Is the Weak Coupling Nonlinear SDM Channel Worse Than the Strong Coupling One?

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ABSTRACT

The two working regimes of strong (SCR) and weak coupling (WCR) for multimode fibers (MMF) for space-division multiplexing are compared in terms of nonlinearity and achievable data rates. Leveraging recently developed perturbative models and considerations on MMF parameters, hints are provided that the WCR can achieve similar or higher rates than the SCR. Some considerations on power allocation over frequency channels and modes are provided.

Keywords: fiber optical communication, space-division multiplexing, nonlinear coupling, power optimization.

1. INTRODUCTION

In the field of space-division multiplexing (SDM) the most common fiber structures are multicore fibers (MCFs) and multimode fibers (MMFs). Based on the level of linear coupling among the fiber modes, two common operational regimes are distinguished: the weak coupling regime (WCR) (coupling only within modal groups) and the strong coupling regime (SCR) (coupling among all modes) [1]. Compared to MCFs with same number of modes, MMFs tend to have a faster accumulation of delay spread and mode-dependent loss (MDL) with fiber length [2], [3], [4], higher frequency-dependence in the delay spread [4], and higher mode-averaged attenuation and MDLs [1], [4], [5]. Hence, the receiver digital signal processing (DSP) complexity tends to be also higher. However, MMFs support a significantly larger number of modes (even above 1000) in the same cross-sectional area of a single mode fiber (SMF), thus they exhibit a potentially higher spatial-spectral efficiency than MCFs. This motivates investigating the use of MMFs in long-haul communications, which requires considering Kerr nonlinearity. For the SCR it has already been pointed out that the scaling of the data rates in the SCR is significantly impacted by fiber design through the nonlinear coupling coefficient $\gamma\kappa$ [6]. In this paper we compare the data rates achievable by optimized MMFs when operating in the SCR and in the WCR in the presence of nonlinearity, showing that the second regime can in some cases reach higher date rates than the first one. The WCR would have the further advantage of per-group DSP compensation, which reduces the DSP computational burden compared to full coupling among modes, being considered as one of the main limiting factors in the exploitation of MMFs for long distances [1].

2. CHANNEL MODELING

The WCR is defined by the following Manakov equation [7]

$$\frac{\partial \boldsymbol{A}_m}{\partial z} = -\alpha \boldsymbol{A}_m + \mathrm{j}\boldsymbol{B}_m^{(0)}\boldsymbol{A} - \boldsymbol{B}_m^{(1)}\frac{\partial \boldsymbol{A}_m}{\partial t} - \mathrm{j}\frac{\beta_m^{(2)}}{2}\frac{\partial^2 \boldsymbol{A}_m}{\partial t^2} + \mathrm{j}\gamma\sum_{i=1}^{G}\kappa_{mi}\|\boldsymbol{A}_i\|^2\boldsymbol{A}_m \tag{1}$$

where $\boldsymbol{A}_m = [A_1, \ldots, A_{M_m}]^{\mathrm{T}}$ is the column vector of modal envelopes (T is the transpose operator), M_m is the total number of modes in the m-th modal group (i.e., including polarizations), G is the total number of modal groups, α is the mode-independent fiber loss, $\beta_{m,n}^{(k)} := \frac{\mathrm{d}^k\beta_{m,n}}{\mathrm{d}\omega^k}(\omega_0)$ is the k-th order derivative of $\beta_{m,n}(\omega)$ with respect to ω, $\beta_{m,n}(\omega)$ is the propagation constant of the n-th mode of the m-th group, $\beta_m^{(k)}(\omega) = (1/M_m)\sum_{n=1}^{M_m}\beta_{m,n}^{(k)}(\omega)$ is the average value of $\beta_{m,n}^{(k)}$ per group, ω_0 is the central angular frequency, $\boldsymbol{B}_m^{(0)}$ is the matrix accounting for $\beta_{m,n}^{(0)}$ and random strong intra-group linear coupling, $\boldsymbol{B}_m^{(1)}$ is the diagonal matrix accounting for $\beta_{m,n}^{(1)}$, γ and κ_{mi} are nonlinearity coupling coefficients relative to the Kerr effect (expressions in [7]). Observe that no linear coupling is assumed among modes belonging to different groups.

Note also that the interaction between intra-group coupling and the propagation delays $\beta_{m,n}^{(1)}$ within a group gives rise to the random phenomenon of spatial mode dispersion (SMD). On the opposite, the difference in propagation delays between modes belonging to different groups $\beta_m^{(k)}$ gives rise to the deterministic phenomenon

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of differential mode group delay (DMGD). In particular, we define the DMGD between group a and b as $\mathrm{DMGD}_{ab} = \beta_b^{(1)} - \beta_a^{(1)}$.

In the SCR all modes are assumed to belong to the same modal group, hence (1) reduces to a single vector equation.

To simplify the analysis of (1) we exploit a GN-like perturbative model developed first for the SCR [8] and then extended to the WCR [9]. Such approach models the Kerr-induced nonlinear interference (NLI) as an equivalent additive circularly-symmetric complex Gaussian (CSCG) noise independent from the transmitted signal. That is, the received signal after matched filter detection, sampling, compensation of chromatic dispersion and intra-group coupling, and average carrier phase recovery is [8]

$$y_{\mathbf{i}} = a_{\mathbf{i}} + w_{\mathbf{i}} + n_{\mathbf{i}} \tag{2}$$

where $a_{\mathbf{i}}$ is the transmitted symbol at the $(\mathrm{time, frequency, mode})$ slot $\mathbf{i}$, $w_{\mathbf{i}} \sim \mathrm{CSCG}(0, \sigma^2_{\mathrm{ASE}})$ accounts for the amplified spontaneous emission (ASE) noise, $n_{\mathbf{i}} \sim \mathrm{CSCG}(0, \sigma^2_{\mathrm{NLI},m})$ accounts for the equivalent NLI noise assumed to be independent from both $a_{\mathbf{i}}$ and $w_{\mathbf{i}}$. The NLI noise term can account for both self-phase modulation (SPM) and cross-phase modulation (XPM), or only XPM, if it is assumed that techniques are available at the receiver side to compensate for SPM, like digital back-propagation (DBP) on the channel of interest (COI).

Hence, the rates in $\mathrm{bits/s/Hz/mode}$ achievable by a receiver under the mismatched channel model (2) when transmitting a CSCG modulated signal under an average power constraint can be computed as

$$R = \sum_{c=1}^{N_{\mathrm{WDM}}} \sum_{m=1}^{G} M_m \log\left(1 + \frac{P_m[c]}{\sigma^2_{\mathrm{ASE}} + \sigma^2_{\mathrm{NLI},m}[c]}\right) \tag{3}$$

where $P_m[c]$ and $\sigma^2_{\mathrm{NLI},m}[c]$ are the power and the NLI noise variance, respectively, for the m-th mode at the c-th frequency channel of the wavelength-division multiplexing (WDM) spectrum; N_{WDM} is the total number of WDM channels.

In the WCR, neglecting the frequency dependence of SMD within the bandwidth of a channel (inter-channel SMD approximation), the variance of the NLI noise for any mode within the m-th modal group can be expressed as [9] (adapted to a non-uniform WDM power spectral density (PSD) as in [10, Eq.41])

$$\sigma^2_{\mathrm{NLI},m}[c] = \sum_{c_2=1}^{N_{\mathrm{WDM}}} \eta_{mm}[c,c_2] P_i^2[c_2] P_m[c] + \sum_{\substack{i=1\\ i\neq m}}^{G} \sum_{c_2=1}^{N_{\mathrm{WDM}}} \eta_{mi}[c,c_2] P_i^2[c_2] P_m[c] + \sum_{\substack{i=1\\ i\neq m}}^{G} \sum_{c_2=1}^{N_{\mathrm{WDM}}} \tilde{\eta}_{mi}[c,c_2] P_i[c_2] P_i[c] P_m[c_2] \tag{4}$$

where $\eta_{mm}[c, c_2]$ is the normalized NLI variance accounting for self-group modulation (SGM); $\eta_{mi}[c, c_2]$ and $\tilde{\eta}_{mi}[c, c_2]$ are the normalized NLI variances accounting for cross-group modulation (XGM). The terms of (4) for which $c_2 = c$ account for SPM, the ones for which $c_2 \neq c$ account for XPM. The normalized variances have units of $1/\mathrm{W}^2$.

In the SCR only a single group of degenerate modes is present. Thus, the XGM terms disappear and the NLI variance 4 simplifies to

$$\sigma^2_{\mathrm{NLI}}[c] = \sum_{c_2=1}^{N_{\mathrm{WDM}}} \eta_{11}[c,c_2] P^2[c_2] P[c] \tag{5}$$

where it is clear that in the SCR the power $P[c]$ is mode-independent, conversely to the WCR.

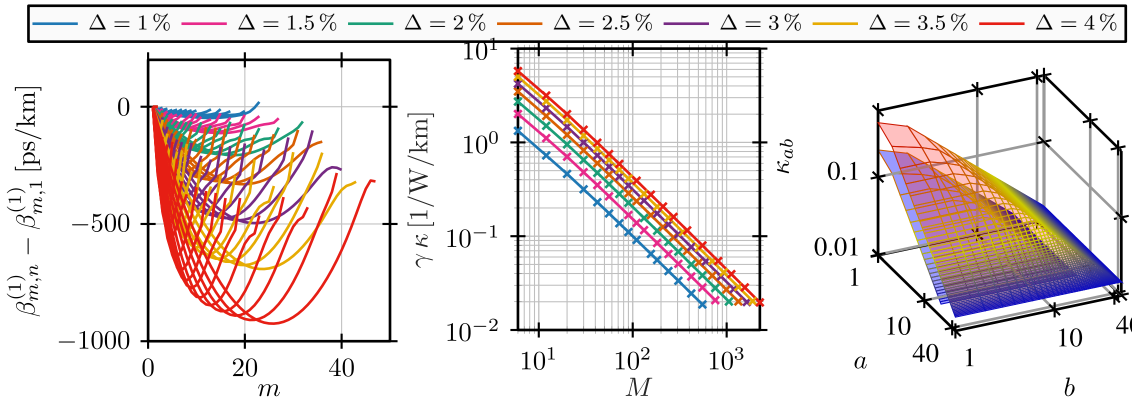
The terms η_{mm}, η_{mi}, and $\tilde{\eta}_{mi}$ depend on the fiber design through several parameters, among which the delays DMGD_{mi} and the nonlinearity coefficients $\gamma\kappa_{mi}$ are particularly important, as it will be clear in the following.

Note from Eqs. (3), (4) and (5) that the various nonlinear interactions depend on the power distribution over frequencies and modes. Hence, data rates can be maximized through power allocation optimization. To do so, as the problem is nonlinear, for the results in the next section we resorted to the interior point algorithm, a numerical nonlinear constrained optimization method, to carry out the maximization.

3. COMPARISON BETWEEN THE STRONG AND THE WEAK COUPLING REGIMES

Cases have been discussed in the literature in which the WCR is shown to suffer more from nonlinearity than the SCR [11], [12]. An indication that this is not a generally valid rule can be drawn by observing that for large enough DMGDs (in combination with SMD) several terms of (4) are rendered negligible [9], [13]. Furthermore, the nonlinear coupling coefficients scale differently with number of modes in the two coupling regimes, in particular for MMF with large number of modes [14], [6]. The combination of these two effects is such that there can exist MMFs for which the NLI in the WCR can be lower than that in the SCR and, thus, the rates can be higher.

To verify that, we computed the rates with (3), (4), and (5) (assuming only XPM) exploiting values of $\Delta\beta_1^{(m)}$ and $\gamma\kappa_{mi}$ (see Fig. 1) computed with a mode solver for sets of optimized trench-assisted graded-index MMFs



(a) (b) (c)

Figure 1: Parameters computed from the optimized graded-index MMFs designed as in [5]: a) $\beta^{(1)}_{m,n} - \beta^{(1)}_{m,1}$, b) $\gamma\,\kappa$, c) κ_{ab}. In a) and b) the line colors refer to the fiber sets with the core-cladding contrast Δ indicated in the legend. In a) each line refers to the relative delays of the various mode groups of a specific fiber within a set. In c) the lower surface refers to the 552-modes MMF of the set with $\Delta = 1\,\%$, the upper one refers to the 2256-modes MMF of the set with $\Delta = 4\,\%$.

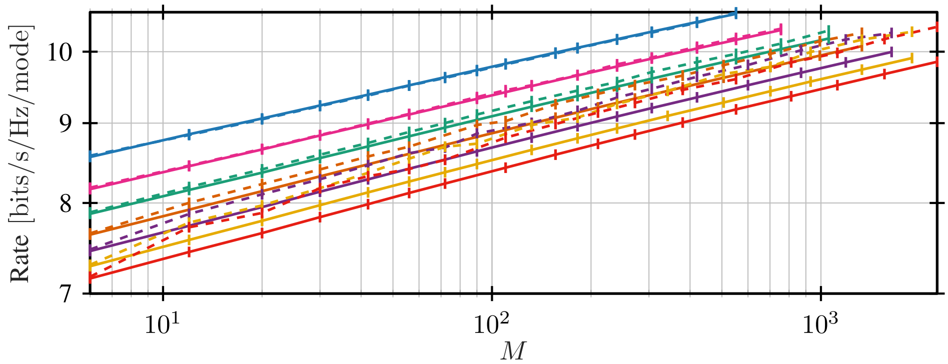


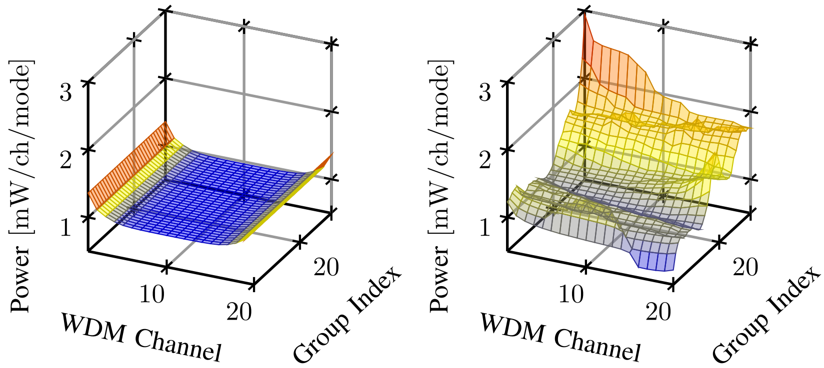
Figure 2: Peak rates for the same fibers of Fig. 1; —+— refers to the SCR (optimized power allocation over frequencies), - + to the WCR (optimized power allocation over frequencies and modes). Line colors refer to the same fiber sets as in Fig. 1.

designed as in [5] for reduced delay spread within acceptable power losses. Every set is characterized by a one common value of core-cladding contrast Δ (from $1\,\%$ to $4\,\%$), and increasing core radius R to support an increasing number of modes. The following other relevant transmission parameters have been assumed: fiber loss $\alpha = 0.2\,\mathrm{dB/km}$, $\beta^{(2)}_m \approx -21.7\,\mathrm{ps^2/km}$ for all modal groups, nonlinear refractive index $n_2 = 2.5 \cdot 10^{-20}\,\mathrm{m^2/W}$, SMD coefficient of $8\,\mathrm{ps}\sqrt{\mathrm{km}}$, a single fiber span of length $100\,\mathrm{km}$ followed by an erbium-doped fiber amplifier (EDFA) compensating the fiber loss with spontaneous emission factor $n_{\mathrm{sp}} = 1.5$, $N_{\mathrm{WDM}} = 20$ WDM channels with rectangular spectral density with bandwidth of $49\,\mathrm{GHz}$ and channel spacing of $50\,\mathrm{GHz}$. In Fig. 2 the peak data rates for the uniform and nonuniform optimized power allocations schemes (over frequency channels for the SCR, over both frequency and modes for the WCR) have been plotted for every fiber. It can be seen from Fig. 2 that, when Δ is large enough, the WCR achieves higher data rates than the SCR. The rates reduce with increasing Δ mainly due to the increase in γ, while the κ_{mi} are substantially independent on Δ [6].

A sample of the optimized power distributions is given in Fig. 3 for both coupling regimes. For the SCR the power distribution is symmetric around the central WDM channel, where a minimum is located as it is the channel that suffers most from the interference from the other channels. For the WCR the power tends to be allocated to higher order modes, which suffer from relatively less NLI (thanks to the favorable scaling of the Manakov coefficients), and to lower frequency WDM channels (because of the distributions of DMGDs in Fig. 1a) which are less affected by (and affect less the other channels through) the shifted four-wave mixing (FWM) efficiency peaks [15].

4. SIDE REMARKS

A number of limiting assumptions has been made in the previous analysis. We list some of the main here. Firstly, the SCR and the WCR are not necessarily the regimes in which a MMF intrinsically operates, with the intermediate one being more realistic [11]. However, there exist techniques to enhance either the SCR, like the use of fiber gratings as mode scramblers [16], or the WCR, like suitable fiber design [1]. The presence of potential uncompensated inter-group linear coupling, which we neglected, would worsen the rates in the



(a) **(b)**

Figure 3: Optimized power distribution for the 1122-modes MMF of the fiber set with $\Delta = 4\,\%$ for: a) the SCR, b) the WCR.

WCR and should be accounted for. MDLs have not been taken into account, while they are expected to play a non-negligible role in the WCR [5]. The approximated closed-form GN-like models exploited here suffer from a lower accuracy compared to the integral expressions for the so-called complete model [9]. Finally, if it is assumed that DBP is employed to compensate SPM, then the WCR might have a latency penalty due to the need to wait for all the modes before processing them.

5. CONCLUSIONS

We compared the SCR and the WCR nonlinear SDM channels in terms of data rates through a perturbation model and optimized fiber parameters. The WCR tends to achieve higher rates when a MMF has a relatively large core-cladding contrast Δ. Power allocation optimization over frequency and modes has been shown to be able to exploit the difference in the nonlinear impairments among the various frequency channels and modes to enhance the data rates, in particular for the WCR. Even though the present analysis needs to be extended to model more accurately a real SDM scenario, the current results encourage the investigation of the design and exploitation of MMFs in the presence of nonlinearity not only in the SCR, but in the WCR as well.

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