Extending the Closed Form Approximation of the ISRS GN model in the Zero-Dispersion Regime for Arbitrary Modulation Format, Span Length, and Fiber Loss

Filippos Balasis[1], Mindaugas Jarmolovičius[2], Daniel J. Elson[1], Eric Sillekens[2], Han Wang[1,*], Robert I. Killey[2], Polina Bayvel[2], Noboru Yoshikane[1], Takehiro Tsuritani[1], and Yuta Wakayama[1]

[1] *KDDI Research, Inc., 2-1-15 Ohara, Fujimino, 356-8502 Japan*
[2] *Optical Networks Group, UCL (University College London), UK*

[*] *ha-wang@kddi.com*

Abstract: *We provide a closed form approximation of the four-wave mixing interference in O-band accounting for arbitrary modulation format, span length and fiber loss. The expressions are validated via comparisons with the split-step Fourier method.*
Keywords: *Transmission systems and subsystems, modeling of transmission*

I. Introduction

Coherent transmission in O-band is expected to play a pivotal role in facilitating the explosive growth of the inter-data center traffic [1, 2]. The advent of bismuth doped fiber amplifiers (BDFAs) combined with the absence of chromatic dispersion compensation renders coherent transmission in O-band an appealing choice over conventional intensity modulation-direct detection (IM-DD) and S+C+L-band transmission systems [2]. At the same time, an accurate estimation of the nonlinear interference in O-band becomes crucial since it can be much stronger compared to other bands [1, 2]. The prior work on Gaussian noise (GN) model [3] was further extended in [4], to be applicable in the zero-dispersion regime, accurately accounting for the four-wave mixing in the presence of inter-band stimulated Raman scattering (ISRS). Nevertheless, the case study considered Gaussian modulation format and 80 km spans. Accuracy for short span links is a feature that can be very important and can find applications to existing infrastructure since the inter-data center links can be as short as 10 km. On the other hand, ultra-low loss is an expected feature of future hollow core fibers [5]. In this paper, a two-fold extension of [4] is provided as shown in Fig. 1. Firstly, it includes a modulation format correction for a fraction of the four-wave mixing (FWM). Secondly, it supports short spans and ultra-low fiber loss.

II. The Closed Form for Arbitrary Modulation Format, Span Length, and Fiber Loss

One of the most common metrics for evaluating the quality of transmission is the generalized signal-to-noise ratio (SNR). It can be broken down into the various sources of impairments as $\mathrm{SNR}^{-1} = \mathrm{SNR}^{-1}_{\mathrm{TRX}} + \mathrm{SNR}^{-1}_{\mathrm{ASE}} + \mathrm{SNR}^{-1}_{\mathrm{NLI}}$, and $\mathrm{SNR}^{-1}_{\mathrm{TRX}}$, $\mathrm{SNR}^{-1}_{\mathrm{ASE}}$, and $\mathrm{SNR}^{-1}_{\mathrm{NLI}}$ are the SNRs due to the transceiver noise, the fiber amplifier noise, and the nonlinear interference (NLI), respectively. The total NLI power can be expressed as the sum of three types of interferences $\mathrm{NLI} = \mathrm{SPM} + \mathrm{XPM} + \mathrm{FWM}$ where SPM, XPM, and FWM are the self-phase modulation, cross-phase modulation and four-wave mixing, respectively. In this paper, due to space limitations, we will provide the

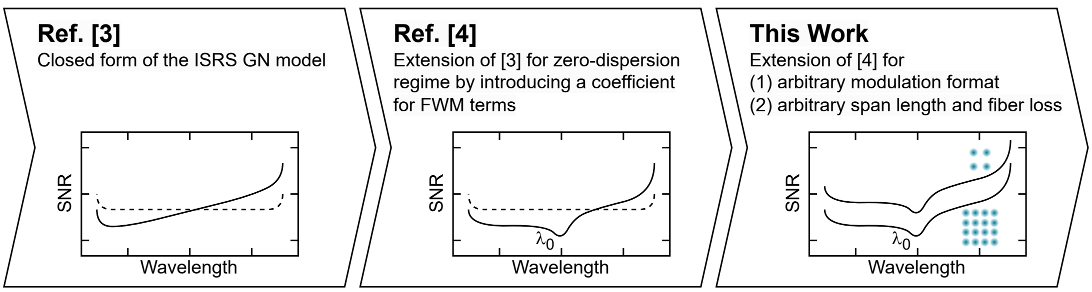

Fig. 1: Evolution of the ISRS GN model.

closed form model (CFM) expressions only for the case of FWM. The SPM and XPM parts can be calculated as shown in [6]. As in [4], each FWM interfering term, caused by two or three channels, is assumed to have its own power profile that is described by parameters $\hat{a}$, $\tilde{a}$ and $C^{'}$ as

$$\rho(z) = e^{g(z)},$$
$$g(z) = -\hat{a}z + C^{'}\left(\frac{1-e^{-\tilde{a}z}}{\tilde{a}}\right).$$

The equations used for calculating these parameters are shown in [4] [Eq. (12)-(14)]. Herein, additional parameters are introduced to support arbitrary span length and fiber loss and are derived with the same methodology as in [5] and [6]

$$\hat{A} = \hat{a} + \tilde{a},$$
$$\hat{T} = \frac{C^{'}}{\tilde{a}},$$
$$b_1 = \frac{\hat{a}\left[1-\exp\left(-\hat{a}L\right)\right]}{1-\left(1+\hat{a}L\right)\exp\left(-\hat{a}L\right)},$$
$$b_2 = \frac{\hat{A}\left[1-\exp\left(-\hat{A}L\right)\right]}{1-\left(1+\hat{A}L\right)\exp\left(-\hat{A}L\right)},$$
$$A_1 = \frac{\left(1+\hat{T}\right)\left[1-\exp\left(-\hat{a}L\right)\right]^2}{1-\left(1+\hat{a}L\right)\exp\left(-\hat{a}L\right)},$$
$$A_2 = -\frac{\hat{T}\left[1-\exp\left(-\hat{A}L\right)\right]^2}{1-\left(1+\hat{A}L\right)\exp\left(-\hat{A}L\right)},$$
$$B_1 = \frac{\left(A_1 b_2 + A_2 b_1\right)^2 - b_1^2\left(A_1+A_2\right)^2}{\left(b_2^2-b_1^2\right)},$$
$$B_2 = \left(A_1+A_2\right)^2 - B_1.$$

Then, the power of a FWM term caused by channels i, j and k such that $f_i + f_j - f_k = f$ is given as

$$\mathrm{FWM}_{i,j,k} = \left(\frac{16}{27} - \frac{32}{81}n\Phi\right)\frac{\gamma^2 K P_i P_j P_k}{R_s 4\pi^3 \beta_3 S_2 S_3},$$

$$\begin{aligned} K = &\frac{B_1}{b_1}\left\{\arctan\left[\frac{4\pi^3\beta_3 S_2 S_3}{b_1}\left(S_1+\frac{R_s}{2}\right)\right] - \arctan\left[\frac{4\pi^3\beta_3 S_2 S_3}{b_1}\left(S_1-\frac{R_s}{2}\right)\right]\right\} \\ &+ \frac{B_2}{b_2}\left\{\arctan\left[\frac{4\pi^3\beta_3 S_2 S_3}{b_2}\left(S_1+\frac{R_s}{2}\right)\right] - \arctan\left[\frac{4\pi^3\beta_3 S_2 S_3}{b_2}\left(S_1-\frac{R_s}{2}\right)\right]\right\}, \end{aligned}$$

where f is the center frequency of the channel under test, f_i, f_j, and f_k are the respective center frequencies of the interfering channels, S_1,S_2, and S_3 are elements of the sorted set $\{|\Delta f_i + \Delta f_j - 2f_0|,\ |\Delta f_i|,\ |\Delta f_j|\}$ such that $S_1 \leq S_2 \leq S_3$, Δf_i and Δf_j are the frequency separations between the CUT and the corresponding interfering channels, f_0 is the spectral distance from the zero-dispersion frequency, P_i, P_j, and P_k are the respective launch powers of the interfering channels, R_s is the symbol rate of the channels, γ is the nonlinear coefficient, β_3 is the third order dispersion, and Φ is the excess kurtosis of the transmitted modulation format. The parameter n takes values 0 or 1 depending on whether a modulation format correction is applied or not. It is equal to 1 only for the cases where $f_i = f_j$ and modulation format correction is required and 0 for all the rest. The total FWM interference in a given channel with center frequency f is the sum of all $\mathrm{FWM}_{i,j,k}$ terms.

III. Accuracy of Closed Form Approximation

For the evaluation of the extended closed form approximation's accuracy, comparisons were made with data from a split step Fourier method (SSFM) simulation. The parameters used in the simulation are given in Table 1. Three modulation formats were examined, QPSK, 16 QAM and 64 QAM for three different launch powers of 0, 2 and 4 dBm. The span length was set at 20 km. The plotted results which display the $\mathrm{SNR}_{\mathrm{NLI}}$ are shown in Fig. 2. Results show excellent accuracy of the provided closed form approximation. The maximum errors were in the range from 4 to 6 dB and appeared at the edge of the transmitted WDM spectrum where the SSFM displays a sudden increase of the SNR. The reason for that large discrepancy in the longest wavelength is under investigation. The mean channel-wise errors in dB were 0.84, 0.68 and 0.66 for the cases of QPSK, 16 QAM, and 64 QAM, respectively.

Table 1: System parameters.

Parameter	Value
Nonlinear coefficient γ [1/W/km]	2
Dispersion slope [ps/nm^2/km]	0.087
Attenuation at 1305 nm [dB/km]	0.33
Attenuation slope [dB/km/nm]	-0.001
Symbol rate [GBd]	96
Channel spacing [GHz]	100
Number of channels	101
Zero-dispersion wavelength [nm]	1303
Center wavelength of WDM [nm]	1303
Length of span [km]	20

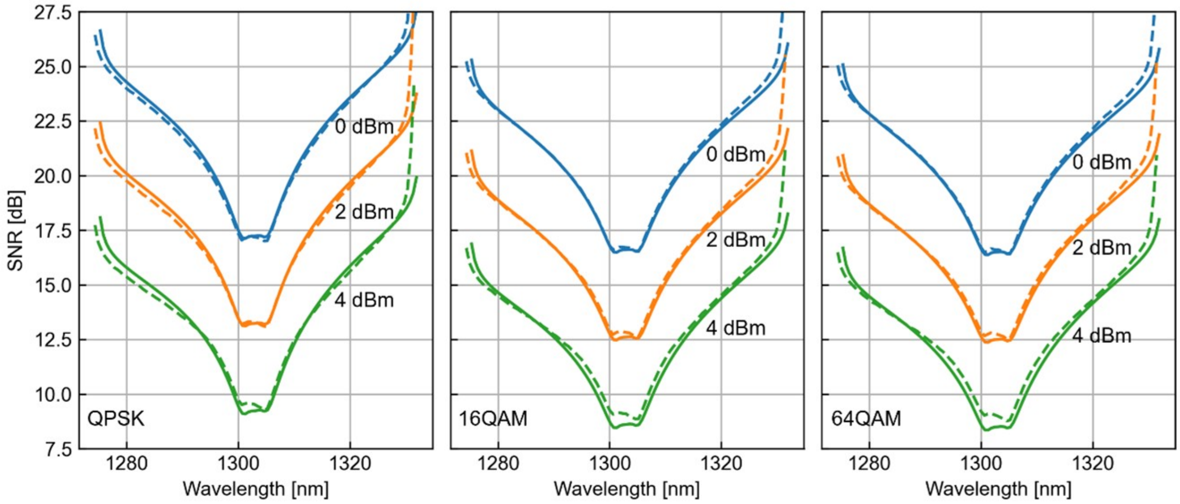

Fig. 2: Results of the comparison between the closed form and the SSFM for three modulation formats. The data of the closed form is displayed as a continuous line and the SSFM as a dashed one.

IV. Conclusions

We presented a novel extension of the closed form of the inter-band stimulated Raman scattering Gaussian noise model in the zero-dispersion regime accounting for arbitrary modulation format, span length, and fiber loss. Comparison with the simulation results showed excellent overall agreement with the split step Fourier method which validates its accuracy and proves that it can be a reliable tool in modelling the fiber's nonlinearity in O-band. The next step would be to explore its accuracy in experiments.

Acknowledgements

EPSRC Programme Grant TRANSNET (EP/R035342/1) and EWOC (EP/W015714/1).

References

1. D. J. Elson, Y. Wakayama, V. Mikhailov, J. Luo, N. Yoshikane, D. Inniss, and T. Tsuritani, "9.6-THz single fibre amplifier O-band coherent DWDM transmission," *Journal of Lightwave Technology*, vol. 42, pp. 1203–1208, Feb. 2024.
2. D. J. Elson, M. Jarmolovičius, V. Mikhailov, J. Luo, D. Inniss, S. Beppu, G. Baxter, R. Stolte, L. Stewart, S. Takasaka, E. Sillekens, R. I. Killey, P. Bayvel, N. Yoshikane, T. Tsuritani, and Y. Wakayama, "Continuous 16.4-THz bandwidth coherent DWDM transmission in O-band using a single fibre amplifier system," *Journal of Lightwave Technology*, vol. 43, no. 4, pp. 1813–1818, 2025.
3. D. Semrau, R. I. Killey, and P. Bayvel, "A closed-form approximation of the Gaussian noise model in the presence of inter-channel stimulated raman scattering," *Journal of Lightwave Technology*, vol. 37, pp. 1203–1208, May 2019.

4. F. Balasis, D. J. Elson, M. Jarmolovičius, H. Buglia, E. Sillekens, R. I. Killey, P. Bayvel, N. Yoshikane, T. Tsuritani, and Y. Wakayama, “An extended closed form of the ISRS GN model for the zero-dispersion regime,” in *2024 European Conference on Optical Communication (ECOC), 2024*, W1B.2.
5. M. R. Zefreh and P. Poggiolini, “A GN-model closed-form formula supporting ultra-low fiber loss and short fiber spans,” *arXiv:21111.04584*, 2021.
6. H. Buglia, M. Jarmolovičius, A. Vasylchenkova, E. Sillekens, L. Galdino, R. I. Killey, and P. Bayvel, “A closed-form expression for the Gaussian noise model in the presence of inter-channel stimulated raman scattering extended for arbitrary loss and fibre length,” *Journal of Lightwave Technology*, vol. 41, pp. 3577–3586, June 2023.