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The (Non-)equivalence of dividends and share buybacks

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Abstract
We study a one-period equilibrium model in which a firm optimally determines its equity distribution level to maximize the expected utility of a representative shareholder. Dividends and share buybacks, the two most prevalent payout methods, are compared and contrasted. In our baseline setup, we demonstrate a Modigliani-Miller-style equivalence, where both dividend payouts and share buybacks result in the same shareholder welfare, distribution ratio and firm's investment level. However, share buybacks lead to a higher firm price in equilibrium. This conclusion is robust to alternative modeling specifications, such as heterogeneous beliefs among investors and endogenous riskfree rates. We also provide examples where this equivalence breaks down due to distortions in managerial incentives or market frictions. For example, firm managers endowed with employee stock options strictly prefer buybacks over dividends. In the presence of trading constraints, the relative attractiveness of dividends versus buybacks becomes ambiguous.

Keywords Dividends · Share buybacks · Equilibrium

Mathematics Subject Classification D53 · G30 · G35

1 Introduction

A central problem in corporate finance is how much cash a firm should distribute to its shareholders, and how such distributions should be implemented. While dividends have historically been the dominant payout mechanism, share buybacks have recently gained widespread usage and attention. Indeed, the Wall Street Journal reports that *"Analysts at Goldman Sachs project total S&P 500 repurchases to reach \$925 billion this year* (2024)

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and $1.075 trillion in 2025" [20], whereas the S&P 500 firms only paid out $630 billion of regular dividends in 2024 [12]. Share buybacks have in fact been on the rise for the past two decades, with more than half of the S&P 500's earnings from 2002 to 2013 allocated to repurchases [35]. Whence, buybacks have undeniably become as significant as - if not already more so than - dividends.

Despite the extensive literature on the motivations behind payouts to shareholders (outlined in our literature review in Section 2), existing theoretical frameworks often fail to distinguish between dividends and buybacks. Many models of payouts following the seminal work of Miller and Modigliani [38] view (or imply) dividends and buybacks as interchangeable. This lack of differentiation between dividends and buybacks has made it difficult to understand the distinct economic outcomes yielded by each method and the incentives driving firms to choose one method over the other.

To start filling this gap in the literature, the present study examines the equivalence (or the lack thereof) between dividends and share buybacks as an equity distribution policy in a one-period general equilibrium model. Our framework jointly describes shareholders' investment decisions, formation of firm prices, and the optimal distribution policy of the firm. More specifically, in our model, a firm announces how much cash to distribute to investors via dividends or share buybacks; any residual capital is in turn invested in a risky technology. Risk averse shareholders then determine their optimal portfolio choice in response to the given dividends or buybacks policy announced by the firm. Anticipating how investors react to an exogenous payout policy and how equilibrium share prices form under market clearing, managers set the optimal dividend yield or the buyback ratio to maximize the expected utility of the shareholders. Our baseline model retains many of the underlying assumptions of Miller and Modigliani [38]: i) Frictionless markets without transaction costs, taxes or trading constraints; ii) Perfect information, where shareholders have accurate knowledge about the risk-return profile of the firm's technology; iii) Selfless managers (i.e., no agency frictions) who maximize shareholder welfare.

By solving this model, we compare several important equilibrium quantities when equity distribution happens via dividends or share buybacks, respectively. We prove a Miller-Modigliani-style equivalence, which shows that both distribution methods lead to identical shareholder welfare, total values of equity distributed, and fractions of equity distribution. Managers therefore are indifferent towards whether dividends or buybacks are adopted as a channel to return cash to investors. Moreover, since the total value of equity distributed is the same, dividends and buybacks will result in the same level of free capital owned by the firm which will eventually be invested in the risky technology. These phenomena echo the seminal insights of Miller and Modigliani [38]: payout decisions are simply a residual policy and shareholders value is maximized via choosing an optimal investment strategy.

However, a novel result is that dividends and buybacks do give rise to different firm share prices in equilibrium – which clearly cannot be deduced from partial equilibrium framework prevalent in much of the extant literature. More specifically, the share price under buybacks is always higher than the corresponding equilibrium price under dividends provided that it is optimal for the firm to return a positive fraction of equity to the shareholders. The economic intuition is natural: buybacks reduce the supply of shares in the market while the investment in the risky technology is held at the same (optimal) level. Whence, the average payoff per share increases and each share in turn becomes more attractive. These insights derived in our baseline model remain valid with additional features such heterogeneous beliefs among shareholders and when the riskfree rate is endogenized as well.

In contrast, one distribution method may be more advantageous than the other if, in addition to shareholders value maximization, the firm managers also have additional incentives directly tied to the firm's share price. A notable example are employee stock options: we demonstrate that if managers are vested with call options, then they strictly prefer share buybacks over dividend payouts, because share prices can be increased by reducing the supply of shares through repurchases, which leaves shareholders indifferent but increases the value of the manager's options.

Trading frictions can also break the equivalence between dividends and buybacks. In another extension, we consider two types of shareholders, where one cannot rebalance their risky holdings. This may for example represent a passive investor who only trades sparingly or a fund manager who faces asset allocation constraints. The choice of dividends versus buybacks now matters because the firm can only repurchase shares from the unrestricted investor, which then forcibly increases the relative stake and in turn risk exposure of the restricted investor. The trading friction thereby prevents optimal risk sharing between the two types of investors so they cannot coordinate to achieve the optimal ownership composition, regardless whether dividends or buybacks are considered. The forced change in ownership composition induced by buybacks may therefore either create or destroy welfare depending on which ownership structure delivers a higher aggregate value.

The rest of the paper is organized as follows. We provide a detailed literature review in Section 2. Our model then is presented in Section 3. The main results of the paper are given and discussed in Section 4, where we highlight in what ways dividends and share buybacks are (not) equivalent to each other. Two extensions of the baseline model are presented in Section 5 to demonstrate the robustness of our key results with respect to heterogenous shareholder belief and endogenous riskfree rates. In Section 6, we consider two different extensions with managerial incentives and trading frictions under which the equivalence between dividends and buybacks does not hold. Section 7 concludes. For better readability, all proofs are deferred to the appendix.

2 Literature review

One of the most classical insights of corporate payout policy is the dividend irrelevance principle of Miller and Modigliani [38], which asserts that under a perfect and frictionless market the value of a firm is solely determined by its investment strategy but not the dividend policy. Once the investment policy is fixed, dividend policy is merely a manifestation of the difference between the firm's retained earnings and the desirable investment level. A vast literature has emerged since then to shed light on the driving factors and economic trade-offs behind corporate payout policies, usually via relaxing one or more modeling assumptions of the Miller and Modigliani model. The landscape of payout policies research can be found in the comprehensive survey by Allen and Michaely [3]. To the best of our knowledge, the comparison of dividends and share buybacks has not been addressed in the context of a general equilibrium model. The existing literature typically treats dividends and buybacks as interchangeable or, if not, studies the differences in partial equilibrium models. Those works therefore cannot speak to how different methods of equity distribution affect firm's investment, share price, payout policy and shareholders' welfare simultaneously.

Concerning why a firm opts to disperse cash to investors (dividends or buybacks), one consideration is asymmetric information between shareholders and firm managers. If the outsiders (public shareholders) cannot directly observe certain characteristics of the firm such as the available investment opportunities or its fundamentals, then payout decisions can serve as a signaling device for the insiders (managers) to convey information about the firm's profitability to the outsiders [6, 30, 33, 39]. Another example are agency frictions: When managers' actions cannot be controlled, leaving too much capital inside the firm may be detrimental to shareholders because selfish managers might divert value to their personal pockets rather than to stipulate growth of the firm [26, 27]. Corporate payout decision also matter when the insiders can use them as a tool to manage the relationship with, and to reduce the threat of intervention by, the outsiders [19, 34, 40]. Our model does not feature informational frictions, but agency will be incorporated via consideration of employee stock options in an extension.[1]

As mentioned previously, the term "payout" used in the literature often encapsulates all forms of equity distribution to shareholders. Existing theoretical frameworks capable of disentangling dividends and buybacks focus on partial equilibrium frameworks. The differentiation can then originate from, for example, the clientele effect due to heterogeneous tax treatments across income and capital gains as well as across retail and institutional clients [2, 30], adverse selection cost of buybacks to less informed shareholders [11, 13, 36], additional financial flexibility brought by buybacks to control cash spending by self-interested managers [41], or disagreement between insiders and outsiders on the firm's prospect [5]. In parallel, there is also an extant literature addressing some unique aspects of dividends and buybacks in isolation. On the one hand, share buybacks can serve as a tool to manage earnings per share [24], to deter the threat of takeover by rival firms [4], or to counter the dilution effect from employee options [31]. On the other hand, the stable passive cash flows generated by (smooth) dividends can be attractive to consuming investors without self-control [42] and can substitute the needs for selling stocks [22].

More generally, there is a large body of theoretical literature on optimal corporate payout, financing and investment decisions. In addition to the previously cited papers, also cf. Jin and Myers [29], Albuquerue and Wang [1], Bolton et al. [9], Décamps et al. [17], Tse [44], and the references therein. Most of these works again focus on partial equilibrium model; one notable exception is Albuquerue and Wang [1], but they do not distinguish dividends and buybacks. The firm managers then do not internalize the potential impact of dividends/buybacks on investors' demand for shares, and in turn asset prices. In a recent working paper, Delao [18] considers a dynamic equilibrium model of a production economy where firms can adopt dividends and buybacks. The analysis in turn focuses on the cost of adjusting dividends but abstracts away from the effect that buybacks have on the supply of shares (and in turn asset prices).

Finally, on the empirical front, the extant literature offers mixed evidence over whether dividends and buybacks are perfect substitutes for each other. Some studies document negative relationship between dividends usage and share buybacks activity, which supports the substitution hypothesis [21, 28, 43], while others do not find strong evidence that buybacks have replaced dividends [16] and dividends and buybacks can be complimentary in nature depending on the business cycle and cash liquidity of the firm [25]. There is little contemporary literature that comprehensively justifies the recent surge in buybacks and decline in dividends. Michaely and Moin [37] show that the systemic shift in historical dividends trend can be attributed to changes in firms' characteristics (especially profitability and earnings volatility) and their proclivity to pay dividends. Cook and Zhang [14] document a positive relationship between CEO option incentive and buybacks activity, suggesting that the uptick in corporate incentive schemes usage can be a driving force, consistent with our theoretical results.

[1] Design of executive compensation schemes under general equilibrium models is considered in Bianchi et al. [7, 8]. In this paper, we focus on how an exogenously given compensation scheme (employee stock options) influences mangers' propensity to adopt buybacks versus dividends.

3 The baseline model

We work with a one-period model with time points indexed by $t \in \{0, 1\}$. A representative investor is initially endowed with the shares of a firm. At $t = 0$, the firm distributes some of its equity back to the investor in form of dividends and/or share buybacks. Immediately after the distribution, the investor rebalances their portfolio across the stock holdings in the firm and the cash position in a riskfree bank account. Meanwhile, all residual capital of the firm post-distribution is invested in a risky technology which grows stochastically between $t = 0$ and $t = 1$. Finally, the firm is liquidated at $t = 1$ and its entire outstanding capital is paid to the investor in form of a lump-sum cash dividend.

3.1 The firm

At time $t = 0$, just before dividend payments and buyback of shares, the firm has an initial capital of $K_0 > 0$ and the number of outstanding shares is normalized to unity. The market price of each share of the firm is denoted by P_0 (which will be endogenized in Sections 3.3 and 3.4).

Distribution via dividends and buybacks

A *distribution strategy* implemented by the firm at $t = 0$ is a pair $(\theta_D, \theta_{BB}) \in [0, 1)^2$. Here, θ_D denotes the *dividend payout ratio*, where a cash dividend amount of $\theta_D K_0$ per unit share is paid to the shareholders. The *buyback ratio* θ_{BB} is the fraction of outstanding shares repurchased by the firm. We assume the buyback of shares is performed at the prevailing market price P_0 via an *open market buyback program*,[2] and that this operation is entirely financed by the firm's capital. Recalling that the total number of outstanding shares has been normalized to one, the total cash value paid out by the firm is $\theta_D K_0 + \theta_{BB} P_0$. Hence the residual capital of the firm post-distribution is $K_0 - \theta_D K_0 - \theta_{BB} P_0$, while the number of outstanding shares is reduced to $1 - \theta_{BB}$.

Throughout the paper, we we will focus on distribution strategies of the form $(\theta_D, \theta_{BB} = 0)$ (*(pure) dividend strategies*) and $(\theta_D = 0, \theta_{BB})$ (*(pure) buyback strategies*).[3]

Remark 3.1 We only consider values of θ_D and θ_{BB} in the range of $[0, 1)$. A choice of $\theta_D > 1$ means the firm is paying out a large dividend that cannot be entirely financed by the existing capital which is typically prohibited in practice due to, for example, debt covenants. We also do not allow $\theta_{BB} > 1$ such that the firm cannot repurchase shares in excess of the total outstanding number of shares. The corner cases $\theta_D = 1$ or $\theta_{BB} = 1$ can be understood as voluntary liquidation or privatization of the firm. While it is possible to view a negative dividend payout $\theta_D < 0$ as capital injection, it is much less clear how one should interpret $\theta_{BB} < 0$ in the context of open market operation. We whence focus on $\theta_D \in [0, 1)$ and $\theta_{BB} \in [0, 1)$ as we are primarily interested in ordinary dividends/buybacks policy (rather than liquidation/privatization or capital injection/equity issuance decision) in

[2] In an open market buyback program, a firm repurchases its shares directly on an exchange. While several other alternatives like fixed-price tender offer or Dutch auction do exist (see Allen and Michaely [3] for an overview), open market operation is the predominant form of buyback programs accounting for around 97% of all buyback transactions in the US market in the recent decade [10].

[3] It is not difficult to extend the current model by allowing the firm to choose θ_D and θ_{BB} simultaneously. However, this extra flexibility is economically redundant in that investors' welfare cannot be improved by mixing dividends and buybacks.

this paper. When we eventually endogenize θ_D and θ_{BB} in Section 3.4, a restriction on the model parameters (Assumption 3.3) is needed to ensure that the endogenous dividend payout or buyback ratios fall into the desired range of values.

The firm's investment opportunities and liquidation payout

The firm has access to a risky investment opportunity, for which each unit of available capital post-distribution at $t = 0$ will grow stochastically to Π at $t = 1$. In our baseline model, we focus on the Gaussian case $\Pi \sim N(\mu, \sigma^2)$ with some given mean $\mu \in \mathbb{R}$ and standard deviation $\sigma > 0$. In an extension presented in Section 5.1, this will be extended to shareholders with heterogeneous beliefs over the profitability of the risky technology.

Since the capital of the firm immediately after distribution is $K_0 - \theta_D K_0 - \theta_{BB} P_0$, the terminal net worth of the firm at $t = 1$ is $(K_0 - \theta_D K_0 - \theta_{BB} P_0)\Pi$. In turn, the liquidation dividend per unit share at $t = 1$ is

$$D_1 := \frac{(K_0 - \theta_D K_0 - \theta_{BB} P_0)\Pi}{1 - \theta_{BB}}.$$

3.2 The representative investor

The population of shareholders is summarized by a single representative investor with initial cash endowment of zero. Their initial share holding is equal to the number of the outstanding shares, which we have normalized to one.

Investor's portfolio dynamics

When the firm distributes equity at $t = 0$, the representative investor receives a total cash dividend of $\theta_D K_0$. In parallel, the investor is allowed to rebalance their positions at the share price P_0. Suppose they choose $\theta_0 \in \mathbb{R}$ as the number of shares to be held after rebalancing, i.e., they opt to sell $1 - \theta_0$ shares. After these adjustments at time $t = 0$, the investor's portfolio consists of cash holdings of

$$W_0 := \theta_D K_0 + (1 - \theta_0) P_0$$

and θ_0 risky shares. The cash holdings W_0 grow at a riskfree rate R_f. We assume R_f to be exogenously fixed in the baseline model, but consider an extension in Section 5.2 where R_f is also endogenized.

At time $t = 1$, the firm pays a liquidating dividend of D_1 per share. Hence the terminal portfolio value of the investor is given by

$$\begin{aligned} W_1 &:= W_0(1 + R_f) + \theta_0 D_1 \\ &= (\theta_D K_0 + (1 - \theta_0) P_0)(1 + R_f) + \theta_0 \frac{(K_0 - \theta_D K_0 - \theta_{BB} P_0)\Pi}{1 - \theta_{BB}}. \end{aligned}$$

Remark 3.2 Since buybacks are performed via an open market program, the investor does not need to make an explicit decision whether they accept the firm's buyback offer or not. Such a decision is instead implied by their choice of θ_0, where $1 - \theta_0 > 0$ means the representative investor is selling shares back to the firm (i.e., they participate in the buyback). When the market clears in equilibrium, the share price P_0 is adjusted such that the representative investor sells exactly θ_{BB} units of shares back to the firm. See the second part of Definition 3.4.

Investor's preferences and goal function

The representative investor has an exponential utility function

$$U(x) := -\exp(-\gamma x),$$

where $\gamma > 0$ is their constant absolute risk aversion coefficient.[4] The certainty equivalent of the terminal portfolio value for a given trading strategy θ_0, the market price of the firm P_0 and the distribution strategy (θ_D, θ_{BB}) is

$$\begin{aligned}\mathrm{CE}(\theta_0;\, P_0, \theta_D, \theta_{BB}) &:= U^{-1}(\mathbb{E}[U(W_1)]) \\ &= (\theta_D K_0 + (1-\theta_0)P_0)\,(1+R_f) \\ &\quad + \left\{ \frac{\theta_0(K_0 - \theta_D K_0 - \theta_{BB} P_0)\mu}{1-\theta_{BB}} - \frac{\sigma^2\gamma}{2} \frac{\theta_0^2 (K_0 - \theta_D K_0 - \theta_{BB} P_0)^2}{(1-\theta_{BB})^2} \right\}. \end{aligned} \tag{3.1}$$

We impose the following standing assumption on the model parameters for the rest of this paper:

Assumption 3.3 The model parameters satisfy

$$0 < \max(K_0\gamma\sigma^2, 1+R_f) < \mu < 1 + R_f + K_0\gamma\sigma^2. \tag{3.2}$$

Assumption 3.3 requires that the expected return of the firm's risky investment opportunities cannot be too high nor too low. If the condition $\mu > 1 + R_f$ does not hold, then the firm is incentivized to "short sell' its investment assets and pay out the proceeds to shareholders – a degenerate scenario that we rule out, cf. Remark 3.1. Additionally, the condition $\mu > K_0\gamma\sigma^2$ ensures the existence of a competitive equilibrium in our model for any buyback policy. See also Remark 3.7. Finally, the condition $\mu < 1 + R_f + \sigma^2\gamma K_0$ is necessary and sufficient to ensure that the firm will pay out a strictly positive fraction of equity to the shareholders. Otherwise, the firm would want to raise capital to further finance its very profitable investment opportunities.

3.3 Competitive equilibrium under exogenously given distribution strategy

We now introduce the concept of equilibrium in the economy under an exogenously given distribution strategy of the firm. The representative investor represents a continuum of small price takers, who do not take into the account the impact on the share price of the firm caused by their portfolio choice. They also view the equity distribution strategy of the firm as fixed.[5] The solution of the representative investor's portfolio choice problem leads to an expression for the optimal share holdings. The equilibrium price of the firm is then determined by the usual condition that the aggregate demand for the shares equals the number of outstanding shares in the market ex-dividend or ex-buyback.

[4] We can extend the analysis to an economy with multiple representative investors with CARA preferences of different absolute risk aversion levels. The problem can be reduced to the case with a single representative investor if aggregate welfare is measured by an equally weighted sum of individual investors' certainty equivalents. See also Section 5.1.

[5] In contrast, the firm will take into account the impact of its distribution strategy on the share price, see Section 3.4.

Definition 3.4 Given the firm's distribution strategy $(\theta_D, \theta_{BB}) \in [0, 1)^2$, a competitive equilibrium is a pair of the investor's trading strategy and market price of the firm $(\hat{\theta}_0, \hat{P}_0)$ such that:

1. The representative investor maximizes – for the fixed share price and distribution strategy – the certainty equivalent of their terminal portfolio value:

$$\mathrm{CE}(\hat{\theta}_0; \hat{P}_0, \theta_D, \theta_{BB}) = \sup_{\theta_0 \in \mathbb{R}} \mathrm{CE}(\theta_0; \hat{P}_0, \theta_D, \theta_{BB}).$$

2. The market clears, in that the total demand for the shares of the firm is equal to the number of outstanding shares post-distribution:

$$1 = \hat{\theta}_0 + \theta_{BB}.$$

We call $\hat{\mathrm{CE}}(\theta_D, \theta_{BB}) := \mathrm{CE}(\hat{\theta}_0; \hat{P}_0, \theta_D, \theta_{BB})$ an *equilibrium value function* under the distribution strategy (θ_D, θ_{BB}). We further define $\hat{\mathrm{CE}}_D(\theta_D) := \hat{\mathrm{CE}}(\theta_D, \theta_{BB} = 0)$ and $\hat{\mathrm{CE}}_{BB}(\theta_{BB}) := \hat{\mathrm{CE}}(\theta_D = 0, \theta_{BB})$ as the equilibrium value functions under a (pure) dividend strategy and a (pure) buyback strategy, respectively.

In this paper, we are interested in the two polar cases of the distribution strategy: (pure) dividend strategies $(\theta_D, \theta_{BB} = 0)$ and (pure) buyback strategies $(\theta_D = 0, \theta_{BB})$. The competitive equilibrium share prices under a dividend and a buyback strategy are denoted by $\hat{P}_0(\theta_D)$ and $\hat{P}_0(\theta_{BB})$ respectively to stress their dependence on the given distribution ratio. The two propositions below characterize the forms of the competitive equilibrium in these two cases.

Proposition 3.5 (Equilibrium under exogenously given dividend strategy). *If the firm adopts a dividend distribution strategy in form of* $(\theta_D, \theta_{BB} = 0)$ *with* $\theta_D \in [0, 1)$*, then the unique competitive equilibrium is given by*

$$\hat{\theta}_0 = 1, \qquad \hat{P}_0(\theta_D) = \frac{K_0(1 - \theta_D)}{1 + R_f} \left(\mu - \sigma^2 \gamma K_0 (1 - \theta_D)\right). \tag{3.3}$$

Note that the expression in Proposition 3.5 is just a simple variation of standard one-period asset pricing result with CARA preferences and normally-distributed dividends.

Proposition 3.6 (Equilibrium under exogenously given buyback strategy). *Suppose the firm adopts a buyback strategy in form of* $(\theta_D = 0, \theta_{BB})$ *with* $\theta_{BB} \in [0, 1)$*. Then there exists at most two competitive equilibria. Moreover, for any competitive equilibrium* $(\hat{\theta}_0, \hat{P}_0(\theta_{BB}))$*, we must have* $\hat{\theta}_0 = 1 - \theta_{BB}$ *and*

$$\hat{P}_0(\theta_{BB}) \in \left\{ \frac{2\sigma^2 \gamma \theta_{BB} K_0 - \mu \theta_{BB} - (1 + R_f)(1 - \theta_{BB}) \pm \sqrt{\Delta(\theta_{BB})}}{2\sigma^2 \gamma \theta_{BB}^2} \right\} \tag{3.4}$$

when $\theta_{BB} \neq 0$*, with*

$$\Delta(\theta_{BB}) := \left[\mu \theta_{BB} + (1 + R_f)(1 - \theta_{BB})\right]^2 - 4\sigma^2 \gamma (1 + R_f) K_0 \theta_{BB} (1 - \theta_{BB}). \tag{3.5}$$

In the corner case of $\theta_{BB} = 0$*, there is only one competitive equilibrium given by* $\hat{\theta}_0 = 1$ *and*

$$\hat{P}_0(\theta_{BB} = 0) = \frac{K_0}{1 + R_f} (\mu - \sigma^2 \gamma K_0). \tag{3.6}$$

Remark 3.7 The requirement of $\mu > K_0\gamma\sigma^2$ in Assumption 3.3 ensures $\Delta(\theta_{BB})$ stays positive for all θ_{BB} such that the expressions in (3.4) are well-defined. Relaxing this condition will result in $\hat{P}_0(\theta_{BB})$ being ill-defined for some values of θ_{BB}, where then the optimization with respect to θ_{BB} (to be discussed in Section 3.4) needs to be performed over a more complicated action space of $\{\theta_{BB} \in [0, 1) : \Delta(\theta_{BB}) \geq 0\}$.

In view of Proposition 3.6, there are potentially two equilibria for the case of buybacks. To single out the economically sensible one, we impose the following additional criterion.

Definition 3.8 A map $f : [0, 1) \to \mathbb{R}^2$ is a *consistent family of competitive equilibria under pure buyback strategies* if f is continuous and $(f_1(\theta_{BB}), f_2(\theta_{BB}))$ is a competitive equilibrium under a given buyback strategy $(\theta_D = 0, \theta_{BB})$ for any $\theta_{BB} \in [0, 1)$.

From Proposition 3.6, we know that the competitive equilibrium price under a trivial buyback strategy $\theta_{BB} = 0$ is uniquely given by (3.6), while there are two possible equilibria when $\theta_{BB} \neq 0$. The idea behind Definition 3.8 is that we want to select a version of the competitive equilibrium which varies continuously with the buyback policy θ_{BB}. This can be understood as a stability requirement such that a small shock to the current buyback policy will not result in a large jump of the corresponding equilibrium quantities.

Corollary 3.9 *The unique consistent family of competitive equilibrium under pure buyback strategy is* $\theta_{BB} \mapsto (\hat{\theta}_0(\theta_{BB}), \hat{P}_0^+(\theta_{BB}))$*, where* $\hat{\theta}_0(\theta_{BB}) := 1 - \theta_{BB}$ *and*

$$\hat{P}_0^+(\theta_{BB}) := \begin{cases} \frac{2\sigma^2\gamma\theta_{BB}K_0 - \mu\theta_{BB} - (1+R_f)(1-\theta_{BB}) + \sqrt{\Delta(\theta_{BB})}}{2\sigma^2\gamma\theta_{BB}^2}, & \theta_{BB} \in [0, 1) \setminus \{0\}; \\ \frac{K_0}{1+R_f}(\mu - \sigma^2\gamma K_0), & \theta_{BB} = 0. \end{cases} \tag{3.7}$$

Here, $\Delta(\theta_{BB})$ *is defined as in* (3.5).

Corollary 3.9 implies that the larger equilibrium share price in (3.4) should be considered in the case of buybacks, as it will preserve continuity with respect to the buyback policy θ_{BB}. Moreover, it is also straightforward to verify that

$$\lim_{\theta_{BB} \to 0} \theta_{BB} \hat{P}_0^+(\theta_{BB}) = 0. \tag{3.8}$$

Note that $\theta_{BB} \hat{P}_0^+(\theta_{BB})$ represents the amount of cash spent by the firm on buyback, when the repurchase ratio is chosen to be θ_{BB}. When the amount of shares to be repurchased vanishes to zero, it is natural to expect that the cost of the buyback will also tend to zero. (3.8) shows that our choice of equilibrium indeed has this sensible property. However, if we would pick the smaller possible competitive equilibrium share price $P_0^-(\theta_{BB})$ in (3.4), then this would lead to

$$\lim_{\theta_{BB} \downarrow 0} \theta_{BB} \hat{P}_0^-(\theta_{BB}) = -\infty.$$

This would result in an absurd scenario, where the firm is willing to repurchase an infinitesimal amount of shares at an unboundedly large negative share price.

Whence, throughout this paper, we only focus on the consistent family of competitive equilibrium induced by pure buyback strategies, i.e., the equilibrium holdings and prices (3.7). This in turn uniquely defines *the* equilibrium value function $\hat{\text{CE}}_{BB}(\theta_{BB})$ via

$$\hat{\text{CE}}_{BB}(\theta_{BB}) := \hat{\text{CE}}(\theta_D = 0, \theta_{BB}) = \text{CE}(\hat{\theta}_0 = 1 - \theta_{BB};\ \hat{P}_0^+(\theta_{BB}), \theta_D = 0, \theta_{BB}),$$

where $\hat{P}_0^+(\theta_{BB})$ is given by (3.7). From here onwards, we simply write $\hat{P}_0(\theta_{BB}) = \hat{P}_0^+(\theta_{BB})$ for brevity.

3.4 Optimally endogenized dividend payout and buyback strategy

Any choice of dividend strategy $\theta_D \in [0, 1)$ or buyback strategy $\theta_{BB} \in [0, 1)$ induces a unique pair of (consistent) equilibrium share price $\hat{P}_0$ and investor's share holdings $\hat{\theta}_0$, which in turn uniquely determines the welfare level of the investor measured by their certainty equivalent $\hat{\mathrm{CE}}_D(\theta_D)$ or $\hat{\mathrm{CE}}_{BB}(\theta_{BB})$. We assume the firm maximizes the shareholder value, in that they maximize the certainty equivalent of the representative investor:

$$V_k := \sup_{\theta_k \in [0,1)} \hat{J}_k(\theta_k), \qquad k \in \{D, BB\}. \tag{3.9}$$

The corresponding optimizer θ_k^* is the *optimal dividend strategy* (for $k = D$) or the *optimal buyback strategy* (for $k = BB$). The corresponding equilibrium share price under the optimal policy θ_k^* is denoted by

$$\hat{P}_0^k := \hat{P}_0(\theta_k^*).$$

Remark 3.10 We assume that the firm and the representative investor have identical beliefs concerning the profitability of the firm's risky technology. In Appendix B, we consider an extension in which investor holds a wrong subjective belief over the firm's profitability, while the firm maximizes the investor's objective certainty equivalent evaluated against the ground truth. In this extension, most of the conclusions of the baseline model remain valid.

4 Equivalence of dividends and buybacks in the baseline model

The primary focus of this paper is to compare the equilibria resulting from the optimal endogenous dividend and optimal buyback policies. Our first main result provides this for the baseline version of our model:

Theorem 4.1 (Comparison of the optimal dividend and buyback strategies). *Dividend and buyback are economically equivalent in terms of shareholders' welfare, the ratio of distribution and the equity value distributed. Specifically:*

1. *Investor's maximized certainty equivalents coincide: $V_D = V_{BB}$.*
2. *The optimal dividend yield and buyback ratio are the same:*

$$\theta_D^* = \theta_{BB}^* = 1 - \frac{\mu - (1 + R_f)}{\sigma^2 \gamma K_0} \in (0, 1).$$

3. *The total values of equity distributed coincide:*

$$K_0 \theta_D^* = \theta_{BB}^* \hat{P}_0^{BB}.$$

However, the equilibrium share prices are not necessarily the same. More specifically, $\hat{P}_0^{BB} = K_0$ (recall that K_0 is the initial capital of the firm), but

$$\hat{P}_0^D = K_0(1 - \theta_D^*) = \hat{P}_0^{BB}(1 - \theta_{BB}^*) = \frac{\mu - (1 + R_f)}{\sigma^2 \gamma}. \tag{4.1}$$

If firm managers only care about the welfare of the shareholders, then dividend payout and share buybacks both allow to achieve the same optimal certainty equivalents, so that managers are indifferent towards the mode of equity distribution. The rightmost expression in (4.1) is precisely the *Merton level* under a portfolio optimization problem faced by a CARA investor, while the second and the third expression in (4.1) are the firm's residual capital post-distribution under dividends and buybacks, respectively. This result is not surprising: The optimization of payout policy is done via a "first best" criterion where the interests of managers and investors align. These two parties can be viewed as a sole risk averse decision maker, and the optimal investment amount in the risky asset (i.e., the firm's risky investment opportunity) is therefore exactly the Merton level. The optimal distribution policy must be designed such that the residual capital left at the firm, $K_0(1-\theta^*_D)$ under dividends or $K_0-\theta^*_{BB}\hat{P}_0^{BB}$ under buybacks, exactly coincides with this Merton level. This observation is precisely the insight of Miller and Modigliani [38] that the payout decision is merely a residual policy, which is entirely guided by the optimal investment decision. As a trivial consequence, the value of equity distributed back to investors must be identical as well (and is equal to $K_0-(\mu-(1+R_f))/\sigma^2\gamma$). We therefore have $K_0\theta^*_D=\theta^*_B\hat{P}_0^{BB}$.[6]

A surprising result here is the equality $\theta^*_D=\theta^*_{BB}$: the optimal dividend yield and buyback ratio are identical. While both θ^*_D and θ^*_{BB} represent some concept of fraction, they do not correspond to the same measurement of financial value. Instead, the former is a fraction of the firm's available capital (i.e., K_0 or equivalently the firm's book value) but the latter is a fraction of the outstanding shares (i.e., market value). The equality $\theta^*_D=\theta^*_{BB}$ is a consequence of the fact that the firm's book value and equilibrium market value coincide under the optimal buyback policy such that $\hat{P}_0^{BB}=K_0$.

An important distinction between dividends and buybacks is that the corresponding equilibrium share prices are not the same. Under our choice of model parameters, it is optimal to pay out dividends or buy back shares (i.e., $\theta^*_D>0$ and $\theta^*_{BB}>0$), the last part of Theorem 4.1 establishes that $\hat{P}_0^{BB}>\hat{P}_0^D$. By reducing the supply of shares through buybacks while maintaining the same optimal level of investment in the risky investment opportunity, the share price of the firm is increased. However, more expensive shares will be compensated by the higher terminal liquidating dividend per share, as the same payoff is now shared by fewer shareholders.

Despite the non-equivalence in share prices, dividends and buybacks do lead to identical *rates of return* per share in equilibrium. Recall the liquidation dividend (i.e. terminal share price of the firm at $t=1$) is D_1 in the case of dividend or $D_1/(1-\theta^*_B)$ in the case of buyback. Then the equality $\hat{P}_0^D=\hat{P}_0^{BB}(1-\theta^*_{BB})$ immediately leads to

$$\frac{D_1}{\hat{P}_0^D}=\frac{D_1/(1-\theta^*_B)}{\hat{P}_0^{BB}}, \tag{4.2}$$

suggesting that the equilibrium (random) returns of the share are the same under the two distribution methods across all the possible states of the world. Put differently, the equality $\hat{P}_0^D=\hat{P}_0^{BB}(1-\theta^*_{BB})$ itself is a manifestation of equivalence between dividend and buyback at the level of returns on the share.

Even though dividend and buyback both lead to the same level of *returns*, the fact that buybacks results in a higher price *levels* is still an economically important consequence. This is especially true when the firm managers have incentives tied to the absolute level of the firm price (rather than its scale-independent rate of return only). In Section 6.1, we will explore an example involving employee stock options.

[6] If the smaller candidate $\hat{P}_0^-(\theta_{BB})$ in (3.4) was used as the equilibrium price under buybacks, then there does not exist $\theta_{BB}\in[0,1)$ such that $\theta_{BB}\hat{P}_0^-(\theta_{BB})=K_0\theta^*_D$. Indeed, the right-hand-side is strictly positive while the left-hand-side is non-positive. Therefore, equivalence between dividends and buybacks does not hold under this improper choice of the equilibrium share price. This conclusion remains the same even if we endogenize the riskfree rate.

5 Extensions for which dividend-buyback equivalence holds

In this section, we consider some extensions of the baseline model for which the equivalence of dividend and buybacks established in Theorem 4.1 remains valid.

5.1 Shareholders with heterogeneous beliefs

In the baseline model, investors have homogeneous beliefs such that the population can be summarized by a representative investor. Now suppose different groups of shareholders have different beliefs about the profitability of the firm as well as risk preferences. To simplify the setup, we consider two representative investors only, indexed by $i \in \{1, 2\}$. Investor i believes that the firm's capital will grow at a Gaussian stochastic rate of $\Pi_i \sim N(\mu_i, \sigma_i^2)$.[7]

The rest of the setup is largely the same as in Section 3. We assume investor i is initially endowed with zero cash and $n_i \geq 0$ units of shares. The total number of outstanding shares is normalized to one such that $n_1 + n_2 = 1$. When the firm pays out dividend of yield θ_D and repurchases a fraction of θ_{BB} shares via open market operation, investor i receives a cash dividend of $n_i K_0 \theta_D$. The number of shares held by investor i after portfolio rebalancing is denoted by θ_0^i, such that the cash amount available to investor i is

$$W_0^i := n_i \theta_D K_0 + (n_i - \theta_0^i) P_0.$$

The final wealth of each investor is then given by

$$W_1^i := W_0(1 + R_f) + \theta_0^i D_1^i,$$

where D_1^i is the liquidating dividend per share, and the distribution of the rate Π_i depends on agent i's beliefs:

$$D_1^i := \frac{(K_0 - \theta_D K_0 - \theta_{BB} P_0)\Pi_i}{1 - \theta_{BB}}.$$

The utility function of investor i is $U_i(x) := -\exp(-\gamma_i x)$ where we may have $\gamma_1 \neq \gamma_2$. The certainty equivalent of investor i under their given trading strategy θ_0^i, the market price of the firm P_0 and the firm's distribution strategy (θ_D, θ_{BB}) is defined as

$$\begin{aligned}
\mathrm{CE}_i(\theta_0^i; P_0, \theta_D, \theta_{BB}) &:= U_i^{-1}(\mathbb{E}[U_i(W_1^i)|\Pi_i \sim N(\mu_i, \sigma_i^2)]) \\
&= \left(n_i \theta_D K_0 + (n_i - \theta_0^i) P_0\right)(1 + R_f) \\
&\quad + \frac{\theta_0^i (K_0 - \theta_D K_0 - \theta_{BB} P_0)\mu_i}{1 - \theta_{BB}} \\
&\quad - \frac{\sigma_i^2 \gamma_i}{2} \frac{(\theta_0^i)^2 (K_0 - \theta_D K_0 - \theta_{BB} P_0)^2}{(1 - \theta_{BB})^2}.
\end{aligned}$$

With the above notations, we can define a competitive equilibrium featuring heterogeneous investors in analogy to Definition 3.4:

[7] When analyzing an equilibrium model featuring agents with heterogeneous beliefs, it is sometimes possible to aggregate the beliefs and reduce the problem to that with a representative agent. See, for example, Jouini and Napp [32] and Cvitanić et al. [15]. This aggregation is possible in our model at the investor level, but not at the firm level unless the firm treats each investor equally when deciding on the optimal dividend/buyback strategy. See Remark 5.4 and the discussion after Theorem 5.5.

Definition 5.1 Given the firm's distribution strategy $(\theta_D, \theta_{BB}) \in [0,1)^2$, in the case with heterogeneous investors a competitive equilibrium is a tuple of the investors' trading strategies and market price of the firm $(\hat{\theta}_0^1, \hat{\theta}_0^2, \hat{P}_0)$ such that:

1. Both representative investors maximize – for a fixed share price and distribution strategy – the certainty equivalents of their terminal portfolio values:
$$\mathrm{CE}_i(\hat{\theta}_0^i; \hat{P}_0, \theta_D, \theta_{BB}) = \sup_{\theta_0^i \in \mathbb{R}} \mathrm{CE}_i(\theta_0^i; \hat{P}_0, \theta_D, \theta_{BB}), \qquad i \in \{1,2\}.$$
2. The market clears, in that the total demand for the shares is equal to the number of outstanding shares post-buyback:
$$1 = \hat{\theta}_0^1 + \hat{\theta}_0^2 + \theta_{BB}.$$

For $i \in \{1,2\}$, we call $\hat{\mathrm{CE}}_i(\theta_D, \theta_{BB}) := \mathrm{CE}_i(\hat{\theta}_0^i; \hat{P}_0, \theta_D, \theta_{BB})$ the *equilibrium value function of investor i* under the distribution strategy (θ_D, θ_{BB}). We also write $\hat{\mathrm{CE}}_{i,D}(\theta_D) := \hat{\mathrm{CE}}_i(\theta_D, \theta_{BB} = 0)$ and $\hat{\mathrm{CE}}_{i,BB}(\theta_{BB}) := \hat{\mathrm{CE}}_i(\theta_D = 0, \theta_{BB})$ for investor i's equilibrium value functions under a (pure) dividend strategy and a (pure) buyback strategy, respectively.

Like for the baseline model, we focus without loss of generality on the two polar cases where the firm implements a pure dividend strategy $(\theta_D, \theta_{BB} = 0)$ or a pure buyback strategy $(\theta_D = 0, \theta_{BB})$. As before, $\hat{P}_0(\theta_D)$ and $\hat{P}_0(\theta_{BB})$ denote the equilibrium share prices under a given dividend and buyback strategy, respectively.

Define the constants
$$\phi := \frac{\frac{\mu_1}{\sigma_1^2\gamma_1} + \frac{\mu_2}{\sigma_2^2\gamma_2}}{\frac{1}{\sigma_1^2\gamma_1} + \frac{1}{\sigma_2^2\gamma_2}}, \qquad \lambda := \frac{2}{\frac{1}{\sigma_1^2\gamma_1} + \frac{1}{\sigma_2^2\gamma_2}}.$$
Here, $\lambda > 0$ can be interpreted as the (harmonic) average of the perceived risk of the firm, while ϕ represents the perceived risk-weighted profitability. As for the single-agent model in Assumption 3.3, we now introduce an assumption on the model parameters for the current two-investor model as to ensure that an equilibrium exists with endogenized dividend/buyback ratio between zero and one:

Assumption 5.2 The model parameters are such that
$$0 < \max\left(\frac{\lambda}{2}K_0, 1 + R_f\right) < \phi < \frac{\lambda K_0}{2} + 1 + R_f. \tag{5.1}$$

The unique competitive and consistent[8] equilibrium can be characterized for both distributions methods, where the proposition below is analogous to Proposition 3.5 and Corollary 3.9.

Proposition 5.3 *Suppose Assumption 5.2 holds.*

1. *Under a dividend strategy* $(\theta_D, \theta_{BB} = 0)$ *with* $\theta_D \in [0,1)$, *the unique competitive equilibrium is given by*
$$\hat{P}_0(\theta_D) = \frac{K_0(1-\theta_D)}{1+R_f}\left(\phi - \frac{\lambda K_0(1-\theta_D)}{2}\right)$$

[8] In the multi-investor case, the definition of consistent family of competitive equilibrium under buyback can be defined in the same way as in Definition 3.8.

and

$$\hat{\theta}_0^i = \frac{\mu_i - \phi}{\sigma_i^2 \gamma_i K_0 (1-\theta_D)} + \frac{\lambda}{2\sigma_i^2 \gamma_i}, \qquad i \in \{1, 2\}.$$

2. *Under a buyback strategy* $(\theta_D = 0, \theta_{BB})$ *with* $\theta_{BB} \in [0, 1)$*, the unique consistent family of competitive equilibrium is given by* $\theta_{BB} \mapsto (\hat{\theta}_0^1(\theta_{BB}), \hat{\theta}_0^2(\theta_{BB}), \hat{P}_0^+(\theta_{BB}))$ *where*

$$\hat{P}_0^+(\theta_{BB}) = \begin{cases} \frac{\lambda\theta_{BB}K_0 - \phi\theta_{BB} - (1+R_f)(1-\theta_{BB}) + \frac{1}{2}\sqrt{\Delta(\theta_{BB})}}{\lambda\theta_{BB}^2}, & \theta_{BB} \in (0, 1); \\ \frac{K_0}{1+R_f}\left(\phi - \frac{\lambda K_0}{2}\right), & \theta_{BB} = 0, \end{cases} \tag{5.2}$$

and

$$\hat{\theta}_0^i(\theta_{BB}) = \frac{1-\theta_B}{\sigma_i^2 \gamma_i (K_0 - \hat{P}_0^+(\theta_{BB})\theta_{BB})}\left[\mu_i - \frac{\hat{P}_0^+(\theta_{BB})(1+R_f)(1-\theta_{BB})}{K_0 - \hat{P}_0^+(\theta_{BB})\theta_{BB}}\right], i \in \{1, 2\},$$

where

$$\Delta(\theta_{BB}) := 4\left\{\left[\phi\theta_{BB} + (1+R_f)(1-\theta_{BB})\right]^2 - 2\lambda(1+R_f)K_0\theta_{BB}(1-\theta_{BB})\right\}.$$

With two representative investors, we can consider the aggregate welfare as a *weighted sum* of their certain equivalents defined via

$$\overline{\text{CE}}_k(\theta_k) := \alpha_1 \hat{\text{CE}}_{1,k}(\theta_k) + \alpha_2 \hat{\text{CE}}_{2,k}(\theta_k) \tag{5.3}$$

under distribution method $k \in \{D, BB\}$. Here, $\alpha_1 > 0$ and $\alpha_2 > 0$ are two exogenously given constants representing the importance of each investor. Without loss of generality, we assume $\alpha_1 \leq \alpha_2$. A natural choice is $\alpha_1 = \alpha_2$ such that each investor has equal importance to the firm, but the firm can also assign a larger weight to one particular investor (e.g., the major shareholder who can exert higher influence on the firm decisions.)

As in the baseline case, the firm can then maximize $\overline{\text{CE}}_k(\theta_k)$ by varying the optimal dividend payout or shares buyback ratio $\theta_k \in [0, 1)$. We denote the corresponding optimizer by θ_k^* and the associated equilibrium share price by $\hat{P}_0^k = \hat{P}_0(\theta_k^*)$ for $k \in \{D, BB\}$. The following result shows that the equivalence of dividend and buyback still holds under heterogeneous beliefs of the investors.

Remark 5.4 In the analysis of partial equilibrium under an exogenously given distribution strategy in Proposition 5.3, the two heterogeneous investors can be summarized by a single investor with utility function $U(x) = -\exp(-x)$ whose belief over the risky technology return is $N(\phi, \lambda/2)$. Indeed, one can see that the expressions of $\hat{P}_0(\theta_D)$ and $\hat{P}_0^+(\theta_{BB})$ in Proposition 5.3 agree with those in Proposition 3.5 and Corollary 3.9 upon replacing μ by ϕ and $\sigma^2\gamma$ by $\lambda/2$. When the firm endogenizes dividend/buyback decision using (5.3) as the objective function, however, the optimal strategy θ_k^* in general depends on each investor's characteristics such that many economically important quantities under general equilibrium cannot be simply summarized by ϕ and $\lambda/2$ (unless we are in the special case of $\alpha_1 = \alpha_2$). However, the equivalence between optimal buybacks and dividends nevertheless remains true.

Theorem 5.5 *Suppose Assumption 5.2 holds, and the weighting parameters* $0 < \alpha_1 \leq \alpha_2$ *satisfies*

$$1 \leq \frac{\alpha_2}{\alpha_1} < \min\left(2 - \frac{1+R_f}{\phi}, \frac{\lambda K_0 + \phi}{\frac{\lambda K_0}{2} + 2\phi - 1 - R_f}\right). \tag{5.4}$$

Then with heterogeneous investor beliefs, dividend and buyback are still economically equivalent in terms of (aggregate and individual) attained welfare, the composition of shareholders and the equity value distributed. Specifically,

1. *The aggregate welfare levels are identical:*

$$\overline{\mathrm{CE}}_D(\theta_D^*) = \overline{\mathrm{CE}}_B(\theta_{BB}^*).$$

2. *Individual welfare levels of both investors also coincide:*

$$\hat{\mathrm{CE}}_{i,D}(\theta_D^*) = \hat{\mathrm{CE}}_{i,BB}(\theta_{BB}^*), \qquad i \in \{1,2\}.$$

3. *The equilibrium compositions (i.e., the fractions of outstanding shares owned by the two types of shareholders) are identical:*

$$\hat{\theta}_0^{i,D,*} = \frac{\hat{\theta}_0^{i,BB,*}}{1-\theta_{BB}^*}, \qquad i \in \{1,2\}.$$

   Here, $\hat{\theta}_0^{i,k,*} := \hat{\theta}_0(\hat{P}_0^k, \theta_k^*)$ denotes the optimal shares held by investor i when the firm adopts the optimal payout policy θ_k^* for $k \in \{D, BB\}$.

4. *The total equity values redistributed coincide:*

$$K_0\theta_D^* = \theta_B^* \hat{P}_0^{BB}.$$

However, the equilibrium share prices are not necessarily the same where

$$\hat{P}_0^{BB} = \hat{P}_0^D + K_0\theta_D^*.$$

In the special case of $\alpha_1 = \alpha_2$, the optimal dividend yield and buyback ratio are the same such that

$$\theta_D^* = \theta_{BB}^* = 1 - \frac{2(\phi - 1 - R_f)}{\lambda K_0},$$

and the equilibrium share prices are given by $\hat{P}_0^D = \frac{2(\phi-1-R_f)}{\lambda}$ and $\hat{P}_0^{BB} = K_0$.

The condition (5.4) suggests that the firm cannot impose strong differential treatment across the two investors. When $\alpha_1 \neq \alpha_2$ are too different, the firm's optimization problem may not yield a finite value function, and (5.4) is a simple sufficient condition to ensure that a well-defined optimizer θ_k^* exists for each distribution method k. Note that, under Assumption 5.2, the rightmost expression in (5.4) is strictly above one. Thus (5.4) holds when $\alpha_1 = \alpha_2$, and this special case leads to $\theta_D^* = \theta_D^*$ as well as $\hat{P}_0^D$ and $\hat{P}_{BB}^0$ being simple functions of $(\phi, \lambda, R_f, K_0)$ only. The expressions of these equilibrium quantities also coincide with those in Theorem 4.1 upon replacing ϕ by μ and $\lambda/2$ by $\sigma^2\gamma$. The two investors can then be effectively summarized by a single representative investor as per Remark 5.4 but this is no longer true when $\alpha_1 \neq \alpha_2$. See the analytical expressions of θ_D^* in equation (A.9) in the Appendix.

The results collected in Theorem 5.5 generally mirror those in Theorem 4.1, except that we do not necessarily have $\theta_D^* = \theta_{BB}^*$ nor $\hat{P}_{BB}^0 = K_0$ when $\alpha_1 \neq \alpha_2$. A new and interesting observation here is that optimal dividends and buybacks not only lead to the same welfare in the aggregate, but also for each individual type of investor. Although the firm determines the optimal dividend/buyback policy by solely maximizing the aggregate welfare across the two types of investors, the optimal distribution strategy is such that both investors are indifferent whether it is implemented using dividends or buybacks. Another new result here is that the choice of dividend versus buyback also has no impact on the equilibrium composition of the shareholders, in the sense that investor i always represents the same proportion among all shareholders regardless of whether dividend or buyback is adopted. The only difference again is the equilibrium share price, where buyback results in a higher price in general because $\theta_D^* > 0$ under the current assumptions on the model parameters. The economic interpretation of this observation is the same as that in the baseline model in Section 4.

5.2 Consumption and endogenous riskfree rate

In this section, we revert to a single-investor setup but now also endogenize the riskfree rate, by letting the investor not only optimize their portfolio but also the split of their consumption between periods $t = 0$ and $t = 1$. To this end, we first describe how the portfolio dynamics in Section 3 change in presence of consumption. In addition to choosing the risky holdings $\theta_0 \in \mathbb{R}$, the representative investor now also consumes a cash amount of $C_0 \in \mathbb{R}$. Their residual cash position at time $t = 0$ is then given by

$$W_0 := \theta_D K_0 + (1 - \theta_0) P_0 - C_0,$$

which grows at the riskfree rate R_f (to be endogenized).

At $t = 1$, the investor receives the terminal dividend and consumes all the available cash. The consumption level at $t = 1$ is hence equal to

$$\begin{aligned} C_1 = W_1 &:= W_0(1 + R_f) + \theta_0 D_1 \\ &= (\theta_D K_0 + (1 - \theta_0) P_0 - C_0)(1 + R_f) + \theta_0 \frac{(K_0 - \theta_D K_0 - \theta_{BB} P_0)\Pi}{1 - \theta_{BB}}. \end{aligned}$$

The representative investor now derives utility from intertemporal consumption (C_0, C_1). Their discounted expected utility under a given investment-consumption strategy (θ_0, C_0), the market price of firm P_0, the prevailing riskfree rate R_f and the firm's distribution strategy (θ_D, θ_{BB}) is now defined as

$$J(\theta_0, C_0; P_0, R_f, \theta_D, \theta_{BB}) := U(C_0) + e^{-\beta} \mathbb{E}[U(C_1)], \tag{5.5}$$

where $\beta > 0$ is the one-period subjective discount rate of the investor and as before we take $U(x) := -\exp(-\gamma x)$.

We now introduce a slightly different assumption on the model parameters which will only be used for this subsection. The motivation behind again is to ensure the existence of an equilibrium and that the firm adopt a distribution ratio between zero and one.

Assumption 5.6 For the model with consumption, the model parameters satisfy

$$0 < \max(e^{\beta - \gamma K_0}, K_0 \gamma \sigma^2) < \mu, \qquad e^{\beta} > (\mu - \gamma \sigma^2 K_0) e^{-\gamma K_0 \mu + \frac{\sigma^2 \gamma}{2} K_0^2 + \gamma K_0}. \tag{5.6}$$

We now give the definition of the competitive equilibrium under which the riskfree rate is endogenized as well by matching the investors' demand to the zero net supply of savings account:

Definition 5.7 Given the firm's distribution strategy $(\theta_D, \theta_{BB}) \in [0, 1)^2$, a competitive equilibrium with endogenized riskfree rate is the collection $(\hat{\theta}_0, \hat{C}_0, \hat{P}_0, \hat{R}_f)$ such that:

1. The representative investor maximizes (under fixed share price, interest rate and distribution strategy) their expected utility, i.e.:

$$J(\hat{\theta}_0, \hat{C}_0; \hat{P}_0, \hat{R}_f, \theta_D, \theta_{BB}) = \sup_{(\theta_0, C_0) \in \mathbb{R}^2} J(\theta_0, C_0; \hat{P}_0, \hat{R}_f, \theta_D, \theta_{BB}),$$

   where J is defined in (5.5).
2. The stock market clears:

$$1 = \hat{\theta}_0 + \theta_{BB}.$$

3. The money market clears:

$$0 = W_0 = \theta_D K_0 + (1 - \hat{\theta}_0)\hat{P}_0 - \hat{C}_0.$$

We once again call $\hat{J}(\theta_D, \theta_{BB}) := J(\hat{\theta}_0, \hat{C}_0; \hat{P}_0, \hat{R}_f, \theta_D, \theta_{BB})$ the *equilibrium value function* under the distribution strategy (θ_D, θ_{BB}). $\hat{J}_D(\theta_D)$ and $\hat{J}_{BB}(\theta_{BB})$ are defined similarly as in Definition 3.4.

Proposition 5.8 *Suppose Assumption 5.6 holds. For the problem with consumption and endogenous riskfree rate:*

1. *Under a dividend strategy* $(\theta_D, \theta_{BB} = 0)$ *with* $\theta_D \in [0, 1)$*, the unique competitive equilibrium is given by* $(\hat{\theta}_0, \hat{C}_0, \hat{P}_0, \hat{R}_f) = (\hat{\theta}_0(\theta_D), \hat{C}_0(\theta_D), \hat{P}_0(\theta_D), \hat{R}_f(\theta_D))$*, where* $\hat{\theta}_0(\theta_D) = 1$*,*

$$\hat{C}_0(\theta_D) = \theta_D K_0, \qquad \hat{R}_f(\theta_D) = \exp\left(\beta + \gamma K_0 \left(\mu(1 - \theta_D) - \theta_D - \frac{\sigma^2 \gamma}{2} K_0 (1 - \theta_D)^2\right)\right) - 1,$$

   and

$$\hat{P}_0(\theta_D) = \frac{K_0(1 - \theta_D)}{1 + \hat{R}_f(\theta_D)} \left(\mu - \sigma^2 \gamma K_0 (1 - \theta_D)\right).$$

2. *Consider a buyback strategy* $(\theta_D = 0, \theta_{BB})$ *with* $\theta_{BB} \in [0, 1)$*. Subject to an additional technical assumption that a certain equation admits a unique solution (see Assumption A.3 in the appendix), there exists a unique solution* $(\hat{\theta}_0, \hat{C}_0, \hat{P}_0, \hat{R}_f) = (\hat{\theta}_0(\theta_{BB}), \hat{C}_0(\theta_{BB}), \hat{P}_0(\theta_{BB}), \hat{R}_f(\theta_{BB}))$ *to the system of equations*

$$\hat{\theta}_0 = 1 - \theta_{BB},$$
$$\hat{P}_0 = \frac{2\sigma^2 \gamma \theta_{BB} K_0 - \mu \theta_{BB} - (1 + \hat{R}_f)(1 - \theta_{BB}) + \sqrt{\Delta}}{2\sigma^2 \gamma \theta_{BB}^2}, \qquad \hat{C}_0 = \theta_{BB} \hat{P}_0,$$

   and

$$\hat{R}_f = \exp\left(\beta + \gamma \mu (K_0 - \theta_{BB} \hat{P}_0) - \gamma \theta_{BB} \hat{P}_0 - \frac{\gamma^2 \sigma^2}{2} (K_0 - \theta_{BB} \hat{P}_0)^2\right) - 1,$$

   where

$$\Delta := \left[\mu \theta_{BB} + (1 + \hat{R}_f)(1 - \theta_{BB})\right]^2 - 4\sigma^2 \gamma (1 + \hat{R}_f) K_0 \theta_{BB} (1 - \theta_{BB}).$$

Moreover, $\theta_{BB} \mapsto (\hat{\theta}_0(\theta_{BB}), \hat{C}_0(\theta_{BB}), \hat{P}_0(\theta_{BB}), \hat{R}_f(\theta_{BB}))$ *is the unique consistent family of competitive equilibrium under pure buyback strategies.*[9] *For the special case* $\theta_{BB} = 0$*, this leads to the explicit formulas*

$$\hat{\theta}_0(\theta_{BB} = 0) = 1, \qquad \hat{C}_0(\theta_{BB} = 0) = 0, \qquad \hat{R}_f(\theta_{BB} = 0)$$
$$= \exp\left(\beta + \gamma K_0 \left(\mu - \frac{\sigma^2 \gamma}{2} K_0\right)\right) - 1,$$

and

$$\hat{P}_0(\theta_{BB} = 0) = \frac{K_0}{1 + \hat{R}_f} \left(\mu - \sigma^2 \gamma K_0\right).$$

Unlike for the baseline model, we now require an additional technical assumption to establish the uniqueness of the competitive consistent equilibrium under buybacks. In particular, we cannot theoretically confirm the construction of the equilibrium proposed in Part 2 of Proposition 5.8 is always unique. However, we have strong numerical evidence that the additional Assumption A.3 in the appendix required for uniqueness holds in all parameter constellations we have explored. Subject to this caveat, we can again define *the* equilibrium value function under buybacks as

$$\hat{J}_{BB}(\theta_{BB}) := J(\hat{\theta}_0(\theta_{BB}), \hat{C}_0(\theta_{BB}), \hat{P}_0(\theta_{BB}), \hat{R}_f(\theta_{BB}), \theta_D = 0, \theta_{BB}).$$

The firm can now determine the optimal distribution policy $\theta_k \in [0, 1)$ to maximize $\hat{\text{CE}}_k(\theta_k)$ for $k \in \{D, BB\}$. Let the optimizer be θ_k^*, and denote the corresponding equilibrium share price and riskfree rate by $\hat{P}_0^k := \hat{P}_0(\theta_k^*)$ and $\hat{R}_f^k := \hat{R}_f(\theta_k^*)$. The next theorem shows that the equivalence of dividend and buyback still holds even if we endogenize the riskfree rate as well.

Theorem 5.9 *Suppose Assumption 5.6 and Assumption A.3 in the appendix hold. In an economy with endogenous riskfree rate, dividends and buybacks are economically equivalent in terms of shareholder welfare, the ratio of distribution, the equity value distributed and the endogenous riskfree rate:*

1. *The representative shareholder's optimal certainty equivalents are the same:*

$$\sup_{\theta_D \in [0,1)} \hat{\text{CE}}_D(\theta_D) = \sup_{\theta_{BB} \in [0,1)} \hat{\text{CE}}_{BB}(\theta_{BB}).$$

2. *The optimal dividend yield and share buyback ratio coincide:*

$$\theta_D^* = \theta_{BB}^*,$$

where θ_D^* *is given by the unique solution to the equation*

$$1 - \left[\mu - \gamma \sigma^2 K_0 (1 - \theta_D)\right] \exp\left(-\beta - \gamma K_0 \mu (1 - \theta_D) + \frac{\gamma^2 \sigma^2}{2} K_0^2 (1 - \theta_D)^2 + \gamma K_0 \theta_D\right) = 0. \qquad (5.7)$$

3. *The total values of equity distributed are the same:*

$$K_0 \theta_D^* = \theta_B^* \hat{P}_0^{BB}.$$

[9] Consistency is again defined in analogy to Definition 3.8.

4. *The equilibrium riskfree rates coincide:*

$$\hat{R}_f^D = \hat{R}_f^{BB}.$$

However, the equilibrium share prices are not necessarily the same: $\hat{P}_0^B = K_0$ *but*

$$\hat{P}_0^D = \hat{P}_0^{BB}(1-\theta_D^*) = \hat{P}_0^{BB}(1-\theta_{BB}^*).$$

With an endogenized riskfree rate, we are no longer able to derive the optimal dividend yield or buyback ratio explicitly but it is still conveniently characterized by the unique solution to the scalar equation (5.7). The equilibrium interest rate is also not available in closed-form. Nonetheless, the key insights of Theorem 4.1 are still applicable even when the investor's objective is a more complex consumption-investment problem.

6 Extensions for which dividend-buyback equivalence breaks down

6.1 Managers with stock options

We have shown (for the baseline model and a few extensions) that dividends and share buybacks are equivalent in terms of shareholder welfare. When the managers of the firm only consider shareholder value, they are therefore indifferent towards either method of distribution.

In this section, we consider again our baseline model in Section 3, but the firm managers are now initially endowed with $\zeta > 0$ units of employee call options. We assume the number of options is small relative to the total number of outstanding shares of the firm. Managers therefore do not internalize the impact on the supply of shares caused by the exercise of these options. Similarly, investors' portfolio decision and equilibrium price formation do not take the firm's issuance costs for the options into account.

These options mature at $t = 1$, with a payoff that is contingent on the terminal share price of the firm (i.e., the liquidating dividend per share). The strike price of each option is denoted by κ. Under the optimal pure dividend policy, the terminal share price is $D_1 = K_0(1-\theta_D^*)\Pi$ in equilibrium. Under the optimal pure buyback policy, the terminal share price instead is

$$\frac{D_1}{1-\theta_{BB}^*} = \frac{(K_0 - \theta_{BB}^*\hat{P}_0^{BB})\Pi}{1-\theta_{BB}^*} = K_0\Pi,$$

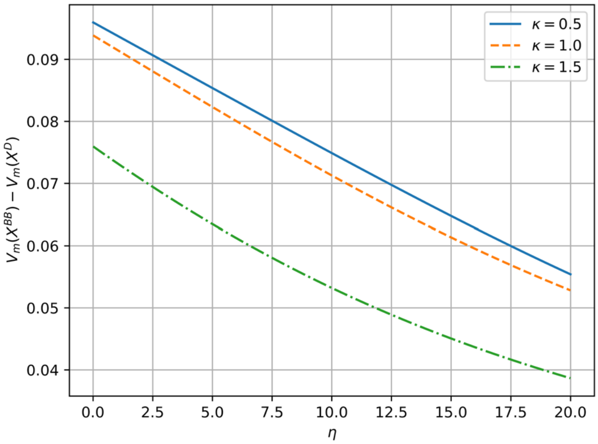
where we have used the results in Theorem 4.1 that $\theta_D^* = \theta_{BB}^*$ and $\theta_{BB}^*\hat{P}_0^{BB} = K_0\theta_D^*$. The total payoff of these ζ units of call option under distribution method $k \in \{D, BB\}$ is therefore

$$X^k := \begin{cases} \zeta(K_0(1-\theta_D^*)\Pi - \kappa)^+, & k = D; \\ \zeta(K_0\Pi - \kappa)^+, & k = BB. \end{cases} \tag{6.1}$$

Suppose managers value these options by some functional $\mathcal{V} : \mathcal{L} \to \mathbb{R}$, where $\mathcal{L}$ denotes a set of random variables. Since shareholders are indifferent between optimal dividends or buybacks, the managers will select the distribution method that gives rise to the higher value of $\mathcal{V}(X^D)$ or $\mathcal{V}(X^{BB})$.

Recall that $\theta_D^* \in (0, 1)$. Hence, we have $\mathbb{P}(X^{BB} > X^D) = 1$ provided that $\kappa > 0$. As a consequence:

Fig. 1 Differences of managers' utility indifference valuation of the options between buyback and dividend as a function of managerial risk aversion η under several strike levels κ. The parameters used are $\mu = 2$, $\sigma = 0.5$, $R_f = 0.05$, $\gamma = 5$, $K_0 = 1$ and $\zeta = 0.2$



Corollary 6.1 *Suppose $\kappa > 0$ and the managers' option valuation rule $\mathcal{V}(\cdot)$ is strictly monotone.*[10] *Then $\mathcal{V}(X^{BB}) > \mathcal{V}(X^D)$.*

The managers' payoff under optimal dividends and buybacks is represented by some call options written on $K_0(1-\theta_D^*)\Pi$ and $K_0\Pi$, respectively. In the realistic case of a strictly positive strike, the payoff under buybacks therefore unanimously dominates the one with dividends. Corollary 6.1 in turn suggests that any sensible valuation rule used by the managers which satisfies strict monotonicity (e.g., risk neutral valuation or utility indifference pricing with non-degenerate utility function) renders buybacks more attractive.

As a more concrete illustration, we specialize the general setup to a particular managerial option valuation rule. To wit, suppose the managers' utility function is $U_m(x) := -\exp(-\eta x)$ where $\eta > 0$ is their absolute risk aversion coefficient. Suppose they have some reference wealth of w_0 and they have no access to trading in the financial market. Then, the utility indifference price of a payoff X is defined as the constant c which solves the equation

$$U_m(w_0 + c) = \mathbb{E}[U_m(w_0 + X)].$$

Their valuation rule is then given by

$$\mathcal{V}_m(X) := c = U_m^{-1}\left[\mathbb{E}[U_m(w_0 + X)]\right] - w_0 = -\frac{1}{\eta}\ln \mathbb{E}[e^{-\eta X}]. \tag{6.2}$$

Lemma 6.2 *Consider the valuation rule in (6.2). For a contingent claim in form of $X := (Y-\kappa)^+$ where Y is a normally distributed random variable with mean $a \in \mathbb{R}$ and standard deviation $b > 0$, and $\kappa \in \mathbb{R}$ is a constant, we have*

$$\mathcal{V}_m(X) = -\frac{1}{\eta}\ln\left\{\exp\left(\frac{\eta^2 b^2}{2} - \eta(a-\kappa)\right)\Phi\left(-\frac{\kappa - a}{b} - \eta b\right) + \Phi\left(\frac{\kappa - a}{b}\right)\right\},$$

where $\phi(\cdot)$ and $\Phi(\cdot)$ are respectively the pdf and cdf of a standard normal distribution.

Using Lemma 6.2 in conjunction with (6.1), we can compute the values of the vested options given optimal buybacks and dividend,s denoted by $\mathcal{V}_m(X^{BB})$ and $\mathcal{V}_m(X^D)$, respectively. In Figure 1, we show the values of $\mathcal{V}_m(X^{BB}) - \mathcal{V}_m(X^D)$ for several values of managerial

[10] That is, $\mathcal{V}(Y_2) > \mathcal{V}(Y_1)$ for any random variables Y_1 and Y_2 with $\mathbb{P}(Y_2 \geq Y_1) = 1$ and $\mathbb{P}(Y_2 > Y_1) > 0$.

risk aversion η and option strike level $\kappa > 0$. We always observe $\mathcal{V}_m(X^{BB}) > \mathcal{V}_m(X^D)$ such that managers (with any size of endowed options) always prefer buybacks to dividends. This observation is consistent with Corollary 6.1. The economic intuition behind is simple: The current share price can be boosted by reducing the supply of shares via a buyback program (whereas dividend payout does not change of the supply of shares), which in turn translates to a higher terminal share price and produces a strictly better payoff to the option holders. Figure 1 also shows that the difference $\mathcal{V}_m(X^{BB}) - \mathcal{V}_m(X^D)$ decreases as η increases. This suggests that the attractiveness of buybacks (relative to dividends) becomes less pronounced to managers with higher risk aversion.

Instead of considering identical dollar strike κ across the two distribution methods, one can also consider at-the-money option such that we take $\kappa = \hat{P}_0^D$ for dividends and $\kappa = \hat{P}_0^{BB}$ for buybacks. Using (6.1) and the fact that $\hat{P}_0^D = (1-\theta_D^*)K_0 = (1-\theta_D^*)\hat{P}_0^{BB}$, one can easily deduce

$$X^D = (1-\theta_D^*)X^{BB}$$

such that the option payoff under dividends is always a strictly smaller fraction of that under buybacks. Hence the managers will also unanimously prefer buybacks to dividends if options are issued at-the-money (struck at the endogenized share price).

The simple analysis above highlights the benefit of buybacks as a mechanism to boost the stock price and in turn the value of employee options. There is scope for future research to extend the current model which might reveal a more non-trivial tradeoff between dividends and buybacks. As an example, one could explore a setup in which managers determine the optimal dividends (and/or) buybacks policy by directly maximizing the expected payoff from their endowed options, while investors' utility is only required to stay above some threshold as a soft governance constraint.

6.2 Investors with trading constraints

In this section, we assume some of the investors are facing institutional trading constraints and therefore cannot rebalance their share holdings throughout the investment horizon. More specifically, we assume there are two representative investors, labeled by $i \in \{1, 2\}$. Investor 1 is required to hold some fixed number of shares at all time points, while Investor 2 is unconstrained. As discussed in the introduction, Investor 1 may represent a fund manager who needs to follow a specific portfolio allocation rule.

We follow the notations introduced in Section 5.1 except we specialize to $\mu = \mu_1 = \mu_2$, $\sigma = \sigma_1 = \sigma_2$, $\gamma = \gamma_1 = \gamma_2$ and $\alpha_1 = \alpha_2 = 1$, i.e., the constrained and unconstrained investors agree on the firm's profitability, have the same risk aversion, and are treated equally by the firm (for simplicity). For the ease of exposition, we also assume investor 1's share holding is fixed at $n_1 = \theta_0^1 = 1/2$ throughout the entire horizon.

Definition 6.3 Given the firm's distribution strategy $(\theta_D, \theta_{BB}) \in [0, 1)^2$, in the case with a constrained investor a competitive equilibrium is a pair of the unconstrained investor's trading strategy and the market price of the firm $(\hat{\theta}_0^2, \hat{P}_0)$ such that:

1. Investor 2 (the unconstrained investor) maximizes the certainty equivalent of their terminal portfolio value:

$$\mathrm{CE}_2(\hat{\theta}_0^2; \hat{P}_0, \theta_D, \theta_{BB}) = \sup_{\theta_0^2 \in \mathbb{R}} \mathrm{CE}_2(\theta_0^2; \hat{P}_0, \theta_D, \theta_{BB}).$$

2. The market clears, i.e., the total demand for the shares is equal to the number of outstanding shares post-distribution, or equivalently

$$1 = \frac{1}{2} + \hat{\theta}_0^2 + \theta_{BB}.$$

Investor i's equilibrium value functions $\hat{\text{CE}}_i(\theta_D, \theta_{BB})$, $\hat{\text{CE}}_{i,D}(\theta_D)$ and $\hat{\text{CE}}_{i,BB}(\theta_{BB})$ are defined as in Definition 5.1.

Proposition 6.4 *In the setup with a constrained investor, a unique (and consistent) competitive equilibrium exists under any exogenously given pure dividend and buyback strategy.*

1. *Under a dividend strategy $(\theta_D, \theta_{BB} = 0)$ with $\theta_D \in [0, 1)$, the unique competitive equilibrium is given by*

$$\hat{\theta}_0^2 = \frac{1}{2}, \qquad \hat{P}_0(\theta_D) = \frac{K_0(1-\theta_D)}{1+R_f}\left(\mu - \frac{\sigma^2\gamma}{2}K_0(1-\theta_D)\right).$$

Moreover, the optimal dividend level which maximizes the sum of certainty equilvanet of the two investors is given by

$$\theta_D^* = 1 - 2\frac{\mu - (1+R_f)}{K_0\sigma^2\gamma}$$

such that $\overline{\text{CE}}_D(\theta_D^*) = \sup_{\theta_D \in [0,1)} \hat{\text{CE}}_D(\theta_D)$.

2. *Under a buyback strategy $(\theta_D = 0, \theta_{BB})$ with $\theta_{BB} \in [0, \infty)$, the unique consistent family of competitive equilibrium is given by $\theta_{BB} \mapsto (\hat{\theta}_0^2(\theta_{BB}), \hat{P}_0(\theta_{BB}))$ where*

$$\hat{\theta}_0^2(\theta_{BB}) = \frac{1}{2} - \theta_{BB},$$
$$\hat{P}_0(\theta_{BB}) = \frac{2\gamma\sigma^2 K_0\theta_{BB}(1-2\theta_{BB}) - 2(1-\theta_{BB})[\mu\theta_{BB} + (1+R_f)(1-\theta_{BB})] + \sqrt{\Delta(\theta_{BB})}}{2\gamma\sigma^2\theta_{BB}^2(1-2\theta_{BB})},$$

with

$$\Delta(\theta_{BB}) := 4(1-\theta_{BB})^2[\mu\theta_{BB} + (1+R_f)(1-\theta_{BB})]^2 - 8K_0\gamma\sigma^2(1+R_f)\theta_{BB}(1-2\theta_{BB})(1-\theta_{BB})^2.$$

As in Section 5.1, the firm can optimize the decision of dividend payout or shares buyback via maximizing $\overline{\text{CE}}_k(\theta_k) := \hat{\text{CE}}_{1,k}(\theta_k) + \hat{\text{CE}}_{2,k}(\theta_k)$ the sum of certainty equivalents of the two investors (recall that we have chosen $\alpha_1 = \alpha_2 = 1$). While analytical solutions are not available for the firm's maximization problem under buybacks, it is numerically straightforward to compute $\sup_{\theta_i} \hat{\text{CE}}_i(\theta_i)$ and the associated optimizer for $i \in \{D, BB\}$.

Unlike what we have established in Theorem 4.1, Theorem 5.5 and Theorem 5.9, with a constrained investor we actually observe that, generally,

$$\sup_{\theta_D \in [0,1)} \overline{\text{CE}}_D(\theta_D) \neq \sup_{\theta_{BB} \in [0,1)} \overline{\text{CE}}_B(\theta_{BB}).$$

In Figure 2, we plot the values of $\hat{\text{CE}}_{i,k}(\theta_k)$ and $\overline{\text{CE}}_k(\theta_k)$ as functions of θ_k for $k \in \{D, BB\}$ and $i \in \{1, 2\}$. There are several important observations. First, in contrast to Part 2 of Theorem 5.5, the constrained and unconstrained investor in the current setup disagree about the optimal buyback ratio. For example, the unconstrained investor would prefer a buyback ratio of 22%, whereas the constrained one would prefer a buyback ratio of 2.5%. The "social planner"

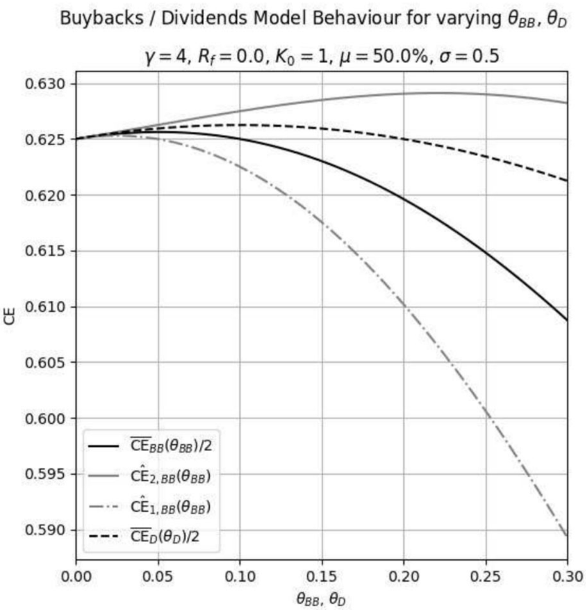


Fig. 2 The individual and average certainty equivalents ($\hat{\mathrm{CE}}_{i,k}(\theta_k)$ and $\frac{1}{2}\overline{\mathrm{CE}}_k(\theta_k)$) under different distribution methods. Recall that Investor 1 faces trading constraints while Investor 2 can trade freely. The parameters used are $\mu = 1.5$, $\sigma = 0.5$, $R_f = 0$, $\gamma = 4$ and $K_0 = 1$. Note that we have $\hat{\mathrm{CE}}_{1,D}(\theta_D) \equiv \hat{\mathrm{CE}}_{2,D}(\theta_D)$ for all θ_D such that a single plot of $\frac{1}{2}\overline{\mathrm{CE}}_D(\theta_D)$ is sufficient to summarize both the individual and combined welfare under dividends

(i.e., the firm) therefore needs to coordinate the preferences of the two types of investors by choosing a middle ground (5.1% in this example) between the two preferred values of the individuals. If dividends are adopted instead, then both investors will unanimously agree that the optimal dividend payout ratio is 10%. Second, the attained maximal aggregate welfare is not the same across the two methods of distribution. In particular, for this numerical example buyback does lead to a lower aggregate welfare. Nonetheless, there also exists configuration in which the ordering is reversed.

The trading constraint shuts down the risk sharing channel between the two investors. If the firm executes buybacks, then market clearing implies all the shares must be bought back from the unconstrained investors in equilibrium. This subsequently increases the constrained investor's relative stake in the firm, so that they become more exposed to the firm's risky investment. The only way to reduce the risk faced by the constrained investor (if this is desirable) is that the firm actively deleverages, but this will create a larger capital surplus such that the value of equity to be distributed might overshoot the optimum level (from the perspective of the unconstrained investor). This creates a non-trivial tradeoff. In contrast, dividends payout facilitate transfer of wealth from the firm back to the shareholders without altering the relative ownership structure of the firm. Whether this feature is desirable or not depends on the model parameters and the required holdings of the constrained investor.

7 Conclusion

We present a novel general equilibrium model that integrates investors' portfolio choice, asset pricing, and firms' optimal payout policies. Our framework provides a theoretical basis for distinguishing between dividends and share buybacks as payout methods. While many classical insights from the Miller-Modigliani irrelevance principle remain valid within our model, an important new finding emerges: buybacks generally lead to a higher equilibrium market price of the firm. When managers have incentives tied directly to the firm's share price, they may exhibit a strict preference for one payout method over the other, breaking the equivalence posited by the irrelevance principle. Employee stock option is considered as an example under which managers prefer buybacks to dividends. When some of the shareholders cannot freely rebalance their portfolios, buybacks might suboptimally change the relative ownership structure of the firm in which case dividends can be the preferred distribution method.

Appendix

A Proofs

Proof of Proposition 3.5 For fixed P_0 and $(\theta_D, \theta_{BB} = 0)$ with $\theta_D \in [0, 1)$, (3.1) is a strictly concave, quadratic function of θ_0. Then we have

$$\mathrm{CE}(\theta_0;\, P_0, \theta_D, \theta_{BB} = 0) \le \mathrm{CE}(\theta_0^*(P_0, \theta_D);\, P_0, \theta_D, \theta_{BB} = 0)$$

for all $\theta_0 \in \mathbb{R}$, where

$$\theta_0^*(P_0, \theta_D) := \frac{1}{\gamma\sigma^2 K_0(1-\theta_D)}\left(\mu - \frac{P_0(1+R_f)}{K_0(1-\theta_D)}\right).$$

Hence the competitive equilibrium $(\hat{\theta}_0, \hat{P}_0)$ must satisfy $\hat{\theta}_0 = \theta_0^*(\hat{P}_0, \theta_D)$. But then the market clearing condition $\hat{\theta}_0 = \theta_0^*(\hat{P}_0, \theta_D) = 1$ implies that the equilibrium share price $\hat{P}_0 = \hat{P}_0(\theta_D)$ satisfies

$$\frac{1}{\gamma\sigma^2 K_0(1-\theta_D)}\left(\mu - \frac{\hat{P}_0(1+R_f)}{K_0(1-\theta_D)}\right) = 1.$$

Hence, the equilibrium price $\hat{P}_0(\theta_D)$ is indeed uniquely given by

$$\hat{P}_0(\theta_D) = \frac{K_0(1-\theta_D)}{1+R_f}\left(\mu - \sigma^2\gamma K_0(1-\theta_D)\right).$$

□

Proof of Proposition 3.6 To ease notation, we simply write $\hat{P}_0 = \hat{P}_0(\theta_{BB})$ throughout this proof. We first argue that it is not possible to have an equilibrium share price given by $\hat{P}_0 = K_0/\theta_{BB}$. When $P_0 = K_0/\theta_{BB}$, (3.1) becomes

$$\mathrm{CE}\left(\theta_0;\, P_0 = \frac{K_0}{\theta_{BB}}, \theta_{BB}, \theta_D = 0\right) = \left(W_{-1} + (1-\theta_0)\frac{K_0}{\theta_{BB}}\right)(1+R_f),$$

in which case the supremum of CE over θ_0 becomes infinite and hence it cannot represent an equilibrium.

It is therefore sufficient to search for an equilibrium price such that $\hat{P}_0 \neq K_0/\theta_{BB}$. For $K_0 \neq \theta_{BB} P_0$ and $\theta_D = 0$, (3.1) is a strictly concave and quadratic function in θ_0 such that

$$\mathrm{CE}(\theta_0;\, P_0, \theta_D = 0, \theta_{BB}) \le \mathrm{CE}(\theta^*(P_0, \theta_{BB});\, P_0, \theta_D = 0, \theta_{BB})$$

for all $\theta_0 \in \mathbb{R}$, where

$$\theta_0^*(P_0, \theta_{BB}) := \frac{1-\theta_{BB}}{\gamma\sigma^2(K_0 - \theta_{BB}P_0)}\left(\mu - \frac{P_0(1+R_f)(1-\theta_{BB})}{K_0 - \theta_{BB}P_0}\right).$$

Therefore, any competitive equilibrium $(\hat{\theta}_0, \hat{P}_0)$ must satisfy $\hat{\theta}_0 = \theta_0^*(\hat{P}_0, \theta_{BB})$. The market clearing condition $\hat{\theta}_0 = 1 - \theta_{BB}$ further leads to an implicit expression for the equilibrium share price:

$$\frac{1}{\gamma\sigma^2(K_0 - \theta_{BB}\hat{P}_0)}\left(\mu - \frac{\hat{P}_0(1+R_f)(1-\theta_{BB})}{K_0 - \theta_{BB}\hat{P}_0}\right) = 1. \tag{A.1}$$

In the case of $\theta_{BB} = 0$, the solution is explicitly given by

$$\hat{P}_0 = \frac{K_0}{1+R_f}(\mu - \sigma^2\gamma K_0).$$

Otherwise if $\theta_{BB} \neq 0$, (A.1) can be expressed as a quadratic equation in $P_0 = \hat{P}_0$ as

$$\theta_{BB}^2\gamma\sigma^2P_0^2 + [\mu\theta_{BB} + (1-\theta_{BB})(1+R_f) - 2K_0\gamma\sigma^2\theta_{BB}]P_0 + [\gamma\sigma^2K_0^2 - \mu K_0] = 0. \tag{A.2}$$

This quadratic equation yields a solution if and only if its discriminant

$$\begin{aligned}\Delta = \Delta(\theta_{BB}) &:= [\mu\theta_{BB} + (1-\theta_{BB})(1+R_f) - 2K_0\gamma\sigma^2\theta_{BB}]^2 - 4\theta_{BB}^2\gamma\sigma^2[\gamma\sigma^2K_0^2 - \mu K_0] \\ &= \left[\mu\theta_{BB} + (1+R_f)(1-\theta_{BB})\right]^2 - 4\sigma^2\gamma(1+R_f)K_0\theta_B(1-\theta_{BB})\end{aligned}$$

is non-negative. Now, we can write $\Delta(\theta_{BB})$ as

$$\begin{aligned}\Delta(\theta_{BB}) &:= \left[\mu\theta_{BB} + (1+R_f)(1-\theta_{BB})\right]^2 - 4\sigma^2\gamma(1+R_f)K_0\theta_{BB}(1-\theta_{BB}) \\ &= a\theta_{BB}^2 + b\theta_{BB} + c,\end{aligned}$$

where

$$\begin{aligned}a &:= (\mu - 1 - R_f)^2 + 4K_0\gamma\sigma^2(1+R_f) > 0, \\ b &:= 2(1+R_f)(\mu - 1 - R_f) - 4K_0\gamma\sigma^2(1+R_f), \\ c &:= (1+R_f)^2.\end{aligned}$$

Under our standing assumption (3.2) such that $\mu > K_0\gamma\sigma^2$ and $1 + R_f > 0$,

$$b^2 - 4ac = 16(1+R_f)^2K_0\gamma\sigma^2(K_0\gamma\sigma^2 - \mu) < 0$$

and hence $\Delta(\theta_{BB})$ must be strictly positive for all θ_{BB}. Therefore, the two expressions in (3.4) are well-defined and are the solutions to (A.2). They therefore constitute the two possible competitive equilibrium prices. □

Proof of Corollary 3.9 For a fixed $\theta_{BB} \neq 0$, Proposition 3.6 show that the only two possible competitive equilibrium share prices are

$$\hat{P}_0^{\pm}(\theta_{BB}) := \frac{2\sigma^2\gamma\theta_{BB}K_0 - \mu\theta_{BB} - (1+R_f)(1-\theta_{BB}) \pm \sqrt{\Delta(\theta_{BB})}}{2\sigma^2\gamma\theta_{BB}^2}.$$

Using L'Hôpital's rule (in conjunction with the standing assumption such that $1 + R_f > 0$), one can verify that

$$\lim_{\theta_{BB}\to 0} \hat{P}_0^+(\theta_{BB}) = \frac{K_0}{1+R_f}(\mu - \sigma^2\gamma K_0), \qquad \lim_{\theta_{BB}\to 0} \hat{P}_0^-(\theta_{BB}) = -\infty.$$

Hence the map $\theta_{BB} \mapsto (\hat{\theta}_0(\theta_{BB}), \hat{P}_0(\theta_{BB}))$ defined in (3.7) is the only possible consistent family of competitive equilibrium. □

Lemma A.1 *For $\theta_{BB} \in \mathbb{R}$, define $g(\theta_{BB}) := \theta_{BB}\hat{P}_0(\theta_{BB})$ where $\hat{P}_0(\theta_{BB})$ is the consistent family of competitive equilibria defined in (3.7). Then $g(\theta_{BB})$ is continuous and strictly increasing on $\mathbb{R}$.*

Proof Although we have insisted $\theta_{BB} \in [0, 1)$ as a part of the modeling assumptions, it is clear that $g(\theta_{BB})$ is well-defined for all $\theta_{BB} \in \mathbb{R}$. The continuity of g is obvious because $\hat{P}_0(\theta_{BB})$ is continuous by its property of consistency.

It is also clear that g is differentiable away from zero. In particular,

$$g'(\theta_{BB}) = \frac{1 + R_f - \Delta(\theta_{BB})^{-\frac{1}{2}}\left(\Delta(\theta_{BB}) - \frac{\theta_{BB}}{2}\Delta'(\theta_{BB})\right)}{2\sigma^2\gamma\theta_{BB}^2}$$

for $\theta_{BB} \neq 0$ where Δ is defined in (3.5). Recalling the notations a, b and c introduced in the proof of Proposition 3.6, we have

$$\begin{aligned}
&\Delta(\theta_{BB})^{\frac{1}{2}}(1 + R_f) - (\Delta(\theta_{BB}) - \frac{\theta_{BB}}{2}\Delta'(\theta_{BB})) > 0\\
&\iff (1 + R_f)\Delta(\theta_{BB})^{\frac{1}{2}} > \left(\Delta(\theta_{BB}) - \frac{\theta_{BB}}{2}\Delta'(\theta_{BB})\right)\\
&\Longleftarrow (1 + R_f)^2\Delta(\theta_{BB}) > \left(\Delta(\theta_{BB}) - \frac{\theta_{BB}}{2}\Delta'(\theta_{BB})\right)^2\\
&\iff c(a\theta_{BB}^2 + b\theta_{BB} + c) > \left(\frac{b}{2}\theta_{BB} + c\right)^2\\
&\iff \left(ac - \frac{b^2}{4}\right)\theta_{BB}^2 > 0\\
&\iff (\mu - K_0\gamma\sigma^2)\theta_{BB}^2 > 0.
\end{aligned}$$

Therefore, $g'(\theta_{BB}) > 0$ for all $\theta_{BB} \neq 0$ because $\mu > K_0\gamma\sigma^2$ by the standing assumption. Hence g is strictly increasing on $\mathbb{R}$. □

Proof of Theorem 4.1 We first solve for the optimal dividend policy and the corresponding equilibrium share price. Using Proposition 3.5 and Definition 3.4, we have

$$\begin{aligned}
\hat{\text{CE}}_D(\theta_D) &= \text{CE}(\hat{\theta}_0; \hat{P}_0(\theta_D), \theta_D, \theta_{BB} = 0)\\
&= \text{CE}\left(1; \frac{K_0(1 - \theta_D)}{1 + R_f}\left(\mu - \sigma^2\gamma K_0(1 - \theta_D)\right), \theta_D, \theta_{BB} = 0\right)\\
&= (W_{-1} + \theta_D K_0)(1 + R_f) + K_0(1 - \theta_D)\mu - \frac{\sigma^2\gamma}{2}K_0^2(1 - \theta_D)^2.
\end{aligned}$$

This is simply a strictly concave quadratic function of θ_D and hence the optimizer is uniquely given by

$$\theta_D^* = 1 - \frac{\mu - (1 + R_f)}{\sigma^2\gamma K_0}.$$

Note that $\theta_D^* \in (0, 1)$ thanks to the standing assumption (3.2) and hence θ_D^* is a feasible optimizer of $\hat{\text{CE}}_D(\theta_D)$ over the range of $\theta_D \in [0, 1)$.

Finally, the corresponding equilibrium price is

$$\hat{P}_0^D = \hat{P}(\theta_D^*) = \frac{K_0(1 - \theta_D^*)}{1 + R_f}\left(\mu - \sigma^2\gamma K_0(1 - \theta_D^*)\right) = \frac{\mu - (1 + R_f)}{\sigma^2\gamma}.$$

Next, recall that the equilibrium value function under buybacks is given by

$$\hat{\text{CE}}_{BB}(\theta_{BB}) = (W_{-1} + \theta_{BB}\hat{P}_0(\theta_{BB}))(1+R_f) + (K_0 - \theta_{BB}\hat{P}_0(\theta_{BB}))\mu - \frac{\sigma^2\gamma}{2}(K_0 - \theta_{BB}\hat{P}_0(\theta_{BB}))^2.$$

We now show that $V_D = V_{BB}$. For any θ_{BB}, we have

$$\begin{aligned} V_D = \hat{\text{CE}}_D(\theta_D^*) &= \sup_{\theta_D \in [0,1)} \hat{\text{CE}}_D(\theta_D) \\ &= \sup_{\theta_D \in \mathbb{R}} \hat{\text{CE}}_D(\theta_D) \\ &\geq \hat{\text{CE}}_D\left(\frac{\theta_{BB}\hat{P}_0(\theta_{BB})}{K_0}\right) \\ &= (\theta_{BB}\hat{P}_0(\theta_{BB}))(1+R_f) + (K_0 - \theta_{BB}\hat{P}_0(\theta_{BB}))\mu \\ &\quad - \frac{\sigma^2\gamma}{2}(K_0 - \theta_{BB}\hat{P}_0(\theta_{BB}))^2 \\ &= \hat{\text{CE}}_{BB}(\theta_{BB}). \end{aligned}$$

Taking the supremum over θ_{BB} on both sides leads to

$$\hat{\text{CE}}_D(\theta_D^*) = V_D \geq \sup_{\theta_{BB} \in [0,1)} \hat{\text{CE}}_{BB}(\theta_{BB}) = V_{BB}. \tag{A.3}$$

To demonstrate that equality holds in (A.3), it is sufficient to show that there exists some $\theta_{BB} \in [0, 1)$ such that

$$\theta_D^* = \frac{\theta_{BB}\hat{P}_0(\theta_{BB})}{K_0}$$

or, equivalently,

$$g(\theta_{BB}) := \theta_{BB}\hat{P}_0(\theta_{BB}) = K_0\theta_D^* = K_0 - \frac{\mu - (1+R_f)}{\sigma^2\gamma}, \tag{A.4}$$

where we have used the closed-form expression of θ_D^* in Proposition 3.5. By Lemma A.1, the solution (if exists) to the equation $g(\theta_{BB}) = K_0\theta_D^*$ must be unique. Hence such solution must also be the unique maximizer of $\hat{\text{CE}}_{BB}(\theta_{BB})$.

We now show that $\theta_{BB}^* = \theta_D^*$. Since $\theta_D^* \in [0, 1)$, it is sufficient to show that $\theta_{BB} = \theta_D^*$ solves (A.4). If $\theta_D^* = 0$, then (A.4) obviously holds for $\theta_{BB} = 0 = \theta_D^*$. Otherwise if $\theta_D^* \neq 0$, it becomes sufficient to show $\hat{P}_0(\theta_{BB} = \theta_D^*) = K_0$. By construction of $\hat{P}_0(\theta_{BB})$, it is the unique positive solution to (A.1). Hence we just need to verify that $\hat{P}_0 = K_0 > 0$ solves equation (A.1) under $\theta_{BB} = \theta_D^*$. The result immediately follows as

$$\frac{1}{\gamma\sigma^2(K_0 - \theta_D^* K_0)}\left(\mu - \frac{K_0(1+R_f)(1-\theta_D^*)}{K_0 - \theta_D^* K_0}\right) = \frac{\mu - 1 - R_f}{\gamma\sigma^2 K_0(1-\theta_D^*)} = 1$$

upon recalling that $\theta_D^* = 1 - \frac{\mu-(1+R_f)}{\sigma^2\gamma K_0}$. We therefore conclude $\theta_{BB}^* = \theta_D^*$ and $V_D = V_{BB}$ which establish Part 1 and Part 2 of the theorem. This also immediately implies

$$\hat{P}_0^{BB} := \hat{P}_0(\theta_{BB}^*) = K_0$$

and

$$\hat{P}_0^D = \frac{\mu - (1+R_f)}{\sigma^2 \gamma K_0} = K_0 \frac{\mu - (1+R_f)}{\sigma^2 \gamma} = \hat{P}_0^{BB}(1-\theta_D^*) = \hat{P}_0^{BB}(1-\theta_{BB}^*).$$

This also establishes Part 3 of the theorem. □

Proof of Proposition 5.3 The proof is largely the same as those of Proposition 3.5, Proposition 3.6 and Corollary 3.9. Under a pure dividend strategy $(\theta_D, \theta_{BB} = 0)$ with $\theta_D \in [0, 1)$, investor's i optimal shares holding is

$$\theta_0^{i,*}(P_0, \theta_D) := \frac{1}{\gamma_i \sigma_i^2 K_0(1-\theta_D)} \left(\mu_i - \frac{P_0(1+R_f)}{K_0(1-\theta_D)} \right).$$

Hence any competitive equilibrium $(\hat{\theta}_0^1, \hat{\theta}_0^2, \hat{P}_0(\theta_D))$ must satisfy $\hat{\theta}_0^i = \theta_0^{i,*}(\hat{P}_0(\theta_D), \theta_D)$ for $i = 1, 2$. The market clearing condition $\hat{\theta}_0^1 + \hat{\theta}_0^2 = 1$ therefore implies

$$\frac{1}{\gamma_1 \sigma_1^2 K_0(1-\theta_D)} \left(\mu_1 - \frac{\hat{P}_0(\theta_D)(1+R_f)}{K_0(1-\theta_D)} \right) + \frac{1}{\gamma_2 \sigma_2^2 K_0(1-\theta_D)} \left(\mu_2 - \frac{\hat{P}_0(\theta_D)(1+R_f)}{K_0(1-\theta_D)} \right) = 1 \tag{A.5}$$

such that the equilibrium price $\hat{P}_0(\theta_D)$ is uniquely given by

$$\hat{P}_0(\theta_D) = \frac{K_0(1-\theta_D)}{1+R_f} \left(\frac{\frac{\mu_1}{\sigma_1^2 \gamma_1} + \frac{\mu_2}{\sigma_2^2 \gamma_2}}{\frac{1}{\sigma_1^2 \gamma_1} + \frac{1}{\sigma_2^2 \gamma_2}} - \frac{K_0(1-\theta_D)}{\frac{1}{\sigma_1^2 \gamma_1} + \frac{1}{\sigma_2^2 \gamma_2}} \right) = \frac{K_0(1-\theta_D)}{1+R_f} \left(\phi - \frac{\lambda K_0(1-\theta_D)}{2} \right). \tag{A.6}$$

The corresponding portfolio rules in equilibrium are then give by

$$\hat{\theta}_0^i = \theta_0^{i,*}(\hat{P}_0(\theta_D), \theta_D) = \frac{1}{\gamma \sigma^2 K_0(1-\theta_D)} \left(\mu_i - \frac{\hat{P}_0(\theta_D)(1+R_f)}{K_0(1-\theta_D)} \right) = \frac{\mu_i - \phi}{\sigma_i^2 \gamma_i K_0(1-\theta_D)} + \frac{\lambda}{2\sigma_i^2 \gamma_i}.$$

Now we consider the case of buybacks. Similarly to the proof of Proposition 3.6, investor i's optimal shares holding is found to be

$$\theta_0^{i,*}(P_0, \theta_{BB}) := \frac{1-\theta_{BB}}{\gamma_i \sigma_i^2 (K_0 - \theta_{BB} P_0)} \left(\mu_i - \frac{P_0(1+R_f)(1-\theta_{BB})}{K_0 - \theta_{BB} P_0} \right).$$

Hence any competitive equilibrium $(\hat{\theta}_0^1, \hat{\theta}_0^2, \hat{P}_0(\theta_{BB}))$ must satisfy $\hat{\theta}_0^i = \theta_0^{i,*}(\hat{P}_0(\theta_{BB}), \theta_{BB})$ for $i = 1, 2$. Then the market clearing condition $\hat{\theta}_0^1 + \hat{\theta}_0^2 = 1 - \theta_{BB}$ leads to

$$\sum_{i=1}^{2} \frac{1}{\gamma_i \sigma_i^2 (K_0 - \theta_{BB} \hat{P}_0(\theta_{BB}))} \left(\mu_i - \frac{\hat{P}_0(\theta_{BB})(1+R_f)(1-\theta_{BB})}{K_0 - \theta_{BB} \hat{P}_0(\theta_{BB})} \right) = 1. \tag{A.7}$$

If $\theta_{BB} = 0$, then a simple rearrangement of (A.7) leads to

$$\hat{P}_0(\theta_{BB}) = \frac{K_0}{1+R_f} \left(\phi - \frac{\lambda K_0}{2} \right).$$

Otherwise, for $\theta_{BB} \neq 0$, $\hat{P}_0(\theta_{BB})$ can be identified as a solution to the quadratic equation

$$\theta_{BB}^2 \lambda P_0^2 + 2[\phi\theta_{BB} + (1-\theta_{BB})(1+R_f) - K_0\lambda\theta_{BB}]P_0 + [\lambda K_0^2 - 2\phi K_0] = 0. \quad \text{(A.8)}$$

Equation (A.8) is identical to that of (A.2) upon replacing μ by ϕ and $\sigma^2\gamma$ by $\lambda/2$. Then we can identify a unique family of consistent equilibrium using the same arguments in the proof of Corollary 3.9. The corresponding unique equilibrium share holdings are then given by, for $i = 1, 2$,

$$\begin{aligned} \hat{\theta}_0^i &= \theta_0^{i,*}(\hat{P}_0(\theta_{BB}), \theta_{BB}) \\ &= \frac{1-\theta_{BB}}{\gamma_i\sigma_i^2(K_0 - \theta_{BB}\hat{P}_0(\theta_{BB}))}\left(\mu_i - \frac{\hat{P}_0(\theta_{BB})(1+R_f)(1-\theta_{BB})}{K_0 - \theta_{BB}\hat{P}_0(\theta_{BB})}\right). \end{aligned}$$

□

Proof of Theorem 5.5 Using Proposition 5.3 and the definition of $\hat{\text{CE}}_{i,D}(\theta_D)$,

$$\begin{aligned} &\hat{\text{CE}}_{i,D}(\theta_D) \\ &= \text{CE}_i(\hat{\theta}_0^i; \hat{P}_0, \theta_D, \theta_{BB} = 0) \\ &= \text{CE}_i\left(\frac{\mu_i - \phi}{\sigma_i^2\gamma_i K_0(1-\theta_D)} + \frac{\lambda}{2\sigma_i^2\gamma_i}; \frac{K_0(1-\theta_D)}{1+R_f}\left(\phi - \frac{\lambda K_0(1-\theta_D)}{2}\right), \theta_D, \theta_{BB} = 0\right) \\ &= (n_i\theta_D K_0)(1+R_f) \\ &\quad + \left(n_i - \left(\frac{\mu_i - \phi}{\sigma_i^2\gamma_i K_0(1-\theta_D)} + \frac{\lambda}{2\sigma_i^2\gamma_i}\right)\right)(1+R)\left[\frac{K_0(1-\theta_D)}{1+R_f}\left(\phi - \frac{\lambda K_0(1-\theta_D)}{2}\right)\right] \\ &\quad + \mu_i K_0(1-\theta_D)\left[\frac{\mu_i - \phi}{\sigma_i^2\gamma_i K_0(1-\theta_D)} + \frac{\lambda}{2\sigma_i^2\gamma_i}\right] \\ &\quad - \frac{\sigma_i^2\gamma_i}{2}K_0^2(1-\theta_D)^2\left[\frac{\mu_i - \phi}{\sigma_i^2\gamma_i K_0(1-\theta_D)} + \frac{\lambda}{2\sigma_i^2\gamma_i}\right]^2 \\ &= n_i K_0(1+R_f)\theta_D + \frac{(\mu_i - \phi)^2}{2\sigma_i^2\gamma_i} + \left[n_i\phi + \frac{\lambda(\mu_i - \phi)}{2\sigma_i^2\gamma_i}\right]K_0(1-\theta_D) \\ &\quad - \frac{\lambda}{2}\left(n_i - \frac{\lambda}{4\sigma_i^2\gamma_i}\right)K_0^2(1-\theta_D)^2. \end{aligned}$$

Hence $\overline{\text{CE}}_D(\theta_D) := \alpha_1\hat{\text{CE}}_{1,D}(\theta_D) + \alpha_2\hat{\text{CE}}_{2,D}(\theta_D)$ is a quadratic function of θ_D. Note that

$$\sum_{i=1}^2 \alpha_i\left(n_i - \frac{\lambda}{4\sigma_i^2\gamma_i}\right) \geq \alpha_1\sum_{i=1}^2 n_i - \alpha_2\frac{\lambda}{4}\sum_{i=1}^2\frac{1}{\sigma_i^2\gamma_i} = \alpha_1 - \frac{\alpha_2}{2} > 0$$

under condition (5.4). Thus $\overline{\text{CE}}_D(\theta_D)$ is strictly concave with a unique maximizer given by

$$\theta_D^* = 1 - \frac{\sum_{i=1}^2\alpha_i\left[n_i(\phi - 1 - R_f) + \frac{\lambda(\mu_i - \phi)}{2\sigma_i^2\gamma_i}\right]}{\lambda K_0\sum_{i=1}^2\alpha_i\left(n_i - \frac{\lambda}{4\sigma_i^2\gamma_i}\right)}. \quad \text{(A.9)}$$

Moreover, condition (5.4) further implies

$$\sum_{i=1}^2\alpha_i\left[n_i(\phi - 1 - R_f) + \frac{\lambda(\mu_i - \phi)}{2\sigma_i^2\gamma_i}\right]$$

$$
\begin{aligned}
&\geq \alpha_1 \sum_{i=1}^{2}[n_i(\phi - 1 - R_f) + \frac{\lambda \mu_i}{2\sigma_i^2\gamma_i}] - \alpha_2 \frac{\lambda\phi}{2} \sum_{i=1}^{2} \frac{1}{\sigma_i^2\gamma_i} \\
&= \alpha_1(2\phi - 1 - R_f) - \alpha_2\phi \\
&> \frac{\alpha_2}{2 - \frac{1+R_f}{\phi}}(2\phi - 1 - R_f) - \alpha_2\phi = 0,
\end{aligned}
$$

and

$$
\begin{aligned}
&\lambda K_0 \sum_{i=1}^{2} \alpha_i \left(n_i - \frac{\lambda}{4\sigma_i^2\gamma_i}\right) - \sum_{i=1}^{2} \alpha_i \left[n_i(\phi - 1 - R_f) + \frac{\lambda(\mu_i - \phi)}{2\sigma_i^2\gamma_i}\right] \\
&\geq \lambda K_0 \left(\alpha_1 - \frac{\alpha_2}{2}\right) - \left[\left(2\phi - 1 - R_f\right)\alpha_2 - \phi\alpha_1\right] \\
&= (\lambda K_0 + \phi)\alpha_1 - \left(\frac{\lambda K_0}{2} + 2\phi - 1 - R_f\right)\alpha_2 > 0,
\end{aligned}
$$

from which we deduce that $\theta_D^* \in (0, 1)$. Then $V_D := \sup_{\theta_D \in [0,1)} \overline{\mathrm{CE}}_D(\theta_D) = \sup_{\theta_D \in \mathbb{R}} \overline{\mathrm{CE}}_D(\theta_D) = \overline{\mathrm{CE}}_D(\theta_D^*)$. The corresponding equilibrium share price is

$$
\hat{P}_0^D = \frac{K_0(1 - \theta_D^*)}{1 + R_f}\left(\phi - \frac{\lambda K_0(1 - \theta_D^*)}{2}\right).
$$

Next, we show that $V_D = V_{BB}$ where $V_{BB} := \sup_{\theta_{BB} \in [0,1)} \overline{\mathrm{CE}}_{BB}(\theta_{BB})$. By definition of $\hat{\mathrm{CE}}_{i,BB}(\theta_{BB})$, we have

$$
\begin{aligned}
&\hat{\mathrm{CE}}_{i,BB}(\theta_{BB}) \\
&= \mathrm{CE}_i(\hat{\theta}_0^i; \hat{P}_0(\theta_{BB}), \theta_D = 0, \theta_{BB}) \\
&= \mathrm{CE}_i\left(\frac{1 - \theta_B}{\sigma_i^2\gamma_i(K_0 - \hat{P}_0(\theta_{BB})\theta_{BB})}\left[\mu_i - \frac{\hat{P}_0(\theta_{BB})(1 + R_f)(1 - \theta_{BB})}{K_0 - \hat{P}_0(\theta_{BB})\theta_{BB}}\right]; \hat{P}_0(\theta_{BB}), \theta_D = 0, \theta_{BB}\right) \\
&= \left(n_i - \left(\frac{1 - \theta_B}{\sigma_i^2\gamma_i(K_0 - \hat{P}_0(\theta_{BB})\theta_{BB})}\left[\mu_i - \frac{\hat{P}_0(\theta_{BB})(1 + R_f)(1 - \theta_{BB})}{K_0 - \hat{P}_0(\theta_{BB})\theta_{BB}}\right]\right)\right)(1 + R_f)\hat{P}_0(\theta_{BB}) \\
&\quad + \frac{(K_0 - \theta_B\hat{P}_0(\theta_{BB}))\mu_i}{1 - \theta_B} \times \frac{1 - \theta_B}{\sigma_i^2\gamma_i(K_0 - \hat{P}_0(\theta_{BB})\theta_{BB})}\left[\mu_i - \frac{\hat{P}_0(\theta_{BB})(1 + R_f)(1 - \theta_{BB})}{K_0 - \hat{P}_0(\theta_{BB})\theta_{BB}}\right] \\
&\quad - \frac{\sigma_i^2\gamma_i}{2}\frac{(K_0 - \theta_B\hat{P}_0(\theta_{BB}))^2}{(1 - \theta_B)^2} \\
&\quad \times \left[\frac{1 - \theta_B}{\sigma_i^2\gamma_i(K_0 - \hat{P}_0(\theta_{BB})\theta_{BB})}\left[\mu_i - \frac{\hat{P}_0(\theta_{BB})(1 + R_f)(1 - \theta_{BB})}{K_0 - \hat{P}_0(\theta_{BB})\theta_{BB}}\right]\right]^2 \\
&= n_i(1 + R_f)\hat{P}_0(\theta_{BB}) + \frac{(\mu_i - \phi)^2}{2\sigma_i^2\gamma_i} + \frac{\lambda(\mu_i - \phi)}{2\sigma_i^2\gamma_i}(K_0 - \theta_{BB}\hat{P}_0(\theta_{BB})) \\
&\quad + \frac{\lambda^2}{8\sigma_i^2\gamma_i}(K_0 - \theta_{BB}\hat{P}_0(\theta_{BB}))^2 \\
&= n_i(1 + R_f)\theta_{BB}\hat{P}_0(\theta_{BB}) + \left[n_i\phi - \frac{\lambda(\mu_i - \phi)}{2\sigma_i^2\gamma_i}\right](K_0 - \theta_{BB}\hat{P}_0(\theta_{BB})) \\
&\quad - \frac{\lambda}{2}\left(n_i - \frac{\lambda}{4\sigma_i^2\gamma_i}\right)(K_0 - \theta_{BB}\hat{P}_0(\theta_{BB}))^2
\end{aligned}
$$

$$+n_i\left\{\frac{\lambda}{2}(K_0-\theta_{BB}\hat{P}_0(\theta_{BB}))^2-\phi(K_0-\theta_{BB}\hat{P}_0(\theta_{BB})+(1+R_f)(\hat{P}_0(\theta_{BB})-\theta_{BB}\hat{P}_0(\theta_{BB}))\right\}$$
$$=n_i(1+R_f)\theta_{BB}\hat{P}_0(\theta_{BB})+\left[n_i\phi-\frac{\lambda(\mu_i-\phi)}{2\sigma_i^2\gamma_i}\right](K_0-\theta_{BB}\hat{P}_0(\theta_{BB}))$$
$$-\frac{\lambda}{2}\left(n_i-\frac{\lambda}{4\sigma_i^2\gamma_i}\right)(K_0-\theta_{BB}\hat{P}_0(\theta_{BB}))^2.$$

In the above, the last equality holds due to (A.8). On comparing the expressions of $\hat{\text{CE}}_{i,D}(\theta_D)$ and $\hat{\text{CE}}_{i,BB}(\theta_{BB})$, we deduce

$$\hat{\text{CE}}_{i,D}\left(\frac{\theta_{BB}\hat{P}_0(\theta_{BB})}{K_0}\right)=\hat{\text{CE}}_{i,BB}(\theta_{BB}) \tag{A.10}$$

and hence

$$V_D=\sup_{\theta_D\in[0,1)}\overline{\text{CE}}_D(\theta_D)=\sup_{\theta_D\in\mathbb{R}}\overline{\text{CE}}_D(\theta_D)\geq\overline{\text{CE}}_D\left(\frac{\theta_{BB}\hat{P}_0(\theta_{BB})}{K_0}\right)=\overline{\text{CE}}_{BB}(\theta_{BB})$$

for any θ_{BB}, which in turn implies $V_D\geq V_{BB}=\sup_{\theta_{BB}\in[0,1)}\overline{\text{CE}}_{BB}(\theta_{BB})$.

To show that $V_D=V_{BB}$, it is sufficient to show that there exists $\theta^*_{BB}\in[0,1)$ such that $\frac{\theta^*_{BB}\hat{P}_0(\theta^*_{BB})}{K_0}=\theta^*_D$ or equivalently

$$f(\theta_{BB}):=\theta_{BB}\hat{P}_0(\theta_{BB})=K_0\theta^*_D.$$

By a trivial extension of Lemma A.1, such θ^*_{BB} is unique provided that it exists. Then θ^*_{BB} is the unique maximizer of $\sup_{\theta_{BB}\in[0,1)}\overline{\text{CE}}_B(\theta_{BB})$.

We now show that the unique solution to $f(\theta_{BB})=K_0\theta^*_D$ is $\theta^*_{BB}=\frac{K_0\theta^*_D}{\hat{P}^D_0+K_0\theta^*_D}$. We first note from (A.6) that $\hat{P}^D_0(\theta_D)$ is a concave quadratic function of θ_D. Then since $\theta^*_D\in(0,1)$ we must have $\hat{P}^D_0=\hat{P}^D_0(\theta^*_D)>\max(\hat{P}^D_0(\theta_D=0),\hat{P}^D_0(\theta_D=1))\geq 0$. Using (A.5), one can check that $\hat{P}_0=\hat{P}^D_0+K_0\theta^*_D>0$ is a solution to (A.7) under $\theta_{BB}=\frac{K_0\theta^*_D}{\hat{P}^D_0+K_0\theta^*_D}\in(0,1)$. Hence $\hat{P}^{BB}_0(\theta_{BB}=\frac{K_0\theta^*_D}{\hat{P}^D_0+K_0\theta^*_D})=\hat{P}^D_0+K_0\theta^*_D$. It is now straightforward to see $f(\theta_{BB})=K_0\theta^*_D$ for $\theta_{BB}=\frac{K_0\theta^*_D}{\hat{P}^D_0+K_0\theta^*_D}$. Thus we conclude $\theta^*_{BB}=\frac{K_0\theta^*_D}{\hat{P}^D_0+K_0\theta^*_D}$ and $\hat{P}^{BB}_0=\hat{P}^D_0+K_0\theta^*_D$.

This establishes $\overline{\text{CE}}_D(\theta^*_D)=\overline{\text{CE}}_{BB}(\theta^*_{BB})$ in Part 1 of the theorem. Part 4 of the theorem is now trivial. Moreover, Part 2 of the theorem immediately follows as well due to (A.10).

Part 3 of the theorem can be verified on recalling that

$$\theta_0^{i,BB,*}=\hat{\theta}^i_0(\hat{P}_{BB},\theta^*_{BB})=\frac{1-\theta^*_{BB}}{\sigma_i^2\gamma_i(K_0-\theta^*_{BB}\hat{P}^{BB}_0)}\left(\mu_i-\frac{\hat{P}^{BB}_0(1+R_f)(1-\theta^*_{BB})}{K_0-\theta^*_{BB}\hat{P}^{BB}_0}\right).$$

and

$$\theta_0^{i,D,*}=\hat{\theta}^i_0(\hat{P}_D,\theta^*_D)=\frac{1}{\sigma_i^2\gamma_iK_0(1-\theta^*_D)}\left(\mu_i-\frac{\hat{P}^D_0(1+R_f)}{K_0(1-\theta^*_D)}\right).$$

Finally, the remarks about the special case of $\alpha_1=\alpha_2$ can be established by computing θ^*_D explicitly. □

Proof of Proposaition 5.8 Under a general distribution strategy (θ_D, θ_{BB}), the investor's objective function is

$$
\begin{aligned}
&J(\theta_0, C_0; P_0, R_f, \theta_D, \theta_{BB}) := U(C_0) + e^{-\beta}\mathbb{E}[U(C_1)] \\
&\quad = U(C_0) + e^{-\beta}\mathbb{E}\Big[U\Big((W_{-1} + \theta_D K_0 + (1-\theta_0)P_0 - C_0)(1+R_f) \\
&\qquad + \theta_0 \frac{(K_0 - \theta_D K_0 - \theta_{BB} P_0)\Pi}{1-\theta_{BB}}\Big)\Big] \\
&\quad = -\exp(-\gamma C_0) - \exp\Bigg(-\beta - \gamma(W_{-1} + \theta_D K_0 + (1-\theta_0)P_0 - C_0)(1+R_f) \\
&\qquad - \gamma \frac{(K_0 - \theta_D K_0 - \theta_{BB} P_0)\mu}{1-\theta_{BB}}\theta_0 \\
&\qquad + \frac{\sigma^2\gamma^2}{2}\frac{(K_0 - \theta_D K_0 - \theta_{BB} P_0)^2}{(1-\theta_{BB})^2}\theta_0^2\Bigg) \\
&\quad = -\exp(-\gamma C_0) - \exp(-\beta - \gamma(W_{-1} + \theta_D K_0 - C_0)(1+R_f)) \\
&\qquad \times \exp\Bigg(-\gamma P_0(1+R_f)(1-\theta_0) - \gamma\frac{(K_0 - \theta_D K_0 - \theta_{BB} P_0)\mu}{1-\theta_{BB}}\theta_0 \\
&\qquad + \frac{\sigma^2\gamma^2}{2}\frac{(K_0 - \theta_D K_0 - \theta_{BB} P_0)^2}{(1-\theta_{BB})^2}\theta_0^2\Bigg).
\end{aligned}
$$

Hence the maximization problem of J with respect to (θ_0, C_0) is decoupled, in that one can find the optimal θ_0 first via minimizing

$$
\begin{aligned}
f(\theta_0) &= f(\theta_0; P_0, R_f) \\
&:= -\gamma P_0(1+R_f)(1-\theta_0) - \gamma\frac{(K_0 - \theta_D K_0 - \theta_{BB} P_0)\mu}{1-\theta_{BB}}\theta_0 \\
&\quad + \frac{\sigma^2\gamma^2}{2}\frac{(K_0 - \theta_D K_0 - \theta_{BB} P_0)^2}{(1-\theta_{BB})^2}\theta_0^2. \qquad \text{(A.11)}
\end{aligned}
$$

One can then solve for the optimal C_0 by maximizing an expression in form of

$$
h(C_0) := -\exp(-\gamma C_0) - M\exp(-\beta - \gamma(W_{-1} + \theta_D K_0 - C_0)(1+R_f)), \qquad \text{(A.12)}
$$

where $M > 0$ is some constant deduced from the optimization problem of θ_0.

Provided that $K_0 - \theta_D K_0 - \theta_{BB} P_0 \neq 0$, the function f in (A.11) is a strictly convex function of θ_0 and hence its unique minimizer is given by

$$
\theta_0^* = \frac{1-\theta_{BB}}{K_0 - \theta_D K_0 - \theta_{BB} P_0}\left[\mu - \frac{(1-\theta_{BB})(1+R_f)P_0}{K_0 - \theta_D K_0 - \theta_{BB} P_0}\right].
$$

Then we proceed to solve for the optimal C_0 which maximizes $h(C_0)$ in (A.12) with $M := \exp(f(\theta_0^*))$. It is straightforward to check that h is a strictly concave function of C_0 and the unique maximiser is given by the first order condition $h'(C_0) = 0$ or equivalently

$$
\gamma\exp(-\gamma C_0) - M\gamma(1+R_f)\exp(-\beta - \gamma(W_{-1} + \theta_D K_0 - C_0)(1+R_f)) = 0,
$$

such that

$$
C_0^* = \frac{\beta + \gamma(1+R_f)(W_{-1} + \theta_D K_0) - f(\theta_0^*) - \ln(1+R_f)}{\gamma(2+R_f)}.
$$

Now we further specialize to the two sub-cases of interest:

1. Under a pure dividend strategy $(\theta_D, \theta_{BB} = 0)$, the optimizer of $J(\theta_0, C_0; P_0, R_f, \theta_D, \theta_{BB} = 0)$ can be further simplified to

$$\theta_0^* = \theta_0^*(\theta_D) = \frac{1}{K_0(1-\theta_D)}\left[\mu - \frac{(1+R_f)P_0}{K_0(1-\theta_D)}\right].$$

The market clearing condition for the risky asset therefore requires the equilibrium share price $\hat{P}_0$ to satisfy $\theta_0^* = 1$, which leads to

$$\hat{P}_0 = \hat{P}_0(\theta_D) = \frac{K_0(1-\theta_D)}{1+\hat{R}_f}\left(\mu - \sigma^2\gamma K_0(1-\theta_D)\right).$$

Moreover, the market clearing condition for the money market in conjunction with $\hat{\theta}_0 = 1$ implies that the equilibrium consumption rate has to satisfy $\hat{C}_0 = \theta_D K_0$. Hence the equilibrium interest rate $\hat{R}_f$ needs to be chosen such that

$$\begin{aligned}\theta_D K_0 = C_0^* &= \frac{\beta + \gamma(1+\hat{R}_f)(0+\theta_D K_0) - f(\hat{\theta}_0 = 1; \hat{P}_0, \hat{R}_f) - \ln(1+\hat{R}_f)}{\gamma(2+\hat{R}_f)}\\ &= \frac{\beta + \gamma(1+\hat{R}_f)\theta_D K_0 + \gamma K_0(1-\theta_D)\mu - \frac{\sigma^2\gamma^2}{2}K_0^2(1-\theta_D)^2 - \ln(1+\hat{R}_f)}{\gamma(2+\hat{R}_f)}.\end{aligned}$$

After rearrangement, we arrive at

$$\hat{R}_f = \hat{R}_f(\theta_D) = \exp\left(\beta + \gamma K_0\left(\mu(1-\theta_D) - \theta_D - \frac{\sigma^2\gamma}{2}K_0(1-\theta_D)^2\right)\right) - 1.$$

2. Under a pure buyback strategy, the optimizer of $J(\theta_0, C_0; P_0, R_f, \theta_D = 0, \theta_{BB})$ is now given by

$$\theta_0^* = \frac{1-\theta_{BB}}{K_0 - \theta_{BB}P_0}\left[\mu - \frac{(1-\theta_{BB})(1+R_f)P_0}{K_0 - \theta_{BB}P_0}\right].$$

The market clearing condition requires $\theta_0^* = 1 - \theta_{BB}$ and hence the equilibrium price and interest rate $(\hat{P}_0, \hat{R}_f)$ must satisfy

$$1-\theta_{BB} = \frac{1-\theta_{BB}}{K_0 - \theta_{BB}\hat{P}_0}\left[\mu - \frac{(1-\theta_{BB})(1+\hat{R}_f)\hat{P}_0}{K_0 - \theta_{BB}\hat{P}_0}\right]$$

or, equivalently,

$$\theta_{BB}^2\gamma\sigma^2\hat{P}_0^2 + [\mu\theta_{BB} + (1-\theta_{BB})(1+\hat{R}_f) - 2K_0\gamma\sigma^2\theta_{BB}]\hat{P}_0 + [\gamma\sigma^2K_0^2 - \mu K_0] = 0. \tag{A.13}$$

Similarly to the pure dividend case, the market clearing condition for the money market implies that the equilibrium consumption rate has to satisfy $\hat{C}_0 = (1-\hat{\theta}_0)\hat{P}_0 = \theta_{BB}\hat{P}_0$. Thus:

$$\begin{aligned}\theta_{BB}\hat{P}_0 = C_0^* &= \frac{\beta - f(\hat{\theta}_0 = 1-\theta_{BB}; \hat{P}_0, \hat{R}_f) - \ln(1+\hat{R}_f)}{\gamma(2+\hat{R}_f)}\\ &= \frac{\beta + \gamma\hat{P}_0(1+\hat{R}_f)\theta_{BB} + \gamma\mu(K_0 - \theta_{BB}\hat{P}_0) - \frac{\sigma^2\gamma^2}{2}(K_0-\theta_{BB}\hat{P}_0)^2 - \ln(1+\hat{R}_f)}{\gamma(2+\hat{R}_f)},\end{aligned}$$

leading to

$$\hat{R}_f = \exp\left(\beta - \gamma \hat{P}_0\theta_{BB} + \gamma\mu(K_0 - \theta_{BB}\hat{P}_0) - \frac{\sigma^2\gamma^2}{2}(K_0 - \theta_{BB}\hat{P}_0)^2\right) - 1. \quad (A.14)$$

Hence, the pair $(\hat{P}_0, \hat{R}_f)$ represents an equilibrium if and only if it simultaneously satisfies (A.13) and (A.14).
When $\theta_{BB} = 0$, we have $\hat{\theta}_0 = 1$, $\hat{C}_0 = 0$, and equation (A.14) simplifies to

$$\hat{R}_f = \exp\left(\beta + \gamma\mu K_0 - \frac{\sigma^2\gamma^2}{2}K_0^2\right) - 1. \quad (A.15)$$

In addition, (A.13) then has the explicit solution

$$\hat{P}_0 = \frac{K_0}{1+\hat{R}_f}(\mu - \sigma^2\gamma K_0). \quad (A.16)$$

We therefore obtain the closed-form expressions of the unique competitive equilibrium when $\theta_{BB} = 0$.
For the more general case of $\theta_{BB} \neq 0$, under the assumption $\mu > K_0\gamma\sigma^2$, (A.13) as an equation in $\hat{P}_0$ admits two real roots of opposite signs under for given $\hat{R}_f$. For $R_f > -1$ (and fixed $\theta_{BB} \neq 0$), define

$$P_0^+(R_f) = P_0^+(R_f;\theta_{BB}) := \frac{2\sigma^2\gamma\theta_{BB}K_0 - \mu\theta_{BB} - (1+R_f)(1-\theta_{BB}) + \sqrt{\Delta}}{2\sigma^2\gamma\theta_{BB}^2}, \quad (A.17)$$

where

$$\Delta := \left[\mu\theta_{BB} + (1+R_f)(1-\theta_{BB})\right]^2 - 4\sigma^2\gamma(1+R_f)K_0\theta_{BB}(1-\theta_{BB}).$$

We extend the definition of $P_0^+(R_f;\theta_{BB})$ to $\theta_{BB} = 0$ by the continuous extension

$$P_0^+(R_f;\theta_{BB}=0) := \lim_{\theta_{BB}\to 0} P_0^+(R_f;\theta_{BB}) = \frac{K_0}{1+R_f}\left(\mu - \sigma^2\gamma K_0\right).$$

Let $(\hat{P}_0^\dagger, \hat{R}_f^\dagger)$ be a solution to the system of equations (in (P_0, R_f)) given by

$$\begin{cases} P_0 = P_0^+(R_f); \\ R_f = \Xi^+(R_f), \end{cases} \quad (A.18)$$

where

$$\begin{aligned} &\Xi^+(R_f) \\ &\quad = \Xi^+(R_f;\theta_{BB}) \\ &\quad := \exp\big(\beta + \gamma\mu(K_0 - \theta_{BB}P_0^+(R_f;\theta_{BB})) - \gamma\theta_{BB}P_0^+(R_f;\theta_{BB}) \\ &\qquad - \frac{\gamma^2\sigma^2}{2}(K_0 - \theta_{BB}P_0^+(R_f;\theta_{BB}))^2\Big) - 1. \end{aligned} \quad (A.19)$$

Note that $[0, 1) \ni \theta_{BB} \mapsto \Xi^+(R_f;\theta_{BB})$ is continuous for any fixed R_f. By construction, $(\hat{P}_0^\dagger, \hat{R}_f^\dagger)$ therefore is a competitive equilibrium for $\theta_{BB} \neq 0$.

Lemma A.2 *There exists $\hat{R}_f$ such that $\hat{R}_f = \Xi^+(\hat{R}_f)$. Moreover, any solution $\hat{R}_f$ to this equation must satisfy*

$$-1 < \hat{R}_f \leq \exp\left(\beta + \gamma\mu K_0 - \frac{\sigma^2\gamma^2}{2}K_0^2 + \frac{(\sigma^2\gamma^2 K_0 - \gamma\mu - \gamma)^2}{2\sigma^2\gamma^2}\right) - 1.$$

Proof The exponent in $\Xi^+(R_f)$ is a strictly concave quadratic function of $\theta_{BB}\hat{P}_0^+(R_f)$ such that $\Xi^+(R_f)$ is bounded from the above by a constant independent of θ_{BB} and $\hat{R}_f$. In particular,

$$-1 < \Xi^+(R_f) \leq \exp\left(\beta + \gamma\mu K_0 - \frac{\sigma^2\gamma^2}{2}K_0^2 + \frac{(\sigma^2\gamma^2 K_0 - \gamma\mu - \gamma)^2}{2\sigma^2\gamma^2}\right) - 1,$$

for all R_f and θ_{BB}. The boundedness of $\Xi^+(R_f)$ implies the existence of some $\hat{R}_f$ which solves the equation $R_f = \Xi^+(R_f)$. Moreover, the bound of $\hat{R}_f$ now simply follows from the fact that $\hat{R}_f = \Xi^+(\hat{R}_f)$. □

We now impose a technical assumption concerning the uniqueness of the equation $R_f = \Xi^+(R_f)$.

Assumption A.3 The solution to the equation $R_f = \Xi^+(R_f; \theta_{BB})$ is unique for all $\theta_{BB} \in [0, 1)$.

Returning to the proof of Proposition 5.8, Assumption A.3 now implies that there exists a unique $R_f = \hat{R}_f^\dagger(\theta_{BB}) > -1$ which solves $R_f = \Xi^+(R_f; \theta_{BB})$, and in turn $(\hat{P}_0^\dagger(\theta_{BB}), \hat{R}_f^\dagger(\theta_{BB}))$ is the unique solution to (A.18). This also implies the system of equations displayed in Part 2 of Proposition 5.8 admits a unique solution given by $(1-\theta_{BB}, \theta_{BB}\hat{P}_0^\dagger, \hat{P}_0^\dagger, \hat{R}_f^\dagger)$. Moreover, when $\theta_{BB} = 0$, the system of equations (A.18) simplifies to

$$\begin{cases} P_0 = P_0^+(R_f; \theta_{BB} = 0) = \frac{K_0}{1+R_f}\left(\mu - \sigma^2\gamma K_0\right) \\ R_f = \Xi^+(R_f; \theta_{BB} = 0) = \exp\left(\beta + \gamma\mu K_0 - \frac{\gamma^2\sigma^2}{2}K_0^2\right) - 1 \end{cases}$$

and hence we conclude

$$\hat{R}_f^\dagger(\theta_{BB} = 0) = \exp\left(\beta + \gamma\mu K_0 - \frac{\sigma^2\gamma^2}{2}K_0^2\right) - 1,$$

$$\hat{P}_0^\dagger(\theta_{BB} = 0) = \frac{K_0}{1 + \hat{R}_f^\dagger(\theta_{BB} = 0)}\left(\mu - \sigma^2\gamma K_0\right),$$

which coincide with the expressions of the equilibrium quantities under $\theta_{BB} = 0$ as previously derived in (A.15) and (A.16), i.e., $\hat{P}_0^\dagger(\theta_{BB} = 0)$ and $\hat{R}_f^\dagger(\theta_{BB} = 0)$ indeed represent a competitive equilibrium under $\theta_{BB} = 0$ as well. Finally, it is easy to see that $\hat{R}_f^\dagger(\theta_{BB})$ is continuous in θ_{BB} because $\Xi^+(R_f; \theta_{BB})$ is continuous in θ_{BB} while $\hat{R}_f^\dagger(\theta_{BB})$ is the unique solution of $R_f = \Xi^+(R_f; \theta_{BB})$ taking values in a compact set. Consequently, $\theta_{BB} \mapsto \hat{P}_0^\dagger(\theta_{BB}) = \hat{P}_0^+(\hat{R}_f^\dagger(\theta_{BB}); \theta_{BB})$ is also continuous and hence the family of equilibrium is consistent.

To complete the proof, we need to establish the uniqueness of

$$\theta_{BB} \mapsto f(\theta_{BB}) := (\hat{\theta}_0(\theta_{BB}) = 1 - \theta_{BB}, \hat{C}_0(\theta_{BB}) = \theta_{BB}\hat{P}_0^\dagger(\theta_{BB}), \hat{P}_0^\dagger(\theta_{BB}), \hat{R}_f^\dagger(\theta_{BB}))$$

as a consistent family of equilibria under pure buyback strategies. Suppose there exists a different consistent family of equilibria of the form $\theta_{BB} \mapsto \tilde{f}(\theta_{BB}) := (\tilde{\theta}_0(\theta_{BB}) = 1-\theta_{BB}, \tilde{C}_0(\theta_{BB}) = \theta_{BB}\tilde{P}_0(\theta_{BB}), \tilde{P}_0(\theta_{BB}), \tilde{R}_f(\theta_{BB}))$ with $\tilde{f} \neq f$. When $\theta_{BB} = 0$, there is only one possible competitive equilibrium given by (A.14) and (A.13) so we must have

$$\begin{aligned}\tilde{R}_f(\theta_{BB}=0) &= \exp\left(\beta + \gamma\mu K_0 - \frac{\sigma^2\gamma^2}{2}K_0^2\right) - 1,\\ \tilde{P}_0(\theta_{BB}=0) &= \frac{K_0}{1+\tilde{R}_f}(\mu - \sigma^2\gamma K_0).\end{aligned} \tag{A.20}$$

Otherwise, for $\theta_{BB} \neq 0$, by the expressions in (A.13) and (A.14) we must have $(\tilde{P}_0(\theta_{BB}), \tilde{R}_f(\theta_{BB}))$ being the solution to either system (A.18), or to the below system of equations

$$\begin{cases} P_0 = P_0^-(R_f;\theta_{BB});\\ R_f = \Xi^-(R_f;\theta_{BB}),\end{cases} \tag{A.21}$$

with

$$\begin{aligned}&\Xi^-(R_f)\\ &\quad = \Xi^-(R_f;\theta_{BB})\\ &\quad := \exp\big(\beta + \gamma\mu(K_0 - \theta_{BB}P_0^-(R_f;\theta_{BB})) - \gamma\theta_{BB}P_0^-(R_f;\theta_{BB})\\ &\qquad - \frac{\gamma^2\sigma^2}{2}(K_0 - \theta_{BB}P_0^-(R_f;\theta_{BB}))^2\Big) - 1\end{aligned}$$

and

$$P_0^-(R_f) = P_0^-(R_f;\theta_{BB}) := \frac{2\sigma^2\gamma\theta_{BB}K_0 - \mu\theta_{BB} - (1+R_f)(1-\theta_{BB}) - \sqrt{\Delta}}{2\sigma^2\gamma\theta_{BB}^2}.$$

Since $\Delta > 0$ for any θ_{BB}, solutions to (A.18) and (A.21) must be distinct. In particular, $\hat{P}_0^+$ and $\hat{P}_0^-$ have opposite signs. By the continuity of f and $\tilde{f}$ together with the fact that $\tilde{f} \neq f$, each $(\tilde{P}_0(\theta_{BB}), \tilde{R}_f(\theta_{BB}))$ must be a solution to (A.21) for all $\theta_{BB} \neq 0$. Moreover, the consistency of $\tilde{f}$ further implies $\tilde{R}_f(\theta_{BB})$ and $\tilde{P}_0(\theta_{BB})$ are continuous functions of θ_{BB} with $\lim_{\theta_{BB}\to 0}\tilde{R}_f(\theta_{BB})$ and $\lim_{\theta_{BB}\to 0}\tilde{P}_0(\theta_{BB})$ coinciding with the values given by (A.20). But then

$$\begin{aligned}&\frac{K_0(\mu - \sigma^2\gamma K_0)}{\exp\left(\beta + \gamma\mu K_0 - \frac{\sigma^2\gamma^2}{2}K_0^2\right)}\\ &\quad = \tilde{P}_0(\theta_{BB}=0)\\ &\quad = \lim_{\theta_{BB}\to 0}\tilde{P}_0(\theta_{BB})\\ &\quad = \lim_{\theta_{BB}\to 0}P_0^-(\tilde{R}_f(\theta_{BB});\theta_{BB})\\ &\quad = \lim_{\theta_{BB}\to 0}\frac{2\sigma^2\gamma\theta_{BB}K_0 - \mu\theta_{BB} - (1+(\tilde{R}_f(\theta_{BB}))(1-\theta_{BB}) - \sqrt{\Delta(\theta_{BB},\tilde{R}_f(\theta_{BB}))}}{2\sigma^2\gamma\theta_{BB}^2}.\end{aligned}$$

Using the known value of $\lim_{\theta_{BB}\to 0}\tilde{R}_f(\theta_{BB})$, the numerator of the final expression converges to $-2\exp\left(\beta + \gamma\mu K_0 - \frac{\sigma^2\gamma^2}{2}K_0^2\right)$ and hence the last term cannot admit a well-defined limit.

We arrive at the required contradiction. This establishes the uniqueness of the family of consistent equilibrium. □

Proof of Theorem 5.9 We first show that the optimal pure dividend strategy is given by the unique solution to (5.7). Using Proposition 5.8, the equilibrium value function under a pure dividend strategy is

$$\begin{aligned}\hat{J}_D(\theta_D) &= J(\hat{\theta}_0, \hat{C}_0; \hat{P}_0, \hat{R}_f, \theta_D, \theta_{BB} = 0)\\ &= J(1, \theta_D K_0; \hat{P}_0(\theta_D), \hat{R}_f(\theta_D), \theta_D, \theta_{BB} = 0)\\ &= -\exp(-\gamma\theta_D K_0) - \exp\left(-\beta - \gamma K_0(1-\theta_D)\mu + \frac{\sigma^2\gamma^2}{2}K_0^2(1-\theta_D)^2\right).\end{aligned}$$

Its first order derivative is

$$\begin{aligned}\hat{J}_D'(\theta_D) &= \gamma K_0 e^{-\gamma K_0\theta_D} - \left[\gamma K_0\mu - \gamma^2\sigma^2 K_0^2(1-\theta_D)\right]\\ &\quad \exp\left(-\beta - \gamma K_0\mu(1-\theta_D) + \frac{\gamma^2\sigma^2}{2}K_0^2(1-\theta_D)^2\right)\\ &= \gamma K_0 e^{-\gamma K_0\theta_D}\left\{1 - \left[\mu - \gamma\sigma^2 K_0(1-\theta_D)\right]\right.\\ &\quad \left.\exp\left(-\beta - \gamma K_0\mu(1-\theta_D) + \frac{\gamma^2\sigma^2}{2}K_0^2(1-\theta_D)^2 + \gamma K_0\theta_D\right)\right\}.\end{aligned}$$

It is clear that $\hat{J}_D'(\theta_D) > 0$ if $\mu - \gamma\sigma^2 K_0(1-\theta_D) \le 0$ or equivalently $\theta_D \le 1 - \frac{\mu}{\gamma\sigma^2 K_0}$, i.e. $\hat{J}_D$ must be increasing on $\theta_D \le 1 - \frac{\mu}{\gamma\sigma^2 K_0}$. Meanwhile, the exponent inside

$$\exp\left(-\beta - \gamma K_0\mu(1-\theta_D) + \frac{\gamma^2\sigma^2}{2}K_0^2(1-\theta_D)^2 + \gamma K_0\theta_D\right)$$

is a convex quadratic function in θ_D. Hence it is easy to see that this function is increasing on $\theta_D \ge 1 - \frac{\mu+1}{\gamma\sigma^2 K_0}$. Then, for $\theta_D \ge 1 - \frac{\mu}{\gamma\sigma^2 K_0} > 1 - \frac{\mu+1}{\gamma\sigma^2 K_0}$, the expression

$$\begin{aligned}f(\theta_D) &:= 1 - \left[\mu - \gamma\sigma^2 K_0(1-\theta_D)\right]\\ &\exp\left(-\beta - \gamma K_0\mu(1-\theta_D) + \frac{\gamma^2\sigma^2}{2}K_0^2(1-\theta_D)^2 + \gamma K_0\theta_D\right)\end{aligned}$$

is strictly decreasing in θ_D because its second term is the product of two strictly positive increasing functions. Moreover, Assumption 5.6 implies $f(1) < 0$ and $f(0) > 0$. We therefore conclude that $\hat{J}_D'(\theta_D)$ must change sign exactly once from positive to negative at some $\theta_D^* \in (0, 1)$. This turning point is the unique maximiser of the function $\hat{J}_D(\theta_D)$, i.e.

$$V_D := \sup_{\theta_D\in[0,1)} \hat{J}_D(\theta_D) = \sup_{\theta_D\in\mathbb{R}} \hat{J}_D(\theta_D) = \hat{J}_D(\theta_D^*).$$

Note that θ_D^*, and in turn $\hat{P}_0^D = \hat{P}_0(\theta_D^*)$ and $\hat{R}_f^D = \hat{R}_f(\theta_D^*)$, are not available in closed-form. Nonetheless, we can still derive a connection between θ_D^* and $\hat{R}_f^D$. Since θ_D^* solves (5.7), we have

$$\exp\left(\beta + \gamma K_0\left(\mu(1-\theta_D^*) - \frac{\gamma\sigma^2}{2}K_0(1-\theta_D^*)^2 - \theta_D\right)\right) = \mu - \gamma\sigma^2 K_0(1-\theta_D^*).$$

But we also have

$$\hat{R}_f = \hat{R}_f(\theta_D^*) = \exp\left(\beta + \gamma K_0 \left(\mu(1-\theta_D^*) - \theta_D^* - \frac{\sigma^2\gamma}{2} K_0 (1-\theta_D^*)^2\right)\right) - 1.$$

Hence we deduce

$$1 + \hat{R}_f^D = \mu - \gamma\sigma^2 K_0 (1-\theta_D^*)$$

such that

$$\theta_D^* = 1 - \frac{\mu - 1 - \hat{R}_f^D}{\sigma^2\gamma K_0}. \tag{A.22}$$

The expression is identical to the optimal dividend rate in our baseline model, expect that the exogenous riskfree rate is now replaced by an endogenous one. Note that $\theta_D^* < 1$ implies

$$\mu - 1 - \hat{R}_f^D > 0. \tag{A.23}$$

We now establish the equivalence between dividend and buyback. The equilibrium value function under a pure buyback strategy is given by

$$\begin{aligned}
\hat{J}_{BB}(\theta_{BB}) &= J(\hat{\theta}_0, \hat{C}_0; \hat{P}_0, \hat{R}_f, \theta_D = 0, \theta_{BB}) \\
&= J(1-\theta_{BB}, \theta_{BB}\hat{P}_0(\theta_{BB}); \hat{P}_0(\theta_{BB}), \hat{R}_f, \theta_D = 0, \theta_{BB}) \\
&= -\exp(-\gamma\theta_{BB}\hat{P}_0(\theta_{BB})) \\
&\quad - \exp\left(-\beta - \gamma(K_0 - \theta_{BB}\hat{P}_0(\theta_{BB}))\mu + \frac{\sigma^2\gamma^2}{2}(K_0 - \theta_{BB}\hat{P}_0(\theta_{BB}))^2\right) \\
&= \hat{J}_D(\theta_{BB}\hat{P}_0(\theta_{BB})/K_0) \\
&\leq \sup_{\theta_D \in \mathbb{R}} \hat{J}_D(\theta_D) = \sup_{\theta_D \in (-1,\infty)} \hat{J}_D(\theta_D) = \hat{J}_D(\theta_D^*) = V_D.
\end{aligned}$$

Hence we have $V_{BB} := \sup_{\theta_{BB}\in[0,1)} \hat{J}_B(\theta_{BB}) \leq V_D$. To show that $V_{BB} = V_D$, it is sufficient to demonstrate the existence of some $\theta_{BB}^* \in (-1, \infty)$ such that $\theta_D^* = \theta_{BB}\hat{P}_0(\theta_{BB})/K_0$ or equivalently

$$\theta_{BB}\hat{P}_0(\theta_{BB}) = K_0\theta_D^*.$$

We now show the following: If the firm adopts a buyback strategy with distribution ratio coinciding with θ_D^*, i.e. a choice such that $\theta_{BB} = \theta_D^*$, then the corresponding equilibrium price and interest rate are such that $\hat{P}_0(\theta_{BB}) = K_0$ and $\hat{R}_f(\theta_{BB}) = \hat{R}_f^D$. It is sufficient to show that $(P_0 = K_0, R_f = \hat{R}_f^D)$ solves the system of equations (A.18) under $\theta_{BB} = \theta_D^*$. Using (A.22), we obtain

$$\begin{aligned}
\Delta(\theta_{BB} = \theta_D^*; R_f = \hat{R}_f^D) &= \left[\mu\theta_D^* + (1+\hat{R}_f^D)(1-\theta_D^*)\right]^2 - 4\sigma^2\gamma(1+\hat{R}_f^D)K_0\theta_D^*(1-\theta_D^*) \\
&= \left[\mu\left(1 - \frac{\mu - (1+\hat{R}_f^D)}{\sigma^2\gamma K_0}\right) + (1+\hat{R}_f^D)\frac{\mu - (1+\hat{R}_f^D)}{\sigma^2\gamma K_0}\right]^2 \\
&\quad - 4\sigma^2\gamma(1+\hat{R}_f^D)K_0\left(1 - \frac{\mu - (1+\hat{R}_f^D)}{\sigma^2\gamma K_0}\right)\frac{\mu - (1+\hat{R}_f^D)}{\sigma^2\gamma K_0}
\end{aligned}$$

$$= \left(\frac{(\mu - (1 + \hat{R}_f^D))^2}{\gamma K_0 \sigma^2} - (\mu - 2(1 + \hat{R}_f^D)) \right)^2.$$

As a consequence,

$$\sqrt{\Delta(\theta_{BB} = \theta_D^*; R_f = \hat{R}_f^D)} = \left| \frac{(\mu - (1 + \hat{R}_f^D))^2}{\gamma K_0 \sigma^2} - (\mu - 2(1 + \hat{R}_f^D)) \right|$$
$$= \frac{(\mu - (1 + \hat{R}_f^D))^2}{\gamma K_0 \sigma^2} - (\mu - 2(1 + \hat{R}_f^D))$$

due to the assumption $\mu > \gamma \sigma^2 K_0 > 0$. Moreover,

$$\hat{P}_0^+(R_f = \hat{R}_f^D, \theta_{BB} = \theta_D^*) = \frac{2\sigma^2 \gamma \theta_D^* K_0 - \mu \theta_D^* - (1 + \hat{R}_f^D)(1 - \theta_D^*) + \sqrt{\Delta(\theta_{BB} = \theta_D^*; R_f = \hat{R}_f^D)}}{2\sigma^2 \gamma (\theta_D^*)^2}$$
$$= \frac{2\sigma^2 \gamma (1 - \frac{\mu - (1+\hat{R}_f^D)}{\sigma^2 \gamma K_0}) K_0 - \mu (1 - \frac{\mu - (1+\hat{R}_f^D)}{\sigma^2 \gamma K_0}) - (1 + \hat{R}_f^D) \frac{\mu - (1+\hat{R}_f^D)}{\sigma^2 \gamma K_0} + \frac{(\mu - (1+\hat{R}_f^D))^2}{\gamma K_0 \sigma^2} - (\mu - 2(1 + \hat{R}_f^D))}{2\sigma^2 \gamma (1 - \frac{\mu - (1+\hat{R}_f^D)}{\sigma^2 \gamma K_0})^2}$$
$$= K_0,$$

and

$$\Xi^+(R_f = \hat{R}_f^D; \theta_{BB} = \theta_D^*)$$
$$= \exp\left(\beta + \gamma \mu (K_0 - \theta_D^* P_0^+(\hat{R}_f^D; \theta_D^*)) - \gamma \theta_D^* P_0^+(\hat{R}_f^D; \theta_D^*) - \frac{\gamma^2 \sigma^2}{2} (K_0 - \theta_D^* P_0^+(\hat{R}_f^D; \theta_D^*))^2 \right) - 1$$
$$= \exp\left(\beta + \gamma \mu (K_0 - \theta_D^* K_0) - \gamma \theta_D^* K_0 - \frac{\gamma^2 \sigma^2}{2} (K_0 - \theta_D^* K_0)^2 \right) - 1$$
$$= \hat{R}_f^D.$$

Hence $(P_0 = K_0, R_f = \hat{R}_f^D)$ is indeed a solution to (A.18) when $\theta_{BB} = \theta_D^*$, i.e. $\theta_{BB} \hat{P}_0(\theta_{BB}) = K_0 \theta_D^*$ holds when $\theta_{BB} = \theta_D^* \in [0, 1)$.

To demonstrate the uniqueness of θ_{BB} such that $\theta_{BB} \hat{P}_0(\theta_{BB}) = K_0 \theta_D^*$, suppose there exists another $\tilde{\theta}_{BB}$ such that $\tilde{\theta}_{BB} \hat{P}_0(\tilde{\theta}_{BB}) = K_0 \theta_D^*$ but $\tilde{\theta}_{BB} \neq \theta_D^*$. By construction of the equilibrium,

$$\hat{R}_f(\tilde{\theta}_{BB}) = \exp\left(\beta + \gamma \mu (K_0 - \tilde{\theta}_{BB} \hat{P}_0(\tilde{\theta}_{BB})) - \gamma \tilde{\theta}_{BB} \hat{P}_0(\tilde{\theta}_{BB}) - \frac{\gamma^2 \sigma^2}{2} (K_0 - \tilde{\theta}_{BB} \hat{P}_0(\tilde{\theta}_{BB}))^2 \right) - 1$$
$$= \exp\left(\beta + \gamma \mu (K_0 - K_0 \theta_D^*) - \gamma K_0 \theta_D^* - \frac{\gamma^2 \sigma^2}{2} (K_0 - K_0 \theta_D^*)^2 \right) - 1$$
$$= \hat{R}_f^D.$$

Then we have $\tilde{\theta}_{BB} \hat{P}_0(\tilde{\theta}_{BB}) = g(\tilde{\theta}_{BB}; \hat{R}_f^D)$, where

$$g(\theta; \hat{R}_f^D) := \frac{2\sigma^2 \gamma \theta K_0 - \mu \theta - (1 + \hat{R}_f^D)(1 - \theta) + \sqrt{\left[\mu \theta + (1 + \hat{R}_f^D)(1 - \theta)\right]^2 - 4\sigma^2 \gamma (1 + \hat{R}_f^D) K_0 \theta (1 - \theta)}}{2\sigma^2 \gamma \theta}.$$

Similarly, we also have $\theta_D^* \hat{P}_0(\theta_D^*) = g(\theta_D^*; \hat{R}_f^D)$. But a trivial extension of Lemma A.1 shows that $g(\theta; R_f)$ is strictly increasing in θ for any fixed $R_f > -1$. Hence the solution to the equation $g(\theta; \hat{R}_f^D) = K_0 \theta_D^*$ must be unique, contracting the hypothesis that $g(\tilde{\theta}_{BB}; \hat{R}_f^D) = \tilde{\theta}_{BB} \hat{P}_0(\tilde{\theta}_{BB}) = K_0 \theta_D^* = \theta_D^* \hat{P}_0(\theta_D^*) = g(\theta_D^*; \hat{R}_f^D)$ with $\tilde{\theta}_{BB} \neq \theta_D^*$.

Consequently, $\theta^*_{BB} = \theta^*_D$ is the unique maximizer of $\sup_{\theta_{BB}\in(-1,\infty)} \hat{J}_{BB}(\theta_{BB})$, and we have shown that $\hat{R}^B_f := \hat{R}_f(\theta^*_{BB}) = \hat{R}^D_f$ as well as $\hat{P}^{BB}_0 := \hat{P}_0(\theta^*_{BB}) = K_0$. The conclusion that $\hat{P}^D_0 = \hat{P}^{BB}_0(1-\theta^*_D)$ now follows from (A.22). □

Proof of Lemma 6.2 By direct computation,

$$\begin{aligned}
\mathbb{E}[e^{-\eta X}] = \mathbb{E}[e^{-\eta(Y-\kappa)^+}] &= \int_{-\infty}^{\infty} e^{-\eta(a+bz-\kappa)^+}\phi(z)dz \\
&= \int_{(\kappa-a)/b}^{\infty} e^{-\eta(a+bz-\kappa)}\phi(z)dz + \int_{-\infty}^{(\kappa-a)/b}\phi(z)dz \\
&= e^{-\eta(a-\kappa)}\int_{(\kappa-a)/b}^{\infty} e^{-\eta bz}\frac{1}{\sqrt{2\pi}}e^{-\frac{z^2}{2}}dz + \Phi\left(\frac{k-a}{b}\right) \\
&= e^{\frac{\eta^2 b^2}{2}-\eta(a-\kappa)}\int_{(\kappa-a)/b}^{\infty}\frac{1}{\sqrt{2\pi}}e^{-\frac{(z+\eta b)^2}{2}}dz + \Phi\left(\frac{k-a}{b}\right) \\
&= \exp\left(\frac{\eta^2 b^2}{2} - \eta(a-\kappa)\right)\Phi\left(-\frac{k-a}{b} - \eta b\right) + \Phi\left(\frac{k-a}{b}\right),
\end{aligned}$$

leading to the stated expression of $\mathcal{V}_m(\cdot)$. The expression of $v_m(\cdot)$ follows immediately by taking limits. □

Proof of Proposition 6.4 The proof is omitted since it follows from a very similar line of arguments as used in the proof of Proposition 5.3. □

B Different beliefs between investors and firm managers

In the baseline model, the representative investor and the firm managers share identical belief over the profitability of the firm's risky technology $\Pi \sim N(\mu, \sigma^2)$. We now consider an extension in which their beliefs diverge: The representative investor thinks the firm's risky technology will deliver a stochastic return of $\Pi \sim N(\mu, \sigma^2)$, but the firm (a paternalistic social planner) knows that the actual distribution of the return shall be $\Pi \sim N(\tilde{\mu}, \tilde{\sigma}^2)$. The firm understands that investor's portfolio selection and equilibrium price formation will materialize under the wrong belief of (μ, σ^2), and the firm will determine the optimal dividends/buybacks policy by maximizing the investor's certainty equivalent evaluated under the ground truth $(\tilde{\mu}, \tilde{\sigma}^2)$. Throughout this section, we assume that Assumption 3.3 holds after replacing (μ, σ^2) with $(\tilde{\mu}, \tilde{\sigma}^2)$.

Since the investor will act based on the belief of $\Pi \sim N(\mu, \sigma^2)$, the equilibrium under a given policy θ_D or θ_{BB} is still given by either Proposition 3.5 or Corollary 3.9. Specifically, the competitive equilibrium under a given dividend strategy θ_D is

$$\hat{\theta}_0 = 1, \qquad \hat{P}_0(\theta_D; \mu, \sigma) = \frac{K_0(1-\theta_D)}{1+R_f}\left(\mu - \sigma^2\gamma K_0(1-\theta_D)\right),$$

while its counterpart for a given buyback strategy θ_{BB} is

$$\begin{aligned}
\hat{\theta}_0 = 1 - \theta_{BB}, \qquad & \hat{P}_0(\theta_{BB}; \mu, \sigma) \\
:= & \begin{cases} \frac{2\sigma^2\gamma\theta_{BB}K_0 - \mu\theta_{BB} - (1+R_f)(1-\theta_{BB}) + \sqrt{\Delta(\theta_{BB})}}{2\sigma^2\gamma\theta_{BB}^2}, & \theta_{BB} \in [0,1)\setminus\{0\}; \\ \frac{K_0}{1+R_f}(\mu - \sigma^2\gamma K_0), & \theta_{BB} = 0. \end{cases}
\end{aligned}$$

Recall from (3.1) that the investor's perceived certainty equivalent is

$$
\begin{aligned}
&\mathrm{CE}(\theta_0; P_0, \theta_D, \theta_{BB}; \mu, \sigma) := U^{-1}(\mathbb{E}[U(W_1)]|\Pi \sim N(\mu, \sigma^2)) \\
&= (\theta_D K_0 + (1-\theta_0) P_0)(1+R_f) \\
&\quad + \left\{ \frac{\theta_0 (K_0 - \theta_D K_0 - \theta_{BB} P_0)\mu}{1-\theta_{BB}} - \frac{\sigma^2 \gamma}{2} \frac{\theta_0^2 (K_0 - \theta_D K_0 - \theta_{BB} P_0)^2}{(1-\theta_{BB})^2} \right\},
\end{aligned}
$$

where we stress the dependence of the certainty equivalent on the perceived statistical parameters of the risky return. From the paternalistic social planner's perspective, the investor's equilibrium value function under dividends as evaluated against the ground truth $(\tilde{\mu}, \tilde{\sigma}^2)$ is now given by

$$
\begin{aligned}
\hat{\mathrm{CE}}_D(\theta_D) &:= \mathrm{CE}(\theta_0 = 1; P_0 = \hat{P}_0(\theta_D; \mu, \sigma), \theta_D, \theta_{BB} = 0; \tilde{\mu}, \tilde{\sigma}) \\
&= \theta_D K_0 (1+R_f) + K_0 (1-\theta_D)\tilde{\mu} - \frac{\tilde{\sigma}^2 \gamma}{2} K_0^2 (1-\theta_D)^2.
\end{aligned}
$$

Likewise, the corresponding value function under buybacks is

$$
\begin{aligned}
\hat{\mathrm{CE}}_{BB}(\theta_{BB}) &:= \mathrm{CE}(\theta_0 = 1-\theta_{BB}; P_0 = \hat{P}_0(\theta_{BB}; \mu, \sigma), \theta_D = 0, \theta_{BB}; \tilde{\mu}, \tilde{\sigma}) \\
&= \theta_{BB} \hat{P}_0(\theta_{BB}; \mu, \sigma)(1+R_f) + (K_0 - \theta_{BB} \hat{P}_0(\theta_{BB}; \mu, \sigma))\tilde{\mu} \\
&\quad - \frac{\tilde{\sigma}^2 \gamma}{2} (K_0 - \theta_{BB} \hat{P}_0(\theta_{BB}; \mu, \sigma))^2.
\end{aligned}
$$

One can in turn easily deduce that the optimal dividend level is given by

$$
\theta_D^* = \theta_D^*(\tilde{\mu}, \tilde{\sigma}) = 1 - \frac{\tilde{\mu} - (1+R_f)}{\tilde{\sigma}^2 \gamma K_0} \in (0, 1).
$$

Moreover, following the same arguments as in the proof of Theorem 4.1, it will hold that

$$
\sup_{\theta_D \in [0,1)} \hat{\mathrm{CE}}_D(\theta_D) = \sup_{\theta_{BB} \in [0,1)} \hat{\mathrm{CE}}_{BB}(\theta_{BB})
$$

if one can demonstrate the existence of some $\theta_{BB} \in [0, 1)$ such that

$$
\theta_{BB} \hat{P}_0(\theta_{BB}; \mu, \sigma) = K_0 \theta_D^*(\tilde{\mu}, \tilde{\sigma}).
$$

Furthermore, any value of $\theta_{BB} = \theta_{BB}^*$ satisfying the above equality is an optimal buyback level. The existence and uniqueness of such θ_{BB}^* follow immediately upon verifying that $\theta_{BB} \mapsto \theta_{BB} \hat{P}_0(\theta_{BB}; \mu, \sigma)$ is continuous and strictly increasing, and that $\hat{P}_0(\theta_{BB} = 1; \mu, \sigma) = K_0 > K_0 \theta_D^*(\tilde{\mu}, \tilde{\sigma})$. Note that θ_{BB}^* depends on $(\mu, \sigma, \tilde{\mu}, \tilde{\sigma})$ in general, while θ_D^* only depends on $(\tilde{\mu}, \tilde{\sigma})$.

To conclude, the equivalence of dividends and buybacks still holds in this extension where both distribution methods can attain the same maximal (objective) certainty equivalent of the investor. It is also easy to verify that the optimal dividend and buyback level are still related via $\theta_{BB}^* \hat{P}_0^{BB} = K_0 \theta_D^*$, i.e., the same amount of equity will be distributed. Nonetheless, some results in Theorem 4.1 may not hold anymore. For example, we do not expect $\theta_D^* = \theta_{BB}^*$ and $\hat{P}_0^{BB} = K_0$.

Data Availability There is no new data created nor analyzed in this study.

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