $\Delta_1(1) = -d_1\theta < 0$. Second, when $\Delta_1$ is evaluated at $p_1^{int}$ and $\bar{\theta}_{2,0}$, which are given by (22) and (23), we obtain $\Delta_1\left(p_1^{int};\bar{\theta}_{2,0}\right) = 0$. Since $\Delta_1$ falls in $\theta$ we have $\Delta_1\left(p_1^{int}\right) > 0$ for $\theta < \bar{\theta}_{2,0}$. The function $\Delta_1$ is continuous; thus, by the Intermediate Value Theorem there exists a $p_{1,1}$ between $p_1^{int}$ and 1 satisfying $\Delta_1\left(p_{1,1}\right) = 0$. Remaining prices are pinned down through (25); i.e.

$$p_{i,1} = 1 - \sqrt{p_{1,1}d_1\theta/d_i} \text{ for } i = 2, ..., n.$$

Since $d_2 < ... < d_n$ it is easy to see that $p_{2,1} < ... < p_{n,1}$. Furthermore, since $p_{1,1}$ is feasible, i.e. $p_{1,1} \in (0,1)$, all other prices are also feasible.

Per Lemma 4 the relevant constraint when $k = 1$ is $p_{2,1}d_2\theta \geq \Omega$. When the constraint binds, we have $p_{2,1}d_2\theta = \Omega$ and when it is slack we have $\left(1 - p_{2,1}\right)^2 d_2 = \Omega$. Thus the critical value of $\theta$ satisfies

$$p_{2,1}d_2\theta = \left(1 - p_{2,1}\right)^2 d_2 \Rightarrow \theta = \bar{\theta}_{2,1} := \frac{\left(1 - p_{2,1}\right)^2}{p_{2,1}}.$$

The constraint is slack when $\theta \geq \theta_1$; thus regime-1 obtains when $\theta \in [\bar{\theta}_{2,1}, \bar{\theta}_{2,0})$. This establishes the claims of the Proposition when $k = 1$.

Now, for the inductive step, suppose the claims in the body of the proposition are valid for the case $k-1$, i.e. when $\theta \in [\bar{\theta}_{2,k-1}, \bar{\theta}_{2,k-2})$ there exists $p_{1,k-1} \in (0,1)$ satisfying $\Delta_{k-1}\left(p_{1,k-1}\right) = 0$. Note when $p_{1,k} = 1$, we have $\Delta_k(1) < 0$. Indeed if $p_{1,k} = 1$ then the expression $\sum_{i=1}^{k}\frac{\sigma_i d_i}{d_i - p_{1,k}d_1}$ in the denominator tends to infinity for $i = 1$, which means that $\Delta_k(1) = -d_1\theta < 0$. Next, we will show that $\Delta_k\left(p_{1,k-1};\bar{\theta}_{2,k-1}\right) = 0$. Per the inductive step we have

$$\Delta_{k-1}\left(p_{1,k-1}\right) = \frac{p_{1,k-1}d_1\sum_{i=1}^{k-1}\sigma_i + \sum_{i=k}^{n}\sigma_i d_i - \sqrt{p_{1,k-1}d_1\theta}\sum_{i=k}^{n}\sigma_i\sqrt{d_i}}{\sum_{i=1}^{k-1}\frac{\sigma_i d_i}{d_i - p_{1,k-1}d_1} + \frac{1}{\sqrt{p_{1,k-1}d_1\theta}}\sum_{i=k}^{n}\sigma_i\sqrt{d_i}} - p_{1,k-1}d_1\theta = 0.$$

The numerator of the first expression can be written as follows

$$p_{1,k-1}d_1\sum_{i=1}^{k}\sigma_i + \sum_{i=k+1}^{n}\sigma_i d_i - \sqrt{p_{1,k-1}d_1\theta}\sum_{i=k+1}^{n}\sigma_i\sqrt{d_i} + \left\{\sigma_k d_k - \sigma_k p_{1,k-1}d_1 - \sqrt{p_{1,k-1}d_1\theta}\,\sigma_k\sqrt{d_k}\right\}.$$

Similarly, the denominator is equal to

$$\sum_{i=1}^{k}\frac{\sigma_i d_i}{d_i - p_{1,k-1}d_1} + \frac{1}{\sqrt{p_{1,k-1}d_1\theta}}\sum_{i=k+1}^{n}\sigma_i\sqrt{d_i} + \left\{\frac{\sigma_k\sqrt{d_k}}{\sqrt{p_{1,k-1}d_1\theta}} - \frac{\sigma_k d_k}{d_k - p_{1,k-1}d_1}\right\}.$$

Per the inductive step when $\theta = \bar{\theta}_{2,k-1}$, where $\bar{\theta}_{2,k-1} = \left(1 - p_{k,k-1}\right)^2/p_{k,k-1}$ we have $p_{1,k-1}d_1 =$

$p_{k,k-1}d_k$; thus

$$\sqrt{p_{1,k-1}d_1\theta} = (d_k - p_{1,k-1}d_1)/\sqrt{d_k}.$$

Using this relationship, we note that the expressions in curly brackets in the numerator and the denominator are both zero. Once these terms vanish, it is easy to check that the remaining expressions in $\Delta_{k-1}(p_{1,k-1})$ are as in $\Delta_k(p_{1,k-1})$, which means that $\Delta_{k-1}\left(p_{1,k-1};\bar{\theta}_{2,k-1}\right) = \Delta_k\left(p_{1,k-1};\bar{\theta}_{2,k-1}\right) = 0$. Since $\Delta_k$ falls in $\theta$, we have $\Delta_k(p_{1,k-1}) > 0$ whenever $\theta < \bar{\theta}_{2,k-1}$. Since $\Delta_k(1) < 0$, the Intermediate Value Theorem guarantees existence of a $p_{1,k} \in (p_{1,k-1}, 1)$ satisfying $\Delta_k(p_{1,k}) = 0$.

The remaining prices are pinned down through (25). Since $p_{1,k}$ is feasible, i.e. since $p_{1,1} \in (0,1)$, all other prices are also feasible. Per Lemma 4, the relevant constraint is $p_{k+1,k}d_{k+1}\theta \geq \Omega$. When the constraint binds, we have $p_{k+1,k}d_{k+1}\theta = \Omega$ and when it is slack we have $(1-p_{k+1,k})^2 d_{k+1} = \Omega$. Thus the critical value of $\theta$ satisfies

$$p_{k+1,k}d_{k+1}\theta = (1-p_{k+1,k})^2 d_{k+1} \Rightarrow \theta = \bar{\theta}_{2,k} := \frac{(1-p_{k+1,k})^2}{p_{k+1,k}}.$$

The constraint is slack when $\theta \geq \bar{\theta}_{2,k}$; thus regime$-k$ obtains when $\theta \in [\bar{\theta}_{2,k}, \bar{\theta}_{2,k+1})$. This establishes the proof of existence for a feasible $p_{1,k}$. To characterize it, start with equation (26), which can be rewritten as

$$p_{1,k}^2 d_1 \sum_{i=1}^k \sigma_i - 2p_{1,k}^{3/2}\sqrt{d_1\theta}\sum_{i=k+1}^n \sigma_i\sqrt{d_i} - (1-\theta)\,p_{1,k}d_1\sum_{i=1}^k \sigma_i$$

$$+\, p_{1,k}\sum_{i=k+1}^n \sigma_i d_i + 2p_{1,k}^{1/2}\sqrt{d_1\theta}\sum_{i=k+1}^n \sigma_i\sqrt{d_i} - \sum_{i=k+1}^n \sigma_i d_i = 0\ .$$

To obtain an approximate solution, we impose the relationship $p_{1,k} = 1-\kappa\theta$ and linearize the higher order terms as follows: $p_{1,k}^2 \approx 1-2\kappa\theta$, $\quad p_{1,k}^{3/2} \approx 1-\frac{3}{2}\kappa\theta$, $\quad p_{1,k}^{1/2} \approx 1-\frac{1}{2}\kappa\theta$. Substituting these expressions into the above equation and solving for $\kappa$, we have

$$\kappa = \frac{d_1\sum_{i=1}^k \sigma_i}{d_1\sum_{i=1}^k \sigma_i + \theta d_1\sum_{i=1}^k \sigma_i + \sum_{i=k+1}^n \sigma_i d_i + 2\sqrt{d_1\theta}\sum_{i=k+1}^n \sigma_i\sqrt{d_i}}, \tag{27}$$

which characterizes $p_{1,k}$. Remaining prices can be obtained via (25). Finally we turn to the lower bound for prices. In regime-$k$ we have

$$(1-p_{k,k})^2 d_k < \Omega \quad \text{and} \quad p_{k+1,k}d_{k+1}\theta > \Omega.$$

The first inequality is due to the fact that the constraint binds at location $k$, whereas the second one obtains because the constraint is slack at location $k+1$. Furthermore $p_{k,k}$ and $p_{k+1,k}$ satisfy

$$p_{k,k}d_k\theta = d_{k+1}(1-p_{k+1,k})^2 = \Omega.$$

Substituting these relationships into the inequalities above yields $p_{k,k} > p_{\min}$ and $p_{k+1,k} > p_{\min}$ where $p_{\min}$ is given in the body of the Proposition. ■

**Proof of Lemma 5.** The commission vector **c** is incentive compatible if it satisfies (5), which, after substituting for $m_i$ and $x_i$ is equivalent to

$$(1-p)\,p\sum_{i=1}^{n}\sigma_i c_i d_i = w\left[(1-\varphi_l)\,\theta-(\varphi_h-\varphi_l)\,(1-p)\right].$$

Furthermore, recall that $r_i \leq x_i \Leftrightarrow w\,(1-\varphi_h) \leq pc_i d_i$. Per our conjecture, under $\mathbf{c}$ we have $pc_i d_i > w\,(1-\varphi_h)$ for $i \leq k$ and $pc_i d_i = w\,(1-\varphi_h)$ for $i \geq k+1$.[27] We will construct a new $\hat{\mathbf{c}}$ by marginally shaving off the rates of $\mathbf{c}$ at locations where the constraint is slack (but without rendering any of these constraints binding) and marginally increasing the rates at locations where the constraint is binding. Let

$$\hat{c}_i = c_i - \varepsilon_i \text{ for } i \leq k \quad \text{and} \quad \hat{c}_i = c_i + \varepsilon_i \text{ for } i \geq k+1,$$

where $(\varepsilon_1, ..., \varepsilon_n) \in (0,1)^n$ is an arbitrarily small tuple satisfying

$$\sum_{i=1}^{k}\sigma_i d_i \varepsilon_i = \sum_{i=k+1}^{n}\sigma_i d_i \varepsilon_i.$$

Note that

$$\sum_{i=1}^{n}\sigma_i d_i \hat{c}_i = \sum_{i=1}^{k}\sigma_i d_i\,(c_i-\varepsilon_i) + \sum_{i=k+1}^{n}\sigma_i d_i\,(c_i+\varepsilon_i) = \sum_{i=1}^{n}\sigma_i d_i c_i,$$

thus $\hat{\mathbf{c}}$, too, is incentive compatible. Since $(\varepsilon_1, ..., \varepsilon_n)$ can be picked arbitrarily small, the inequality $p_i \hat{c}_i d_i > w\,(1-\varphi_h)$ can be satisfied for all $i$. ■

**Proof of Proposition 4.** The platform solves

$$\max_{p}\; p\,(1-p)\,\mathbb{E}_\sigma\,(d) - w\,(1-\varphi_l)\,\theta + w\,(\varphi_h-\varphi_l)\,(1-p) \;\text{ s.t. }\; w\,(1-\varphi_h) \leq pc_i d_i.$$

The objective function is concave in $p$; thus, ignoring the constraints, the first-order condition yields the interior price

$$p^{int} = \frac{1}{2} - \frac{(\varphi_h-\varphi_l)\,w}{2\mathbb{E}_\sigma\,(d)}.$$

The commission rates must satisfy (12), which after re-arranging becomes

$$\sum_{i=1}^{n}\sigma_i c_i d_i = \frac{w\,(1-\varphi_l)\,\theta}{(1-p)\,p} - \frac{(\varphi_h-\varphi_l)\,w}{p}. \tag{28}$$

Since there are $n$ commission rates, there exists a continuum of solutions to (28), i.e., the commission rates are indeterminate in the interior equilibrium. Note that if the constraints are active, i.e., if $pc_i d_i = w\,(1-\varphi_h)$, then

$$\sum_{i=1}^{n}\sigma_i c_i d_i = w\,(1-\varphi_h)\,/p. \tag{29}$$

[27] For ease of exposition, we assume that the constraints are slack at locations $1, ..., k$ and that they bind at the remaining locations; however, this is without loss of generality. The proof can be recast when the constraints are slack/binding at some randomly selected locations.

Substituting this and $p^{int}$ into (28) we see that if $\theta > \bar{\theta}_3$, where

$$\bar{\theta}_3 = \frac{\mathbb{E}_\sigma(d) + (\varphi_h - \varphi_l) w}{2\mathbb{E}_\sigma(d)},$$

then $w(1-\varphi_h) < pc_i d_i$ i.e. the constraints are slack and we have an interior equilibrium. If, on the other hand, $\theta \leq \bar{\theta}_3$ then the constraints bind, i.e. $w(1-\varphi_h) = pc_i d_i$, and therefore (28) and (29) imply that

$$p^{cor} = 1 - \theta \quad \text{and} \quad c_i^{cor} = \frac{w(1-\varphi_h)}{(1-\theta) d_i}. \tag{30}$$

This concludes the proof of Proposition 4. ■

**Proof of Proposition 5.** The platform solves

$$\max_{\mathbf{p}} \pi(\mathbf{p}) = \max_{\mathbf{p}} [g(\mathbf{p}) + w(\varphi_h - \varphi_l)] h(\mathbf{p})^{-1} - w(1-\varphi_l)\theta,$$

$$\text{s.t. } r_i \leq x_i \Leftrightarrow w(1-\varphi_h) \leq p_i c_i d_i \text{ for all } i.$$

Suppose the constraints are slack. The platform's problem is the same as the unconstrained problem in Model 2; thus $p_i^{int}$ is the same as the interior price there, i.e.

$$p_i^{int} = 1 - \frac{\mathbb{E}_\sigma(d) + w(\varphi_h - \varphi_l)}{2\sqrt{d_i}\mathbb{E}_\sigma(\sqrt{d})}. \tag{31}$$

The commission rates must satisfy (14) which, after re-arranging becomes

$$\sum_{i=1}^{n} \sigma_i p_i c_i d_i = w[h(\mathbf{p})(1-\varphi_l)\theta - (\varphi_h - \varphi_l)].$$

Since there are $n$ commission rates, there exists a continuum of solutions satisfying this equality. Now suppose the constraints are active, i.e. suppose $p_i c_i d_i = w(1-\varphi_h)$, for all $i$. Substituting these equalities into (14) yields $h(\mathbf{p}) = 1/\theta$. The platform, therefore, solves

$$\max_{\mathbf{p}} \ \pi(\mathbf{p}) \quad \text{s.t.} \quad h(\mathbf{p}) = 1/\theta,$$

while the commission rates are uniquely pinned down via $p_i c_i d_i = w(1-\varphi_h)$, for all $i$. Letting $\lambda$ denote the Lagrange multiplier, the first order condition with respect to $p_i$ is given by (recall that $\pi(\mathbf{p})$ is strictly concave and $h(\mathbf{p})$ is strictly convex)

$$d_i(1-p_i)^2 = [g(\mathbf{p}) + w(\varphi_h - \varphi_l)]h(\mathbf{p})^{-1} + \lambda h(\mathbf{p}).$$

Since the right hand side is not indexed by $i$, we have $d_i(1-p_i)^2 = d_j(1-p_j)^2$. Combining this relationship with the constraint $h(\mathbf{p}) = 1/\theta$ yields

$$p_i^{cor} = 1 - \frac{\mathbb{E}_\sigma(\sqrt{d})}{\sqrt{d_i}}\theta. \tag{32}$$

Substituting $p_i^{cor}$ into $p_i c_i d_i = w(1-\varphi_h)$ yields

$$c_i^{cor} = \frac{w(1-\varphi_h)}{d_i - \sqrt{d_i}\mathbb{E}_\sigma(\sqrt{d})\theta}. \tag{33}$$

Finally the threshold $\bar{\theta}_4$ can be obtained via $\theta = 1/h\left(\mathbf{p}^{int}\right)$, yielding

$$\bar{\theta}_4 = \frac{\mathbb{E}_\sigma(d) + w(\varphi_h - \varphi_l)}{2\mathbb{E}_\sigma^2(\sqrt{d})}. \tag{34}$$

This completes the proof of Proposition 5. ■

**Proof of Proposition 6**. Start with Model 4. The platform's profit function is given by (13). Recall that if $\theta > \overline{\theta}_4$ then the (interior) prices are given by (31) and if $\theta \leq \overline{\theta}_4$ then the (corner) prices are given by (32). First we show that the optimal $\theta$ cannot exceed $\overline{\theta}_4$. By contradiction, suppose it does, i.e., focus on the region where $\theta > \overline{\theta}_4$. Substituting the interior prices (31) and the labor supply relationship $w = \theta/\mu$ into the profit function (13), we have

$$\pi = \frac{\left[\mathbb{E}_\sigma(d) + \theta(\varphi_h - \varphi_l)/\mu\right]^2}{4\mathbb{E}_\sigma^2(\sqrt{d})} - \frac{(1-\varphi_l)\theta^2}{\mu}.$$

The platform solves $\max_\theta \pi$. The first order condition yields

$$\hat{\theta} = \frac{(\varphi_h - \varphi_l)\mathbb{E}_\sigma(d)}{4\mathbb{E}_\sigma^2(\sqrt{d})(1-\varphi_l) - (\varphi_h - \varphi_l)^2/\mu}.$$

Per our conjecture we must have $\hat{\theta} > \overline{\theta}_4$. The inequality holds if $\varphi_h + \varphi_l > 2$, which is impossible because both $\varphi_h$ and $\varphi_l$ are less than 1; a contradiction. Thus the optimal $\theta$ must be less than $\overline{\theta}_4$. Conjecturing this to be the case, and now substituting the corner prices (32) into the profit function (13), we have

$$\pi = \mathbb{E}_\sigma(d)\theta - \mathbb{E}_\sigma^2(\sqrt{d})\theta^2 - \theta^2(1-\varphi_h)/\mu.$$

The profit function is strictly concave in $\theta$. The first-order condition yields

$$\theta_4^* = \frac{\mathbb{E}_\sigma(d)}{2(1-\varphi_h)/\mu + 2\mathbb{E}_\sigma^2(\sqrt{d})}.$$

Basic algebra shows that indeed $\theta_4^* < \overline{\theta}_4$, verifying our conjecture. This establishes the optimal entry under Model 4. Going through the same procedure, one can obtain optimal entries $\theta_1^*$ for Model 1 and $\theta_3^*$ for Model 3, which are on display in (16).

Turning to Model 2, equation (10) implies

$$\frac{g(\mathbf{p}_k)}{h(\mathbf{p}_k)} = p_{1,k}d_1\theta\frac{1-\varphi_l}{1-\varphi_h} - \frac{p_{1,k}d_1}{h(\mathbf{p}_k)}\frac{\varphi_h - \varphi_l}{1-\varphi_h},$$

where we take $p_{1,k}$ as reference. Recall that when $\varphi_h$ and $\varphi_l$ are close to each other, $p_{1,k}$ is approximated by $p_{1,k} = 1-\kappa\theta$. Using this relationship and substituting $w = \theta/\mu$ into (9), the profit is approximately equal to

$$\pi_2 = d_1\theta - \kappa d_1\theta^2 - \frac{1-\varphi_l}{\mu}\theta^2. \tag{35}$$

Maximizing $\pi_2$ with respect to $\theta$ yields the expression for $\theta_2^*$, given by (17).

Now we will show that (i) $\theta_4^* > \theta_3^*$, (ii) $\theta_1^* > \theta_3^*$ and $\theta_1^* > \theta_4^*$ unless $\mu$ is too small, and finally (iii) $\theta_2^* > \theta_3^*$ unless $\mu$ is too small. Starting with (i), we note that

$$\theta_4^* > \theta_3^* \Leftrightarrow \mathbb{E}_\sigma^2(\sqrt{d}) < \mathbb{E}_\sigma(d),$$

which, in turn, is equivalent to

$$\sum_{i=1}^{n} \sigma_i d_i > \left[\sum_{i=1}^{n} \sigma_i \sqrt{d_i}\right]^2.$$

Letting $t_i \equiv \sqrt{\sigma_i d_i}$ and $s_i \equiv \sqrt{\sigma_i}$ and noting that $\sum \sigma_i = 1$, the inequality can be rewritten as

$$\sum_{i=1}^{n} t_i^2 \sum_{i=1}^{n} s_i^2 > \left[\sum_{i=1}^{n} t_i s_i\right]^2.$$

The result follows from Cauchy-Schwarz and completes the proof of $\theta_4^* > \theta_3^*$. Turning to (ii), note that the inequality $\theta_1^* > \theta_3^*$ holds if

$$\mu d_1 > 1 - \varphi_h + d_1 (\varphi_h - \varphi_l) / \mathbb{E}_\sigma(d).$$

The expression on the right-hand side is less than 1 because $d_1 < \mathbb{E}_\sigma(d)$. The parameter $d_1$ typically exceeds 1; therefore the inequality holds unless $\mu$ is too small. It is straightforward to verify that $\theta_1^* > \theta_4^*$ holds under a similar condition. Now turn to (iii). First we establish the inequality $\kappa < d_1/\mathbb{E}_\sigma(d)$. After substituting for $\kappa$, while noting that the most restrictive case involves $k = 1$, we need

$$\frac{d_1\sigma_1}{d_1\sigma_1(1+\theta) + \sum_{i=2}^{n} \sigma_i d_i + 2\sqrt{d_1\theta}\sum_{i=2}^{n} \sigma_i \sqrt{d_i}} < \frac{d_1}{\mathbb{E}_\sigma(d)}.$$

Noting that $\mathbb{E}_\sigma(d) = \sum_{i=1}^{n} \sigma_i d_i$, the inequality holds. Now compare $\theta_2^*$ and $\theta_3^*$:

$$\theta_2^* > \theta_3^* \Leftrightarrow \mathbb{E}_\sigma(d)\, d_1 (1-\kappa) > \frac{1}{\mu}\left[\mathbb{E}_\sigma(d)(1-\varphi_l) - d_1(1-\varphi_h)\right].$$

The right hand side is positive since $d_1 < \mathbb{E}_\sigma(d)$, whereas the left hand side is positive since $\kappa < d_1/\mathbb{E}_\sigma(d)$. The comparison between $\theta_2^*$ and $\theta_3^*$, therefore, depends primarily on $\mu$. If $\mu$ is large then $\theta_2^*$ exceeds $\theta_3^*$, whereas if $\mu$ is small then the opposite is true. ■

**Proof of Proposition 7.** In Model 4 $\theta_4^*$ is given by (16). Since $w = \theta/\mu$, we have

$$w_4^* = \frac{\mathbb{E}_\sigma(d)}{2(1-\varphi_h) + 2\mu\mathbb{E}_\sigma^2(\sqrt{d})}.$$

Note that $\theta_4^*$ rises while $w_4^*$ falls in $\mu$. Substituting $\theta_4^*$ into (32) yields

$$\frac{dp_i^*}{d\mu} = -\frac{\mathbb{E}_\sigma(\sqrt{d})}{\sqrt{d_i}}\frac{d\theta_4^*}{d\mu},$$

establishing that $p_i^*$ decreases with $\mu$, as $\theta_4^*$ increases in $\mu$. Turning to the commission rate from

(33)

$$\frac{dc_i^*}{d\mu} = -\frac{c_i^*}{\mu} + \frac{\mu c_i^{*2} d_i}{(1-\varphi_h)\theta_4^{*2}} \frac{d\theta_4^*}{d\mu}.$$

Substituting for $d\theta_4^*/d\mu$, this expression implies

$$\frac{dc_i^*}{d\mu} < 0 \Leftrightarrow \mathbb{E}_\sigma(d) < 2\sqrt{d_i}\mathbb{E}_\sigma(\sqrt{d}),$$

which is equivalent to $p_i^* > 0$; thus implying $dc_i^*/d\mu < 0$. Now consider the expressions for matches ($M_4$), profits ($\pi_4$), and consumer surplus ($CS_4$), given by (36), (38), and (39), respectively. Since $d\theta_4^*/d\mu > 0$, all three outcomes increase with $\mu$.

In Model 2, optimal entry $\theta_2^*$ is given by (17), which increases in $\mu$. Using $w = \theta/\mu$, we have

$$w_2^* = \frac{d_1}{2(1-\varphi_l + \kappa\mu d_1)},$$

which decreases in $\mu$. Given the price relation $p_{1,k} = 1 - \kappa\theta$, we obtain

$$\frac{dp_{1,k}}{d\mu} = -\kappa \frac{d\theta_2^*}{d\mu} < 0.$$

The commission rate satisfies (24), which implies

$$\frac{dc}{d\mu} < 0 \Leftrightarrow \frac{dw_2^*}{d\mu} p_{1,k} < \frac{dp_{1,k}}{d\mu} w_2^*.$$

Substituting for $w_2^*$ and $p_{1,k}$ confirms that the inequality on the right-hand side holds, so the commission rate decreases in $\mu$. Turning to profits, given by (41), we have

$$\frac{d\pi_2}{d\mu} = \frac{1}{4} \frac{d_1^2(1-\varphi_l)}{(\mu\kappa d_1 + 1 - \varphi_l)^2},$$

which is positive. Furthermore, both the number of matches and the amount of consumer surplus, as given in (42), increase in $\mu$ because $\theta_2^*$ increases in $\mu$. This confirms the stated claims for Models 2 and 4. Analogous steps can be used to verify the corresponding results for Models 1 and 3. ■

**Proof of Remark 2**. Consider Model 3, and recall the relationship

$$pc_i d_i = w(1-\varphi_h).$$

Totally differentiating with respect to $\mu$ yields

$$\underbrace{\frac{\mu}{w}\frac{dw}{d\mu}}_{\epsilon(w,\mu)} = \underbrace{\frac{\mu}{p}\frac{dp}{d\mu}}_{\epsilon(p,\mu)} + \underbrace{\frac{\mu}{c_i}\frac{dc_i}{d\mu}}_{\epsilon(c_i,\mu)},$$

i.e. the elasticity of the total driver compensation $w$ with respect to $\mu$ is equal to the sum of the elasticities of the price and commission rates with respect to $\mu$. Noting that $p = 1 - \theta_3^*$, where $\theta_3^*$

is given by (16), we have

$$\epsilon(w,\mu) = -\frac{\mathbb{E}_\sigma(d)\,\mu}{(1-\varphi_h)+\mathbb{E}_\sigma(d)\,\mu} \quad \text{and} \quad \epsilon(p,\mu) = \frac{(1-\varphi_h)\,\epsilon(w,\mu)}{2(1-\varphi_h)+\mu\mathbb{E}_\sigma(d)}.$$

Elasticities $\epsilon(p,\mu)$, $\epsilon(c_i,\mu)$ and $\epsilon(w,\mu)$ are all negative. It is straightforward to verify that

$$|\epsilon(p,\mu)| < 0.5\,|\epsilon(w,\mu)|,$$

i.e. $\epsilon(p,\mu)$ constitutes less than 50% of $\epsilon(w,\mu)$, which implies that $\epsilon(c_i,\mu)$ exceeds 50%. In other words, the platform responds more strongly through commissions than through prices. This confirms the Remark under Model 3.

Now consider Model 4. The process is the same, but the relevant equations are now

$$\epsilon(w,\mu) = -2\theta_4^*\frac{\mathbb{E}_\sigma^2(\sqrt{d})}{\mathbb{E}_\sigma(d)} \quad \text{and} \quad \epsilon(p_i,\mu) = -2\theta_4^{*2}\frac{(1-\varphi_h)}{\mu\mathbb{E}_\sigma(d)}\frac{\mathbb{E}_\sigma(\sqrt{d})}{\sqrt{d_i}-\mathbb{E}_\sigma(\sqrt{d})\theta_4^*},$$

where $p_i$ is given by (32) and $\theta_4^*$ is given by (16). The inequality $|\epsilon(p_i,\mu)| < 0.5\,|\epsilon(w,\mu)|$ boils down to

$$\frac{2(1-\varphi_h)}{\mu}\left[\mathbb{E}_\sigma(d) - \sqrt{d_i}\mathbb{E}_\sigma(\sqrt{d})\right] < \mathbb{E}_\sigma^2(\sqrt{d})\left[2\mathbb{E}_\sigma(\sqrt{d})\sqrt{d_i} - \mathbb{E}_\sigma(d)\right],$$

which typically holds true (unless $d_i$, $\varphi_h$ and $\mu$ are all too small).

Turning to Model 2, we have $p_{1,k} = 1-\kappa\theta_2^*$ where $\theta_2^*$ is given by (17). The relevant equations are

$$\epsilon(w,\mu) = -\frac{\kappa\mu d_1}{\kappa\mu d_1+1-\varphi_l} \quad \text{and} \quad \epsilon(p_{1,k},\mu) = -\frac{(1-\varphi_l)\,d_1\mu\kappa}{2(\kappa\mu d_1+1-\varphi_l)^2(1-\kappa\theta_2^*)}.$$

It is straightforward to show that as long as $\kappa > 0$ the inequality $|\epsilon(p_{1,k},\mu)| < 0.5\,|\epsilon(w,\mu)|$ holds true. Finally, the process for Model 1 is the same as the one for Model 2, except $\kappa$ is replaced with $d_1/\mathbb{E}_\sigma(d)$. Since this expression is positive, the inequality holds under Model 1 as well. ■

**Proof of Proposition 8.** The total number of matches is equal to $M = \sum_{i=1}^n m_i$. With Model 4 $m_i = \sigma_i h(\mathbf{p})^{-1}$, whereas with Model 3 $m_i = \sigma_i(1-p)$. Prices in Model 4 are given by (32) and in Model 3 by (30). Substituting for prices, we have

$$M_4 = \theta_4^* \text{ and } M_3 = \theta_3^*. \tag{36}$$

The inequality $M_4 > M_3$ follows from the fact that $\theta_4^* > \theta_3^*$ (Proposition 6). In Model 1 $m_i = \sigma_i(1-p)$, where $p$ is given by (21). After substituting for the price

$$M_1 = \frac{d_1(1-\varphi_l)\,\theta_1^*}{\mathbb{E}_\sigma(d)(1-\varphi_h)+d_1(\varphi_h-\varphi_l)}.$$

Basic algebra establishes that $M_3 > M_1$ if

$$(\mathbb{E}_\sigma(d)-d_1)\left[(\mathbb{E}_\sigma(d)+d_1)(1-\varphi_h)+d_1(\varphi_h-\varphi_l)\right] > 0. \tag{37}$$

The first term is positive because $d_1 < d_2 < ... < d_n$. The expression inside the square brackets is positive because $1 > \varphi_h > \varphi_l$. The inequality holds true, thus $M_3 > M_1$ follows. Now turn to profits. Substituting for prices

$$\pi_3 = \frac{1}{4}\frac{\mathbb{E}_\sigma(d)^2\mu}{1-\varphi_h+\mu\mathbb{E}_\sigma(d)} \quad \text{and} \quad \pi_4 = \frac{1}{4}\frac{\mathbb{E}_\sigma(d)^2\mu}{1-\varphi_h+\mu\mathbb{E}_\sigma^2(\sqrt{d})}. \tag{38}$$

The inequality $\pi_4 > \pi_3$ follows from the fact that $\mathbb{E}_\sigma(d) > \mathbb{E}_\sigma^2$. Turning to Model 1, we have

$$\pi_1 = \frac{1}{4}\frac{\mu d_1^2\mathbb{E}_\sigma(d)(1-\varphi_l)}{\mu d_1^2(1-\varphi_l)+\mathbb{E}_\sigma(1-\varphi_h)^2+d_1(\varphi_h-\varphi_l)(1-\varphi_h)}.$$

Routine algebra shows that the inequality $\pi_3 > \pi_1$ reduces to (37), which holds. Now, turn to the consumer surplus. After substituting for prices and $\theta_3^*$ ad $\theta_4^*$ we have

$$CS_3 = \frac{1}{8}\frac{\mathbb{E}_\sigma^3(d)}{[(1-\varphi_h)/\mu+\mathbb{E}_\sigma(d)]^2} \quad \text{and} \quad CS_4 = \frac{1}{8}\frac{\mathbb{E}_\sigma^2(\sqrt{d})\mathbb{E}_\sigma^2(d)}{[(1-\varphi_h)/\mu+\mathbb{E}_\sigma^2(\sqrt{d})]^2}. \tag{39}$$

Similarly, for Model 1

$$CS_1 = \frac{1}{8}\frac{d_1^4\mu^2(1-\varphi_l)^2\mathbb{E}_\sigma(d)}{[\mu d_1^2(1-\varphi_l)+\mathbb{E}_\sigma(d(1-\varphi_h)^2+d_1(1-\varphi_h)(\varphi_h-\varphi_l)]^2}. \tag{40}$$

The inequality $CS_3 > CS_1$ reduces to (37), which holds true. Finally $CS_4 > CS_3$ is equivalent to

$$\mathbb{E}_\sigma^2(\sqrt{d})\mathbb{E}_\sigma(d) > (1-\varphi_h)^2/\mu^2,$$

which is typically true, unless $\varphi_h$ and $\mu$ are both too small.

Substituting the expression for $\theta_2^*$ into (35) yields the profit in Model 2

$$\pi_2 = \frac{1}{4}\frac{\mu d_1^2}{\mu\kappa d_1+1-\varphi_l}. \tag{41}$$

Note that

$$\pi_3 > \pi_2 \Leftrightarrow \frac{1}{\mu}\left[\mathbb{E}_\sigma^2(d)(1-\varphi_l)-d_1^2(1-\varphi_h)\right] > \mathbb{E}_\sigma(d)\left[d_1-\mathbb{E}_\sigma(d)\kappa\right].$$

The expression on the left is positive because $\mathbb{E}_\sigma(d) > d_1$. The expression on the right is positive as $\kappa < d_1/\mathbb{E}_\sigma(d)$. The comparison between $\pi_2$ and $\pi_3$, therefore, hinges primarily on the value of $\mu$: when $\mu$ is small, we have $\pi_3 > \pi_2$; when $\mu$ is large, the inequality reverses. Note that this observation aligns with the simulations in Figure 4.

The number of matches and the amount of consumer surplus in Model 2 are given by

$$M_2 = h(\mathbf{p})^{-1} \quad \text{and} \quad CS_2 = \frac{1}{2}\sum_{i=1}^{n}\frac{\sigma_i d_i}{h(\mathbf{p})}(1-p_i).$$

Recall that prices in Model 2 are bounded below by $p_{\min}$, given in Proposition 3. Upper bounds for $M_2$ and $CS_2$ can be obtained by substituting $p_{\min}$ for prices. We have

$$\overline{M}_2 = \sqrt{\theta_2^{*2}/4+\theta_2^*}-\theta_2^*/2 \quad \text{and} \quad \overline{CS}_2 = \frac{1}{2}\mathbb{E}_\sigma(d)\left[\sqrt{\theta_2^{*2}/4+\theta_2^*}-\theta_2^*/2\right]^2. \tag{42}$$

$\overline{M}_2$ is close to $\theta_2^*$. Similarly $\overline{CS}_2$ is close to $\frac{1}{2}\mathbb{E}_\sigma(d)\,\theta_2^{*2}$. In comparison, under Model 3 we have

$$M_3 = \theta_3^* \text{ and } CS_3 = \frac{1}{2}\mathbb{E}_\sigma(d)\,\theta_3^{*2}.$$

Recall that $\theta_3^*$ exceeds $\theta_2^*$ when $\mu$ is small, while the reverse holds when $\mu$ is large (see the proof of Proposition 6). It follows that, Model 3 generates more matches and higher consumer surplus when $\mu$ is small, whereas Model 2 performs better on both dimensions when $\mu$ is large. ■

### A.4 Transition and Distance Matrices

In what follows we provide the transition and distance matrices for NYC.

Table 1: Transition Matrix for NYC

| | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | $\sigma_i$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Battery Park | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 10.4 | 10.3 | 8.3 | 6.5 | 17.2 | 9.9 | 3.8 | 5.2 | 5 | 0 | 5 | 3.5 | 4.4 | 3.3 | 0 | 0 | 7 | *1.11* |
| Carnegie Hill | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 10.4 | 12.1 | 0 | 0 | 30.9 | 10.4 | 12.4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 23.4 | *0.64* |
| Central Park | 3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 8.6 | 13.7 | 7.9 | 0 | 24.4 | 15.4 | 9.9 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 20.1 | *0.73* |
| Harlem | 4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 20 | 0 | 0 | 36.9 | 18.7 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 24.2 | *0.25* |
| N. Sutton Area | 5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 12.2 | 8.8 | 6 | 6.9 | 25.5 | 11.6 | 6.8 | 6.8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 15.3 | *0.54* |
| Chinatown | 6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 16.7 | 7.9 | 8.7 | 10.1 | 18 | 14.6 | 0 | 0 | 0 | 0 | 9 | 0 | 7.5 | 0 | 0 | 0 | 7.3 | *0.24* |
| Yorkville | 7 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 100 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | *0.12* |
| East Harlem | 8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 100 | *0.04* |
| West Village | 9 | 1.4 | 0.8 | 0.8 | 0 | 0.7 | 0.6 | 0 | 0 | 6.6 | 8.2 | 6.3 | 6.2 | 16.3 | 10.5 | 4.3 | 3.6 | 3.9 | 2.5 | 4.6 | 3.7 | 3.6 | 3.2 | 2.4 | 1.8 | 8.3 | *7.71* |
| Gramercy | 10 | 1.6 | 0.7 | 1.1 | 0.7 | 0.5 | 0.5 | 0 | 0 | 7.5 | 7.2 | 5.2 | 6.6 | 16.3 | 9.1 | 5.2 | 4.5 | 4.6 | 2.5 | 3.5 | 3.3 | 3.9 | 2.8 | 1.9 | 2.8 | 8.1 | *8.17* |
| Soho | 11 | 1.2 | 0.4 | 0.7 | 0.4 | 0.6 | 0.4 | 0 | 0 | 10.7 | 8.1 | 6.3 | 6 | 14.5 | 11.7 | 3.6 | 4.2 | 3.4 | 2.7 | 5.2 | 3.4 | 3.5 | 2.8 | 1.1 | 2.6 | 6.5 | *5.22* |
| Greenwich Vil. | 12 | 1.7 | 0.8 | 0.8 | 0.4 | 0.7 | 0.5 | 0.3 | 0 | 7.7 | 7.9 | 4.9 | 6.4 | 14.8 | 10.1 | 4.9 | 4 | 3.7 | 3.5 | 5.1 | 3.3 | 3.8 | 2.6 | 2.4 | 1.9 | 8 | *6.38* |
| Midtown | 13 | 1.3 | 0.9 | 1 | 0.5 | 0.7 | 0.3 | 0.3 | 0.2 | 7.6 | 8.8 | 4.7 | 6.5 | 12.7 | 9.2 | 8.1 | 4.6 | 3.1 | 2.6 | 4 | 2.7 | 2.1 | 3.2 | 1.6 | 1 | 12.1 | *16.1* |
| Chelsea | 14 | 1.1 | 0.7 | 0.7 | 0.5 | 0.6 | 0.4 | 0.3 | 0.2 | 8 | 7 | 5.6 | 6.8 | 15.9 | 10.3 | 5.1 | 3.5 | 4.4 | 3.1 | 3.7 | 3.2 | 3.4 | 4.1 | 1.9 | 1.6 | 7.8 | *9.95* |
| U. West Side | 15 | 1 | 1.3 | 1.2 | 0 | 0 | 0 | 0 | 0 | 5.1 | 7.5 | 4.7 | 4.7 | 23.7 | 9.8 | 9.3 | 4.6 | 2.5 | 2.8 | 2.5 | 2.4 | 1.8 | 2.8 | 1 | 1.2 | 10.1 | *5.51* |
| Garment Dist. | 16 | 1.4 | 1 | 0.8 | 0 | 0.9 | 0 | 0 | 0 | 6.5 | 8.4 | 4.7 | 6.2 | 17.9 | 9.1 | 5.3 | 2.9 | 4.1 | 2.6 | 3.7 | 2.8 | 2.1 | 2.4 | 1.8 | 1.8 | 13.2 | *4.26* |
| East Village | 17 | 0.9 | 0 | 0.6 | 0 | 0 | 0 | 0 | 0 | 6.4 | 10.7 | 7.1 | 6.8 | 12.9 | 11.5 | 3.6 | 6 | 4.5 | 2.6 | 3.1 | 4.5 | 4.2 | 2.6 | 2.3 | 3.1 | 6.4 | *3.82* |
| Murray Hill | 18 | 0.9 | 0 | 0.9 | 0 | 0.8 | 0 | 0 | 0 | 8.2 | 7.3 | 5.6 | 7.9 | 15 | 11.5 | 5.7 | 4.9 | 5.2 | 3.7 | 3.6 | 2.8 | 2.7 | 2.5 | 1.7 | 1.3 | 7.9 | *2.55* |
| Tribeca | 19 | 1.3 | 0 | 0 | 0 | 0 | 0.6 | 0 | 0 | 10.6 | 7.6 | 6.2 | 8.1 | 12.6 | 10.5 | 3.3 | 3.9 | 4 | 1.9 | 5.4 | 3.8 | 3.8 | 2.4 | 3.6 | 2.6 | 7.7 | *3.93* |
| Financial Dist. | 20 | 1 | 0 | 0 | 0 | 1.1 | 0 | 0 | 0 | 11.5 | 7.3 | 5.4 | 7.6 | 12.9 | 9.9 | 4.1 | 3.7 | 5.4 | 2.6 | 5.2 | 5.1 | 4.3 | 2.8 | 1.8 | 2.2 | 6 | *3.09* |
| L. East Side | 21 | 1.5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 9 | 11.2 | 5 | 7.3 | 11.3 | 10.6 | 3.5 | 4 | 5 | 3.4 | 3.5 | 4.1 | 4.9 | 2.9 | 1.8 | 3.7 | 7.1 | *3.09* |
| Clinton | 22 | 0.8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 7.6 | 7.9 | 5.4 | 6.2 | 18.3 | 12.1 | 5.1 | 4.5 | 4 | 2.7 | 4 | 2.5 | 3.1 | 5.7 | 1.7 | 1.8 | 6.5 | *2.91* |
| Little Italy | 23 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 9.4 | 8.1 | 3.4 | 5.7 | 17.4 | 11.6 | 3.6 | 5.4 | 5 | 2.5 | 4.7 | 3.9 | 3.1 | 2.5 | 2.3 | 3.9 | 7.5 | *1.78* |
| Williamsburg | 24 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 7.6 | 9.4 | 6.3 | 8.2 | 9.7 | 8.6 | 3.5 | 2.4 | 10 | 0 | 4.7 | 5 | 6.7 | 2.5 | 3.5 | 7.2 | 4.5 | *1.86* |
| U. East Side | 25 | 0.7 | 1.2 | 1.2 | 0.3 | 1.1 | 0 | 0.4 | 0 | 5.9 | 7 | 3.8 | 6 | 20.5 | 7.6 | 5.6 | 5.7 | 2.5 | 2.3 | 3.3 | 2 | 2 | 2.3 | 1.2 | 0.7 | 16.7 | *9.79* |

The transition matrix $T$ provides the likelihood of rides across locations. Consider, for instance, location 1 (Battery Park): 10.4% of rides originating from this location are directed towards West Village, 10.3% are directed towards Gramercy, 8.3% towards Soho and so on. Given $T$, we calculate its steady state distribution $\sigma_i$ (last column), which reveals the "centrality" of each location. Midtown appears to be the most active location as it sends (and receives) 16.1% of the total ride-sharing traffic. It is followed by Chelsea (9.95%), Upper East Side (9.79%), Gramercy (8.17%) and West Village (7.71%).

Table 2: Distance Matrix for NYC

| | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | $d_i$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Battery Park | 1 | 0.5 | 9.32 | 10 | 11.6 | 6.44 | 2.58 | 9.18 | 9.99 | 1.97 | 5.22 | 1.57 | 2.39 | 7.01 | 3.01 | 7.35 | 3.98 | 4.32 | 5.81 | 0.93 | 1.66 | 2.81 | 3.73 | 2.03 | 7.79 | 7.73 | *4.13* |
| Carnegie Hill | 2 | 9.06 | 0.5 | 1.03 | 2.67 | 3.2 | 8.07 | 0.94 | 1.75 | 6.99 | 4.67 | 7.53 | 4.32 | 2.33 | 6.26 | 1.95 | 3.35 | 6.45 | 4.11 | 9.79 | 8.06 | 6.92 | 5.15 | 7.08 | 8.56 | 1.29 | *3.22* |
| Central Park | 3 | 9.01 | 1 | 0.5 | 3.39 | 3.14 | 8.01 | 1.66 | 2.47 | 6.76 | 4.62 | 7.47 | 4.14 | 2.15 | 6.04 | 0.87 | 3.17 | 6.4 | 4.05 | 9.73 | 8.01 | 6.86 | 4.92 | 7.03 | 8.51 | 1.21 | *3.59* |
| Harlem | 4 | 11.1 | 2.54 | 3.4 | 0.5 | 5.3 | 10.1 | 3.26 | 1.39 | 8.61 | 6.77 | 9.63 | 8.1 | 6.25 | 7.42 | 3.66 | 7.02 | 8.56 | 6.21 | 11.8 | 10.1 | 9.02 | 6.31 | 9.19 | 11 | 3.87 | *5.98* |
| N. Sutton Area | 5 | 7.2 | 3.15 | 2.68 | 5.5 | 0.5 | 6.21 | 3.01 | 3.82 | 4.64 | 2.81 | 5.67 | 4.14 | 1.34 | 3.25 | 3.28 | 2.35 | 4.59 | 1.73 | 7.93 | 6.2 | 5.06 | 2.53 | 5.22 | 6.7 | 1.47 | *2.77* |
| Chinatown | 6 | 2.13 | 8.23 | 8.98 | 10.5 | 5.35 | 0.5 | 8.09 | 8.9 | 1.7 | 4.13 | 0.86 | 1.71 | 3.61 | 2.87 | 7.22 | 3.21 | 1.47 | 4.72 | 0.9 | 1.13 | 0.88 | 3.6 | 0.5 | 2.74 | 6.64 | *2.56* |
| Yorkville | 7 | 9.55 | 1.04 | 1.78 | 2.31 | 3.68 | 8.55 | 0.5 | 0.8 | 6.99 | 5.16 | 8.02 | 6.49 | 4.64 | 6.61 | 2.64 | 5.41 | 6.94 | 4.59 | 10.2 | 8.55 | 7.41 | 5.68 | 7.57 | 9.05 | 1.77 | *4.64* |
| East Harlem | 8 | 10 | 1.81 | 2.56 | 1.97 | 4.2 | 9.07 | 1.36 | 0.5 | 7.51 | 5.68 | 8.53 | 7.01 | 5.15 | 7.13 | 3.31 | 5.93 | 7.46 | 5.11 | 10.7 | 9.07 | 7.92 | 12.2 | 8.09 | 10.6 | 2.17 | *2.17* |
| West Village | 9 | 1.94 | 6.57 | 6.31 | 7.91 | 8.2 | 1.82 | 6.83 | 7.64 | 0.5 | 2.55 | 1.35 | 0.92 | 3.17 | 1.35 | 5.7 | 2.33 | 2.4 | 3.11 | 1.47 | 3.43 | 2.31 | 2.08 | 1.53 | 4.18 | 9.5 | *2.97* |
| Gramercy | 10 | 5.04 | 5.04 | 5.79 | 7.39 | 1.83 | 1.92 | 4.9 | 5.71 | 1.82 | 0.5 | 2.14 | 1.32 | 2.15 | 1.44 | 5.26 | 1.73 | 1.29 | 0.92 | 2.5 | 4.04 | 2.9 | 2.41 | 1.71 | 3.72 | 3.46 | *2.35* |
| Soho | 11 | 1.97 | 7.68 | 7.2 | 10 | 4.8 | 1.06 | 7.55 | 8.36 | 0.97 | 2.13 | 0.5 | 0.85 | 2.75 | 2.24 | 6.59 | 2.34 | 1.66 | 2.68 | 0.87 | 2.01 | 1.32 | 2.97 | 0.51 | 3.03 | 6.1 | *2.38* |
| Greenwich Vil. | 12 | 2.3 | 6.29 | 4.82 | 8.63 | 3.07 | 1.42 | 6.15 | 6.96 | 0.92 | 1.34 | 1.3 | 0.5 | 2.43 | 1.73 | 6.56 | 2.02 | 1.17 | 1.9 | 1.63 | 2.42 | 1.8 | 2.7 | 0.99 | 3.48 | 4.7 | *2.29* |
| Midtown | 13 | 7.09 | 2.53 | 2.39 | 4.92 | 1.21 | 3.81 | 3.19 | 4 | 2.49 | 2.15 | 3.38 | 2.44 | 0.5 | 1.9 | 2.99 | 1.01 | 4.48 | 1.32 | 3.63 | 6.09 | 4.95 | 1.19 | 3.41 | 7.44 | 1.92 | *2.42* |
| Chelsea | 14 | 3.44 | 6.25 | 5.91 | 7.51 | 2.93 | 2.59 | 6.11 | 6.92 | 1.27 | 1.3 | 2.16 | 1.49 | 1.82 | 0.5 | 5.3 | 0.98 | 2.5 | 1.86 | 2.41 | 4.93 | 4.11 | 1.41 | 2.34 | 4.83 | 4.66 | *2.37* |
| U. West Side | 15 | 7.89 | 1.77 | 0.82 | 4.06 | 3.75 | 8.62 | 2.34 | 3.15 | 6.29 | 5.22 | 7.49 | 4.74 | 2.75 | 5.57 | 0.5 | 5.35 | 7 | 4.66 | 7.42 | 8.61 | 7.47 | 4.45 | 7.64 | 9.11 | 1.81 | *4.21* |
| Garment Dist. | 16 | 6.34 | 3.37 | 3.23 | 7.36 | 1.95 | 3.06 | 4.88 | 5.69 | 1.74 | 1.41 | 2.63 | 1.69 | 0.84 | 1.15 | 5 | 0.5 | 3.73 | 0.78 | 2.88 | 5.34 | 4.2 | 1.11 | 2.67 | 6.72 | 2.8 | *2.25* |
| East Village | 17 | 4.3 | 5.85 | 6.6 | 8.2 | 2.64 | 1.39 | 5.72 | 6.53 | 2.14 | 1.29 | 1.67 | 1.04 | 3.21 | 2.54 | 6.08 | 2.83 | 0.5 | 2.02 | 2.03 | 3.3 | 0.96 | 4.13 | 1.2 | 2.77 | 4.27 | *2.46* |
| Murray Hill | 18 | 6.16 | 2.89 | 3.03 | 6.58 | 1.17 | 5.16 | 4.1 | 4.91 | 2.48 | 1.02 | 2.87 | 1.98 | 1.19 | 1.9 | 3.63 | 1 | 3.55 | 0.5 | 6.88 | 5.16 | 4.01 | 1.49 | 2.56 | 6.31 | 1.99 | *2.36* |
| Tribeca | 19 | 1 | 8.74 | 7.58 | 11 | 5.86 | 0.77 | 8.6 | 9.41 | 1.45 | 2.79 | 0.66 | 1.51 | 3.41 | 2.62 | 6.97 | 3.01 | 2.33 | 3.35 | 0.5 | 1.4 | 1.4 | 3.35 | 0.92 | 3.37 | 7.16 | *2.64* |
| Financial Dist. | 20 | 1.12 | 9.1 | 9.85 | 11.4 | 6.22 | 2.37 | 8.96 | 9.77 | 2.83 | 5 | 2.01 | 2.86 | 6.79 | 3.87 | 8.21 | 6.41 | 4.1 | 5.6 | 1.79 | 0.5 | 2.59 | 4.59 | 2.69 | 4.67 | 7.51 | *4.26* |
| L. East Side | 21 | 3.58 | 7.21 | 7.96 | 9.55 | 4.33 | 0.8 | 7.07 | 7.88 | 2.27 | 3.11 | 1.29 | 1.88 | 4.9 | 3.57 | 7.43 | 4.52 | 1.09 | 3.71 | 1.4 | 1.93 | 0.5 | 5.21 | 0.88 | 2.06 | 5.63 | *3.16* |
| Clinton | 22 | 3.8 | 4.75 | 4.49 | 6.09 | 2.4 | 3.69 | 5.01 | 5.82 | 2.29 | 2.35 | 3.4 | 2.88 | 1.19 | 1.39 | 3.88 | 0.94 | 4.68 | 1.52 | 3.34 | 5.29 | 5.14 | 0.5 | 3.55 | 7.64 | 3.13 | *2.46* |
| Little Italy | 23 | 2.01 | 7.21 | 7.96 | 9.56 | 4.33 | 0.4 | 7.07 | 7.88 | 1.64 | 1.69 | 0.74 | 1.18 | 3.39 | 2.9 | 7.25 | 2.99 | 1.19 | 2.24 | 1.24 | 1.62 | 0.81 | 3.63 | 0.5 | 2.52 | 5.63 | *2.61* |
| Williamsburg | 24 | 7.54 | 8.88 | 7.67 | 12.6 | 8.4 | 3.98 | 8.74 | 11.7 | 5.58 | 7.92 | 4.74 | 3.92 | 8.55 | 6.75 | 8.27 | 8.15 | 2.86 | 7.33 | 4.69 | 4.72 | 2.88 | 8.82 | 2.93 | 0.5 | 6.64 | *5.29* |
| U. East Side | 25 | 8.11 | 1.07 | 1.2 | 3.74 | 1.19 | 7.11 | 1.43 | 2.24 | 5.55 | 3.71 | 6.57 | 5.04 | 1.97 | 5.17 | 1.8 | 3.96 | 5.5 | 3.15 | 8.83 | 7.11 | 5.96 | 3.42 | 6.13 | 7.61 | 0.5 | *3.43* |

The distance matrix $D$ is obtained by querying Google Maps API with the lat-long coordinates of the above locations. The API's result reflects the distance along the fastest route at the time of querying, which depends on the traffic at that time, as such $\delta_{i,j}$ is not necessarily equal to $\delta_{j,i}$. A second issue is the trip lengths for rides starting and ending in the same neighborhood. We assigned 0.5 mile for such rides; however, the results remain the same with other (sensible) trip lengths. Given $T$ and $D$, we can calculate the expected distances $d_i$ associated with each location (last column). In this dimension, Harlem is the leading location: trips originating from Harlem are expected to take 5.98 miles. For the majority of other locations this number is less than 3 miles. Given $\mathbf{d}$ and $\boldsymbol{\sigma}$, we can calculate the equilibrium prices, commissions, and profits under each operating model, which we discuss in the main text. The corresponding tables for Los Angeles are available upon request.