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Fluid Antenna Enabled Space-Time Block Coding With Achievable Rate Optimization

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***Abstract*—The conventional space-time block coding (STBC) technique offers advantages such as low detection complexity and full-diversity transmission while suffering from a reduced achievable rate. Inspired by the novel concepts of fluid antennas (FAs) exploiting extra spatial flexibility, we propose to maximize the STBC achievable rate by jointly optimizing the positions of transmit and receive FAs. In order to solve the corresponding highly non-convex problem, an alternating optimization (AO) algorithm based on successive convex approximation (SCA) is employed to obtain a locally optimal solution. Simulation results reveal the superiority of the FA-enabled STBC (FA-STBC) system over its fixed-position antenna (FPA) and STBC counterparts within limited FA regions.**

***Index Terms*—Achievable rate, space-time block coding (STBC), fluid antenna (FA), alternating optimization (AO), successive convex approximation (SCA).**

I. Introduction

THE space-time block coding (STBC) technique [1]–[3], as a pivotal paradigm of multiple-input multiple-output (MIMO) systems, fundamentally enhances wireless communication reliability by capitalizing on both spatial and temporal diversity. Its advantages—specifically, the ability to achieve full diversity gain coupled with the inherent linearity in detection complexity—render it a highly suitable candidate for deployment in communication scenarios where stringent reliability requirements are paramount, such as ultra-reliable low-latency communications (uRLLC) and massive machine-type communications (mMTC). However, in light of the exponential growth of Internet of Everything (IoE) services, the need for significantly higher achievable rates in future wireless systems has become increasingly urgent [4], [5]. Despite having a plethora of studies on STBC systems, their practical constraint of degradation in achievable rate when compared to the conventional Vertical Bell Labs Space-Time (V-BLAST) architecture [6], which ultimately constrains its applicability in high-throughput contexts and hindering its ability to meet the evolving demands of next-generation communication systems, have received limited attention, despite the many-fold benefits of STBC systems.

Meanwhile, the concept of fluid antennas (FAs) was recently introduced in [7] as a novel means to fully exploit the spatial variability of wireless channels by enabling antenna movement. Unlike conventional fixed-position antennas (FPAs) confined to static configurations, FAs possess the ability to traverse a restricted area on the panel, thereby offering a significantly higher degree of adaptability and operational flexibility [8]. Distinct from reconfigurable antenna techniques altering internal antenna states (e.g., patterns/polarization) for goals like diversity [9] or instantaneous SNR [10], FA's adaptability stems fundamentally from optimizing their physical position. This spatial flexibility is made possible through various enabling technologies, such as motors, fluid-based systems, microelectromechanical systems (MEMS) [11], pixel-based antenna arrays and pinching antennas [12]. Consequently, the integration of FA technology holds the potential to substantially enhance both the adaptability and performance of next-generation wireless communication systems, particularly in dynamic environments where traditional antenna systems struggle to maintain the optimal performance.

Building upon these aforementioned advantages, FA systems have garnered significant attention in the academic field of wireless communications, prompting a series of studies aiming at advancing spatial diversity and enhancing multiplexing capabilities. For instance, in [13], the authors explored the application of position index modulation (PIM) within FA systems, proposing a strategy to mitigate bit error rate (BER) while simultaneously capitalizing on the rate gains inherent to index modulation. Furthermore, Ref. [14] introduced a method to boost the achievable rate of FA-enabled MIMO systems by jointly optimizing both the positions of transmit and receive FAs and the covariance of the transmit signals, thereby facilitating more efficient communication. Additionally, in [15], it was demonstrated that FA systems could not only simplify the processing of reconfigurable intelligent surfaces (RIS) but also enhance their effectiveness in scenarios where only statistical channel state information (CSI) is available, offering a promising approach for reducing system complexity without compromising performance.

Drawing inspiration from the preceding discussion, this paper explores the potential enhancement of the achievable rate of STBC systems through the integration of FAs. Specifically, we address the problem of maximizing the achievable rate in STBC systems by concurrently optimizing the positions of both transmit and receive FAs. To solve this, we adopt an alternating optimization (AO) framework, decomposing

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the problem into two distinct sub-problems. Subsequently, second-order Taylor expansions are employed to derive a sub-optimal solution for the position optimization of both transmit and receive FAs. To the best of the authors' knowledge, this study marks the first attempt to exploit the dynamic positioning capabilities of FA systems as a means to enhance the performance of STBC, offering a novel perspective in the optimization of communication systems.

II. System Model and Problem Formulation

A. Channel Model of FA Systems

We consider an FA-enabled MIMO system with N_t transmit and N_r receive FAs, respectively, and resort to the field-response-based channel model [13].

More specifically, the numbers of transmit paths and receive paths are denoted as L_t and L_r, respectively, and the signal propagation difference between the transimit FA position $\mathbf{t} = [x_t, y_t]^T \in \mathcal{C}_t$ with $\mathcal{C}_t$ being the region for the movement of transmit FAs, and the reference point $\mathbf{o}_t = [0,0]^T$ for the p-th ($p = 1, 2, \ldots, L_t$) transmit path can be calculated as $\rho_t^p(\mathbf{t}) = x_t \sin\theta_t^p \cos\phi_t^p + y_t \cos\theta_t^p$, where θ_t^p and ϕ_t^p denote the elevation and azimuth angle of departures (AoDs), respectively. Therefore, the phase difference of the p-th transmit path between the position of the transmit FA and reference point $\mathbf{o}_t$ can be further represented as $2\pi\rho_t^p(\mathbf{t})/\lambda$, where λ is the wavelength of the signal. Thus, the transmit field response vector of the transmit FA is given by

$$\mathbf{g}(\mathbf{t}) \triangleq \left[e^{j\frac{2\pi}{\lambda}\rho_t^1(\mathbf{t})}, e^{j\frac{2\pi}{\lambda}\rho_t^2(\mathbf{t})}, \ldots, e^{j\frac{2\pi}{\lambda}\rho_t^{L_t}(\mathbf{t})}\right]^T \in \mathbb{C}^{L_t}. \quad (1)$$

To facilitate subsequent derivations, we define the field response matrix of all N_t transmit FAs as

$$\mathbf{G}(\tilde{\mathbf{t}}) \triangleq [\mathbf{g}(\mathbf{t}_1), \mathbf{g}(\mathbf{t}_2), \ldots, \mathbf{g}(\mathbf{t}_{N_t})] \in \mathbb{C}^{L_t \times N_t}. \quad (2)$$

Similarly, at the receiver side, the elevation and azimuth angle of arrival (AoAs) of the q-th ($q = 1, 2, \ldots, L_r$) receive path are represented as $\theta_r^q \in [0, \pi]$ and $\phi_r^q \in [0, \pi]$, respectively. The field response vector of the receive FA is given by

$$\mathbf{f}(\mathbf{r}) \triangleq \left[e^{j\frac{2\pi}{\lambda}\rho_r^1(\mathbf{r})}, e^{j\frac{2\pi}{\lambda}\rho_r^2(\mathbf{r})}, \ldots, e^{j\frac{2\pi}{\lambda}\rho_r^{L_r}(\mathbf{r})}\right]^T \in \mathbb{C}^{L_r}, \quad (3)$$

where $\rho_r^q(\mathbf{r}) = x_r \sin\theta_r^q \cos\phi_r^q + y_r \cos\theta_r^q$ is the propagation difference for the q-th receive path between the receive FA $\mathbf{r} = [x_r, y_r]^T \in \mathcal{C}_r$ with $\mathcal{C}_r$ being the region for the movement of receive FAs and the reference point $\mathbf{o}_r = [0,0]^T$. Then, the corresponding receive field response matrix is

$$\mathbf{F}(\tilde{\mathbf{r}}) \triangleq [\mathbf{f}(\mathbf{r}_1), \mathbf{f}(\mathbf{r}_2), \ldots, \mathbf{f}(\mathbf{r}_{N_r})] \in \mathbb{C}^{L_r \times N_r}. \quad (4)$$

The matrix describing the path response from the reference point of the transmit region $\mathbf{o}_t$ to the reference point of the receive region $\mathbf{o}_r$ is established as $\boldsymbol{\Sigma} \in \mathbb{C}^{L_r \times L_t}$, where $\boldsymbol{\Sigma}_{q,p}$ represents the field response between the p-th transmit path and the q-th receive path. Hence, the channel matrix between the transmitter and receiver is given by

$$\mathbf{H}(\tilde{\mathbf{t}}, \tilde{\mathbf{r}}) = \mathbf{F}(\tilde{\mathbf{r}})^H \boldsymbol{\Sigma} \mathbf{G}(\tilde{\mathbf{t}}). \quad (5)$$

The construction of $\boldsymbol{\Sigma}$ with varied structures facilitates the characterization of different types of channels, including line-of-sight (LoS), geometric, Rayleigh, and Rician channels.

B. MIMO and STBC Achievable Rate of FPA Systems

1) MIMO Achievable Rate: Assuming that channel state information (CSI) is available at the transmitter side, the MIMO achievable rate can be written as

$$C = \log_2 \det\left(\mathbf{I}_{N_r} + \frac{P}{N_0 N_t}\mathbf{H}\mathbf{Q}\mathbf{H}^H\right), \quad (6)$$

where $\mathbf{I}_{N_r}$ is an identity matrix of dimension N_r, $\mathbf{Q}$ is transmit covariance matrix which can be obtained by water-filling technique, P is the average transmit power, and N_0 is the noise power. It can be lower bounded by

$$\begin{aligned} C &\geq \log_2 \det\left(\mathbf{I}_{N_r} + \frac{P}{N_0 N_t}\mathbf{H}\mathbf{H}^H\right) \\ &= \log_2\left(1 + \frac{P}{N_0 N_t}\|\mathbf{H}\|_F^2 + S\right), \end{aligned} \quad (7)$$

where $S = \left(\frac{P}{N_0 N_t}\right)^2 \sum_{i_1 \neq i_2}^{i_1 < i_2} \sigma_{i_1}^2 \sigma_{i_2}^2 + \left(\frac{P}{N_0 N_t}\right)^3 \sum_{i_1 \neq i_2 \neq i_3}^{i_1 < i_2 < i_3} \sigma_{i_1}^2 \sigma_{i_2}^2 \sigma_{i_3}^2 + \cdots + \left(\frac{P}{N_0 N_t}\right)^R \prod_{i=1}^R \sigma_i^2$, R is the rank of $\mathbf{H}$, and $\{\sigma_i\}_{i=1}^R$ are the non-zero singular values of $\mathbf{H}$.

2) STBC Achievable Rate: According to [6], the achievable rate of STBC systems can be obtained as

$$C_{\mathrm{STBC}} = \frac{K}{T}\log_2\left(1 + \frac{P}{N_0 N_t}\|\mathbf{H}\|_F^2\right), \quad (8)$$

where K represents the number of distinct baseband symbols contained within the STBC codeword, T denotes the number of time slots occupied by the STBC, and $\frac{K}{T}$ signifies the corresponding code rate.

By comparing (7) and (8), the reduction of the achievable rate can be calculated as

$$\begin{aligned} &C - C_{\mathrm{STBC}} \\ &\geq \tfrac{T-K}{T}\log_2\left(1 + \tfrac{P}{N_0 N_t}\|\mathbf{H}\|_F^2\right) + \log_2\left(1 + \tfrac{S}{1 + \frac{P}{N_0 N_t}\|\mathbf{H}\|_F^2}\right), \end{aligned} \quad (9)$$

which is greater than 0. As will be demonstrated in the subsequent analysis, even when the size of the movable region is limited to a scale on the order of the square of the wavelength, small adjustments in antenna positions can yield substantial improvements, effectively compensating for the inherent rate degradation typically associated with such constraints.

C. Problem Formulation

FAs endow the transceiver with the ability to actively improve channel conditions. By leveraging the degrees of freedom induced by antenna movement, the STBC achievable rate can be enhanced, thereby overcoming the inherent limitations of the STBC scheme. This approach is capable of achieving higher achievable rate while maintaining low-complexity detection and full-diversity transmission.

To explore the theoretical achievable rate limit of STBC systems enhanced by FAs, it is assumed that perfect knowledge of the multi-path channel components is available. Consequently, the achievable rate of the FA-enabled STBC (FA-STBC) channel with flexible positions can be expressed as

$$C_{\mathrm{STBC}}(\tilde{\mathbf{t}}, \tilde{\mathbf{r}}) = \frac{K}{T}\log_2\left(1 + \frac{P}{N_0 N_t}\left\|\mathbf{H}(\tilde{\mathbf{t}}, \tilde{\mathbf{r}})\right\|_F^2\right). \quad (10)$$

Unlike conventional MIMO channels with FPAs, the achievable rate of FA-STBC systems in (10) relies on the positions of the transmit and receive FAs $\tilde{\mathbf{t}}$, $\tilde{\mathbf{r}}$ as they directly impact the channel matrix $\mathbf{H}(\tilde{\mathbf{t}}, \tilde{\mathbf{r}})$.

To prevent the coupling effect among antennas within the transmit/receive region, it is necessary to maintain a minimum separation distance D between each antenna pair, such that the Euclidean distance between any two transmit antennas $\mathbf{t}_k$ and $\mathbf{t}_l$ satisfies $\|\mathbf{t}_k-\mathbf{t}_l\|_2 \geq D$ for $k,l=1,2,...,N_t$, $k \neq l$, and the distance between any two receive antennas $\mathbf{r}_k$ and $\mathbf{r}_l$ satisfies $\|\mathbf{r}_k-\mathbf{r}_l\|_2 \geq D$ for $k,l=1,2,...,N_r$, $k \neq l$. Consequently, our objective is to enhance the achievable rate of an FA-STBC channel by simultaneously optimizing the positions of the FAs $\tilde{\mathbf{t}}$, $\tilde{\mathbf{r}}$ within the constraints imposed by the minimum distance requirements for the FA positions.

Based on the above discussion, the optimization problem can be formulated as

$$\text{(P1)} \quad \max_{\tilde{\mathbf{t}},\tilde{\mathbf{r}}} \quad \frac{K}{T}\log_2\left(1+\frac{P}{N_0 N_t}\left\|\mathbf{H}(\tilde{\mathbf{t}},\tilde{\mathbf{r}})\right\|_F^2\right) \tag{11a}$$

$$s.t. \quad \tilde{\mathbf{t}} \in \mathcal{C}_t, \tag{11b}$$

$$\tilde{\mathbf{r}} \in \mathcal{C}_r, \tag{11c}$$

$$\|\mathbf{t}_k-\mathbf{t}_l\|_2 \geq D, \quad k,l=1,2,\ldots,N_t, \quad k \neq l, \tag{11d}$$

$$\|\mathbf{r}_k-\mathbf{r}_l\|_2 \geq D, \quad k,l=1,2,\ldots,N_r, \quad k \neq l. \tag{11e}$$

Given the positive correlation between STBC achievable rate and channel gain $\left\|\mathbf{H}(\tilde{\mathbf{t}},\tilde{\mathbf{r}})\right\|_F^2$, maximizing STBC achievable rate is equivalent to maximizing the corresponding channel gain. Therefore, the optimization problem (P1) can be reformulated as

$$\text{(P2)} \quad \max_{\tilde{\mathbf{t}},\tilde{\mathbf{r}}} \quad \left\|\mathbf{H}(\tilde{\mathbf{t}},\tilde{\mathbf{r}})\right\|_F^2 \quad s.t. \quad \text{(11b), (11c), (11d), (11e)}. \tag{12}$$

III. Proposed Algorithm

In this section, we present an AO algorithm to solve (P2). Two subproblems are solved in the sequel, which respectively optimize the transmit FA position $\mathbf{t}_n$ and the receive FA position $\mathbf{r}_m$, with all the other variables being fixed. The AO algorithm devised can guarantee a solution that is at least locally optimal for (P2).

A. Optimization of $\mathbf{t}_n$ given $\{\mathbf{t}_k, k \neq n\}_{k=1}^{N_t}$ and $\{\mathbf{r}_m\}_{m=1}^{N_r}$

In this subproblem, we will consider the optimization of $\mathbf{t}_n$ in (P2) given $\{\mathbf{t}_k, k \neq n\}_{k=1}^{N_t}$ and $\{\mathbf{r}_m\}_{m=1}^{N_r}$. Initially, we consider isolating the impact of $\mathbf{t}_n$ on channel gain from the objective function. In particular, this can be achieved through some mathematical derivations as

$$\left\|\mathbf{F}(\tilde{\mathbf{r}})^H\mathbf{\Sigma}\mathbf{G}(\tilde{\mathbf{t}})\right\|_F^2 = \mathbf{g}(\mathbf{t}_n)^H\mathbf{\Sigma}^H\mathbf{F}(\tilde{\mathbf{r}})\mathbf{F}(\tilde{\mathbf{r}})^H\mathbf{\Sigma}\mathbf{g}(\mathbf{t}_n)+C_1^t, \tag{13}$$

where $C_1^t = \sum_{i=1,i\neq n}^{N_t}\left\|\mathbf{F}(\tilde{\mathbf{r}})^H\mathbf{\Sigma}\mathbf{g}(\mathbf{t}_i)\right\|_2^2$ is a constant independent of $\mathbf{t}_n$. By defining

$$\mathbf{D} = \mathbf{\Sigma}^H\mathbf{F}(\tilde{\mathbf{r}})\mathbf{F}(\tilde{\mathbf{r}})^H\mathbf{\Sigma}, \tag{14}$$

the subproblem for optimizing $\mathbf{t}_n$ can be expressed as

$$\text{(P3-n)} \quad \max_{\mathbf{t}_n} \quad \mathbf{g}(\mathbf{t}_n)^H\mathbf{D}\mathbf{g}(\mathbf{t}_n) \tag{15a}$$

$$s.t. \quad \mathbf{t}_n \in \mathcal{C}_t, \tag{15b}$$

$$\|\mathbf{t}_n-\mathbf{t}_k\|_2 \geq D, \quad k=1,2,\ldots,N_t, \quad k \neq n. \tag{15c}$$

It is evident that the objective function of (P3-n) is neither convex nor concave with respect to $\mathbf{t}_n$, and the minimum distance constraints exhibit the same characteristics. Consequently, (P3-n) remains a highly non-convex optimization problem, posing significant challenges to its solution.

To address the issue, we adopt successive convex approximation (SCA) for the optimization of $\mathbf{t}_n$. A lower bound for the objective function of (P3-n) is given by

$$\begin{aligned} g\left(\mathbf{t}_n\right) \triangleq& \mathbf{g}(\mathbf{t}_n)^H\mathbf{D}\mathbf{g}(\mathbf{t}_n) \\ \geq& g\left(\mathbf{t}_n^i\right)+2\,\mathrm{Re}\left\{\mathbf{g}\left(\mathbf{t}_n^i\right)^H\mathbf{D}\left(\mathbf{g}\left(\mathbf{t}_n\right)-\mathbf{g}\left(\mathbf{t}_n^i\right)\right)\right\} \\ =& 2\mathrm{Re}\left\{\mathbf{g}\left(\mathbf{t}_n^i\right)^H\mathbf{D}\mathbf{g}\left(\mathbf{t}_n\right)\right\}+C_2^t, \end{aligned} \tag{16}$$

where $\mathbf{t}_n^i$ is the position of $\mathbf{t}_n$ in the i-th iteration and $C_2^t = -\mathbf{g}\left(\mathbf{t}_n^i\right)^H\mathbf{D}\mathbf{g}\left(\mathbf{t}_n^i\right)$ is a constant that is independent of $\mathbf{t}_n$. Thus, maximizing $g\left(\mathbf{t}_n\right)$ is equivalent to maximizing $\bar{g}\left(\mathbf{t}_n\right) \triangleq \mathrm{Re}\{\mathbf{g}(\mathbf{t}_n^i)^H\mathbf{D}\mathbf{g}(\mathbf{t}_n)\}$, which can be lower-bounded by its second order expansion as

$$\begin{aligned} \bar{g}\left(\mathbf{t}_n\right) \geq& -\frac{\delta_n^t}{2}\mathbf{t}_n^T\mathbf{t}_n+\left(\nabla\bar{g}\left(\mathbf{t}_n^i\right)+\delta_n^t\mathbf{t}_n^i\right)^T\mathbf{t}_n \\ &+C_3^t, \end{aligned} \tag{17}$$

where δ_n^t is a positive real number satisfying $\delta_n^t\mathbf{I}_2 \succeq \nabla^2\bar{g}(\mathbf{t}_n)$ and $C_3^t = \bar{g}\left(\mathbf{t}_n^i\right)-\frac{\delta_n^t}{2}\left(\mathbf{t}_n^i\right)^T\mathbf{t}_n^i$ is a constant term independent of $\mathbf{t}_n$. Evidently, maximizing $\bar{g}\left(\mathbf{t}_n\right)$ can be converted to maximizing $\tilde{g}\left(\mathbf{t}_n\right) \triangleq -\frac{\delta_n^t}{2}\mathbf{t}_n^T\mathbf{t}_n+\left(\nabla\bar{g}\left(\mathbf{t}_n^i\right)+\delta_n^t\mathbf{t}_n^i\right)^T\mathbf{t}_n$. To this end, in the i-th iteration of SCA, the optimization subproblem for determining the n-th transmit FA position $\mathbf{t}_n$ can be relaxed as

$$\text{(P4-n)} \quad \max_{\mathbf{t}_n} \quad -\frac{\delta_n^t}{2}\mathbf{t}_n^T\mathbf{t}_n+\left(\nabla\bar{g}\left(\mathbf{t}_n^i\right)+\delta_n^t\mathbf{t}_n^i\right)^T\mathbf{t}_n \quad s.t. \quad \text{(15b), (15c)}. \tag{18}$$

The closed-form global optimum can be obtained by ignoring the non-convex constraints (15b), (15c) since (18) is concave over $\mathbf{t}_n$. By setting $\nabla\tilde{g}\left(\mathbf{t}_n\right) = -\delta_n^t\mathbf{t}_n+\nabla\bar{g}\left(\mathbf{t}_n^i\right)+\delta_n^t\mathbf{t}_n^i$ to zero, we can obtain the optimal solution as

$$\mathbf{t}_{n,i+1}^* = \frac{1}{\delta_n^t}\nabla\bar{g}\left(\mathbf{t}_n^i\right)+\mathbf{t}_n^i. \tag{19}$$

If $\mathbf{t}_{n,i+1}^*$ satisfies (15b) and (15c), it is the global optimum for (P4-n). Otherwise, no optimal solution can be obtained due to the non-convex constraint (15c), and we alternatively resort to obtaining a sub-optimal local maximum by applying convex relaxation to constraint (15c). More precisely, by conducting the first-order Taylor expansion of the convex function with respect to $\mathbf{t}_n$, we can derive a lower bound of $\|\mathbf{t}_n-\mathbf{t}_k\|_2$ as

$$\begin{aligned} \|\boldsymbol{t}_n-\boldsymbol{t}_k\|_2 \geq& \left\|\boldsymbol{t}_n^i-\boldsymbol{t}_k\right\|_2+\left(\nabla\left\|\boldsymbol{t}_n^i-\boldsymbol{t}_k\right\|_2\right)^T\left(\boldsymbol{t}_n-\boldsymbol{t}_n^i\right) \\ =& \frac{1}{\left\|\boldsymbol{t}_n^i-\boldsymbol{t}_k\right\|_2}\left(\boldsymbol{t}_n^i-\boldsymbol{t}_k\right)^T\left(\boldsymbol{t}_n-\boldsymbol{t}_k\right), \end{aligned} \tag{20}$$

thereby allowing convex relaxation of the constraint to

$$\frac{1}{\left\|\boldsymbol{t}_n^i - \boldsymbol{t}_k\right\|_2}\left(\boldsymbol{t}_n^i - \boldsymbol{t}_k\right)^T\left(\boldsymbol{t}_n - \boldsymbol{t}_k\right) \geq D, \tag{21}$$

and furthermore, (P4-n) can be transformed into

$$\text{(P5-n)} \quad \max_{\mathbf{t}_n} \quad -\frac{\delta_n^t}{2}\mathbf{t}_n^T\mathbf{t}_n + \left(\nabla\bar{g}\left(\mathbf{t}_n^i\right) + \delta_n^t\mathbf{t}_n^i\right)^T\mathbf{t}_n \tag{22a}$$

$$s.t. \quad \frac{1}{\left\|\boldsymbol{t}_n^i - \boldsymbol{t}_k\right\|_2}\left(\boldsymbol{t}_n^i - \boldsymbol{t}_k\right)^T\left(\boldsymbol{t}_n - \boldsymbol{t}_k\right) \geq D, \quad k = 1, 2, \ldots, N_r, \quad k \neq n, \tag{22b}$$

(15b),

which is a convex quadratic programming (QP) problem and can be efficiently solved.

B. Optimization of $\mathbf{r}_m$ given $\{\mathbf{r}_l, l \neq m\}_{l=1}^{N_r}$ and $\{\mathbf{t}_n\}_{n=1}^{N_t}$

In this subproblem, we will consider the optimization of $\mathbf{r}_m$ in (P2) given $\{\mathbf{r}_l, l \neq m\}_{l=1}^{N_r}$ and $\{\mathbf{t}_n\}_{n=1}^{N_t}$. Similar to the approach in Section III-A, we first isolate the impact of the receive antenna positions on the objective function of (P2) as

$$\left\|\mathbf{G}(\tilde{\mathbf{t}})^H\mathbf{\Sigma}\mathbf{F}(\tilde{\mathbf{r}})\right\|_F^2 = \mathbf{f}(\mathbf{r}_m)^H\mathbf{\Sigma}^H\mathbf{G}(\tilde{\mathbf{t}})\mathbf{G}(\tilde{\mathbf{t}})^H\mathbf{\Sigma}\mathbf{f}(\mathbf{r}_m) + C_1^r, \tag{23}$$

where $C_1^r = \sum_{i=1, i\neq m}^{N_r}\left\|\mathbf{G}(\tilde{\mathbf{t}})^H\mathbf{\Sigma}\mathbf{f}(\mathbf{r}_i)\right\|_2^2$ is a constant independent of $\mathbf{r}_m$. It can be observed that the effective terms in (23) share the same structure as that in (15a). Therefore, the optimization sub-problem for $\mathbf{r}_m$ can be formulated as

$$\text{(P6-m)} \quad \max_{\mathbf{r}_m} \quad \mathbf{f}(\mathbf{r}_m)^H\mathbf{B}\mathbf{f}(\mathbf{r}_m) \tag{24a}$$

$$s.t. \quad \mathbf{r}_m \in \mathcal{C}_r, \tag{24b}$$

$$\left\|\mathbf{r}_m - \mathbf{r}_l\right\|_2 \geq D, \quad l = 1, 2, \ldots, N_r, \quad l \neq m, \tag{24c}$$

where

$$\mathbf{B} = \mathbf{\Sigma}^H\mathbf{G}(\tilde{\mathbf{t}})\mathbf{G}(\tilde{\mathbf{t}})^H\mathbf{\Sigma}. \tag{25}$$

Similarly, the formulations of problems (P6-m) and (P3-n) have the same form. Consequently, by adhering to the same procedure in Section III-A, a locally optimal solution can be obtained, a detailed exposition of which is omitted herein for brevity. The overall algorithm is summarized in Algorithm 1.

C. Complexity Analysis

The computational complexity of Algorithm 1 is analyzed as follows. In Step 2, the computational complexity for obtaining $\mathbf{D}$ is $\mathcal{O}\left(N_r L_t\right)$ and from Steps 3 to Step 5, the corresponding complexity to determine all positions of the transmit FAs is $\mathcal{O}\left(N_t L_t \gamma_t^1 + N_t^{2.5}\ln\left(1/\beta\right)\gamma_t^2\right)$, where γ_t^1 and γ_t^2 represent the maximum numbers of inner iterations for Step 1 and Step 5, respectively, and β is the accuracy of the inner-point method. Similarly, the complexities for calculating $\mathbf{B}$ and determining the positions of all receive FAs are $\mathcal{O}\left(N_t L_r\right)$ and $\mathcal{O}\left(N_r L_r \gamma_r^1 + N_r^{2.5}\ln\left(1/\beta\right)\gamma_r^2\right)$ and thus the overall computational complexity is $\mathcal{O}(N_r L_t + N_t L_t \gamma_t^1 + N_t^{2.5}\ln\left(1/\beta\right)\gamma_t^2 + N_t L_r + N_r L_r \gamma_r^1 + N_r^{2.5}\ln\left(1/\beta\right)\gamma_r^2)$.

Algorithm 1 Alternating Optimization for Solving Problem (P1)

Input: $\mathbf{\Sigma}$, N_t, N_r, L_r, L_t, $\{\theta_t^p\}_{p=1}^{L_t}$, $\{\phi_t^p\}_{p=1}^{L_t}$, $\{\theta_r^q\}_{q=1}^{L_r}$, $\{\phi_r^q\}_{q=1}^{L_r}$, $\mathcal{C}_t$, $\mathcal{C}_r$, D, ϵ.
Output: $\{\mathbf{t}_n\}_{n=1}^{N_t}$, $\{\mathbf{r}_m\}_{m=1}^{N_r}$.

1: **while** Increase of the STBC achievable rate is above ϵ **do**
2: Obtain $\mathbf{D}$ via (14).
3: **for** $n = 1 \to N_t$ **do**
4: Given $\{\mathbf{t}_k, k \neq n\}_{k=1}^{N_t}$ and $\{\mathbf{r}_m\}_{m=1}^{N_r}$,
5: **while** Increase of the objective value in (P3-n) is above ϵ **do**
6: Update δ_n^t and obtain $\mathbf{t}_{n,i+1}^*$ via (19).
7: **if** $\mathbf{t}_{n,i+1}^*$ statisfies (15b) and (15c) **then**
8: $\mathbf{t}_n^{i+1}$=$\mathbf{t}_{n,i+1}^*$.
9: **else**
10: Obtain $\mathbf{t}_n^{i+1}$ by solving (P5-n).
11: **end if**
12: **end while**
13: **end for**
14: Obtain $\mathbf{B}$ via (25).
15: **for** $m = 1 \to N_r$ **do**
16: Given $\{\mathbf{r}_l, l \neq m\}_{l=1}^{N_r}$ and $\{\mathbf{t}_n\}_{n=1}^{N_t}$, solve (P6-m) with similar procedure as above.
17: **end for**
18: **end while**

IV. Simulation Results

In this section, computer simulations are carried out to demonstrate the effectiveness of the algorithms for maximizing the 1% outage achievable rate of FA-STBC systems. In the simulations, we consider Alamouti's STBC with $N_t = 2$ transmit FAs and the code in [3] with $N_t = 4$ transmit FAs. The regions for moving are square with a size of $A \times A$ and $A = 2\lambda$ unless otherwise specified. The numbers of transmit and receive paths are set as $L_t = L_r = 5$ and the path response matrix is assumed to be diagonal with $\mathbf{\Sigma}_{1,1} \sim \mathcal{CN}\left(0, 1/2\right)$ and $\mathbf{\Sigma}_{l,l} \sim \mathcal{CN}\left(0, 1/(2L_t - 2)\right), l = 1, 2, \ldots, L_t$. We assume that the elevation and azimuth AoDs/AoAs $\{\theta_t^p\}_{p=1}^{L_t}, \{\phi_t^p\}_{p=1}^{L_t}, \{\theta_r^q\}_{q=1}^{L_r}, \{\phi_r^q\}_{q=1}^{L_r}$ to be independent and identically distributed (i.i.d.) variables with the uniform distribution over $[0, \pi]$. The minimum distance required between FAs is set as $D = \lambda/2$ and the signal-to-noise ratio (SNR) is defined as $\rho = P/N_0$. For baselines of close-loop STBC and FPA systems, we consider both the transmitter and receiver equipped with a uniform linear array of antenna spacing in $\lambda/2$.

Fig. 1 depicts the convergence behavior of the proposed algorithm for FA-STBC systems with different numbers of receive FAs at SNR $= 15$ dB. The results demonstrate that the algorithm consistently achieves rapid convergence within three iterations across all scenarios. Notably, for $N_r = 1$, the converged achievable rate exhibits an increase of 39.46% compared to the initial value.

Fig. 2 illustrates the performance comparison of FA-STBC, conventional FPA and STBC systems equipping $N_r = 1$ receive antenna with respect to SNR. STBCs with different

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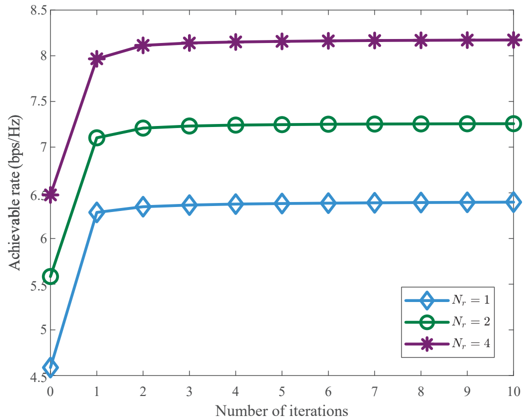

Fig. 1. Convergence of Algorithm 1 under different values of N_r.

transmit FAs are considered. It is revealed that the achievable rate of FA-STBC not only exceeds that of the original STBC but also outperforms FPA over the entire SNR range in both cases. Moreover, for the baseline FPA and STBC schemes, STBC exhibits a loss in achievable rate compared to FPA, which aligns with the analysis presented in Section II-B2.

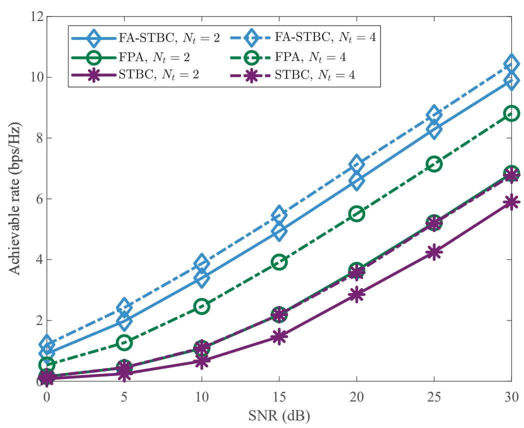

Fig. 2. Achievable rate versus SNR.

Fig. 3 shows the achievable rates of the FA-STBC system and baselines with $N_r = 2$ receive antennas at $\mathrm{SNR} = 5$ dB with respect to the region size. Even within a limited region, FA-STBC achieves substantial performance gains. While the achievable rate increases with the size of the region, this trend gradually levels off. This implies that a region size with merely $A = 2\lambda$ can yield near-optimal performance. Specifically, for a region of $2\lambda \times 2\lambda$, FA-STBC attains gains of 35.91% and 61.25% compared to FPA and STBC, respectively.

V. Conclusion

This paper studied the achievable rate maximization problem for point-to-point FA-STBC systems via jointly optimizing the positions of transmit and receive FAs. An AO algorithm with a rapid convergence rate was developed to obtain a locally optimal solution by iterative optimization. Simulation results demonstrated the performance gains of the FA-STBC system over the conventional FPA and STBC counterparts and

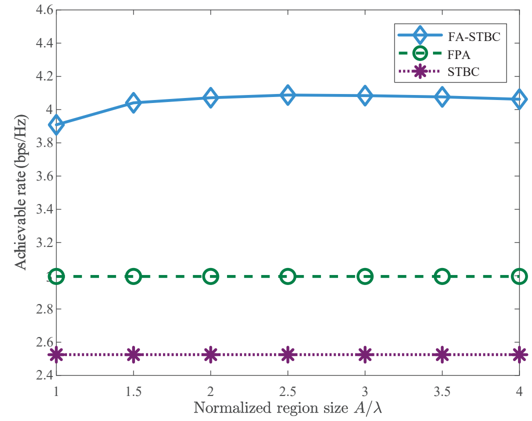

Fig. 3. Achievable rate versus normalized region size.

revealed that the maximum achievable rate can be achieved within a limited region for FAs.

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