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Revealing higher-order interactions through multimodal irreversibility in flood-affected transportation networks

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ABSTRACT

Climate change and extreme weather events increasingly threaten urban transportation systems, challenging their ability to maintain essential mobility services. Current analytical approaches primarily focus on individual modes or simplified interactions, failing to capture the complex, non-equilibrium dynamics that emerge when multiple transportation modes interact under stress. This research introduces a novel Multi-modal Visibility Graph Irreversibility (MmVGI) framework for analysing transportation system behaviour during extreme weather events. By integrating concepts from non-equilibrium dynamics with visibility graph analysis, our approach quantifies complex interactions between different transportation modes and reveals the underlying mechanisms driving system non-equilibrium characteristics. Through a case study in the City of London during an extreme rainfall event, we demonstrate that transportation system adaptation exhibits clear hierarchical patterns across different road types. While primary roads maintain stable dynamics dominated by motorised transport, secondary networks show complex patterns of modal interaction, with cycling emerging as a crucial component in system adaptation. The strong correlation between unique and combined irreversibility measurements provides evidence for genuine higher-order interactions that cannot be reduced to simpler modal combinations. These findings advance both theoretical understanding of urban system dynamics and practical approaches to transportation management, offering valuable insights for urban planners and policymakers in developing more resilient, adaptive transportation systems for future climate challenges.

1. Introduction

Climate change has emerged as one of the most pressing challenges facing modern cities, with extreme weather events becoming increasingly frequent and intense [1,2]. Among various urban infrastructure systems, surface transportation networks are particularly vulnerable to climate-related disruptions, yet they are critical for maintaining urban mobility and economic activities [3]. Understanding how multi-modal surface transportation systems - including buses, private cars, bicycles, and pedestrians - respond to and recover from extreme weather events has become crucial for urban resilience. The complexity of multi-modal surface transportation systems under extreme conditions stems from their intricate patterns of interaction and interdependence [4]. During severe weather events, these interactions become more pronounced as travellers adapt their mode choices and routes in response to disruptions. For instance, when heavy rainfall affects road conditions, the interactions between different modes intensify as they compete for limited usable road space, potentially leading to system-wide instability [5]. Traditional approaches to analysing transportation systems behaviours have primarily focused on normal operating conditions, employing static network properties or simplified flow models [6]. However, these methods prove inadequate when studying system behaviour under extreme weather conditions, where non-linear interactions and complex adaptation patterns dominate [7–9]. Transportation networks during extreme events operate far from equilibrium, continuously dissipating energy and producing entropy as they struggle to maintain functionality [10]. The second law of thermodynamics provides a fundamental framework for understanding such non-equilibrium systems. In the absence of entropy sinks, a system's average entropy increases as time flows forward [11,12]. This principle becomes particularly relevant in

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transportation systems under stress, where the detailed balance condition breaks down [13], leading to asymmetric transition probabilities between system states. The entropy production rate (EPR) emerges as a natural measure of the degree of non-equilibrium [14,15], offering insights into system stability and potential vulnerabilities during extreme events.

Despite these advances, several critical research gaps remain in our understanding of transportation systems under extreme conditions. Current approaches to analysing system behaviour during extreme weather events primarily focus on individual modes [16] or simplified interactions [17], failing to capture the complex, non-linear dynamics that emerge when multiple modes interact under stress. Furthermore, while existing research has examined transportation systems under extreme conditions, these studies have not fully revealed the fundamental non-equilibrium characteristics that drive system behaviour during such events [3,18,19]. Understanding these non-equilibrium properties is crucial because they represent the underlying mechanisms through which transportation systems adapt to and recover from extreme disruptions. Current approaches typically focus on describing observed phenomena rather than uncovering the physical principles that govern system evolution during stress [20,21]. Thus, a fundamental gap exists in understanding how irreversibility manifests across different organisational levels in transportation systems, particularly during extreme weather events. To address these gaps, this study develops a comprehensive framework based on non-equilibrium dynamics for analysing multi-modal transportation systems under extreme weather conditions. We introduce the Multi-modal Visibility Graph Irreversibility (MmVGI) framework, which enables the quantification of complex interactions between different transportation modes during extreme events. By mapping multi-modal traffic patterns to visibility graphs, our approach captures both the temporal evolution of traffic states and the spatial organisation of modal interactions. Additionally, we develop methods for measuring unique contributions to system irreversibility, enabling the identification of genuine higher-order interactions that cannot be reduced to simpler modal combinations.

The remainder of this paper is organised as follows. Section 2 reviews relevant literature on transportation system analysis, non-equilibrium dynamics, and their applications. Section 3 presents our theoretical foundation and methodology for quantifying multimodal irreversibility. Section 4 demonstrates the application of our approach through real-world case studies. Section 5 discusses the implications of our findings and their practical significance. Finally, Section 6 concludes the paper with a summary of key contributions and future research directions.

2. Literature review

2.1. Transportation system dynamics

The theoretical foundations of traffic flow analysis were established in the 1950s and 1960s through seminal works by Lighthill and Whitham and Richards, who developed the fundamental hydrodynamic theory of traffic flow [22]. These studies introduced wave propagation concepts to traffic analysis, establishing the LWR model that remains influential today. Subsequent work by Greenshields provided empirical foundations for understanding the relationship between traffic density and flow [23]. During the 1970s and 1980s, researchers expanded these foundations to incorporate network-level analysis. Daganzo [24] developed the cell transmission model, while Newell [25] introduced simplified theories of traffic flow that balanced theoretical rigor with practical applicability. These developments enabled better understanding of network-wide traffic phenomena, though they primarily focused on vehicular traffic in isolation. The 1990s saw increasing attention to multi-modal transportation analysis. Ben-Akiva and Lerman [26] developed comprehensive frameworks for analysing travel behaviour across different modes, while cellular automata models [27] is introduced to represent multiple vehicle types. These studies began to address the complexity of modal interactions, though often under simplified assumptions. Recent decades have witnessed significant advances in understanding transportation system dynamics. Mishra et al. [28] developed methods for analysing cross-modal interactions in urban networks, while Xiong et al. [29], Gallotti and Barthelemy [30] explored the dynamics of mode switching behaviour. Zhang et al. [31] introduced frameworks for studying system-wide responses to disruptions, particularly focusing on the propagation of congestion across different modes. Studies of transportation system resilience have emerged as a crucial research direction. Sohouenou et al. [32] and Bucar et al. [33] analysed system responses to extreme events, while Huang [34] developed frameworks for quantifying system adaptability. These studies have highlighted the importance of understanding both structural and dynamic aspects of transportation system resilience.

However, several significant limitations in current approaches have become apparent. First, while existing models can effectively describe individual mode behaviour, they struggle to capture the complex interactions between different modes, particularly during disruptions [35–37]. As noted by Lynn et al. [38], traditional approaches often fail to account for the non-linear nature of these interactions. In this context, nonlinearity refers both to disproportionate system responses, such as tipping points in congestion [39,40], and more importantly, to emergent higher-order effects where the dynamics of a multi-modal group cannot be explained by simply summing its constituent pairwise interactions [41]. Second, most existing frameworks assume near-equilibrium conditions or simple steady states. The work of Borowska-Stefańska et al. [13] highlighted how these assumptions break down during extreme events, when transportation systems operate far from equilibrium. Pan et al. [42] and Gao et al. [43] demonstrated the need for new theoretical approaches that can better handle non-equilibrium dynamics. Third, there remains a significant gap between theoretical models and practical applications. While researchers like Assaad [44] have attempted to bridge this gap, the complexity of real-world transportation systems often exceeds the capabilities of current analytical frameworks. These frameworks primarily include large-scale simulation models (e.g., agent-based models) [45] and equilibrium-based Dynamic Traffic Assignment (DTA) models [46]. Despite their power, their reliance on pre-defined behavioural rules and, most critically, on near-equilibrium assumptions, limits their ability to quantify the fundamental non-equilibrium dynamics that dominate during severe, transient disruptions. This limitation becomes particularly apparent when studying system behaviour under stress [47].

Recent efforts by Song et al. [48] and Duan et al. [49] have begun to address these limitations by developing more comprehensive frameworks that integrate multiple analytical approaches. However, a complete understanding of transportation system dynamics, particularly under extreme conditions, remains elusive. These gaps highlight the need for new theoretical frameworks that can better capture the complexity of modern multi-modal transportation systems while remaining practically applicable.

2.2. Non-equilibrium dynamics theory

The study of non-equilibrium dynamics has evolved significantly since Onsager's pioneering work in the 1930s. Onsager established the fundamental reciprocal relations in near-equilibrium systems, providing the first rigorous framework for understanding non-equilibrium processes [50]. This foundation was extended by Prigogine [51] in the 1960s, who introduced the concept of dissipative structures and developed systematic approaches to analysing systems far from equilibrium. Significant advances in EPR (Entropy Production Rate) theory emerged in the 1990s through the work of Evans et al. [52] and Gallavotti and Cohen [53], who developed the fluctuation theorem for non-equilibrium steady states. These developments were complemented by Jarzynski [11], who established the relationship between non-equilibrium work and equilibrium free energy differences. Crooks further extended these concepts, providing a more general framework for understanding non-equilibrium processes [54]. The application of these theories to complex systems has seen remarkable progress. Seifert [55] developed stochastic thermodynamics, providing tools for analysing small systems subject to thermal fluctuations. Van den Broeck and Esposito extended these approaches to coupled systems and developed frameworks for analysing information flows in non-equilibrium processes [56,57]. In recent years, researchers have focused increasingly on applying non-equilibrium concepts to real-world systems. Skinner et al. [58] developed methods for measuring entropy production in biological systems, while Nartallo-Kaluarachchi et al. [14] applied these approaches to neuroscience fields. These studies have demonstrated the broad applicability of non-equilibrium frameworks while also revealing the challenges in adapting them to specific contexts.

However, several significant limitations in current approaches have become apparent. First, most applications of non-equilibrium theory focus on relatively simple systems or idealised models. The work of Farsi et al. [59] highlighted the difficulties in extending these frameworks to complex, real-world systems with multiple interacting components. Second, while theoretical frameworks exist for analysing steady-state behaviour, understanding transient dynamics and responses to extreme perturbations remains challenging [60]. The gap between theoretical developments and practical applications is particularly evident in transportation research. While researchers like Li et al. [61] have attempted to apply non-equilibrium concepts to traffic flow analysis, and Zhou et al. [62] have explored entropy production in transportation networks. However, their application to large-scale transportation systems, particularly under extreme conditions, remains limited. These gaps highlight the need for new theoretical approaches that can better bridge the gap between fundamental non-equilibrium physics and the practical challenges of analysing complex transportation systems.

2.3. Multilevel interaction analysis methods

Recent decades have witnessed extensive research on analysing multilevel interactions in complex systems. Scholars in network science have made significant contributions to understanding the structural aspects of multilevel systems. Watts and Strogatz [63] pioneered the study of small-world networks, while Barabási and Albert [64] introduced scale-free network concepts, laying the groundwork for analysing complex network structures. Building on these foundations, Newman [65] and others developed methods for community detection and hierarchical structure analysis in networks, enabling deeper insights into system organisation across multiple scales. The application of network analysis to transportation systems has evolved significantly. Gallotti and Barthelemy [30] developed multilayer network models for urban transportation, while Wang et al. [66] extended these approaches to incorporate temporal dynamics. These studies revealed important patterns in mode interactions, though they primarily focused on structural rather than dynamic aspects. Lin et al. [3] integrated network theory with traffic flow analysis, providing new insights into how network topology influences system performance. Time series analysis has emerged as another crucial approach for studying multilevel interactions. Traditional methods based on correlation analysis have been enhanced by more sophisticated techniques. Shamsan et al. [67] developed nonlinear time series analysis methods, while Lynn et al. [68] and Braun et al. [69] introduced recurrence quantification analysis, providing tools for identifying complex temporal patterns. In transportation research, these methods have been applied by Laval [70], Zeng and Tang [71]to analyse traffic flow dynamics. A significant advance in the field came with the development of multiscale analysis techniques. Lacasa et al. [72] introduced multiscale entropy analysis, while Smith et al. [73] developed wavelet-based methods for analysing hierarchical temporal structures. These approaches have been adapted to transportation studies by researchers such as Liu et al. [74], who studied the temporal complexity of airport air traffic flow and Harrou et al. [75], who analysed traffic patterns across multiple time scales.

However, several limitations in current approaches have become apparent. First, most existing studies focus on either spatial or temporal aspects in isolation, failing to capture the complex spatiotemporal interactions characteristic of transportation systems. Second, while methods exist for analysing individual modes or simple mode pairs, techniques for understanding higher-order interactions among multiple modes remain underdeveloped. Third, current approaches struggle to account for the fundamental non-equilibrium nature of transportation systems, particularly under extreme conditions. The integration of multiple analytical approaches remains a significant challenge. While researchers have attempted to combine network and time series analysis [76], and Yin et al. [77] have worked to incorporate multiscale permutation mutual information with traditional traffic flow analysis, a comprehensive framework that can capture all relevant aspects of multilevel interactions in transportation systems remains elusive. This limitation becomes particularly apparent when studying system behaviour under stress, where existing methods often fail to capture the complex adaptation patterns that emerge. Thus, these gaps in current methodological approaches highlight the need for new frameworks that can better capture the complexity of multilevel interactions in transportation systems.

Through a comprehensive review of existing literature across transportation dynamics, non-equilibrium theory, and multilevel analysis methods, we identify three fundamental research gaps that warrant investigation.

1. **Non-equilibrium Nature of Transportation Systems**: Current research has not fundamentally revealed what causes the non-equilibrium characteristics in transportation systems during extreme events. While studies have separately explored transportation networks and non-equilibrium theory, they have failed to establish the underlying mechanisms that drive transportation systems away from equilibrium. Understanding these mechanisms is crucial because it would enable us to quantitatively measure and evaluate how different components of the transportation system contribute to its overall non-equilibrium behaviour.
2. **Higher-order Modal Interaction Analysis**: Existing analytical frameworks fail to capture the full complexity of multi-modal interactions in transportation systems. Most studies focus on analysing individual modes or simple pairwise relationships, overlooking the critical higher-order interactions that emerge during extreme conditions. This limitation is particularly significant because transportation system adaptation often involves complex, synchronised changes across multiple modes that cannot be understood through simplified analysis of individual components or mode pairs.
3. **Unique Contribution Identification:** Current methodological approaches lack the capability to distinguish between combined effects and unique contributions in multi-modal interactions. While researchers have observed complex behavioural patterns in transportation systems during extreme events, existing methods cannot effectively identify which interactions represent genuine higher-order effects versus those that merely reflect the accumulation of simpler interactions. This methodological gap has prevented a deeper understanding of how different transportation modes truly influence each other during system stress.

These research gaps highlight the need for a new theoretical and analytical framework that can reveal the fundamental causes of non-equilibrium behaviour while capturing the complex nature of multi-modal interactions. Such a framework must be capable of not only measuring overall system behaviour but also identifying genuine higher-order interactions that emerge during extreme conditions, thereby capturing the complex **spatiotemporal dynamics** of multi-modal adaptation. This understanding is crucial for developing more effective strategies to enhance transportation system resilience in the face of increasing climate-related challenges.

3. Methodology

The complex interactions within multi-modal transportation systems during extreme weather events require a comprehensive analytical framework that can capture both their non-equilibrium characteristics and multi-level modal interactions. In this section, we present our methodological approach that progresses from theoretical foundations to practical measurements. First, we establish the theoretical basis for analysing transportation systems as non-equilibrium systems. Transportation networks under extreme weather conditions exhibit clear non-equilibrium characteristics, manifested through multivariate time series data across different modes. The irreversibility of these time series serves as a natural measure of the system's deviation from equilibrium, providing a quantitative approach to assess system behaviour during extreme events.

Building on this theoretical foundation, we develop a three-step analytical process to quantify system irreversibility and modal interactions, as illustrated in Fig. 1. The first step involves constructing visibility graphs from multivariate time series data. For each transportation mode, we map temporal patterns to network structures where nodes represent time points and edges capture visibility relationships between these points (Fig. 1a). This transformation preserves crucial dynamic features while enabling network-based analysis. The second step focuses on analysing the degree distribution patterns within these visibility graphs. We compute both in-degree and out-degree distributions for individual modes and their combinations. These distributions capture the fundamental asymmetry in system evolution, reflecting how different modes interact and influence each other over time. As shown in Fig. 1b, the distinctly different patterns between in-degree and out-degree distributions provide evidence of system irreversibility. The final step quantifies system irreversibility through Jensen-Shannon divergence (JSD) calculations on these distributions. We measure irreversibility at multiple organisational levels, from individual modes to higher-order modal combinations. This multi-level analysis reveals both combined effects and unique contributions of different modal interactions. Fig. 1c illustrates how these measurements distinguish between overall system behaviour (combined measurements) and genuine higher-order interactions (unique contributions). Through this methodological framework, we can systematically analyse how different transportation modes contribute to system non-equilibrium characteristics and reveal the complex adaptation patterns that emerge during extreme weather events.

3.1. Theoretical foundations

3.1.1. Applicability of non-equilibrium dynamics to urban transportation systems

Urban transportation systems are archetypal complex adaptive systems, exhibiting intricate patterns of behaviour that emerge from the interactions of individual components [78,79]. Their evolution is constrained by small-world and scale-free network topologies [40], which support the emergence of non-trivial collective dynamics where local interactions between individual vehicles aggregate into macroscopic traffic patterns that cannot be predicted from individual behaviours alone. Small perturbations such as localised weather disruptions can cascade through the network, creating system-wide effects that are disproportionate to the initial disturbance, exhibiting the characteristic sensitivity of far-from-equilibrium systems. A deeper commonality lies in the path-dependent or memory effects inherent in the evolution of transportation systems [80–82]. During extreme events, the system's future evolution depends not only on its current state but also strongly on the historical trajectory it took to arrive there. This is formally known as a non-Markovian stochastic process, which has been approved by previous studies [83–87]. For example, a traffic jam that formed due to a slow, gradual accumulation of vehicles will dissipate with very different dynamics than a jam that formed instantaneously from a multi-lane accident [87]. This memory is a key characteristic that distinguishes traffic flow from simple, memoryless Markovian processes, a complexity that has been increasingly documented in various traffic flow studies [88,89].

This path-dependent, directional evolution in transportation systems

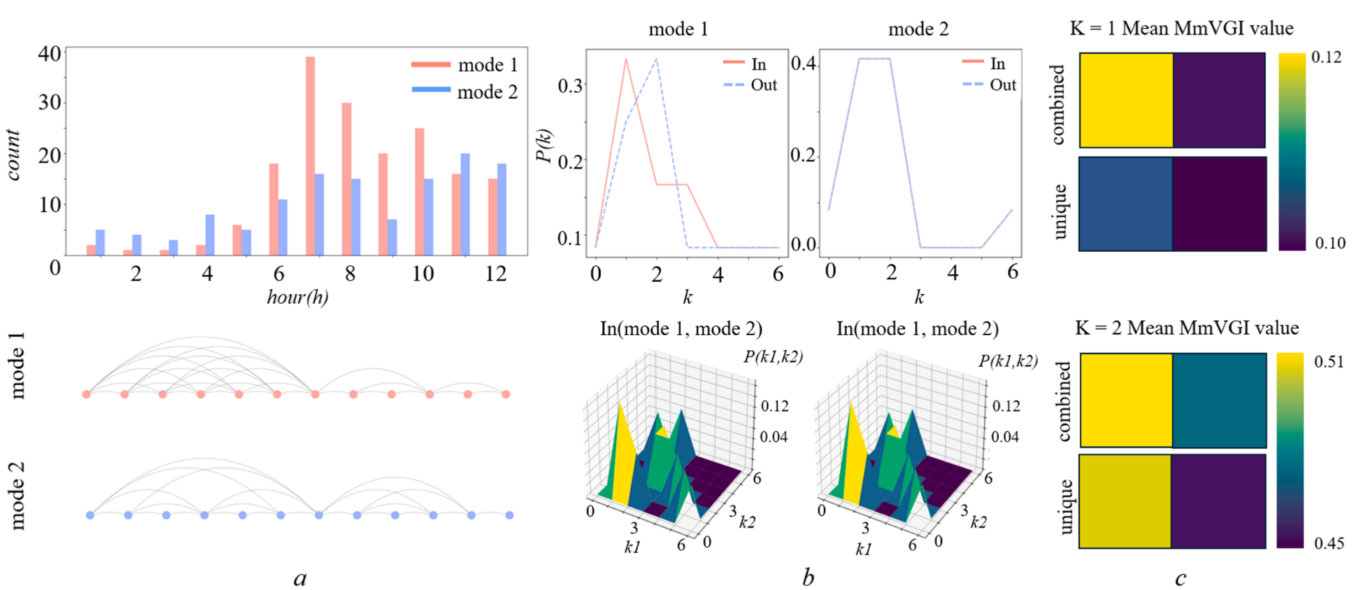

Fig. 1. Illustration of the MmVGI Framework. (a) illustrates the visibility graph construction process, showing the transformation of temporal data into network structures for two distinct transportation modes. The upper histogram displays the degree distribution characteristics for both modes, while the lower network visualisations demonstrate how temporal patterns are mapped to visibility relationships. (b) presents the probability distributions of in-degree and out-degree for both modes individually (shown in 2D plots) and their joint distributions (shown in 3D surface plots), revealing the asymmetric evolution patterns in the system. (c) compares the combined and unique irreversibility measurements across different organisational levels ($K = 1$ and $K = 2$), using color-coded heatmaps to represent the strength of irreversibility values.

manifests a fundamental concept that appears across diverse complex systems: the 'arrow of time' that Eddington described to characterize processes with inherent directional bias [90]. The mathematical framework for understanding such directional evolution can be traced to statistical mechanics and information theory, where entropy serves as a measure of system asymmetry and uncertainty [12,58]. Building on the conceptual foundation established by the second law of thermodynamics for characterizing irreversible processes [91,92], transportation networks during extreme events exhibit preferential directions in their evolution [93–96] that can be quantified using corresponding information-theoretic entropy measures [97]. In transportation systems, this directional bias manifests as asymmetric probabilistic pathways - the transition from normal flow to congestion follows different statistical dynamics than the recovery process, even though both may ultimately reach similar states [88,98]. This asymmetry is fundamentally information-theoretic: while a traffic jam may eventually dissipate and the system may return to baseline conditions, the statistical patterns governing jam formation and dissipation are distinctly different, creating measurable directional characteristics in the system's evolution [88,99,100]. The 'irreversibility' and 'entropy' in this research quantify therefore represent information-theoretic measures of statistical asymmetry in probabilistic transitions, borrowing the mathematical structure from thermodynamics but applying it to quantify information rather than energy dissipation. This analogy is not merely metaphorical—it points to deeper physical principles that govern systems operating far from equilibrium. The core mechanism underlying these irreversible behaviours is the systematic violation of detailed balance [101,102]. In equilibrium systems, microscopic transitions are statistically reversible—the probability of transitioning from state A to state B is balanced by the probability of transitioning from B back to A, mathematically expressed as:

$$P(X_i \to X_j) \cdot P_{eq}(X_i) \approx P(X_j \to X_i) \cdot P_{eq}(X_j) \tag{1}$$

where $P_{eq}(X_i)$ represents the equilibrium probability distribution of traffic states [55]. However, under the significant stress of an extreme event, this fundamental balance is broken systematically in a transportation network, creating net probability fluxes between different system states and generating measurable temporal asymmetry in the system's evolution [103,104]. This systematic violation of detailed balance marks the system's transition from an equilibrium state, characterised by symmetric interactions and balanced transition probabilities (conceptually illustrated in Fig. 2a and c), to a non-equilibrium state, defined by directed interactions and asymmetric transition probabilities (Fig. 2b and d). For instance, the probabilistic path from free-flow to gridlock is highly asymmetric to the path from gridlock back to free-flow [100,105,106]. This observable, irreversible evolution—the systematic breaking of detailed balance—is a definitive signature of a system operating far from equilibrium. Modern non-equilibrium physics provides rigorous, quantitative tools for measuring this departure from equilibrium through information-theoretic approaches. The primary such tool is the rate of entropy production [12,58], which, in this context, is reinterpreted as a pure information measure. It mathematically quantifies the degree of irreversibility by measuring the statistical divergence between the probabilities of a system's forward and time-reversed evolutionary paths. Therefore, the EPR provides a direct, quantitative link between the foundational mechanism of broken detailed balance and a measurable, information-based quantity that signals the system's distance from equilibrium. It is mathematically equivalent to the Kullback-Leibler divergence between the probability of a forward trajectory γ and its time-reversal $\tilde{\gamma}$:

$$\Sigma = \left\langle \log \frac{P[\gamma]}{P[\tilde{\gamma}]} \right\rangle \tag{2}$$

This measure provides a direct, quantitative indicator of how far the system operates from equilibrium conditions, rooted in the information generated by the system's temporal evolution rather than any thermodynamic interpretation [14,15]. The mathematical rigor of this approach ensures that the irreversibility measurements reflect genuine system properties rather than artifacts of the analytical method.

Recent empirical studies validate this theoretical perspective by documenting specific non-equilibrium characteristics in urban transportation systems [107]. Traffic systems exhibit clear temporal asymmetries during disruptions, manifesting as faster collapse than recovery during extreme events, hysteresis in system response to weather conditions, and path-dependent recovery patterns that depend on the specific sequence of disruptions experienced [88,108,109]. These systems dissipate efficiency through multiple mechanisms including increased travel times, sub-optimal routing choices, and modal coordination breakdowns. While not involving thermal dissipation, this loss of organisational efficiency creates measurable effects that are mathematically analogous to entropy production in physical systems [10,110]. The entropy production rate framework is applicable to transportation systems through several fundamental justifications. The stochastic evolution of traffic states follows the same probabilistic structure as other complex systems studied in non-equilibrium physics [111,112], allowing EPR calculation methods to be applied without modification to the underlying mathematical framework. Moreover, EPR can be reinterpreted as measuring information production rate in any stochastic system, providing a foundation that is independent of physical energy considerations [14]. This information-theoretic interpretation means that EPR captures the rate at which the system generates new information about its state, which is a meaningful concept for any complex adaptive system regardless of its physical substrate.

While strong mathematical and phenomenological parallels exist between transportation systems and non-equilibrium physical systems [113], important distinctions must be acknowledged to maintain scientific rigor. Transportation systems share fundamental characteristics with non-equilibrium physics including stochastic evolution governed by probability flux relationships, systematic detailed balance breaking under stress conditions, measurable information-theoretic irreversibility, and multi-scale emergent behaviour that spans organisational levels [111,113,114]. However, transportation systems fundamentally differ from classical thermodynamic systems in that they do not exhibit microscopic energy conservation in the thermodynamic sense, lack well-defined thermodynamic temperature concepts, do not follow Boltzmann distribution equilibria, and do not involve classical heat dissipation mechanisms [115–117]. These distinctions define the scope within which the analogy remains valid and useful, ensuring that the mathematical tools are applied appropriately rather than overextended beyond their theoretical foundations. Therefore, it is precisely this observable, non-Markovian, and irreversible evolution in transportation

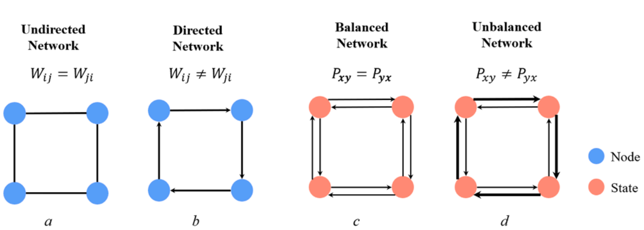

Fig. 2. Conceptual Illustration of Equilibrium and Non-Equilibrium System Dynamics. (a) and (c) together represent an equilibrium system. (a) depicts a network with symmetric, reciprocal interactions ($W_{ij} = W_{ji}$). This underlying structure leads to the dynamic property shown in panel (c), where the transition probabilities between any two states are equal in both directions ($P_{xy} = P_{yx}$). This condition is known as detailed balance. (b) and (d) together represent a non-equilibrium system. (b) depicts a network with asymmetric, directed interactions ($W_{ij} \neq W_{ji}$). This structure leads to the dynamic property shown in (d), where transition probabilities are unbalanced ($P_{xy} \neq P_{yx}$). This condition represents a state of broken detailed balance.

systems that provides a solid and mathematically precise foundation for applying the analytical framework of non-equilibrium statistical mechanics, while acknowledging the information-theoretic rather than thermodynamic nature of the entropy measures employed. This theoretical foundation establishes that non-equilibrium dynamics concepts are not merely metaphorical when applied to transportation systems but represent mathematically rigorous tools for analysing stochastic systems that operate far from equilibrium. The framework provides genuine predictive and analytical power for understanding transportation system adaptation during extreme weather events, rooted in the fundamental breaking of detailed balance and the information-theoretic quantification of temporal irreversibility.

3.1.2. Problem formulation

Multi-modal transportation systems in urban environments share road space and resources, creating complex interactions between different modes such as buses, private vehicles, bicycles, and walks. During extreme weather events, these interactions intensify as travellers adapt their mode choices and routes in response to adverse conditions [118]. Recent empirical studies have documented specific manifestations of this complexity: when severe rainfall or flooding occurs, traffic flows exhibit persistent directional patterns and asymmetric evolution [93,96] - the transition from normal operations to disrupted states follows different patterns than the recovery process. These observations provide concrete evidence of the non-equilibrium dynamics predicted by theory [13].

As we established in the preceding section (3.1.1), these observable, asymmetric dynamics are definitive signatures of a system operating far from equilibrium. The theoretical framework of non-equilibrium physics provides a rigorous way to quantify this deviation via the EPR, which captures the fundamental breaking of detailed balance through the statistical divergence between forward and reverse system trajectories. For a transportation system, the EPR is formally defined as:

$$\Phi = k\lim_{\tau\to\infty}\left(\frac{1}{\tau}\right)\cdot D_{KL}\left[P\left(\{X(t)\}_{t=0}^{\tau}\right) \parallel P\left(\{X(\tau-t)\}_{t=0}^{\tau}\right)\right] \tag{3}$$

where $\{X(t)\}_{t=0}^{\tau}$ represents the system trajectory and $\{X(\tau-t)\}_{t=0}^{\tau}$ its time reversal. Here, $X(t)$ captures the full system state, including traffic flows, densities, and speeds across different transportation modes. $P(\cdot)$ denotes the path probability, k is the Boltzmann constant, and D_{KL} measures the Kullback-Leibler divergence [119] between the probability distribution of forward trajectories $P\left(\{X(t)\}_{t=0}^{\tau}\right)$, and the probability distribution of time-reversed trajectories, $P\left(\{X(\tau-t)\}_{t=0}^{\tau}\right)$, defined as:

$$D_{KL}(P \parallel Q) = \int p(x)\log\frac{p(x)}{q(x)}dx \tag{4}$$

where P and Q are probability distributions with densities p and q respectively [120]. This measure quantifies the statistical distance between two probability distributions, providing a foundation for analysing system asymmetry. The fundamental characteristic of non-equilibrium behaviour—asymmetric transition probabilities $P\left(X^1\to X^2\right) \neq P\left(X^2\to X^1\right)$. lies at the heart of this theoretical framework. In the context of a transportation network, these asymmetries manifest as emergent directional flows, where certain modes or road segments act as sources of displaced traffic while others become sinks. However, while this theoretical foundation is powerful, its direct application presents a major practical hurdle. For a system as complex and high-dimensional as a real-world multi-modal transportation network, computing the path probabilities $P\left(\{X(t)\}_{t=0}^{\tau}\right)$ required for the EPR calculation is computationally intractable. The continuous state space of transportation systems, combined with the high-dimensional nature of multi-modal interactions, creates a fundamental gap between our theoretical understanding of the system's non-equilibrium nature and our ability to practically measure and analyse it.

This computational challenge intersects with the fundamental research gaps identified in our literature review. While the theoretical foundation establishes that transportation systems exhibit non-equilibrium characteristics during extreme events, current research has not fundamentally revealed what causes these characteristics or established the underlying mechanisms that drive transportation systems away from equilibrium. Existing analytical frameworks fail to capture the full complexity of multi-modal interactions, with most studies focusing on individual modes or simple pairwise relationships while overlooking the critical higher-order interactions that emerge during extreme conditions. Furthermore, current methodological approaches lack the capability to distinguish between combined effects and unique contributions in multi-modal interactions, preventing identification of which interactions represent genuine higher-order effects versus those that merely reflect the accumulation of simpler interactions.

These limitations create a significant barrier to understanding how transportation systems truly adapt during extreme weather events. The inability to quantify system irreversibility directly, combined with the lack of methods for analysing genuine higher-order modal interactions and distinguishing unique contributions from combined effects, prevents the development of effective strategies for enhancing transportation system resilience. Motivated by these challenges, we propose the MmVGI framework. This framework is designed specifically to overcome the computational barrier of direct EPR calculation while simultaneously addressing the fundamental research gaps identified in the literature. By mapping multivariate traffic patterns to visibility graphs, our approach provides a practical and powerful tool for quantifying the complex, hierarchical, and higher-order interactions that drive transportation system adaptation under stress. Through this framework, we can bridge the gap between theoretical understanding and practical analysis, enabling new insights into the mechanisms underlying transportation system non-equilibrium behaviour and the genuine higher-order interactions that emerge during extreme weather events.

3.2. Multi-modal visibility graph irreversibility

We build on the established paradigm of network-based time series analysis, which has proven particularly effective in studying complex dynamical systems [121,122]. The visibility graph (VG) approach has emerged as a powerful model-free tool for transforming continuous-valued time series into network representations, preserving essential dynamical features while enabling network-based analysis [15,122]. Its versatility and assumption-free nature have led to successful applications across various fields, particularly in analysing complex [123,124] and chaotic dynamics [125].

Building on these foundations, we introduce the MmVGI framework to analyse transportation systems with N different modes. For such a system, its state can be represented as an N-dimensional time series $\{X(t)\}_{t=0}^{T}$, where $X(t) = (x_1(t),\ldots,x_N(t))$. For any k_{th} order subsystem $\Gamma\left(x_{i_1},\ldots,x_{i_k}\right)$ on a **given road segment**, its trajectory can be expressed as:

$$\Gamma^{\left(x_{i_1},\ldots,x_{i_k}\right)} = \left\{x_{i_1}(t),\ldots,x_{i_k}(t)\right\}_{t=0}^{T} \tag{5}$$

Theoretically, the irreversibility (ς) of this subsystem is defined as the statistical divergence between its forward and time-reversed trajectories, calculated using path probabilities $P(\Gamma)$:

$$\varsigma^{\left(x_{i_1},\ldots,x_{i_k}\right)} = \sum_{\Gamma^{\left(x_{i_1},\ldots,x_{i_k}\right)}} P(\Gamma^{\left(x_{i_1},\ldots,x_{i_k}\right)})\log\frac{P(\Gamma^{\left(x_{i_1},\ldots,x_{i_k}\right)})}{P(\Gamma'^{\left(x_{i_1},\ldots,x_{i_k}\right)})} \tag{6}$$

where Γ' represents the time-reversed trajectory. While Eq. (5) provides a fundamental theoretical definition, its direct application presents a major practical hurdle: computing the path probabilities $P(\Gamma)$ for a system as complex and high-dimensional as a real-world multi-modal transportation network is computationally intractable. To overcome this challenge, we developed the MmVGI framework. This framework provides a practical and powerful pathway to quantify irreversibility by measuring a direct signature of temporal asymmetry within the time series data itself. It is important to clarify our choice of a component-wise approach over other potential multivariate methods. An alternative could be to construct a single visibility graph from the vector trajectory Γ, for instance, by first reducing the multivariate data to a univariate series. While this approach could capture the irreversibility of the aggregated system state, it would do so at the cost of significant information loss, as the unique dynamics of individual modes would be blended together. Given that the central goal of this research is to reveal the interactions between different modes and to isolate their unique higher-order contributions, a component-wise framework that preserves all modal information is essential.

Therefore, instead of attempting to approximate these probabilities, our MmVGI framework provides an alternative and practical pathway, which is designed to first analyse the modes individually and then to probe their statistical coupling. It quantifies irreversibility by measuring a direct signature of temporal asymmetry within the time series data itself, through a three-step process:

1) Transformation to Visibility Graphs

First, we transform the time series data of each mode into a directed network called a visibility graph (VG). As illustrated in Fig. 3a, a directed edge is formed from a time point t_i to a later time point t_j if and only if a direct line of sight exists between their corresponding data values. This geometric visibility condition is defined as:

$$x_k < x_j + (x_i - x_j)\frac{t_j - t_k}{t_j - t_i} \tag{7}$$

This mapping (as shown in Fig. 3a) preserves key dynamical features of the original signal, including periodicity, fractality, and causality. To capture the temporal direction of evolution, we construct directed edges pointing from earlier to later times, represented by the adjacency matrix:

$$A_{ij}^{[m]} = \begin{cases} 1 \text{ } if \text{ } i \rightarrow j \text{ in mode } m \\ 0 \text{ } otherwise \end{cases} \tag{8}$$

where $A^{[m]}$ represents the adjacency matrix of mode m. This step effectively encodes the temporal dynamics of each mode into a unique network **structure (as shown in** Fig. 3**b).**

2) Capturing Asymmetry in Degree Distributions

The temporal irreversibility of the original time series is fundamentally captured and preserved as a structural asymmetry within its corresponding visibility graph. Specifically, this asymmetry manifests as a difference between the in-degree and out-degree distributions of the graph's nodes. For a perfectly reversible process, these two distributions would be identical. For an irreversible process, they will differ, reflecting the system's directional evolution. We calculate the in-degree and out-degree for each node i in each mode's graph m:

$$d_i^{[m],in} = \sum_j A_{ji}^{[m]}, \quad d_i^{[m],out} = \sum_j A_{ij}^{[m]} \tag{9}$$

These distributions capture the fundamental asymmetry in system evolution. For a k-order subsystem, we compute the joint degree

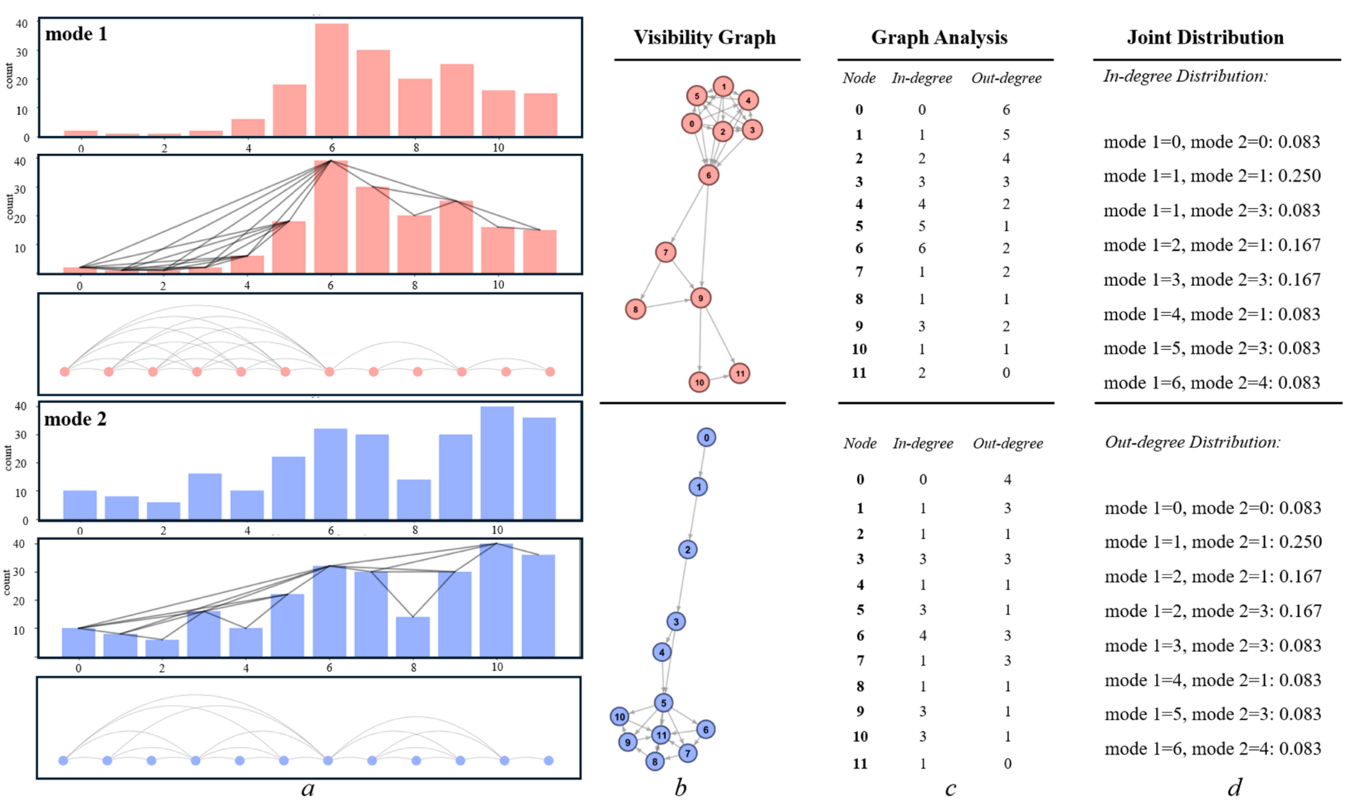

Fig. 3. Construction and Analysis Process of Multi-modal Visibility Graphs. (a) illustrates the original time series data for two distinct modes (shown in red and blue), along with their visibility mapping process. The upper plots show the raw time series, the middle plots display the visibility connections between data points, and the lower plots present the final node arrangement for visibility graph construction. (b) shows the resulting visibility graphs for both modes, where nodes represent temporal data points and edges indicate visibility relationships between these points. (c) presents the detailed graph analysis results, including in-degree and out-degree calculations for each node in both modes. (d) displays the joint distribution analysis, quantifying the relationships between different modes through their degree statistics.

distributions:

$$P_{in}^{(n_1,\dots,n_k)}(d_1,\dots,d_k), P_{out}^{(n_1,\dots,n_k)}(d_1,\dots,d_k) \tag{10}$$

Here, the tuple $(n_1,\dots,n_k)$ is a set of identifiers for the k specific transportation modes being considered in a k-order analysis (e.g., for $k = 2$, the tuple could be ('car', 'bus')).

3) Quantifying Irreversibility with Jensen-Shannon Divergence

Finally, to obtain a single, robust value for the irreversibility, we quantify the statistical distance between the in-degree and out-degree distributions using the Jensen-Shannon Divergence (JSD), as shown in Fig. 3d The JSD is a rigorous, symmetrised measure of the difference between two probability distributions. The irreversibility ς for the k-order subsystem is thus practically calculated as:

$$\varsigma^{(n_1,\dots,n_k)} = JSD\left(P_{in}^{(n_1,\dots,n_k)} \parallel P_{out}^{(n_1,\dots,n_k)}\right) \tag{11}$$

The specific calculation formula is:

$$JSD(P_{\text{in}}\|P_{\text{out}}) = \frac{1}{2}D_{KL}(P_{\text{in}}\| M) + \frac{1}{2}D_{KL}(P_{\text{out}}\| M) \tag{12}$$

where $M = \frac{Pin+Pout}{2}$, and the Kullback-Leibler divergence is defined as Eq. (2).

For finite-length time series, we employ Laplace smoothing to avoid zero probability issues:

$$P^{(n_1,\dots,n_k)}(d_1,\dots,d_k) = \frac{N+1}{M+d_{max}^k} \tag{13}$$

where N is the number of nodes satisfying the degree value conditions, M is the total number of nodes, and d_{max} is the maximum degree in the network. The term d_{max}^k in the Laplace smoothing denominator serves as an approximation for the total number of possible degree-tuple categories.

Through this method, we can systematically analyse temporal irreversibility at different levels: $k = 1$ corresponds to the dynamical characteristics of individual modes, $k = 2$ reflects interactions between mode pairs, and higher order k reveals complex collective behavioural patterns. Strong interactions between modes typically manifest as higher irreversibility values, providing crucial insights for identifying key coupling structures in the system.

3.3. High-order unique contribution measurement

In urban transportation networks under extreme weather conditions, the interactions among different modes (cars, buses, cycles, and walks) exhibit complex nonlinear and non-equilibrium characteristics. These interactions manifest not only in individual modes or simple pairwise combinations but also through higher-order couplings (e.g., three-mode or four-mode interactions), leading to a multi-scale structure of system irreversibility. To systematically analyse these complex interactions, we propose a comparative framework distinguishing between combined irreversibility (ζ), which is the total measured effect, and unique irreversibility (η), which isolates the genuine, emergent higher-order contribution.

3.3.1. Combined irreversibility and the need for unique contribution

Consider a k-order interaction among transportation modes $(m_1,\dots, m_k)$, where m_i represents specific modes corresponding to the general variables xi. Its combined irreversibility $\zeta^{(m_1,\dots,m_k)}$ captures the total non-equilibrium dynamics including all possible sub-interactions:

$$\zeta^{(m_1,\dots,m_k)} = \sum_{\Gamma^{(m_1,\dots,m_k)}} P(\Gamma^{(m_1,\dots,m_k)})\log\frac{P(\Gamma^{(m_1,\dots,m_k)})}{P(\Gamma^{(m_1,\dots,m_k)})} \tag{14}$$

However, while the combined irreversibility provides insights into overall system behaviour, it inherently includes the effects of lower-order interactions. This can obscure genuine higher-order interactions, which are critical for understanding system adaptation under stress. To isolate these higher-order contributions, we define the unique irreversibility $\eta^{(m_1,\dots,m_k)}$ by recursively removing all lower-order contributions based on Lynn's decomposition approach [38,68]:

$$\eta^{(m_1,\dots,m_k)} = \zeta^{(m_1,\dots,m_k)} - \sum_{\Omega\subset\{m_1,\dots,m_k\}} \eta^{\Omega} \tag{15}$$

Where ζ is the combined irreversibility of the k-mode system, and the summation term removes the unique irreversibility (η) of all proper subsets Ω. To illustrate how this recursive decomposition works, consider the calculation for the unique third-order irreversibility η (car, bus, cycle). The process proceeds hierarchically:

1) **Base Case ($k = 1$):** First, the unique irreversibility of each individual mode is equal to its combined irreversibility: $\eta(car) = \zeta(car)$, and so on for the other modes.
2) **Second-Order Calculation ($k = 2$):** Next, the unique pairwise irreversibility are calculated by subtracting the individual contributions from the combined values, for example: $\eta(car, bus) = \zeta(car, bus) - [\eta(car) + \eta(bus)]$. This is repeated for all pairs.
3) **Third-Order Calculation ($k = 3$):** Finally, the unique third-order interaction is isolated by subtracting all lower-order unique contributions from the combined third-order value: $\eta(car, bus, cycle) = \zeta(car,bus,cycle) - [\eta(car,bus) + \eta(car,cycle) + \eta(bus,cycle) + \eta(car) + \eta(bus) + \eta(cycle)]$. The value that remains represents the emergent effect from the simultaneous interaction of all three modes.

This decomposition framework is validated by its behaviour under the condition of statistical independence. As stated in Eq. (15), if two modes m_i and m_j are independent, their combined irreversibility is simply additive:

$$\varsigma^{(m_i,m_j)} = \eta^{(m_i)} + \eta^{(m_j)} \text{and } \eta^{(m_i,m_j)} = 0 \tag{16}$$

The result $\eta^{(m_i,m_j)} = 0$ is a direct mathematical consequence of the decomposition formula. This provides a crucial benchmark: a non-zero value for η is a rigorous indicator of a genuine system interaction beyond the sum of its parts.

3.3.2. Implications for urban transportation networks

Applying this framework to multi-modal transportation systems reveals critical insights into the hierarchical organisation of modal interactions. For example, in analysing the interaction among cars (c), buses (b), and walks (w), the unique irreversibility can be expressed as:

$$\eta^{(c,b,w)} = \zeta^{(c,b,w)} - \left[\eta^{(c,b)} + \eta^{(b,w)} + \eta^{(c,w)}\right] - \left[\eta^{(c)} + \eta^{(b)} + \eta^{(w)}\right] \tag{17}$$

The distinction between unique and combined measurements is particularly crucial for understanding system adaptation to extreme weather conditions. A high unique irreversibility indicates the emergence of genuine higher-order interactions that cannot be reduced to simpler combinations, manifesting as synchronised adaptations across multiple transportation modes. Conversely, when the unique irreversibility is low despite high combined irreversibility, it suggests that the observed complexity primarily stems from the superposition of lower-order interactions.

Through this dual measurement approach, we can uncover the fundamental structure of multi-modal dependencies during extreme weather events. The comparison between unique and combined measurements enables us to identify critical higher-order interactions, understand the hierarchical organisation of system adaptations, and provide quantitative insights for urban resilience planning and emergency response strategies. This deeper understanding of multi-modal interactions under extreme conditions is essential for developing effective, integrated management approaches that account for the complex interdependencies between different transportation modes.

3.4. Validation against conventional traffic performance metrics

To validate the practical relevance of our MmVGI framework, we examine the relationship between irreversibility measurements and conventional traffic performance indicators. We correlate our irreversibility values across different orders (K1, K2, K3) with four established traffic performance metrics: congestion duration, mode switching rate, travel time index, and recovery time.

Congestion Duration [126,127] quantifies the number of hours during which traffic flow exceeds normal operating conditions. For each road segment, we define the baseline flow as the average of flows during early morning hours (0:00–5:00), representing uncongested conditions. The congestion threshold is set as the maximum of either 1.5 times the baseline flow or the daily average flow:

$$\text{Congestion Duration} = \sum_{h=0}^{23} 1[\text{Flow}_h > \max(1.5 \times \text{Baseline}, \text{Daily Average})] \tag{18}$$

where $1[\cdot]$ is the indicator function and h represents hourly time steps.

Mode Switching Rate (Coefficient of Variation, a standard statistical measure of relative variability) captures the temporal variability in modal composition, reflecting traveller adaptation during extreme events. We calculate the coefficient of variation for each mode's share of total hourly traffic:

$$\text{Mode Switching Rate} = \frac{1}{4} \sum_{m \in \{car, bus, cycle, walk\}} \frac{\sigma(S_m)}{\mu(S_m)} \tag{19}$$

where $S_m = \{s_{m,h}\}_{h=0}^{23}$ represents the hourly modal shares for mode m, with $s_{m,h} = \frac{\text{Flow}_{m,h}}{\sum_{m'} \text{Flow}_{m',h}}$, $\sigma(\cdot)$and $\mu(\cdot)$ denote standard deviation and mean respectively.

Travel Time Index [128,129] provides a proxy for congestion severity based on the flow-capacity relationship. We estimate road capacity as the 85th percentile of daily flows and calculate the ratio of hourly flows to this capacity:

$$\text{Travel Time Index} = \frac{1}{24} \sum_{h=0}^{23} min\left(\frac{\text{Flow}_h}{\text{Capacity}}, 3.0\right) \tag{20}$$

where capacity is estimated as Capacity $= P_{85}(\{\text{Flow}_h\}_{h=0}^{23})$ and the ratio is capped at 3.0 to represent extreme congestion conditions.

Recovery Time [130,131] measures the system's ability to return to normal operations after peak disruption. We identify the peak flow hour and calculate the time required for flows to return to near-baseline conditions:

$$\text{Recovery Time} = \min\{t : \text{Flow}_{h^*+t} \le 1.2 \times \text{Baseline}\} \tag{21}$$

where $h^* = \arg\max_h \text{Flow}_h$ represents the peak flow hour, and recovery is defined as returning to within 20 % of baseline flow levels.

These metrics provide comprehensive coverage of transportation system performance aspects: operational efficiency (congestion duration), adaptive behaviours (mode switching), service quality (travel time index), and resilience (recovery time). All metrics are calculated using the same 24-hour traffic flow data employed in the MmVGI analysis, ensuring consistency in the validation framework.

4. Case study

4.1. Data description

Our case study focuses on the City of London, one of the most crucial districts in Greater London, covering an area of 12.727 km^2 (As shown in Fig. 4a, b). This area represents a dense urban environment with complex multi-modal transportation interactions, containing 5200 road segments (Fig. 4c) in the analysed network. The study utilizes comprehensive traffic data provided by University College London's Department of Civil, Environmental and Geomatic Engineering [132], with all spatial data referenced in the British National Grid coordinate system (EPSG:27700).

The traffic dataset encompasses hourly flow measurements across four primary transportation modes: buses, cars, cycle, and walks. For each road segment, the data structure incorporates unique road identification numbers, road classification (Fig. 4d), directional information (Fig. 4e), and hourly traffic flow counts for each mode(car, bus, cycle and walk, as shown in Fig. 4f). This dataset allows us to examine the intricate interactions between different transportation modes across the urban network. Based on this dataset, we selected October 3, 2020, as our primary study period, which represents a significant extreme weather event in London's recent history. According to the UK Met Office [133], this date recorded the highest daily rainfall (31.7 mm area-average) in the UK since records began in 1891. This extreme event provides an ideal case for examining how multi-modal transportation systems respond to and recover from severe weather disruptions.

4.2. Results analysis

4.2.1. Multi-modal irreversibility analysis

Building on the comprehensive dataset, we conducted a detailed analysis of transportation system behaviour across different road hierarchies. Our examination focused particularly on how various transportation modes interact and adapt under extreme raining condition, revealing distinct patterns in both temporal evolution and network structure. Our analysis of traffic patterns across different road hierarchies reveals distinct characteristics in both temporal evolution and network structure. As shown in Fig. 5, we examined representative segments from A Roads, B Roads, and Minor Roads, analysing both their temporal flow patterns and the resulting visibility graph structures. **A Roads (Segment 1)** demonstrate distinct characteristics across all modes. The car flow patterns show smooth, high-volume temporal evolution throughout the day, resulting in densely connected visibility graphs with numerous node interactions. Bus flows exhibit similar stability but with more regular patterns, reflected in their structured visibility graph organisation. Cycle and walk modes show lower volumes but maintain consistent patterns, represented by more sparse but well-organised network structures. These patterns contribute to high irreversibility values that increase from $k = 1$ (0.1037) to $k = 4$ (0.8155), indicating strong multi-modal coordination. **B Roads (Segment 2)** reveal more variable behaviour. The temporal sequences show noticeable fluctuations, particularly evident in car and bus flows. Their visibility graphs display moderate connectivity, with car networks showing scattered clusters and bus networks maintaining some regular structures. Cycle and walk networks exhibit more flexible patterns, reflected in their looser network organisation. This variability is captured in the progression of irreversibility values from $k = 1$ (0.0599) to $k = 4$ (0.7998). **Minor Roads (Segment 3)** exhibit the most dynamic patterns. Their temporal sequences show pronounced local variations across all modes, particularly evident in the cycle and walk patterns. The resulting visibility graphs are notably different: car networks show sparse connections, bus networks display limited structure, while cycle and walk networks reveal localised clustering. This complex behaviour is reflected in their irreversibility measurements, which progress from relatively low first-order values (0.0892) to substantial higher-order values (0.8176).

This hierarchical analysis reveals the complex relationship between road hierarchy and multi-modal interactions during extreme conditions, demonstrating clear transitions from highly structured, integrated patterns on A Roads to more flexible, localised behaviours on Minor Roads.

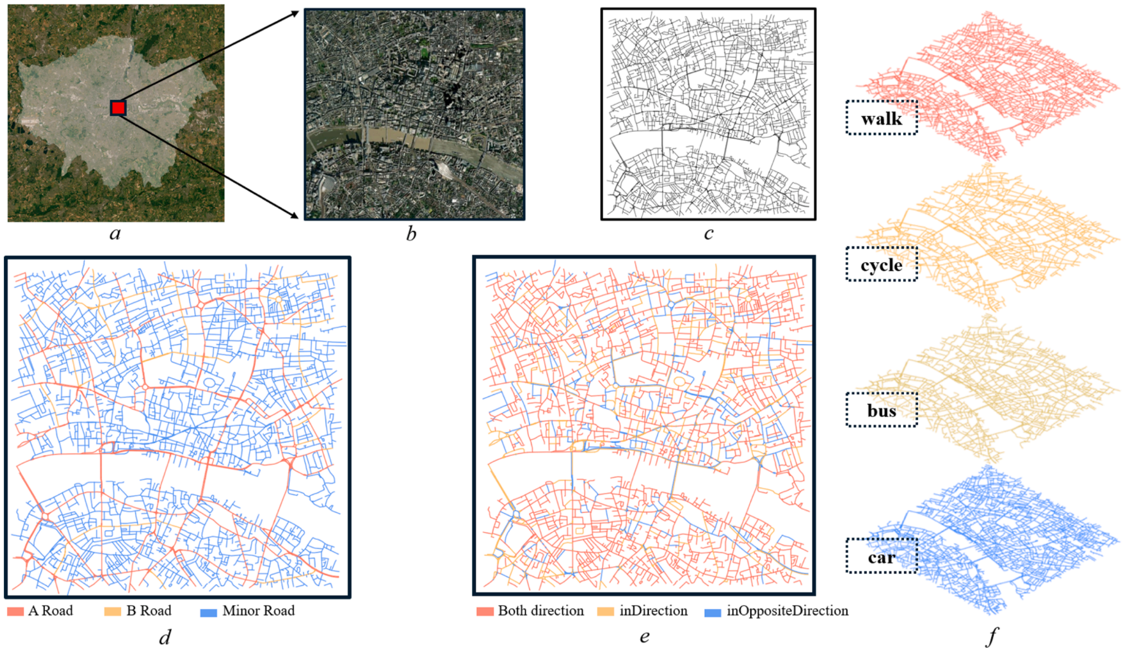

Fig. 4. Multi-modal Transportation Network Structure in the City of London. (a) shows the location of the City of London within Greater London. (b) providing a detailed satellite view of this chosen area. (c) presents the complete road network segments. (d) categorizes the road network by hierarchy, distinguishing among A Roads, B Roads, and Minor Roads. (e) displays the directional attributes of road segments, indicating bidirectional flows, single-direction flows, and opposing direction flows. (f) decomposes the network by transportation mode, showing the distinct spatial distributions of walk, cycle, bus, and car traffic flows.

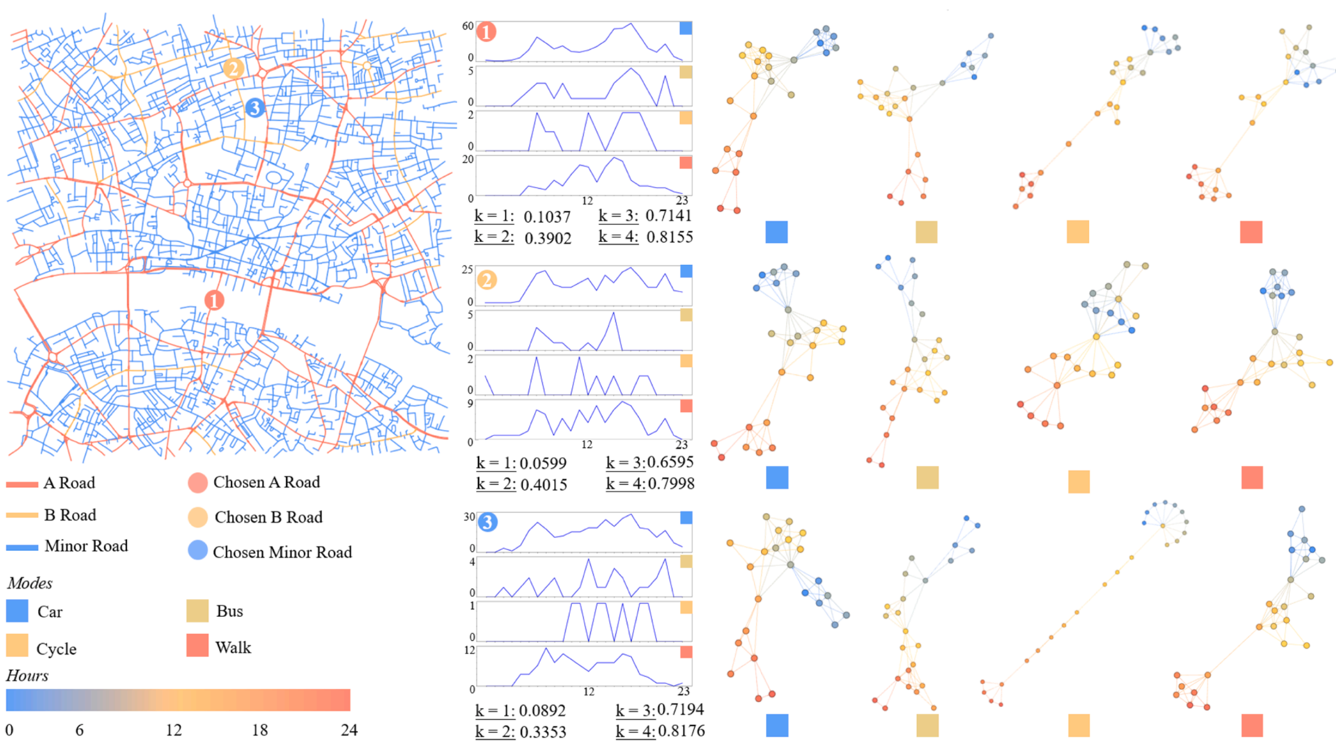

Fig. 5. Hierarchical Analysis of Multi-modal Traffic Patterns Through Network Structure, Temporal Evolution, and Visibility Graphs. 1) The left panel displays the urban road network, with hierarchical classification into A Roads (red), B Roads (orange), and Minor Roads (blue), highlighting three representative segments (labelled 1–3) selected for detailed analysis. 2) The middle panel shows the 24-hour temporal evolution of traffic flows across four transportation modes (car, bus, cycle, and walk) for each selected segment, revealing distinct patterns of stability and variability across different road types. 3) The right panel illustrates the corresponding visibility graph structures for each transportation mode, organised in columns from left to right (car, bus, cycle, walk), demonstrating how temporal patterns translate into network representations. Each road segment's analysis is accompanied by irreversibility measurements across multiple orders ($k = 1$ to $k = 4$), quantifying the complexity of modal interactions.

The progression of visibility graph structures and irreversibility values provides quantitative evidence for these hierarchical differences in network behaviour.

To better understand the higher-order interaction patterns within our selected network area, we conducted a comprehensive statistical analysis, as presented in Fig. 6. The visualisation employs a colour gradient scheme, where lighter shades indicate stronger irreversibility in modal combinations, while darker shades represent more reversible interactions. Each modal combination is represented by icons on the x-axis, with reference to Fig. 6a) showing the four primary modes: cars, buses, cycles, and walks. Our analysis across multiple orders ($k = 1,2,3$) to uncover systematic relationships between different transportation modes.

Single-mode analysis ($\boldsymbol{k = 1}$, Fig. 6b) revealed distinctive behavioural patterns across different transportation modes. Motorised modes, particularly cars and buses, demonstrated moderate irreversibility values, reflecting their operational stability and structured service patterns. This stability appears particularly pronounced in bus services, likely due to their scheduled operations and fixed routes. In contrast, walk flows exhibited the highest irreversibility, suggesting remarkable adaptability to changing conditions. Notably, cycling patterns showed the lowest irreversibility, indicating well-distributed flow patterns that

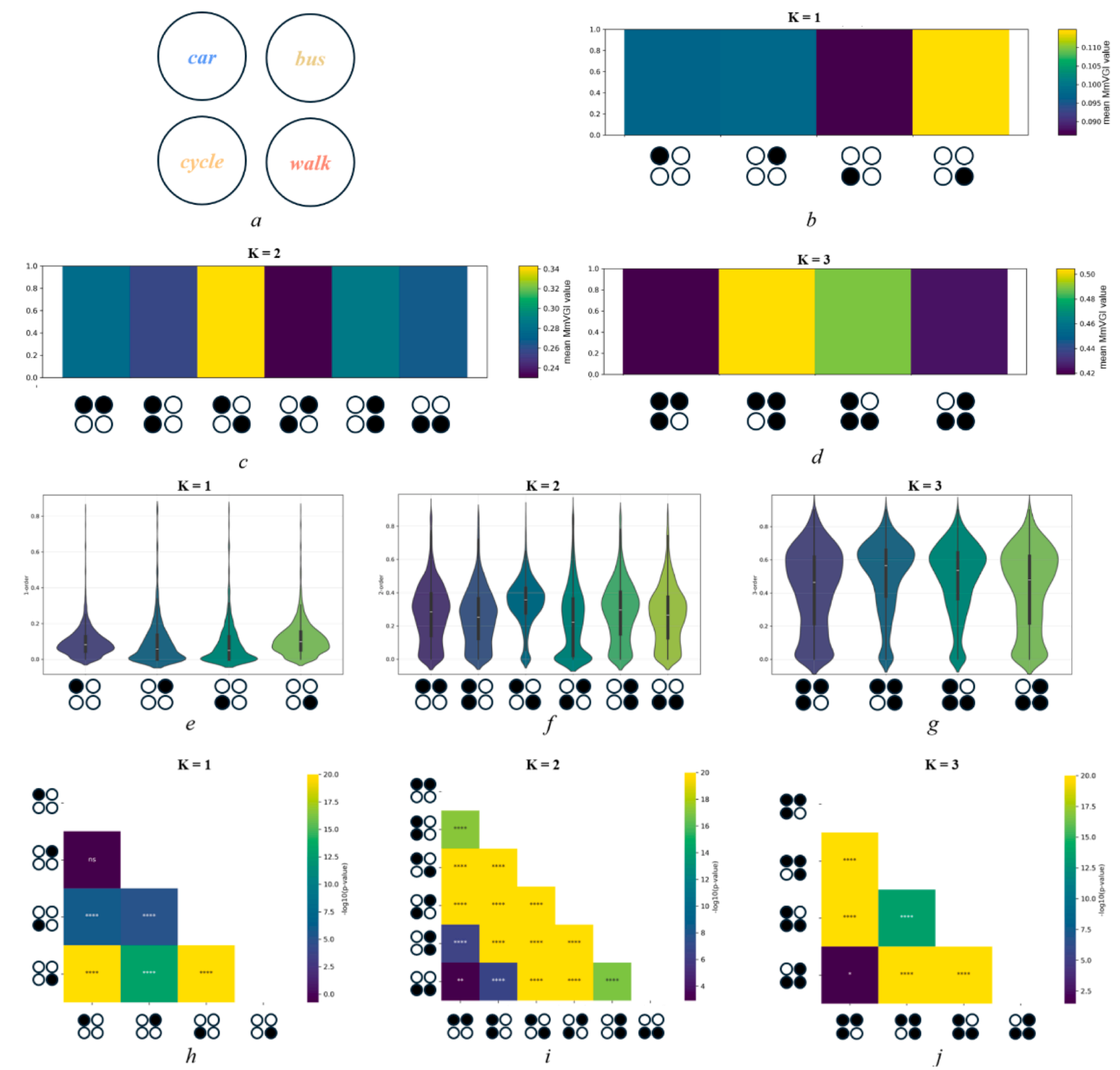

Fig. 6. Multi-order Analysis of Modal Interactions through Irreversibility Measurements and Statistical Testing. (a) introduces the four primary transportation modes analysed: car, bus, cycle, and walk. (b), (c), and (d) display the irreversibility values for different modal combinations at first, second, and third order respectively, using a colour gradient where lighter shades indicate stronger irreversibility. (e), (f), and (g) provide violin plots showing the distribution characteristics of irreversibility values across different modal combinations. (h), (i),(j) presents statistical significance testing results through heatmaps, where colour intensity and asterisk notation (*) indicate the level of statistical significance between different modal combinations. The systematic progression from $k = 1$ to $k = 3$ demonstrates the evolution from simple single-mode characteristics to complex higher-order interactions.

maintain balance across the network. The examination of **pairwise interactions (k = 2,** Fig. 6c**)** highlighted strong coupling between motorised transportation modes. Car-bus combinations displayed particularly high irreversibility values, indicating synchronised traffic flow patterns, especially along major transportation corridors. Interactions between vehicular modes and walks also showed significant irreversibility, suggesting adaptive complementarity where walks flow adjust to accommodate vehicular traffic during adverse weather conditions. Cycling-related pairs, however, maintained lower irreversibility values, indicating more independent behavioural patterns. **The third-order analysis (k = 3,** Fig. 6d**)** revealed sophisticated multi-modal interactions, with the car-bus-walk combination exhibiting notably high irreversibility. This finding suggests the presence of complex adaptation mechanisms during extreme weather events, where these three modes demonstrate coordinated behavioural adjustments. The emergence of such pronounced higher-order interactions emphasizes that transportation system adaptation involves complex multi-modal dependencies that cannot be fully captured through simpler single-mode or pairwise analyses.

Statistical analysis confirms systematic variations in irreversibility across different modal combinations, with evidence drawn from both distribution patterns and significance testing. The violin plots (Fig. 6e-g) reveal the evolving complexity of modal interactions across different orders. At $k = 1$, the distributions show relatively concentrated patterns with distinct medians for each mode, particularly highlighting the contrast between motorised and non-motorised transportation. As we move to $k = 2$ and $k = 3$, the distributions demonstrate increasing spread and complexity, indicating more sophisticated interaction patterns at higher orders. The significance testing results (Fig. 6h-j) provide statistical validation of these observed patterns. At the single-mode level (panel h), the heatmap reveals significant differences between most modal comparisons (indicated by **** and bright colours), though some comparisons, particularly involving bus modes, show no significant differences (ns). This aligns with the concentrated distributions seen in the $k = 1$ violin plot. The pairwise analysis (panel i) demonstrates even more pronounced differences, with most combinations showing high statistical significance (****), particularly in car-bus interactions. This corresponds to the broader distributions observed in the $k = 2$ violin plot, reflecting the emergence of complex pairwise dynamics. Most notably, the $k = 3$ analysis (panels g and j) reveals both the highest variability in distributions and the strongest statistical differences among modal combinations. The violin plot shows distinct spreading patterns for different triple-mode combinations, while the significance heatmap confirms these differences are highly significant (****), especially for combinations involving car-bus-walk interactions. This dual evidence strongly supports the emergence of genuine higher-order interactions in the transportation system during extreme weather events. These statistical results reinforce the importance of considering multi-order interactions in transportation system analysis. They provide quantitative evidence for the hierarchical nature of urban mobility patterns, demonstrating that modal interactions become increasingly complex and statistically distinct at higher orders.

These statistical patterns reflect specific transportation mechanisms during extreme weather events. The high irreversibility observed in car-bus combinations indicates synchronized traffic flow disruptions, where congestion in one mode immediately affects the other due to shared infrastructure constraints. The strong car-bus-walk third-order interactions suggest the emergence of complex adaptation mechanisms where all three modes must coordinate their use of limited road space during adverse conditions. The distinctive behaviour of cycling, showing lower pairwise irreversibility but significant influence on network dynamics, reflects its unique adaptive capacity. Unlike motorized modes, cycling can rapidly shift between different types of infrastructure (roads, cycle lanes, sidewalks) during extreme weather, creating flow redistributions that affect the broader transportation system. Walk flows exhibit the highest individual irreversibility, indicating their role as the most adaptable mode during extreme conditions. Pedestrians can modify routes, timing, and destinations more flexibly than other modes, leading to highly variable temporal patterns that contribute significantly to overall system dynamics.

These mechanisms are not just theoretical; they are directly reflected in the observable transportation behaviours documented in our case study. The temporal sequences in Fig. 5 demonstrate how different modes exhibit distinct adaptation characteristics: A Road segments show coordinated but stressed patterns across all modes, reflecting travellers constrained choices on critical corridors. B Road segments display more variable temporal patterns, particularly in cycling and walking, indicating travellers' ability to exercise greater route and timing flexibility on secondary networks. The progression from individual mode behaviour (K1) to complex multi-modal interactions (K3) captures the evolution from independent traveller decisions to coordinated system-wide adaptations during the extreme weather event.

4.2.2. Spatial pattern analysis of multi-modal interactions

Our spatial analysis of the urban transportation network reveals distinct patterns of irreversibility across different modal combinations, offering deeper insights into how various transportation modes influence network dynamics during extreme weather conditions (Fig. 7). The baseline analysis of single-mode patterns, focusing on vehicular traffic (car mode), shows pronounced irreversibility along A Roads, with MmVGi values reaching 0.8347. This distribution highlights the fundamental structure of urban mobility, where A Roads exhibit high dynamic complexity due to concentrated vehicular flow. In contrast, B Roads and Minor Roads display relatively lower irreversibility, reflecting more stable traffic patterns.

When examining dual-mode interactions, the car-bus combination demonstrates high irreversibility values (0.8907) along A Roads, indicating strong coupled dynamics between these motorised modes. The comparison values suggest that buses moderate the high irreversibility observed in car-only scenarios, particularly along A Roads. This moderation effect is visible through the transition from red to yellow-green patterns in many arterial segments.

The introduction of cycling into the network (car-cycle) shows an irreversibility value of 0.9009, with distinct patterns emerging especially on B Roads and Minor Roads. The comparison maps reveal extensive yellow-green regions in previously low-irreversibility areas, suggesting that cycling significantly influences network dynamics at these scales. This indicates that cycling serves as a redistributive force in the network, particularly where motorised traffic is less dominant.

Pedestrian interactions, as shown in the car-walk combination (0.9149), produce more subtle effects. Unlike cycling, which reshapes B Roads and Minor Roads on a larger scale, pedestrian dynamics primarily affect localised sections of Minor Roads. This is evidenced by scattered yellow regions in these areas, indicating isolated rather than network-wide impacts.

The analysis of higher-order combinations provides additional insights. The transition from car-bus-cycle to car-bus-cycle-walk configurations shows minimal additional changes in irreversibility patterns. This suggests that pedestrian flows integrate into existing modal patterns without introducing substantial new dynamic structures. In contrast, the comparison between car-bus and car-bus-cycle patterns reveals significant reorganisation of network dynamics, particularly across B Roads and Minor Roads.

These spatial patterns demonstrate that transportation system adaptation during extreme weather events operates differently across road hierarchies, revealing underlying transportation mechanisms operating at different scales. The concentration of high irreversibility on A Roads reflects their role as critical infrastructure that cannot be bypassed during extreme events, forcing them to operate under stressed conditions where motorised modes dominate the dynamics. In contrast, cycling emerges as a significant influence on B Roads and Minor Roads, demonstrating how non-motorized modes can significantly affect

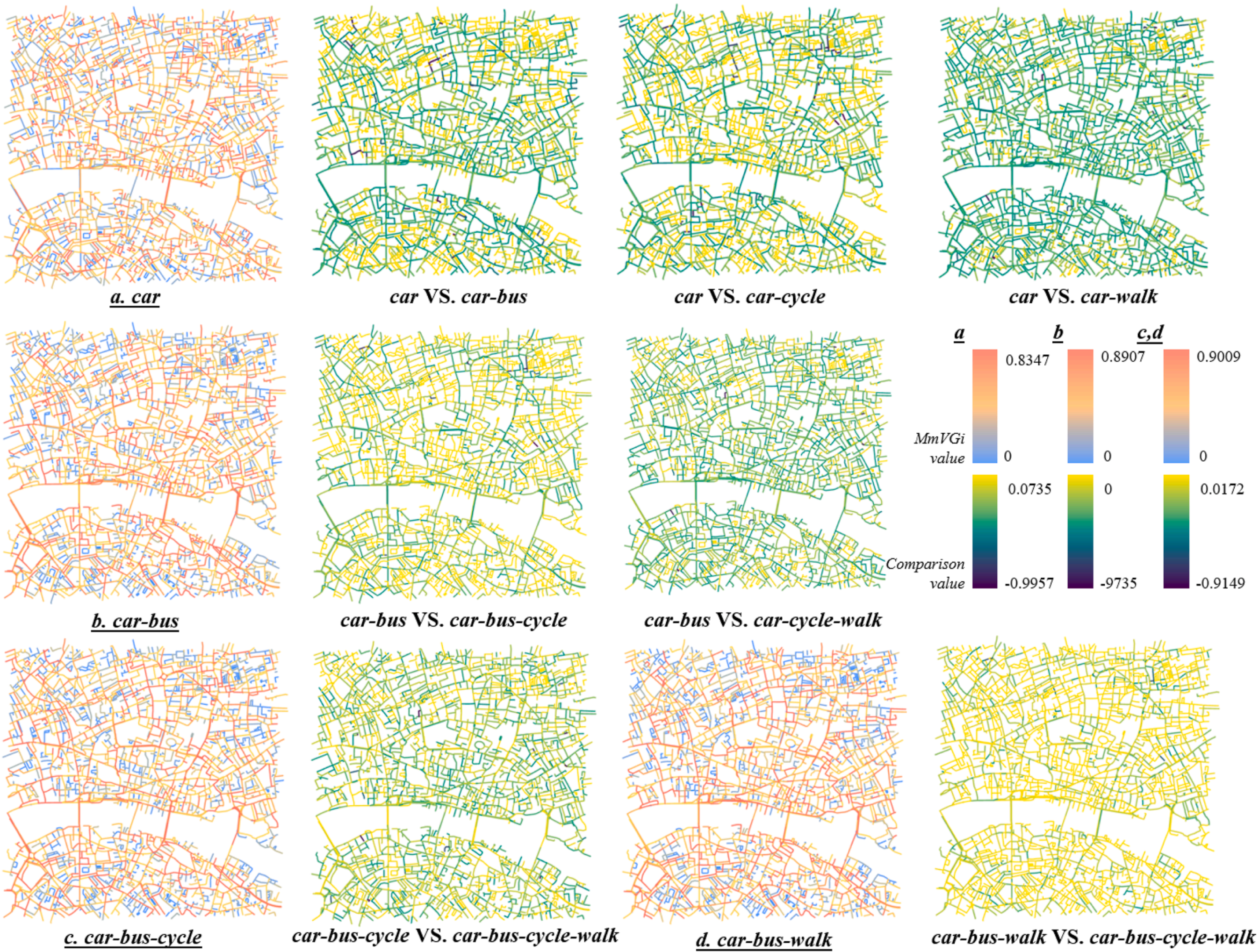

Fig. 7. Spatial Distribution and Comparison of Multi-modal Irreversibility Patterns in Urban Transportation Networks. (a)shows the progression from single-mode analysis (car) to dual-mode comparisons (car versus car-bus, car-cycle, and car-walk), revealing how the addition of different modes influences network behaviour. (b) illustrates the evolution of more complex modal combinations, focusing on car-bus interactions and their relationships with other modes. (c)(d) demonstrates higher-order modal interactions, particularly highlighting the car-bus-cycle and car-bus-walk combinations.

network dynamics through their adaptive route selection behaviour during adverse conditions. This cycling influence introduces dynamic variability and reshapes local irreversibility patterns, creating a hierarchical system where different modes dominate different spatial scales. Understanding these hierarchical patterns is crucial for developing targeted resilience strategies that account for both primary corridor performance under stress and the adaptive flexibility that characterizes local road network dynamics.

4.2.3. Unique measurement compares with combined measurement

In urban transportation networks, the complex interactions between different modes during extreme weather conditions are crucial for understanding system resilience and management. These interactions exhibit distinctive nonlinear and non-equilibrium characteristics that extend beyond what can be captured through traditional single-mode or pairwise analyses. The relationships between motorised transport (cars and buses) and non-motorised modes (cycling and walking) become particularly intricate during adverse weather conditions, necessitating a deeper understanding of their higher-order interactions.

Our analysis reveals an important relationship between the combined and unique irreversibility patterns across both second and third-order interactions (Fig. 8). When examining normalised irreversibility values, we observe a consistent correlation between unique and combined measurements, suggesting that the relative strength of modal interactions is preserved across both measurement approaches. At the second order ($k = 2$), the correlation coefficient between unique and combined measurements reaches $r = 0.966$, indicating a remarkably strong positive relationship. This high correlation suggests that modal combinations showing stronger combined irreversibility also tend to exhibit proportionally higher unique contributions. This pattern reveals that the relative importance of different modal pairs is consistently reflected in both their overall behaviour and their genuine pairwise interactions. The pattern continues at the third order ($k = 3$), maintaining a strong correlation ($r = 0.946$), though slightly lower than the second-order relationship. This consistent pattern suggests that even as interactions become more complex, the relative strength of different modal combinations remains stable across measurement approaches.

The preservation of these strong correlations across different orders reveals a crucial characteristic of multi-modal transportation systems: the presence of genuine higher-order coupling effects that cannot be reduced to simple combinations of individual modes. These higher-order interactions represent emergent system properties rather than mere accumulations of lower-order effects. The consistent relationship between unique and combined measurements demonstrates that the system exhibits coherent, non-linear coupling patterns that emerge specifically at higher orders.

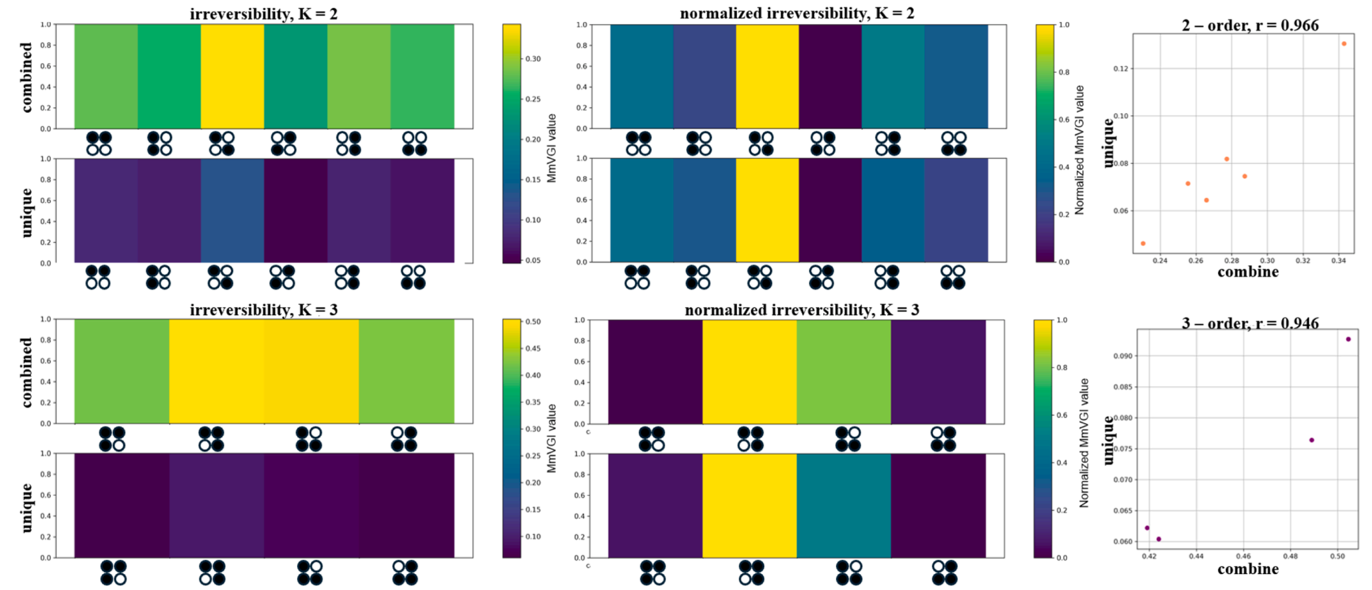

Fig. 8. Comparison of Combined and Unique Irreversibility Measurements Across Modal Combinations. (a) display both combined and unique irreversibility values for second-order ($K = 2$) and third-order ($K = 3$) interactions, using color gradients to represent the strength of irreversibility. (b) show these same measurements after normalisation, providing a standardised comparison across different modal combinations. (c) illustrate the strong correlations between combined and unique measurements.

4.3. Correlation analysis results

We correlate our irreversibility values across different orders (K1, K2, K3) with these four-performance metrics across all 5200 road segments in our study area. Our results reveal a systematic strengthening of correlations as we progress from individual modes (K1) to higher-order interactions (K3), as shown in Fig. 9. At the single-mode level (K1), correlations are moderate but significant, ranging from $r = 0.194$ for recovery time to $r = 0.522$ for mode switching rate. The strongest correlation at K1 is observed with mode switching rate ($r = 0.522$, $p < 0.001$), suggesting that even individual modal irreversibility captures adaptive behavioural responses during extreme events. Moving to pairwise interactions (K2), correlations strengthen substantially across all performance metrics. Travel time index shows the most dramatic improvement, increasing from $r = 0.338$ at K1 to $r = 0.633$ at K2 ($p < 0.001$). Congestion duration correlation nearly doubles from $r = 0.321$ to $r = 0.606$, while recovery time correlation more than doubles from $r = 0.194$ to $r = 0.442$. These improvements indicate that pairwise modal interactions provide significantly better predictive power for conventional traffic performance measures than individual modal analysis. The highest correlations emerge at the third-order level (K3), where travel time index reaches $r = 0.732$ ($p < 0.001$) and congestion duration achieves $r = 0.697$ ($p < 0.001$). Recovery time correlation continues to strengthen to $r = 0.516$, demonstrating that higher-order modal interactions are most closely associated with system recovery characteristics. Interestingly, mode switching rate correlation shows a different pattern, peaking at K1 and declining at higher orders, suggesting that individual modal adaptations are the primary drivers of mode choice changes during extreme events.

The systematic increase in correlations from K1 to K3 provides strong empirical evidence that higher-order irreversibility measurements capture increasingly sophisticated aspects of transportation system performance that are not apparent from traditional single-mode or simple multi-modal analyses. The particularly strong correlations with travel time index and congestion duration at higher orders ($r > 0.6$) demonstrate that our irreversibility framework effectively quantifies the complex coordination challenges that emerge during extreme weather events. The distinct correlation pattern observed for mode switching rate—where individual modal irreversibility (K1) shows the strongest relationship—reveals important insights into traveller adaptation mechanisms. This suggests that mode choice decisions are primarily driven by individual modal performance degradation rather than complex multi-modal interactions, providing valuable guidance for transportation demand management strategies during extreme events. The progressive strengthening of correlations with recovery time across orders (from $r = 0.194$ to $r = 0.516$) indicates that system recovery is fundamentally a multi-modal phenomenon requiring coordination across transportation modes. This finding supports the theoretical framework's emphasis on higher-order interactions as critical factors in transportation system resilience. All correlations achieve high statistical significance ($p < 0.001$) across the complete dataset of 5200 road segments, providing robust evidence for the relationships between irreversibility measurements and conventional performance metrics. The large sample size and comprehensive network coverage ensure that these findings are representative of urban transportation system behaviours during extreme weather conditions.

These validation results demonstrate that the MmVGI framework captures meaningful patterns of system performance that are both theoretically grounded and practically relevant for transportation planning and management applications, directly addressing the reviewer's concern about the practical utility of irreversibility measurements in transportation contexts.

5. Discussion

Our analysis of multi-modal transportation networks during extreme weather conditions reveals several significant findings about system adaptation and modal interactions across different spatial scales. The results provide important insights into both theoretical understanding of transportation system dynamics and practical implications for urban resilience planning.

5.1. Findings

5.1.1. Road hierarchies and modal interactions in transportation systems

The distribution of irreversibility across different road hierarchies demonstrates distinct patterns of modal interactions. A Roads consistently exhibit high irreversibility values across all orders of analysis,

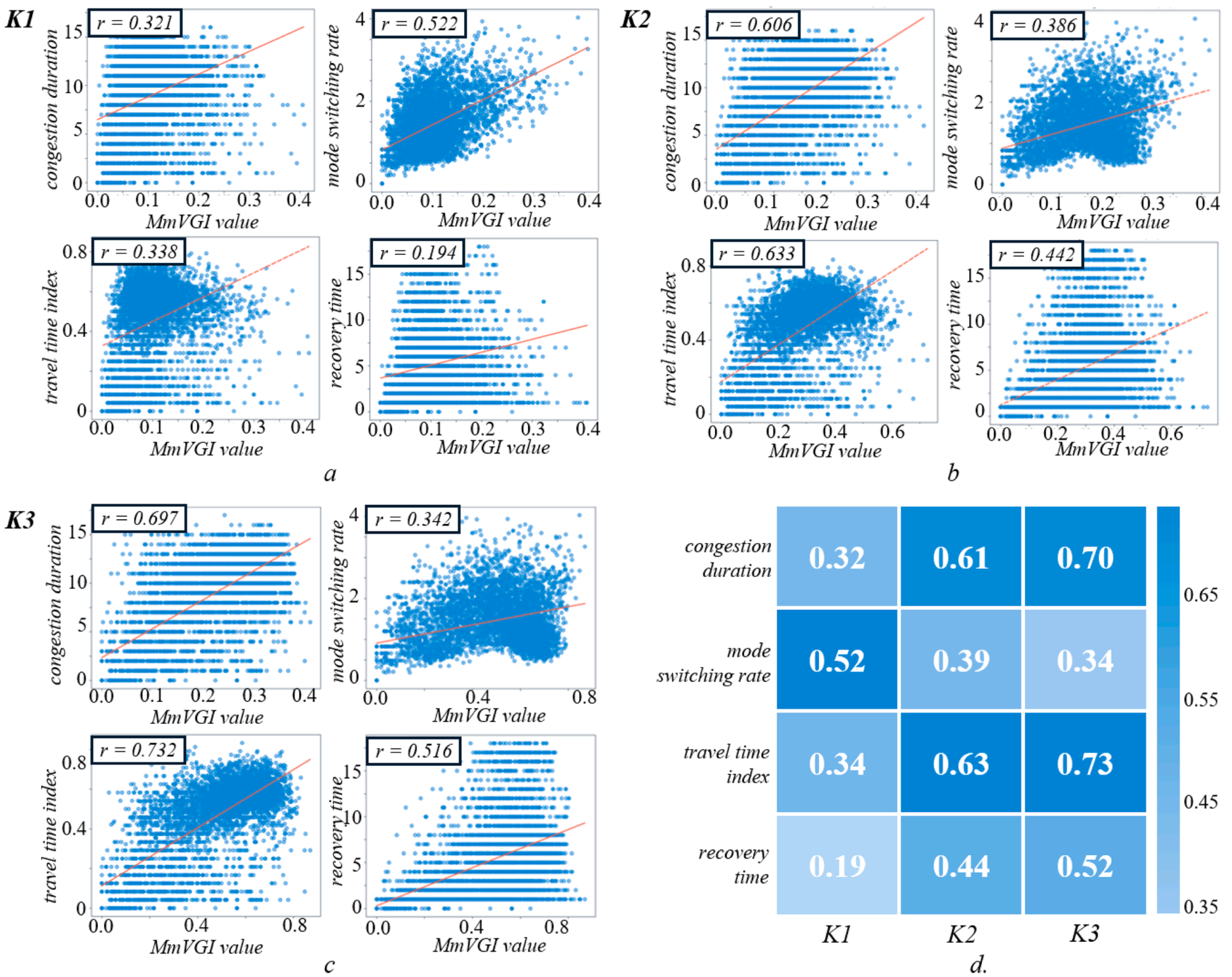

Fig. 9. Validation of MmVGI Framework Against Conventional Traffic Performance Metrics. (a) K1 Scatter Plots displaying correlations for individual mode analysis. (b) K2 Scatter Plots showing pairwise modal interaction correlations. (c) K3 Scatter Plots demonstrating three-mode interaction correlations. Red lines indicate linear trends with correlation coefficients shown in each panel. Analysis covers 5200 road segments with all correlations statistically significant ($p < 0.001$). Progressive strengthening from K1 to K3 demonstrates superior predictive power of higher-order interactions. (d) Correlation Heatmaps showing relationship strength between MmVGI measurements and traffic performance indicators across K1, K2, and K3 interaction orders.

indicating their crucial role as network backbones during extreme conditions. This stability in primary corridors suggests that motorised transport modes (cars and buses) maintain relatively structured interactions even under stress. Particularly noteworthy is the significant influence of cycling on network dynamics, especially in B Roads and Minor Roads. The introduction of cycling leads to substantial reorganisation of local network patterns, evidenced by increased irreversibility values in these areas. These finding challenges traditional perspectives that primarily focus on motorised transport, highlighting cycling's crucial role in system adaptation. Pedestrian flows, while contributing to system dynamics, show more limited influence primarily confined to Minor Roads. This localised impact suggests that pedestrian modes serve as complementary rather than transformative elements in the broader network structure during extreme weather events.

5.1.2. *Dynamic adaptation across modal combinations*

The network's response to different modal combinations reveals a hierarchical pattern of adaptation. A Roads, dominated by car-bus interactions, maintain relatively stable dynamic patterns, suggesting robust infrastructure utilisation despite extreme conditions. In contrast, B Roads demonstrate notable non-linear responses, particularly following the introduction of cycling modes. This differentiated response indicates that secondary networks possess greater adaptive capacity through modal diversification. The limited contribution of pedestrian modes to system irreversibility, particularly in broad network dynamics, suggests a natural segregation of modal influences across spatial scales. This finding has important implications for understanding how different transportation modes contribute to system resilience during extreme events.

5.1.3. *Higher-order interactions and system behaviour*

The relationship between unique and combined contributions reveals fundamental characteristics of multi-modal interactions. The strong correlation between these measurements ($r = 0.966$ for $k = 2$ and $r = 0.946$ for $k = 3$) indicates that higher-order interactions represent genuine system properties rather than mere accumulations of lower-order effects. This finding provides empirical evidence for the emergence of complex adaptive behaviour in urban transportation systems. Particularly significant is the observation that three-mode combinations, such as car-bus-cycle, demonstrate irreversibility patterns that cannot be reduced to simpler modal interactions. This non-linear characteristic suggests that transportation system adaptation operates through sophisticated multi-modal mechanisms rather than simple additive effects.

5.2. Transportation mechanisms behind irreversibility patterns

5.2.1. Infrastructure competition and modal interdependency

The high irreversibility values observed in car-bus-walk combinations reflect fundamental transportation mechanisms operating during extreme weather events. When heavy rainfall reduces effective road capacity, these three modes enter into competitive interactions for limited usable infrastructure. Cars and buses compete directly for road space, while pedestrians seek alternative routes that may conflict with vehicular traffic. This competition creates cascading effects where disruption in one mode triggers adaptations in others. When car traffic becomes severely congested, passengers shift to bus services, increasing bus dwell times and reducing schedule reliability. Delayed buses subsequently force more travellers to walk, creating additional pedestrian-vehicle conflicts at intersections and crosswalks.

5.2.2. Differential recovery dynamics

The temporal asymmetries captured by our irreversibility measurements reflect the different recovery characteristics of each transportation mode. Cars can resume normal speeds relatively quickly once weather conditions improve and surface water drains. Bus services require longer recovery periods due to accumulated schedule delays and the need to redistribute passengers who concentrated at stops during disruptions. Pedestrian flows exhibit the most gradual recovery patterns, as route choice preferences normalize slowly and weather risk perceptions change over extended periods. These differential recovery rates create the prolonged temporal asymmetries that manifest as higher-order irreversibility in our measurements.

5.2.3. Spatial adaptation mechanisms

The prominence of cycling influences on B Roads and Minor Roads reflects specific spatial adaptation mechanisms. During extreme weather, cyclists possess unique route flexibility, utilizing alternative infrastructure types and creating temporary connections between normally separated route segments. This adaptive behaviour generates complex flow redistributions that significantly affect secondary network dynamics. A Roads maintain high irreversibility across all modal combinations because they serve as critical bottlenecks that cannot be easily bypassed during extreme events. These corridors must accommodate diverted traffic from compromised secondary routes while maintaining essential connectivity, operating in highly stressed conditions that generate the persistent non-equilibrium dynamics we observe

5.2.4. Behavioural validation through performance metrics

The transportation mechanisms identified through our irreversibility analysis are validated by their correlations with conventional traffic performance indicators (Fig. 9). The strengthening correlations from K1 ($r = 0.522$ for mode switching) to K3 ($r = 0.732$ for travel time index) demonstrate that higher-order irreversibility measurements capture increasingly complex behavioural adaptations that directly impact system performance. The strong correlation with mode switching rates at the individual level (K1) confirms that irreversibility captures travellers' adaptive mode choice behaviours during extreme weather. The progression to higher correlations with congestion duration and travel time index at higher orders (K2, K3) indicates that multi-modal interactions reflect coordinated behavioural responses that determine overall system performance during extreme events. These empirical relationships validate that our analytical framework captures the same transportation phenomena that operational practitioners monitor through conventional metrics, while providing additional insights into the behavioural coordination mechanisms that drive system-wide performance outcomes.

5.3. Contributions

The MmVGI framework introduced in this study offers several methodological advances in analysing multi-modal transportation systems. By combining visibility graph analysis with irreversibility measurements, our approach captures both global network patterns and localised modal interactions. This dual perspective provides a more comprehensive understanding of system dynamics than traditional methods focused on individual modes or simple flow analysis.

Our framework's ability to quantify both unique and combined contributions to system irreversibility represents a significant methodological innovation. This differentiation enables the identification of genuine higher-order interactions, providing a quantitative basis for understanding how different modal combinations contribute to system adaptation. The framework's application across spatial scales demonstrates its versatility in capturing both network-wide patterns and local dynamic responses.

In doing so, the MmVGI framework offers a new perspective that complements traditional network science metrics. While conventional metrics like betweenness centrality or clustering coefficients describe the static, topological structure of a network, our irreversibility measure quantifies its **dynamic, functional properties**. By focusing on the temporal asymmetries derived from information theory, our approach provides insights into how the network actually behaves and adapts under stress, revealing emergent properties that are invisible to purely structural analysis.

5.4. Practical applications and management implications

5.3.1. Mechanism-based network diagnostics

Understanding the transportation mechanisms behind irreversibility patterns enables more effective system diagnostics. High irreversibility hotspots indicate specific operational problems: car-bus interactions on A Roads suggest needs for transit priority systems during extreme weather, while high cycling influences on secondary roads indicate opportunities for weather-responsive infrastructure management. The spatial distribution of higher-order irreversibility serves as a diagnostic tool for identifying network vulnerabilities that are not apparent from traffic volume data alone. Areas showing strong three-mode interactions (car-bus-walk) require integrated management strategies that account for modal interdependencies rather than treating each mode independently.

5.3.2. Adaptive infrastructure management

The identified mechanisms suggest specific operational strategies. The strong car-bus coupling on A Roads indicates the need for coordinated management approaches, such as adaptive traffic signal systems that prioritize bus operations during extreme weather events. Dynamic lane allocation strategies could help manage the competition for road space that drives high irreversibility in these corridors. The significant cycling influence on B Roads and Minor Roads suggests implementing weather-responsive cycling infrastructure that can accommodate modal shifts during extreme conditions. This might include temporary route modifications or enhanced information systems that help cyclists adapt their travel patterns more effectively.

5.3.3. Recovery process optimization

Understanding the differential recovery dynamics of different modes enables targeted interventions to accelerate system restoration. Since bus services have longer recovery periods due to schedule coordination challenges, priority should be given to restoring transit operations and redistributing accumulated passenger loads. The gradual recovery patterns observed in pedestrian flows suggest the need for proactive information systems that help travellers adapt their route choices more efficiently as conditions improve. This could reduce the prolonged temporal asymmetries that contribute to extended system recovery periods.

5.5. Limitations and future research

Our study, while providing valuable insights into multi-modal transportation dynamics, faces several limitations that suggest promising directions for future research. A primary limitation lies in the scope of transportation modes analysed. While our framework successfully captures interactions between cars, buses, cycles, and pedestrians, urban mobility encompasses a broader spectrum of transportation modes. Modern cities increasingly feature diverse mobility options, including e-bikes, scooters, ride-sharing services, and various forms of micro-mobility. The exclusion of these emerging transportation modes, due to data availability constraints, potentially limits our understanding of the full complexity of urban mobility systems during extreme weather events. The temporal scope of our analysis presents another limitation, as we focus on a single extreme weather event. Transportation system behaviour might vary significantly across different types of weather conditions or during different seasons, potentially revealing additional patterns of modal interactions not captured in our current analysis. Furthermore, the computational complexity of analysing higher-order interactions (beyond $k = 4$) presents technical challenges that constrain our ability to explore more complex modal combinations.

Future research could address these limitations in several ways. Expanding the analysis to include emerging transportation modes would provide a more comprehensive understanding of urban mobility adaptation. For instance, incorporating e-bikes and micro-mobility services could reveal new patterns of system resilience, particularly in areas where traditional modes face limitations. This expanded analysis could help identify how different transportation modes complement each other during extreme conditions and inform the development of more robust mobility systems. Additionally, future studies should examine temporal variations in system behaviour across multiple extreme events and different seasonal conditions. This broader temporal perspective would help establish more robust patterns of adaptation and reveal how modal interactions evolve under varying environmental challenges. In addition to expanding the analysis to include emerging transportation modes and multiple weather events, several exciting methodological avenues are now open. One key direction is the development of new "irreversibility-based centrality" measures to identify nodes that are critical not just structurally, but dynamically. Another important step will be to integrate the MmVGI framework with traffic simulation models to test its predictive power, potentially creating early warning systems for network stress. Finally, the principles of this framework could inform new network optimisation problems that aim to design more resilient systems by minimising irreversibility under stress scenarios.

Furthermore, a promising theoretical direction for future research involves formally connecting our framework with the mathematical language of topological data analysis and advanced network theory. Conceptually, our analysis of k-order interactions is analogous to identifying and assigning a dynamic property—irreversibility—to the higher-order structures in a multilayer network (where each mode is a layer) or the hyperedges in a hypergraph. This perspective, related to the study of simplicial complexes, could yield deeper insights into the topological structure of multi-modal adaptation and bridge our non-equilibrium framework with other cutting-edge analytical tools.

5.6. Broader impacts

This research has significant implications for urban resilience planning in the context of climate change. The identification of hierarchical adaptation patterns suggests the need for differentiated resilience strategies across spatial scales. Understanding how different modal combinations contribute to system resilience could inform the development of more integrated multi-modal transportation systems.

The demonstrated importance of higher-order interactions contributes to our broader understanding of complex urban systems. This insight suggests that effective climate adaptation strategies must consider the emergent properties of multi-modal interactions rather than focusing on individual mode improvements in isolation.

In conclusion, our findings advance both theoretical understanding of non-equilibrium dynamics in urban systems and practical approaches to transportation system management by revealing the specific mechanisms through which different transportation modes interact and adapt under stress. The framework and insights developed here provide a foundation for future research into urban resilience and adaptation strategies in the face of increasing environmental challenges.

6. Conclusion

Transportation networks in urban environments face increasing challenges from extreme weather events, necessitating a deeper understanding of how different transportation modes interact and adapt under stress. Traditional approaches to analysing transportation system dynamics have primarily focused on individual modes or simple pairwise interactions, potentially overlooking critical higher-order relationships that emerge during extreme conditions. This study introduced the MmVGI framework to analyse the complex interactions between different transportation modes during extreme weather events. By examining the behaviour of cars, buses, cycles, and pedestrians across different road hierarchies, we revealed distinct patterns of system adaptation and modal cooperation that emerge under stress.

Our analysis yielded several key findings. First, we demonstrated that transportation system adaptation exhibits clear hierarchical patterns, with A Roads showing consistent high irreversibility dominated by motorised transport, while B Roads and Minor Roads display more complex patterns of modal interaction. Second, we identified cycling as a crucial component in system adaptation, particularly in secondary networks where it significantly influences local dynamics. Third, our comparison of unique and combined measurements revealed strong correlations, providing evidence for genuine higher-order interactions that cannot be reduced to simpler modal combinations. These findings have significant implications for urban transportation planning and management. The identification of distinct adaptation patterns across road hierarchies suggests the need for tailored resilience strategies that account for both network-wide stability and local dynamic responses. The demonstrated importance of cycling in secondary networks challenges traditional infrastructure planning approaches that prioritize motorised transport, suggesting the need for more balanced, multi-modal development strategies.

While our study was limited by the number of transportation modes analysed and its focus on a single extreme weather event, it establishes a foundation for understanding how urban mobility systems adapt to environmental challenges. Future research should expand this analysis to include emerging transportation modes and examine system behaviour across different types of extreme events, potentially revealing additional patterns of adaptation and resilience. This research contributes to both theoretical understanding of complex urban systems and practical approaches to transportation management. As cities face increasing environmental challenges, the insights and methodological framework developed here provide valuable tools for building more resilient, adaptive transportation systems that can maintain functionality during extreme weather events.

CRediT authorship contribution statement

Xuhui Lin: Writing – original draft, Visualization, Validation, Resources, Methodology, Conceptualization. **Long Chen:** Writing – review & editing, Validation, Supervision, Investigation. **Qiuchen Lu:** Writing – review & editing, Supervision, Methodology, Funding acquisition, Conceptualization. **Pengjun Zhao:** Validation, Supervision, Resources.

Tao Cheng: Writing – review & editing, Resources.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

References

[1] Aerts JCJH, Botzen WJ, Clarke KC, Cutter SL, Hall JW, Merz B, Michel-Kerjan E, Mysiak J, Surminski S, Kunreuther H. Integrating human behaviour dynamics into flood disaster risk assessment. Nature Clim Change 2018;8:193–9. https://doi.org/10.1038/s41558-018-0085-1.

[2] Othman A, El-Saoud WA, Habeebullah T, Shaaban F, Abotalib AZ. Risk assessment of flash flood and soil erosion impacts on electrical infrastructures in overcrowded mountainous urban areas under climate change. Reliab Eng Syst Saf 2023;236:109302. https://doi.org/10.1016/j.ress.2023.109302.

[3] Lin X, Lu Q, Chen L, Brilakis I. Assessing dynamic congestion risks of flood-disrupted transportation network systems through time-variant topological analysis and traffic demand dynamics. Adv Eng Informatics 2024;62:102672. https://doi.org/10.1016/j.aei.2024.102672.

[4] Sun J, Chow A, Madanat S. Multimodal transportation system protection against sea level rise. Transp Res Part D 2020;88. https://doi.org/10.1016/j.trd.2020.102568.

[5] Yang Z, Liu X, Wang J, Yan X, Shen R, Huo Z. Assessment on resilience of urban agglomeration transportation system considering passenger choice and load-capacity factor. Reliab Eng Syst Saf 2025;253:110527. https://doi.org/10.1016/j.ress.2024.110527.

[6] Zhou Yaoming, Wang J, Yang H. Resilience of transportation systems: concepts and comprehensive review. IEEE Trans Intell Transp Syst 2019;20:4262–76. https://doi.org/10.1109/TITS.2018.2883766.

[7] Barahimi AH, Eydi A, Aghaie A. Multi-modal urban transit network design considering reliability: multi-objective bi-level optimization. Reliab Eng Syst Saf 2021;216:107922. https://doi.org/10.1016/j.ress.2021.107922.

[8] Huang W, Li L, Liu H, Zhang R, Xu M. Defense resource allocation in road dangerous goods transportation network: a self-contained Girvan-Newman algorithm and Mean Variance Model combined approach. Reliab Eng Syst Saf 2021;215:107899. https://doi.org/10.1016/j.ress.2021.107899.

[9] Chen D, Fang X, Li Y, Ni S, Zhang Q, Sang CK. Three-level multimodal transportation network for cross-regional emergency resources dispatch under demand and route reliability. Reliab Eng Syst Saf 2022;222:108461. https://doi.org/10.1016/j.ress.2022.108461.

[10] Sleiman M, Bouyekhf R, El Moudni A. Urban transportation network analysis from a thermodynamic perspective. J Franklin Inst 2019;356:351–81. https://doi.org/10.1016/j.jfranklin.2018.09.033.

[11] Jarzynski C. Nonequilibrium equality for free energy differences. Phys Rev Lett 1997;78:2690–3. https://doi.org/10.1103/PhysRevLett.78.2690.

[12] Crooks GE. Entropy production fluctuation theorem and the nonequilibrium work relation for free energy differences. Phys Rev E 1999;60:2721–6. https://doi.org/10.1103/PhysRevE.60.2721.

[13] Borowska-Stefańska M, Bartnik A, Dulebenets MA, Kowalski M, Sahebgharani A, Tomalski P, Wiśniewski S. Changes in the equilibrium of the urban transport system of a large city following an urban flood. Reliab Eng Syst Saf 2025;253:110473. https://doi.org/10.1016/j.ress.2024.110473.

[14] R. Nartallo-Kaluarachchi, M. Asllani, G. Deco, M.L. Kringelbach, A. Goriely, R. Lambiotte, Broken detailed balance and entropy production in directed networks, (2024). http://arxiv.org/abs/2402.19157 (accessed October 29, 2024).

[15] R. Nartallo-Kaluarachchi, L. Bonetti, G. Fernández-Rubio, P. Vuust, G. Deco, M.L. Kringelbach, R. Lambiotte, A. Goriely, Multilevel irreversibility reveals higher-order organisation of non-equilibrium interactions in human brain dynamics, (2024). https://doi.org/10.1101/2024.05.02.592195.

[16] Wang L, Zhang S, Szűcs G, Wang Y. Identifying the critical nodes in multi-modal transportation network with a traffic demand-based computational method. Reliab Eng Syst Saf 2024;244:109956. https://doi.org/10.1016/j.ress.2024.109956.

[17] J. Hackl, B.T. Adey, Generation of spatially embedded random networks to model complex transportation networks, (2017). https://doi.org/10.1007/978-3-319-47886-9_15.

[18] He R, Zhang L, Tiong RLK. Flood risk assessment and mitigation for metro stations: an evidential-reasoning-based optimality approach considering uncertainty of subjective parameters. Reliab Eng Syst Saf 2023;238:109453. https://doi.org/10.1016/j.ress.2023.109453.

[19] Wang W, Yang S, Stanley HE, Gao J. Local floods induce large-scale abrupt failures of road networks. Nat Commun 2019;10. https://doi.org/10.1038/s41467-019-10063-w.

[20] B. Abdulla, A. Mostafavi, B. Birgisson, Characterization of the vulnerability of road networks to fluvial flooding using network percolation approach, in: Y. Cho, F. Leite, A. Behzadan, C. Wang (editors), 2019: pp. 428–35.

[21] Yang Y, Ng S, Zhou S, Xu F, Li H. Physics-based resilience assessment of interdependent civil infrastructure systems with condition-varying components: a case with stormwater drainage system and road transport system. Sustain Cities Soc 2020;54. https://doi.org/10.1016/j.scs.2019.101886.

[22] Gazis DC. The origins of traffic theory. Oper Res 2002;50:69–77. https://doi.org/10.1287/opre.50.1.69.17776.

[23] Ni D. Traffic flow theory: characteristics, experimental methods, and numerical techniques. Butterworth-Heinemann; 2015.

[24] Daganzo CF. The cell transmission model: a dynamic representation of highway traffic consistent with the hydrodynamic theory. Transp Res Part B 1994;28:269–87. https://doi.org/10.1016/0191-2615(94)90002-7.

[25] Newell GF. Control of pairing of vehicles on a public transportation route, two vehicles, one Control point. Transp Sci 1974;8:248–64. https://doi.org/10.1287/trsc.8.3.248.

[26] F.S. Koppelman, M.E. Ben-Akiva, Aggregate forecasting with dissaggregate travel demand models using normally available data, in: E.J. Visser (editor), Transport Decisions in an Age of Uncertainty: Proceedings of the Third World Conference on Transport Research Rotterdam, 26 –28 April 1977, Springer Netherlands, Dordrecht, 1977: pp. 159–66. https://doi.org/10.1007/978-94-009-9707-3_26.

[27] Knospe W, Santen L, Schadschneider A, Schreckenberg M. CA models for traffic flow: comparison with empirical single-vehicle data. arXiv: Stat Mech 2000:431–6. https://doi.org/10.1007/978-3-642-59751-0_46.

[28] Mishra S, Welch TF, Jha MK. Performance indicators for public transit connectivity in multi-modal transportation networks. Transp Res Part A 2012;46:1066–85. https://doi.org/10.1016/j.tra.2012.04.006.

[29] Xiong C, Yang D, Ma J, Chen X, Zhang L. Measuring and enhancing the transferability of hidden Markov models for dynamic travel behavioral analysis. Transportation (Amst) 2020;47:585–605. https://doi.org/10.1007/s11116-018-9900-9.

[30] Gallotti R, Barthelemy M. The multilayer temporal network of public transport in Great Britain. Sci Data 2015;2:140056. https://doi.org/10.1038/sdata.2014.56.

[31] Zhang L, Wen H, Lu J, Lei D, Li S, Ukkusuri SV. Exploring cascading reliability of multi-modal public transit network based on complex networks. Reliab Eng Syst Saf 2022;221:108367. https://doi.org/10.1016/j.ress.2022.108367.

[32] Sohouenou P, Neves L, Christodoulou A, Christidis P, Presti DLo. Using a hazard-independent approach to understand road-network robustness to multiple disruption scenarios. Transp Res Part D 2021;93. https://doi.org/10.1016/j.trd.2020.102672.

[33] Bucar R, Hayeri Y. Quantitative assessment of the impacts of disruptive precipitation on surface transportation. Reliabil Eng Syst Saf 2020;203. https://doi.org/10.1016/j.ress.2020.107105.

[34] Huang X, Gong J, Chen P, Tian Y, Hu X. Towards the adaptability of coastal resilience: vulnerability analysis of underground gas pipeline system after hurricanes using LiDAR data. Ocean Coast Manag 2021;209. https://doi.org/10.1016/j.ocecoaman.2021.105694.

[35] Zhang F, Zheng N, Yang H, Geroliminis N. A systematic analysis of multimodal transport systems with road space distribution and responsive bus service. Transp Res Part C 2018;96:208–30. https://doi.org/10.1016/j.trc.2018.09.009.

[36] Li X, Ha J, Lee S. Unveiling the roles of public bike systems: from leisure to multimodal transportation. Travel Behav Soc 2024;34:100705. https://doi.org/10.1016/j.tbs.2023.100705.

[37] B. Wang, Y. Leng, G. Wang, Y. Wang, FusionTransNet for Smart Urban Mobility: spatiotemporal traffic forecasting through multimodal network integration, (2024). https://doi.org/10.48550/arXiv.2405.05786.

[38] Lynn CW, Holmes CM, Bialek W, Schwab DJ. Emergence of local irreversibility in complex interacting systems. Phys Rev E 2022;106:034102. https://doi.org/10.1103/PhysRevE.106.034102.

[39] Gabrielli A, Garlaschelli D, Patil SP, Serrano MÁ. Network renormalization. Nat Rev Phys 2025:1–17. https://doi.org/10.1038/s42254-025-00817-5.

[40] Lin X, Lu Q, An Y, Yang T, Tang J. Decomposing traffic flow patterns under extreme weather: a novel field-theoretic framework using discrete differential operators. In: CIB Conferences. 1; 2025. https://doi.org/10.7771/3067-4883.1729.

[41] Strogatz SH. Nonlinear dynamics and chaos: with applications to physics, biology, chemistry, and engineering. 3rd ed. Boca Raton: Chapman and Hall/CRC; 2024. https://doi.org/10.1201/9780429398490.

[42] Pan Y, Li T, Zhu Q. Is stochastic mirror descent vulnerable to adversarial delay attacks? A traffic assignment resilience study. In: 2023 62nd IEEE Conference on Decision and Control (CDC); 2023. p. 8328–33. https://doi.org/10.1109/CDC49753.2023.10384003.

[43] S. Gao, Z. She, Q. Liang, N. Zheng, D. Li, Urban traffic resilience control - an ecological resilience perspective, (2024). http://arxiv.org/abs/2404.11082 (accessed October 27, 2024).

[44] Assaad R, Dagli C, El-adaway IH. A system-of-systems model to simulate the complex emergent behavior of vehicle traffic on an Urban transportation infrastructure network. Procedia Comput Sci 2020;168:139–46. https://doi.org/10.1016/j.procs.2020.02.277.

[45] Nguyen J, Powers ST, Urquhart N, Farrenkopf T, Guckert M. An overview of agent-based traffic simulators. Transp Res Interdiscip Perspect 2021;12:100486. https://doi.org/10.1016/j.trip.2021.100486.

[46] Yu Y, Han K, Ochieng W. Day-to-day dynamic traffic assignment with imperfect information, bounded rationality and information sharing. Transp Res Part C 2020;114:59–83. https://doi.org/10.1016/j.trc.2020.02.004.

[47] Pi X, Ma W, Qian Z(Sean). A general formulation for multi-modal dynamic traffic assignment considering multi-class vehicles, public transit and parking. Transp Res Part C 2019;104:369–89. https://doi.org/10.1016/j.trc.2019.05.011.

[48] Song X, Qi L, Liu S, Ding S, Li D. Simple analysis of complex system safety based on Finite State Machine Network and phase space theory. Reliab Eng Syst Saf 2024;249:110205. https://doi.org/10.1016/j.ress.2024.110205.

[49] Duan J, Li D, Huang H-J. Reliability of the traffic network against cascading failures with individuals acting independently or collectively. Transp Res Part C 2023;147:104017. https://doi.org/10.1016/j.trc.2023.104017.

[50] Callen HB. The application of Onsager's reciprocal relations to thermoelectric, thermomagnetic, and galvanomagnetic effects. Phys Rev 1948;73:1349–58. https://doi.org/10.1103/PhysRev.73.1349.

[51] Prigogine I. Time, structure, and fluctuations. Science (1979) 1978;201:777–85. https://doi.org/10.1126/science.201.4358.777.

[52] Evans DJ, Searles DJ, Rondoni L. Application of the Gallavotti-Cohen fluctuation relation to thermostated steady states near equilibrium. Phys Rev E 2005;71: 056120. https://doi.org/10.1103/PhysRevE.71.056120.

[53] Cohen EGD, Rondoni L. Note on phase space contraction and entropy production in thermostatted hamiltonian systems. Chaos 1998;8:357–65. https://doi.org/10.1063/1.166317.

[54] Halpern NY, Garner AJP, Dahlsten OCO, Vedral V. Introducing one-shot work into fluctuation relations. New J Phys 2015;17:095003. https://doi.org/10.1088/1367-2630/17/9/095003.

[55] Seifert U. Stochastic thermodynamics, fluctuation theorems and molecular machines. Rep Prog Phys 2012;75:126001. https://doi.org/10.1088/0034-4885/75/12/126001.

[56] Esposito M, den Broeck CV. Second law and Landauer principle far from equilibrium. EPL 2011;95:40004. https://doi.org/10.1209/0295-5075/95/40004.

[57] Horowitz JM, Esposito M. Thermodynamics with continuous information flow. Phys Rev X 2014;4:031015. https://doi.org/10.1103/PhysRevX.4.031015.

[58] Skinner DJ, Dunkel J. Estimating entropy production from waiting time distributions. Phys Rev Lett 2021;127:198101. https://doi.org/10.1103/PhysRevLett.127.198101.

[59] Farsi M, Erkoyuncu JA, Steenstra D, Roy R. A modular hybrid simulation framework for complex manufacturing system design. Simul Model Pract Theory 2019;94:14–30. https://doi.org/10.1016/j.simpat.2019.02.002.

[60] Moxnes E, Davidsen PI. Intuitive understanding of steady-state and transient behaviors. Syst Dyn Rev 2016;32:130–55. https://doi.org/10.1002/sdr.1561.

[61] Li J, Zhang HM. The variational formulation of a non-equilibrium traffic flow model: theory and implications. Procedia - Social Behav Sci 2013;80:327–40. https://doi.org/10.1016/j.sbspro.2013.05.019.

[62] Zhou H, Bouyekhf R, El Moudni A. Modeling and entropy based control of urban transportation network. Control Eng Pract 2013;21:1369–76. https://doi.org/10.1016/j.conengprac.2013.06.007.

[63] Watts DJ, Strogatz SH. Collective dynamics of 'small-world9 networks. Nature 1998;393:440–2. https://doi.org/10.1038/30918.

[64] Barabási A-L, Albert R. Emergence of scaling in random networks. Science (1979) 1999;286:509–12. https://doi.org/10.1126/science.286.5439.509.

[65] Newman MEJ. The structure and function of complex networks. SIAM Rev 2003; 45:167–256. https://doi.org/10.1137/S003614450342480.

[66] Wang M, Wang Y, Ding J, Yu W. Interaction aware and multi-modal distribution for ship trajectory prediction with spatio-temporal crisscross hybrid network. Reliab Eng Syst Saf 2024;252:110463. https://doi.org/10.1016/j.ress.2024.110463.

[67] Shamsan A, Wu X, Liu P, Cheng C. Intrinsic recurrence quantification analysis of nonlinear and nonstationary short-term time series. Chaos 2020;30(9). https://doi.org/10.1063/5.0006537.

[68] Lynn CW, Holmes CM, Bialek W, Schwab DJ. Decomposing the local arrow of time in interacting systems. Phys Rev Lett 2022;129:118101. https://doi.org/10.1103/PhysRevLett.129.118101.

[69] Braun T, Unni VR, Sujith R, Kurths J, Marwan N. Detection of dynamical regime transitions with lacunarity as a multiscale recurrence quantification measure. Nonlinear Dyn 2020;104:3955–73. https://doi.org/10.1007/s11071-021-06457-5.

[70] Laval JA. Self-organized criticality of traffic flow: implications for congestion management technologies. Transp Res Part C 2023;149:104056. https://doi.org/10.1016/j.trc.2023.104056.

[71] Zeng J, Tang J. Modeling dynamic traffic flow as visibility graphs: a network-scale prediction framework for lane-level traffic flow based on LPR data. IEEE Trans Intell Transp Syst 2023;24:4173–88. https://doi.org/10.1109/TITS.2022.3231959.

[72] Lacasa L, Nicosia V, Latora V. Network structure of multivariate time series. Sci Rep 2015;5:15508. https://doi.org/10.1038/srep15508.

[73] Smith RX, Jann K, Ances B, Wang DJJ. Wavelet-based regularity analysis reveals recurrent spatiotemporal behavior in resting-state fMRI. Hum Brain Mapp 2015; 36. https://doi.org/10.1002/hbm.22865.

[74] Liu H, Zhang X, Zhang X. Multiscale complexity analysis on airport air traffic flow volume time series. Physica A 2020;548:124485. https://doi.org/10.1016/j.physa.2020.124485.

[75] Harrou F, Zeroual A, Kadri F, Sun Y. Enhancing road traffic flow prediction with improved deep learning using wavelet transforms. Results Eng 2024;23:102342. https://doi.org/10.1016/j.rineng.2024.102342.

[76] Wen T, Chen H, Cheong KH. Visibility graph for time series prediction and image classification: a review. Nonlinear Dyn 2022;110:2979–99. https://doi.org/10.1007/s11071-022-08002-4.

[77] Yin Y, Wang X, Li Q, Shang P, Gao H, Ma Y. Multiscale permutation mutual information quantify the information interaction for traffic time series. Nonlinear Dyn 2020;102:1909–23. https://doi.org/10.1007/s11071-020-05981-0.

[78] Liu X, Li D, Ma M, Szymanski BK, Stanley HE, Gao J. Network resilience. Phys Rep 2022;971:1–108. https://doi.org/10.1016/j.physrep.2022.04.002.

[79] E.J. Oughton, W. Usher, P. Tyler, J.W. Hall, Infrastructure as a complex adaptive system, (n.d.). https://doi.org/10.1155/2018/3427826.

[80] Rosvall M, Esquivel AV, Lancichinetti A, West JD, Lambiotte R. Memory in network flows and its effects on spreading dynamics and community detection. Nat Commun 2014;5. https://doi.org/10.1038/ncomms5630.

[81] Delvenne J-C, Lambiotte R, Rocha LEC. Diffusion on networked systems is a question of time or structure. Nat Commun 2015;6. https://doi.org/10.1038/ncomms8366.

[82] Williams OE, Lacasa L, Millán AP, Latora V. The shape of memory in temporal networks. Nat Commun 2022;13:499. https://doi.org/10.1038/s41467-022-28123-z.

[83] Feng M, Cai S-M, Tang M, Lai Y-C. Equivalence and its invalidation between non-Markovian and Markovian spreading dynamics on complex networks. Nat Commun 2019;10:3748. https://doi.org/10.1038/s41467-019-11763-z.

[84] Walter B. Field theory of survival probabilities, extreme values, first-passage times, and mean span of non-Markovian stochastic processes. Phys Rev Res 2022; 4. https://doi.org/10.1103/PhysRevResearch.4.043197.

[85] Chowdhury SNag, Ray A, Dana SK, Ghosh D. Extreme events in dynamical systems and random walkers: a review. Phys Rep 2022;966:1–52. https://doi.org/10.1016/j.physrep.2022.04.001.

[86] Feng M, Tian L, Lai Y-C, Zhou C. Validity of Markovian modeling for transient memory-dependent epidemic dynamics. Commun Phys 2024;7:86. https://doi.org/10.1038/s42005-024-01578-w.

[87] Hu J, Hu C, Yang J, Bai J, Lee JJ. Do traffic flow states follow Markov properties? A high-order spatiotemporal traffic state reconstruction approach for traffic prediction and imputation. Chaos Solitons Fractals 2024;183:114965. https://doi.org/10.1016/j.chaos.2024.114965.

[88] Duan J, Zeng G, Serok N, Li D, Lieberthal EB, Huang H-J, Havlin S. Spatiotemporal dynamics of traffic bottlenecks yields an early signal of heavy congestions. Nat Commun 2023;14:8002. https://doi.org/10.1038/s41467-023-43591-7.

[89] Ma G, Li K, Sun H. Modeling and simulation of traffic flow based on memory effect and driver characteristics. Chin J Phys 2023;81:144–54. https://doi.org/10.1016/j.cjph.2022.11.008.

[90] S.A. Eddington, The nature of the physical World: the Gifford Lectures 1927, BoD – Books on demand, 2019.

[91] Cooper JLB. The foundations of thermodynamics. J Math Anal Appl 1967;17: 172–93. https://doi.org/10.1016/0022-247X(67)90174-6.

[92] Lieb EH, Yngvason J. The physics and mathematics of the second law of thermodynamics. Phys Rep 1999;310:1–96. https://doi.org/10.1016/S0370-1573(98)00082-9.

[93] Bucar RCB, Hayeri YM. Quantitative flood risk evaluation to improve drivers' route choice decisions during disruptive precipitation. Reliab Eng Syst Saf 2022; 219:108202. https://doi.org/10.1016/j.ress.2021.108202.

[94] Ouyang M. Review on modeling and simulation of interdependent critical infrastructure systems. Reliab Eng Syst Saf 2014;121:43–60. https://doi.org/10.1016/j.ress.2013.06.040.

[95] Mattsson L-G, Jenelius E. Vulnerability and resilience of transport systems – a discussion of recent research. Transp Res Part A 2015;81:16–34. https://doi.org/10.1016/j.tra.2015.06.002.

[96] K.A. Salvi, M. Kumar, A.M. Hainen, Sensitivity of traffic speed to rainfall, (2022). https://doi.org/10.1175/WCAS-D-22-0024.1.

[97] Lin X, Lu Q, Zhao P, Chen L, Tang J, Guan D, Broyd T. Field-theory inspired physics-informed graph neural network for reliable traffic flow prediction under urban flooding. Reliab Eng Syst Saf 2025:111487. https://doi.org/10.1016/j.ress.2025.111487.

[98] Saifuzzaman M, Zheng Z, Haque MdM, Washington S. Understanding the mechanism of traffic hysteresis and traffic oscillations through the change in task difficulty level. Transp Res Part B 2017;105:523–38. https://doi.org/10.1016/j.trb.2017.09.023.

[99] Kim Y, Yeo H. Asymmetric repulsive force model: a new car-following model with psycho-physical characteristics. Transp Res Part C 2024;161:104571. https://doi.org/10.1016/j.trc.2024.104571.

[100] Schönhof M, Helbing D. Empirical features of congested traffic states and their implications for traffic modeling. Transp Sci 2007;41:135–66.

[101] Gnesotto FS, Mura F, Gladrow J, Broedersz CP. Broken detailed balance and non-equilibrium dynamics in living systems: a review. Rep Prog Phys 2018;81: 066601. https://doi.org/10.1088/1361-6633/aab3ed.

[102] Vaidya T, Chotibut T, Piliouras G. Broken detailed balance and non-equilibrium dynamics in noisy social learning models. Physica A 2021;570:125818. https://doi.org/10.1016/j.physa.2021.125818.

[103] Zia RKP, Schmittmann B. Probability currents as principal characteristics in the statistical mechanics of non-equilibrium steady states. J Stat Mech 2007;2007: P07012. https://doi.org/10.1088/1742-5468/2007/07/P07012.

[104] Gao J, Buldyrev SV, Stanley HE, Havlin S. Networks formed from interdependent networks. Nature Phys 2012;8:40–8. https://doi.org/10.1038/nphys2180.

[105] Kerner BS. Empirical breakdown phenomenon: phase transition from free flow to synchronized flow. In: Kerner BS, editor. The physics of traffic: empirical freeway pattern features, engineering applications, and theory. Berlin, Heidelberg: Springer; 2004. p. 269–304. https://doi.org/10.1007/978-3-540-40986-1_10.

[106] Kerner BS. Physics of traffic gridlock in a city. Phys Rev E 2011;84. https://doi.org/10.1103/PhysRevE.84.045102.

[107] Schadschneider A, Chowdhury D, Nishinari K. Chapter one - introduction to nonequilibrium systems and transport phenomena. In: Schadschneider A, Chowdhury D, Nishinari K, editors. Stochastic transport in complex systems. Amsterdam: Elsevier; 2011. p. 3–25. https://doi.org/10.1016/B978-0-444-52853-7.00001-4.

[108] Schadschneider A, Chowdhury D, Nishinari K. Chapter four - asymmetric simple exclusion process – exact results. In: Schadschneider A, Chowdhury D, Nishinari K, editors. Stochastic transport in complex systems. Amsterdam: Elsevier; 2011. p. 109–205. https://doi.org/10.1016/B978-0-444-52853-7.00004-X.

[109] Qin Y, Wu C, Hu Y. Traffic flow prediction by an ensemble framework with data denoising and functional principal components analysis. Transp Res Rec 2024. https://doi.org/10.1177/03611981241242375.

[110] Zhou H, Bouyekhf R, El Moudni A. Modeling and entropy based control of urban transportation network. Control Eng Pract 2013;21:1369–76. https://doi.org/10.1016/j.conengprac.2013.06.007.

[111] Mahnke R, Kaupužs J, Lubashevsky I. Probabilistic description of traffic flow. Phys Rep 2005;408:1–130. https://doi.org/10.1016/j.physrep.2004.12.001.

[112] Sugar L, Kennedy C. Thermodynamics of urban growth revealed by city scaling. Physica A 2020;557:124971. https://doi.org/10.1016/j.physa.2020.124971.

[113] Chowdhury D, Santen L, Schadschneider A. Statistical physics of vehicular traffic and some related systems. Phys Rep 2000;329:199–329. https://doi.org/10.1016/S0370-1573(99)00117-9.

[114] Helbing D. Traffic and related self-driven many-particle systems. Rev Mod Phys 2001;73:1067–141. https://doi.org/10.1103/RevModPhys.73.1067.

[115] Daganzo CF. Requiem for second-order fluid approximations of traffic flow. Transp Res Part B 1995;29:277–86. https://doi.org/10.1016/0191-2615(95)00007-Z.

[116] Ván P. Nonequilibrium thermodynamics: emergent and fundamental. Philos Trans R Soc A 2020. https://doi.org/10.1098/rsta.2020.0066.

[117] Laval JA. Traffic flow as a simple fluid: toward a scaling theory of urban congestion. Transp Res Rec 2024;2678:376–86. https://doi.org/10.1177/03611981231179703.

[118] Liu E, Barker K, Chen H. A multi-modal evacuation-based response strategy for mitigating disruption in an intercity railway system. Reliab Eng Syst Saf 2022;223:108515. https://doi.org/10.1016/j.ress.2022.108515.

[119] Feng EH, Crooks GE. Length of Time's Arrow. Phys Rev Lett 2008;101:090602. https://doi.org/10.1103/PhysRevLett.101.090602.

[120] Roldán É, Parrondo JMR. Estimating dissipation from single stationary trajectories. Phys Rev Lett 2010;105:150607. https://doi.org/10.1103/PhysRevLett.105.150607.

[121] Luque B, Lacasa L, Ballesteros F, Luque J. Horizontal visibility graphs: exact results for random time series. Phys Rev E 2009;80:046103. https://doi.org/10.1103/PhysRevE.80.046103.

[122] Lacasa L, Luque B, Ballesteros F, Luque J, Nuño JC. From time series to complex networks: the visibility graph. Proc Natl Acad Sci USA 2008;105:4972–5. https://doi.org/10.1073/pnas.0709247105.

[123] Varley TF, Sporns O. Network analysis of time series: novel approaches to network neuroscience. Front Neurosci 2022;15. https://doi.org/10.3389/fnins.2021.787068.

[124] Sulaimany S, Safahi Z. Visibility graph analysis for brain: scoping review. Front Neurosci 2023;17. https://doi.org/10.3389/fnins.2023.1268485.

[125] Lacasa L, Luque B, Luque J, Nuño JC. The visibility graph: a new method for estimating the Hurst exponent of fractional Brownian motion. EPL 2009;86:30001. https://doi.org/10.1209/0295-5075/86/30001.

[126] Laflamme EM, Ossenbruggen PJ. Effect of time-of-day and day-of-the-week on congestion duration and breakdown: a case study at a bottleneck in Salem, NH. J Traffic Transp Eng (English Edition) 2017;4:31–40. https://doi.org/10.1016/j.jtte.2016.08.004.

[127] OKI, Additional performance measures, OKI performance measures (2024). https://performance.oki.org/mobility-congestion/additional-performance-measures/ (accessed July 18, 2025).

[128] Jalali S. Travel time reliability: how to measure and why it is important? SMATS 2020. https://www.smatstraffic.com/2020/07/16/travel-time-reliability-how-to-measure-and-why-it-is-important/. accessed July 18, 2025.

[129] Office of Operations, Traffic congestion and reliability: linking solutions to problems, (2017). https://ops.fhwa.dot.gov/congestion_report_04/appendix_c.htm (accessed July 18, 2025).

[130] Jeihani M, James P, Saka AA, Ardeshiri A. Traffic recovery time estimation under different flow regimes in traffic simulation. J Traffic Transp Eng (English Edition) 2015;2:291–300. https://doi.org/10.1016/j.jtte.2015.08.001.

[131] Loder A, Ambühl L, Menendez M, Axhausen KW. Understanding traffic capacity of urban networks. Sci Rep 2019;9:16283. https://doi.org/10.1038/s41598-019-51539-5.

[132] Zhang X, Cheng T. The impacts of the COVID-19 pandemic on multimodal human mobility in London: a perspective of decarbonizing transport. Geo-Spatial Inf Sci 2023;26:703–15. https://doi.org/10.1080/10095020.2022.2122876.

[133] Kendon M. Storm Alex and heavy rain 2 to 4 October 2020. Met Office 2020. https://www.metoffice.gov.uk/binaries/content/assets/metofficegovuk/pdf/weather/learn-about/uk-past-events/interesting/2020/2020_09_storm_alex_1.pdf.