"To BEM or not to BEM?": Does Modelling Sound Scattering Improve Ultrasonic Mid-Air Haptics?

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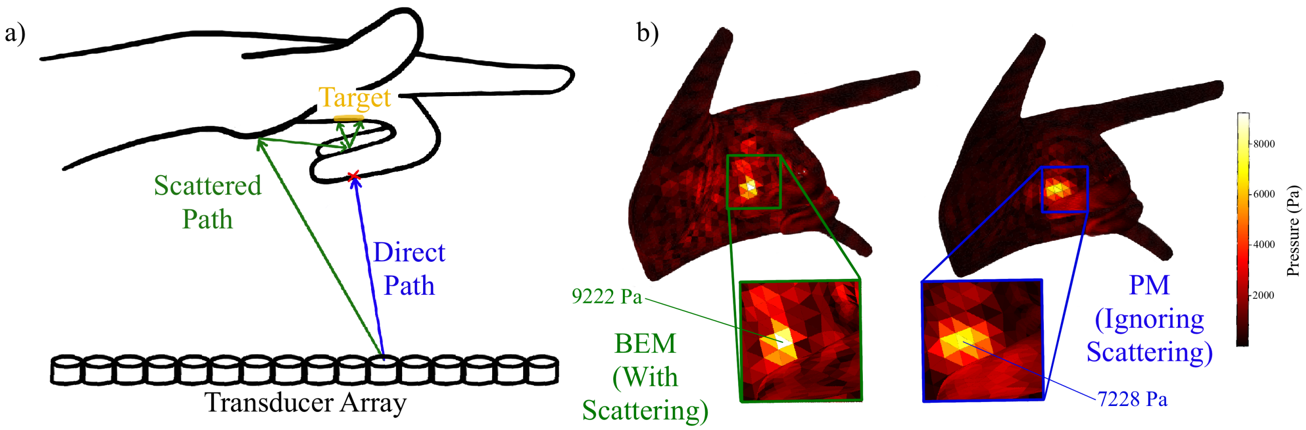

Figure 1: Mid-air ultrasonic haptics uses focused ultrasound to create tactile sensation on a user's hand. Incorporating scattering to the acoustic focusing gives focal points higher pressures, providing more intense stimulus while maintaining user's stimulus shape recognition rate. a) When considering scattered paths, transducers without a direct line of sight to a focus can still contribute pressure. b) Focusing using the boundary element method (BEM), uses scattered waves to provide higher pressure focal points compared to the piston model (PM). Both hands were rendered using scattering effects but only the BEM hand considered scattering in the focusing calculations.

ABSTRACT

Haptic interfaces have promised to bring touch feedback to computer interaction and control of ultrasound fields can facilitate non-contact haptic stimulus. To create these mid-air ultrasound sensations, it is essential to model acoustic transmission at the points of interest. However, current methods assume an empty working volume where ultrasound waves are propagated to the point of interest without obstructions, limiting accuracy in realistic scenarios. We show more intense stimuli are created by taking into account scattering with no reduction in user's shape identification. Particularly significantly, this work is a case study highlighting the issues that arise when considering scattering effects for real time applications. The two most significant issues we identify are computation speeds when modelling scattering and the accuracy of real-world object-to-mesh construction. We urge that solving these two problems should be a major focus of future acoustic research, facilitating a new era of interactive devices and beyond.

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CCS CONCEPTS

• **Human-centered computing** → **Haptic devices**; **Empirical studies in HCI**; • **Computing methodologies** → *Simulation types and techniques.*

KEYWORDS

Human-Computer Interaction, Boundary Element Method, Mid-Air Haptics, Acoustic Holography

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1 INTRODUCTION

Touch feedback for computer systems enables a new way to interact with users, which up until recently has been largely underused [15]. Haptic stimulation could enhance many different domains, with applications from car interfaces [13, 52] to gaming [19] and interactive signs [23]. The methods for achieving haptic feedback range from simple vibration [46, 47] and brushing [43] to thermal or electrostatic effects [34, 45] as well as more complex methods such as stimulating the brain directly [44] or using small drones [2]. However, these methods require hardware that, largely, needs to be in contact with the user [34, 45–47] which limits the spontaneity with which the system can be used.

In order to produce non-contact haptics, acoustic holography aims to control and manipulate ultrasonic sound fields [26, 35]. This is normally achieved through an array of ultrasound transducers, with a delay between their emission to build up sound waves in the desired way (e.g., forming a region of high pressure - a focal point) [26]. Many of the most common methods for computing these delays rely on a simple propagation method which assumes no reflection or scattering of the field, known as the piston model (PM). This model works well when there are no obstructions in the way [25, 26, 35]. However, when scattering objects block the path of incoming waves, or the reflected waves are a significant factor that affect focal point pressure, scattering effects cannot be ignored. In order to take scattered waves into account, the Boundary Element Method (BEM) can be used to replace PM. BEM models real-world objects as a mesh and computes how the waves scatter in space based on their interactions with the boundary of this mesh [5]. However, due to the additional complexity of BEM, it is significantly slower to compute when compared with PM propagation [16].

In this work, we investigated the deployment of scattering techniques in users' haptic experiences and highlight the potential pitfalls for real-time and interactive use for ultrasonic mid-air haptics (UMH). Previous haptic studies have shown higher pressure stimulus can be theoretically generated when using BEM compared to PM. But, that work was limited to simulated results [27]. We apply BEM based haptics to real world stimuli - rendering numerals on a user's hand. This allows us to directly compare PM and BEM derived stimulus and show that BEM gives higher intensity with the same shape identification rates. This demonstrates that BEM is a useful method and should be applied where it can be for UMH applications. However, we also discuss some drawbacks with BEM which may limit where it can be used. BEM is a computational expensive method, with many elements of a mesh required to be considered in computations. We discuss that for a real time application where fast computation is needed, the mesh may need to be made more coarse, and thus, accuracy may be limited [5]. We also point out that the mesh being used does not accurately reflect the real-world object then the simulated sound field will not reflect the real world either. From this work we hope that future work will be directed towards improving BEM's computation speed, allowing finer meshes to be used in real-time applications and opening the door to these more intense UMH stimulus.

2 RELATED WORK

2.1 Mid-Air Haptics

Mid-air haptics uses ultrasound from a grid of ultrasound sources [25, 35] to create stimulus on a user's skin. Normally, many ultrasound sources are arranged into an array with the waves from each source delayed such that they converge at a specified point in space at the same time [27, 28, 35]. This causes a region of high pressure to be produced but the ultrasound's high frequency pressure cannot be felt directly. Instead, some modulation at a perceivable frequency is required. (e.g., moving the point across the users skin to form shapes [1]).

This allows for interactive devices that can be used spontaneously, without wearing or touching a delivery device [25]. A museum exhibit where visitors can interact with virtual objects [8], a car interface that guides a driver with gentle inputs [13] and immersive virtual reality games [14, 19] could all be possible with this technology. Since there is no need for contact with the user, this technology is far more suited for deployment in real-life situations where bulky and complex contact devices are to be avoided [45, 46].

2.2 Boundary Element Method (BEM)

When solving for the signals that drive ultrasound sources to create a UMH stimulus, in most cases, only the direct paths from the sound source to the point of interest are taken into account. The waves are assumed not to reflect, using a propagation model known as the piston model (PM) [25, 35]. However, in the presence of scattering or occluding objects, such as a user's hand, this assumption becomes more inaccurate and the scattering waves need to be considered [5]. These scattered waves can be considered using a propagator known as the boundary element method (BEM). This method models the contributions from an ultrasound source to a point in the presence of a scattering object as a three step model as seen in Fig.1.a. The contributions from the source directly to the point is combined with the contributions from the source to the object and the object to the point, allowing for efficient covering and computation of BEM problems [16]. Therefore, many acoustic problems can be addressed, from levitation of objects larger than the acoustic wavelength [17], to levitation and manipulation of small objects around complex scattering objects [16] and self-assembly of objects for 3D printing applications [36].

For haptic applications, BEM can be used to model complex scenarios such as simulating soft materials [29], allowing a reflector to focus sound on a hand without direct line of sight to the source [28]. Additionally, it has been shown to be capable of taking into account scattered waves from the hand itself and theoretically generates higher pressure focal points compared to other methods [27]. However, [27] did not validate if such higher pressure in simulation actually corresponds to a stronger stimulus in reality. In addition, this previous work use an approximation to BEM instead of taking into account the full set scattered waves [27]. To the best of our knowledge, there is no work comparing BEM and PM haptic stimuli in reality, to evaluate if BEM provides any benefit over PM when used in the UMH context.

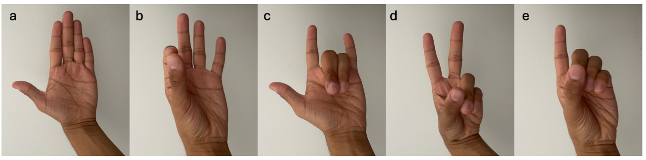

Figure 2: Different hand poses the user was asked to imitate. a) flat hand with all fingers extended, b) OK where the tips of the thumb and index finger touch with the remaining fingers extended , c) 'Spider-man' with the thumb, index and little fingers extended with the ring and middle fingers curled back. There is a gap between the surface of the palm and the curled fingers, d) peace, where the index and middle fingers are extended and the rest are brought together. The fingers under the palm do not touch the palm e) point where all but the index finger are together with the index finger extended. The fingers under the palm do not touch the palm

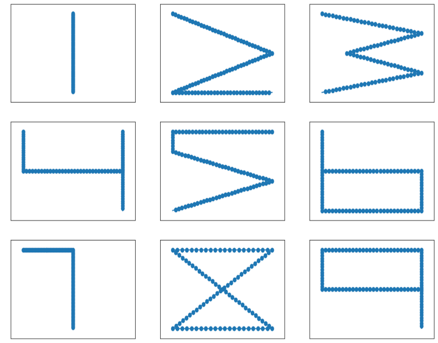

Figure 3: Shape of numerals 1-9 to be used, each sampled to 100 points. These numerals were scaled using the scan of users hand so that the numerals fit inside the palm.

3 USER STUDY

3.1 Study Design

3.1.1 Study Pipeline. In order to examine the impact of using BEM on users' tactile sensations, we conducted a between group study where tactile numerals were rendered either using BEM and PM in two groups of users. An Ultraleap Leap Motion Controller 2 [48] was used to track user's hand position, since this controller (and its family of devices) is the standard hand tracking device in many mid-air haptic studies [9, 10, 14, 25, 30, 31] due to its speed, relatively low price and ease of use compared to other more complex scanners [38, 53]. Such advantages made it the ideal tool to examine state-of-the-art hardware with the general utility of BEM. The hand data was converted to a mesh using Unity (version 2022.3.9f1) and the `Leap.Unity` API, with the "Low Poly Hands" being used [49]. The resulting mesh was then loaded using Python, centred and sampled to ensure consistent computation speeds.

The tracked mesh was then used to create unique path for drawing a random numeral between 1-9, see Fig.3, each sampled to 100 points. The numbers were scaled such that they fit within a square on the palm of the users hand, roughly a 5cm square. These numeral paths were then generated using the weighted Gerchberg-Saxton (WGS) algorithm [6, 21] either with BEM or PM as the propagator and using single-point spatio-temporal modulation (STM) with an average speed of $0.24ms^{-1}$. WGS will produce a hologram that generates a single high pressure focal point, which can then be sent to the physical device to create this stimulus. STM was chosen as it tends to give superior sensation compared to other methods of modulation [1, 42]. For both BEM and PM conditions, we conducted the whole BEM computation so the time to complete the did not vary depending on which method was being used, and for the PM cases it was then discarded. In order to keep the study as close to a real-world application, the computation time was constrained to around 10s. This reflects the realistic limit on using BEM in real time computation, and in turn a realistic limit on the resolution of the scanned mesh that can be used.

3.1.2 Study Procedure. We chose the following hand poses from literature to reduce complexity and ensure good coverage of gestures in UMH: a flat hand [18, 52], an OK gesture where the thumb touches the index finger [52], 'Spider-man' where the two central fingers are curled in [18], peace with the index and middle fingers extended with the rest curled [39], and a point where only the index finger is extended [39, 52], all shown in Fig.2. This set of poses gives a range of 'occlusion' between the sound sources and the palm, with some poses having no fingers covering the palm, and others have all the fingers in the way, allowing for a range of cases to be compared. The flat hand has no occlusion, increasing with the OK and point and more again for the peace and spider man poses. This occlusion will not be considered when using PM propagation and as such should highlight where BEM and PM differ [16]. Because the scattering is dependant on the geometry of the scattering object, with scattered waves interacting with one another, different poses highlight differences due to different levels of scattering [5].

Similarly to [50], we used numerals 1-9 as this provides a wide range of shapes that users will all be familiar with and facilitate comparison. Additionally, since there are 9 different shapes, the chance of a false positive results is much lower than if only a few shapes were used. While [50] uses 0-9, we did not include the 0 numeral because we felt it could be easily confused for the other rounded numbers and to keep the study as short as possible for the users comfort (as the number of samples is the number of numerals used multiplied by the number of poses). This is why a between-groups study was used to allow for each participant to feel all combinations of hand pose and numerals for the propagation method they were assigned. Using a within-group method would have meant the study would need to be twice as long (or have half as many stimuli) which was deemed unfeasible. Deploying a between-group design would significantly reduce the study duration, and thus reduce carry-over effects such as fatigue or learning effects that would bias study results.

The set-up used for study can be seen in Fig.4. We used a single transducer board of 256 (16x16) transducers, powered with 20V. Both PM and BEM uses the same starting paths, sampled to the

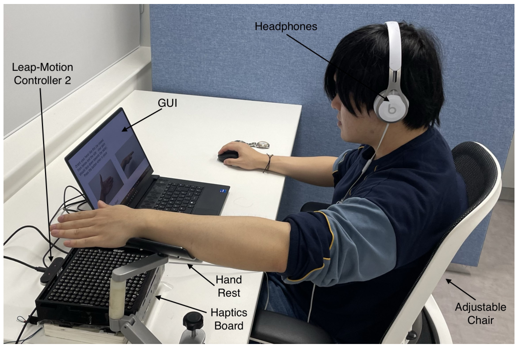

Figure 4: The set-up for the user study. The user sit on an adjustable chair and were provided with an arm rest to position the hand comfortably above the haptics board. The Leap-motion controller 2 tracks the hand and a python-based GUI allows the user to interact with the study. While the user is conducting the study, pink noise is played to avoid any device noise impacting them.

same number of points and mapped to the users hand, with transducer amplitudes being set to the maximum (amplitude = 1) for all stimuli. For hand scanning, the leap-motion controller was positioned behind the haptic device as seen in Fig.4. Users interacted and recorded their responses using a python-based graphical user interface (GUI). This study received ethics approval from the institution's Ethics Review Board and all participants gave consent, and were compensated for their time.

3.2 Study Procedure

24 participants were involved in the study (10 Female, mean age 26.5 years, standard deviation 6.3 years). The users were evenly split into two groups, one was given the BEM-derived stimulus and one the PM-derived stimulus. The first participant was given BEM stimuli and then the second PM stimuli and so on, therefore, ensured an even split between the BEM and PM groups. A video was presented to participants before the study to help participants fully understand the study. Initially, each user was given the same three training examples to gain familiarity with the system. Later, they felt all 45 combinations of the 5 poses and 9 numbers in a random order, using their assigned propagation method (PM or BEM) for all cases.

The users interacted with the Python-based graphical interface, receiving instructions as to which pose to make and informed when the stimulus would begin. After perceiving the stimuli, users were asked to identify which number they had felt and to rank the intensity of the sensation using an unbounded scale (0-infinity) [1, 12, 37, 41]. Both of these values were recorded by entering a numerical value into a corresponding box in the GUI, with the input validated to be in the correct ranges. The ratings were later normalised for each user between 0-10 to allow comparison. A 1-minute rest were provided every 10 stimuli to avoid fatigue. During the study, the user heard pink noise played through headphones to avoid any effects from device noise [42].

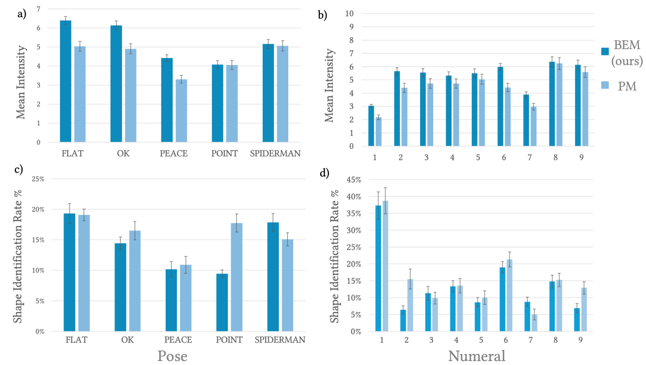

Figure 5: Results of the user study conducted. a) the normalised intensity by pose . b) the normalised intensity by numeral. c) The shape identification rate broken down by pose. d) the shape identification rate by numeral. For each point, the respective result is split into BEM cases and PM cases. For all cases of intensity, BEM gives stronger results while for shape identification rate, there is no significant difference in the rankings when compared pairwise across all cases for a given pair using a Wilcoxon signed rank test. Error bars show the standard error

4 RESULTS

The results of the user study are presented in Fig.5. The data is displayed as shape identification rate and intensity for each hand pose (with all numerals for that pose aggregated) and for each numeral (with all poses for that numeral aggregated). For each condition, the results are split again as the BEM cases and the PM cases.

Given the data was normally distributed (using a Shapiro–Wilk test [40], $p = 0.075$ and $p = 0.976$ for intensity and $p = 0.709$ and $p = 0.434$ for shape identification rate when separated by pose and numeral respectively) with equality of variance (using a Brown–Forsythe test [3], $p = 0.737$ and $p = 0.510$ for intensity rankings and $p = 0.340$ and $p = 1$ for accuracy, separated by pose and numeral for each) but with a small number of outliers (Number of outliers = 2 and 3 for intensity separated by pose and numeral respectively and 1 for both shape identification rate aggregations), we conducted both student's t-test and the non-parametric Wilcoxon signed-rank test [51] using JASP 0.19 [20], finding little difference between the two, the following p-values are derived from the Wilcoxon signed-rank test due to the outliers present. For each metric (combination of intensity or shape identification rate and numeral or pose) the two methods are compared pairwise. The pairwise rankings across each data point are then used as the statistic to test, with the probability of the two methods being the same.

For every hand pose and numeral, BEM was ranked as more intense (See Fig 5.a & b) when compared to PM stimulus. This represents significantly higher intensity for BEM when using a one sided test ($p = 0.032$ when aggregated by hand poses and $p = 0.002$ for numerals). Such results indicates that BEM does provide stronger stimulus when compared to not considering scattered waves with a very strong effect size (Cohen's $d = 1.183$ and 1.847 for pose and

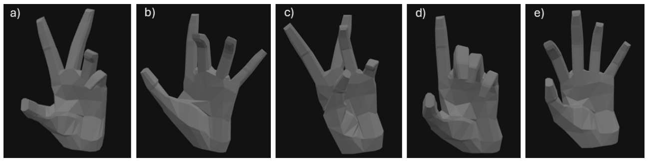

Figure 6: Examples of poorly scanned hand meshes, each is meant to be one of the poses in Fig.2. The scanner has misplaced some of the fingers leading to an inaccurate mesh which will lead to the simulated scattering not reflecting reality. a) peace pose with last two fingers out of place. b) spiderman pose with middle two fingers incorrectly places. c) peace pose with overlapping middle and ring fingers. d) point pose with insufficient curl of the last three fingers. e) OK pose with insufficient bend in the index finger

numerals [7]). On the other hand, for shape identification rate of users guessing which numeral they felt, we did not find significant difference between the two propagation methods (See Fig 5.a & b) when using a two sided test ($p = 0.461$ when aggregated by pose and $p = 0.181$ for numerals). Therefore, modelling scattered waves can give more intense stimulus while the shape identification rate remains unaffected.

Additionally, the numeral one had a much higher rate of recognition compared to the other numerals. This has been shown in previous work [50] and the simplicity of the numeral means this is not a surprising result. Somewhat surprising though is that BEM seems to perform much better than PM for hand-poses that might be considered low-occlusion while the gap is much smaller for high-occlusion poses. For the 'flat' and the 'OK' poses, with a repetitively small degree of occlusion, BEM has a much higher intensity but for the 'peace' and 'spiderman' pose where there is much more occlusion the methods are much more similar. The reason for this is not entirely clear but could be down to the added complexity of the task means BEM struggles to capture the whole scattering with the coarse meshes used. This emphasises faster BEM computation is needed, as a more fine mesh could be used without compromising the computation time.

5 DISCUSSION

In this study, we systematically compared BEM with PM when rendering mid-air haptic shapes. We found that BEM provides more intense stimuli while maintaining shape identification rates. This means that there is a trade-off between computation speed and the intensity of the stimuli delivered. Longer computation speeds associated with BEM can lead to more intense stimuli while shorter computation speeds of PM gives less a intense focus. Since the identification rate of users was not improved nor was it hindered, PM derived stimulus may as well be used over BEM when the goal is only identification rates. We believe that this work can be seen as a case study, highlighting sources of error in BEM across all applications. These, largely, come down to computation speed limiting the mesh size that can be used as well as inaccuracy in hand-to-mesh generation. We urge that work should be focused on faster BEM computation allowing more fine meshes to be used and more intense results obtained.

In the real world, say for gaming or interactive media, developers could incorporate BEM to create highly immersive and emotionally charged interactions during key moments, such as combat or critical decisions, while using the simpler PM for less intense moments. This approach would allow for a more dynamic and engaging user experience tailored to the context of interaction. Or in consumer electronics, where tactile feedback is increasingly being integrated into devices such as smartphones and wearables, understanding the role of perceptual intensity can guide the design of more customized haptic feedback. Devices could vary the intensity based on the task at hand—using higher-intensity feedback to alert users to important notifications or lower-intensity feedback for less critical interactions—thereby enhancing user satisfaction and usability.

5.1 Limitations and Future Work

5.1.1 Computation. As discussed, computation of BEM is slow due to the very large matrices involved and the corresponding inverse problems. This means the speed of computation is mostly defined by the number of elements in the mesh [16]. Because a hand is a relatively large object when compared to the roughly 8mm wavelength and the number of mesh divisions should be on the order of half to a quarter of this wavelength [5, 16], the number of sub-divisions in the mesh should be very large. Subsequently, the computation will become extremely slow. Therefore, in order to have reasonable computation time during the study, a more coarse mesh was used. This subsequently reduced the scattering accuracy of the computation.

In order to resolve this, faster methods of BEM computation should be explored, which would ideally enable both faster computation and finer mesh sizes. For example, machine learning could be used with mesh based learning techniques to predict the BEM scattering matrix directly [11, 24, 32]. Alternatively, a hierarchical method could aggregate mesh elements to reduce the complexity of the task [4]. Focus should also be maintained on accuracy to the true BEM results. As seen in existing fast-BEM approximations, they will not take into account the full scattering picture and so may be less accurate than full BEM [27]. For a truly real-time interactive system, such real-time computation would be essential. Gains in computation speed of BEM will have wide ranging benefits from staring a similar shift from pre-computation to interactivity. Example scenarios include scattering-conscious levitation displays [35], and potentially enabling levitation of large objects in real time [17], and of course real time BEM haptics [27]. Because this development would benefit not just haptics but almost all aspects of acoustic holography, such investigation should be a primary focus of future research.

5.1.2 Hand Tracking. The hand tracking for this study uses Ultraleap leap-motion controller 2. This device detects the positions of the hand's joints, then a mesh is rigged using these positions. This method of retrieving meshes is common across many of the state-of-the-art for hand pose estimation methods [22, 33, 54] and so they share the same flaws. Firstly, the mesh is a generic hand mesh, meaning that it may lack smaller details such as finger widths or palm size. Secondly, these scanned hands are sometimes significantly different to the generated mesh because these methods do not have perfect positional accuracy, Fig.6 shows some examples of poorly scanned hands (compare with the targets in Fig.2). This will lead to simulated scattering effects that do not correspond to the real-life effects and as such the stimulus may give poor sensation.

Finally, some bio-mechanically unlikely situations are generated by these methods. As an example, in the 'spiderman pose', a gap arises between middle and ring fingers when the hand is bent, which does not reflect reality. A gap between fingers that does not exist in reality will provide a path for waves to the palm in simulation that does not exist in reality. This demonstrates that despite leap-motion controller family's widespread use for hand tracking and particularly mid air-haptics, such technology may not be viable in the current form for the step from simple tracking to full hand reconstruction, and thus, BEM haptic rendering. We hope that showing these limitations may help direct future research into hand tracking that would be more appropriate while maintaining a low price and ease of use.

5.1.3 Study Design. Because of the large number of stimuli needed (hand positions times numerals), there was not scope in this study to have each user feel each stimulus to allow for a within-group study. This may introduce some bias for individual perception, however, intensity was normalised for each user which should reduce this effect. Additionally, because the participants were randomly assigned to each group, any inconsistencies would be reduced. Additionally, the length of the study meant repeats were not possible, which may also reduce the strength of the findings but because all participants felt all stimuli for their propagation method, each stimulus was repeated across the entire study. In this way our design prioritizes broad coverage of shapes and gestures within a single session, ensuring comprehensive data collection while preserving engagement and response reliability.

6 CONCLUSION

Despite the limitations, our findings are crucial for haptic designers and researchers. The results of our study reveal that Boundary Element Method leads to a higher perceptual intensity at the focal point compared to the Piston Model, indicating that BEM provides a more immersive and engaging haptic experience. Even with this difference in perceived intensity, both the BEM and PM yielded equivalent shape recognition results, suggesting that perceptual intensity does not impact the ability to accurately identify shapes in mid-air haptic feedback. To achieve this improved performance, the most pressing issue is the speed of computation. A slow computation will, of course, limit the development of real time applications and coarse meshes limit accuracy. Ideally, high speed and high accuracy would lead itself to the best-of-both-worlds situation. Additionally, many applications require improved object-to-mesh scanning using low cost and fast methods. We suggest that these are limiting factors for BEM across all applications and we believe that future acoustic research should focus on this exciting and promising problem to open a new era of interactive devices.

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