Play My Math: Second development cycle of an EdTech tool supporting the teaching and learning of fractions through music in algebraic notation

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Abstract. In this paper, we present an enhanced version of an EdTech tool called Play My Math (PMM) that aims to make the teaching and learning of fractions more engaging, fun, and meaningful through music. Our tool, accounting for music and mathematics notation tensions, has been designed with a gamified narrative that brings momentum to a proposed mathematical musician identity mindset and has been informed by both teachers' and students' feedback for increasing its educational value. Following a design-based research methodology, we capitalize on quantitative (questionnaire) and qualitative (focus group, observations) student and teacher feedback derived from the testing phase of PMM's first development cycle ($N = 67$ students, $N = 4$ teachers). In the current building phase of PMM's second development cycle, we address students' and teachers' expectations and suggestions. Regarding students, they expect the tool to allow them to learn and use different instruments, make music beyond rhythm-based capabilities, and facilitate their social interaction among peers. Regarding teachers, they expect the tool to allow them to cover the subtopics of fractions exhaustively and to have authoring capabilities to design custom learning paths and assignments. The feedback obtained in our first development cycle's testing phase suggested PMM to be a promising tool for engaging students in fractions-based mathematics while promoting skills beyond the academic domain. In future work, we will test PMM's second prototype version and assess whether PMM has fulfilled users' expectations and its impact on students' learning outcomes.

Keywords: Educational Technology, Educational Games, Fractions Teaching and Learning, Design-Based Research, Music and Mathematics, Mathematical Music Theory, Mathematical Musician Identity.

1. Introduction

Findings from PISA 2022 indicate a striking contrast in mathematical proficiency among students worldwide: while 69% can tackle basic math problems, fewer than 5% excel in complex tasks [1]. The issue of student disengagement in math is multifaceted, with a significant factor being the difficulty in understanding abstract concepts and their practical applications, which diminishes motivation [2-3]. Additionally, stress from exams and societal pressures exacerbate mathematical anxiety and undermine students' confidence in their math abilities [4-5], further deterring their interest. Among the concepts that pose a challenge from an early age is fractions, a fundamental topic that proves difficult for many young learners. Addressing this challenge is imperative, as a solid grasp of fractions lays the groundwork for learning more advanced mathematical skills, such as algebra [6].

In response to these problems, we designed and developed an Edtech tool to support teachers' practice and to help struggling math learners understand the magnitude, equivalency, and representations of fractions with an innovative, meaningful, and age-appropriate narrative through music [7]. To inform the define, conceive, and build phases of our second design-based iterative cycle, we capitalized on quantitative (questionnaire) and qualitative (focus group, observations) student and teacher feedback derived from PMM's first development cycle testing phase ($N = 67$ students, $N = 4$ teachers), and took inspiration from similar tools found in the literature.

2. Theoretical background

In the interdisciplinary field of music and mathematics, mathematics can be used as a tool for music creation, music can be leveraged to enhance mathematical cognition, and music can be adopted as a pedagogical strategy in mathematics education [8]. While the first two dimensions have been extensively explored, there is a growing advocacy for a more integrated pedagogical approach that not only combines music and mathematics but also includes technology, fostering a fertile interdisciplinary ground for creative and artistic expression [9].

In exploring the nexus between mathematics and music in K-12 education, significant attention has been paid to employing multimodal teaching approaches, integrating music (sonic representations) [10-13] and physical activities like dance (motor/kinesthetic representations) [14] to illustrate mathematical concepts. Among these, music-themed activities [10] have shown promise, enhancing both academic outcomes [15] and attitudes toward math [16-17]. Notably, an advantage of music-themed activities is that they inherently include movement when music is actively performed. For instance, one music-themed activity can involve aligning the values of musical notes (whole,

half, quarter, eighth, and sixteenth) with their mathematical counterparts (1/1, 1/2, 1/4, 1/8, 1/16), facilitating an intuitive grasp of fractions [11-12].

Research has demonstrated varied impacts of music-themed activities on learning outcomes, depending on the degree of engagement with different musical dimensions (i.e., rhythm, melody, harmony) and applications (i.e., listening, playing, creating music). Studies, such as one by Libby et al. [11], highlight the benefits of integrating multiple musical aspects, showing improved math performance when students engage with both rhythm and melody through listening and playing, compared to focusing on rhythm alone without a broader musical context [12].

However, challenges arise in maintaining fidelity between musical and mathematical representations, as highlighted by Lovemore et al. [18]. Issues include discrepancies in symbol usage between music and mathematics, potential confusion due to mismatched starting points and spatial divisions in notation, and the practicality of translating musical measures directly into mathematical notation. These tensions extend to the representation of silence, the varying significance of note values across different musical time signatures, and the dynamic interpretation of fractional values in music versus their static counterparts in mathematics [7].

Digital educational technologies have sought to bridge music note values and fractions [18-20], with some researchers opting to bypass these notation tensions by focusing solely on mathematical representation to aid fraction learning. Frisina [20], for example, developed a prototype that visually represented fractions, percentages, and decimals through graphical bars associated with musical sounds, though this approach did not fully accommodate musical accuracy when referencing a fraction notation. In other words, when referring to a mathematical notation both sound and silence must be notated to accurately reflect the desired musical output.

As mentioned above, the combination of mathematics, music, and technology for K-12 education is a fertile interdisciplinary ground. This holistic perspective suggests a shift towards a mathematical musician identity paradigm, where individuals do not perceive mathematics, music, and technology as isolated disciplines but as a unified toolkit for personal expression and creating artistic or scientific works. This approach positions individuals at the core, with the disciplines serving as orbiting tools for innovation and expression, thereby encouraging a more inclusive and engaging learning environment.

3. Method

We have been following a design-based research approach that we depict in Figure 1. The key characteristic of this methodology is that it considers multiple iterations to generate useful products aiming at solving individual or collective problems in education following these phases: focus, understand, define, conceive, build, and test [21].

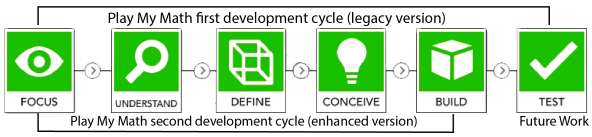

Fig. 1. Phases of the design-based research process based on Easterday et al.

3.1. Focus phase – targeted audience, topic, and scope of the project.

We focused our project on addressing an early and pervasive problem in mathematics education related to the sense-making of the abstract concept of fractions. Hence, our audience (hereafter jointly referred to as users) are elementary teachers and students ranging from 3rd- to 6th-grade levels, which are the levels where fraction topics are usually taught in different regions.

3.2. Understand phase – context and users' needs.

Using PMM's legacy prototype, we conducted a series of workshops with 67 Mexican primary school students. We collected students' feedback using a short survey where students reflected on their experience and expressed what they would like to change from the program and the tool. We also took notes on students' engagement during the workshop sessions. Finally, we conducted a follow-up focus group [22] with 4 Mexican teachers. Two authors conducted a thematic analysis [23] from a database of 45 participants' anonymized utterances.

Play My Math concept and legacy prototype. In PMM, students learn mathematical skills while using them to write, read, and perform musical compositions. The educational game starts with a mathematical musician introductory course on writing and reading music using solely numbers, arithmetic, fractions, algebraic expressions, and bar graphical representations. In the game, students aim to keep composing and performing music, for which completing courses, solving mathematical puzzles, and overcoming mathematical musical challenges is the key. In our first development cycle, we developed a Composition Tool that allows students to configure bar graphs with customizable percussive sounds and then listen to the rhythms they compose jointly. As the graphical rhythms are manipulated, an algebraic notation reference updates so students connect what they see and hear to the underlying mathematics. Once the users are satisfied with the artistic output (i.e., composition), the tool renders the algebraic notation in a printable file for further interaction with the creative work. In section 4.2, Figure 2, we have depicted the Composition Tool.

Students' feedback. Students (n=67) who participated in 10 consecutive PMM workshops reported a high level of enjoyment, averaging a score of 9.37 out of 10 (SD=1). The enjoyment variance was linked to individual challenges and nervousness, whereas positive feedback emphasized the engaging and enjoyable combination of music and math, particularly benefiting those less confident in math. The workshops also had a positive impact on students' attitudes toward mathematics, with an average appreciation score of 8.06 out of 10 (SD=2.59). Negative feedback was primarily due to a pre-existing disinterest in math or difficulties in comprehension, while positive responses praised the music-math integration for simplifying and enhancing the learning experience, especially with topics like integers and fractions. The perceived difficulty of the sessions was moderate, with an average score of 3.19 out of 10 (SD=2.31), reflecting varied experiences from finding the sessions accessible and engaging through music to

challenges related to the fast-paced nature of the activities or personal obstacles in following along.

Observation notes. In the workshops, introducing essentially unfamiliar tasks, students initially exhibited a blend of curiosity and confusion, leading to behaviors such as seeking clarification, mimicking peers, and sometimes disengaging to seek distractions. This confusion, stemming from either the tasks' novelty or a lack of interest, was mitigated as instructors provided further guidance or as peer engagement increased, helping students regain focus. Despite these early challenges, integrating music into learning activities significantly boosted enthusiasm and engagement, with students eagerly participating in musical tasks, showing joy in their achievements, and continuing their engagement with music even after lesson completion. While the music-themed activities generally enhanced student interest and collaboration, not all students were equally captivated, with some engaging only to follow peers and others remaining less involved or distracted.

Teachers' feedback. Teachers valued the tool designed for teaching fractions, highlighting its practical application in aiding students' comprehension, comparison, and meaningful use of fractions, as well as facilitating visual comparisons and social interactions. The tool's ability to customize lessons for enhanced student engagement and creativity was particularly valued, alongside its role in fostering enjoyable and collaborative classroom dynamics. Despite these benefits, teachers identified areas needing enhancement, such as extending the tool's capacity beyond four bars and eighth divisions for wider coverage of fraction topics and related concepts like percentages, mixed fractions, and least-common-multiple different cases. There was a unified call for the tool to support broader mathematical practice and assessment capabilities without necessarily incorporating the musical element, including automated exercise generation and grading to streamline teaching processes. Additionally, teachers expressed a desire for improved authoring capabilities, allowing for the preparation and preservation of lesson plans within the tool to ensure structured, focused learning experiences and to prevent student distractions, thereby enhancing overall classroom management and lesson delivery.

4. Results

4.1. Define phase – setting goals and assessments.

Based on the insights derived from the *understand phase*, we adapted our design goals to meet the following key points with their respective assessment strategy.

To overcome the disadvantages teachers spotted in PMM, we set our new design goals to meet the following aspects, PMM should firstly consider features for teachers to work on and discuss fraction topics exhaustively. Secondly, the tool should enable the teachers to create lesson plans or courses using the features PMM provides. Thirdly, PMM should enable teachers to generate exercises for students to practice. Fourthly,

the teachers should be able to control in what sections of PMM students can work during the lesson. To assess our goals, we are planning a study where we will train teachers in using PMM so that they can later apply PMM in their practice. We will analyze the teachers' lesson planning, observe their practice when using PMM, and conduct follow-up focus groups or ask them to fill out surveys.

Capitalizing on students' feedback, we are upgrading PMM so that they can learn and use other musical instruments to compose beyond rhythm-based music and facilitate cooperative and collaborative affordances within PMM. Regarding our goals assessment, in future work, we will capitalize on the envisioned study mentioned above, where we will ask the students to give their feedback. Moreover, we will analyze students' interaction with the Composition Tool and take notes on students' engagement during the workshop sessions.

4.2. Conceiving phase – Play My Math enhanced concept and prototype.

To address our users' expectations and needs, we expanded PMM's educational concept to have six main sections: Tools, Studio, Courses, Puzzles, Challenges, and Profile. The Tools section hosts all modules that are used in the rest of the sections except the Profile section. In the Tools section, users can easily access the following modules: Music Lab, Music Generator, Fraction Lab, Fraction Generator, and Metronome. The Studio section still hosts the Composition tool that allows users to visualize, create, modify, and listen to fractions by manipulating graphical bars that are dynamically translated to algebraic, percentage (new), and decimal notation (new). Users can save, print, co-create, and share their compositions with the PMM community. The courses section hosts in-house-produced PMM courses with video lessons, composition tasks, and formative assessments to help users learn how to use PMM and perform their mathematical compositions using different instruments. Moreover, teachers can also create their own courses using all components of the Tools section plus additional modules for organizing their courses or lesson plans for the students to follow (e.g. notes, videos). The Puzzles section presents mathematical riddles, which teachers can customize and assign to their students so that they can practice their mathematical skills. The Challenges section features special composition tasks for community competitions and the students. The Profile section displays users' data, performance, and progress in their Mathematical Musician artistic career (i.e., PMM's game narrative).

4.3. Build phase – second version of PMM prototype.

In the PMM enhanced version (Fig. 2), we have implemented the Tools, Studio, and Courses sections as those will help us to later validate if our users' feedback and expectations have been met.

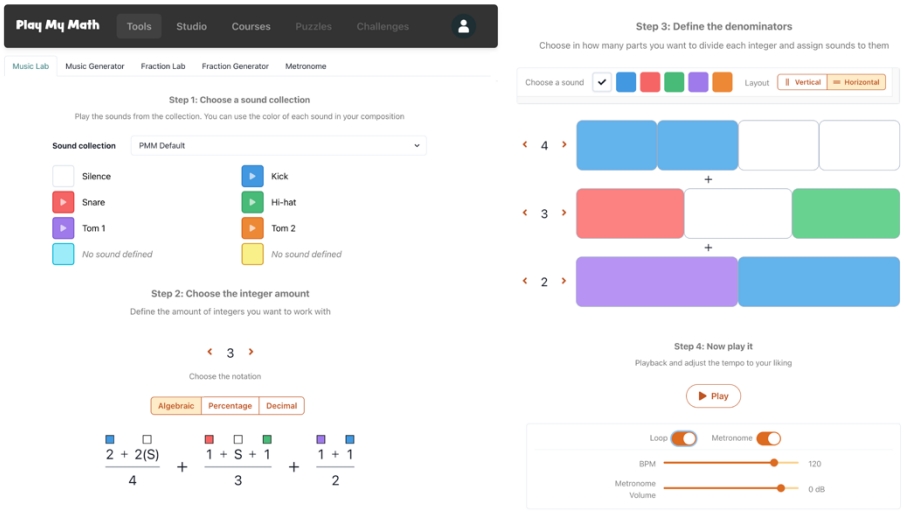

Fig. 2. Second prototype of the Play My Math Composition Tool. In this version, the Tools, Studio, and the Courses section have been implemented.

Teachers' targeted goals. *PMM should consider features for teachers to work on and discuss fraction topics exhaustively.* We have included in our Composition Tool (also the main component of the new Music Lab in the Tools section) a new notations tab where teachers can reference algebraic, percentage, and decimal notation. We built the Fractions Lab module where teachers can decide to work with up to eight graphical bars (wholes) and divide them into up to 50 parts. This will help the teachers to work more flexibly with bigger denominators and to discuss more challenging least-common-multiple examples. Users can now link active fractions of two graphical bars to represent mixed fractions for both the Music and the Fractions Labs.

The tool should enable the teachers to create lesson plans or courses using the features PMM provides. We have implemented the Course section with authoring capabilities so teachers can flexibly use the modules of the Tool section to create their courses and assign them to their students. All modules have a snapshot feature that enables the teacher to "freeze" modules' settings so students follow the desired instructional design. Additionally, when creating a course, they can use extra modules to design a coherent narrative, such as text, video, or audio modules.

PMM should enable teachers to generate exercises for students to practice. We have built a module called Fraction Generator where teachers can configure and generate exercises for different subtopics of fractions. These exercises can be configured to practice equivalence of fractions (<, >, =), translation between algebraic, percentage, decimal notations, and three cases of least-common-multiple (e.g., same denominator, different denominator but a direct multiple, and different denominator with no direct mul-

tiple). Teachers can adjust difficulty levels mainly by configuring the number of graphical bars (wholes), denominators, the number of exercises, and tempo when using the Composition Tool.

Teachers should be able to control in what sections of PMM students can work during the lesson. We have included a "lock" mode in the courses section that teachers can activate for their students. This "lock" mode will block access to other sections for students enrolled in a particular teacher's course.

Students' targeted goals. *Be able to learn and use other musical instruments to compose beyond rhythm-based music.* We have extended the Composition tool capabilities to support mono- and multi-timbre playback. This means that students can now decide in their rhythm-based compositions if they would like to have a single rhythmic line or up to eight different simultaneously playing bars (e.g., sounds from the sound bank). Additionally, we have expanded the Sound Bank to include an instrument mode where students can compose for piano. In this case, in the current version, melody and harmony can be written in algebraic notation with up to 4 simultaneous notes (notes are coded using module 12 and a subscript specifying the octave).

Facilitate cooperative and collaboration affordances within PMM. We have expanded the Studio section with a collaboration option where students can invite their peers to co-create musical compositions. The current version enables up to three collaborators.

5. Discussion, Conclusion, and future work

The development, testing, and refinement journey of PMM underscores the essential role of educational technology in bridging music and mathematics to enhance learning experiences and cover curriculum topics exhaustively. We have capitalized on the interplay between technological innovation and pedagogical practices, where feedback from both teachers and students plays a pivotal role in shaping the PMM's evolution. Teachers have sought a platform that enables a comprehensive exploration of fractions and the creation of customized learning paths, which has been achieved through introducing the Fractions Lab and an enhanced Course section with authoring capabilities. This development marks a significant stride towards creating a tool that allows flexibility for educators and comprehensive learning for students, addressing diverse needs within the classroom.

In parallel, students' desire for a more interactive and engaging experience has led to incorporating multi-timbre playback and collaboration features, transitioning PMM from a purely educational tool to a platform that fosters high-level musical creativity and peer learning. This shift aligns with modern educational paradigms prioritizing engagement and the innovative integration of music and mathematics, opening new avenues for exploration in educational environments.

In conclusion, PMM represents a forward-thinking approach to engaging students with fractions through the universal language of music combined with the interactive capabilities of educational technology. The enhancements made to PMM, driven by

user feedback, showcase its potential to significantly transform the learning experience for students and teachers. The next phase involves rigorous testing of PMM's enhanced version to validate its effectiveness and to refine it further based on empirical evidence. This continuous cycle of feedback and improvement is crucial for maintaining PMM's edge as an innovative, engaging, and pedagogically sound educational technology.

Finally, the implications of this study extend well beyond PMM, offering valuable insights into the design and implementation of educational technologies at large. By exploring the synergies between different fields of knowledge, such as music and mathematics, we are opening up new possibilities for enriching educational experiences and outcomes.

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