Essays on Macroeconomics with Heterogeneous Agents

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Declaration

I, Yongsoo Kim, confirm that the work presented in my thesis is my own. Where information has been derived from other sources, I confirm that this has been indicated in the thesis.

Abstract

This thesis comprises three independent essays.

Chapter 1 examines a new type of risk that creates unavoidable expenditure needs. Using data from the Panel Study of Income Dynamics, I show that financially constrained households face greater difficulty in increasing consumption of goods affected by expenditure shocks and tend to make larger reductions in other spending categories. I develop a heterogeneous agent New Keynesian model with incomplete markets, which predicts that expenditure risks reshape wealth distribution and marginal propensities to consume, compared to standard models. Policy simulations reveal that redistributive fiscal policies are more effective in stimulating the economy under expenditure risks but are less welfare-enhancing in the long run due to general equilibrium effects.

Chapter 2 builds a HANK model to study the effects of the furlough scheme implemented in the UK during the COVID-19 pandemic. To model lockdowns, I introduce inactivity shock which generates a state of neither employment nor complete unemployment. The model shows that inactivity shocks have recessionary effects on the economy. While the furlough scheme effectively mitigates declines in aggregate demand, its overall impact is partially offset by higher tax rates required to finance the policy. The results suggest that the furlough scheme provides benefits to those directly affected by the pandemic.

Chapter 3 examines the effects of sector-specific job separation shocks on labor reallocation and macroeconomic dynamics using a two-sector heterogeneous agent New Keynesian model with discrete labor choices. In response to a separation shock, we find that the model produces puzzling results: the directly affected sector experiences a milder recession and faster recovery due to increased labor demand, while the indirectly affected sector suffers a deeper recession caused by negative demand effects. These findings suggest that incorporating additional mechanisms is necessary to better capture observed economic patterns.

Impact statement

This thesis investigates the impacts of economic shocks on household behavior, labor markets, and macroeconomic outcomes.

The first chapter makes a contribution to the macroeconomic literature by investigating the underexplored role of expenditure risks in shaping household consumption and saving behaviors. While traditional macroeconomic models primarily focus on income risks, this study introduces a novel approach by modeling expenditure shocks as preference shocks within a heterogeneous-agent general equilibrium framework. This methodological innovation broadens the scope of analysis, capturing how expenditure risks create additional layers of uncertainty that disproportionately affect constrained households. This study also deepens the understanding of consumption responses by empirically demonstrating how expenditure risks influence household consumption-saving behavior across different income and wealth groups. By integrating these insights into the analysis, the study provides a more comprehensive view of the factors driving inequality and the mechanisms through which they operate. These results have important implications for policymakers, emphasizing the effectiveness of targeted fiscal policies that address not only income volatility but also expenditure uncertainties. The model in this chapter shows that such policies are shown to play a critical role in stimulating aggregate demand. This work not only enhances the theoretical understanding of consumption-savings dynamics under multiple risk factors but also provides practical guidance for policymakers aiming to address the complex interplay between household behavior and broader economic performance.

The second chapter provides a novel analysis of the UK's Coronavirus Job Retention Scheme (CJRS) using a heterogeneous-agent New Keynesian framework with search and matching frictions. By modeling inactivity shocks to represent pandemic-induced furloughs, the study highlights the scheme's role in mitigating aggregate demand contractions and providing support to those most affected by the crisis. The findings underscore the effectiveness of income support policies in stabilizing household consumption during crises, while revealing general equilibrium effects such as higher tax burdens that dampen policy's efficacy. These results highlight the importance of considering indirect general equilibrium effects when designing and implementing policies.

The third chapter analyzes sectoral labor reallocation during economic disruptions, such as the COVID-19 pandemic, by developing a two-sector heterogeneous-agent New Keynesian model with labor market frictions. By modeling sectoral-specific job separation shocks, the study captures the uneven impacts of the pandemic on labor market dynamics and macroeconomic outcomes. The findings reveal counterintuitive sectoral responses, with faster recoveries in the directly affected sector due to increased labor demand and slower recoveries in the indirectly affected sector due to demand spillovers. This research contributes to the literature by identifying limitations in current modeling approaches and suggesting the incorporation of additional mechanisms, such as demand reallocation shocks, to better explain observed labor market adjustments. The insights provide valuable guidance for enhancing policy responses to sector-specific disruptions and improving the design of labor market interventions.

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Chapter 1

The Pass-through Effects of Expenditure Risks and Redistribution

1.1 Introduction

Traditionally, macroeconomic literature has heavily focused on income risks—uncertainties related to changes in wages and unemployment risk. These risks, whether idiosyncratic or aggregate, significantly influence household consumption and saving behaviors. However, it is not hard to recognize that income is not the sole source of uncertainty that households face. Expenditure risks, such as unexpected medical bills, auto repairs, or other substantial unavoidable expenses, also play a critical role in shaping consumption-savings decisions. This paper aims to shed light on the importance of these expenditure risks and their implications for both household behavior and macroeconomic policy.

In recent years, research has increasingly recognized that households save not only in response to income uncertainties but also due to the anticipation of unavoidable expenditures sometime in the future. For instance, Fulford (2015) provides evidence that households prioritize precautionary savings to buffer against such expenditure shocks rather than solely focusing on income risks.[1] Despite this recognition, the macroeconomic impact of expenditure risks remains underexplored.

This paper introduces a new dimension of heterogeneity to an otherwise standard incomplete markets model by incorporating idiosyncratic expenditure risks. While the concept of expenditure risk is not entirely novel, most studies have limited their scope to the impact of unexpected increases in expenses on household finance, modeling expenditure shocks as negative wealth shocks. These studies primarily focus on how uncertainties in the budget constraint affect saving decisions across the life cycle and their consequences on labor supply and wealth distribution.

In contrast, this paper models expenditure risks as a form of preference shock, considering that expenditure risks affect not only the budget constraint but also individual preferences. For instance, if someone is injured, medical expenses for treatment are considered as an expenditure shock to this individual. These costs spent for the treatment reduce the available resources for other consumption, but at the same time, they increase the individual's utility by helping recovery from the injury. Foregoing treatment to save the associated costs would result in a significant welfare loss. To capture this mechanism, I assume a two-goods utility function in which one of the goods is subject to expenditure shocks. The way expenditure shocks enter the utility function is similar to how a demand shifter affects the consumption of a specific good in other literature. Using this framework, I explore how changes in expenditure risks alter individual consumption behaviors between the two goods and extend the analysis to further investigate the broader macroeconomic impacts of expenditure risks and their policy implications within a general equilibrium environment.

[1] Using the questions from the Survey of Consumer Finances, he argues that households rarely select unemployment as a motive for saving. Instead, he suggests that their precautionary savings are more closely tied to uncertainties related to expenditures.

To gain insights into the implications of expenditure shocks, I begin with an illustrative consumption-savings choice model. This simple model allows me to analytically explore how an individual allocates their consumption between two goods when one good is affected by an expenditure shock. My findings reveal that households adjust their consumption differently in response to the shock, depending on their financial constraints. Specifically, unconstrained households can optimize by flexibly increasing the consumption of the affected good and slightly reducing the consumption of the other good, thereby satisfying both intertemporal and intratemporal optimal conditions. However, constrained households must make a larger cut in the consumption of the other good just to increase the consumption of the affected good, as they are unable to redistribute consumption across periods.

In the empirical analysis, this paper makes a methodological contribution to the literature by introducing a novel approach to estimating consumption responses to expenditure shocks. By treating these shocks as preference shifts, this approach allows for exploring the ways in which households adjust their consumption to the shocks but also uncovers the significant role of heterogeneity in responses across income and wealth distributions. Specifically, I use data from the Panel Study of Income Dynamics (PSID) to provide evidence supporting the findings discussed above. First, I split expenditure categories into two groups based on their volatilities to maintain the consistency with the two-good consumption structure. I then exploit the optimal conditions of the model to estimate household-level expenditure shocks. With these estimated results, I explore the pass-through effects of expenditure shocks on consumption across various household characteristics.

The results reveal substantial heterogeneity in how households respond to expenditure shocks, consistent with the findings from the simple model. Low-wealth and low-income, and more constrained households, face higher expenditure uncertainties, tend to make larger cuts in the other consumption to finance the increase in the consumption of the good affected by the expenditure shocks. Additionally, I find that the increase in total consumption, the sum of the two goods, is larger for wealthier households, as they can better afford unexpected expenditure increases.

Next, to deepen the understanding on the macroeconomic impacts of the expenditure shock and its implications for policy, I construct a quantitative general equilibrium model allowing for expenditure risks, alongside income risks. Two economies, one with and without expenditure risks, are considered for comparision. The steady-state results predict that households facing expenditure risks exhibit a stronger precautionary saving motive compared to those facing only income risks. The stochastic nature of expenditure shocks means that they may need to deplete their assets to cover increased spending during large expenditure shocks. Consequently, households have a stronger incentive to save more assets as a buffer against such situations. Overall, this difference in consumption-saving behaviors leads to a more dispersed wealth distribution, with a larger share of constrained households near the constraint and wealthier households at the top of the distribution.

The interplay between expenditure and income risks provides useful insights into policy design. The distribution of the marginal propensity to consume (MPC) by income and expenditure shocks suggests the potential effectiveness of targeted transfer policies aimed at stimulating demand. To examine this, I conduct two counterfactual redistributive transfer policies: an insurance policy and an income redistribution policy. The results from both policies highlight that targeted transfer policies, which redistribute wealth from the rich to the poor, can have larger expansionary effects, especially in the presence of expenditure shocks. These policies are particularly effective because financially constrained households, especially those facing high expenditure risks, are more likely to spend additional income, thereby stimulating economic activity. However, in terms of welfare, I find that these policies may not be as welfare-improving in the long run.

Related literature This paper is most related to a literature which studies how an unexpected increase in expenditure induces households to adjust their consumption and saving decision. Telyukova (2013) adds a new type of idiosyncratic cash consumption uncertainty in the model and suggests the need for liquidity as a possible explanation for "credit card debt puzzle".[2] Melcangi and Sterk (2024) also introduces an infrequent consumption good, which creates an additional saving motive. They show that the presence of the infrequent consumption can help replicate a fat-tailed wealth distribution, consistent with my findings. Miranda-Pinto et al. (2023) develop a heterogeneous agent model where households are subject to time-varying consumption thresholds below which they suffer a large utility loss. Using that framework, they show that their model is capable of matching some key moments observed in the data, such as the U-shaped MPC distribution documented in the recent empirical literature. Bhutta et al. (2015) and Huang (2023) attempt to understand why poor households tend to exhibit the demand for high-cost credits, such as payday loans, and find that it is the occurrence of unexpected expenses that causes this irrational borrowing behavior and prevents them from accumulating wealth. Chatterjee et al. (2022) and Livshits et al. (2007) also include expenditure shock in their model, with a focus on its impact on households' choice on default and the implications for bankruptcy policies. Chetty and Szeidl (2007) discuss the impacts of consumption commitments on risk preferences and rationalize why individuals might simultaneously engage in both risk-averse and risk-seeking behaviors. While the consumption commitments discussed in their paper lack the stochastic component of expenditure risks considered in my study, my research is related in that both focus on how non-adjustable changes in expenditure affect households' portfolio choices and consumption behaviors.

[2] The expenditure uncertainty is modelled as a form of a preference shock, which induces individuals to increase the consumption of good affected by the shock. This way of modelling bears a great resemblance to mine, which will be described in detail in the following section.

Broadly, this paper contributes to a strand of literature that emphasizes the role of heterogeneity in preferences in analyzing consumption-savings behaviors. Aguiar et al. (2024) highlight that persistent differences in preferences, such as higher impatience and higher intertemporal elasticity of substitution (IES), are crucial in understanding why some households are HtM. Similarly, Pfäuti et al. (2024) allow for heterogeneity in cognitive skills and overconfidence of households to account for observed HtM shares and wealth distribution. Krueger et al. (2016) demonstrate that incorporating permanently different time discount factors improves the model's ability to match wealth inequality. Additionally, Kekre and Lenel (2022) show that differences in risk aversion can explain the observed diversity in portfolio choices.

The empirical analysis in this paper is motivated by the literature on consumption insurance, which focuses on the degree of consumption smoothing with respect to changes in income. The seminal paper by Blundell et al. (2008) develops a semi-structure estimation method that can separately measure the pass-through of transitory and permanent income shocks to consumption. Commault (2022) relaxes some assumptions required for the method in Blundell et al. (2008) and obtains a more robust estimate. Blundell et al. (2024) extend the methodology to study the effect of health shocks on consumption among households over age 65. Gourinchas and Parker (2002) and Kaplan and Violante (2010) measure the degree of consumption insurance against income fluctuations in a life-cycle model with incomplete markets. To the best of my knowledge, none of these papers examine the pass-through effects of expenditure shocks on consumption. This paper contributes to the consumption insurance literature by broadening the analysis to include expenditure risks in addition to income shocks. This expanded focus allows for a more comprehensive understanding of how households adjust consumption in the face of financial uncertainties.

Lastly, this paper also contributes to a vast literature evaluating the effects of distributive fiscal policy with incomplete markets. Most relevant to my research is

Oh and Reis (2012), who examine the effectiveness of targeted government transfers during the Great Recession and find lump-sum transfers to liquidity-constrained households have significant expansionary effects. Their findings provide a crucial perspective on the design of effective fiscal policies. In a similar vein, Bilbiie et al. (2013) explore how revenue-neutral redistribution policies favoring borrowers can positively impact consumption and economic activity when prices are sticky. Heathcote (2005) also finds that temporary income tax cuts have substantial real effects in an incomplete markets setting. Kaplan and Violante (2014) develop a model to analyze household consumption responses to fiscal stimulus payments, emphasizing the role of liquid and illiquid assets. In addition, there exists a substantial body of research that demonstrates similar findings. This includes Guo et al. (2023), Boutros (2023), Galí et al. (2007), Alonso-Ortiz and Rogerson (2010). This literature collectively suggests that the effect of fiscal policy is amplified in the presence of financially constrained households as they tend to exhibit higher marginal propensities to consume (MPCs). This is consistent with the findings from my model, which allows for idiosyncratic uncertainties in both expenditure and income.

Layout The remainder of this paper is structured as follows. Section 1.2 presents a simple model to analytically explore the role of expenditure shocks in a standard consumption-saving framework. Section 1.3 provides empirical evidence supporting the model's findings, using data from the PSID to estimate and analyze how expenditure and income shocks impacts consumption behaviors. In section 1.4, a quantitative general equilibrium model is developed to provide deeper insights into the implications of expenditure risks. Section 1.5 discusses the calibration of the model, and section 1.6 explores how the presence of expenditure risks alters the model's outcomes and discusses the policy implications. Lastly, section 1.7 concludes.

1.2 Illustrative Model

This section provides an analytical discussion on the implications of expenditure shocks in the standard household optimization problem. Specifically, the focus is mainly on the pass-through effect of an unanticipated expenditure shock on consumption decision among two goods. I then extend the model in section 1.4 to quantitatively evaluate the effect of the shock in an environment where agents are heterogeneous along income and wealth. For motivational purpose, however, this section only presents a simple consumption-savings model in partial equilibrium and compares the behaviors of unconstrained and constrained households, respectively.

1.2.1 Unconstrained Households

Consider first an optimization problem of an unconstrained household who has preferences over two consumption goods A, B.

$$\max_{c_{At}, c_{Bt}, a_{t+1}} \sum_{t=0}^{\infty} \beta^t u(c_t) \tag{1.1}$$

$$\text{s.t. } c_{At} + c_{Bt} + a_{t+1} = y_t + (1+r)a_t, \tag{1.2}$$

$$a_{t+1} \geq 0,$$

where

$$u(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma} \text{ and } c_t = \left(\alpha^{\frac{1}{\eta}} \theta_t c_{At}^{\frac{\eta-1}{\eta}} + (1-\alpha)^{\frac{1}{\eta}} c_{Bt}^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}. \tag{1.3}$$

$\beta \in (0,1)$ is the discount factor and σ is the inverse of elasticity of inter-temporal substitution (EIS). α is the consumption share for good A. η denotes the elasticity of substitution between the two goods. For simplicity I assume the price for each good is fixed at 1, i.e., $P_{At} = P_{Bt} = 1$. Unlike other literature, which often assumes that the two goods are highly substitutable, this paper assumes that goods A and B are less substitutable. Therefore, I assume throughout the paper that $0 < \eta < 1$, indicating that changes in the consumption of one good cannot be fully replaced by changes in the consumption of the other good.

More importantly, the expenditure shock is modelled as a preference shock to good A, denoted by θ_t. A change in θ_t reflects a shift in the household's preferences toward good A. Therefore, the expenditure shock differs from the income shock in that it directly affects households' utility while the income shock represents a change in the household's constraint. In normal times, when θ_t equals 1, the overall consumption bundle, c_t, simplifies to the standard CES consumption bundle. When $\theta_t > 1$, consuming one more unit of good A gives a higher utility, hence the household reallocates consumption toward good A according to the intra-temporal and inter-temporal optimization conditions. Specifically, within the period, in response to an increase in the relative preference for good A (i.e. $\theta_t > 1$), the household adjusts each consumption such that

$$u_{At} = u_{Bt} \tag{1.4}$$

where

$$u_{At} = \alpha^{\frac{1}{\eta}} \theta_t c_t^{\eta - \frac{1}{\sigma}} c_{At}^{-\frac{1}{\eta}}$$

and

$$u_{Bt} = (1-\alpha)^{\frac{1}{\eta}} c_t^{\eta - \frac{1}{\sigma}} c_{Bt}^{-\frac{1}{\eta}}$$

are the marginal utilities of consumption with respect to each good. Equation (1.4) implies the following optimal intra-temporal allocation between good A and B in each period

$$\frac{c_{At}}{c_{Bt}} = \left(\frac{\alpha}{1-\alpha}\right) \theta_t^{\eta}. \tag{1.5}$$

Equation (1.5) provides some intuitive interpretation. A larger θ_t means a stronger desire for the household to spend on good A. Thus, the greater θ_t is, the higher ratio of c_{At} to c_{Bt} becomes in the equilibrium. Additionally, households with a higher η are less averse to substitution between goods within the period. This also indicates a higher consumption level of good A compared to good B for a given level of $\theta_t > 1$.

Turning to the inter-temporal condition, I combine the first-order conditions to derive the typical Euler equation between t and $t+1$,

$$u'(c_t) = \beta(1+r)u'(c_{t+1}) \quad \Leftrightarrow \quad c_{t+1} = [\beta(1+r)]^{\frac{1}{\sigma}} c_t. \tag{1.6}$$

Assuming that initial wealth $a_0 = 0$ for simplicity, the intertemporal budget constraint is given as

$$\sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t c_t = \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t y_t. \tag{1.7}$$

Substituting (1.6) into (1.7) recursively and rearranging terms gives

$$c_0 = \left[\frac{1+r-[\beta(1+r)]^{\frac{1}{\sigma}}}{1+r}\right] \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t y_t, \tag{1.8}$$

where c_0 denotes the overall consumption level in the steady state. It is not difficult to realize that the consumption level in equation (1.8) is the same as one predicted by the Permanent Income Hypothesis model.

To make the analysis more tractable, I further assume that

$$\beta(1+r) = 1, \quad \eta = \frac{1}{\sigma} \quad \text{and} \quad y_t = y.$$

The first assumption equalizes the marginal utilities of consumption over time, while the second one reflects the household's symmetric approach to consumption choices over time and across goods. I believe that these assumptions are not too strong, and lead to more straightforward analytical solutions, providing clear insights into households' consumption behavior.

The goal in this section is to find analytical expressions for c_{At}, c_{Bt} when there is a one-time shock to θ_t at t. Since unconstrained households are assumed to have access to financial markets, they are capable of smoothing their consumption over time through borrowings and savings. Specifically, in response to a one-time increase in θ_t, they adjust each consumption in a way that 1) equation (1.5) is satisfied at all times, and 2) the consumption of good B, c_{Bt}, is immediately adjusted to a new level.[3]

Defining x_{At}^{uncon} and x_{Bt}^{uncon} as the amount of adjustment in consumption A and B, respectively, from the pre-shock equilibrium level, in response to the shock at t, I have the following conditions,

$$\frac{\alpha c_0 + x_{At}^{\text{uncon}}}{(1-\alpha)c_0 + x_{Bt}^{\text{uncon}}} = \left(\frac{\alpha}{1-\alpha}\right)\theta_t^{\eta} \tag{1.9}$$

$$(1-\alpha)c_0 + x_{Bt}^{\text{uncon}} = (1-\alpha)\left(\frac{r}{1+r}\right)\left[\Phi_t - (1+r)(x_{At}^{\text{uncon}} + x_{Bt}^{\text{uncon}})\right], \tag{1.10}$$

where $\Phi_t \equiv \sum_{t=0}^{\infty}\left(\frac{1}{1+r}\right)^t y$ is the discounted sum of lifetime wealth. Equation (1.9) is associated with the intra-temporal condition. The left-hand side is the ratio of consumption A to B after the adjustment. Note that $\alpha c_0 + x_{At}^{\text{uncon}}$ represents the new consumption level for good A, while $(1-\alpha)c_0 + x_{Bt}^{\text{uncon}}$ represents that for good B. Equation (1.9) reflects that the optimizing households adjust their consumption to ensure that the intra-temporal reallocation between the goods still remains valid even after the shock. Equation (1.10) corresponds to the inter-temporal condition for good B. The right hand side represents the new level of consumption for good B after the shock dissipates at $t+1$ onwards. Note that this is a $(1-\alpha)$ share of total consumption, calculated based on the newly evaluated lifetime income (i.e. $\Phi_t - (1+r)(x_{At}^{\text{uncon}} + x_{Bt}^{\text{uncon}})$), which follows the same allocation rule.

[3] A more detailed discussion is provided in Appendix 1.A.

Given that there are two equations and two unknowns, x_{At}^{uncon} and x_{Bt}^{uncon} can be pinned down as

$$x_{At}^{\text{uncon}} = \frac{\alpha c_0(1+\mathcal{M})(\theta_t^{\eta}-1)}{1+\mathcal{M}+\left(\frac{\alpha}{1-\alpha}\right)\theta_t^{\eta}\mathcal{M}} \tag{1.11}$$

$$x_{Bt}^{\text{uncon}} = -\left(\frac{\mathcal{M}}{1+\mathcal{M}}\right)x_{At}^{\text{uncon}}, \tag{1.12}$$

where $\mathcal{M} = (1-\alpha)(1+r)\left[\frac{1+r-[\beta(1+r)]^{\frac{1}{\sigma}}}{1+r}\right] = (1-\alpha)r$. The derivation process is relegated to the Appendix 1.B. The amount of adjustments in consumption in (1.11) and (1.12) immediately lead to the analytical solutions for each consumption,

$$c_{At}^{\text{uncon}} = \alpha c_0 + \frac{\alpha c_0(1+\mathcal{M})(\theta_t^{\eta}-1)}{1+\mathcal{M}+\left(\frac{\alpha}{1-\alpha}\right)\theta_t^{\eta}\mathcal{M}} \tag{1.13}$$

$$c_{Bt}^{\text{uncon}} = (1-\alpha)c_0 - \frac{\alpha c_0\mathcal{M}(\theta_t^{\eta}-1)}{1+\mathcal{M}+\left(\frac{\alpha}{1-\alpha}\right)\theta_t^{\eta}\mathcal{M}} \tag{1.14}$$

$$c_t^{\text{uncon}} = c_{At}^{\text{uncon}} + c_{Bt}^{\text{uncon}} = c_0 + \frac{\alpha c_0(\theta_t^{\eta}-1)}{1+\mathcal{M}+\left(\frac{\alpha}{1-\alpha}\right)\theta_t^{\eta}\mathcal{M}}. \tag{1.15}$$

Equation (1.13) and (1.14) denote the responses of each consumption to an unanticipated expenditure shock. For $\theta_t > 1$, unconstrained households are able to increase the consumption of good A (i.e. $\frac{\partial c_{At}}{\partial \theta_t} > 0$), while reducing the other consumption (i.e. $\frac{\partial c_{Bt}}{\partial \theta_t} < 0$). Since the increase in consumption of good A outweighs the decrease in consumption of good B, the total consumption in equation (1.15) rises. This illustrates how those unconstrained adjust their consumption in response to an expenditure shock. By reallocating resources both intertemporally and intratemporally, they optimize consumption by prioritizing the good with higher marginal utility (i.e., good A), even though it necessitates a reduction in the other consumption. Notably, the reduction in good B is smoothed over time, with an adjustment occurring only once at the time of the shock.

1.2.2 Constrained Households

A constrained household faces the same problem as described in equation (1.1)-(1.3). When hit by an expenditure shock, the constrained household can still adjust consumption within period across goods. However, one key difference is that she cannot smooth out the consumption path for B across periods, due to limited access to borrowing and savings. The only option left for them is to cut back on the consumption of good B to finance the increase in consumption of A. Therefore, the two conditions in equation (1.9) and (1.10) change to

$$\frac{\alpha c_0 + x_{At}^{\text{con}}}{(1-\alpha)c_0 + x_{Bt}^{\text{con}}} = \left(\frac{\alpha}{1-\alpha}\right)\theta_t^{\eta}$$

$$x_{At}^{\text{con}} = -x_{Bt}^{\text{con}}.$$

Given these conditions, the analytical expressions for consumption of the constrained household are obtained as

$$c_{At}^{\text{con}} = \alpha c_0 + \frac{\alpha c_0(\theta_t^{\eta} - 1)}{1 + \left(\frac{\alpha}{1-\alpha}\right)\theta_t^{\eta}}, \tag{1.16}$$

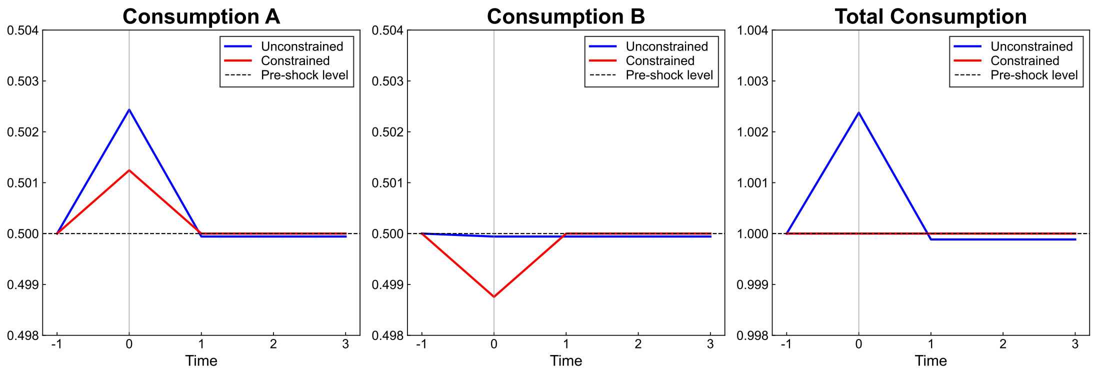
$$c_{Bt}^{\text{con}} = (1-\alpha)c_0 - \frac{\alpha c_0(\theta_t^{\eta} - 1)}{1 + \left(\frac{\alpha}{1-\alpha}\right)\theta_t^{\eta}}. \tag{1.17}$$

Due to the inability to substitute over time, the constrained reduce their consumption of B by the same amount that they increase the consumption of A. As a result, the total consumption $c_t^{\text{con}} = c_{At}^{\text{con}} + c_{Bt}^{\text{con}}$ for the constrained households remains unchanged, while that for the unconstrained increases.

1.2.3 Comparison between the Two Households

Comparing equation (1.13) with (1.16) and (1.14) with (1.17) clearly shows that the unconstrained substitute consumption in a more flexible and utility-maximizing

Figure 1.1: Time path of consumption



Note: The figures are drawn using $\alpha = 0.5$, $\eta = 0.5$, $\sigma = 2$, $y = 1$, $r = 0.05$, and $\beta = 0.9524$ such that $\beta(1 + r) = 1$. Given these parameters, I simulate a one-time shock to θ where it increases to 1.01 at $t = 0$ and reverts back to 1 for $t \geq 1$.

way. They increase c_{At} by more when necessary while decreasing c_{Bt} by less than the constrained do. As they cannot reallocate consumption inter-temporally and intra-temporally as the unconstrained do, they suffer some utility losses over lifetime.

$$x_{At}^{\text{uncon}} = \frac{\alpha c_0 (1 + \mathcal{M})(\theta_t^\eta - 1)}{1 + \mathcal{M} + \left(\frac{\alpha}{1-\alpha}\right)\theta_t^\eta \mathcal{M}} > \frac{\alpha c_0 (\theta_t^\eta - 1)}{1 + \left(\frac{\alpha}{1-\alpha}\right)\theta_t^\eta} = x_{At}^{\text{con}}$$

$$x_{Bt}^{\text{uncon}} = -\frac{\alpha c_0 \mathcal{M}(\theta_t^\eta - 1)}{1 + \mathcal{M} + \left(\frac{\alpha}{1-\alpha}\right)\theta_t^\eta \mathcal{M}} > -\frac{\alpha c_0 (\theta_t^\eta - 1)}{1 + \left(\frac{\alpha}{1-\alpha}\right)\theta_t^\eta} = x_{Bt}^{\text{con}}.$$

Figure 1.1 visually demonstrates the time path of each consumption after a one-time shock to θ at $t = 0$, exploiting the results derived in the previous subsection.

As discussed in the preceding two subsections, unconstrained households immediately choose the optimal allocation between the two goods at $t = 0$, once there is a change in θ_t. They redistribute by allocating more of their lifetime resources to $t = 0$, allowing them to increase consumption of A while slightly reducing consumption of B. In the next period, however, they realize that their lifetime wealth is now diminished due to the frontloading of consumption at $t = 0$. Consequently, both consumption from $t = 1$ onwards are permanently lower than the pre-shock level. In contrast, constrained households are unable to redistribute resources across periods due to their constraints. They have to reduce consumption B significantly to increase the consumption of A, resulting in the unchanged overall consumption level over time.[4]

The discussion in this section sheds light on how expenditure shocks influence consumption behavior of households across the distribution. Unlike income shocks, the expenditure shocks elicit larger consumption responses from those unconstrained, while the responses from the constrained remain muted. This suggests that the constrained households tend to exhibit a higher marginal propensity to consume compared to what is typically observed in standard incomplete markets models without expenditure shocks. This is because constrained households, especially those experiencing high expenditure shocks, benefit more from additional income, which enables them to allocate more resources to the affected good. This discussion continues in section 1.4 where I develop a fully-fledged quantitative general equilibrium model and further explore the implications of expenditure shocks.

1.3 Empirical Analysis

This section presents empirical evidence which gives support to the results from the discussion in section 1.2. In section 1.3.1, I briefly describe data and variables and explain how I classify households. In section 1.3.2 I discuss estimation strategies to estimate expenditure and income shocks. I then document some features about the estimated expenditure shock in relation to income shock in section 1.3.3. Here, I empirically show that the consumption responses observed in the data, with respect to the expenditure shock, are actually consistent with Figure 1.1. The aim of this section is to motivate that the expenditure shock is not negligible when households make consumption/savings decisions.

[4]Clearly, the discounted sum of lifetime utility is greater for the unconstrained even with the permanently lower consumption from $t \geq 1$ onwards.

1.3.1 Data

The data for empirical analysis in this paper are from the Panel Study of Income Dynamics (PSID), a nationally representative longitudinal survey conducted biennially in the U.S. In fact, there are other large panel datasets that provide information on households' expenditure. For instance, the Consumer Expenditure Survey (CE) is the primary dataset for cross-sectional studies regarding consumption expenditures. It collects considerably detailed and comprehensive expenditure data on a quarterly basis, allowing highly accurate analysis on changes in expenditure across the population. However, the CE is a rotating panel, which interviews different households every four quarters. For this reason, it is not easy to construct a panel dataset at household level for a long period of time.

In contrast, the PSID have kept track of the same households and their split-offs since the first wave of survey in 1968, which enables me to construct a dataset at household level. Furthermore, the PSID not only provides a very good approximation of the consumption measures provided by the CE, but also contains rich information on income and wealth, and covers many dimensions of socioeconomic status of each individual household surveyed.

Among the 41 waves of surveys conducted since 1968, I focus on the time period from 2005 to 2019. 2005 is the latest year when the PSID expanded its expenditure data collection. Since then the categories of consumption expenditures in the questionnaire have remained unchanged.[5] For the analysis in the following section, I exclude from the sample households with changes in household head between 2005 to 2019. I also drop households whose head is younger than 25 or older than 65. Following Blundell et al. (2008), households with annual income less than $100 are

[5]The PSID broadened its coverage of expenditure data in 1999 and 2005. In 1999, new expenditure questions are added to collect information on spending on medical-related expenses, education, childcare, and transportation. The questions were further expanded in 2005 to capture spending on items such as home repairs/maintenance, household furnishing, clothing, trips and vacation, and entertainment.

Table 1.1: Expenditure categories

		Items
c_A	Food away	food away from home, food delivered
	Transportation	gas, transportation fares, parking fees, taxi, other transportation
	Medical	hospital/nursing home, doctor, prescription drugs, health insurance
	Recreation	trips/vacation, entertainment
	Others	clothing, education, child care, housing repairs, auto repairs
c_B	Food home	food at home
	Utilities	electricity, heating, water, telecommunication, other utilities

dropped. Given the paper's focus on expenditure, I further filter out households with missing reports on essential expenses such as food expenditure. The resulting final sample contains 12,152 observations, originating from 1,519 households.

As for the measure of consumption, I focus on non-durable expenditures. Table 1.1 presents the expenditure classification used for analysis. In line with the two-goods CES utility function as in equation (1.3), I split non-durable expenditures into two broad consumption groups, c_A and c_B, by aggregating the subcategories based on their volatilities. The coefficient of variation and expenditure shares for each subcategories are reported in Table 1.2. Specifically, I classify as c_A expenditures with relatively high coefficient of variations and those with low coefficient of variations as c_B. In other words, c_A includes expenditures that are more likely to be exposed to expenditure risks, while c_B consists of expenditures that rarely fluctuate to a great extent over time. Later in this section, I estimate how the pass-through effect of expenditure shocks to c_A, on c_B and the total consumption, c, differs for household's characteristics such as wealth and income.

Table 1.2: Volatilities of consumption goods and expenditure shares

	Subcategory	Avg household coef. of variation	Avg expenditure share of goods
c_A	Food away	0.60	0.08
	Medical	0.82	0.11
	Recreation	0.86	0.08
	Transportation	0.47	0.16
	Others	0.87	0.19
c_B	Food home	0.38	0.20
	Utilities	0.32	0.19
	Total	0.32	1.00

Note: All values are computed by taking average over each household's CV and expenditure shares over 2005-2019.

Assets are classified into liquid/illiquid assets based on their level of liquidity. Liquid assets include certificates of deposit, treasury bonds, checking and savings balances, and stocks. Illiquid assets consist of equity, annuity/IRA holdings, and the net value of other real estate equity, business, farm and vehicles. To group households by assets, I first compute the average of total assets for each household throughout the sample period. I then classfiy as low-wealth households with averages falling within the bottom 50th percentile and high-wealth, otherwise. The measure of income includes labor income, government transfers of household head and spouse. I apply the same approach to split households into low-income and high-income groups. Descriptive statistics for the classifications are given in Table B1.

Additionally, to broaden the analysis, I categorize households as hand-to-mouth by assets and income as in Aguiar et al. (2024). They identify a household as hand-to-mouth based on net worth if its net worth is less than two months of its labor earnings. Also, they define a household as constrained if its liquid wealth is equal to or less than a week of earnings. Following the same approach, I define those who satisfy each criterion as poor hand-to-mouth and wealthy-hand-to-mouth, respectively. The effects of expenditure shock on consumption for different household groups are presented in section 1.3.3.

1.3.2 Estimating Shock Processes

Expenditure Shock

As is already discussed in section 1.2, the expenditure shock is modelled as a preference shock. One problem is that changes in preferences are clearly not observable. However, the changes in consumption due to the change in preferences can be observed in the data. Therefore, I exploit one of the optimality conditions of the household problem in section 1.2 and directly estimate θ from the data. Specifically, from the household's problem in section 1.2, the first-order condition for good A of household i is given as[6]

$$\frac{c_{i,At}}{c_{i,Bt}} = \theta_{i,t}^{\eta} \left(\frac{\alpha_i}{1-\alpha_i} \right) \left(\frac{P_{At}}{P_{Bt}} \right)^{-\eta}. \tag{1.18}$$

Taking logs of both sides gives

$$\log \left(\frac{c_{i,At}}{c_{i,Bt}} \right) = \varphi_i + \delta_t + \gamma' \mathbf{X}_{i,t} + u_{i,t}^{\theta}, \tag{1.19}$$

where φ_i is the individual fixed effect of household i, δ_t is the time fixed effect at t. $\mathbf{X}_{i,t}$ is a vector of household i's demographics, such as age, employment, number of kids, marital status, education and other variables. The idea in (1.19) is that household i's unobserved individual characteristics are captured by φ_i and the changes in aggregate variables, such as changes in prices which affect all households equally, are controlled for by the term δ_t. $\mathbf{X}_{i,t}$ is included to capture the variations in log consumption ratio of household i, being influenced by its demographic factors.

[6] The relative price term is now included as fluctuations in prices need to be controlled for in order to estimate expenditure risk.

The variable of interest in (1.19) is the residual term $u^{\theta}_{i,t}$. It is an unexplained part of the variations in $\log \frac{c_{i,At}}{c_{i,Bt}}$ due to the change in $\theta_{i,t}$, conditional on other demographic characteristics. Thus, $u^{\theta}_{i,t}$ can be interpreted as the expenditure shocks to $c_{i,At}$, unexpected changes in household i's preferences for good A. Once the shock process is obtained, the next step is to characterize the persistence of the shock and its volatility. For this I simply assume an AR(1) process for $u^{\theta}_{i,t}$.

$$u^{\theta}_{i,t} = \rho_{\theta} u^{\theta}_{i,t-1} + \epsilon^{\theta}_{i,t}, \quad \epsilon^{\theta}_{i,t} \sim N(0, \sigma^2_{\theta}) \tag{1.20}$$

Income Shock

As opposed to the expenditure shock, there is a vast literature that explores the estimation of income shock and its relationship with consumption. One canonical method for estimation is to separate log income from the impact of demographic characteristics by retrieving the residuals in the regression

$$\log y_{i,t} = \varphi_i + \delta_t + \gamma' \mathbf{X}_{i,t} + u^{y}_{i,t}. \tag{1.21}$$

The traditional approach in the literature considers $u^{y}_{i,t}$ as the sum of a permanent shock and a transitory shock.[7] To simplify the discussion, however, I assume an AR(1) process for $u^{y}_{i,t}$ as (1.20).

$$u^{y}_{i,t} = \rho_{y} u^{y}_{i,t-1} + \epsilon^{y}_{i,t}, \quad \epsilon^{y}_{i,t} \sim N(0, \sigma^2_{y}) \tag{1.22}$$

Persistence and Volatility

In this section, I estimate the persistence and volatilities of the two shock processes. In (1.20) and (1.22), There are four parameters to be estimated: two for persistence ρ_{θ},

[7] See Blundell et al. (2008), Commault (2022), Floden and Lindé (2001), Kaplan and Violante (2014).

Table 1.3: Persistence and volatility of the shock processes

	Inc shock	Exp shock
ρ	0.631***	0.535***
	(0.020)	(0.011)
σ^2	0.236***	0.334***
	(0.011)	(0.009)

Note: Standard errors in parentheses. *** $p<0.01$, ** $p<0.05$, * $p<0.1$.

ρ_y and another two for standard deviation σ_θ, σ_y. For this I simply regress the shock processes on their lagged values and then back out $\hat{\epsilon}^{y}_{i,t}$, $\hat{\epsilon}^{\theta}_{i,t}$. The estimated parameters are given in Table 1.3. Consistent with predictions, the estimation results show that expenditure shocks are less persistence and more volatile than income shocks. These parameters will be used to discretize the idiosyncratic shock processes in section 1.6.

Table 1.4 shows the volatilities of uncertainties in income and expenditure households face. The variances of $u^{y}_{i,t}$ and $u^{\theta}_{i,t}$ are estimated by wealth and income using a generalized method of moments (GMM). It shows that expenditure uncetainty is relatively larger than income uncertainty. This is also in line with Table 1.3. Comparing by wealth and income reveals that households in the low wealth or low income group face larger uncertainties in both income and expenditure. Higher volatilities in income shock among the poor may be due to the fact that the unemployed and retiree experience relatively substantial fluctuations in their income.

As for expenditure shocks, the difference in the level of uncertainties between the two groups can be attributed to the difference in their average consumption levels. Households experience unexpected fluctuations in expenditures when unanticipated events occur. The way these events happen resembles the one i.i.d shocks normally happen. In other words, the magnitude and likelihood of these expenditure shocks are independent of wealth or income levels or other variables that characterize the households. Consequently, expenditure uncertainties households face are on average greater for those in the low wealth or low income group.[8]

Table 1.4: Volatility of income and expenditure uncertainty ($\text{var}(u^y_{i,t})$, $\text{var}(u^\theta_{i,t})$)

	All	by wealth		by income	
		low	high	low	high
Income shock	0.156***	0.181***	0.131***	0.202***	0.110***
	(0.008)	(0.012)	(0.008)	(0.013)	(0.008)
Expenditure shock	0.498***	0.522***	0.414***	0.552***	0.383***
	(0.015)	(0.026)	(0.014)	(0.026)	(0.013)

Note: Standard errors in parentheses. *** $p<0.01$, ** $p<0.05$, * $p<0.1$. Note that expenditures and income of a household are not in the same level. Normally, income levels are much higher than expenditure levels. Therefore, I scale both variables by each household's average income, making them comparable to each other.

1.3.3 Pass-through Effects of the Shocks

I now turn my attention to how households' consumption responds to the shocks estimated in the previous section. To this end, I present a framework motivated by the one proposed in Blundell et al. (2008).[9] Specifically, I measure the degree of transmission of income and expenditure shock to consumption by estimating

$$\Delta \log \tilde{\mathbf{c}}_{i,t} = \beta_y \epsilon^y_{i,t} + \beta_\theta \epsilon^\theta_{i,t} + \nu_{i,t}, \tag{1.23}$$

[8] For instance, a medical treatment cost of \$1,000 may not be considered a rare and extreme event for the wealthy, but it can be a significant expenditure shock for the poor.

[9] In their paper, they posit a specification for consumption growth with respect to permanent and transitory income shocks:

$$\Delta c_{i,t} = \phi_{i,t}\zeta_{i,t} + \psi_{i,t}\varepsilon_{i,t} + \xi_{i,t},$$

where $\zeta_{i,t}$, $\varepsilon_{i,t}$ denote the permanent and the transitory income shock, respectively. Based on this, they develop a method to measure the extent of consumption insurance to the two income shocks, exploiting the idea that the degree to which income growth varies with consumption growth reflects the strength of the transmission of income shocks into consumption.

where $\tilde{\mathbf{c}}_{i,t} = \{\tilde{c}_{i,Bt}, \tilde{c}_{i,t}\}$ is a set of consumption of good B and total consumption net of its demographic characteristics.[10] $\epsilon^y_{i,t}$ and $\epsilon^\theta_{i,t}$ are the innovations to the income and expenditure shock process, respectively, from equation (1.20) and (1.22). The term $\nu_{i,t}$ reflects innovations in consumption that are not explained by the two shocks. This may capture measurement errors, persistent impacts of income and expenditure shocks from the past, or other types of shocks such as a shock to wealth. The underlying idea in (1.23) is that each coefficient, β_y and β_θ, represents the degree of transmission of income and expenditure shock to $\tilde{\mathbf{c}}_{i,t}$, respectively. If households are fully insured against those shocks, both coefficients, β_y, β_θ, would be statistically insignificant from zero.[11] In the case of partial or no insurance, the coefficients deviate from zero, being closer to 1 or -1 (i.e., $0 < \beta_y, \beta_\theta < 1$ for $\tilde{c}_{i,t}$ and $0 < \beta_y < 1$, $-1 < \beta_\theta < 0$ for $\tilde{c}_{i,Bt}$), with difference depending on assets and income.

Given that all the variables in equation (1.23) are known from the data, the standard OLS method is used to estimate the coefficients. The results are given in Table 1.5. It reports the response of the other consumption, c_B, and the total consumption, c_t, by households' wealth and income level.

First, looking at the first row of each panel reveals that households with relatively low wealth and income exhibit a more pronounced positive response to the income shock in both expenditures, implying higher MPCs. In constrast, when faced with an increase in expenditure on good A, they tend to reduce the other expenditure on good B by more than those with higher wealth and income, although the coefficients for the income groups are not substantially different from each other. These responses are largely in line with the results shown in the middle panel of Figure 1.1. Those

[10] I remove the influence of demographic factors from consumption in order to focus solely on the impact of shocks on consumption.

[11] For simplicity and comparability, I abstract away from the assumption that a shock consists of both transitory and the permanent components. Instead, I assume AR(1) processes, as in (1.20) and (1.22), which focus primarily on the transitory component. This approach may not adequately capture the pass-through effects of the permanent component, particularly in the income shock process, as explored in the consumption insurance literature. In that case, the coefficients could still deviate from zero even under perfect insurance against transitory shocks.

Table 1.5: Pass-through effects of income and expenditure shocks, by wealth and income

		All	by wealth		by income	
			low	high	low	high
$\Delta\log\tilde{c}_{i,Bt}$	β_y	0.125***	0.153***	0.083***	0.134***	0.105***
		(0.025)	(0.040)	(0.010)	(0.036)	(0.011)
	β_θ	−0.256***	−0.270***	−0.237**	−0.257***	−0.254***
		(0.007)	(0.010)	(0.010)	(0.010)	(0.009)
	Adj.R^2	0.225	0.249	0.194	0.234	0.209
$\Delta\log\tilde{c}_{i,t}$	β_y	0.152***	0.180***	0.113***	0.159***	0.134***
		(0.025)	(0.041)	(0.011)	(0.037)	(0.012)
	β_θ	0.332***	0.274***	0.415***	0.279***	0.424***
		(0.008)	(0.011)	(0.011)	(0.011)	(0.010)
	Adj.R^2	0.320	0.274	0.402	0.269	0.416

Note: Standard errors in parentheses. * $p<0.1$, ** $p<0.05$, *** $p<0.01$

in the low wealth and low income group are more likely to be constrained, leading to insufficient savings to serve as a buffer against expenditure shocks affecting specific components of their consumption bundle (i.e. c_A). Consequently, they are forced to make larger cuts in the other consumption (i.e. c_B) to meet their budget constraint. However, the response of total consumption moves in the opposite direction. As illustrated in the third panel of Figure 1.1, the lower panel of Table 1.5 indicates a greater increase in the total consumption for those in the high wealth or high income group in comparison to their counterparts. The underlying reason for this outcome has already been discussed in section 1.2. Relatively richer households hold substantial resources and can efficiently reallocate them within or across the periods in response to an expenditure shock, whereas the poorer households cannot do the same due to limited resources. Therefore, the rich have the ability to absorb the expenditure shock without having to make cuts on the other consumption.

For robustness, I run the same regressions in (1.23) by the hand-to-mouth groups, defined previously in section 1.3.1. The results shown in Table 1.6 are similar to those in Table 1.5. Households hand to mouth reduce the consumption of B to a

Table 1.6: Pass-through effects of income and expenditure shocks, by hand-to-mouth

		All	Non HtM	Poor HtM	Wealthy HtM
	β_y	0.125***	0.090***	0.119***	0.208***
		(0.025)	(0.009)	(0.020)	(0.085)
$\Delta\log\tilde{c}_{i,Bt}$	β_θ	−0.256***	−0.248***	−0.281**	−0.255***
		(0.007)	(0.009)	(0.017)	(0.016)
	Adj.R^2	0.225	0.210	0.269	0.236
	β_y	0.152***	0.116***	0.150***	0.234***
		(0.025)	(0.010)	(0.021)	(0.087)
$\Delta\log\tilde{c}_{i,t}$	β_θ	0.332***	0.398***	0.210***	0.299***
		(0.008)	(0.010)	(0.017)	(0.015)
	Adj.R^2	0.320	0.387	0.205	0.310

Notes: Households are classified as poor HtM if their net worth is less than two months of labor earnings, and as wealthy HtM if their liquid wealth is equal to or less than a week of earnings. The rest are defined as non HtM. Standard errors in parentheses. * p<0.1, ** p<0.05, *** p<0.01

greater extent relative to non HtM households in response to an unexpected increase in the expenditure on A. Among those hand to mouth, the reduction is larger for the poor hand to mouth households. In contrast, as expected, the increase in the total consumption is the largest for the non HtM households and the smallest for the poor HtM.

In this section, I empirically validate the theoretical insights from section 1.2, by providing evidence of heterogeneity in consumption adjustments in response to expenditure and income shocks across different household groups. Specifically, I find that low-wealth households make larger cuts to the other consumption reflecting tighter constraints, while wealthier households reallocate resources more flexibly. By identifying heterogeneity in consumption responses, this section contributes to understanding how these differences influence the effectiveness of redistributive policies analyzed with a general equilibrium model in the next section.

1.4 Model

In this section, I develop a heterogeneous-agent New Keynesian model with incomplete markets to study the implications of the expenditure risk and its impact on the effectiveness of redistributive fiscal policy. A novel feature is that I introduce a new state variable representing the magnitude of expenditure shocks each household faces. The expenditure shock process, estimated in section 3, is discretized and modelled as an idiosyncratic shock, similar to the standard income risks.

I begin by describing the setup of the model and then conduct a steady-state comparison with a standard incomplete model without expenditure risk. The comparison is focused mainly on the difference in the distribution of households, wealth inequality and the marginal propensities to consume (MPC) distribution. Following this, I further explore how the presence of expenditure risk alters the consequences of redistributive fiscal policies in a general equilibrium.

1.4.1 Model Setup

Households

The basic structure of the household block remains unchanged from that described in section 2. However, to make the model more compatible with a general equilibrium environment, I have made slight adjustments to the problem, (1.1)-(1.3), by introducing additional elements.

The economy is populated by a continuum of households. Each household is subject to idiosyncratic income risk in the form of productivity shocks, z. Additionally, they also face idiosyncratic expenditure risk, θ. I assume that both the expenditure and income risk follow a first-order Markov chain. They insure themselves by investing their assets in a mutual fund that pays a return of r^a.

Denote by $V_t(\theta_t, z_t, a_t)$ the value of a household with asset holdings a, productivity level z, and expenditure shock θ. Given these, each household solves the following dynamic problem by choosing consumption, savings and labor supply,

$$V_t(\theta_t, z_t, a_t) = \max_{c_{At}, c_{Bt}, a_{t+1}, n_t} u(c_t, n_t) + \beta \mathbb{E}_t V_{t+1}(\theta_{t+1}, z_{t+1}, a_{t+1}), \tag{1.24}$$

subject to the budget,

$$c_{At} + c_{Bt} + a_{t+1} = (1 - \tau_t) w_t z_t n_t + (1 + r_t^a) a_t + \mathcal{T}_t \tag{1.25}$$

as well as the no-borrowing constraint,

$$a_{t+1} \geq 0$$

where

$$u(c_t, n_t) = \log c_t - \varphi \frac{n_t^{1+\nu}}{1+\nu}.$$

In the budget constraint, c_{At}, c_{Bt} denote each consumption expenditure for each good, respectively, $(1 - \tau_t) w_t$ denotes after-tax real wage, n_t represents labor hours supplied by the household. $\mathcal{T}_t$ denotes lump-sum transfers received from the government. In the steady state, I assume that $\mathcal{T}_t = 0$ for all households. Lastly, in the utility function, the parameter φ controls the disutility from supplying labor, and ν is the inverse Frisch elasticity of labor supply.

Mutual Fund

Following Auclert et al. (2020), I assume that there exists a mutual fund that raises deposits from households and invests in firm shares and government bonds. The goal of the mutual fund is to maximize the expected return on the deposits it pays to households, $\mathbb{E}_t[r^a_{t+1}]$. Therefore, the mutual fund's problem is

$$\max_{s_{jt},\, B^g_t} \mathbb{E}_t[1 + r^a_{t+1}]$$

$$\text{s.t. } (1 + r^a_t)A_t = (1 + r_t)B^g_{t-1} + \int (p^E_{jt} + d_{jt})s_{jt-1}dj$$

$$A_{t+1} = \int p^E_{jt}s_{jt}dj + B^g_t$$

where s_{jt} denotes the shares the mutual fund holds in firm j with price p^E_{jt}. d_{jt} represents the sum of dividends from the intermediate firm and captial firms, discussed below. The first constraint shows that, at the beginning of the period, the outstanding liabilities (LHS) of the mutual fund must be equal to the liquidation value of its assets (RHS). The second constraint tells that, at the end of the period, the value of newly collected deposits is equal to the value of newly purchased shares and bonds.

The first-order conditions lead to

$$\mathbb{E}_t\left[1 + r^a_{t+1}\right] = \frac{\mathbb{E}_t\left[p^E_{t+1} + d_{t+1}\right]}{p^E_t}$$

where $\int s_{jt}dj = 1$, $\int d_{jt}dj = d_t$ and $p^E_{jt} = p^E_t$ in the equilibrium. By allowing the mutual fund to invest also in nominal reserves that pay a promised return, i_t, the no-arbitrage condition requires that

$$\mathbb{E}_t\left[1 + r^a_{t+1}\right] = \frac{\mathbb{E}_t\left[p^E_{t+1} + d_{t+1}\right]}{p^E_t} = (1 + i_{t-1})\frac{1}{\pi_t} = 1 + r_t \tag{1.26}$$

where the last equality holds by the Fisher equation.

Final Good Firms

A representative final goods producer operates in a competitive market by combining a continuum of intermediate goods y_{jt} indexed by $j \in [0, 1]$ to produce the final goods

Y_t. The production technology is

$$Y_t = \left(\int_0^1 y_{jt}^{\frac{1}{\mu_p}} dj \right)^{\mu_p},$$

where μ_p is the steady-state price markup. Given a level of aggregate demand Y_t, profit maximization for the final good firms reads

$$\max_{y_{jt}} P_t Y_t - \int_0^1 p_{jt} y_{jt} dj,$$

where $P_t = \left(\int_0^1 p_{jt}^{\frac{1}{1-\mu_p}} dj \right)^{1-\mu_p}$ denotes the nominal price index of the final good. The optimization implies that final good firms' demand for the intermediate good j is given by

$$y_{jt} = \left(\frac{p_{jt}}{P_t} \right)^{-\frac{\mu_p}{\mu_p - 1}} Y_t.$$

Intermediate Good Firms

Each monopolistically competitive intermediate good firm indexed by $j \in [0, 1]$ produces using inputs of capital k_{jt} and labor n_{jt}, with the standard Cobb-Douglas production function:

$$y_{jt} = Z_t k_{jt}^{\gamma} n_{jt}^{1-\gamma},$$

where γ is the capital share in the intermediate good production process. The intermediate good firms set p_{jt} taking as given the real rental rate of labor and capital, w_t, r_t^k, respectively, subject to a quadratic price adjustment cost Θ_p as in Rotemberg (1982).[12] Denote by $J_{jt}^F(p_{jt-1})$ the (maximal attainable) value of an intermediate firm

[12] Cost minimization problem implies that the marginal cost, the rental rate of capital and labor are given respectively by

$$mc_t = \frac{1}{Z_t} \left(\frac{r_t^k}{\gamma} \right)^{\gamma} \left(\frac{w_t}{1-\gamma} \right)^{1-\gamma}$$

$$r_t^k = \gamma mc_t Z_t K_t^{\gamma-1} N_t^{1-\gamma}$$

$$w_t = (1-\gamma) mc_t Z_t K_t^{\gamma} N_t^{-\gamma}.$$

j at t that posted price p_{jt-1} in $t-1$. The bellman equation is then

$$J_{jt}^F(p_{jt-1}) = \max_{p_{jt},k_{jt},n_{jt}} \frac{p_{jt}}{P_t} y_{jt} - w_t n_{jt} - r_t^k k_{jt} - \Theta_p(p_{jt}, p_{jt-1}) + \frac{1}{1+r_{t+1}} \mathbb{E}_t J_{jt+1}^F(p_{jt})$$

$$\text{s.t. } y_{jt} = k_{jt}^\gamma n_{jt}^{1-\gamma}$$

$$y_{jt} = \left(\frac{p_{jt}}{P_t}\right)^{-\frac{\mu_p}{\mu_p-1}} Y_t$$

$$\Theta_p(p_{jt}, p_{jt-1}) = \frac{1}{2\kappa_p(\mu_p - 1)} \left(\frac{p_{jt} - p_{jt-1}}{p_{jt-1}}\right)^2 Y_t,$$

where $\kappa_p \geq 0$ indexes the nominal rigidities in price setting. Since all intermediate goods firms are identical, they make the same choices when they produce. Thus, $k_{jt} = K_t$, $n_{jt} = N_t$ and $p_{jt} = P_t$ in equilibrium. Combining the first-order conditions and the envelope condition, with the symmetry assumption leads to the following forward-looking New Keynesian Phillips curve

$$\pi_t(\pi_t + 1) = \kappa_p(\mu_p \cdot mc_t - 1) + \frac{1}{1+r_{t+1}} \frac{Y_{t+1}}{Y_t} \pi_{t+1}(1 + \pi_{t+1}), \tag{1.27}$$

where mc_t is the real marginal cost of production and $1 + \pi_t = \frac{P_t}{P_{t-1}}$ is the inflation rate.

Capital Firms

A capital firm owns capital and rents it to the intermediate firms at rate r_t^k. The capital firm's decision characterizes aggregate investment I_t and aggregate captial stock for the next period K_{t+1}. It chooses investment to maximize its value, $J_{jt}^K(k_{jt})$, by solving

$$J_{jt}^k(k_{jt}) = \max_{i_{jt}} r_t^k k_{jt} - i_{jt} - \Theta_k(k_{jt+1}, k_{jt}) + \frac{1}{1+r_{t+1}} \mathbb{E}_t J_{jt+1}^k(k_{jt+1})$$

$$\text{s.t. } i_{jt} = k_{jt+1} - (1-\delta)k_{jt}$$

$$\Theta_k(k_{jt+1}, k_{jt}) = \frac{\phi_k}{2}\left(\frac{k_{jt+1} - k_{jt}}{k_{jt}}\right)^2 K_t.$$

By defining $Q \equiv \frac{\partial p_t^E}{\partial K_{t+1}}$, the capital firm's optimal decision implies the following investment dynamics

$$\frac{I_t}{K_t} - \delta = \frac{1}{\phi_k}(Q_t - 1)$$

$$Q_t = \frac{1}{1+r_{t+1}}\left[r_{t+1}^k - \frac{I_{t+1}}{K_{t+1}} - \frac{\phi_k}{2}\left(\frac{K_{t+2} - K_{t+1}}{K_{t+1}}\right)^2 + \frac{K_{t+2}}{K_{t+1}}Q_{t+1}\right].$$

Labor Unions

There is no canonical way to model labor market frictions in HANK literature. To allow for sticky wages in an environment with heterogeneous households, I extend the model with wage rigidities in representative agent models. While having sticky wages in the model is not essential, it is useful to produce more realistic macroeconomic quantity dynamics. Regarding the specific setup, I follow Auclert et al. (2021).

I assume that each household i provides differentiated labor services to a labor union k. The objective of the labor union k is then to set hours n_{ikt} and nominal wages W_{kt} to maximize the average utility of households it hires, taking as given the consumption-saving decision made by its members. Adjusting the nominal wage W_{kt} incurs quadratic adjustment costs,

$$\Theta_W(W_{kt}, W_{kt-1}) = \frac{\mu_w}{\mu_w - 1}\frac{1}{2\kappa_w}\left(\frac{W_{kt} - W_{kt-1}}{W_{kt-1}}\right)^2 N_t.$$

The labor union produces a union-specific labor service $N_{kt} = \int z_{it} n_{ikt} di$, which will be then packaged into aggregate labor services by a competitive labor packer using the following CES technology,

$$N_t = \left(\int N_{kt}^{-\mu_w} dk\right)^{\mu_w},$$

where μ_w is the steady-state wage markup. I assume that each labor union allocates its labor uniformly across all households, so that $n_{ikt} = N_{kt}$.

In each period, the labor union k maximizes

$$\sum_{s\geq 0} \beta^{t+s} \left(\int \{u(c_{it+s}) - v(n_{it+s})\}\, dD_{it+s} - \Theta_W(W_{kt}, W_{kt-1}) \right),$$

taking as given the demand for labor services

$$N_{kt} = \left(\frac{W_{kt}}{W_t}\right)^{-\frac{\mu_w}{\mu_w - 1}} N_t,$$

where $W_t = \left(\int W_{kt}^{-\frac{1}{\mu_w-1}} dk\right)^{1-\mu_w}$. Using the first-order conditions with respect to W_{kt} with the assumption that all labor unions charge the same wage rate, so $W_{kt} = W_t$, leads to following the New Keynesian wage Phillips curve

$$\pi_t^w (1 + \pi_t^w) = \kappa_w N_t \left(v'(n_{it}) - \frac{(1-\tau_t) w_t}{\mu_w} \int z_{it} u'(c_{it}) di \right) + \beta \pi_{t+1}^w (1 + \pi_{t+1}^w), \quad (1.28)$$

where $\pi_t^w = \frac{W_t}{W_{t-1}} - 1$. Note that the labor unions adjust nominal wages upwards if the marginal rates of substitution between labor and consumption exceeds the marked-down after-tax income, $\frac{(1-\tau)w}{\mu_w}$.

Fiscal Policy

The government issues bonds B_t^g, obtains tax revenue from imposing labor income tax τ_t, and spends on goods and services G_t. It also follows a simple rule for income tax rate that adjusts to the deviation of government debt from its steady state level as

$$\tau_t = \tau_{ss} + \zeta_\tau \left(B_t^g - B_{ss}^g \right),$$

where the coefficient ζ_τ determines the speed at which the tax rate is returned to its target level. For simplicity, I assume that the government spendings are held constant

$$G_t = G.$$

Its budget constraint is then given as

$$G + (1 + r_t)B^g_{t-1} + \int \mathcal{T}_{it} di = \tau_t w_t N_t + B^g_t.$$

Monetary Policy

The monetary authority follows the standard Taylor rule for the nominal interest rate

$$i^{CB}_t = \bar{r} + \phi_\pi \pi_t, \quad \phi_\pi > 1,$$

where $\bar{r}$ is the steady-state real interest rate, ϕ_π governs the sensitivity of the central bank to deviation of inflation from its target $\bar{\pi} = 0$.

Equilibrium

A competitive equilibrium is a path of prices $\{P_t, W_t, \pi_t, \pi^w_t, w_t, r^k_t, mc_t, r_t, r^a_t, p^E_t, \tau_t, Q_t, i^{CB}_t\}$, aggregate quantities $\{Y_t, C_t, A_t, N_t, I_t, K_t, d_t\}$, individual policy rules $\{c_t(\theta, z, a), a_{t+1}(\theta, z, a)\}$, and joint distribution of agents $D_t(\theta, z, a)$ such that:

1. Given the associated prices w_t, r^a_t, τ_t, aggregate N_t and idiosyncratic shocks θ_t, z_t, households optimize with the individual policy rules $c_t(\theta, z, a)$, $a_{t+1}(\theta, z, a)$.

2. The mutual fund optimizes by maximizing the expected real return on savings r^a_{t+1}.

3. Firms maximize profits taking prices P_t, r^k_t, w_t as given.

4. Nominal wages W_t and hours worked N_t are set by labor unions.

5. All markets clear.

   (a) Asset market: $A_t = p_t^E + B^g$

   (b) Labor market: $N^s = N^d$

   (c) Goods market: $Y_t = C_t + I_t + G + \Theta_{p,t} + \Theta_{k,t}$[13]

6. The government follows its fiscal rule to set τ_t and B_t^g and the nominal interest, i_t^{CB}, is given by the monetary policy rule.

1.5 Calibration

I calibrate the model at a quarterly frequency. The main focus of the analysis is on the implications that expenditure risk poses for an otherwise standard incomplete markets model with heterogeneous agents. To this end, I solve the model for two different steady states: one without expenditure risk (hereafter, baseline model) and one with expenditure risk (hereafter, extended model).

Households First, I convert the persistence and the volatility parameters of the AR(1) process for income and expenditure risk, estimated in Table 1.3, to a quarterly frequency. This conversion yields $\rho_z = 0.94$, $\sigma_z = 0.318$ for the income risk and $\rho_\theta = 0.92$, $\sigma_\theta = 0.398$ for the expenditure risk.[14] I then discretize each into a Markov chain with 5 states for income and 3 states for expenditure, using the Rouwenhorst's method, so that each shock becomes idiosyncratic from the households' perspective. Note that the estimates for income risk are equally used in both models, while those for expenditure risk are applied only to the extended model.

For a reasonable comparison, it is essential to ensure that, across the two economies, households have the same discount factor, β. Therefore, I set the discount rate β to

[13] As long as the asset and labor markets clear, the goods market clearing condition must automatically follow from Walras's law.

[14] For more details on the conversion, see Appendix A.2 in Chang and Kim (2006).

0.987 across the two models. This implies an annual interest rate of 5.0% in the baseline model and 0.8% in the extended model.[15] The difference in equilibrium interest rates reflects the difference in the degree of precautionary savings by households. Given a constant bond supply, the presence of expenditure risks, in addition to income risks, induces households to increase their precautionary savings, leading to a lower equilibrium interest rate. In other words, due to the stronger precautionary motive, households in the extended model are willing to hold the same amount of bonds through savings even at a much lower rate of return.

I consider the standard values of $\sigma = 1$ for the elasticity of intertemporal to assume log utility, and $\nu = 1$ for the inverse of the Frisch elasticity of labor supply. The disutility of labor supply, φ, is set such that $N = 1$ in the steady state. Lastly, the share of good A is set to $\alpha = 0.6$, as measured from the PSID.

Firms As standard in other New Keynesian literature, the depreciation rate of capital (δ) is set to 0.04 and the slope of Phillips curve, κ_p, is set to 0.1. Following Ferriere and Navarro (2024), I assume that the capital adjustment cost $\phi_k = 15$. The steady state markup is given as $\mu_p = \frac{1}{mc}$ where mc is the real marginal costs. The capital share (γ) is calibrated such that $\gamma = (r + \delta)\frac{K}{Y \cdot mc}$. It is 0.32 and 0.25 in the baseline and extended model, respectively.

Labor Unions Regarding the wage Phillips curve, I set the slope parameter κ_w to 0.1 and the steady state wage markup to $\mu_w = 1.1$.

Policy The government spending G is set to match the spending-to-output ratio of 20%. The tax rate response parameter to debt ζ_τ is set to 0.1 following Auclert et al. (2020). I set the government debt B^g to 2.8. The steady-state income tax rate, τ, is determined to finance the government expenditures including transfers, $\mathcal{T}$, which

[15] Specifically, I calibrate β in the baseline model first, targeting 5.0% of annual interest rate. Then I solve the extended model using the same β to find the implied equilibrium interest rate.

Table 1.7: Calibration

Parameter	Description	Baseline (w/o exp risk)	Extended (w/. exp risk)	Targets/Source
1. Households				
β	Discount factor	0.987	0.987	$r = 0.05$ (Baseline)
r	Interest rate (annually)	0.05	0.008	$\beta = 0.987$
ρ_z	Persistence of inc shock	0.94	0.94	PSID
σ_z	std of inc shock	0.318	0.318	PSID
ρ_θ	Persistence of exp shock	0.0	0.92	PSID
σ_θ	std of exp shock	0.0	0.398	PSID
σ	Inverse of the EIS	1	1	log utility
ν	Inverse of Frisch elasticity	1	1	Standard
φ	Disutility of labor	0.725	0.921	$N = 1$
2. Firms				
δ	Depreciation rate	0.04	0.04	
κ_p	Slope of NKPC	0.1	0.1	Standard
ϕ_k	Capital adj cost	15	15	
γ	Capital share	0.32	0.25	$\gamma = (r+\delta)\frac{K}{Y \cdot mc}$
μ_p	ss markup	1.015	1.003	$\mu_p = {}^{1}/{mc}$
3. Labor unions				
κ_w	Slope of wage NKPC	0.1	0.1	
μ_w	ss wage markup	1.1	1.1	
N	Hours worked	1	1	
4. Policy				
G	Gvt spending	0.2	0.2	Spending-to-GDP
ζ_τ	Sensitivity of tax rate	0.1	0.1	
B^g	Gvt debt	2.8	2.8	Debt-to-GDP
τ	ss inc tax rate	0.35	0.28	Gvt budget constraint
π	ss inflation rate	0	0	Net zero inflation rate
ϕ_π	Taylor rule coefficient	1.5	1.5	Standard

depend on the distribution of households in the stationary equilibrium. As a result, τ is set to 0.35 in the baseline model and 0.28 in the extended model.

Turning to monetary policy, I assume the central bank targets a steady-state rate of inflation of zero percent. The Taylor rule coefficient with respect to inflation, ϕ_π, is set to 1.5, in line with other New Keynesian literature and empirical estimates. Table 3.1 summarizes the calibration.

1.6 Quantitative Results

In this section, I first examine the steady-state properties of the model, focusing on how the introduction of expenditure risk alters a standard incomplete markets model. I then explore the potential implications of expenditure risk on fiscal policy. Through this analysis, I will demonstrate that expenditure risk is crucial for the design and effectiveness of redistributive fiscal policies.

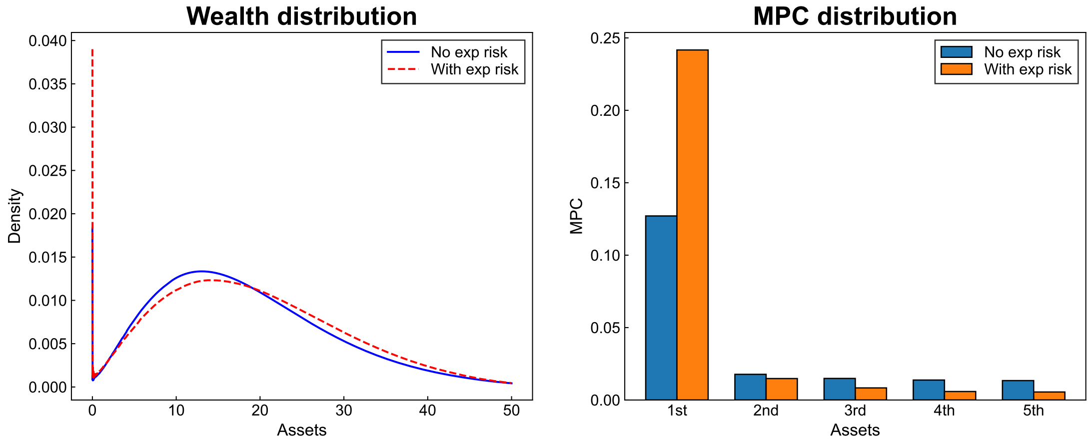
1.6.1 Steady-state Properties

Since the expenditure risk significantly influences households' consumption and saving decisions, the two models feature different steady-state properties. As shown in Section 5, there is a relatively large difference in the equilibrium interest rates for the same discount factor. This suggests that the expenditure risk increases the precautionary savings motive among households, making them more willing to save. In other words, households are willing to hold bonds even at a lower interest rate due to the stronger need for savings as a buffer.

The interplay between the expenditure risk and heterogeneity in income and asset holdings among households results in a more dispersed wealth distribution. The left panel of Figure 1.2 illustrates how the distributions of households differ between the models. Notably, there is a relatively larger share of households located at the very bottom of the distribution in the model with expenditure risk. This indicates that expenditure risk can exacerbate financial constraints for households, particularly those experiencing high expenditure shocks (i.e. high realization of θ), which forces them to deplete their assets to sustain higher consumption levels. This is consistent with the findings by Huang (2023) and Bhutta et al. (2015).

Conversely, the upper range of the distribution also sees a relatively larger share of households, consisting of those lucky enough to avoid high expenditure shocks.

Figure 1.2: Wealth and MPC distribution



Note: The blue solid line denotes the distribution from the baseline model (w/o exp risk) while the red dashed line represents one from the extended model (w/. exp risk). Each bar in the right panel represents the average MPC of households in the respective wealth quintile groups.

These households, experiencing the low-θ state, have more incentives to accumulate more assets to self-insure themselves away from the borrowing constraint, resulting in their concentration in the higher range of the distribution. Consequently, the overall distribution of the extended model is more dispersed compared to that of the baseline model. The marginal distributions of households conditional on θ, provided in Figure B1, confirms this discussion. Those affected by high expenditure shocks are more likely to be constrained while those affected by low expenditure shocks are relatively well self-insured.

The right panel of Figure 1.2 compares the distribution of average marginal propensity to consume (MPC) of the two models. Due to the concavity in the consumption function, the MPCs decrease with assets in both models. However, consistent with the more dispersed wealth distribution, the negative correlation between the MPC and assets is more pronounced in the extended model.[16] This model has a

[16] The negative covariance between MPCs and assets is larger in the extended model. I find that it is -0.479 while -0.229 in the baseline model. The covariance is computed as

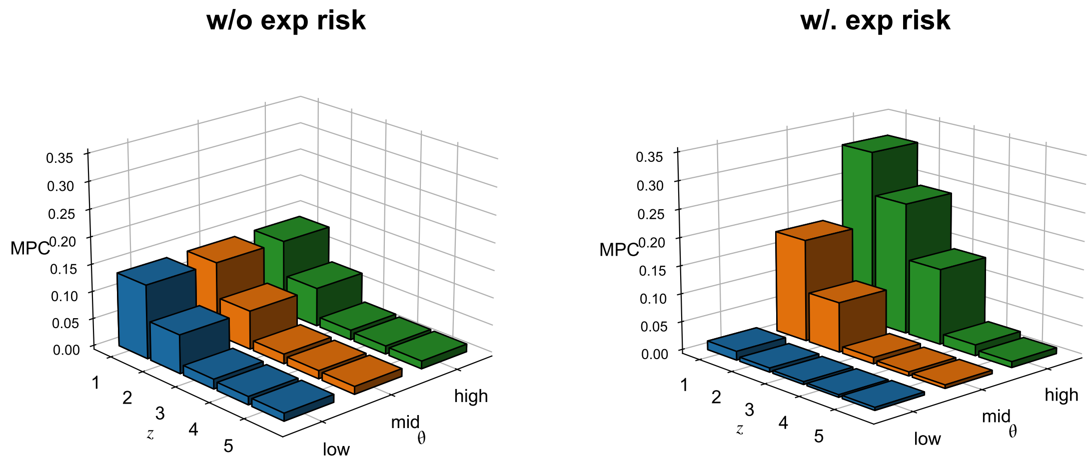
$$\operatorname{cov}(MPC_i, a_i) = \int \left(MPC_i - M\bar{P}C\right)(a_i - \bar{a})\, dD(\theta, z, a),$$

where $M\bar{P}C = \int MPC_i dD(\theta, z, a)$ and $\bar{a} = \int a_i dD(\theta, z, a)$.

larger share of constrained households at the lower end of the distribution, as shown in Figure 1.2, which corresponds to higher MPCs near the borrowing constraint. Beyond the left-tail of the distribution, the extended model exhibits lower MPCs, suggesting stronger precautionary savings. This indicates that households above the left-tail of the distribution are more likely to delay consumption until they have accumulated sufficients assets to mitigate the impact of potential expenditure shocks.

Plotting the MPCs against income and expenditure shock provides further insight into how the expenditure risk affects households' consumption-savings policies. Figure 1.3 demonstrates that incorporating expenditure shocks significantly alters the MPC distribution. Specifically, the right panel of Figure 1.3 reveals that households facing high expenditure shocks tend to exhibit higher MPCs, conditional on their income levels. This finding is intuitive; for example, an individual facing \$10,000 in medical bills is more likely to spend a larger share of financial transfers, if given, than those with only \$100 in bills, assuming their income levels are identical. Particularly, those in the low-income and high-expenditure state display the highest MPC. The model predicts that they would spend on average more than 30% of an additional increase in their income, which is significantly higher than that of their counterpart in the baseline model. In a standard HANK model, it is constrained households that usually exhibit the highest MPC. In the model with expenditure risk, however, this is not always the case. Figure 1.3 suggests that households may display high MPCs when faced with large expenditure shocks, even if they are not necessarily financially constrained. The model can show that in the steady state, there exist households near the constraint— but not completely constrained— showing a MPC close to 1, especially when hit by large expenditure shocks. The presence and the distribution of these households have significant implications for redistribution policies, which I will discuss in the following section.

Figure 1.3: MPC distribution



Note: Each bar represents the average MPC (Marginal Propensity to Consume) of households in their respective income and expenditure states. In the baseline model (without expenditure risk), the expenditure state is a dummy variable that does not affect the outcomes of the model. It is included solely to ensure that the two diagrams are comparable to each other. Note that the MPC distributions are identical across different expenditure states.

On the other side of the distribution, those households with low incomes but low expenditure shocks tend to exhibit very low MPCs, compared to their counterparts in the baseline model. This indicates that low expenditure shocks influence the consumption and savings behavior by increasing precautionary savings motive of these low-income households who would otherwise display the typical hand-to-mouth behavior, represented by high MPCs.

1.6.2 Policy Implications

Now I turn my attention to how the expenditure risk influences the design and effectiveness of policies. Particularly, I restrict my focus to redistributive fiscal policies. Exploring the implications for other types of policies, such as monetary policy or changes in government spending, could also be an interesting approach. However, as is well known in other HANK literature, those policies do not necessarily lead to significant differences in the response of aggregate variables even with a richer household heterogeneity in the model.[17] In Figure B2 in the appendix, I show that my model also produces the results that align with these findings. Therefore, this section will specifically focus on two types of fiscal policies that involve redistribution among households: insurance policy and income redistribution policy.

Insurance Policy

Under this policy, redistribution occurs from households with low expenditure shocks to those with high expenditure shocks. The goal of this policy is to explore how the effect of the redistribution can be amplified when the presence of expenditure risk is taken into account, compared to the baseline model.

However, there is an important caveat to this exercise. Unlike income data, which is publicly available, data on expenditure shocks are not readily observable. Therefore, the implementation of this policy is hypothetical. Specifically, I assume that the government has the ability to accurately observe the magnitude of expenditure shocks of each individual household and then targets the redistribution to those suffering large expenditure shocks. Under this policy, households with low expenditure shocks would finance the policy. They would contribute a certain amount as an insurance premium and, in return, receive a lump-sum transfer as compensation in the event of high expenditure shocks, functioning similarly to an insurance scheme. Therefore, the budget constraint of households in (1.25) now changes to

$$c_{At} + c_{Bt} + a_{t+1} = (1 - \tau_t) w_t z_t n_t + (1 + r_t^a) a_t + \mathcal{T}_t(\theta_t)$$

[17] Kaplan et al. (2018) show that while the indirect effects of monetary policy on aggregate consumption are dominant in HANK models, the overall response of aggregate consumption to a cut in interest rates may be larger or smaller than in RANK models, depending on various factors that are neutral in RANK. See also Broer et al. (2020), Kopiec (2022) for more discussions.

where

$$\mathcal{T}_t(\theta_t) = \begin{cases} -\Delta & \text{for low } \theta \\ 0 & \text{for mid } \theta \\ \Delta & \text{for high } \theta \end{cases}, \tag{1.29}$$

where Δ denotes the amount of transfer. For more simplicity, the population shares for low and high θ households are kept the same in calibration.[18] Therefore, this is an expenditure-neutral policy in which $\int \mathcal{T}_t(\theta_t) dD_t(\theta, z, a) = 0$ for all t.

Short-run Analysis First, I explore how aggregate variables respond to a redistribution shock between households. Figure 1.4 shows the impulse responses to a one-time increase in Δ. I find that the insurance policy is successful in the extended model in stimulating the aggregate demand, while it generates no effects on the economy in the baseline model.[19] In Figure B3(a), decomposing the consumption responses, following Kaplan et al. (2018), reveals that the direct effect of transfers contributes the most to the increase in consumption.

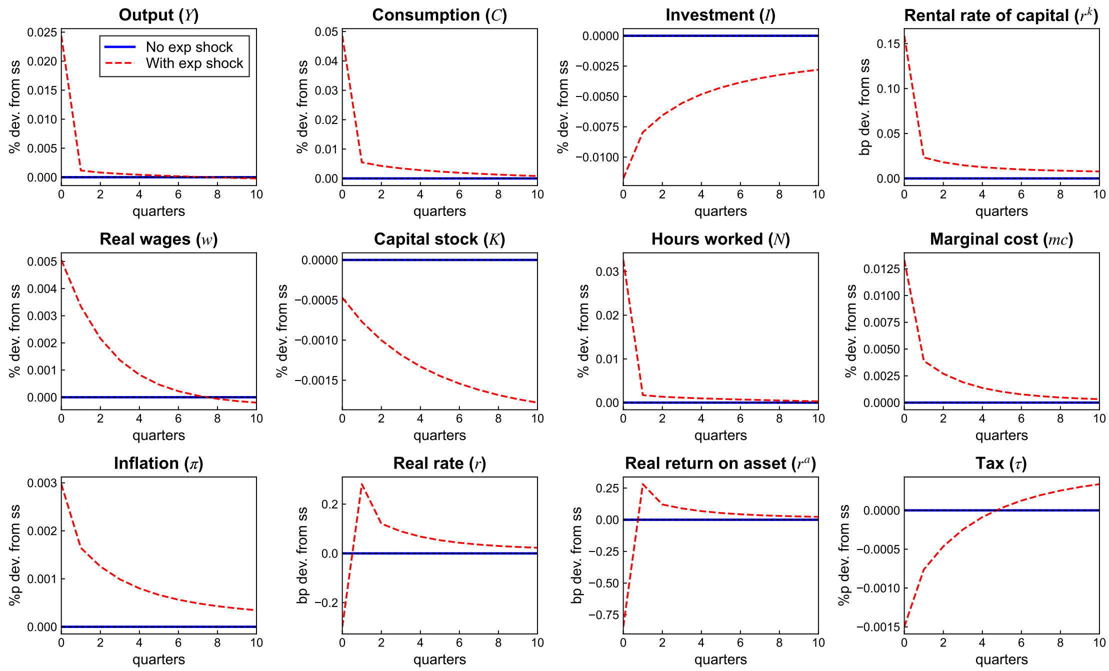
As the redistribution boosts aggregate demand, firms respond by increasing production due to sticky prices, which results in higher hours worked and output. Consequently, the prices of inputs for production, real wages, and the rental rate of capital also rise, leading to an increase in the inflation rate. To stabilize the economy, the central bank raises its policy rate. However, due to the immediate increase in inflation, the ex-post real interest rate does not increase at the time of the shock, according to the Fisher relation.

On the downside, the insurance policy reduces aggregate investment. The policy redistributes wealth from higher-saving households (i.e. households with low expen-

[18] The Markov process for θ is symmetric. This means the population shares are 0.25, 0.5, and 0.25, respectively, from the lowest expenditure shocks to the highest.

[19] Redistribution based on θ in the baseline model is equivalent to redistribution within identical households of the same population share. Thus, the two constrasting responses offset each other, resulting in no impacts on the economy.

Figure 1.4: Impulse responses to a one-time insurance policy shock



Note: The solid blue line denotes the responses in the baseline model and the dashed red line represents the extended model. The size of Δ is set to 1% of the steady-state output level.

diture shocks) to lower-saving households (i.e. households with high expenditure shocks). Since savings are the primary source of investment funds, this redistribution leads to a decrease in available funds for investment, causing investment to fall. Lastly, tax rates decrease due to the general equilibrium effects, although the policy itself does not directly affect the tax rates.

At the household's level, as predicted in Figure 1.3, the targeted transfers in the extended model enable households facing high expenditure shocks to increase their consumption more than the reduction in consumption of those experiencing low expenditure shocks. Heterogeneous responses in consumption by income and expenditure shock, shown in Figure B4, quantitatively confirm this.

Long-run Analysis Now, I take the analysis to a long-term perspective and discuss welfare changes resulting from the implementation of the insurance policy.

The key question here is whether transitioning to an economy with this policy can improve household welfare.

To determine new steady states, I re-solve the model for both the baseline and extended economies, assuming that the insurance policy is implemented as a permanent measure. The welfare measure used in this analysis is consumption equivalence. Therefore, for a household with expenditure shock θ, productivity z, and asset holdings a, the consumption equivalence, $\lambda(\theta, z, a)$, is computed by

$$E_0\left[\sum_{t=0}^{\infty}\beta^t u\left[(1+\lambda(\theta,z,a))\,c_t, n_t\right]\right] = E_0\left[\sum_{t=0}^{\infty}\beta^t u\left(\tilde{c}_t,\tilde{n}_t\right)\right], \tag{1.30}$$

where c_t, n_t are the consumption and hours worked in the benchmark economy, $\tilde{c}_t$, $\tilde{n}_t$ are the corresponding counterparts in the counterfactual economy with the insurance policy, respectively.[20] In equation (1.30), the consumption equivalence, λ, represents the level of welfare, expressed in terms of a percentage change in consumption, that would make the household indifferent between the two steady states. Therefore, $\lambda > 0$ suggests that policy improves welfare, while $\lambda < 0$ indicates it reduces welfare.

The welfare changes due to the insurance policy are presented in Table 1.8. Each column shows the consumption equivalences by different level of expenditure shocks, θ, between the two steady states for each model. Across different levels of θ, the welfare changes show that transfer recipients, those with high θ, gain, whereas the payers of transfer suffer as they have to bear the cost of the policy. The aggregate λ indicates that the insurance policy reduces the aggregate welfare, particularly more in the extended model where the expenditure risks are considered. Compared to the

[20] The welfare measure $\lambda(\theta, z, a)$ can be backed out from

$$\lambda(\theta,z,a) = \exp\left((V_1 - V_0)(1-\beta)\right) - 1$$

where $V_1 = E_0\left[\sum_{t=0}^{\infty}\beta^t u\left(\tilde{c}_t,\tilde{n}_t\right)\right]$ and $V_0 = E_0\left[\sum_{t=0}^{\infty}\beta^t u\left(c_t,n_t\right)\right]$. Then, the aggregate λ is given as

$$\text{agg } \lambda = \int \lambda(\theta,z,a)dD_0(\theta,z,a).$$

Table 1.8: Welfare change: Insurance policy

		λ	
		baseline	extended
by θ	low θ	-0.0296	-0.1039
	mid θ	-0.0038	-0.0768
	high θ	0.0186	0.1429
	Agg λ	-0.0148	-0.0379

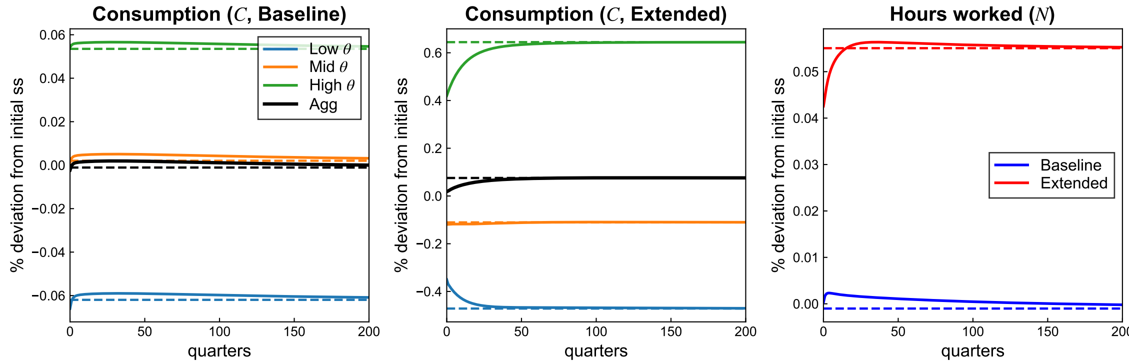
Notes: Each value represents the consumption equivalence of welfare changes between the two steady states, expressed as a percentage of consumption. The size of Δ is set to 1% of the steady-state output level.

case without the insurance policy, the aggregate welfare losses amount to 0.0148% for the baseline and 0.0379% for the extended model. This larger welfare loss with the expenditure shocks is mainly due to a larger increase in hours worked, resulting from general equilibrium.[21] This targeted transfer policy stimulates aggregate demand, which in turn induces firms to increase production. Achieving a higher production level in the new steady state requires more labor supply, thereby reducing household welfare.

Figure 1.5 displays the transition paths of consumption and hours worked from the initial steady state to the new one. I focus on these variables because changes in consumption and hours worked are primary factors determining welfare. Following a one-time permanent increase in Δ, the aggregate consumption in the extended model converges to a higher level, driven by the substantial increase in consumption of those with high θ. Hours worked also converge to a higher level relative to the initial level. In contrast, in the baseline model, the new equilibrium consumption and hours worked

[21] To isolate the effect of hours worked on welfare, I compute a counterfactual consumption equivalence, assuming that welfare depends only on consumption. In this scenario, I find that the policy improves welfare in the extended model but still reduces welfare in the baseline model. Specifically, in the extended model, the significant increase in the consumption of transfer recipients results in a welfare gain of 0.0129. In contrast, the absence of such an increase in the baseline model leads to a welfare reduction of -0.0154.

Figure 1.5: Transition paths following a permanent increase in Δ: Insurance policy



Note: The first two panels show the transition paths of consumption by θ in the baseline and extended model respectively, while the transition paths of hours worked for both models are plotted in the last panel together. Note that, in the baseline model (w/o expenditure risk), expenditure state, θ, is a dummy state variable included only to make the two models comparable. The solid lines denote the transition paths of variables, and the dashed lines represent the new steady state level relative to the initial one. The size of Δ is set to 1% of the steady-state output level.

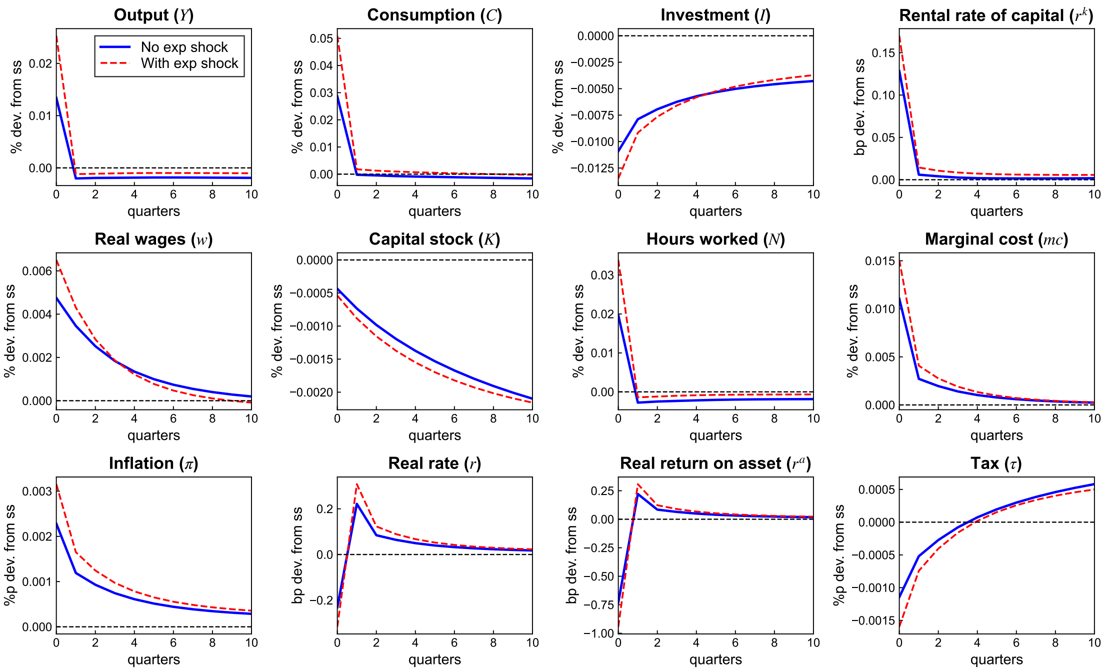
remain largely unchanged from the initial equilibrium level.[22] Notably, the minimal change in hours worked in the baseline model results in negligible general equilibrium effects on welfare. Consequently, while the insurance policy allows the aggregate consumption to increase through redistribution by θ, it also increases disutilities from supplying more labor, ultimately reducing overall welfare.

Income Redistribution Policy

To better understand the implications of expenditure risks, I now explore another policy that redistributes wealth from high-income households to low-income households. This policy, like the previously discussed insurance policy, also redistributes wealth from households with low MPC to those with higher MPC. However, the income redistribution policy is more realistic and feasible in that the government can observe each household's income level. Under this policy, I assume that the lump-sum transfers are now determined based on the productivity level, z. To maintain the expenditure-neutral feature of the policy, I assume that the transfer system is specified

[22] This is in line with the findings in Figure 1.4 where a one-time transitory shock does not generate any responses in the baseline model.

Figure 1.6: Impulse responses to a one-time income redistribution policy shock



Note: The solid blue line denotes the responses in the baseline model and the dashed red line represents the extended model. The size of Δ is set to 1% of the steady-state output level.

as[23]

$$\mathcal{T}(z_t) = \begin{cases} \Delta & \text{for } z_1 \\ \Delta & \text{for } z_2 \\ 0 & \text{for } z_3 \\ -\Delta & \text{for } z_4 \\ -\Delta & \text{for } z_5 \end{cases} .$$

Short-run Analysis Figure 1.6 displays the responses of aggregate variables to a one-time shock to Δ. This one-time income redistribution generates expansionary effects on the economy in both models. The overall shape of the impulse responses does not change much from those of the insurance policy. However, the

[23] The Markov process for z is also symmetric. This means the population shares are 0.0625, 0.25, 0.375, 0.25, and 0.0625, respectively, from the lowest productivity to the highest.

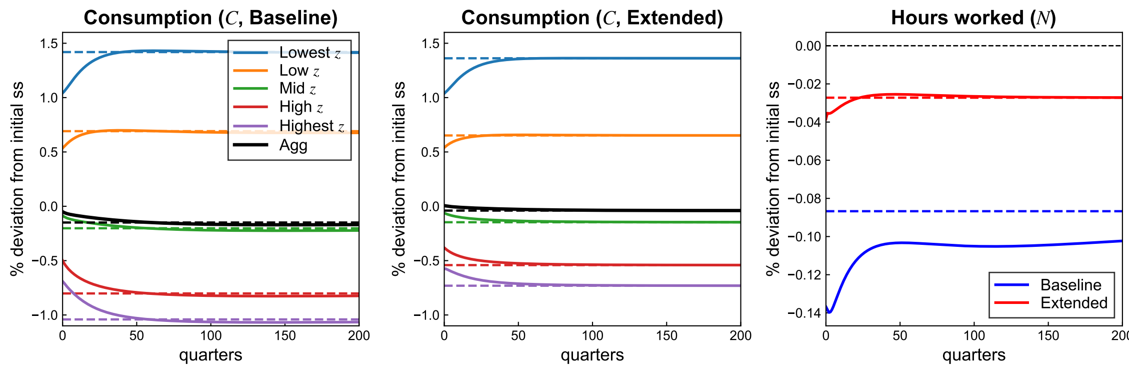
magnitudes of the responses are still more pronounced in the extended model with expenditure risks. Specifically, there is a larger increase in output, consumption and hours worked compared to the baseline model. The decomposition of consumption in Figure B3(b) shows that, as similar to the insurance policy, those differences can be explained mainly by the stimulative effects of transfers. The heterogeneous responses of consumption by income and expenditure shocks are also given in Figure B5. As simliar to the insurance policy, it confirms that households with low income and high expenditure shock in the extended model increase their consumption the most.

Long-run Analysis Table 1.9 compares the welfare changes from the transition to an economy with the income redistribution policy. Across the income levels, the welfare of the recipients of transfer (low income households) improves while that of the payers of transfer (high income ones) deteriorates. The result shows that although the policy enhances the aggregate welfare in both models, the aggregate increase is smaller in the extended model. This difference in welfare increase can also be explained by changes in hours worked.[24]

Transitioning to an economy with income redistribution has different implications compared to transitioning to one with the insurance policy. A permanent increase in transfers from high-income households to low-income households generates contractionary effects because it hinders aggregate capital accumulation in the long-term. High-income households usually save a larger portion of their income, which is the primary source of funds for investment. Therefore, a permanent redistribution away from these households can lead to a recession in the long-term. Figure 1.7 shows that both consumption and hours worked decrease following a permanent increase in Δ. The decrease in hours worked contributes to an increase in welfare, but its impact on

[24] When excluding the effect of hours worked on welfare, welfare gains are larger for the extended model. Specifically, the welfare gain is 0.0693 in the baseline model and 0.0793 in the extended model.

Figure 1.7: Transition paths following a permanent increase in Δ: Income redistribution policy



Note: The first two panels show the transition paths of consumption by θ in the baseline and extended model respectively, while the transition paths of hours worked for both models are plotted in the last panel together. The solid lines denote the transition paths of variables, and the dashed lines represent the new steady state level relative to the initial one. The size of Δ is set to 1% of the steady-state output level.

welfare is more significant in the baseline model because the new equilibrium hours worked are lower.

In this section, I discuss policy implications of expenditure risks using two redistribution policies. The policies are intentionally designed with a simple structure to ensure clarity of the mechanism. Other redistribution policies with more realistic and complex structures may be also further explored. However, as long as the fundamental principle of redistributing from wealthy households to poor households is maintained, those policies would likely yield similar results to mine.

1.7 Conclusion

This paper examines the impact of expenditure risks on household consumption and savings, extending beyond the traditional focus on income risks. Using an analytical model, I show that expenditure shocks disproportionately affect low-wealth, low-income households, forcing them to reduce other consumption more significantly than wealthier households. Empirical evidence from the Panel Study of Income

Table 1.9: Welfare change: Income redistribution policy

		λ	
		baseline	extended
by z	lowest z	0.0946	0.0842
	low z	0.1920	0.1638
	mid z	0.0014	−0.0161
	high z	−0.1127	−0.0930
	highest z	−0.0402	−0.0345
	Agg λ	0.1351	0.1044

Notes: Each value represents the consumption equivalence of welfare changes between the two steady states, expressed as a percentage of consumption. The size of Δ is set to 1% of the steady-state output level.

Dynamics (PSID) supports these findings, revealing that resource-constrained households struggle to adjust to unexpected expenditures, unlike wealthier households who manage shocks with less impact on overall consumption.

Using a quantitative HANK model, I find that expenditure risks alter wealth distribution and marginal propensities to consume (MPCs). Higher expenditure risks result in a more fat-tailed wealth distribution, with more households becoming financially constrained near the borrowing limit, while others accumulate buffer assets at the upper end of the distribution. Policy simulations reveal that redistributive fiscal policies targeting liquidity-constrained households effectively boost demand in the short term but have limited long-term welfare benefits, as they require increased hours worked. By broadening the analysis to include expenditure risks, this paper provides a more comprehensive understanding of how households adjust their consumption behavior and its implications for policy.

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Appendix

1.A Details on Equation (1.10)

Assume that the expenditure shock hits at $t = 0$ and dissipates at $t = 1$. Examining the Euler equation for each good then sheds light on the optimal consumption decisions between $t = 0$ and $t = 1$. The Euler equations for good A and B are respectively

$$\theta_0 c_0^{\frac{1}{\eta}-\sigma} c_{A0}^{-\frac{1}{\eta}} = \beta(1+r)\theta_1 c_1^{\frac{1}{\eta}-\sigma} c_{A1}^{-\frac{1}{\eta}}$$

$$c_0^{\frac{1}{\eta}-\sigma} c_{B0}^{-\frac{1}{\eta}} = \beta(1+r) c_1^{\frac{1}{\eta}-\sigma} c_{B1}^{-\frac{1}{\eta}}.$$

Since $\beta(1+r) = 1$ and $\theta_1 = 1$, these simplify to

$$\frac{c_{A1}}{c_{A0}} = \frac{1}{\theta_0^{\eta}} \left(\frac{c_0}{c_1}\right)^{\eta\sigma-1},$$

$$\frac{c_{B1}}{c_{B0}} = \left(\frac{c_0}{c_1}\right)^{\eta\sigma-1}.$$

Additionally, if $\eta = \frac{1}{\sigma}$, these further reduce to

$$c_{A0} = \theta_0^{\eta} c_{A1}$$

$$c_{B0} = c_{B1},$$

where the last equation implies that the consumption of good B is adjusted only once at the time of the shock (i.e., $t = 0$), as shown in (1.10).

1.B Proof of Equation (1.11) and (1.12)

From (1.9) and (1.10), solving for x_{At}^{uncon} and x_{Bt}^{uncon}, respectively, gives

$$x_{At}^{\text{uncon}} = \left(\frac{\alpha}{1-\alpha}\right)\theta^{\eta}\left[(1-\alpha)c_0 + x_{Bt}^{\text{uncon}}\right] - \alpha c_0, \tag{1.31}$$

$$x_{Bt}^{\text{uncon}} = \frac{(1-\alpha)\left(\frac{r}{1+r}\right)\Phi_t - (1-\alpha)r x_{At}^{\text{uncon}} - (1-\alpha)c_0}{1+(1-\alpha)r}. \tag{1.32}$$

Substituting (1.32) into (1.31),

$$x_{At}^{\text{uncon}} = \left(\frac{\alpha}{1-\alpha}\right)\theta^{\eta}\left[(1-\alpha)c_0 + \underbrace{\frac{(1-\alpha)\left(\frac{r}{1+r}\right)\Phi_t - (1-\alpha)r x_{At}^{\text{uncon}} - (1-\alpha)c_0}{1+(1-\alpha)r}}_{=x_{Bt}^{\text{uncon}}}\right] - \alpha c_0.$$

For ease of notation, I define $\mathcal{M} \equiv (1-\alpha)r$. Multiplying both sides by $1+\mathcal{M}$ and solving for x_{At}^{uncon} yields

$$x_{At}^{\text{uncon}} = \frac{\left(\frac{\alpha}{1-\alpha}\right)\theta_t^{\eta}\left[(1-\alpha)c_0\mathcal{M} + \frac{\mathcal{M}\Phi_t}{1+r}\right] - (1+\mathcal{M})\alpha c_0}{1+\mathcal{M}+\left(\frac{\alpha}{1-\alpha}\right)\theta_t^{\eta}\mathcal{M}}, \tag{1.33}$$

which leads to

$$x_{Bt}^{\text{uncon}} = \frac{(1-\alpha)\left(\frac{r}{1+r}\right)\left[\Phi_t - (1+r)x_{At}^{\text{uncon}}\right] - (1-\alpha)c_0}{1+\mathcal{M}}. \tag{1.34}$$

For more simplification, notice that, since $y_t = y$,

$$\Phi_t = \sum_{t=0}^{\infty}\left(\frac{1}{1+r}\right)^t y_t = \left(\frac{1+r}{r}\right)y, \tag{1.35}$$

and by equation (1.7),

$$c_0 = y. \tag{1.36}$$

Also notice that

$$\frac{\mathcal{M}\Phi_t}{1+r} = \frac{(1-\alpha)r\left(\frac{1+r}{r}\right)y}{1+r} = (1-\alpha)y. \tag{1.37}$$

Using the conditions in equation (1.35), (1.36), and (1.37), the expression in (1.33) and (1.34) can be further reduced to

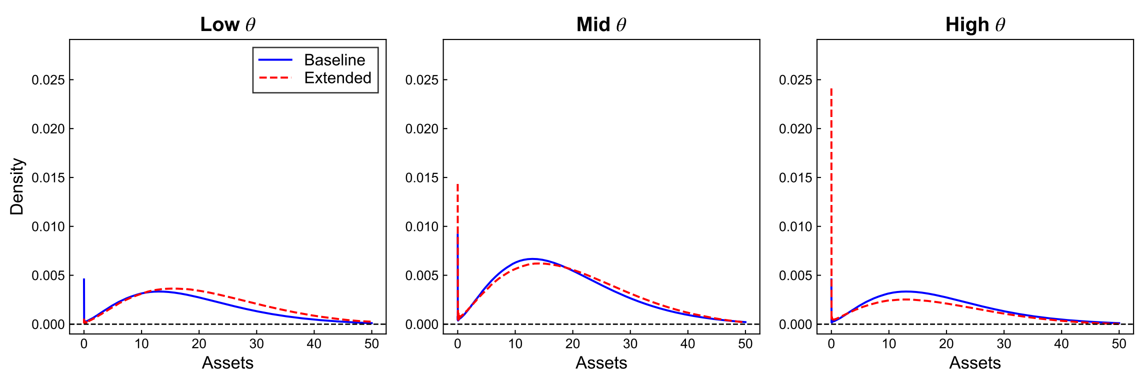
$$x_{At}^{\text{uncon}} = \frac{\alpha c_0(1+\mathcal{M})(\theta_t^\eta - 1)}{1+\mathcal{M}+\left(\frac{\alpha}{1-\alpha}\right)\theta_t^\eta\mathcal{M}}$$

$$x_{Bt}^{\text{uncon}} = -\left(\frac{\mathcal{M}}{1+\mathcal{M}}\right)x_{At}^{\text{uncon}},$$

which are (1.11) and (1.12), respectively.

1.C Supplementary Figures

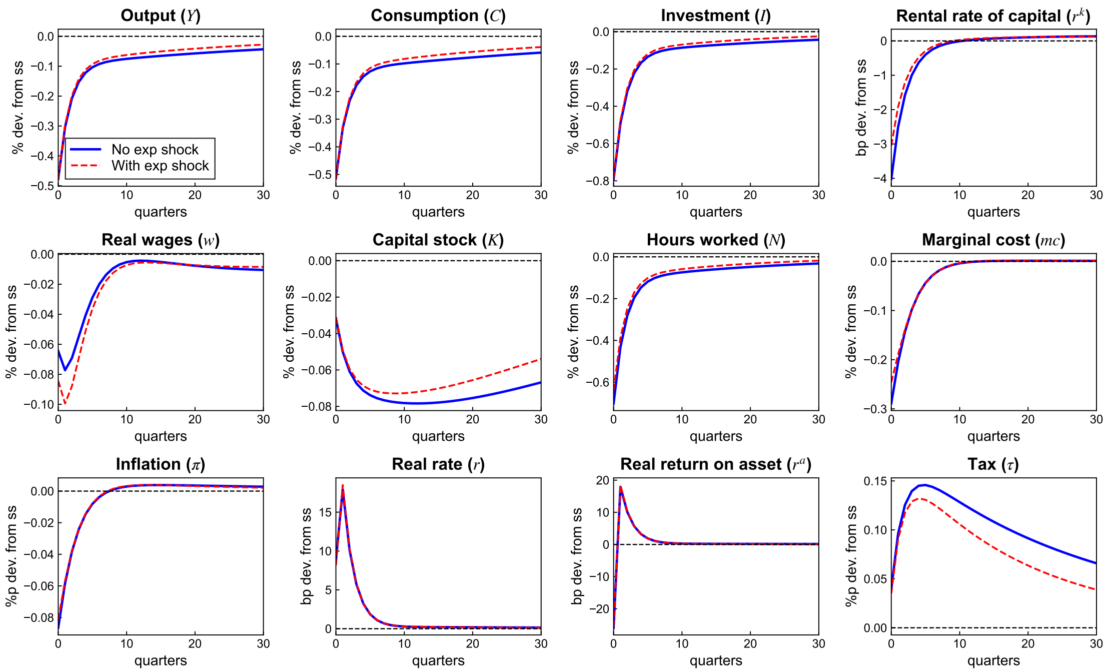
Figure B1: Marginal distribution of households, by θ



Note: The blue solid line denotes the distribution from the baseline model (w/o exp risk) while the red dashed line represents one from the extended model (w/. exp risk).

Figure B2: Impulse responses

(a) Monetary policy shock (+25bp)



(b) Government spending shock (1%)

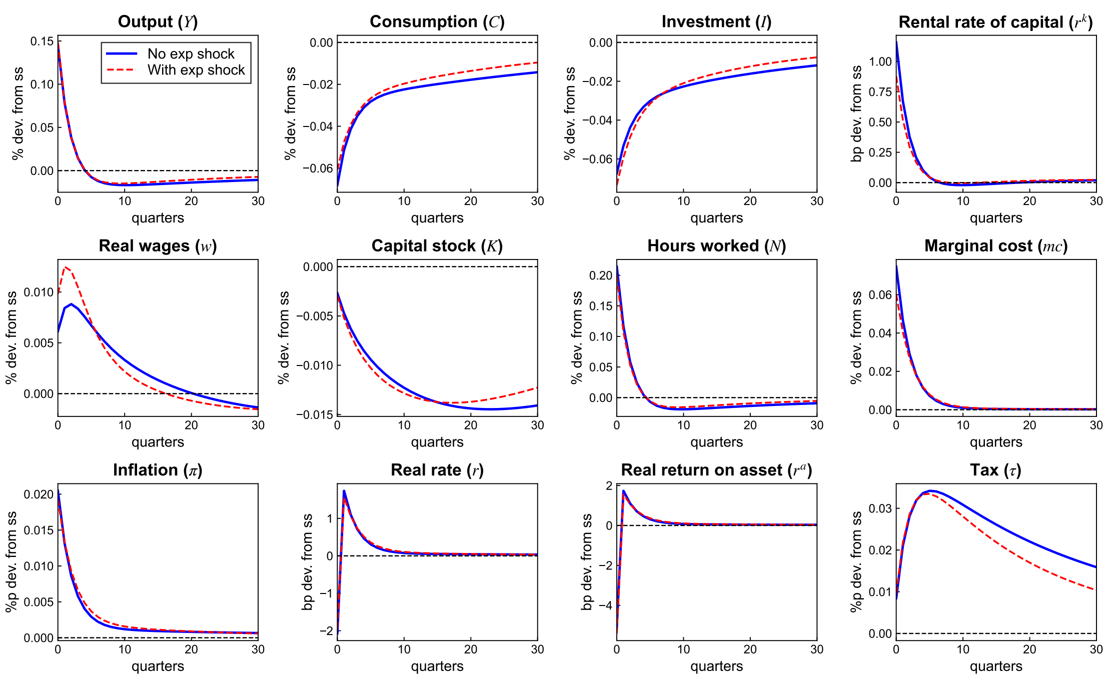
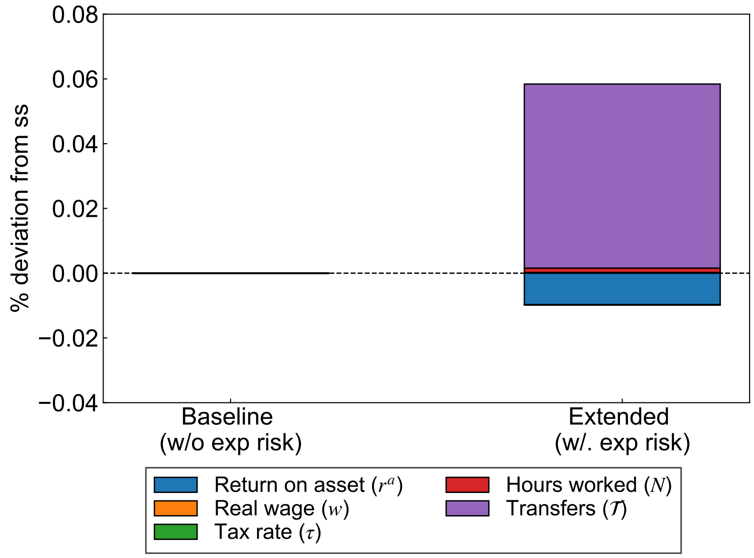
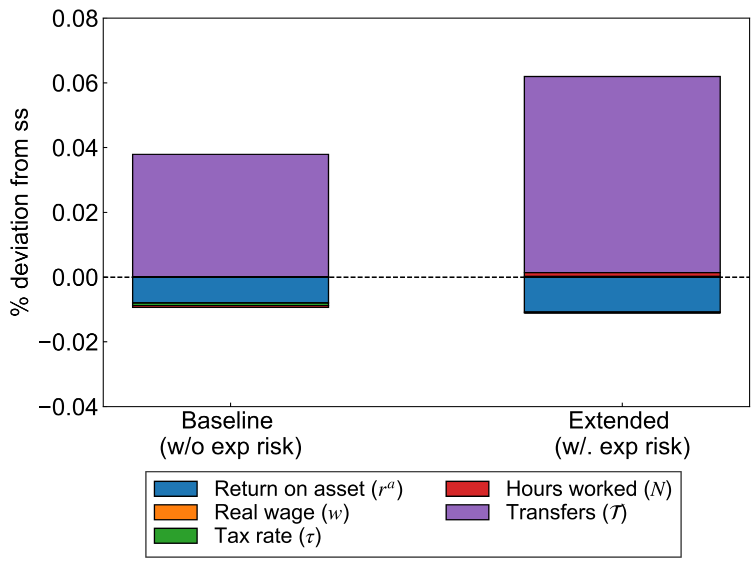


Figure B3: Decomposition of consumption

(a) Insurance policy

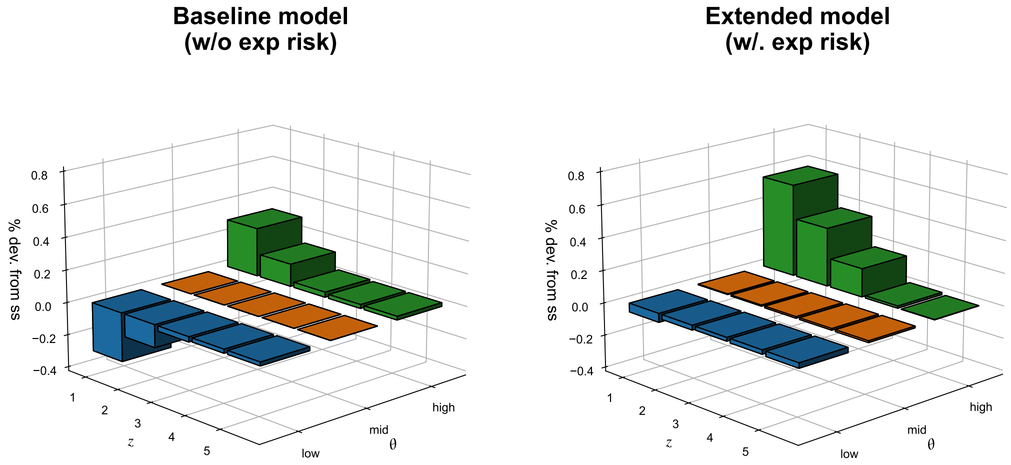


(b) Income redistribution policy



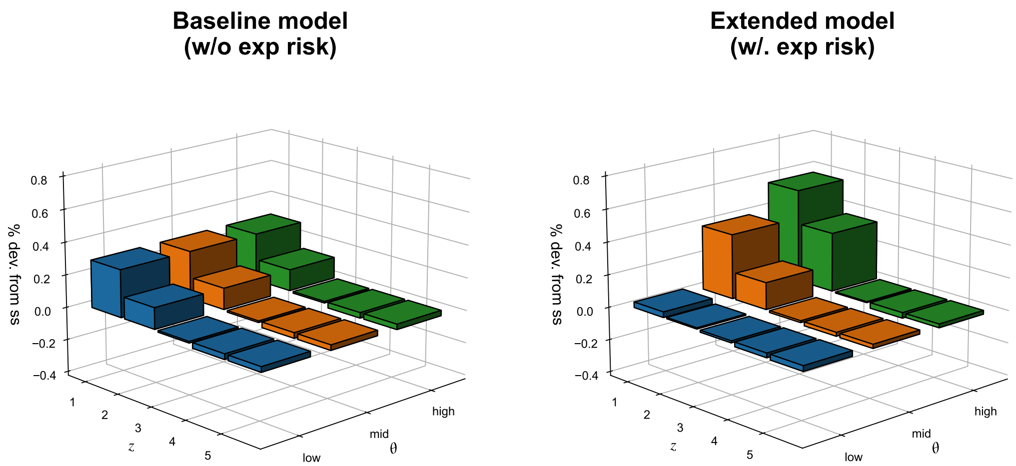
Note: Each bar represents the on-impact percentage deviation of consumption from the steady-state due to the change in the respective variable, while keeping the other variables constant at their steady-state values.

Figure B4: On-impact consumption responses: Insurance policy



Note: Each bar represents the average on-impact response of consumption of households in the respective income and expenditure state.

Figure B5: On-impact consumption responses: Income redistribution policy



Note: Each bar represents the average on-impact response of consumption of households in the respective income and expenditure state.

Table B1: Summary statistics of households by asset and income

	All	by wealth		by income	
		low	high	low	high
Age	45.6	43.6	47.5	45.2	45.9
Family size	3.0	2.9	3.1	2.8	3.3
Non-durables	15,573	11,883	19,267	11,001	20,150
consumption A, c_A	10,328	7,286	13,374	6,590	14,071
consumption B, c_B	5,245	4,597	5,893	4,411	6,079
Assets	135,200	18,771	251,781	44,491	226,028
liquid	30,798	2,235	59,399	6,096	55,532
illiquid	104,402	16,537	192,383	38,394	170,496
Income	42,502	27,296	57,727	21,973	63,057

Notes: All figures except for age and familiy size are in dollars and are deflated by the CPI index. Age is the average age of household head.

Chapter 2

Evaluation of the UK Furlough Scheme During the Pandemic Using a HANK Framework

2.1 Introduction

The COVID-19 pandemic had adverse impacts on national economies and businesses worldwide. As the governments struggled with lockdown measures to prevent the spread of the virus, firms faced challenges in maintaining their usual levels of production for goods and services. Moreover, a significant portion of consumers either temporarily or permanently lost their jobs, causing them to postpone their spending while waiting out the situation. As a result, since the onset of the pandemic, the majority of countries worldwide have witnessed substantial declines in their GDP and an increase in unemployment rates.

In response to the pandemic, most countries have introduced substantial stimulus policies, which include increased expenditure on goods and services as well as extensions to unemployment insurance (UI) benefits. The UK launched the Coronavirus

Job Retention Scheme (CJRS) in 2020 to protect households' employment stability and to sustain their income. The main feature of this scheme is that the government supports up to 80% of previous wages (capped at £2,500 per month) of affected employees who might otherwise have been laid off by employers. Despite the controversy over its extremely expensive cost, the scheme was implemented to slow down a sudden increase in unemployment and thereby prevent the economy from spiralling down into a more severe economic downturn.

The goal of this paper is to examine the effects of the pandemic on the UK economy and see how stimulative the furlough scheme was during the period. To this end, I construct a heterogeneous agent New Keynesian model. The model is also augmented with search and matching frictions, following Ravn and Sterk (2017b), to allow for the unemployment risks. As well is known, including employment status transitions among households enables the consideration of endogenous income risks. With these uninsurable unemployment risks in play, the model demonstrates increased precautionary savings, thereby leading to amplifying feedback loop effects following aggregate shocks. Moreover, similar to Kaplan et al. (2018), I adopt a two-asset structure in which each household has the option to invest either in illiquid assets or liquid assets. While investing in illiquid assets provides higher returns than investing in liquid assets, it also incurs adjustment costs. Integrating the two-asset portfolio framework allows the model to generate a realistic distribution of marginal propensity to consume (MPC) among households, a factor that bears relevance to the analysis of the distributive impacts of shocks and policies.

The most novel feature of this paper is the introduction of a new type of shock and a state that represents the pandemic and the subsequent large-scale (temporary) layoffs. These are termed in this paper as "inactivity shock" and "inactive state," respectively. In the model, households face this inactivity shock with a certain probability.[1] Once hit by the shock, the household's employment status is forcibly switched to the inactive state, which becomes the third employment status alongside the employed and unemployed state. Households in the inactive state cannot work, yet they are not completely laid off. The inclusion of these inactive workers aims to simulate households who have been temporarily furloughed by their employers and receive furlough scheme payments throughout the pandemic period. They are similar to unemployed households in the sense that they do not supply labor, but differ in that they have better prospects for future employment status. This will be discussed in more detail in the following section.[2] This modeling approach not only parsimoniously captures the UK economy's adoption of the furlough scheme during the pandemic but also addresses the interplay between the pandemic and the policy, ultimately yielding implications for the UK economy.

In order to understand to what extent the furlough scheme was successful in preventing a collapse of aggregate demand, it is important to understand what the inactivity shock and the furlough scheme mean to households. The inactivity shock places employed households in the inactive state, subject to an inevitable decline in their labor incomes. Consequently, higher inactivity risks induce households to adopt a more cautious approach towards saving. In response to this, the furlough scheme, which offers support amounting to 80% of households' previous labor income, mitigates the escalation of precautionary savings and prevents a drastic decline in aggregate demand.

To quantitatively demonstrate this, I begin by solving the model and investigating the impacts of the pandemic on the economy. As expected, the pandemic generates recessionary effects. The model reveals that both consumption and investment fall,

[1] I assume that the inactivity risk is given exogenously in the model. Obviously, it is the firm's decision whether or not to furlough its worker. However, to focus more on the demand side of the economy, I abstract from the model environment in which the transition from employment to the inactive is endogeneously determined in the model.

[2] It is worth noting that the concept of inactive households in this paper is similar to recalled workers in Fujita and Moscarini (2017a).

and output also declines, suggesting that the pandemic acts as a negative shock on both the demand and supply aspects of the economy. Next, I compare the general equilibrium responses of aggregate variables to an increase in inactivity risk under scenarios with and without the furlough scheme. I find that the furlough scheme is effective in mitigating the reduction in aggregate demand. A higher inactivity risk strengthens households' incentives for precautionary saving, leading to a reduction in consumption. The extent to which they cut their consumption is indeed larger in the absence of the furlough scheme, indicating that the policy functions as intended. However, the effectiveness of the policy is dampened by equilibrium effects stemming from the shock, such as an increase in the tax rates. The model predicts that the government increases income tax rates to a greater extent in the presence of the furlough scheme to finance the policy. This results in higher tax burdens placed on households, which in turn significantly counteracts the intended stimulating effects of the policy.

Using a HANK model allows me to explore another critical aspect of the policy: assessing the how the furlough scheme effectively redistribute consumption among households. To explore this question, I focus on heterogeneous impacts of the furlough scheme on consumption patterns of households with different employment status. This experiment predicts that the furlough scheme is an effective redistribution policy, which benefits those (potentially) affected by the pandemic, even though its impact on the aggregate level remains relatively unchanged.

Lastly, I conduct a counterfactual experiment to evaluate the model's ability to explain the real economy in the aftermath of the pandemic. Throughout the paper, my primary focus centers around the inactivity shock. However, in this experiment, I assume that the COVID-19 pandemic is characterized by two additional types of shocks along with the inactivity shock: preference shock, matching efficiency shock. The three shock processes are estimated from the model by exactly matching the targeted time series in the data. The estimated shock processes are then fed back into the model to generate counterfactual responses of macroeconomic variables. I find that the model overall fits the data observed reasonably well following the onset of the pandemic.

Related literature This paper relates to several strands of the literature. First, obviously, this paper belongs to the literature related to the COVID-19 epidemic and its economic implications. Overall, there are two main sub-branches of the literature. One of the branches exploits both SIR (epidemiological) and macroeconomic models to study the interactions between pandemic and macroeconomic variables depending on lockdown policies. Kaplan et al. (2020) combine a very rich heterogeneous-agent macro model with the COVID-19 infection dynamics and study the responses to various policies to contain the pandemic. Christiano et al. (2011) extend the canonical epidemiology model to show that there is a tradoff between the severity of recession and the spread of the virus.[3]

The other branch of literature explores the macroeconomic consequences of lockdown measures and how policy makers should react in order to mitigate the negative effects caused by the pandemic. Most of them have focused on the extensions of unemployment insurance benefits as a way to mitigate the effects of unemployment risk (Ravenna and Walsh (2022), Auray and Eyquem (2020b)). Unlike other literature, Gertler et al. (2022a) document the role of temporary layoffs in the business cycle using a structural model. They show that a surge of “loss-of-recall”, a phenomenon in which temporarily laid-off employees lost their job permanently, can enhance the destabilizing effect in the labor market. Although my work shares a common theme with this paper, the focus is different. They focus more on labor market dynamics such as stocks and flows for the labor market states while my focus is more on the

[3] See also Bradley et al. (2021), Glover et al. (2023), Krueger et al. (2022), and Piguillem and Shi (2022).

implications of the furlough scheme for aggregate consumption demand. In addtion to this, Dengler and Gehrke (2022) examine the effect of short term work in a New Keynesian model with matching and searching frictions. Bayer et al. (2023a) simulate the lockdowns in a heterogeneous agent New Keynesian model which features a quarantine state for households and quantify the effects of the US stimulus packages. They find that the quarantine shock generates a large recession and that conditional transfers is more effective than unconditional transfers in terms of transfer multiplier. Auray and Eyquem (2020a) model lockdown policies as a shock to the extensive margin of labor and argue that it gives rise to large and persistent depressive effects on output, and raises unemployment and precautionary savings. Aiyar and Dao (2021) find that Germany's short term work policy (Kurzarbeit) played a crucial role in mitigating unemployment and strengthening the demand.

Second, my paper also contributes to the literature on HANK & SAM. The related literature includes Challe (2020), Den Haan et al. (2018a), Gornemann et al. (2016b), Lee (2020), McKay and Reis (2021), and Ravn and Sterk (2017b). On top of the Diamond-Mortensen-Pissarides (DMP) search and matching frictions, my paper also features endogenous job separation as in Mortensen and Pissarides (1994) and Zanetti (2011). Especially, Zanetti (2011) studies the effects of labor market institutions, such as changes in firing costs and unemployment benefits, on macroeconomic stability, offering insights relevant to furlough schemes during crises. The search and matching framework used in this paper also relates to Fujita and Ramey (2012) and Broer et al. (2023).

Third, there exists an extensive body of empirical research that investigates the effectiveness of furlough scheme policies across many countries during the pandemic period. Bennedsen et al. (2020) use a Danish firm-level micro data to evaluate policies that financially aid firms. They empirically show that those policies were effective in preserving the matches between firms and households.[4] When it comes to studies involving quantitative models, the closest to mine is a paper by Mohimont et al. (2024). They employ a DSGE model with heterogeneous households categorized by employment status - employed, unemployed, and furloughed - and demonstrate that the Job Retention Scheme implemented across the euro area effectively prevented job destructions and benefited all households. While my paper shares many similarities with theirs, there are key distinctions. My study adopts a HANK framework, which features an infinite-dimensional distribution of wealth and income distribution of households. This allows for a richer analysis by quantifying the relative contributions of different channels to the overall results. Furthermore, while Mohimont et al. (2024) focuses on the welfare effects of job retention schemes, my paper emphasizes the heterogeneous impacts of the furlough scheme on consumption behaviors and highlights the general equilibrium feedback mechanisms that mitigate its effectiveness.

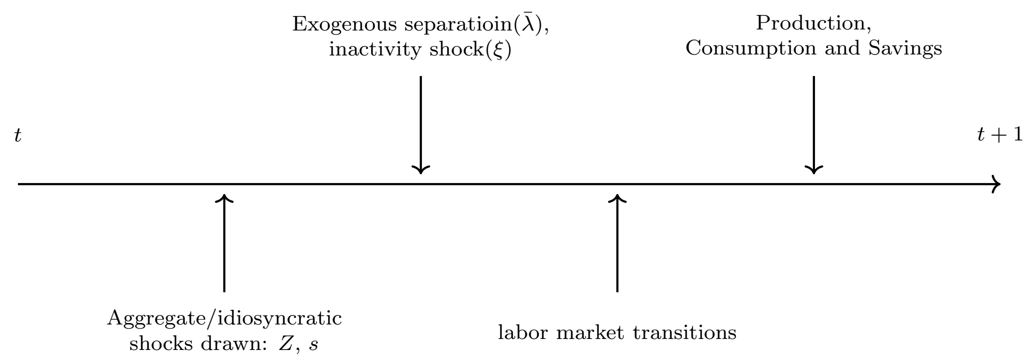
The remainder of this paper is organized as follows. Section 2.2 develops a heterogeneous agent New Keynesian model, and section 2.3 describes the calibration strategies for the steady-state. Section 2.4 explores the macroeconomic impacts of the pandemic predicted by the model and assesses the effect of the furlough scheme by comparing with a counterfactual case without the furlough scheme. It also provides detailed analysis on the transmission mechanisms of the shock to better understand the results. In addition, this section discusses the implications for redistribution among households. Section 2.5 presents a counterfactual experiment to examine how well the model captures the pandemic. Lastly, section 2.6 concludes.

2.2 Model

To study the economy in the COVID-19 pandemic situation, I build a dynamic model of heterogeneous households in a New Keynesian environment with incomplete

[4] See also Ando et al. (2022), Lam and Solovyeva (2023)

Figure 2.1: Timing of the model



markets and labor market frictions. The economy consists of households, firms, labor agencies, finanicial intermediary, and policies. The household sector is composed of households, who supply labor to firms and are subject to market incompleteness and searching and matching frictions. There are monopolistic competitive firms, subject to nominal rigidities, that produce differentiated outputs. Labor agencies hire labor from households and rent it out to the firms. The financial intermediary collects assets from households and invests them in government bonds and firm equity. It also transforms illiquid asset into liquid assets. Lastly, the central bank and the government constitute the policy sector. I extend Gornemann et al. (2016b) and Auclert et al. (2021) to establish the basic structure of model.

2.2.1 Timings

The timings of the model are presented in Figure 2.1. Aggregate shocks and shocks to households' productivity s_t are drawn at the beginning of a period and become known immediately. Then, exogenous inactivity and job separation shock arrive. Following this, labor agencies post vacancies, and matchings take place in the labor market, endogenously determining the job-finding rate, f_t, and the vacancy-filling rate, q_t, together with endogenous job separation rate, λ_t^n. Thereafter, firms produce and households make decisions on their cosnumption and savings.

2.2.2 Employment Status Transition

I assume that there are three types of households in terms of employment status, e_t; the employed ($e_t = 1$), unemployed ($e_t = 2$), and the inactive ($e_t = 3$). Given that there are multiple household types, it is essential to understand how the transitions between these statuses take place.

Figure 2.2: Employment status transition

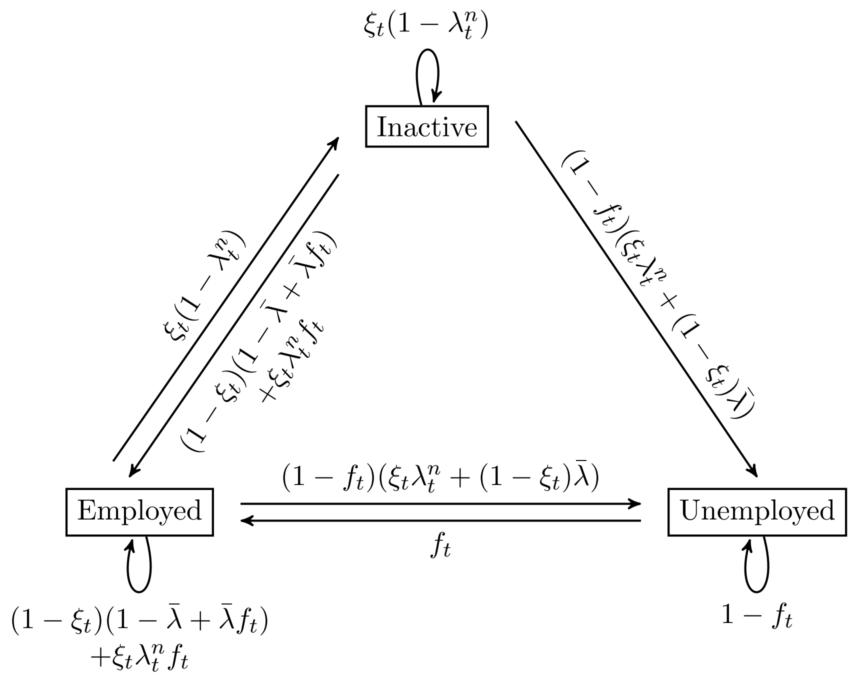


Figure 2.2 provides an overview of the employment status transition process.[5] Initially, households face the inactivity shocks, denoted by ξ_t. Additionally, they experience exogenous job separation shock, $\bar{\lambda}$. Furthermore, they are exposed to other risks that are determined endogeneously in the labor market, such as the job-finding rate, f_t, and endogenous job separation rate, λ_t^n. Once households have information about all of these risks, they make their decisions, taking them as given.

[5] An alternative representation of the employment transition is shown in D.1

The inactivity shock is assumed to consist of two components,

$$\xi_t = \bar{\xi} + \xi_t^*. \tag{2.1}$$

$\bar{\xi}$ is a fixed component of the probability of becoming inactive which ensures a certain population share for inactive households in the steady state.[6] ξ_t^* is an exogenous shock component. It is assumed to be zero in the steady-state, and turns positive when the lockdowns measures come into effect.

In the beginning of a period, employed and inactive households face both exogenous separation ($\bar{\lambda}$) and inactivity shocks (ξ_t) simultaneously.[7] If hit by the inactivity shock, the household faces subsequent endogenous separation risks in which the match with their employer is completely destroyed with a probability λ_t^n. In cases where the match survives endogenous separation, the household transitions to the inactive state. Households separated from their job—whether due to exogenous or endogenous separation—begin searching for new jobs within the same period. If their search is successful, they become employed; otherwise, they remain unemployed. I assume that those entering the inactive state cannot search for employment but have to wait until the current period ends.

Hence, in summary, employed households stay employed in the next period with a probability of $(1-\xi_{t+1})(1-\bar{\lambda}+\bar{\lambda}f_{t+1})+\xi_{t+1}\lambda_{t+1}^n f_{t+1}$, and become unemployed with a probability of $[\xi_{t+1}\lambda_{t+1}^n + (1-\xi_{t+1})\bar{\lambda}](1-f_{t+1})$. They can also become inactive with a probability of $\xi_{t+1}(1-\lambda_{t+1}^n)$. Inactive households face the same probabilities as the employed households for the future employment transitions. Unemployed households, on the other hand, become employed with probability f_{t+1}, or remain unemployed with the complement probability, $1-f_{t+1}$. Note that unemployed households cannot

[6] If, for example, the steady-state population share for inactive households is zero, $\bar{\xi} = 0$.

[7] For simplicity I assume that there is no correlation between the two shocks.

directly transition to the inactive state because they are not subject to the inactivity shock.

2.2.3 Households

All households maximise the expected present value of their time-separable utilities. For the form of utility function, I assume the CRRA utility function such that

$$u(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma}.$$

Each faces uninsured idiosyncratic income risk and search and matching frictions. I assume that households can save in two assets and made decision on how much to consume and save every period. The Bellman equation of a productivity type s_t employed household ($e_t = 1$) with a_{t-1} and b_{t-1} is given by

$$\begin{aligned} V_t\left(1, s_t, b_{t-1}, a_{t-1}\right) = \max_{c_t, b_t, a_t} \Big\{ & u(c_t) + \beta \mathbb{E}_t \Big[\left[\left(1-\xi_{t+1}\right)\left(1-\bar{\lambda}+\bar{\lambda} f_{t+1}\right) + \xi_{t+1}\lambda_{t+1}^n f_{t+1} \right] V_{t+1}\left(1, s_{t+1}, b_t, a_t\right) \\ & + \left[\xi_{t+1}\lambda_{t+1}^n \left(1-f_{t+1}\right) + \left(1-\xi_{t+1}\right)\bar{\lambda}\left(1-f_{t+1}\right) \right] V_{t+1}\left(2, s_{t+1}, b_t, a_t\right) \\ & + \xi_{t+1}\left(1-\lambda_{t+1}^n\right) V_{t+1}\left(3, s_{t+1}, b_t, a_t\right) \Big] \Big\}. \end{aligned} \tag{2.2}$$

Assets a_t are illiquid in the sense that households need to pay a cost for adjustment. β denotes the time discounting factor, f_t is the job finding rate, $\bar{\lambda}$ and λ_t^n represent the exogenous, endogenous job separation rate, respectively. The employed household optimizes subject to the following budget constraint,

$$c_t + b_t + a_t = (1-\tau_t) w_t^H s_t + (1+r_t^b) b_{t-1} + (1+r_t^a) a_{t-1} - \psi_t(a_t, a_{t-1}), \tag{2.3}$$

$$b_t \geq \underline{b}, \qquad a_t \geq 0, \tag{2.4}$$

where τ_t is the labor income tax rate imposed by the government, w_t^H is the real wage, r_t^b and r_t^a are the ex-post real returns on liquid and illiquid asset, respectively. The employed household uses its resources for consumption c_t, and for purchasing liquid asset b_t and illiquid asset a_t that it carries into the next period. The right hand side of the budget constraint describes the resources of the household. The household's resources are the sum of after-tax labor income, $(1-\tau_t)w_t^H s_t$, and returns from the rental of liquid and illiquid asset of the current period, $(1+r_t^b)b_{t-1}+(1+r_t^a)a_{t-1}$, net of adjustment costs for illiquid asset, $\psi_t(a_t, a_{t-1})$. Following Auclert et al. (2021), the functional form for the adjustment cost function, $\psi_t(a_t, a_{t-1})$, is given by

$$\psi_t(a_t, a_{t-1}) = \frac{\chi_1}{\chi_2}\left|\frac{a_t-(1+r_t^a)\,a_{t-1}}{(1+r_t^a)a_{t-1}+\chi_0}\right|^{\chi_2}\left[(1+r_t^a)a_{t-1}+\chi_0\right], \tag{2.5}$$

where $\chi_0,\ \chi_1>0,\ \chi_2>1$.[8]

The Bellman equation for a type s_t unemployed household ($e_t = 2$) is

$$V_t\left(2, s_t, b_{t-1}, a_{t-1}\right) = \max_{c_t, b_t, a_t}\left\{u(c_t)+\beta\mathbb{E}_t\Big[f_{t+1}\,V_{t+1}\left(1, s_{t+1}, b_t, a_t\right)+(1-f_{t+1})\,V_{t+1}\left(2, s_{t+1}, b_t, a_t\right)\Big]\right\}. \tag{2.6}$$

The unemployed household is subject to the same budget constraint as the employed household except that it receives the UI benefits from the government instead of labor income. That is,

$$c_t+b_t+a_t = (1-\tau_t)\nu w_t^H s_t+(1+r_t^b)b_{t-1}+(1+r_t^a)a_{t-1}-\psi_t(a_t, a_{t-1}) \tag{2.7}$$

[8] χ_1 is the overall scaling factor for the adjustment cost, and it reflects the importance of the adjustment cost when households optimise. Therefore, as χ_1 is higher, households would want to save more (i.e. exhibit a stronger precautionary-saving motive) because they recognise that they will have to pay more costs to liquidate their illiquid assets if χ_1 is high. When parameter χ_2 is higher than one, rich households (with high $(1+r_t^a)a_{t-1}$) see that the adjustment costs become less expensive. Hence, parameter χ_2 is useful to make rich and poor households face different degrees of financial frictions

where ν is the replacement rate of unemployment benefits. I assume that t the unemployment benefits depend on the household's productivity to reflect, in a parsimonious way, that these benefits are based on past earnings.

An inactive household faces the same problem as the employed except for the different labor income in the budget constraint.

$$V_t\left(3, s_t, b_{t-1}, a_{t-1}\right) = \max_{c_t, b_t, a_t} \Big\{ u(c_t) + \beta \mathbb{E}_t \Big[\left[(1-\xi_{t+1}) \left(1-\bar{\lambda}+\bar{\lambda} f_{t+1}\right) + \xi_{t+1} \lambda^n_{t+1} f_{t+1} \right] V_{t+1}\left(1, s_{t+1}, b_t, a_t\right)$$
$$+ \left[\xi_{t+1} \lambda^n_{t+1} \left(1-f_{t+1}\right) + \left(1-\xi_{t+1}\right) \bar{\lambda} \left(1-f_{t+1}\right) \right] V_{t+1}\left(2, s_{t+1}, b_t, a_t\right)$$
$$+ \xi_{t+1} \left(1-\lambda^n_{t+1}\right) V_{t+1}\left(3, s_{t+1}, b_t, a_t\right) \Big] \Big\}. \tag{2.8}$$

$$\text{s.t. } c_t + b_t + a_t = \text{income}_t + (1+r^b_t) b_{t-1} + (1+r^a_t) a_{t-1} - \psi_t(a_t, a_{t-1}). \tag{2.9}$$

$$\text{income}_t = \begin{cases} (1-\tau_t)\left[\nu + \xi^*_t(\varphi-\nu)\right] w^H_t s_t & \text{with the furlough scheme} \\ (1-\tau_t)\nu w^H_t s_t & \text{without the furlough scheme,} \end{cases} \tag{2.10}$$

where $\varphi > \nu$.

I assume that inactive households receive the same unemployment benefits as unemployed households in the steady state. However, when an inactivity shock occurs (i.e. $\xi_t > \bar{\xi}$), they are granted additional benefits of $(\varphi - \nu)$ under the furlough scheme. In the absence of the furlough scheme, they continue to receive the same unemployment benefits as unemployed households, without any additional payments in the inactive state.[9]

[9] This way of assumption is necessary to ensure that the steady states remain identical between scenarios with and without the furlough scheme, even when the population share of inactive households is nonzero. Otherwise, the model would produce two different steady states, making proper comparisons between the scenarios impossible.

2.2.4 Financial Intermediary

I closely follow Auclert et al. (2021) in modeling the financial intermediary block. The financial intermediary in this paper can be interpreted as mutual funds in other literature. It collects illiquid assets from households and invests them in government bonds B^g and firm equity p_t^E. In addition to this activity, it engages in another activity of transforming the collected illiquid assets into liquid assets at a proportional cost ω.

The financial intermediary's problem is to maximize the expected real return on illiquid assets, $E_t[r_{t+1}^a]$, subject to two budget constraints. At the beginning of the period, the value of outstanding illiquid and liquid liabilities must be equal to the sum of the values of government bonds and equity and dividends that the financial intermediary holds, net of the intermediation cost of liquid deposits. Thus, the first budget constraint must be

$$(1 + r_t^a)A_{t-1} + (1 + r_t^b)B_{t-1} + \omega B_{t-1} = (1 + r_t)B_t^g + p_t^E + d_t. \tag{2.11}$$

At the end of the period, the value of newly purchased government bonds and firm equity must be equal to the value of newly issued liquid and illiquid liabilities, so the other budget constraint is[10]

$$A_t + B_t = p_t^E + B_t^g. \tag{2.12}$$

The financial intermediary solves the following problem.

$$\max \mathbb{E}_t \left[1 + r_{t+1}^a\right] = \mathbb{E}_t \left[\frac{(1 + r_{t+1})B_t^g + p_{t+1}^E + d_{t+1} - (1 + r_{t+1}^b + \omega)B_t}{p_t^E + B_t^g - B_t}\right]. \tag{2.13}$$

[10] This is also equivalent to the asset market clearing condition.

The first-order conditions lead to

$$\mathbb{E}_t\left[1+r_{t+1}\right]=\frac{\mathbb{E}_t\left[d_{t+1}+p_{t+1}^E\right]}{p_t}=\mathbb{E}_t\left[1+r_{t+1}^a\right]=\mathbb{E}_t\left[1+r_{t+1}^b\right]+\omega. \tag{2.14}$$

The first equation implies that the ex-ante real return $\mathbb{E}_t\left[1+r_{t+1}\right]$ equals the expected real returns on equity (i.e. no-arbitrage condition). Also, the last equation shows that the competitive intermediary fully passes through the cost of transformation to a lower deposit interest rate.

For simplicity I further assume that the real return on illiquid asset is determined as a weighted average of returns on steady state equity and bonds share. The specific form is given as

$$1+r_t^a=\frac{p^E}{A}\left(\frac{d_t+p_t^E}{p_{t-1}^E}\right)+\left(\frac{B_t^g-B}{A}\right)(1+r_t), \tag{2.15}$$

where $\frac{p^E}{A}$ denotes the steady-state share of equity holdings to the aggregate illiquid asset holdings.[11]

2.2.5 Firms

Let there be identical firms that produce differentiated goods using actively-employed labor n_t^E and capital k_t. Each producer owns capital and hires labor from the labor agencies. I assume that the firm is subject to nominal rigidities, and quadratic adjustment costs for price and capital as in Rotemberg (1982). The firm sets the price p_t to maximise its value in the following problem,

$$J_t^F(k_{t-1})=\max_{n_t,p_t,k_t}\left\{\frac{p_t}{P_t}y_t-w_t^A n_t^E-i_t-\frac{\mu_p}{\mu_p-1}\frac{1}{2\kappa_p}\left[\log(1+\pi_t)\right]^2Y_t-\frac{1}{2\delta\epsilon_I}\left(\frac{i_t}{k_{t-1}}-\delta\right)^2k_{t-1}+\frac{J_{t+1}^F(k_t)}{1+r_{t+1}}\right\},$$

[11] This assumption is to account for surprise inflation and capital gains. When an unexpected shock hits, the no-arbitrage condition fails to hold for one period because of surprise inflation and capital gains. However, this weighted average assumption for r_t^a holds in all periods with and without unexpected shocks.

$$\text{s.t. } y_t = F(k_{t-1}, n_t^E) = Z_t k_{t-1}^{\alpha} \left(n_t^E\right)^{1-\alpha},$$

$$y_t = \left(\frac{p_t}{P_t}\right)^{-\frac{\mu_p}{\mu_p - 1}} Y_t. \tag{2.16}$$

μ_P is the steady-state markup. z_t denotes total factor productivity (TFP). The production function reflects that only those who survive the inactivity shock are practically used as a labor input for production in the presence of the inactivity shock. In aggregate, employed households consist of the previously employed or inactive who are hit neither by an inactivity shock nor an exogenous separation shock and those who are newly employed in the labor market. Therefore, the law of motion for employment is $N_t^E = (1 - \xi_t)(1 - \bar{\lambda})N_{t-1} + M_t$. ϵ_I indexes the extent of nominal rigidities for adjusting capital stock. Firms transfer their profits to equity which is, in turn, distributed to households in the form of returns to illiquid asset by the financial intermediary.

In order to meet demand in (2.16), firms rent labor at a competitive rate w_t^A and make investment decisions.[12] In equilibrium, all firms face the same marginal costs and set the same price and choose the same amount of labor and capital inputs, so that k_t and n_t will be identical for all firms.[13]

[12] Investment follows the standard Tobin's q-theory.

[13] The above problem leads to derive the following aggregate equilibrium conditions.

- Phillips curve

$$\log(1 + \pi_t) = \kappa_p \left(mc_t - \frac{1}{\mu_p}\right) + \mathbb{E}_t \left[\frac{1}{1 + r_{t+1}} \frac{Y_{t+1}}{Y_t} \log(1 + \pi_{t+1})\right]$$

- Valuation

$$(1 + r_{t+1})Q_t = \alpha \frac{Y_{t+1}}{K_t} mc_{t+1} - \left[\frac{K_{t+1}}{K_t} - (1 - \delta) + \frac{1}{2\delta\epsilon_I}\left(\frac{K_{t+1} - K_t}{K_t}\right)^2\right] + \frac{K_{t+1}}{K_t} Q_{t+1}.$$

The left-hand side describes the gross return on investment Q_t in period t evaluated at $t + 1$, and the right-hand side can be interpreted as its (opportunity) costs. In the equilibrium the both sides must be the same.

- Investment

$$Q_t = 1 + \frac{1}{\delta\epsilon_I}\left(\frac{K_t - K_{t-1}}{K_{t-1}}\right).$$

2.2.6 Labor Agencies

I extend the labor agency structure in Gornemann et al. (2016a) by incorporating endogenous job separation. There exist labor agencies that rent out homogeneous labor services to firms. Labor agencies are either matched or not matched, or inactive. Only a matched agency just hit by the inactivity shock faces a stochastic operation cost, x_t.[14] The operation cost is independently and identically distributed according to an exponential distribution, $x_t \overset{iid}{\sim} Exp(\vartheta)$ with CDF $G(.)$. In other words, once a matched labor agency is affected by an inactivity shock, it draws the cost and decides whether to continue or destroy the match, observing the cost.

If the labor agency chooses to break up the match, it immediately becomes unmatched and earns no profits until it is matched again with a new household in the next period, but in that case, the agency is no longer subject to the operation cost. This endogenous separation decision happens with probability λ_t^n. With the complement probability, $1 - \lambda_t^n$, it chooses to continue (or equivalently, it chooses to stay inactive), and pays the cost, x_t, makes no profits in the current period, but can save hiring costs if it escapes from the inactive state in the next period.

Therefore, the value of an inactive agency is

$$J_t^{L,I}(s_t) = \mathbb{E}_t \left[\int^{\bar{x}_t} \max \left\{ \Lambda \left[(1 - \xi_{t+1})(1 - \bar{\lambda}) J_{t+1}^{L,M}(s_{t+1}) + \xi_{t+1}(1 - \lambda_{t+1}^n) J_{t+1}^{L,I}(s_{t+1}) \right] - x_t, 0 \right\} dG(x) \right] \tag{2.17}$$

where $J_t^{L,M}$ denotes the value function of a matched labor agency, and Λ is the discount factor for labor agencies. Note that the value is the same as the expected

- Law of motion for capital

$$I_t = K_t - (1 - \delta) K_{t-1}.$$

- Production

$$Y_t = Z_t K_{t-1}^{\alpha} \left(N_t^E \right)^{1-\alpha}.$$

[14] The operation costs may include costs for employers to maintain connections with their furloughed employees and all relevant administrative expenses even when they are not operating. I assume that this operation cost is a utility cost, thus it does not enter into the aggregate resource constraint.

continuation value of staying inactive in $t+1$ net of the expected operation cost conditional on the cost being less than the expected value of staying inactive. The inactive labor agency may become active and operate as a matched agency with probability $(1-\xi_{t+1})(1-\bar{\lambda})$ or stay inactive if it is hit by the shock again in the next period, but choose to continue the match with probability $\xi_{t+1}(1-\lambda^n_{t+1})$. The right hand side of (2.17) can be analytically rewritten as

$$\left(\Lambda \mathbb{E}_t\left[(1-\xi_{t+1})(1-\bar{\lambda})J^{L,M}_{t+1}(s_{t+1})+\xi(1-\lambda^n_{t+1})J^{L,I}_{t+1}(s_{t+1})\right]-\mathbb{E}_x\left[x \mid x\leq \bar{x}_t\right]\right)G(\bar{x}_t) \tag{2.18}$$

$$\text{where } \mathbb{E}_x\left[x \mid x\leq \bar{x}_t\right]=\vartheta\left[\frac{1-(\frac{\bar{x}_t}{\vartheta}+1)e^{-\frac{\bar{x}_t}{\vartheta}}}{1-e^{-\frac{\bar{x}_t}{\vartheta}}}\right]. \tag{2.19}$$

This also implies that there exists a threshold value, $\bar{x}_t$, above which a labor agency who has been just hit by an inactivity shock will decide to break up the match. It then can be inferred from this that the endogenous separation rate, λ^n_t, is determined as

$$\lambda^n_t = 1-G(\bar{x}_t). \tag{2.20}$$

The value function of a matched labor agency is

$$J^{L,M}_t(s_t)=(w^A_t-w^H_t)s_t+\Lambda \mathbb{E}_t\left[(1-\xi_{t+1})(1-\bar{\lambda})J^{L,M}_{t+1}(s_{t+1})+\xi_{t+1}(1-\lambda^n_{t+1})J^{L,I}_{t+1}(s_{t+1})\right]. \tag{2.21}$$

The matched labor agency produces an amount s of labor services, which are sold to firms at rate w^A_t and pays the matched household real wage w^H_t. The continuation value reflects the fact that a match between the agency and a household continues in the next period with probability $(1-\xi_{t+1})(1-\bar{\lambda})$ (i.e. if the household is hit neither by the inactivity shock nor the exogenous job separation shock) and that the match becomes inactive with probability $\xi_t(1-\lambda^n_{t+1})$. I assume that the value of an agency not matched with a household or whose match is destroyed by the job-separation shocks is zero.

The equilibrium real wage is not uniquely determined in models with search and matching frictions. Following Gornemann et al. (2016a), I assume that wages are determined by an exogenous rule,

$$\log\, w_t^H = \log\, \bar{w}^H + \epsilon_w \left[\log \left(\frac{w_t^A}{\bar{w}^A} \right) \right], \tag{2.22}$$

where $\bar{w}^H$ is the steady-state wage level. ϵ_w is the elasticity of the wage with respect to the competitive rental rate of labor. $\epsilon_w \in [0,1]$ implies that there is stickiness in wages and plays a role in amplifying labor market fluctuations.

Agencies that are not matched with a household post vacancies with unit cost κ according to the following free-entry condition

$$\kappa = \frac{M_t(S_t, V_t)}{V_t} \int J_t^{L,M}(s_t) dD_t. \tag{2.23}$$

The agencies post vacancies up to the point where the cost of posting a vacancy (left-hand side) equals the expected gain (right-hand side). Note that $M_t(S_t, V_t)$ denotes the aggregate measure of matches and V_t represents the aggregate measure of vacancies posted. Matches between workers and firms are formed according to the following standard Cobb-Douglas matching function

$$M_t(S_t, V_t) = \mu S_t^{\gamma} V_t^{1-\gamma}. \tag{2.24}$$

$S_t = 1 - N_{t-1} + \left[(1-\xi_t)\bar{\lambda} + \xi_t \lambda_t^n \right] N_{t-1}$ is the measure of households searching for a new job in the labor market. Note that searchers consist of those previously unemployed and those who were not unemployed but are either exogenously or endogenously separated from job. μ is the matching efficiency factor. It follows that the job-finding rate, f_t, and vacancy-filling rate, q_t, are determined by

$$f_t = \mu \left(\frac{V_t}{S_t}\right)^{1-\gamma}, \tag{2.25}$$

$$q_t = \mu \left(\frac{V_t}{S_t}\right)^{\gamma}. \tag{2.26}$$

2.2.7 Policies

Central Bank

The assumption for the central bank is standard. It implements monetary policy according to

$$i_t^R = \bar{r} + \phi_\pi \pi_t + \phi_Y (Y_t - \bar{Y}), \qquad \phi_\Pi > 1,\ \phi_Y \geq 0. \tag{2.27}$$

ϕ_Π and ϕ_Y are the responsiveness of the nominal rate to inflation and output, respectively. That is, each governs the extent to which the central bank tries to stabilise inflation and output gap. $\bar{r}$ is the (ex-post) real rate targeted by the central bank and is assumed to be the same as the real return r_t in the steady-state.

Government

The government collects taxes and pays the UI benefits to the unemployed and the furlough payments to the inactive. It also issues bonds, B^g, and chooses tax rates to finance its expenditure. I assume that the government spending G is held constant over time. In more detail, the government follows a simple rule for the tax rates:

$$\tau_t = \bar{\tau} + \eta \left(B_t^g - \bar{B}^g\right). \tag{2.28}$$

The parameter η governs the speed at which the government adjusts its tax in response to the deviation of government debt from its steay-state level. It then follows that the government budget constraint is determined as

$$B_t^g + TR_t = (1 + r_t)B_{t-1}^g + T_t + G, \tag{2.29}$$

where

$$TR_t = \begin{cases} \tau_t \left(w_t^H N_t^E + \nu w_t^H u_t + \left[\nu + \xi_t^* (\varphi - \nu)\right] w_t^H N_t^I \right) & \text{with furlough} \\ \tau_t \left(w_t^H N_t^E + \nu w_t^H \left(u_t + N_t^I \right) \right) & \text{without furlough,} \end{cases} \tag{2.30}$$

$$T_t = \begin{cases} \nu w_t^H u_t + \left[\nu + \xi_t^* (\varphi - \nu)\right] w_t^H N_t^I & \text{with furlough} \\ \nu w_t^H \left(u_t + N_t^I \right) & \text{without furlough.} \end{cases} \tag{2.31}$$

N_t^E, N_t^I are the population share for the employed, inactive households, respectively. The share for the unemployed is determined residually as $u_t = 1 - N_t$ where $N_t = N_t^E + N_t^I$.

2.2.8 Equilibrium

A recursive equilibrium is a set of decision rules of households and firms $\{c_t, b_t, a_t, n_t, k_t, i_t\}$, labor market variables $\{f_t, q_t, V_t, \lambda_t^n\}$, input prices for labor $\{w_t^A, w_t^H\}$[15], returns on liquid and illiquid assets $\{r_t^b, r_t^a, r_t\}$, prices $\{P_t, \mathit{\Pi}_t\}$, dividends $\{d_t\}$, values functions $\{V_t,\ J_t^F,\ J_t^{L,M},\ J_t^{L,I}\}$, policies $\{i_t^R, \tau_t, B_t^g\}$, and the distribution of households $\{D_t\}$ such that:

- Given the prices, policies, aggregate states and labor market variables, the decision rules $\{c_t, b_t, a_t\}$ solve the household's problem.
- Given the prices, policies and aggregate states, the decision rules $\{n_t, p_t, i_t\}$ solve the firm's problem.

[15]Since I assume that firms own capital, the input price for capital does not exist in the model.

- Given w_t^A and w_t^H and labor market variables, $J_t^{L,M}$, $J_t^{L,I}$ are the solutions to the problem of labor agencies.

- Aggregate dividends are given by $d_t = Y_t - w_t^A N_t^E - I_t - \Psi^K - \Psi^P + (w_t^A - w_t^H)N_t^E - \kappa V_t$.

- All markets clear.

  - Assets market:

$$A_t + B_t = p_t^E + B^g \tag{2.32}$$

  - Goods market:

$$Y_t = C_t + G_t + I_t + \omega B_{t-1} + \kappa V_t + \Psi_t^K + \Psi_t^p + \Psi_t^A \tag{2.33}$$

  - labor market:

$$N_t^E + N_t^I + u_t = 1 \tag{2.34}$$

- The government follows its fiscal rule and the nominal interest, i_t^R, is given by the monetary policy rule.

2.3 Calibrations

I calibrate the model to the UK economy, using averages over the 2001–2019 period for all targets. The unit of time period in the model is a quarter. Below I provide further details on the calibration.

2.3.1 Households

For parameters in the household problem, I calibrate β such that the annual steady-state real interest rate is $r = 5\%$. I set $\chi_0 = 0.25$, $\chi_2 = 2$ and calibrate the value of χ_1

such that liquid asset savings is 1.04 times the output following Auclert et al. (2021), that is, $B = 1.04Y$.[16] Since I assume that households cannot borrow, the lower limit for the liquid asset is $\underline{b} = 0$. Table 2.1 summarises the calibration for households. For the income process I set three discrete productivity levels, with s_1 being the lowest productivity level and s_3 being the highest. Since transitions in productivity are not affected by the business cycle or employment status, I simply assume that the process of transitions between different productivity levels are governed by AR(1) process, following the Rouwenhorst method. Table 2.2 reports the resulting productivities and their transition matrix used in the model.

Table 2.1: Calibration: Households

Parameter	Description	Value	Target / Source
β	Discout factor	0.978	$r = 0.0125$
σ	Inverse IES	2	standard
χ_0	Portfolio adjustment: cost pivot	0.25	
χ_1	Portfolio adjustment: cost scale	7.52	$B = 1.04Y$
χ_2	Portfolio adjustment: cost curvature	2	
r^a	Real return on illiquid assets	0.0125	annual 5%
r^b	Real return on liquid assets	0.0075	annual 3%
$\underline{b}$	Borrowing limit	0	standard
ρ_e	Productivity persistence	0.966	

Table 2.2: Household productivities (left) and transition matrix (right)

	Values
s_1	0.1832
s_2	0.6728
s_2	0.6728

		Tomorrow		
		s_1	s_2	s_3
Today	s_1	0.9663	0.0334	0.0003
	s_2	0.0167	0.9666	0.0167
	s_3	0.0003	0.0334	0.9663

[16] For simplicity I normalise the steady-state output to 1 (i.e. $Y = 1$).

2.3.2 Labor Market

Table 2.3 reports the parameters for the labor market. I set the probability of exogenous job separation rate at 1.2%, in line with the UK data. For the job separation rate, I use the employment-to-unemployment rate provided by the UK labor Force Survey. The data do not contain information on the difference between endogenous and exogenous separation rates. For simplicity I simply assume that the probability of endogenous separation, conditional on being inactive, is the same as the one of exogenous separation. I set the matching function elasticity with respect to the number of searchers, γ, to 0.65. To target the 5% of steady-state unemployment rate, the matching efficiency factor, μ, is internally calibrated to 0.237. The vacancy cost, κ, is also internally calibrated to ensure the free entry condition. I set the stickiness parameter for the real wage, ϵ_w, to 0.45, following Hagedorn and Manovskii (2008). Lastly, the probability of being affected by the inactivity shock, $\bar{\xi}$, is calibrated by targeting the steady-state population share for inactive households, which is 0.55%.

Table 2.3: Calibration: labor market

Parameter	Description	Value	Target / Source
$\bar{\lambda}$	Exogenous job separation rate	0.012	UK LFS
λ^n	Endogenous job separation rate	0.012	UK LFS
γ	Matching function elasticity	0.65	standard
Λ	Discount factor for labor agency	0.99	standard
μ	Matching efficiency factor	0.237	$u = 0.05$
$\bar{w}^H$	Steady-state wages to household	0.650	
κ	Vacancy posting cost	0.068	Free entry
ϵ_w	Wage elasticity	0.45	Hagedorn and Manovskii (2008)
$\bar{\xi}$	Steady-state probability of being inactive	0.0058	$N^I_{ss} = 0.0055$

2.3.3 Firms

Table 2.4 shows the parameters for the firm and policy block in the model. First, I assume that the capital depreciation rate δ is 0.02 and the slope of Phillips curve

κ_p is set to 0.1 as in other literature. Firms produce goods according to a standard Cobb-Douglas production function $Y_t = Z_t K_{t-1}^{\alpha} \left(N_t^E\right)^{1-\alpha}$. I set $K = 10$ and calibrate the capital share such that $\alpha = (r+\delta)\frac{K}{mc}$, which implies the standard result that $\alpha = 0.34$. Given all the parametised values, the steady-state TFP is determined as $Z = 0.475$.

Table 2.4: Calibration: Firms and Policies

Parameter	Description	Value	Target / Source
δ	Capital depreciation	0.02	standard
κ_p	Slope of Phillips curve	0.1	standard
α	Capital share	0.34	$\alpha = (r+\delta)\frac{K}{mc}$
Z	Total factor productivity	0.475	$Y = 1$
G	Government spending	0.2	
B^g	Government bonds	2.61	ONS
ν	Replacement rate for UI benefits	0.2	OECD
φ	Replacement rate for Furlough payments	0.8	
$\bar{\tau}$	labor income tax rate	0.386	
ϕ_π	Taylor rule coefficient on inflation	1.5	Taylor (1993)
ϕ_Y	Taylor rule coefficient on output	0	

2.3.4 Policies

In the model, government spending G is given exogenously. For government bonds, the data on government debt as a percentage of GDP is used. Therefore B^g is calibrated to 2.61. The replacement rate of UI benefits are 20% of the steady-state real wage. Given that the government balances its budget constraint, the steay-state income tax rate, $\bar{\tau}$, is set to 0.386. The response of the policy rate to inflation in the Taylor rule is set at $\phi_\pi = 1.5$. For simplicity I assume the response parameter to the output gap, ϕ_Y, is 0.

2.4 Quantitative Results

In this section, I present the results of the model in dynamic simulation. First, I show the impulse responses of aggregate variables in the case with and without the furlough scheme to examine the effect of the inactivity shock and the furlough scheme, respectively. Then, I decompose the response of aggregate consumption into direct and indirect effects to understand the results in more detail.

Regarding the solution method, I adopt a novel approach proposed by Auclert et al. (2021). Under this method, I linearize the model around the steady-state and construct sequence space Jacobians matrices, which are then used to compute general equilibrium impulse responses. The algorithm leverages a directed acyclic graph (hereinafter DAG) to exploit the sparsity of system. A key advantage of this method lies in its ability to achieve dimensionality reduction, significantly enhancing computational speed by avoiding redundant matrix recomputations. Further details on the solution method are provided in Appendix 2.A.

2.4.1 Impulse Responses of Aggregate Variables

In this subsection, I focus on how an inactivity shock affects the economy while comparing the effects of the furlough scheme between the cases with and without the furlough scheme. Figure 2.3 illustrates the impulse responses of the economy to a 1% increase in the inactivity shock at $t = 0$. At first glance, and somewhat unexpectedly, there is no substantial difference in the responses between the two cases, with the exception of consumption, tax rates, and government bonds. A more detailed discussion of these differences is provided in the following subsection.[17]

[17] One underlying but technical factor contributing to this relatively minor difference lies in the initially negligible population share of the inactive households in the steady-state. As demonstrated in the calibration, their population share is set at a mere 0.55%, a proportion that restrains the influence of the response from inactive households from significantly impacting the aggregate outcomes. For instance, aggregate consumption C is determined as a weighted average of each household's consumption based on their respective population shares: $C = N^E C^E + uC^U + N^I C^I$.

Looking at the supply side first, the employment rate experiences an immediate drop in response to the shock, as a certain portion of previously employed workers fall into the inactive state, temporarily not being able to supply their labor. This results in a direct reduction in labor input for production, leading to a subsequent contraction in output by about 0.66% on impact in both cases. On the other hand, the decline in employment leads to an increase in the marginal product of labor. This increase, in turn, brings about higher rental rates of labor, w_t^A, accompanied by an increase the real wages, w_t^H, according to (2.22). From the perspective of firms, the increase in the rental rates of labor is equivalent to an increase in marginal costs. Consequently, this upward pressure on the marginal costs pushes up inflation by around 0.16%p in both cases.[18]

On the policy side, the central bank responds to this increase in inflation by raising the nominal interest rate following (2.27). The nominal rate thus rises by approximately 24bp. The government needs to increase its expenditures to sustain the furlough scheme. As the government spending remains constant in (2.29), the necessary resources must be financed through higher tax rates and an increase in bond issuance. As a result, the two fiscal instruments increase by more than in the case without the furlough scheme. Specifically, these initially decline on impact due to the immediate reduction in the ex-post real rate[19], followed by a gradual upward trajectory thereafter. According to the Fisher equation, the ex-post real rate, r, is defined as the difference between the nominal interest rate from the previous period

The response of C to a change in ξ can be expressed as

$$\partial C/\partial \xi = \left(\partial N^E/\partial \xi\right) C^E + \left(\partial C^E/\partial \xi\right) N^E + \left(\partial u/\partial \xi\right) C^U + \left(\partial C^U/\partial \xi\right) u + \left(\partial N^I/\partial \xi\right) C^I + \left(\partial C^I/\partial \xi\right) N^I,$$

where C^E, C^U, C^I represent the steady-state consumption of employed, unemployed and inactive households, respectively. The presence of a relatively small N^I results in the last term approaching zero, which reduces the contribution of the behavior of inactive households to the aggregate consumption.

[18] Interestingly, this suggests that, at least according to the current model, the fiscal expansion through the furlough scheme does not contribute to the recent surge in inflation.

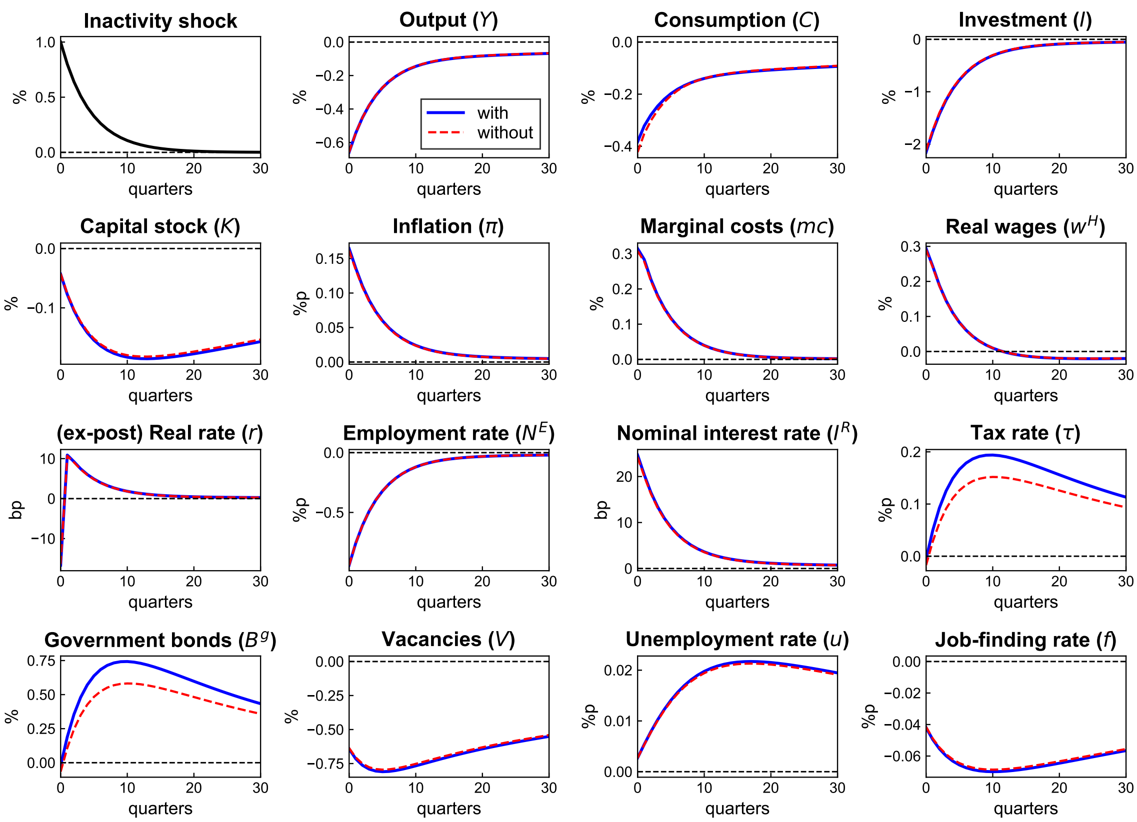
[19] (2.29) shows that both tax rate and bond issuance are influenced by the response of the ex-post real rate as it determines the value of outstanding government liabilities.

and realized inflation. Hence, due to the immediate increase in inflation, the real interest rate experiences a decline on impact at $t = 0$, but increases from $t = 1$ onward due to the rise in the nominal rates.

In the labor market, labor agencies respond to the shock by reducing vacancies. Equation (2.17), (2.21) and (2.23) imply that when labor agencies make decisions on vacancy postings, they take into account not only the mark-ups (i.e., $w_t^A - w_t^H$) they will accrue through intermediation, but also the sum of all their discounted expected values. An inactivity shock generates a greater mark-up for labor agencies as the rental rates of labor increase. This effect, however, is dominated by a fall in the discounted expected values. Upon the shock, these agencies realize that the labor market conditions have deteriorated, and the household attached to them now faces a higher risk of falling into the inactive state, which also increases the probability of the match becoming inactive. As a result, the labor agencies reduce their posting of vacancies. The decline in vacancies translates into a less tight labor market, leading to a fall in the job-finding rate and an increase in the unemployment rate.

Turning to the demand side of the aggregate economy, an increase in ξ adds a new layer of risk for households, which strengthens the incentive for precautionary saving. In response, households reduce their consumption due to this amplified precautionary motive. However, with the furlough scheme in place, the decline in consumption is mitigated, as the scheme partially offsets the income loss caused by the inactivity shock. Specifically, consumption decreases by 0.38% with the furlough scheme in place, compared to a sharper decline of 0.42% in its absence. Investment is associated with households' portfolio choices regarding the allocation of their resources between liquid and illiquid assets. In response to reductions in income and the increase in tax rates, households decide to cut their savings in illiquid assets to smooth their consumption. This adjustment strategy is driven mostly by their incentives to hold a higher proportion of liquid assets. This leads to a fall in the inflow of funds

Figure 2.3: Impulse responses to an inactivity shock



Note: Impulses responses to a 1% increase in the probability of becoming inactive, ξ_t. The blue line denotes the impulse responses in the case with the furlough scheme, while the red line is for the case without.

into financial intermediaries. From the firms' perspective, this decline in available funds within the financial intermediary translates into a corresponding reduction in resources available for them to invest. Consistent with the stylized fact that investment is more volatile, the fall in investment is larger than the declines observed in output and consumption, amounting to 2.15% and 2.12%, respectively.

2.4.2 Understanding Transmission Mechanisms of an Inactivity Shock

Now, I provide an explanation for why the responses of aggregate variables, shown in Figure 2.3, are rather indistinguishable between the cases with and without the furlough scheme. This is due to the presence of spillover effects in general equilibrium. To quantify this, I decompose the response of aggregate consumption into direct and indirect effects. Based on the setups of the model, the aggregate consumption can be expressed as a function of the probability of being inactive, real returns on liquid/illiquid assets, tax rate, real wages, job-finding rate, and endogenous job separation rate. Thus, I have

$$C_t = \mathcal{C}_t \left(\left\{ \xi_s, r_s^a, r_s^b, \tau_s, w_s^H, f_s, \lambda_s^n \right\} \right). \tag{2.35}$$

Following Kaplan et al. (2018), I decompose the total effect of an inactivity shock on consumption by total differentiating (2.35) and obtain

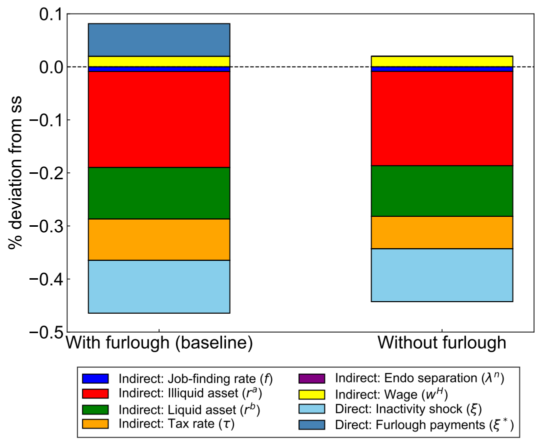
$$\begin{aligned} dC_t = & \underbrace{\sum_s \frac{\partial \mathcal{C}_t}{\partial \xi_t} d\xi_t}_{\text{direct effect}} \\ & + \underbrace{\sum_s \frac{\partial \mathcal{C}_t}{\partial r_t^a} dr_t^a + \sum_s \frac{\partial \mathcal{C}_t}{\partial r_t^b} dr_t^b + \sum_s \frac{\partial \mathcal{C}_t}{\partial \tau_t} d\tau_t + \sum_s \frac{\partial \mathcal{C}_t}{\partial w_t^H} dw_t^H + \sum_s \frac{\partial \mathcal{C}_t}{\partial f_t} df_t + \sum_s \frac{\partial \mathcal{C}_t}{\partial \lambda_t^n} d\lambda_t^n}_{\text{indirect effects}}. \end{aligned} \tag{2.36}$$

As is known, the direct effect denotes the immediate impact stemming from the changes in the inactivity risk, with all other variables being held constant. This direct effect emerges directly from the shift in the likelihood of becoming inactive (i.e., ξ_t). In constrast, the indirect effects are the impacts of changes in the other variables induced by the inactivity shock. As indicated by equation (2.36), the relative influence of the direct and indirect effects is determined by how sensitively households adjust their consumption in response to changes in ξ_t given the other variables, as well as the adjustments made by changes in the other variables when ξ_t changes.

Figure 2.4 focuses only on the immediate direct and indirect effects at $t = 0$.[20] Focusing only on the direct effects across the two cases, it becomes evident that the

[20]The consumption decomposition for the entire time periods is given in Appendix 2.B.

Figure 2.4: Consumption decomposition at $t = 0$



Note: Each bar denotes the percentage deviations of consumption from the steady-state resulted from the change in respective variable while holding the others constant at their steady-state value.

Table 2.5: Consumption decomposition

	With (A)	Without (B)	Difference (A) - (B)
Total	-0.383	-0.423	0.040
f_t	-0.009	-0.009	-0.000
r_t^a	-0.181	-0.178	-0.003
r_t^b	-0.097	-0.095	-0.002
τ_t	-0.078	-0.061	-0.017
w_t^H	0.020	0.020	0.000
λ_t^n	0.000	0.000	0.000
ξ_t	-0.100	-0.100	0.000
ξ_t^*	0.061	0.000	0.061

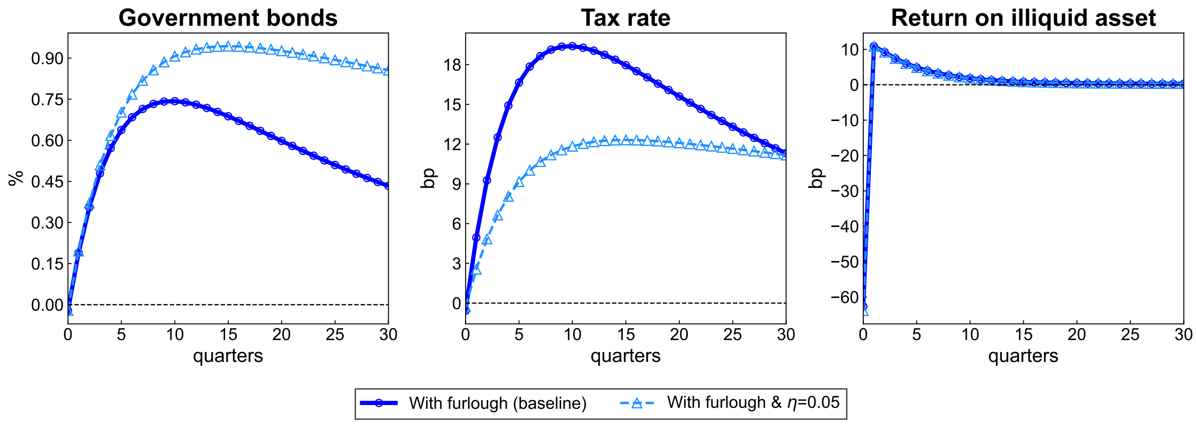
Note: The first two columns report the same results shown in Figure 2.4. The last column shows the difference between the two. Units are percentage.

furlough scheme payment (i.e. dark sky blue bar) offsets the decline in consumption through the additional benefits to those impacted by the shock.[21] This suggests that the furlough scheme is effective at least in partial equilibrium where all other variables are held constant.

However, the introduction of the furlough scheme in the general equilibrium results in an increase in the overall tax burden, which serves to dampen the impact of furlough payments on consumption. Hence, the net difference between the cases with and without the furlough scheme is not as pronounced as it appears in the partial equilibrium. Table 2.5 shows that approximately 30% of the stimulating impact attributed to the furlough scheme is counteracted by the higher tax burderns in general equilibrium.

[21] Equation (2.1) and (2.10) indicate that the inactivity shock generates two channels in households' behavior, which dampen each other. That is, an increase in ξ_t^* leads to an increase in ξ_t, but, at the same time, cancels out some of the consumption changes by directly sustaining the affected households' income.

Figure 2.5: Impulse response of government bonds and the real returns on illiquid assets



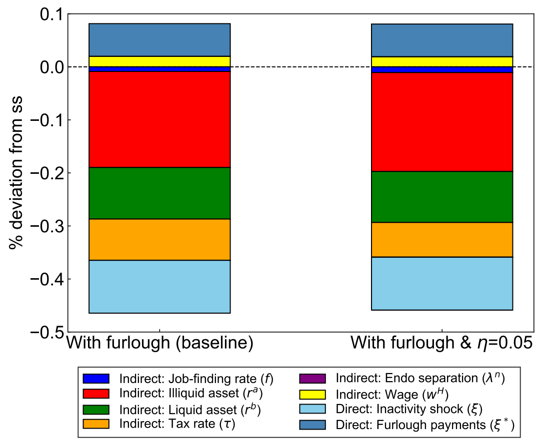
Note: Note that both lines are with the furlough scheme.

I find this dampening effects to be quite robust. To mitigate them, I conducted a simple counterfactual experiment where the sensitivity of the tax rate to changes in government bonds, denoted by η in (2.28), is intentionally set to a lower value (e.g., 0.1 $\rightarrow$ 0.05). The goal of this change is to limit the rise in the tax rate compared to the baseline model. However, the dampening effects persist, albeit through a different channel. Specifically, I find that the dampening impact in the general equilibrium continues to operate via the real return on illiquid assets, represented by r_t^a. With a lower η, the government's ability to raise the tax rate is constrained, and this induces the government to instead rely more on its bond issuance to finance the additional expenditures required by the furlough scheme, as illustrated in the left panel of Figure 2.5. This generates a less demand for firm equity,[22] which leads to a larger on-impact drop in the real return on illiquid assets by (2.14), as shown in the right panel of Figure 2.5.

Figure 2.6 and table 2.6 quantitatively compare the consumption decomposition for different η. They show that, when η is reduced, the negative wealth effect stemming from the illiquid assets channel becomes stronger, despite the reduced

[22]Given the LHS in (2.32) is unchanged, a higher B_t^g implies a lower p_t^E.

Figure 2.6: Consumption decomposition



Note: Each bar denotes the percentage deviations of consumption from the steady-state resulted from the change in respective variable while holding the others constant at their steady-state value.

Table 2.6: Consumption decomposition

	With (A)	$\eta = 0.05$ (B)	Difference (A) - (B)
Total	-0.383	-0.378	-0.005
f_t	-0.009	-0.011	0.002
r_t^a	-0.181	-0.187	0.005
r_t^b	-0.097	-0.096	-0.001
τ_t	-0.078	-0.065	-0.012
w_t^H	0.020	0.019	0.001
λ_t^n	0.000	0.000	0.000
ξ_t	-0.100	-0.100	0.000
ξ_t^*	0.061	0.061	0.000

Note: The first two columns report the same results shown in Figure 2.6. The last column shows the difference between the two. Units are percentage.

tax burden, ultimately making no substantial difference in overall consumption response.[23]

2.4.3 Heterogeneous Effects of Furlough Scheme

In this section, I examine the heterogeneous impact of the inactivity shock on the consumption responses of households with different employment statuses. To this end, I decompose the aggregate consumption response into the responses of employed, unemployed, and inactive households. Each group's consumption is further broken down into the factors, shown in (2.35). By isolating these effects, this experiment allows for a detailed analysis of whether the implementation of the furlough scheme generates heterogeneous effects on the consumption behavior of households across different employment statuses.

[23]A period-by-period budget balancing fiscal rule even strengthens the dampening effect because the required tax rates are higher in order to meet the budget constraint at all times.

Table 2.7: Consumption decomposition by employment status

	With furlough			Without furlough		
	Empl	Unempl	Inac	Empl	Unempl	Inac
Total change	−0.404	−0.420	3.612	−0.423	−0.408	−0.454
Job-finding rate (f_t)	−0.008	−0.032	−0.007	−0.008	−0.031	−0.007
Illiquid asset (r_t^a)	−0.176	−0.279	−0.223	−0.173	−0.274	−0.219
Liquid asset (r_t^b)	−0.099	−0.048	−0.077	−0.098	−0.047	−0.075
Tax rate (τ_t)	−0.078	−0.065	−0.075	−0.061	−0.052	−0.060
Wage (w_t^H)	0.020	0.010	0.012	0.020	0.010	0.012
Endo separation (λ_t^n)	0.000	0.000	0.000	0.000	0.000	0.000
Inactivity shock (ξ_t)	−0.103	−0.014	−0.105	−0.103	−0.014	−0.105
Furlough payment (ξ_t^*)	0.041	0.008	4.087	-	-	-

Notes: Responses of consumption (at $t = 0$) to a 1% increase in the probability of becoming inactive. Each value denotes the percentage deviations of consumption from the steady-state resulted from the change in respective variable while holding the others constant at their steady-state value. Note that the consumption responses explained by the job-finding rate(f_t), endogenous separation(λ_t^n), and inactivity shock(ξ_t) induce changes in employment status among households. For a clear comparison, however, I intentionally do not take into account these changes and focus more on their behavioral changes. The responses that reflect the changes in employment status are provided in Table C1 in Appendix 2.C. Units are percentage.

Table 2.7 presents the decomposition of consumption changes by employment status (employed, unemployed, and inactive) resulting from a 1% increase in the probability of becoming inactive. Each column of the table represents the consumption changes attributed to both direct and indirect effects for households with varying employment statuses.

Starting with employed households, their consumption decreases by 0.423% in the absence of the furlough scheme but only by 0.404% with the scheme in place. The largest difference between the two cases stems from the direct effect of the furlough payments, represented by ξ_t^*, where its stimulating effect contributes to an increase in consumption by 0.041%. This suggests that employed households also benefit from the policy, despite being its primary contributors through taxes. By compensating for potential income losses associated with the risk of falling into the inactive state, the furlough scheme helps limit further reductions in their consumption. Notably, the stabilizing role of the furlough scheme mitigates precautionary savings among employed households, ensuring more consistent consumption patterns.

For unemployed households, the direct effect of the inactivity shock plays a less significant role. Unlike the employed households, the unemployed are not directly impacted by the inactivity shock.[24] This leads to relatively smaller precautionary saving effects from the shock, as reflected in their negligible response (-0.014% in both cases). The indirect effects, however, have more significant impacts on the consumption of the unemployed. Since they rely more on labor market conditions and unearned income, their consumption is particularly sensitive to changes in the job-finding rate (-0.032% with the scheme vs. -0.031% without) and the real return on illiquid assets (-0.279% vs. -0.274%). Interestingly, their total consumption decreases slightly more (-0.420%) when the furlough scheme is in place compared to without it (-0.408%). This is primarily driven by the higher tax rates required to fund the policy.

A striking contrast emerges when comparing the consumption responses of inactive households. With the furlough scheme in place, the inactive households increase their consumption by 3.612% in response to the shock. This is because the furlough scheme directly benefits inactive households by providing additional income, as shown in equation (2.10). As a result, they perceive the inactivity shock as advantageous. While the shock induces a reduction in consumption due to stronger precautionary saving motives, the stimulating impact of the furlough payments ($+4.087\%$) far outweighs this effect. In contrast, without the furlough scheme, inactive households experience a sharp consumption decline of -0.454%, similar to the pattern observed for the employed households. This sharp divergence highlights the critical role of the furlough scheme in redistributing resources to support vulnerable groups, particularly

[24] In order for the unemployed households to be directly affected by the inactivity shock, they need to transition into employment first. Thus, the difference in the direct effect between the two cases for the unemployed is not as large as it is for the employed or inactive.

those who are affected by the shock and temporally cannot participate in the labor market.

In summary, the analysis highlights the varied consumption responses across employment statuses to a 1% increase in the probability of becoming inactive. This quantitatively demonstrates that the furlough scheme effectively mitigates the consumption declines of employed and inactive households, with the latter benefiting most significantly through direct income support.

2.5 Counterfactual Experiments

I now turn my attention to how well the model can account for the real economy following the pandemic. Here, for simplicity, I abstract from modelling the spread of virus in consumptionand its impacts on the economy. Instead, I focus on the model's ability to capture the actual time series observed in the data through a sequence of shocks derived from the model. In this section, I assume that the COVID-19 pandemic is characterized by three types of different shocks: inactivity shock, preference shock, and matching efficiency shock.

Arguably, the most striking feature observed in many countries, including the UK, during the pandemic was unprecedented implementation of lockdowns and a significant rise in the number of workers placed on furlough. This led to a sudden decline in labor utilization on the aggregate supply side and simultaneous impairment of aggregate demand.

In addition to this, the pandemic caused an increase in pessimism among individuals as uncertainties about future economic conditions heightened. Households feared that their employment stability might become less secure and thus suffer losses in labor income. Therefore, households' preferences for consumption might have been adversely impacted. Several studies have already explored the macroeconomic impacts of uncertainties on economies. For instance, Baker et al. (2020) and Altig et al. (2020) use real-time data to measure an enormous spike in uncertainty due to the COVID-19 pandemic and assess its macroeconomic impact on economy. Their findings suggest that the downturns in both production and demand during the pandemic are associated with the COVID-induced uncertainties. In this section, the preference shock is modelled as a shock to discount factor, β.

Lastly, the pandemic has led to a sharp rise in labor market mismatch. With the implementation of extensive lockdown measures, the matching processes in the labor market may have not functioned optimally. Heterogeneity among sectors generates matching frictions in which job creation is reduced in contact-intensive sectors while vacancies in sectors with ability to work remotely are not filled (See Pizzinelli and Shibata (2022)). I assume that an increase in matching inefficiency is modelled as a fall in matching efficiency factor, μ.

In the following subsections, I explain in detail how I derive the sequence of the three shocks mentioned above and examine whether the model can accurately replicate the economic consequences of the pandemic through these mechanisms.

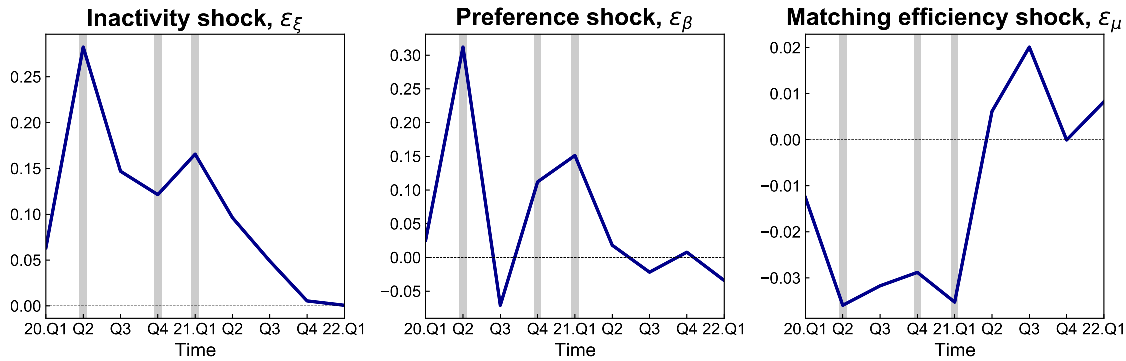
2.5.1 Deriving the Shocks from the Model

For this counterfactual experiment, I date the onset of the pandemic in 2020.Q1 and initialize the model with 2019.Q4 as a baseline economy for comparison. The key idea of estimating the shocks is to guess and verify a sequence of shock processes in a way that allows the model to replicate the (model-implied) response, exactly matching the observed time series.[25]

The first step involves choosing the pertinent variable from the model for each shock to be matched with time series from the data. In the case of the inactivity

[25]For counterfactual experiments of this kind, other literature directly estimates the sequence of shocks from the data and feeds them into the model. In my case, however, as the unit of period is quarter the length of available data series is too short to be estimated.

Figure 2.7: Estimated shock processes



Note: As the CJRS came to an end in 2021.Q3, there is no available data on the number of workers on furlough from 2021.Q4 to 2022.Q1. For these periods, I simply assume that the number decreases at a rate such that it returns to the pre-pandemic level at 2022.Q1. The shaded areas represent the time periods at which the lockdowns are enforced.

shock, I focus on the time series of the proportion of workforce on furlough with the population share for the inactive households from the model (i.e. N_t^I).[26] Given the data is collected on a daily basis, I aggregate it to form a quarterly data set by taking averages. Regarding the preference shocks, I assume that it is a shock to the households' discount factor, β, and aim to match the time series for private consumption with the consumption implied by the model. Lastly, for the matching efficiency shock, a sequence of shocks to matching efficiency parameter, μ, is backed out by matching the observed time series of the job-finding rate.[27]

Once I have sets of the targeted time series and model variables, I jointly guess and verify a sequence of shocks, $\{\epsilon_{\xi,t}, \epsilon_{\beta,t}, \epsilon_{\mu,t}\}_{t=2020.Q1}^{2022.Q1}$, such that, through this combination of shocks, the model excactly matches the associated time series at all time periods.[28] The resulting shock processes are then fed back into the model to produce counterfactual experiments.

[26]That is, the goal is to back out a sequence of shock process, $\epsilon_{\xi,t}$, which delievers the response of the population share for the inactive, N_t^I, identical to the data on the proportion of workforce on furlough.

[27]For the job-finding rate I use the unemployment-to-employment transition rate from the UK labor Force Survey.

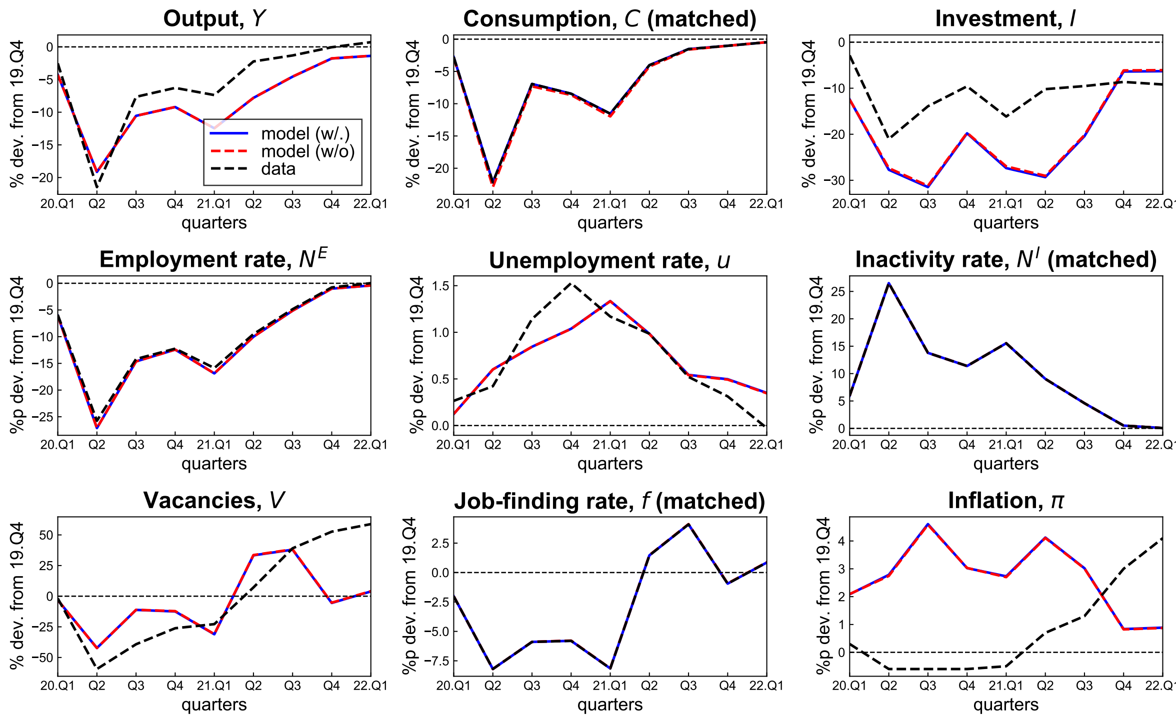
[28]I apply Broyden's method to back out 27 shocks (9 time periods for three shocks) in total matching 27 values in the data.

The shock processes are given in Figure 2.7. Reflecting on the timeline of lockdowns by the UK government, three lockdowns took place in the UK in 2021.Q2, 2020.Q4 and 2021.Q1 since the start of the pandemic. Consistent with the observed time series, the estimated shock processes for $\epsilon_{\xi,t}$ reveal that the probability of becoming inactive rapidly rose during the same periods. Simultaneously, the estimated preference shocks suggest that households delayed consumption and significantly increased precautionary savings during the extensive lockdowns. The matching efficiency shocks also indicate that there was an increase in labor market mismatch that lasted for approximately 4-5 quarters following the outbreak of the pandemic, subsequently recovering as the magnitude of the inactivity shocks diminishes.

2.5.2 Quantitative Results

Figure 2.8 shows the effects of the estimated shocks on the aggregate variables against the actual data. On the whole, both the data and the model indicate that the pandemic had a large recessionary impact on the UK economy in 2020.Q1. A majority of variables implied by the model display similar patterns to the data in terms of magnitude and timing. The model, guided by the estimated inactivity and preference shocks, accurately captures the abrupt declines of more than 20% followed by subsequent recoveries in output and investment. However, the model somewhat overestimates the extent of investment response in terms of magnitude. Nevertheless, its cyclically fluctuating pattern mirrors that of the data. Incorporating the matching efficiency shock enhances the model's ability to capture the labor market dynamics. The unemployment rate implied by the model replicates the gradual rise and decline observed in the data, though the timing of the peaks differs by one period. Moreover, the immediate fall and rapid rebound in vacancies from the data are successfully replicated by the model. The model, however, fails to account for inflation. Upon impact, the model generates a sustained increase in inflation, while in reality, inflation

Figure 2.8: Counterfactuals vs. data



Note: Counterfactual responses of aggregate variables derived from the model (solid blue/dashed red) against the actual data (dashed black line). Note that the response of consumption, inactivity rate (i.e., share of workforce on furlough), and the job-finding rate are exactly matched to the respective time series.

drops initially and remains below pre-pandemic levels until it begins to surge from 2021.Q1 onward.

2.6 Conclusion

In this paper, I develop a heterogeneous-agent New Keynesian model that features incomplete markets and labor market frictions. The main goals are to explore the effect of the COVID-19 pandemic on the UK economy and to analyze the effectiveness of the furlough scheme in preventing the collapse of aggregate demand. The most innovative contribution of this paper is that I introduce a new state for households in which households are temporary inactive.

This new set-up of the model generates negative impacts on aggregate supply and demand simultaneously. The model quantitatively shows that the pandemic has recessionary impacts on the economy. What is rather surprising is that the model suggests that although the furlough scheme might be effective in partial equilibrium, the substantial proportion of its stimulating effects is offset by equilibrium effects such as higher tax rates. Analyzing consumption responses of households in different employment status, however, reveals that the furlough scheme was able to reduce consumption inequality among households implying that it still plays a role as a redistribution policy. Additionally, to test the model's ability to explain the real time series, I run an experiment where I back out from the model a few shock processes that might be relevant to the pandemic and feed them back into the model. With the combination of the shocks, I find that the model is reasonably successful in accounting for most of the macroeconomic data following the pandemic.

In this paper, I focus mostly on the responses from the demand side of the economy, that is, how households, faced with the risk of being inactive, change their consumption, and its spillover effects. I have ignored the interplay with the supply side in which firms can endogenously decide the size of the inactivity shock taking into account the aggregage state of the economy. I have also ignored issues related to the influence of sectoral heterogeneity, which can broaden the scope of the analysis. These would be interesting avenues for future research.

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Appendix

2.A Solving Equilibrium using Auclert et al. (2021)'s Method

I basically follow an algorithm in Auclert et al. (2021) to solve the model. They develop a highly efficient method for solving general equilibrium heterogeneous agent models with aggregate shocks. The first step is to solve for the steady state. That is, I need to solve heterogeneous household's problem. For this I use the endogenous gridpoints method to solve the each household's policy function. After this step I use the non-stochastic simulation method developed by Young (2010) to compute the stationary distribution.

Once the steady-state is pinned down, Auclert et al. (2021)'s method exploits a directed acyclic graph (DAG) to compute the Jacobians of the block. In the main text, I briefly explain the concept of "block". A "block" in the DAG is a function that maps the sequences of inputs $\{x_{1,t}, x_{2,t}, \cdots, x_{n_x,t}\}_{t=0}^{T}$ into the sequences of outputs $\left\{y_{1,t}, y_{2,t}, \cdots, y_{n_y,t}\right\}_{t=0}^{T}$ according to corresponding equilibrium conditions. The Jacobian of a block is a $T \times T$ matrix whose elements are the partial derivative of the outputs with respect to the inputs, that is, $\left\{\frac{\partial y_{j,s}}{\partial x_{i,t}}\right\}$, $1 \leq i \leq n_x, 1 \leq j \leq n_y, 0 \leq s,t \leq T$. For example, the household block takes as inputs the sequence of variables $\left\{r_t^a, r_t^b, f_t, w_t^H, \tau_t\right\}_{t=0}^{T}$ and maps these into the sequence of output variables $\left\{C_t, B_t, A_t, \Psi_t^A\right\}_{t=0}^{T}$ by solving the optimisation problems. Therefore, the Jacobian of the household block is a $T \times T$ matrix consisting of $\left\{\frac{\partial y_s}{\partial x_t}\right\}$, $x \in \left\{r_t^a, r_t^b, f_t, w_t^H, \tau_t\right\}$, $y \in \left\{C_t, B_t, A_t, \Psi_t^A\right\}$, $0 \leq s, t \leq T$. The DAG representation in Figure 2 shows that a variable coming out of a block is used as an input of a later block, allowing me to accumulate the Jacobians along the DAG. In this process, I can take advantage of sparsity, which makes the computation much efficient.

Let U be the sequences of unknowns and Z be the sequences of exogenous variables. If I stack all the equilibrium conditions they can be characterized by a system of equations in sequence space.[29]

$$H(U, Z) = 0.$$

The goal of the solution is to compute the impulse responses of U with respect to Z. Total differentiating to the first-order around the steady-state gives

$$dU = -H_u^{-1} H_z dZ \quad \Leftrightarrow \quad \frac{dU}{dZ} = -H_u^{-1} H_z.$$

Once I have the Jacobians H_u, H_z, all the necessary impulse responses of aggregate variables to shocks Z can be easily obtained.

Lastly, I present my version of the DAG, which encapsulates the entire model's economic structure, illustrated in Figure A1.

[29]In more detail, $H(U, Z)$ consists of $\{H_{1,t}, H_{2,t}, H_{3,t}, H_{4,t}\}_{t=0}^{T}$ in which $H_{1,t}, H_{2,t}, H_{3,t}, H_{4,t}$ are the four targets of the DAG in Figure A1, defined as

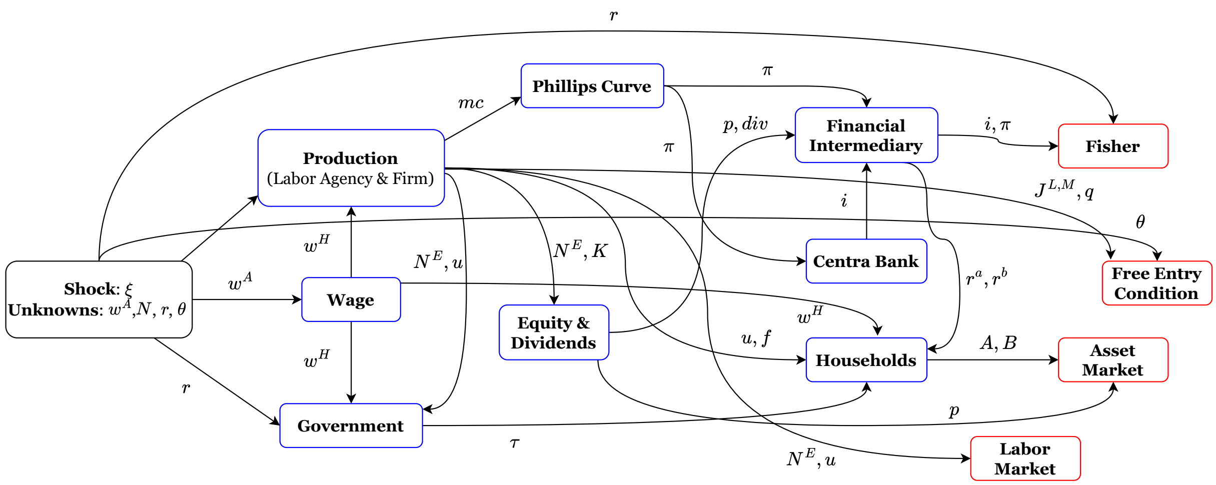
$$\text{Fisher equation: } H_{1,t} = (1 + i_{t-1}) - (1 + r_t)(1 + \pi_t)$$

$$\text{Asset market clearing: } H_{2,t} = A_t + B_t - p_t - B^g$$

$$\text{Labour market clearing: } H_{3,t} = 1 - u_t - N_t$$

$$\text{Free entry condition: } H_{4,t} = \kappa - q_t J_t^{L,M}$$

Figure A1: DAG representation of the model

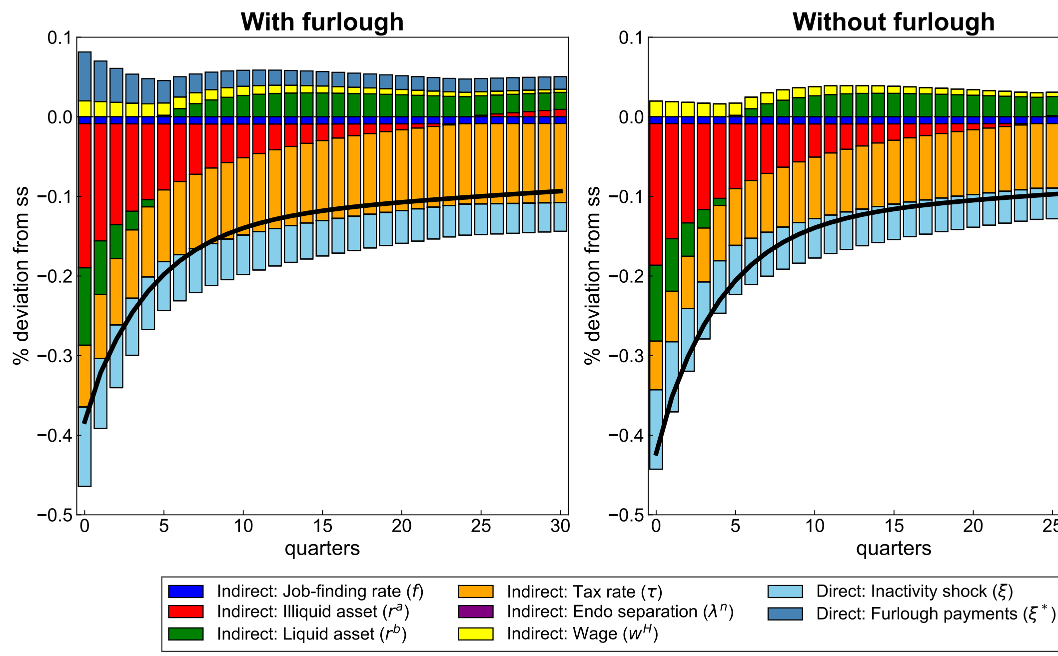


2.B Consumption Decomposition

In this subsection, I provide a further analysis to get a better sense of what drives the households' total consumption response.

Figure B1 illustrates the breakdown of aggregate consumption into the direct and indirect effects. Similar to Kaplan et al. (2018) and Lee (2020), a significant share of consumption response is explained by the indirect effects. While the direct effect does play a role in the changes in consumption, its relative significance is not predominant. A closer examination of each effect reveals that the negative wealth effect and intertemporal substitution effect emerge following an inactivity shock. These effects arise from shifts in the real returns on liquid and illiquid assets, both of which contribute to a decrease in consumption. Figure B2 plots the responses of the real returns on the both assets. The negative wealth effect materializes due to the initial drop in the (ex-post) real returns on assets, r^a, r^b, triggered by an inflation increase. Consequently, households experience a relative reduction in wealth and, in turn, reduce their consumption. The intertemporal substitution effect takes shape as households, operating under perfect foresight, foresee a rise in the future real returns on assets. This induces them to cut back on their current consumption even

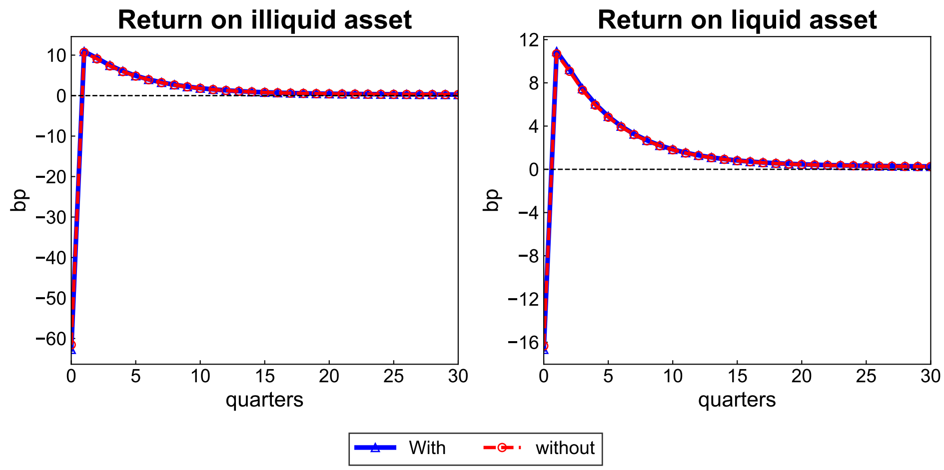
Figure B1: Consumption decomposition



Note: Each bar denotes the percentage deviations of consumption from the steady-state resulted from the change in respective variable while holding the others constant at their steady-state value. The black solid line is the overall response of aggregate consumption as shown in Figure 2.3. The decomposition is almost exact. The contribution of each component adds up to the overall response of consumption with an infinitesimally small error in both cases at all times.

further. Additionally, the anticipated increase in tax rates exerts further downward pressure on consumption. According to the fiscal rule in (2.28), the tax rate becomes a dominant component that slows down the recovery of aggregate consumption over time. Changes in the job-finding rate, real wages and endogenous separation rate also play a role in shaping the aggregate consumption, yet the individual contributions of these factors are relatively modest.

Figure B2: Real returns on assets



Note: Impulse response of the real returns on liquid ans illiquid asset to a 1% increase in the probability of becoming inactive, ξ_t.

2.C Consumption Decomposition by Employment Status

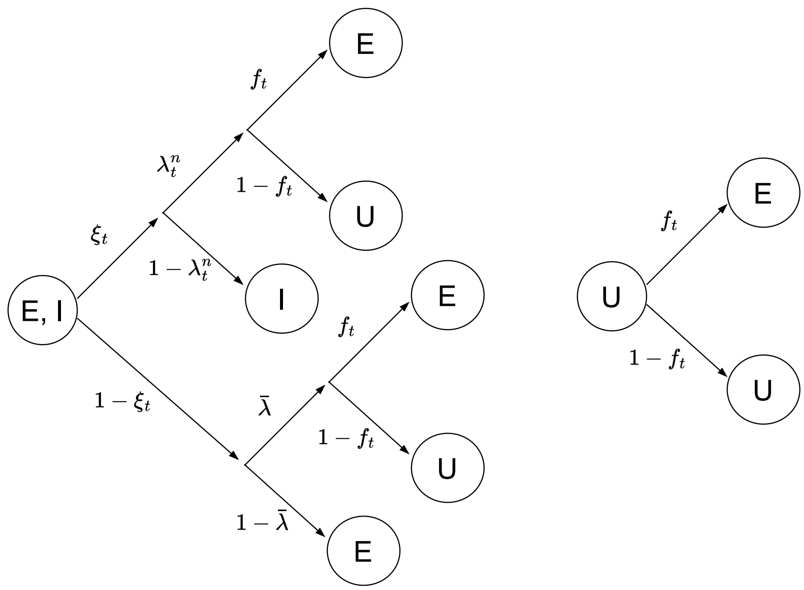
Table C1: Consumption decomposition by employment status

	With furlough			Without furlough		
	Empl	Unempl	Inac	Empl	Unempl	Inac
Total change	−1.361	−0.360	174.3	−1.380	−0.348	170.2
Job-finding rate (f_t)	−0.010	0.025	−0.007	−0.010	0.025	−0.007
Illiquid asset (r_t^a)	−0.176	−0.279	−0.223	−0.173	−0.274	−0.219
Liquid asset (r_t^b)	−0.099	−0.048	−0.077	−0.098	−0.047	−0.075
Tax rate (τ_t)	−0.078	−0.065	−0.075	−0.061	−0.052	−0.060
Wage (w_t^H)	0.020	0.010	0.012	0.020	0.010	0.012
Endo separation (λ_t^n)	0.000	0.003	−0.033	0.000	0.003	−0.033
Inactivity shock (ξ_t)	−1.058	−0.014	170.6	−1.058	−0.014	170.6
Furlough payment (ξ_t^*)	0.041	0.008	4.087	-	-	-

Notes: Responses of consumption (at $t = 0$) to a 1% increase in the probability of becoming inactive. Each value denotes the percentage deviations of consumption from the steady-state resulted from the change in respective variable while holding the others constant at their steady-state value. Units are percentage.

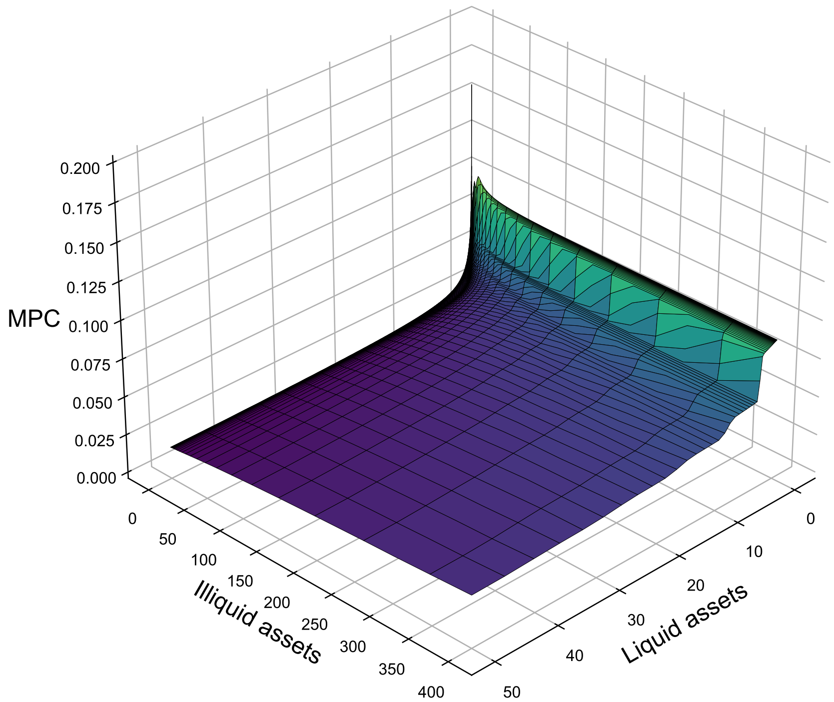
2.D Supplementary Figures

Figure D.1: Employment transition tree



Note: This figure illustrates the employment transition taking place in the beginning of the period. E, U, and I represent the employed, unemployed, and inactive households, respectively.

Figure D.2: MPC distribution



Note: This figure plots the MPC distribution over asset grids in the steady state.

Chapter 3

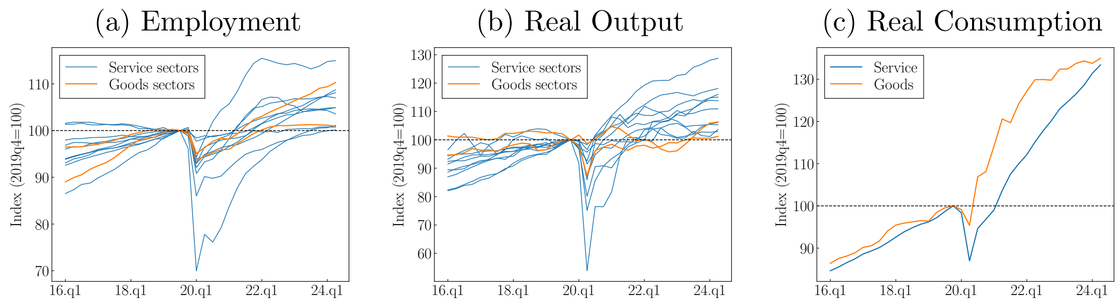
Job Separation Shock and Sectoral Labor Reallocation

3.1 Introduction

During economic recessions, there has been significant increases in labor mobility across sectors, with workers moving from heavily impacted sectors to those less affected. Figure 3.1(a) presents employment trends by industry in the U.S. before and after the COVID-19 pandemic. There is clearly a difference in the magnitude of employment changes across industries. Employment in service-related sectors, which were more significantly affected by the pandemic, decreased substantially, while employment in the relatively less affected good-producing sectors decreased by less. This sectoral dispersion is observed not only in employment but also in production and consumption. Figure 3.1(b) and (c) also illustrate that production and consumption declined in the service-related sectors in overall, more than those in the goods-related sector.

In this paper, we develop a quantitative general equilibrium model to explore whether it can effectively capture the dynamics of the pandemic. Central to this

Figure 3.1: Employment, Output, and Consumption in the Goods and Services Sectors.

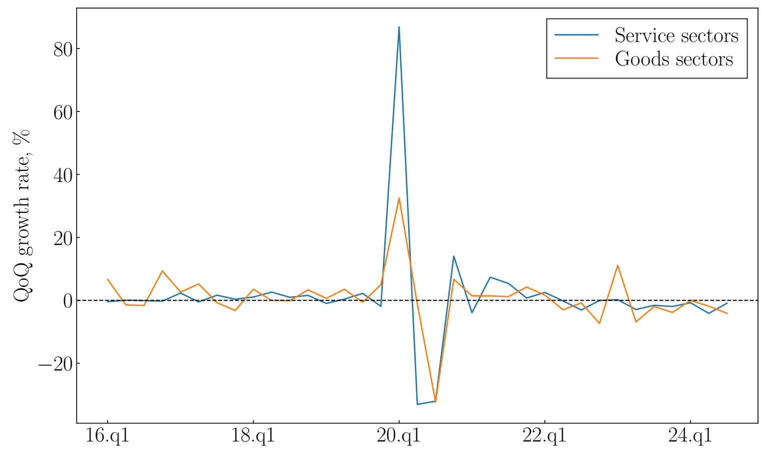


Note: The service sectors include wholesale, retail trade, transportation and warehousing, utilities, finance, information, professional and business services, education and health care, leisure and accommodation. The goods sectors include construction and manufacturing. Goods consumption consists of expenditures on durable and nondurable goods, while service consumption includes all expenditures on service-related activities.

approach is identifying the type of shock that best represents the pandemic. While multiple factors have contributed to the outcomes shown in Figure 3.1, we focus on the fact that the pandemic caused a sudden and massive destruction of firm-worker matches, particularly in service-related sectors, such as hospitality, leisure, and retail, as shown in Figure 3.2. To reflect this distinct nature of the pandemic, we model the pandemic as a sector-specific separation shock, which captures the initial spike in job separations and its consequent effects on the economy.

This framework not only accounts for the uneven impacts observed across sectors, as shown in Figure 3.1 and 3.2, , but also enables a detailed analysis of how sector-specific dynamics interact with broader macroeconomic outcomes. While alternative shocks, such as demand reallocation or productivity shocks, can also generate labor reallocation between sectors, it is more intuitive to use separation shocks to replicate the sharp rise in separations seen at the onset of the pandemic. The use of a separation shock also provides a clear and intuitive framework for emphasizing search and matching frictions, as well as intersectoral reallocation in the labor market. By modeling the abrupt destruction of firm-worker matches, the separation shock highlights the adjustment processes required to re-establish employment relationships,

Figure 3.2: The Growth Rate of Separations by Sectors



Note: The service sectors include wholesale, retail trade, transportation and warehousing, utilities, finance, information, professional and business services, education and health care, leisure and accommodation. The goods sectors include construction and manufacturing.

such as job search, vacancy posting, and the costs associated with switching sectors, which are central to understanding the labor market dynamics after the pandemic.[1]

Based on this assumption, this paper aims to address the following questions: Can a sector-specific separation shock account for the pandemic? How do labor market flows differ across sectors after this separation shock? To explore these questions, we construct a two-sector general equilibrium model featuring heterogeneous agents in an environment with incomplete markets and labor market frictions. Furthermore, given that many countries have implemented substantial job retention measures—such as furlough schemes and wage subsidies—following the COVID-19 pandemic, we also incorporated these features into the model to enhance its realism.

One of the distinguishing feature of our model is that it allows households to make discrete choices regarding their labor participation, particularly in deciding whether or not to switch sectors. In the standard multi-sector New Keynesian literature, inter-sectoral labor allocation is typically determined solely by the relative wage between

[1] Although this paper remains in its preliminary stage, one could argue that modeling the pandemic as a separation shock also offers a valuable perspective for evaluating the effectiveness of policies such as furlough schemes, wage subsidies, and job retention programs implemented during the pandemic, as these policies were implemented to mitigate the adverse effects of job separations.

sectors. Each household allocates a larger share of its household members to a sector that pays a higher wage. However, our model extends this framework by assuming that each household evaluates the expected gains from switching sectors, taking into account not only the relative wage but also other factors such as the job-finding rate between the sectors, switching costs, and their own productivity and wealth levels.[2] This approach better captures the complexities of real-world labor market dynamics, providing a deeper understanding of how individuals' characteristics influence their labor participation decisions. Additionally, it offers deeper insights into the various factors that drive sectoral shifts in the labor market.

Throughout this paper, we focus primarily on how well the model captures sectoral dynamics after a sector-specific separation shock. Specifically, we study the effects of a 1% increase in the separation rate in the service-related sector (sector 1). Unfortunately, we find that the model struggles to replicate sectoral responses following the shock.

At the aggregate level, the model captures patterns consistent with standard models: unemployment rises, employment rates decline, and both production and consumption contract, reflecting a recessionary response to the shock. However, when we focus on the sectoral responses, the model produces puzzling and counterintuitive results. Sector 1, despite being directly impacted by the separation shock, experiences a moderate recession followed by a relatively faster recovery in employment, output, and consumption. This recovery is primarily driven by the tighter labor market in sector 1, as firms quickly increase vacancy postings to restore production levels. The increased tightness leads to improved job-finding rates and higher wages in this sector, incentivizing workers to switch from sector 2 into sector 1. Conversely, sector 2, which is indirectly impacted by the shock, suffers a deeper and more prolonged recession. The negative demand effects from sector 1, caused by a reduction in the production

[2] This idea is in line with the rational forward-looking behavior of workers in Kline (2008).

and consumption in general equilibrium, result in reduced demand for sector 2 goods. This demand contraction leads to a decline in labor demand in sector 2, exacerbating unemployment and lowering wages. As a result, sector 2 experiences deeper and longer-lasting declines in output, consumption and employment compared to sector 1.

This puzzling outcome between the rapid recovery in the directly affected sector and the extended downturn in the indirectly affected sector highlights potential limitations in the current setup of the model. The findings suggest that additional mechanisms, such as more intricate intersectoral linkages or complementary shocks, are required to better capture the dynamics observed in real economy following the economic labor market disruption.

Related literature This paper is closely related to the literature on intersectoral labor reallocation in the labor market. Pilossoph (2012) develops a multi-sector search model to explore whether intersectoral labor mobility can explain fluctuations in aggregate unemployment. Similarly, Carrillo-Tudela and Visschers (2023) and Chodorow-Reich and Wieland (2020) examine the effects of intersectoral mobility among unemployed workers on the overall unemployment rate. While these studies primarily focus on labor market dynamics, our research includes analysis on the broader fluctuations in the overall economy.

Second, our paper shares several modeling aspects with the literature on multi-sector New Keynesian models and general equilibrium models with search and matching (SAM) frameworks. Relevant works in the former include Aoki (2001), Bhattarai et al. (2023), Carvalho et al. (2021), Ferrante et al. (2023). In the latter, key references include Challe (2020), Den Haan et al. (2018), Gornemann et al. (2016), Lee (2020), Ravn and Sterk (2017), Ravn and Sterk (2021), Thomas and Zanetti (2009), and Zanetti (2019). The multi-sector New Keynesian literature primarily explores sectoral heterogeneity and its implications for monetary and fiscal policy. By incorporating the search and matching frictions into a multi-sector framework, we shift the focus to the macroeconomic impacts of labor market policies, extending the traditional analysis on monetary and fiscal policies. Our paper also relates to Guerrieri et al. (2022) who present a theoretical framework with a two-sector NK model and show that a supply shock in one sector can generate a negative demand effect in the other sector.

Lastly, although this version of the paper does not yet include a discussion from a policy perspective, this paper also relates to the literature that evaluates the effect of government policies implemented to mitigate the adverse effects of the pandemic. Most relevent to our paper is a work by García-Cabo et al. (2023). They evaluate two labor market policies—unemployment insurance (UI) and wage subsidy (WS)—in two distinct labor market environments: a flexible labor market and a rigid labor market. Their findings show that following a sector-specific shock such as the COVID-19 recession, the UI policy is preferred in the flexible labor market while the WS policy is preferred in the rigid labor market. Mohimont et al. (2024) use a DSGE model to show that job retention scheme across the euro area was effective in terms of preventing a massive job destruction. Gertler et al. (2022) highlight the destabilizing effect of "loss-of-recall" and show that the Paycheck protection program significantly has reduced loss-of-recall. Auray and Eyquem (2020) demonstrate that extending UI benefits simulates the economy more during lockdowns. Dengler and Gehrke (2022) find that short-time work programs can stabilize employment by reducing the risk of unemployment, which in turn decreases the need for precautionary savings. This body of research also includes work by Bayer et al. (2023), Faria-e-Castro (2021), Elenev et al. (2022), and Mitman and Rabinovich (2021), who have examined various aspects of fiscal and labor market policies during the pandemic.

Layout The rest of the paper proceeds as follows. Section 3.2 provides a detailed description of the model, while section 3.3 explains the calibration strategies. The quantitative, albeit preliminary, results are presented in section 3.4, and section 3.5 concludes the paper. Since we are still in the preliminary stage, this draft primarily focuses on describing the model and presenting its basic results.

3.2 Model

In this section, we present a two-sector heterogeneous agent New Keynesian model with incomplete markets and labor markets frictions. Regarding the two-sector framework, we build on Lucas and Prescott (1974), in which an island refers to a sector. The two sectors are symmetric. The economy consists of households, labor service firms, production firms, the government and the central bank. Households face not only idiosyncratic productivity shocks and borrowing constraints, but also idiosyncratic taste shocks that affect their labor supply decisions, including sector switching. In each sector, labor service firms hire households and provide labor service to intermediate-goods production firms, which produce sector-specific goods. These goods are aggregated into a final consumption composite good by retailers operating in a perfectly-competitive market. The government runs a balanced-budget fiscal policy, while the central bank operates monetary policy. The remainder of this section provides detailed explanations on the model's key blocks.

3.2.1 Households

An infinitely lived household in the model can be employed in one of the two sectors, which are ex-ante symmetric, and has preferences over consumption, so that its expected lifetime utility is

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{c_t^{1-\sigma}}{1-\sigma} \right),$$

where

$$c_t = \left[\alpha^{\frac{1}{\eta}} c_{1,t}^{\frac{\eta-1}{\eta}} + (1-\alpha)^{\frac{1}{\eta}} c_{2,t}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}.$$

c_t is the household's consumption of the composite good, and $c_{1,t}$ and $c_{2,t}$ denote the sectoral consumption good produced from each sector, respectively. η is the elasticity of substitution between the sectoral consumption goods.

Each household purchases the composite consumption goods at price P_t, and trades one-period real bonds, a_t, which pays a return at nominal rate R_t. They receive after-tax labor income depending on its employment status, $(1-\tau)y$, which will be discussed below in more details. Finally, they earn firms' nominal dividends, Div_t. Its budget constraint is therefore given by

$$P_t c_t + P_t a_{t+1} = P_t(1-\tau_t)y_t + R_t a_t + Div_t,$$

and the no-borrowing constraint

$$a_{t+1} \geq 0,$$

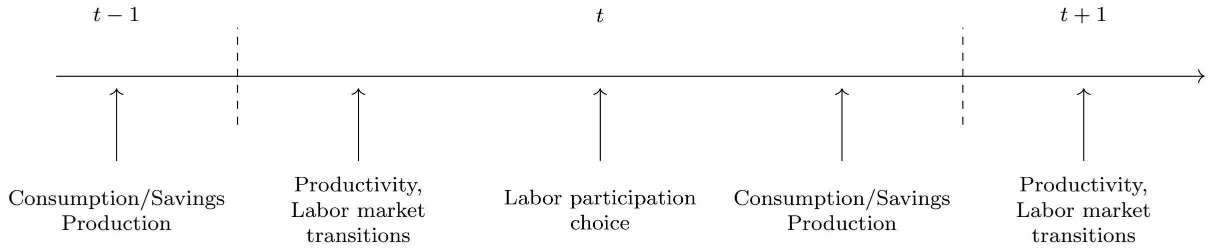
where

$$P_t = \left[\alpha P_{1,t}^{1-\eta} + (1-\alpha)P_{2,t}^{1-\eta}\right]^{\frac{1}{1-\eta}}. \tag{3.1}$$

P_t denotes the aggregate price index associated with the composite consumption good c_t. $P_{1,t}$ and $P_{2,t}$ are the price of the sectoral good, respectively.

Figure 3.3 illustrates the timing of the model. At the beginning of each period, households observe their realizations for productivity (z) and employment status (e). Specifically, there are four different employment states to begin with: Matched (M), Unmatched (UM), Furloughed (F), and Non-participating (N). After observing these realizations, the households make choices regarding their labor market participation. At the end of the period, they make consumption-saving decisions before moving to the next period.

Figure 3.3: Timing of the model



We denote by $V^e(s, z, a)$ the value of a household attached to sector s with productivity z, asset holdings a, and employment status e. The employment status, e, takes values from $\{M, UM, F, N\}$ in the beginning of the period before the labor participation choices, and from $\{E, U_b, U_{nb}, F_{search}, F_{ns}, SW, X\}$ after making the choices. Appendix 3.B provides a visual representation of the employment status transition (Figure B1) and detailed law of motion for each employment status in the beginning of the period. In this section, we focus only on a household's problem in sector s. According to the symmetry assumption across sectors, the problems described below hold same for those in sector $\tilde{s}$.

Matched (M)

Matched households in sector $s \in \{1, 2\}$ are those with an employment offer in hand. They have the option to accept the offer and become employed (E) or switch the sector (SW), or exit the labor force (X). Comparing the three available options, they choose the one that gives the maximum value. Therefore, the value of a matched household is given as

$$V^M(s, z, a) = \max \left\{ V^E(s, z, a),\ V^{SW}(s, z, a),\ V^X(s, z, a) \right\} \tag{3.2}$$

where

$$V^E(s,z,a) = \max_{c,\, a'} u(c) - \varphi + \beta\mathbb{E}\Big[(1-\lambda_s+\lambda_s f_s')V^M(s,z',a') \tag{3.3}$$

$$+ \lambda_s(1-f_s')\left[\xi_s' V^F(s,z',a') + (1-\xi_s')V^{UM}(s,z',a')\right]\Big] \tag{3.4}$$

$$\text{s.t. } c + a' = (1-\tau)w_s z + (1+r)a + div(z)$$

$$V^{SW}(s,z,a) = \max_{c,\, a'} u(c) + \beta\mathbb{E}\left[f_{\tilde{s}}' V^M(\tilde{s},z',a') + (1-f_{\tilde{s}}')V^{UM}(\tilde{s},z',a')\right]$$

$$\text{s.t. } c + a' = tr + (1+r)a + div(z)$$

$$V^X(s,z,a) = \max_{c,\, a'} u(c) + \beta\mathbb{E}\left[V^N(s,z',a')\right]$$

$$\text{s.t. } c + a' = tr + (1+r)a + div(z)$$

Note that all the budget constraints are expressed in real terms in the units of the composite good. The variable with a prime symbol denotes the one in the next period. For ease of notation, we suppress the time subscript t.

λ_s is the job separation rate, and f_s is the job-finding rate in sector s. Working incurs disutilities, denoted by φ. Employed households are paid a real wage w_s and pay a proportional income tax at rate τ. We assume that even if households are separated from a match, they can still find a new employment with probability f_s within the same period. Therefore, currently employed households can stay matched in the next period with probability $1-\lambda_s+\lambda_s f_s$, but may be separated from employment with probability $\lambda_s(1-f_s)$. Among those separated, a ξ_s share are exogenously furloughed, reflecting the fact that the decision to furlough workers is mostly determined by firms.[3] Those not furloughed become unmatched.

We assume that it takes one period before the employment status of a switching household in the new sector is determined. Specifically, once she decides to switch, she gives up her current labor income, instead receives transfers, tr, from the government, and starts searching for a job in the new sector. Those matched can earn labor income as before from the next period, but we assume there is a 20% productivity loss in the first period of the switch. This productivity loss captures financial disutilities experienced during the sector switching process. Therefore, the budget constraint of a household who switched from sector s to $\tilde{s}$ in the previous period and has been just matched with a firm is

$$c + a' = (1 - \tau)w_{\tilde{s}}(1 - loss)z + (1 + r)a + div(z),$$

where $loss = 0.2$. Those who remain unmatched in the first period of switching are treated as equivalent to other existing unmatched households.

Note that the continuation value of those have switched from sector s to $\tilde{s}$ depends on the expected value and the job-finding rate they would face in sector $\tilde{s}$. This indicates that labor market prospects in the other sector, such as the job-finding rate and wages, are crucial factors in the decision to switch. If one chooses to exit the labor force, it becomes a non-participating household in the next period with probability 1. These households also receive the transfers.

Lastly, we assume that households own firms operating in the economy. They receive a lump-sum dividend each period as compensation for their ownership of the

[3] More details on the determination of the probability of being furloughed, ξ, will be discussed later.

firms. Dividends from each sector are aggregated and then distributed to households based on their productivity.[4]

Unmatched (UM)

Unmatched households in sector s decide between searching for jobs within the sector receiving unemployment benefits (U_b), or switching the sector (SW), or leaving the labor force (X).

$$V^{UM}(s,z,a) = \max\left\{V^{U_b}(s,z,a),\, V^{SW}(s,z,a),\, V^{X}(s,z,a)\right\} \tag{3.5}$$

where

$$V^{U_b}(s,z,a) = \max_{c,\,a'} u(c) - \chi + \beta\mathbb{E}\left[f_s' V^{M}(s,z',a') + (1-f_s')V^{UM}(s,z',a')\right]$$

$$\text{s.t. } c + a' = b(1-\tau)w_s z + (1+r)a + div(z)$$

$0 < b < 1$ is the unemployment benefit replacement rate. If an unmatched household chooses to be unemployed eligible for UI benefits, they engage in searching within the same sector.[5] Job searching incurs a utility loss of χ. The household may be matched in the next period with probability f_s. The value of households who choose to switch (V^{SW}) and to exit the labor market (V^X) are the same as ones for those who are matched.

Furloughed (F)

Furloughed households also have three options. They can choose to search for jobs (F_{search}) while being furloughed or to just stay furloughed (F_{ns}) not engaging in search.

[4] The dividend rule specifies that dividends are allocated based on each household's contribution to the economy in terms of productivity. Thus, households with higher productivity receive a larger share of the total dividends.

[5] In order to search for jobs in the other sector, they must first switch to that sector.

Otherwise, they can switch into the other sector (SW). If they stay in the current sector (F_{search} or F_{ns}), all furloughed households can be recalled by their previous employers with probability ω_s. If they exert efforts in searching for jobs, they may increase the chance of being matched in the next period, but suffer from disutilities from searching. If they simply stay furloughed (not searching), they may be either recalled by their previous employers with probability of ω_s, or remain furloughed with the complement probability. Therefore,

$$V^F(s,z,a) = \max\left\{V^{F_{search}}(s,z,a),\, V^{SW}(s,z,a),\, V^{F_{ns}}(s,z,a)\right\} \tag{3.6}$$

where

$$V^{F_{search}}(s,z,a) = \max_{c,\,a'} u(c) - \chi + \beta\mathbb{E}\left[\left(\omega_s + f_s'(1-\omega_s)\right)V^M(s,z',a') + (1-\omega_s)(1-f_s')V^F(s,z',a')\right]$$

$$\text{s.t. } c + a' = \delta(1-\tau)w_s z + (1+r)a + div(z)$$

$$V^{F_{ns}}(s,z,a) = \max_{c,\,a'} u(c) + \beta\mathbb{E}\left[\omega_s V^M(s,z',a') + (1-\omega_s)V^F(s,z',a')\right]$$

$$\text{s.t. } c + a' = \delta(1-\tau)w_s z + (1+r)a + div(z)$$

$0 < \delta < 1$ is the job retention scheme replacement rate, and we assume that $\delta > b$ to effectively capture the purpose of the policy. Note that the value of switchers (V^{SW}) is omitted as it is the same as one for those who are matched.

Non-participating (N)

Non-participating households may join the labor force and start searching for jobs despite being ineligible for UI benefits (U_{nb}), or choose to remain outside the labor force (X).

$$V^N(s,z,a) = \max\left\{V^{U_{nb}}(s,z,a),\, V^X(s,z,a)\right\} \tag{3.7}$$

where

$$V^{U_{nb}}(s,z,a) = \max_{c,\,a'} u(c) - \chi + \beta \mathbb{E}\left[f_s' V^M(s,z',a') + (1-f_s')V^N(s,z',a')\right]$$

$$\text{s.t. } c + a' = tr + (1+r)a + div(z)$$

3.2.2 Labor Service Firms

In each sector, there are representative labor service firms operating in a frictional labor market. The labor service firms hire workers and sell labor services to intermediate firms at a competitive market price h. They pay households a real wage w. For simplicity, we assume that the real wage is determined by a following exogenous rule as in Graves (2020),

$$w_{s,t} = \bar{w}_s \left(\frac{h_{s,t}}{\bar{h}_s}\right)^{\epsilon_w}, \tag{3.8}$$

where ϵ_w captures the degree of stickiness in the wage.

We start by describing the problem faced by a labor service firm on furlough. Depending on whether a furloughed household chooses to search or not, the value of the labor firm can be expressed as a weighted average of the respective two values: $J_t^{F_{search}}$, $J_t^{F_{ns}}$, which represent the value of a labor firm whose furloughed worker is searching for jobs or not, respectively.[6] Therefore,

$$J_{s,t}^F = \Pr(F_{search}|F)J_{s,t}^{F_{search}} + \Pr(F_{ns}|F)J_{s,t}^{F_{ns}},$$

where $\Pr(F_{search}|F)$, $\Pr(F_{ns}|F)$ are the average probabilities of households choosing to search (F_{search}) and not to search (F_{ns}) conditional on being furloughed (F),

[6] If the furloughed household chooses to switch, it is the same as a destruction of the match from the labor service firm's perspective. Therefore, we do not include this case in the discussion as the value of the labor firm is always zero.

respectively.[7] We assume that each furloughed labor firm does not hire or sell labor, and remains inactive, subject to an operation cost. This operation cost, x_t, includes maintenance and administrative costs that may arise during the period the firm is not operating. The cost is independently and identically distributed according to an exponential distribution, $x_t \sim \text{Exp}(\theta)$ with CDF $G(\cdot)$. The labor firm with a furloughed worker who is searching pays this cost every period and is subject to the following three possibilities: it may become matched in the next period if its worker is recalled with probability w_s, or it remains furloughed if the worker is neither recalled nor matched with another employer. If the worker is not recalled but finds a new employer, the match is destroyed ($J^U_{s,t} = 0$). Therefore, we have

$$J^{F_{search}}_{s,t} = -x_t + \Lambda \mathbb{E}\left[\omega_s J^M_{s,t+1}(z_{t+1}) + (1-\omega_s)(1-f_{s,t+1})J^F_{s,t+1}\right],$$

where Λ is the labor service firm's discount factor. The labor firm with a furloughed worker not searching faces a similar recursive problem, except that it can remain furloughed only if the worker is not recalled.

$$J^{F_{ns}}_{s,t} = -x_t + \Lambda \mathbb{E}\left[\omega_s J^M_{s,t+1}(z_{t+1}) + (1-\omega_s)J^F_{s,t+1}\right].$$

The value function of a labor firm in sector s, matched with a household whose productivity is z_t is given by

$$J^M_{s,t}(z_t) = \Pr(E|M) J^E_{s,t}(z_t), \tag{3.9}$$

[7] For instance, the average probability of searching while on furlough is computed by

$$\Pr(F_{search}|F) = \frac{\int p\left(F_{search}|F\right) dD_{s,t+1}(e=F)}{D_{s,t+1}(e=F)},$$

where $p(e'|e)$ denote the probability that a household chooses labor choice e' conditional on e and $D_{s,t}(e)$ denotes the measure of households in period t with employment status e in sector s.

where

$$J^E_{s,t}(z_t) = (h_{s,t} - w_{s,t})\, z_t - \Phi^L_s + \Lambda \mathbb{E}\left[(1-\lambda_s) J^M_{s,t+1}(z_{t+1}) + \lambda_s \int^{\bar{x}} \max\left\{J^F_{s,t+1}, 0\right\} dG(x_t)\right].$$

Equation (3.9) indicates that the matched labor firm becomes active if the paired worker decides to work from (3.2).[8] This matched labor firm makes profits by intermediating labor between production firms and households, subject to a fixed operating cost, Φ^L_s. Its continuation value reflects that the labor firm may remain matched ($J^M_{s,t+1}$) if it is not affected by the exogenous separation shock. When affected by the separation shock, it draws a stochastic operation cost. Based on the drawn cost, it decides whether to furlough the worker or break up the match. A threshold value, $\bar{x}_t$ exists, above which the labor firm will break up the match. Operation costs below this threshold determine the endogenous furlough rate, $\xi_{s,t}$, which enters into (3.3),

$$\xi_{s,t} = 1 - G(\bar{x}_t).$$

An unmatched labor service firm incurs a cost, κ, to post a vacancy. Given that the value of an unmatched labor firm is zero, the following free entry condition holds:

$$\kappa = q_{s,t} J^M_{s,t}(z_t), \tag{3.10}$$

where q_s is the job-filling rate in sector s. This condition implies that a vacant labor service firm posts vacancies until the marginal cost of posting (LHS) equals the expected benefits (RHS).

[8] For the other two choices, the value of the firm is zero.

3.2.3 Labor Market

Search is random. Matches between agencies and households are formed according to the standard Cobb-Douglas matching function

$$M_{s,t} = \mu S_{s,t}^{\gamma} V_{s,t}^{1-\gamma} \tag{3.11}$$

where the measure of searchers, S_s, in the labor market in sector s is the sum of five measures of household.

$$\begin{aligned} S_{s,t} = \lambda \int p(E|M_s) dD_{s,t-1}(M_s) + \int p(U_b|U_s) dD_{s,t-1}(U_s) + (1-\omega_s) \int p(F_{search}|F_s) dD_{s,t-1}(F_s) \\ + \int p(U_{nb}|N_s) dD_{s,t-1}(N_s) + \int p(SW_{\tilde{s}}|M_{\tilde{s}}, U_{\tilde{s}}, F_{\tilde{s}}) dD_{\tilde{s},t-1}(M_{\tilde{s}}, U_{\tilde{s}}, F_{\tilde{s}}). \end{aligned} \tag{3.12}$$

The first term of the RHS in (3.12) represents the measure of households who were previously employed but are separated by the exogenous separation shock, λ, in the beginning of the period. From the second to the fourth terms are unmatched, furloughed, and non-participating households searching for jobs, respectively. The last term denotes the measure of those who have switched from the other sector, $\tilde{s}$.

From the matching function in (3.11) and the measure of searchers in (3.12), the sector-specific job-finding rate, $f_{s,t}$, and job-filling rate, $q_{s,t}$, are computed by

$$f_{s,t} = \frac{M_{s,t}}{S_{s,t}} = \mu \theta_{s,t}^{1-\gamma}$$

$$q_{s,t} = \frac{M_{s,t}}{V_{s,t}} = \mu \theta_{s,t}^{-\gamma},$$

where $\theta_{s,t} = \frac{V_{s,t}}{S_{s,t}}$ is the labor market tightness in sector s.

3.2.4 Production

In the production block, we assume that there are three types of producers: intermediate firms, wholesaler, and retailer. In each sector, there is a continuum of intermediate firms that use labor to produce differentiated varieties of goods. These goods are then packaged into a sectoral consumption good by a representative wholesaler. The sectoral goods are then purchased by a retailer, who aggregates them into a single composite consumption good sold to households.

Intermediate Firms

Each monopolistically competitive intermediate good firm indexed by $j \in [0,1]$ in sector s produces using labor $n_{s,jt}$ only, with a linear production function:

$$y_{s,jt} = z_{s,t} n_{s,jt},$$

where $Z_{s,t}$ is the sector-specific TFP. The intermediate good firms set $p_{s,jt}$ taking as given the real rental rate of labor $h_{s,t}$, subject to a quadratic price adjustment cost Θ_p as in Rotemberg (1982) and a fixed operating cost, Φ_s^I. Denote by $J_{s,jt}^F(p_{s,jt-1})$ the (maximal attainable) value of an intermediate firm j in sector s at t that posted price $p_{s,jt-1}$ in $t-1$. The recursive problem of the firm is then

$$J_{s,jt}^F(p_{s,jt-1}) = \max_{p_{s,jt}, n_{s,jt}} \frac{p_{s,jt}}{P_t} y_{s,jt} - h_{s,t} n_{s,jt} - \Theta_{s,t}^p - \Phi_s^I + \frac{1}{1+r_{t+1}} \mathbb{E}_t J_{s,jt+1}^F(p_{s,jt})$$

$$\text{s.t. } y_{s,jt} = z_{s,t} n_{s,jt}$$

$$y_{s,jt} = \left(\frac{p_{s,jt}}{P_{s,t}}\right)^{-\epsilon_p} Y_{s,t}$$

$$\Theta_{s,t}^p = \frac{\kappa_p}{2} \left(\frac{p_{s,jt} - p_{s,jt-1}}{p_{s,jt-1}}\right)^2 \frac{P_{s,t} Y_{s,t}}{P_t}.$$

The first order conditions to this intermediate firm's problem leads to the following sector-specific New Keynesian Phillips curve:

$$1-\epsilon_p+\epsilon_p mc_{s,t}-\kappa_p(\pi_{s,t}-1)\pi_{s,t}+\kappa_p\frac{1}{1+r_{t+1}}\mathbb{E}_t\left[(\pi_{s,t+1}-1)\frac{\pi_{s,t+1}^2}{\pi_{t+1}}\frac{Y_{s,t+1}}{Y_{s,t}}\right]=0,$$

where ϵ_p measures the elasticity of substitution across intermediate goods. $\pi_{s,t}=\frac{P_{s,t}}{P_{s,t-1}}$ denotes the inflation rate in sector s and $\pi_t=\frac{P_t}{P_{t-1}}$ is the aggregate inflation rate across the sectors.

Wholesaler

A representative wholesaler, operating in a perfectly competitive market, purchases the differentiated goods from the intermediate firms and bundles them into a single final sectoral goods using the following CES production technology,

$$Y_{s,t}=\left(\int_0^1 y_{s,jt}^{\frac{\epsilon_p-1}{\epsilon_p}}\,dj\right)^{\frac{\epsilon_p}{\epsilon_p-1}},$$

where $Y_{s,t}$ is the output of sector s. The price of the sectoral good in sector s is

$$P_{s,t}=\left(\int_0^1 p_{s,jt}^{1-\epsilon_p}dj\right)^{\frac{1}{1-\epsilon_p}}.$$

Therefore, this representative wholesaler in sector s solves

$$\max_{y_{s,jt}}\ P_{s,t}Y_{s,t}-\int_0^1 p_{s,jt}y_{s,jt}dj$$

$$\text{s.t. } Y_{s,t}=\left(\int_0^1 y_{s,jt}^{\frac{\epsilon_p-1}{\epsilon_p}}\,dj\right)^{\frac{\epsilon_p}{\epsilon_p-1}},$$

which implies the optimal demand for the intermediate good j,

$$y_{s,jt} = \left(\frac{p_{s,jt}}{P_{s,t}}\right)^{-\epsilon_p} Y_{s,t}.$$

Retailer

To further simplify the model, I add to the model a representative competitive retailer that produces composite final good, Y_t, by combining a bundle of intermediate goods from each sector, according to the CES aggregator. The retailer therefore solves the following optimization problem

$$\max_{Y_{s,t}} \; P_t Y_t - \sum_s P_{s,t} Y_{s,t}$$

$$\text{s.t. } Y_t = \left[\alpha^{\frac{1}{\eta}} Y_{1,t}^{\frac{\eta-1}{\eta}} + (1-\alpha)^{\frac{1}{\eta}} Y_{2,t}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}, \tag{3.13}$$

where $Y_{s,t}$ is the amount of sectoral goods purchased by the retailer from sector s, and P_t is the aggregate price index defined in (3.1). As is standard, the demand for each sectoral good is given as,

$$Y_{1,t} = \alpha \left(\frac{P_{1,t}}{P_t}\right)^{-\eta} Y_t,$$

$$Y_{2,t} = (1-\alpha) \left(\frac{P_{2,t}}{P_t}\right)^{-\eta} Y_t. \tag{3.14}$$

3.2.5 Government

The government budget constraint is

$$G + rB^g + tr \sum_s \int dD_s(e = SW, X, U_{nb}) + bw_s \sum_s \int zdD_s(e = U_b) + \delta w_s \sum_s \int zdD_s(e = F)$$
$$= \tau \left[w_s \sum_s \int zdD_s(e = E) + bw_s \sum_s \int zdD_s(e = U_b) + \delta w_s \sum_s \int zdD_s(e = F \right.$$

where government debt, B^g, and spending, G, are exogenous.

3.2.6 Central Bank

The monetary authority follows the standard Taylor rule for the nominal interest rate

$$i_t = \bar{r} + \phi_\pi(\pi_t - \bar{\pi})$$

where $\bar{r}$, $\bar{\pi}$ are the steady-state real interest rate and inflation, respectively. ϕ_π governs the responsiveness of the nominal rate to the deviation of inflation from its steady state level.

3.2.7 Equilibrium

A stationary recursive equilibrium is a set of prices $\{\pi_{1,t}, \pi_{2,t}, w_{1,t}, w_{2,t}, r_t, i_t\}$, aggregates $\{Y_{1,t}, Y_{2,t}, C_{1,t}, C_{2,t}, N_{1,t}, N_{2,t}, div_t\}$, labor market variables $\{q_{1,t}, q_{2,t}, f_{1,t}, f_{2,t}V_{1,t}, V_{2,t}, \xi_{1,t}, \xi_{2,t}\}$, individual household policy rules $\{c_t, a_{t+1}, p(e_t|e_{t-1})\}$, and joint distributions of agents $D_t(s, z, a)$ such that:

1. Given aggregate shocks, prices, households solve their optimization problem with value functions $\left\{V^M, V^U, V^F, V^N, V^E, V^{U_b}, V^{U_{nb}}, V^{SW}, V^{F_{search}}, V^{F_{ns}}, V^X\right\}$ and policy rules $\{c_t, a_{t+1}, p(e_t|e_{t-1})\}$;

2. Given aggregate shocks, prices, and quantities, production firms in each sector maximize their profits;

3. Given aggregate shocks, prices, and quantities, labor service firms in each sector optimize;

4. Government budget constraint holds;

5. The monetary authority follows monetary policy rule;

6. The sequence of distributions satisfies aggregate consistency conditions;

7. All markets clear for labor, goods in both sectors, and economy-wide asset market.

   - Asset market:

$$\underbrace{\int a_{t+1}\, dD}_{=A_{t+1}} = B^g$$

   - Labor markets:

$$\underbrace{N_{s,t}}_{\text{demand}} = \underbrace{L_{s,t}}_{\substack{\text{effective}\\ \text{supply}}} \quad \text{for each } s$$

   where

$$L_{s,t} = \int z\, dD_{s,t}(e = E)$$

   - Goods markets:

$$Y_{s,t} = C_{s,t} + G + \kappa V_{s,t} + \Theta^p_{s,t} + \Phi^I_s + \Phi^L_s \quad \text{for each } s$$

3.3 Calibration

The unit of time in the model is a quarter. A summary of our calibration is provided in Table 3.1. Note that the calibration is preliminary and is based on standard values in the literature. We plan to discipline the model in future work.

Households We calibrate the discount factor, β, such that the steady-state annual real interest rate is 5%. The coefficient of relative risk aversion, σ, is set to 2, so that the inverse of the elasticity of intertemporal substitution (EIS) is 0.5. The disutility of work, φ, is chosen so that the effective labor supply across the two sectors amounts to 80% of the population, while the disutility of search, χ, is adjusted to target a steady-state unemployment rate of 4%. The elasticity of substitution between sectoral goods, η, is set at 2, and the utility weight on goods produced in sector 1, α, is set to 0.5. We set the scale of taste shocks, σ_ϵ, to 0.01 for all households.[9] The idiosyncratic productivity parameters, σ_z and ρ_z, are set to 0.5 and 0.95, respectively.

Production On the supply side, we set ϵ_p to 7, which is a standard value in the literature. We set $\kappa_p = 200$ to match a Phillips curve slope, ϵ_p/κ_p, of 0.035, following Ferriere and Navarro (2024). We set the fixed cost of production so that intermediate firms make zero profits in steady state.

Labor Market We set the exogenous separation rate, λ, to 0.1 and the probability of recall, ω, to 0.5. We also set the matching efficiency parameter, μ, to 0.65 and the elasticities of matches with respect to searchers, γ, to 0.5, all of which are in the range of values discussed in the literature. We calibrate the threshold level for the operational cost, $\bar{x}$, such that the probability of being furloughed, ξ, is 50%. This implies that only 50% of separated workers have the opportunity to remain with their employers on furlough. Following the estimate in Fujita and Moscarini (2017), we set the probability of recall, ω, to 0.5. We also calibrate the vacancy posting cost, κ, based on the free-entry condition in equation (3.10).

Policy As for the policy parameters, the supply of government bonds, B^g, is fixed at 5.6 to match the ratio of aggregate liquid assets to annual GDP, as in McKay et al. (2016). The government spending, G, is set to 0.15. The unemployment insurance replacement rate is given as $b = 0.4$ and the job retention scheme replacement rate, δ, is set to 0.5. The transfers, tr, are set to 0.1. Lastly, the parameter in the Taylor rule, ϕ_π, is set to 1.5.

[9] Adjusting the scale of taste shocks induces a mild smoothing of the labor choice policy. Technically, one can match population shares of different employment statuses by varying these values.

Table 3.1: Calibration

Parameter	Description	Baseline (w/o exp risk)	Targets/Source
1. Households			
β	Discount factor	0.974	$r = 0.05$
ρ_z	Persistence of inc shock	0.95	
σ_z	Std of inc shock	0.5	
σ	Inverse of the EIS	2	Standard
φ	Disutility of labor	0.969	$N = 0.8$
χ	Disutility of search	0.590	$u = 0.04$
σ_ϵ	Scale of taste shock	0.01	
2. Production			
ϵ_p	Elasticity of substitution	7	Standard
κ_p	Cost of adjusting prices	200	Ferriere and Navarro (2023)
3. Labor market			
λ	Exogenous separation rate	0.1	JOLTS
ω	Prob of recall	0.5	Fujita and Moscarini (2017)
$\bar{x}$	Threshold for operation cost	0.103	$\xi = 0.5$
ξ	Prob of furlough	0.5	
μ	Matching efficiency	0.65	Standard
γ	Matching elasticity	0.5	Standard
κ	Vacancy posting cost	0.098	Free-entry condition
ϵ_w	Wage stickiness parameter	0.45	Gornemann et al. (2016)
4. Policy			
G	Gvt spending	0.15	Spending-to-GDP
B^g	Gvt debt	5.6	McKay et al. (2016)
τ	Income tax rate	0.313	Gvt budget constraint
b	UI replacement rate	0.4	Standard
δ	Job retention replacement rate	0.5	
ϕ_π	Taylor rule coefficient on inflation	1.5	Standard

Solution Method The household optimization problem in our model involves both discrete choices (labor supply) and continuous choices (consumption-savings). Standard solution methods that rely on linearization are not well-suited for models with discrete-continuous choices, as combining these choices presents computational challenges that the methods cannot handle.[10] Iskhakov et al. (2017) develop an efficient solution method, extending the endogenous gridpoint method (DC-EGM) to handle these problems by including extreme value type I taste shocks. We build on this approach to solve the households' optimization problem for the steady state, and then apply the "fake-news algorithm" by Auclert et al. (2021) for transitional dynamics. For the detailed structure of the household's problem, we extend Bardóczy (2022).

3.4 Results

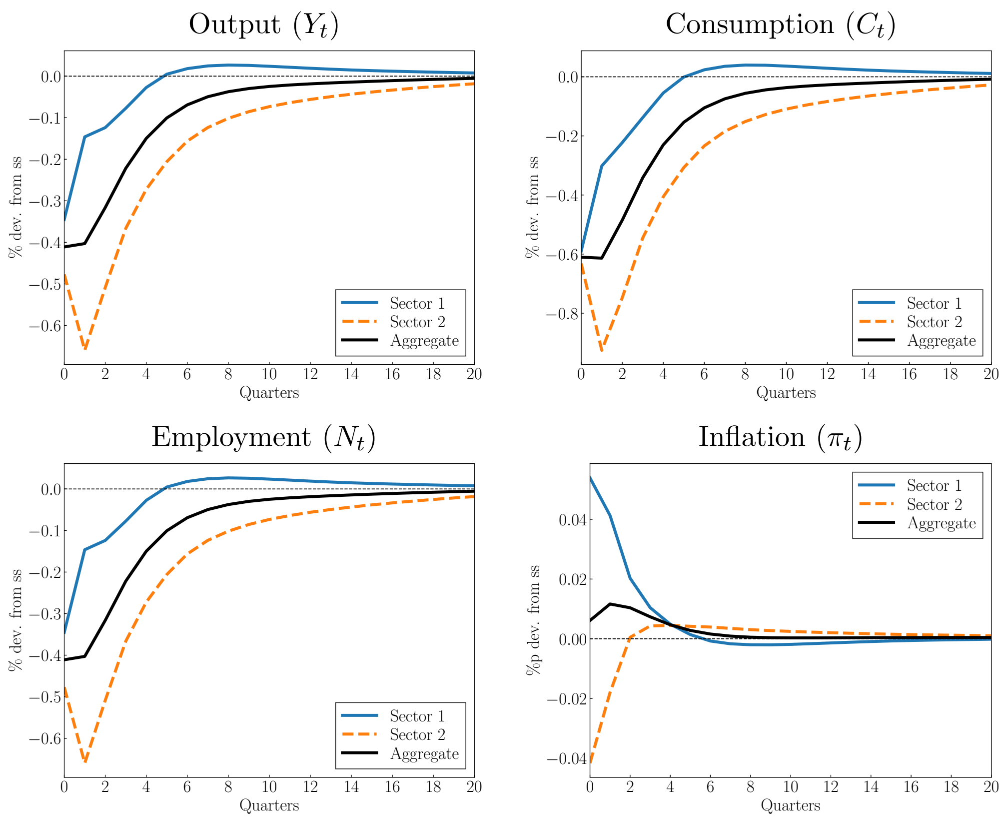
To evaluate how well the model captures the post-pandemic economy through a separation shock, we focus on the impulse responses following such a shock.[11] Specifically, we consider a 1% increase in the exogenous separation rate in sector 1, assuming that the sector 1 is a service-related sector.

Before discussing the results in detail, it is worth mentioning two key channels that operate in response to the separation shock. The first channel arises from the sudden destruction of firm-worker matches, which leads to an increase in labor demand by firms. Specifically, this separation shock creates a scarcity of labor. To maintain their production levels, firms increase vacancy postings to rehire laid-off workers, raising labor market tightness and offering higher wages to attract labor. The second channel, however, operates through aggregate household demand in the opposite direction.

[10] Discrete choices can create kinks or non-concavities in policy functions, leading to jumps and potentially multiple solutions.

[11] In Appendix 3.C, we also present results in the steady-state, focusing on households' labor supply choice policies.

Figure 3.4: Impulse responses to a sector-specific separation shock



Note: Each line represents the sectoral response of its respective variable, while the black line shows the aggregate response.

The unexpected separation results in a loss of household income, leading to reduced demand for goods and a subsequent economic downturn. As a result, firms' labor demand decreases, and wages decline. In standard models with search and matching frictions, the second channel typically dominates the other.[12] Consequently, when job market conditions deteriorate, such as through increased risk of job loss, economic activity is expected to contract.

The responses of the economy from our model to a sector-specific separation shock are given in Figure 3.4. To begin with the aggregate responses (black line), they are in line with predictions from other standard models. Following the sectoral separation shock to sector 1, aggregate employment declines, leading to reductions in output

[12] See Challe and Ragot (2016), Challe et al. (2017), and Ravn and Sterk (2017).

and consumption—a recessionary effect dominated by the second channel discussed above.

Turning to the sectoral responses, however, reveals a surprising outcome. Although the increase in the destruction of matches leads to a recession in both sectors, the impact is less severe in sector 1. The declines in output, consumption, and employment are smaller compared to those in sector 2. This is surprising because sector 1 is directly impacted by the shock and would typically be expected to experience a more severe recession. Figure 3.5 illustrates the mechanism driving this unexpected result. In sector 1, where unexpected match destructions occur, labor demand actually increases, as predicted by the first channel. This rise in labor demand, denoted by an increase in the real competitive wage h_t in Figure 3.5, generates two forces that facilitate a faster recovery from the recession in sector 1. First, as labor demand rises, wages in sector 1 increase accordingly by equation (3.8), making jobs in this sector more attractive to workers. In addition to this, increased labor demand in sector 1 induces firms to post more vacancies, further tightening the labor market and increasing the job-finding rate. Together, these factors improve job prospects in sector 1, encouraging workers to switching from sector 2 in search of higher wages and better employment opportunities. As a result, employment in sector 1 rebounds quickly, mitigating the severity of the recession compared to sector 2.

There still remains the question of why labor demand declines in sector 2. The answer is associated with the second channel in which the separation shock creates negative demand effects in general equilibrium. Specifically, the match destruction in sector 1 reduces the production of sector 1 goods, which in turn lowers the demand for goods produced in sector 2 (see equation (3.13) and (3.14)). Firms in sector 2 respond to the reduced demand by cutting back on labor. Given the symmetry between the two sectors, the mechanisms observed in sector 1 operate in reverse in sector 2. Following a decline in labor demand, wages fall, the job-finding rate declines,

Figure 3.5: Impulse responses to a sector-specific separation shock

Note: Each line represents the sectoral response of its respective variable, while the black line shows the aggregate response. Note that the unemployment and switcher lines illustrate the response of each population measure following the shock. Specifically, the lines for the switcher capture the flow of workers switching from their respective sector to the other sector. For example, the dashed orange line represents the change in the population share of workers who were previously in sector 2 but decided to move to sector 1, and vice versa for the solid blue line.

and employment decreases in sector 2, with workers leaving for sector 1.[13] As a result, this dynamic creates a counterintuitive outcome, where a shock originating in sector 1 spills over to sector 2, resulting in a more pronounced recession in the sector indirectly exposed to the shock.

To gain further insight into this surprising result, we repeat the experiment with a different assumption on price rigidity in sector 2. Specifically, we eliminate price rigidity in sector 2 in order to hypothetically shut down the negative demand effect channel that arise in general equilibrium. However, the results, presented in Figure D1 in the appendix, indicate that this adjustment also fails to account for the puzzling results regarding the sectoral responses. Under fully-flexible prices, labor demand and real wages remain unchanged in sector 2 following the separation shock in sector 1. Instead, flexible prices fully adjust to the reduced demand, resulting in a sharp

[13]Although there are differences in modeling and the type of shock, this result is somewhat related to the findings of Guerrieri et al. (2022), where a supply shock in one sector generates a demand shock in another sector.

decline in inflation. In contrast, sector 1, with relatively rigid prices, responds to the shock by adjusting real variables, such as labor. Consequently, labor demand, wages, and the job-finding rate in sector 1 increase even more than in the case where both sectors had sticky prices. Ironically, this labor market response boosts employment, production, and consumption from period $t = 1$ onward.[14]

Thus, we conclude that relying solely on the separation shock is insufficient to fully explain the dynamics in economy observed after the pandemic. The model predicts that, even after a separation shock, firms still strive to maintain production, which prevents labor demand from declining and instead causes it to increase. However, in reality, rehiring workers was neither as active nor as smooth as the model predicts. This indicates the need to consider additional shocks and improve the model framework in our future work to better account for observed sectoral responses in data.[15]

3.5 Conclusion

In this paper, we develop a two-sector heterogeneous agent New Keynesian model with incomplete markets and labor market frictions to study sectoral labor reallocation and macroeconomic dynamics in response to job separation shocks. Employing this framework, we explore whether such shocks can account for labor market and macroeconomic dynamics observed during the COVID-19 pandemic.

We find that the model predicts unexpected patterns in the transitional dynamics. While aggregate responses, such as rising unemployment and declining production, are consistent with the predictions of standard macroeconomic models and empirical evidence, sectoral outcomes reveal puzzling results. The directly impacted sector

[14] Conversely, if sector 1 is flexible, the negative demand effect in sector 2 intensifies, inducing the sector to further reduce its labor demand and deepening the recession.

[15] For instance, other studies have considered TFP shocks, capturing disruptions to input utilization, or demand reallocation shocks to model the pandemic.

(i.e., sector 1) recovers faster, driven by higher labor market tightness that improves job-finding rates and wages. Conversely, the indirectly affected sector (i.e., sector 2) experiences a more severe and prolonged downturn due to negative demand spillovers, leading to a deeper contraction.

These findings point to limitations in the current model's ability to fully capture the dynamics of sectoral responses during the pandemic. To address these shortcomings, future research will focus on incorporating additional mechanisms, such as demand-side shocks or more intersectoral linkages, to better reflect observed empirical results.

A promising approach involves modeling the pandemic as a demand reallocation shock from the service sector to the goods sector, as studied in Baqaee and Farhi (2022) and Ferrante et al. (2023). Under this framework, the massive separations could be the results of firms in the service sectors laying off their workers as consumers shifted their preferences from services to goods. The puzzling results in this paper arise due to increased labor demand following separations in the directly affected sector, which contrasts with typical recessionary patterns. Modeling the pandemic as a demand reallocation shock may help generate a decrease in the labor demand in the directly affected sector, as firms reduce production. Contrary to this, the indirectly affected sector may experience a less severe recession due to the increased demand for goods. This leads to relatively higher wages, the job-finding rate, and more vacancy postings, all of which contribute to the quick recovery, as observed in data. Therefore, such extensions will enhance the model's ability to explain the labor market adjustments, and ultimately, allow us to evaluate the effectiveness of the policies implemented during the pandemic to mitigate the effects of the sector-specific disruptions.

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Appendix

3.A Walras's Law

We have the following budget constraints from

- Households:

$$C_t + A_{t+1} = (1-\tau_t)\sum_s w_{s,t}\int zdD_{s,t}(e=E) + b(1-\tau_t)\sum_s w_{s,t}\int zdD_{s,t}(e=U_b)$$
$$+\delta(1-\tau_t)\sum_s w_{s,t}\int zdD_{s,t}(e=F) + tr\sum_s\int dD_{s,t}(e=SW, U_{nb}, X) + (1+r_t)A_t + div_t,$$

  where $div_t = d_t^L + d_t^I$, each of which is defined below.

- Labor service firms:

$$\sum_s d_{s,t}^L + \sum_s w_{s,t}\int zdD_{s,t}(e=E) + \kappa\sum_s V_{s,t} + \sum_s \Phi_s^L = \sum_s h_{s,t}\int zdD_{s,t}(e=E)$$

- Intermediate firms:

$$\sum_s d_{s,t}^I + \sum_s h_{s,t}\int zdD_{s,t}(e=E) + \sum_s \Theta_{s,t}^p + \sum_s \Phi_s^I = \sum_s\left(\frac{P_{s,t}}{P_t}\right)Y_{s,t}$$

- Government:

$$G + rB^g + tr\sum_s \int dD_s(e = SW, X, U_{nb}) + bw_s\sum_s \int zdD_s(e = U_b) + \delta w_s \sum_s \int zdD_s(e = F)$$
$$= \tau\left[w_s\sum_s \int zdD_s(e = E) + bw_s\sum_s \int zdD_s(e = U_b) + \delta w_s \sum_s \int zdD_s(e = F)\right]$$

We also have the asset market clearing conditon

$$A_{t+1} = B^g.$$

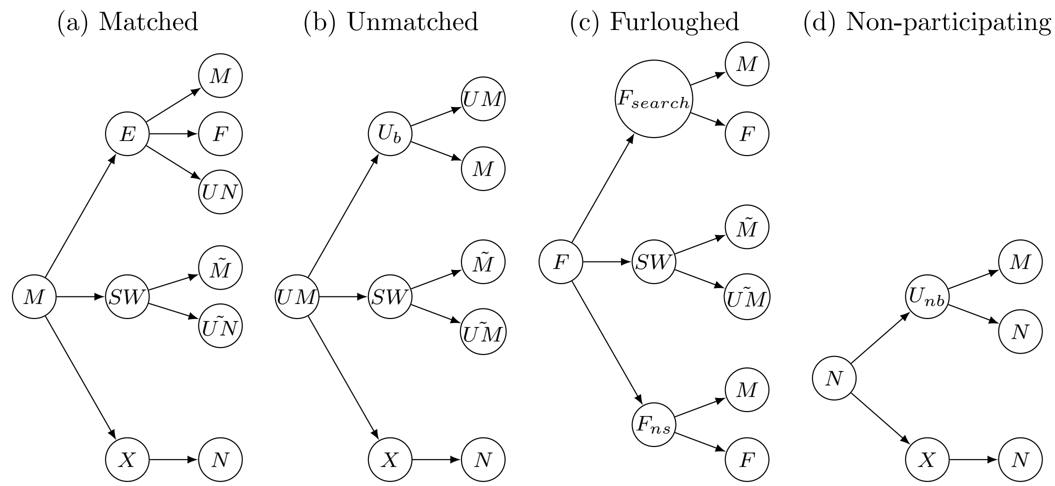
Combining all the budget constraints and the asset market clearing condition should imply the following economy-wide resource constraint.

$$\sum_s C_{s,t} + G + \kappa\sum_s V_{s,t} + \sum_s \Theta^p_{s,t} + \sum_s \Phi^I_s + \sum_s \Phi^L_s = \sum_s Y_{s,t}.$$

3.B Labor Market Transition

3.B.1 Employment Transition

Figure B1: Employment transition



Note: Each diagram illustrates how a household's employment status evolves, depending on the initial status at the beginning of the period. Note that the transition from the first to the second node occurs within the period, while the transition from the second to the third node takes place as the household moves to the next period. Those with a tilde indicate the employment status in the other sector.

3.B.2 Law of Motion for Employment Status

Figure B1 implies the following law of motion for the matched, unmatched, furloughed, and non-participating households.

- Matched (M):

$$M_{s,t} = \underbrace{(1-\lambda_{s,t}+\lambda_{s,t}f_{s,t})\int dD_{s,t-1}(e=E)}_{\text{Previously employed hhs who are matched again}} + \underbrace{f_{s,t}\left(\int dD_{s,t-1}(e=U_b)+\int dD_{s,t-1}(e=U_{nb})\right)}_{\text{Previously unemployed hhs who are matched}}$$
$$+ \underbrace{(\omega_{s,t}+f_{s,t}(1-\omega_{s,t}))\int dD_{s,t-1}(e=F_{search})}_{\text{Previously furloughed hhs who are matched either from recall or searching}} + \underbrace{\omega_{s,t}\int dD_{s,t-1}(e=F_{ns})}_{\text{Previously furloughed hhs not searching who are matched from recall}}$$
$$+ \underbrace{f_{s,t}\int dD_{\tilde{s},t-1}(e=SW)}_{\text{Switched hhs who are matched}}$$

- Unmatched (UM):

$$U_{s,t} = \underbrace{\lambda_{s,t}(1-f_{s,t})(1-\xi_{s,t})dD_{s,t-1}(e=E)}_{\substack{\text{Previously employed hhs who are} \\ \text{separated without being furloughed}}} + \underbrace{(1-f_{s,t})\left(\int dD_{s,t-1}(e=U_b) + \int dD_{s,t-1}(e=U_{nb})\right)}_{\substack{\text{Previously unemployed hhs who are} \\ \text{not matched again}}} + \underbrace{(1-f_{s,t})\int dD_{\tilde{s},t-1}(e=SW)}_{\text{Switched hhs who are not matched}}$$

- Furloughed (F):

$$F_{s,t} = \underbrace{\lambda_{s,t}(1-f_{s,t})\xi_{s,t}\int dD_{s,t-1}(e=E)}_{\substack{\text{Previously employed hhs who are} \\ \text{furloughed}}} + \underbrace{(1-\omega_{s,t})\left((1-f_{s,t})\int dD_{s,t-1}(e=F_{search}) + \int dD_{s,t-1}(e=F_{ns})\right)}_{\substack{\text{Previously furloughed hhs who are} \\ \text{furloughed again}}}$$

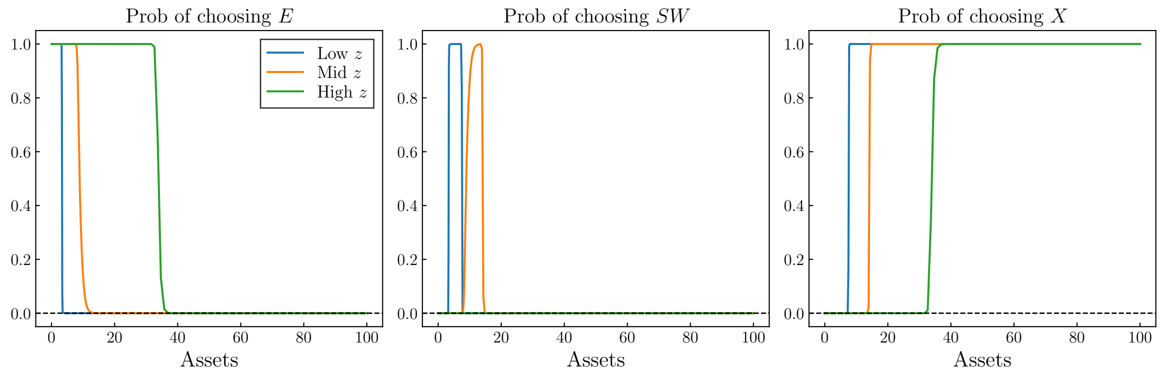
- Non-participating (N):

$$N_{s,t} = \underbrace{\int dD_{s,t-1}(e=X)}_{\text{Previously non-participating hhs}} + \underbrace{(1-f_{s,t})\int dD_{s,t-1}(e=U_{nb})}_{\substack{\text{Previously ineligible unemployed hhs who are} \\ \text{not matched}}}$$

3.C Labor Choices

Figure C1 shows the labor choices of matched households by productivity and assets. The model predicts that the more productive the matched households are, the more likely they are to accept the offer (and become employed) and less likely to exit the labor force. This is because it is more beneficial to accumulate sufficient assets by increasing labor supply when they are more productive. In contrast, among the matched with lower productivity, those close to the budget constraint tend to work in order to avoid becoming constrained, while those with a certain level of assets are less

Figure C1: Labor supply choices: Matched

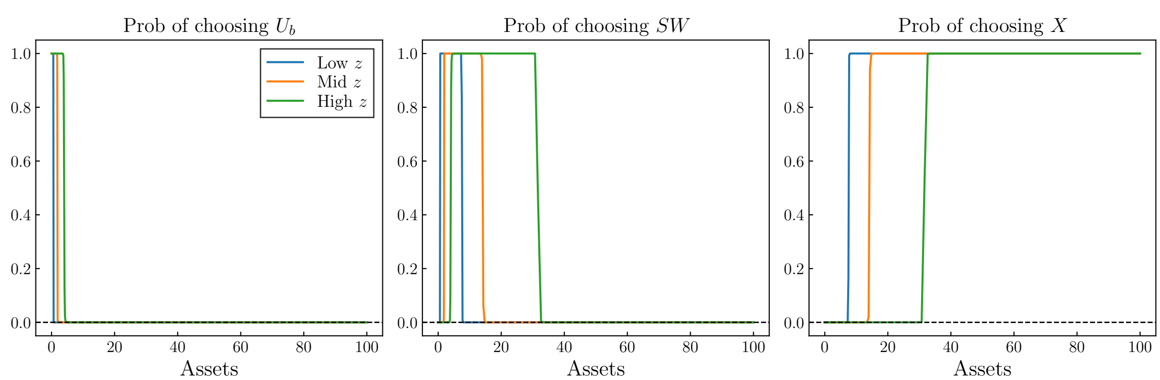


Note: These figures plot the probabilities that each option is selected by matched (M) households with different productivities and asset holdings. As indicated in (3.2), every matched household must choose one option in the beginning of the period. Therefore, connecting the lines representing the same productivity level across the three panels should result in a total of 1 throughout the asset distribution.

likely to work, as they prefer not to incur the disutility of working. Additionally, the matched households rarely switch sectors due to the costs associated with the switch. Only a little share of those with lower productivity move to the other sector, as those with higher productivity have little incentive to do so.

Figure C2 reports the labor choices of the unmatched. The most significant

Figure C2: Labor supply choices: Unmatched

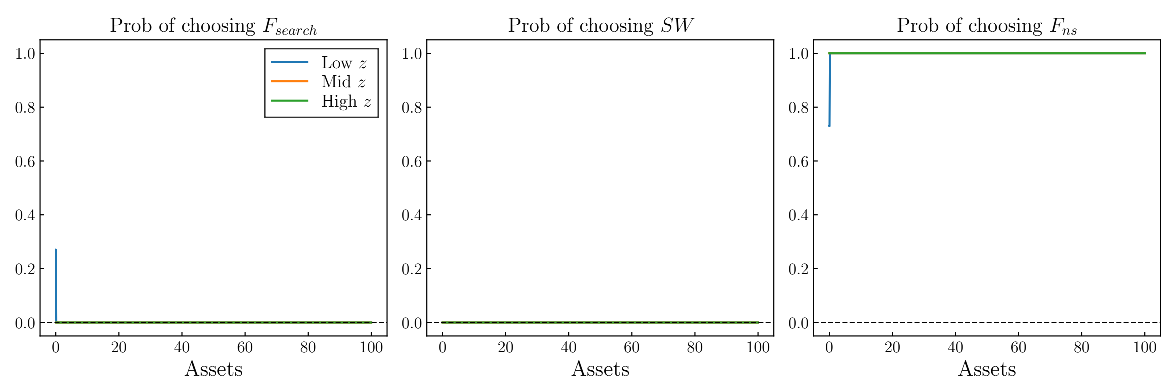


Note: These figures plot the probabilities that each option is selected by unmatched (UM) households with different productivities and asset holdings. As indicated in (3.5), every unmatched household must choose one option in the beginning of the period. Therefore, connecting the lines representing the same productivity level across the three panels should result in a total of 1 throughout the asset distribution.

difference between the unmatched and the matched is that the unmatched switch sectors more frequently. Without a job offer, the opportunity cost of switching is lower, compared to the matched. Therefore, if wages or labor market prospects are more favorable in the other sector, they are more likely to switch. However, the model indicates that those with fewer assets do not switch. In order to make a switch, one has to spend one more period unemployed, forgoing the unemployment benefits they would otherwise receive. As a result, households that cannot even afford this temporary loss stay in the same sector, continuing their job search. Additionally, as the model predicts, more productive unmatched households are more likely to engage in job search (U_b), as the expected gains from searching are larger for those with higher productivity.

Figure C3 illustrates the labor choices of households on furlough in the beginning of the period. According to our model, nearly all furloughed households opt not to

Figure C3: Labor supply choices: Furloughed



Note: These figures plot the probabilities that each option is selected by furloughed (F) households with different productivities and asset holdings. As indicated in (3.6), every furloughed household must choose one option in the beginning of the period. Therefore, connecting the lines representing the same productivity level across the three panels should result in a total of 1 throughout the asset distribution.

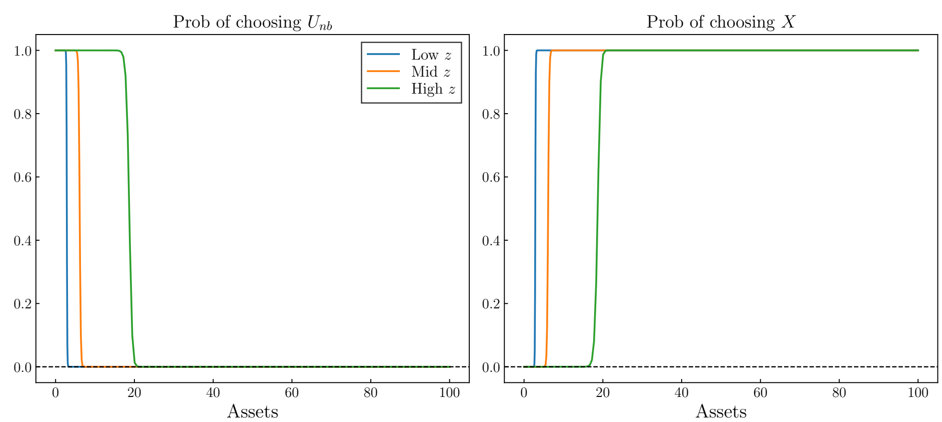
search while on furlough.[16] They prefer to stay with their employers throughout the period, without engaging in any job searching or sector-switching activities. The job

[16] Almost all households along the asset distribution choose F_{ns} under the job retention policy. Fully constrained households, especially those with low productivity, however, choose to search while on furlough in order to avoid hitting the budget constraint, even if it incurs some disutility. This is reflected in a spike at the very bottom of the distribution.

retention scheme preserves their incomes at a higher level than unemployment benefits would, which discourages them from searching for other options, even though those options are available. This is clearly a rational decision under such a generous job retention scheme. With a guaranteed income that exceeds unemployment benefits simply by staying with their employer, there is little incentive for them to search jobs or switch sectors, particularly when it involves disutility or a productivity loss. Contrary to the fact that a large share of the unmatched households choose to switch, this implies that the job retention policy signficantly reduces potential intersectoral labor reallocation that would happen otherwise.

Lastly, Figure C4 shows the choices of the households that are outside the labor market. The decisions made by these households bear similarities to those of both

Figure C4: Labor supply choices: Non-participating

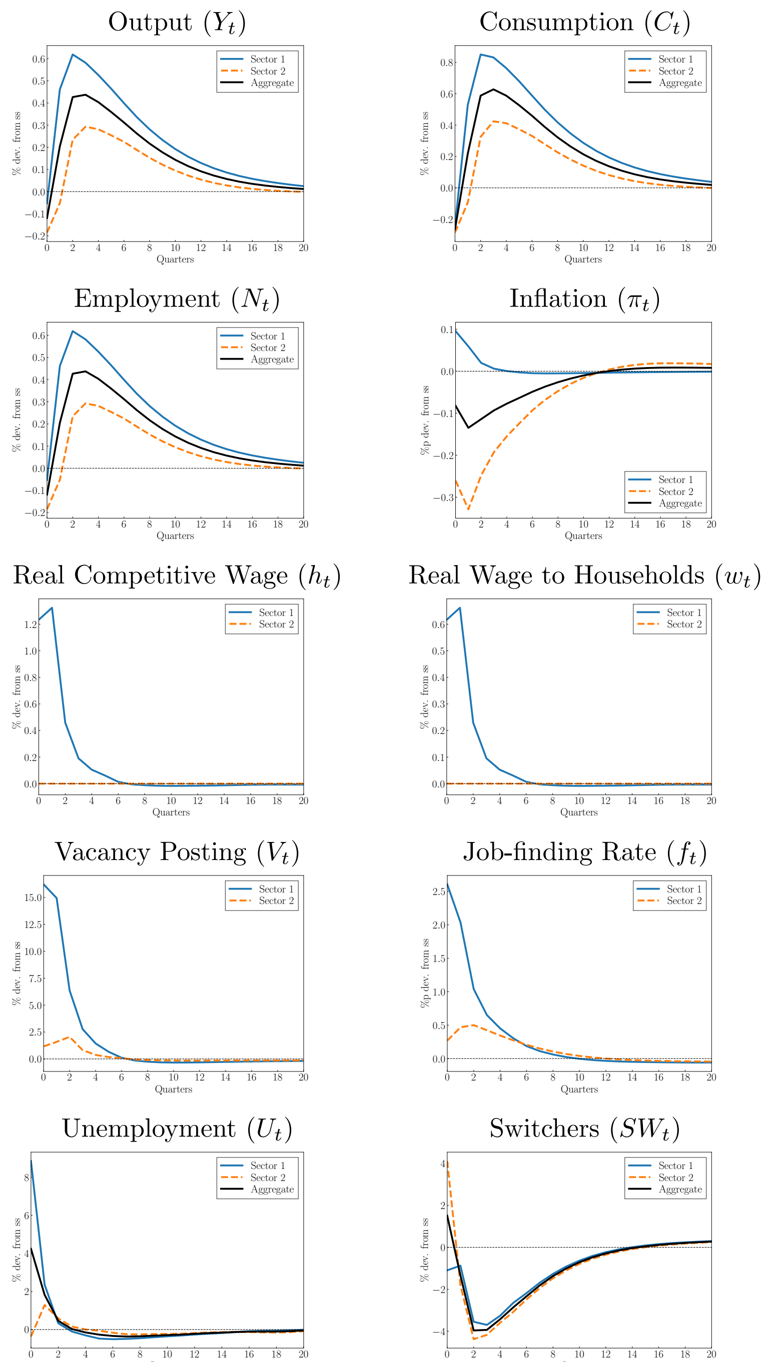


Note: These figures plot the probabilities that each option is selected by non-participating (N) households with different productivities and asset holdings. As indicated in (3.7), every non-participating household must choose one option in the beginning of the period. Therefore, connecting the lines representing the same productivity level across the three panels should result in a total of 1 throughout the asset distribution.

the matched and unmatched households. More productive households are more likely to search for jobs, even if they are not eligible for unemployment benefits, while less productive households tend to remain outside the labor force.

3.D Sticky Sector 1 ($\kappa_p = 200$) vs. Flexible Sector 2 ($\kappa_p = 0$)

Figure D1: Impulse responses to a sector-specific separation shock



Note: Each line represents the sectoral response of its respective variable, while the black line shows the aggregate response.