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ARTICLE TEMPLATE

An arc-based approach for stochastic dynamic traffic assignment

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ABSTRACT
In dynamic traffic assignment (DTA) models, it seems relevant to consider the uncertainty inherent to motorist route choices. Particularly, choices on realistic transport networks are mostly made using motorists' perceived costs of all routes from their origins to their destinations. We present an approach to address stochastic DTA based on nested cost operators, where motorists choose according to the perceived costs of the remaining trip, namely, from current position to destination. We integrate the Markovian traffic equilibrium by Baillon and Cominetti with the DTA formulation by Addison and Heydecker obtaining an arc-based stochastic DTA model, which we denote in short as ABSDTA. The resulting approach accommodates overlapping routes, respecting costs correlation as well as the first-in-first-out rule. We present a solution method for discrete time periods, computational results on an illustrative network, including sensitivity analyses of the parameters, and comparisons with a previous suitable stochastic DTA model from the literature.

KEYWORDS
Stochastic DTA; Arc-based model; Probabilistic Route Choice

1. Introduction

In the context of traffic studies and transport planning analysis, modelling the behavioural principles that lead motorists to choose their routes on a transport network to fulfill their travel necessities constitutes one of the most relevant issues in evaluating strategic and tactical transport investment projects. Static formulations behind traffic assignment models are well established, with known properties and associated methods to calculate high-quality solutions efficiently. However, the use of static formulations that assume steady-state networks precludes appropriate modelling of congestion in peak periods where overloaded networks cannot achieve steady-state conditions.

In the last few decades, due to new methodological and technological advances, much of the research on these topics has focused on the dynamics governing the behaviour behind the assignment of vehicles on transport networks. Dynamic formulations of traffic assignment have challenged researchers and remain the topic of current research for

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several reasons. In this sense, we highlight the technical difficulty of spatiotemporal traffic modelling that plausibly respects the properties of flow conservation. In addition, the dynamic properties of capacity limitations, causality and flow propagation are important features of the dynamic context. These models are required to furnish estimates of travel times that influence route choice, and for the success of a suitable dynamic formulation, it is essential to build an appropriate route choice model.

Thus, interest in the so-called *dynamic traffic assignment* (DTA) problem has grown considerably, as it takes into account temporal variation in demand in the process of assignment, which provides more realistic modelling of congestion in the context of transport network planning and policy studies. Specifically, DTA establishes the relationship between the dynamic route choice and the consequent variation in travel times given the features of the physical network. It is a natural extension of existing static assignment models, in which routing decisions of motorists are assumed constant through the study period. By contrast, DTA respects that traffic conditions change as motorists move through the network. Research on DTA has focused on motorists' behaviour, model formulations, and solution methods to represent the time dependence, consistent with the observed congestion dynamics.

From the formal introduction of the dynamic version of traffic assignment problems (Merchant & Nemhauser 1978), DTA has been addressed through several approaches. Many of them extend the deterministic Wardropian setting. Szeto & Wong (2011) proposed a classification that distinguishes for a DTA model: (1) the choice dimension; (2) the time dimension; and (3) the formulation. Recently, Friesz & Han (2018) proposed the dynamic user equilibrium (DUE) as a differential variational inequality (DVI), suggesting a fixed-point algorithm as a way to compute a DUE solution. They adapted this algorithm in both continuous and discrete time. Work on DTA in the last two decades has been extensive and diverse in terms of formulations and solution methods; as part of a deep analysis of many DTA developments, we have identified some articles, such as Addison & Heydecker (1996), that have stipulated necessary requirements for the appropriate formulation of a suitable DTA model: a demand profile, a traffic model, and a route-choice model. Comparisons among different traffic models to show how they could contribute to a DTA formulation, based on the pursued objectives, have also been addressed (Addison & Heydecker 1998).

In the present work, a major motivation is to study the effects of uncertainty in motorist behaviour as a key aspect influencing their routing decisions while travelling through the network. In this context, the stochastic version of DTA has been studied considering different ways to incorporate uncertainty in routing decisions with the dynamic evolution of traffic during the modelling period, as represented by DTA models. Several modelling approaches consider probabilistic assumptions in route choice decisions or evolution of flows. In this direction, Han (2003) and Szeto *et al.* (2011) presented analytic route-based models with uncertainty in motorists' choice under dynamic assignment schemes, the former under an assignment protocol explained in detail in the next section, while the latter supports their work on a cell-based formulation. Han (2003) developed an extension to general networks and discrete time of a previous work by Heydecker & Addison (1997) where uncertainty is added to their modelling framework by assuming that route cost is perceived differently by different motorists. In this formulation, route choice is performed through a logit model that considers generalized cost as the dominant criterion, which includes an error with an iid Gumbel distribution, generating a stochastic version of the originally deterministic DTA model. Lim & Heydecker (2005) further extended the previous models, considering departure time choice in conjunction with route choice, defining a condition for what the authors called *dynamic departure time/stochastic user equilibrium, DDSUE*, establishing that *no traveller can improve their perceived travel cost by unilaterally changing their departure time and route combination.* Using approaches based on simulation, Long *et al.* (2019) and Barceló *et al.* (1999) incorporated stochasticity in the choice using stochastic simulation to represent the dynamics behind motorists' route choice with fixed demand. Unlike previous works, Waller & Ziliaskopoulos (2006) developed an analytic route-based model in a DTA context in which demand is uncertain.

As outlined, we find stochastic versions of the route-choice model in the literature. Although these approaches vary in how they address the uncertainty of the motorists' choices when making routing decisions, they consider as the choice criterion the individual's perceived cost of travel from their origin to their destination. Baillon & Cominetti (2008) introduced the concept of *Markovian Traffic Equilibrium* (*MTE*) for the static case. This Markovian framework is distinguished by its traffic assignment model, which considers that a motorist advances towards its destination according to a sequential process of selection of arcs, commanded by a discrete choice model activated at each intermediate node belonging to its journey. In fact, MTE permits different discrete choice models at each node, even allowing combinations of deterministic and stochastic rules for assignment at different stages of the trip. Given the flexibility and properties, an attractive discrete choice model to be used is the multinomial logit, in which, from its current node, the motorist chooses the next arc to proceed to based on its perception of what is left of the trip, pursuing the minimization of its expected minimum cost to reach the destination based on an underlying logit modelling structure. This overcomes the limitations of route-based stochastic models in its treatment of routes with common sections. Zimmermann *et al.* (2021) integrated the MTE approach with capacity constraints developed by Marcotte *et al.* (2004), in which some vehicles are not able to enter a link at a rate that exceeds its capacity. In this context, Mai Anh *et al.* (2015) presented a recursive approach for the static case, where the choice of arcs led to the construction of the route that a user follows. While, in the context of a nested operator for traffic assignment, Wie *et al.* (1995) presented a discrete-time approach for the DUE problem. Fosgerau *et al.* (2013) approached the MTE by generating a model that could be interpreted as dynamic under considerations such as deterministic arc costs. They also addressed overlapping routes by a correction of their utilities. Shimamoto & Kondo (2020) extended a static path flow estimation to a semi-dynamic version in a specific context.

Looking to combine the benefits of a good arc-based model like MTE with the generality and simplicity of Addison & Heydecker's DTA framework, the present paper develops an arc-based dynamic model of route choice in which the remainder of each journey is assigned probabilistically to the available routes. This is integrated into a dynamic setting with suitably chosen traffic models to estimate costs depending on the assigned flows. Specifically, the route-choice model proposed by Baillon & Cominetti (2008) in their MTE model is adapted to consider the dynamic features associated with a DTA formulation by following the modelling considerations established by Addison & Heydecker (1996). We denote this framework as the *arc-based stochastic dynamic traffic assignment model (ABSDTA).*

As mentioned above, an important contribution of this research is the integration of these two well-known approaches in the literature to address different but complementary traffic assignment problems, one focused on solving a stochastic static equilibrium problem (Baillon and Cominetti's MTE), and another oriented to searching for a dynamic traffic assignment (Addison and Heydecker's model). The integration covers both the stochastic and the dynamic features, in a quite natural way, by developing a

DTA formulation using the route-choice model, constructed recursively through arc-choice decisions, proposed in Baillon and Cominetti's model, with the traffic model features established in Addison and Heydecker's dynamic modelling framework, respecting the *first-in-first-out* (*FIFO*) traffic property with *non-vehicle holding* (*NVH*). In addition, as a second contribution to highlight, the way the integration of the models was articulated generated a natural manner to find a solution to this stochastic and dynamic problem, through the implementation of a solution algorithm inspired by *Dial's algorithm* (Dial 1971) but repeated at each time increment and with a reversed order of the two passes of network scanning. To validate our modelling framework and algorithm, we compare our approach with the stochastic dynamic user equilibrium (SDUE) model proposed by Han (2003) and, then, we explore examples and sensitivity calculations to show the features of the solutions under different conditions.

Recalling the classification of Szeto & Wong (2011), our model includes (1) pure route choice including en-route adjustment/reactive capability with fixed demand; (2) within-day study with a continuous horizon; and (3) analytical, arc-based treatment of a single-class user, with a non-physical queue. In our framework, we develop the traffic assignment associated with the MTE to a dynamic version while integrating a capacity restraint concept that differs from that of Zimmermann *et al.* (2021) through the deterministic punctual queuing traffic model, according to which queues are formed whenever the service capacity of an arc is exceeded. Unlike Fosgerau *et al.* (2013), our approach directly addresses both dynamic and stochastic aspects. Moreover, our treatment of overlapping routes is straightforward and the arc-based expressions for flows and queues are explicit.

In the following section (section 2), we discuss the foundations in literature of our approach, introducing the concepts that are then elaborated. In section 3, our proposed is model is presented, after the introduction of the concept of *reasonable arc*, to continue with a detailed explanation of the solution algorithm in section 4. In section 5, we compare the performance of our approach with that of a comparable DTA approach, followed by a presentation of computational results in section 5.3. Finally, in section 6 we present our conclusions, comments and insights for further research.

2. Basics for the Model Formulation

The modelling approach by Heydecker and Addison works as follows. Given a transport network represented by the digraph (N, A), where N is the set of nodes and A is the set of arcs ($A \subseteq N \times N$), for each arc $a \in A$, $E_a(t)$ is the inflow rate to arc a at t and $G_a(t)$ is the outflow rate from arc a at time t.

In the case of the deterministic queuing model, let ϕ_a be the free flow travel time of arc a, Q_a be the queue service capacity of arc a, $L_a(t)$ be the amount of traffic in the queue on arc a at time t and $r_a(t)$ be the delay incurred because of the queue on arc a, having joined it at time t. According to this model, the following equations apply:

$$\frac{dL_a}{dt} = E_a(t - \phi_a) - G_a(t), \tag{1}$$

$$G_a(t) = \begin{cases} E_a(t - \phi_a), & \text{if } L_a(t) = 0 \text{ and } E_a(t - \phi_a) < Q_a, \\ Q_a, & \text{otherwise,} \end{cases} \tag{2}$$

$$r_a(t) = \frac{L_a(t + \phi_a)}{Q_a}, \tag{3}$$

and

$$\tau_a(t) = t + \phi_a + r_a(t). \tag{4}$$

The cost $c_a(t)$ of travel on arc a entering it at time t is given by $c_a(t) = \phi_a + r_a(t)$.

In addition, following Papageorgiou (1990), Heydecker & Addison (2005) show that as a consequence of the FIFO rule applied to traffic travelling to different destinations,

$$G_a^d(\tau_a(t)) = \frac{E_a^d(t)}{\frac{d\tau_a(t)}{dt}}, \tag{5}$$

where $E_a^d(t)$ is destination d's-specific inflow rate to arc a at time t and $G_a^d(\tau)$ is destination d's-specific outflow rate from a at time τ.

Han (2003) generalizes this approach incorporating stochasticity through a logit model for the route choice. Note that in stochastic assignment, not all routes have least cost at their time of use. Considering this, the *stochastic dynamic user equilibrium (SDUE)* traffic assignment model is presented, in which, according to the logit specification with positive dispersion parameter θ, the probability $P_p^{od}(t)$ of using route p at time t between origin-destination (O-D) pair (o,d), among the set R_{od} of all routes from o to d, is given by:

$$P_p^{od}(t) = \frac{\exp(-\theta C_p^{od}(t))}{\sum_{q \in R_{od}} \exp(-\theta C_q^{od}(t))}, \ p \in R_{od}, \tag{6}$$

where $C_p^{od}(t)$ is the cost of using route p, starting at time t, to go from o to d. Heydecker & Addison (1997) show that the route choice model (6) is continuous in cost and, as a consequence of the deterministic queue model, costs $C(t)$ are continuous in time, so that route choice is continuous in time. Thus, in a discrete-time (Δt) solution approach the route choice model (6) can be populated with costs $C_p^{od}(t)$ calculated for time t to give assignment proportions P_p for the time interval $[t, t + \Delta t)$ that have error $O(\Delta t)$.

An intuitive explanation of this approach is given in Sheffi (1985) by the following definition for SDUE: *At every instant, no driver believes that they can improve their perceived travel cost by changing routes unilaterally.* For continuous time, this definition is analytically expressed as follows. Let $q^{od}(t)$ be the demand for the O-D pair (o,d) at time t, $f_p^{od}(t)$ be the flow assigned to route $p \in R_{od}$ at time t, and $\hat{C}_p(t)$ be the least-perceived cost among the routes in R_{od} at time t (which depends on the cost pattern of all routes at time t, $\mathbf{C}(t)$). The authors define the probability of choosing route p at t to go from o to d as:

$$P_p^{od}(t) = \mathbb{P}(\hat{C}_p(t) \leq \hat{C}_{p'}(t), \forall p' \in R_{od} | \mathbf{C}(t)); \tag{7}$$

then, the SDUE is given by:

$$f_p^{od}(t) = P_p^{od}(t) q^{od}(t) \forall p \in R_{od}, \forall od, \forall t, \tag{8}$$

so that

$$\sum_{p \in R_{od}} f_p^{od}(t) = q^{od}(t), \forall od, \forall t, \tag{9}$$

and

$$f_p^{od}(t) \geq 0, \forall od \forall t. \tag{10}$$

In another line of work, Baillon & Cominetti (2008) proposed a stochastic, although static, user equilibrium model, which was built by applying notions related to *Markovian chains*, generating what they introduced as the *Markovian traffic equilibrium* (*MTE*). Here, the flow on routes is obtained by assigning flow to the outgoing arcs from each node i according to those arcs' expected minimum costs, to the different destinations. Given the construction of the model under its *arc-based choice* approach, rather than in a *route-based choice* approach, no enumeration of the routes is required and no independence of the route costs is assumed.

For a destination d, the uncertainty is given by the motorists' perception of the travel costs, towards d, on the arcs. Thus, in the case of arc $a \in A$, the perceived cost is modelled as $\hat{c}_a = c_a + \epsilon_a$, with ϵ_a being a random variable with $\mathbb{E}(\epsilon_a) = 0$. From node $n \in N$, the perceived cost of using route $p \in R_n^d$ is $\hat{C}_p = \sum_{a \epsilon p} \hat{c}_a$.

The MTE model relies on the estimation of the expected minimum cost of travelling from node n to destination d, which is $\hat{W}_n^d = \min_{p \in R_n^d} \hat{C}_p$. Thus, the expected cost of taking a route that starts from node n choosing arc $a = (n, m)$ to proceed to destination d is computed as:

$$\hat{Z}_a^d = \hat{c}_a + \hat{W}_m^d. \tag{11}$$

Thus, given a destination $d \in N$ and an arc $a = (n, m) \in A, n \neq d$, the expected flow V_a^d entering arc a travelling towards destination d and the expected flow X_n^d from node n to d satisfy:

$$V_a^d = X_n^d \mathbb{P}(\hat{Z}_a^d \leq \hat{Z}_b^d, \forall b \in A_n^+). \tag{12}$$

Using a logit model where $\hat{Z}_a^d$ are iid Gumbel variables with expected cost Z_a^d and dispersion parameter θ, yields:

$$Z_a^d = c_a + W_m^d = c_a - \frac{1}{\theta} \ln \sum_{b \in A_n^+ \cap R^d} \exp(-\theta Z_b^d), \tag{13}$$

and

$$\mathbb{P}(\hat{Z}_a^d \leq \hat{Z}_b^d, \forall b \in A_n^+) = \frac{\exp\left(-\theta Z_a^d\right)}{\sum\limits_{b \in A_n^+ \cap R^d} \exp\left(-\theta Z_b^d\right)}, \tag{14}$$

where $a = (n, m)$.

3. The arc-based stochastic dynamic traffic assignment model (ABSDTA)

As mentioned in the first section, the core contribution of this paper is the integration of two adequate models from the literature in a combined framework to address the assignment of vehicles taking both the dynamic and the stochastic features in the process, denoted in short as the ABSDTA model for general transport networks. This framework is developed according to the structure for DTA models established by Addison & Heydecker (1996). One important characteristic of the proposed approach is that the choice is performed at an arc level instead of a route level, as suggested by the line of research developed by Baillon and Cominnetti, which results in a more efficient structure to later on develop an ad-hoc algorithm to find a credible assignment in the case of general networks.

In this section, we introduce and develop *the demand profile*, *the traffic model* and *the arc-choice model* (which serves as the route-choice model), in the context of the integrated approach ABSDTA.

Consider a transport network represented by the digraph (N, A), where N is the set of nodes and A is the set of arcs $(A \subseteq N \times N)$; for each $n \in N$, A_n^- and A_n^+ are the sets of incoming arcs to n and outgoing arcs from n, respectively. For each arc $a \in A$, its free flow travel time ϕ_a and its queue service capacity Q_a are parameters assumed to be known. Next, regarding the characteristics of the demand, there are a set of origin nodes $O \subseteq N$, a set of destination nodes $D \subseteq N$, a set of O-D pairs $OD \subseteq O \times D$ and time-dependent demands $q^{od}(t)$ for each O-D pair $(o, d) \in OD$. The analyzed period, represented by the time interval $[0, T]$, is also known.

In what follows, after the introduction of a fundamental concept in the first subsection, and according to the presented definitions and notations, we develop the three main structures of our proposed DTA approach.

3.1. *Reasonable arcs towards a destination*

Following Dial (1971), given an O-D pair, it is usually assumed that a motorist considers both moving further from their origin and closer to their destination as simultaneous conditions to be satisfied to consider a route as an option. To respect this within a model, the flow of motorists that enters a transport network at a given time through the same origin and going to the same destination would need to be labelled by both its origin and its destination.

In this work, we adopt the idea that while travelling to its destination, a motorist considers only the remaining cost of travel to its destination so that only a label associated with its destination is required. Given this, whenever a demand going to a given destination d enters the network through an origin node o at an instant t and meets an already existing flow (from the incoming arcs to o at time t) going to the same destination d, all flows to d are aggregated as a single flow. From this, when the flow is assigned to the outgoing arcs from a current node to move forward to d, the label of the origin is no longer required.

We adapt the "reasonability" concept, now according to destinations and arcs instead of O-D pairs and routes as in Dial (1971). Let us consider a destination node d; then, an arc (i, j) is a *reasonable arc* towards d if the minimum cost from j to d is less than or equal to the minimum cost from i to d. Intuitively, this means that a motorist does not use arcs that bring them farther from their destination if minimum cost routes are meant to be chosen.

In our approach, for each destination node $d \in D$, the set of reasonable arcs towards d, R^d, can be calculated using the free-flow travel times.

3.2. *Demand profile*

For each O-D pair $(o, d) \in OD$, the time-dependent demand $q^{od}(t)$ from origin o to destination d is given as exogenous. Collectively, these constitute the *demand profile*, the first of the three parts of our proposed ABSDTA model.

3.3. *Traffic model*

For the second structure, the *traffic model*, we adopt the *deterministic punctual queuing model* to represent the traffic behaviour within each arc, considering its features referenced in Addison & Heydecker (1998). This part of the model, for each arc, characterizes the relationship between inflows, outflows, and the variable part of travel times, so determining the travel time component of the cost functions.

For each arc $a \in A$ and at each time $t \in [0, T]$, the specific inflow and outflow travelling to destination $d \in D$ are denoted as $E_a^d(t)$ and $G_a^d(t)$, respectively, while the current length of the queue at the arc is denoted as $L_a(t)$.

Because the travel time on each arc $a \in A$ depends on the total inflow $E_a(t)$ and, according to FIFO is experienced equally by all traffic irrespective of destination, the relationship between destination-specific outflows and their corresponding inflows is

$$G_a^d(\tau_a(t)) = \begin{cases} E_a^d(t) \text{ if } L_a(\tau_a(t)) = 0 \text{ and } 0 \le E_a(t) < Q_a, \\ \frac{Q_a}{E_a(t)} E_a^d(t) \text{ otherwise,} \end{cases} \tag{15}$$

$$\frac{dL_a(t)}{dt} = E_a(t - \phi_a) - G_a(t), \tag{16}$$

where

$$E_a(t) = \sum_{d \in D} E_a^d(t), \tag{17}$$

and

$$G_a(\tau) = \sum_{d \in D} G_a^d(\tau). \tag{18}$$

Together, these expressions show how the traffic model can be used to calculate destination-specific link outflows $G_a^d(\tau(t))$ from the corresponding link inflows $E_a^d(t)$ at an earlier time, respecting causality. Notice that, in the uncongested case $\tau_a(t) = t + \phi_a$, so that $G_a^d(\tau_a(t)) = E_a^d(t)$.

We remark that the unloading at the end of arcs obeys the non-vehicle holding traffic behaviour. For link-based traffic models, vehicle holding (VH) holds when some motorists are reluctant to move forward from upstream to downstream links even if vacant spaces exist in the downstream links. This theory has been applied by Long *et al.* (2019) for generating a cell-based system optimum dynamic traffic assignment (SO-DTA) model and has been addressed by Zhu & Ukkusuri (2013) to approach link-based SO-DTA problems. Under our traffic modelling approach, the deterministic punctual queuing model considers no physical capacities and queue discharge is only limited by unloading capacity. Hence, the VH behaviour does not hold.

To determine the *cost function* of an arc (or, indistinctly, *total travel time*, *travel time* or *cost*), first, we need to consider that once entered an arc at a given time, all motorists experience the same cost, independent of their destinations. Now, for each arc $a \in A$ and at each time $t \in [0, T]$, the cost of the arc a, having entered it at t, denoted as $c_a(t)$, is given by the free flow travel time of a plus any delay due to the waiting time in the queue. This can be expressed analytically as:

$$c_a(t) = \phi_a + \frac{L_a(t + \phi_a)}{Q_a}. \tag{19}$$

Given the arc-based formulation of the present model, and in particular the specification of the arc costs, correlation of costs in overlapping routes that have arcs in common is respected implicitly.

We remark on the determinant influence of integrating the deterministic queuing model into our arc-based stochastic dynamic traffic assignment model. As one of the arc parameters is the service capacity, a restriction over the unloading of vehicles is applied; thus, whenever the assigned inflow rate overpasses the service capacity of an arc, a queue is formed by those vehicles that are not able to leave yet. This determines a dynamic term on the cost function, added to the free flow travel time, as a delay is experienced due to the queue. This dynamic cost determines the expected minimum costs to reach each destination, which, in turn, determines the dynamic assignment to each arc. This means that adapting the queue model is fundamental to achieving the dynamic nature of the present approach.

3.4. *Arc-choice model*

In the literature, route choices of motorists are usually built directly as a route-choice model. Under our arc-based approach, we represent these decisions through an arc-choice model. Thus, it is the recursive choices of arcs that end up building the route that motorists follow to their respective destinations. Our model is a dynamic adaptation of the static flow assignment embedded in the *MTE* concept (Baillon & Cominetti 2008), which considers that motorists make their choice decisions following a logit model that considers the expected minimum costs from the current node i to their respective destinations by using i's outgoing arcs. For the specifications, let us consider a constant and known dispersion parameter θ.

For each destination node $d \in D$, for each arc $a = (n, m) \in A$ and at each time $t \in [0, T]$, the expected minimum cost of going from n to d by choosing arc a, entering it at t, denoted $Z_a^d(t)$, is computed as:

$$Z_a^d(t) = c_a(t) - \frac{1}{\theta} \ln \left(\sum_{b \in A_m^+} \exp\left(-\theta Z_b^d\left(\tau_a\left(t\right)\right)\right) \right), \tag{20}$$

while the expected minimum cost of going from node n to destination d, starting at t, denoted as $W_n^d(t)$, is given by:

$$W_n^d(t) = -\frac{1}{\theta} \ln \left(\sum_{a=(n,m)\in A_n^+} \exp\left(-\theta \left(c_a(t) + W_m^d\left(\tau_a(t)\right)\right)\right) \right). \tag{21}$$

Therefore, from expressions (20) and (21), for each destination node $d \in D$, for each arc $a = (n, m) \in A$ and at each time $t \in [0, T]$, the following equations hold:

$$Z_a^d(t) = c_a(t) + W_m^d\left(\tau_a(t)\right), \tag{22}$$

and

$$W_n^d(t) = -\frac{1}{\theta} \ln \left(\sum_{a\in A_n^+} \exp\left(-\theta Z_a^d(t)\right) \right). \tag{23}$$

Now that we have formulated the expected minimum costs (from nodes and from arcs) to the destinations, we can formulate the expressions for the inflow rates associated with the arcs. Let us recall that because of the reasonability concept introduced before, given a destination d and a node n, only outgoing arcs a from node n that are reasonable towards d (arcs a such that $a \in A_n^+ \cap R^d$) are assigned positive inflows; otherwise, no inflow is assigned. Additionally, at a given instant, the flow to be assigned from node n can come from two sources: the aggregate outflow from incoming arcs to node n, and any demand generated at n (if n is an origin).

Analytically, for each destination node $d \in D$ there are two cases: (1) for each node $n \in N$ such that $(n, d) \notin OD$ (not origins for destination d), for each arc $a \in A_n^+$ and at each time $t \in [0, T]$, the inflow to arc a travelling to destination d is given by:

$$E_a^d(t) = \begin{cases} \dfrac{\exp\left(-\theta Z_a^d(t)\right)}{\sum\limits_{b\in A_n^+ \cap R^d} \exp\left(-\theta Z_b^d(t)\right)} \sum\limits_{b\in A_n^-} G_b^d(t) & \text{if } a \in R^d, \\ 0, & \text{otherwise;} \end{cases} \tag{24}$$

and (2), for each $o \in O$ such that $(o, d) \in OD$, for each arc $a \in A_o^+$ and at each time $t \in [0, T]$, the inflow to arc a travelling to destination d is given by:

$$E_a^d(t) = \begin{cases} \dfrac{\exp\left(-\theta Z_a^d(t)\right)}{\sum\limits_{b\in A_o^+ \cap R^d} \exp\left(-\theta Z_b^d(t)\right)} \left(\sum\limits_{b\in A_o^-} G_b^d(t) + q^{(o,d)}(t) \right), & \text{if } a \in R^d, \\ 0 & \text{otherwise.} \end{cases} \tag{25}$$

With this, we have suitably constructed and established the three fundamental structures of a DTA model under the approach developed in this research. This framework defines the ABSDTA model for general transport networks. In section 4, a solution method for this model is presented but, before that, in the following subsection we comment on the influence of correlation in our approach.

3.5. *The effect of the arc-based approach on the assignment*

To easily show the intuition behind this approach in a static context, let us consider Figure 1, where the cost of each arc is shown next to it. Let us suppose a demand going from node 1 to node 3 to be assigned following a logit rule.

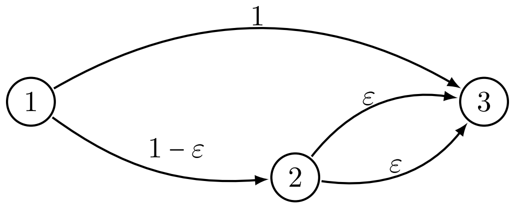

Figure 1. Simple network to compare path-choice versus arc-choice approaches (Baillon & Cominetti 2008).

From a route-choice approach, as independence assumptions are not well suited for overlapping paths and all three paths from node 1 to node 3 have cost 1, the logit model operates by assigning 1/3 of the demand to each route. On the other hand, as the two lower routes differ from one another only in their final links, under the arc-choice model presented in this work, the solution assigns 1/2 of the demand to the upper arc (that defines by itself a route) and 1/2 to arc $(1, 2)$, as the expected minimum cost of using $(1, 2)$ until reaching destination is 1. Then, the flow is split into 1/4 units for each one of the parallel arcs from node 2 to node 3, given that they have the same cost. These outputs are summarized in Table 1.

Table 1. Figure 1's assignments for each approach

arc	route-based approach	**arc-based approach**
(1,3)	1/3	**1/2**
(1,2)	2/3	**1/2**
$(2,3)_{upper}$	1/3	**1/4**
$(2,3)_{lower}$	1/3	**1/4**

We acknowledge that, even though independence of route costs is not needed, there are cases where correlations can impact the assignment that results from this kind of approach, particularly when routes differ only on outgoing arcs from the origin, as further addressed in Maher & Hughes (1997). Consider a modified version of the network in Figure 1, shown in Figure 2. As the expected minimum cost from node 2 to node 3 is $1 - \varepsilon$, the expected minimum costs of both lower routes is 1. Thus, the assignment to each outgoing arc from node 1 is 1/3. These cases result in assignments that are similar to those obtained by applying route-based formulations.

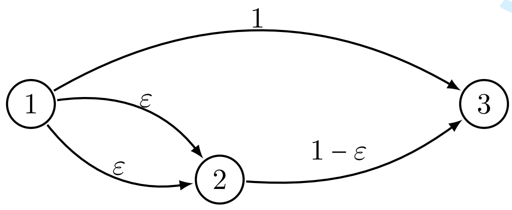

Figure 2. Case where path-choice and arc-choice result in similar assignments.

Different existing choice models can be embedded to our approach, as later addressed in Section 6, that could overcome this issue, but that work falls out of the scope of our current research stage.

4. Solution algorithm

We present a solution algorithm to solve the assignment problem associated with our proposed model. This discrete-time method is heavily motivated by *Dial's algorithm* (Dial 1971) but with a reverse order of its steps, which are repeated in every time increment that results from the time discretisation.

The inputs are: the digraph (N, A) associated with the transport network; the set of origins $O \subseteq N$; the set of destinations $D \subseteq N$; the set of O-D pairs $OD \subseteq O \times D$; the free flow travel time ϕ_a and the queue service capacity Q_a of every arc $a \in A$ (for notation, ordered as the arrays ϕ and Q, respectively); the time-dependent demands $q^{od}(t)$ for each O-D pair $(o, d) \in OD$; the length of the time period, T; the timestep Δt; and, for the logit model The number of intervals is $K = T/\Delta t$, then, given $k \in \{1, ..., K\}$, we refer to $[(k-1)\Delta t, k\Delta t]$ as the k^{th} *time increment*.

The outputs are two hypermatrices $E = \left({}_kE_a^d\right), G = \left({}_kG_a^d\right) (a \in A, d \in D, k = 1, ..., K)$ of size $|A| \times |D| \times K$, and a matrix $L = ({}_kL_a) (a \in A, k = 1, ..., K)$ of size $|A| \times K$. Here, given $a \in A$, $d \in D$, and $k \in \{1, ..., K\}$, ${}_kE_a^d$ and ${}_kG_a^d$ are the inflow and outflow of arc a going to destination d during time increment k, respectively, and ${}_kL_a$ is the queue length on arc a that will be encountered by traffic that enters during k. Note that this temporal reference, which is adopted for notational convenience for the algorithm, differs from that in continuous-time by the arc free-flow time ϕ_a.

In what follows, a summary of the steps of the solution algorithm is presented:

- It sets initial values for an empty network: arc costs equal to free-flow travel times; accordingly, the reasonable arcs to each destination; the average demand that will enter the network at each time increment for each O-D pair.

Then, at each time increment k:

- Backwards, from each destination d, it computes the expected minimum cost of using each node and each arc to reach d, according to discrete versions of equations (20) and (21);
- Forwards, for each destination d and from each node i, it performs the assignment by splitting the aggregated flow rates (going to d) at i among its outgoing arcs (the ones reasonable towards d) as inflows according to discrete versions of equations (24) and (25). It then computes the corresponding outflows and queue lengths according to discrete versions of equations (15) and (16);
- It computes the costs of using each arc having entered it at k, according to a discretized version of equation (19);
- If there are no more flows to assign and no more queues to empty, the algorithm ends, otherwise, it continues to time increment $k + 1$, repeating the process.

The following pseudocode summarizes the solution algorithm. In Appendix A we present a comprehensive formulational description of the whole routine.

Algorithm 1 $(E, G, L) = \text{SolutionAlgorithm}((N, A), O, D, OD, \phi, Q, \mathcal{D}(\cdot), T, \Delta t, \theta)$

```
INITIALIZATION Settings for an empty network
for k=1,...,K do
    BACKWARD
    for all d ∈ D do
        for all n ∈ N do
```

for all $a \in A_n^-$ incoming arcs to n, **do**
Compute expected minimum costs starting from a to d
end for
Compute expected minimum costs starting from n to d
end for
end for
FORWARD
for all $n \in N$ **do**
for all $d \in D$ **do**
for all $a \in A_n^+$ outgoing arcs from n, **do**
Compute the inflow, outflow and queue length going to d through a
end for
end for
end for
COST UPDATES
for all $a \in A$ **do**
Update the cost of a because of the delays in queues
end for
STOP CONDITION
if $k = K$ or there are no more flow rates to assign **then**
End algorithm
end if
end for

This solution algorithm can be extended readily to allow initialization with non-empty transport networks. We do not address this feature in the present paper, but we intend to investigate it in future research.

We remark that our proposed method differs from the *DYNASTOCH* algorithm by Ran & Boyce (1996), also based in Dial's algorithm (Dial 1971). DYNASTOCH and our proposed model both share the existence of an INITIALIZATION step, a BACKWARD step a FORWARD step, but they are far from similar on how they are processed and how they are formulated. Some of the substantial differences are:

- DYNASTOCH solves the assignment for the case of one O-D pair, thus for solving the multiple case it must be run repeatedly one time for each O-D pair. Our proposed algorithm has been conceived for general cases, thus it deals with all O-D pairs of the network simultaneously in one execution.
- In DYNASTOCH the INITIALIZATION is run for all time intervals, then the BACKWARD process is run for all intervals, and then the FORWARD process is run for all intervals; thus DYNASTOCH runs through all time intervals three times for performing its main steps. Our proposed algorithm, after setting initial values, is run just once through all intervals, one time increment at a time for the BACKWARD step and then the FORWARD step.
- In DYNASTOCH, INITIALIZATION computes likelihoods, BACKWARD computes weights, and FORWARD computes the assignments of flows. In our proposed approach, INITIALIZATION sets values for an empty network, BACKWARD computes expected minimum costs to reach destinations, and FORWARD computes the assignments of flow. Even though the FORWARD steps share the objective, their respective routines are different.

4.1. *Illustration of the assignment performed by the algorithm*

To illustrate how the algorithm works, let us consider the transport network represented by the digraph (N, A) in Figure 3. Nodes 1 and 2 are origins and node 6 is the single destination. On every arc a, the pair (ϕ_a, Q_a) shows its free-flow travel time $[sec]$ and its service capacity $[veh/sec]$, respectively.

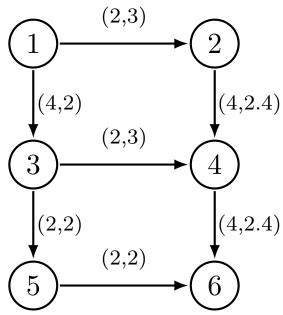

Figure 3. Network (N, A), with $(\phi_a[sec],\ Q_a[veh/sec])$ on each arc a.

Figure 4 shows both demands for a period of $T = 18$ sec with a timestep size of $\Delta t = 1$ sec. For the logit model, the dispersion parameter is θ =0.2 sec^{-1}.

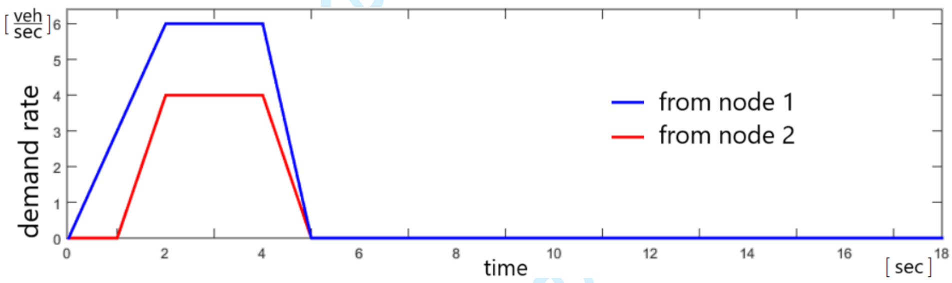

Figure 4. Demands [veh/min] from origin node 1 (blue) and origin node 2 (red), respectively.

Figures 6 and 7 show the evolution of inflows and queues. Figure 5 details the used notations, where at each time k: given an origin o, a) shows the average demand ${}_kD_o$ at o; given an arc $a = (i, j)$, b) and c) show inflow ${}_kE_a$ (blue) traversing arc a, getting behind a previous inflow, as well as the queue ${}_kL_a$ (red); At destination 6, d) shows flows arriving the destination. Outflows are not shown but they can be computed for each arc $a = (i, j)$: if j is an origin, ${}_kG_a = \sum_{b\in A_j^+} {}_kE_b - {}_kD_j$, otherwise ${}_kG_a = \sum_{b\in A_j^+} {}_kE_b$.

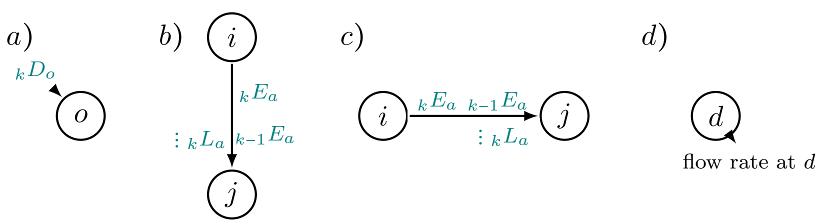

Figure 5. Notations used in Figures 6 and 7

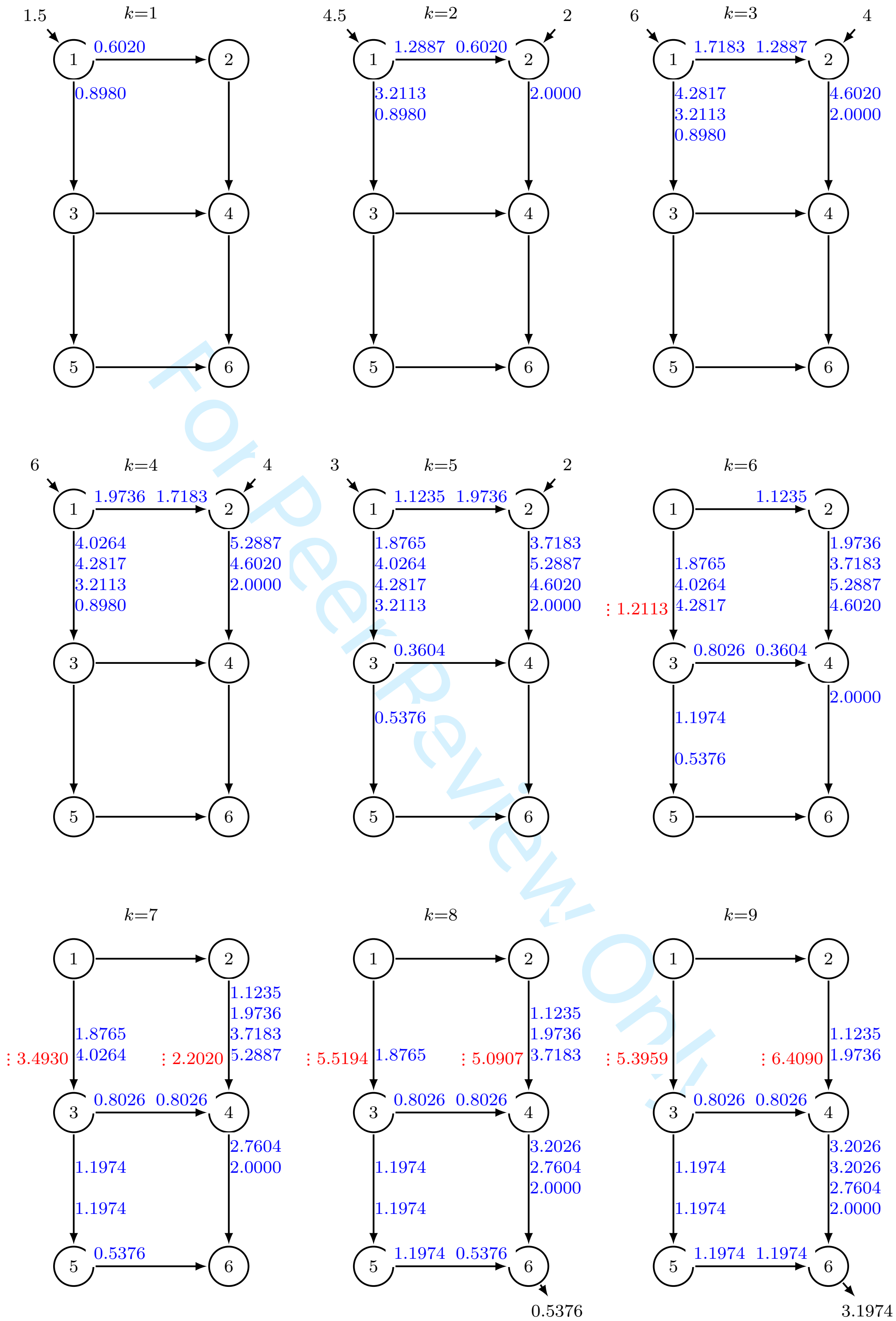

Figure 6. Evolution of inflows and queues from $k = 1$ to $k = 9$.

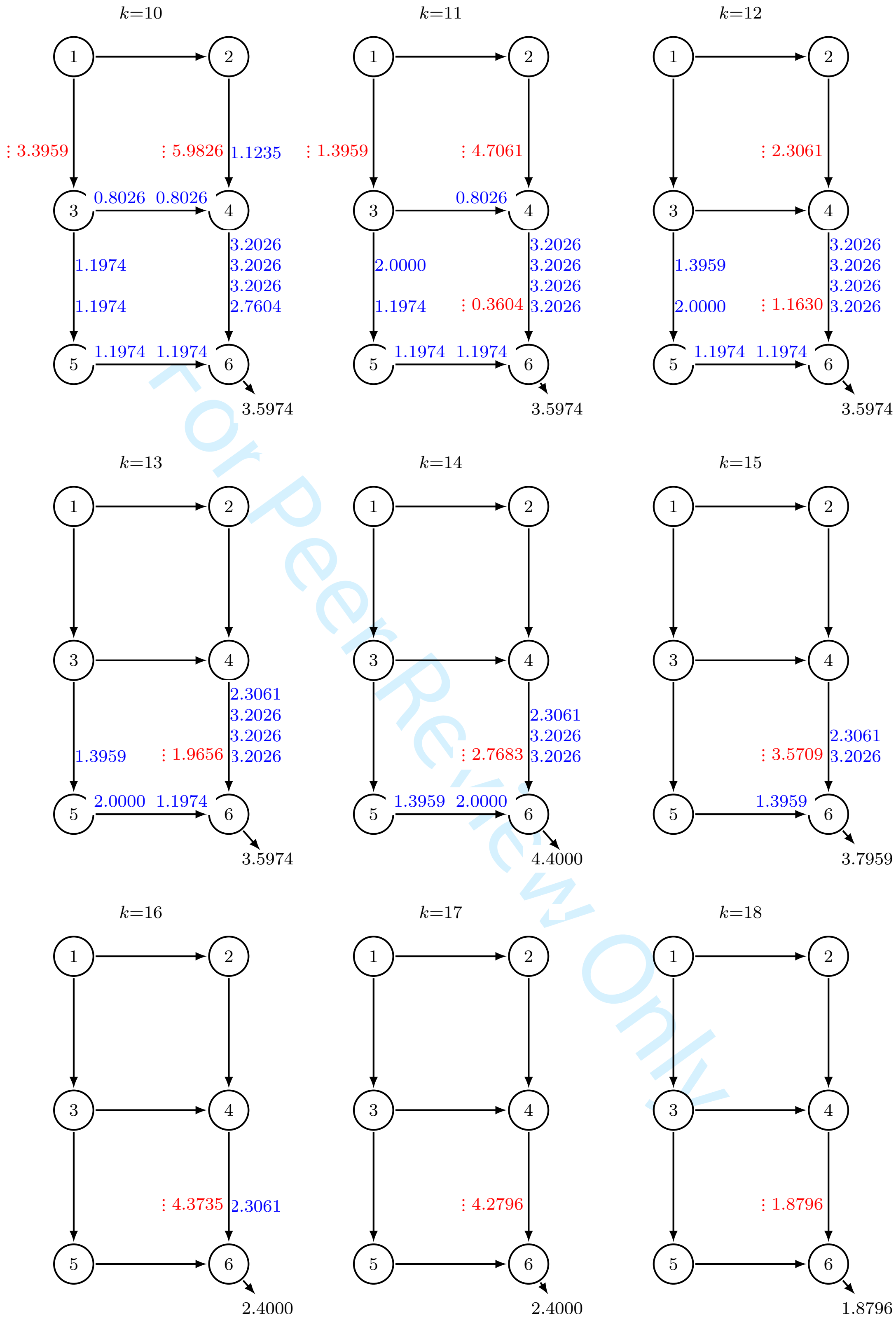

Figure 7. Evolution of inflows and queues from $k = 10$ to $k = 18$.

4.2. *An intuition on the unloading process*

To give an insight on how the unloading subroutine within the algorithm works, we illustrate a simple scenario presented in Fig 8. At a given time, let us consider that arc (i,j) has a free-flow travel time of ϕ and service capacity of Q, and that there are two destinations, d_1 and d_2. There are remaining positive flows, for both destinations, that entered the arc (and have not left it yet) at times k and $k+1$. Those flows, at the end of the arc, are denoted as ${}_{k+\phi}S_{d_1}$, ${}_{k+1+\phi}S_{d_1}$, ${}_{k+\phi}S_{d_2}$, and ${}_{k+1+\phi}S_{d_2}$.

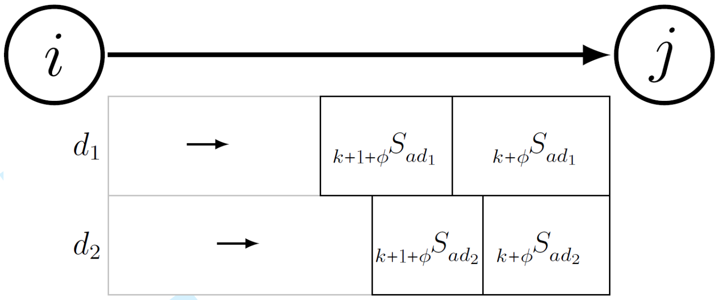

Figure 8. Illustration of arcs' unloading process

Now, there are three cases: 1) if the aggregated flows do not overpass the service capacity (${}_{k+\phi}S_{d_1} + {}_{k+1+\phi}S_{d_1} + {}_{k+\phi}S_{d_2} + {}_{k+1+\phi}S_{d_2} \leq Q$), then all remaining flow is able to exit as outflow rates, thus $G_{d_1} = {}_{k+\phi}S_{d_1} + {}_{k+1+\phi}S_{d_1}$ and $G_{d_2} = {}_{k+\phi}S_{d_2} + {}_{k+1+\phi}S_{d_2}$; 2) if the service capacity overpasses the aggregated flows that entered first (at k) but does not overpass the aggregation of all flows (${}_{k+\phi}S_{d_1} + {}_{k+\phi}S_{d_2} \leq Q < {}_{k+\phi}S_{d_1} + {}_{k+1+\phi}S_{d_1} + {}_{k+\phi}S_{d_2} + {}_{k+1+\phi}S_{d_2}$), then the flows that entered first exit first (and totally) and the remaining capacity is split proportionally into the flows that entered later (at $k+1$), thus $G_{d_1} = {}_kS_{d_1} + \frac{{}_{k+1}S_{d_1}}{{}_{k+1}S_{d_1} + {}_{k+1}S_{d_2}}(Q - {}_kS_{d_1} - {}_kS_{d_2})$ and $G_{d_2} = {}_kS_{d_2} + \frac{{}_{k+1}S_{d_2}}{{}_{k+1}S_{d_1} + {}_{k+1}S_{d_2}}(Q - {}_kS_{d_1} - {}_kS_{d_2})$; 3) if the service capacity is less than the aggregated flows that entered first (at k), then part of those flows exit splitting proportionally the capacity, thus $G_{d_1} = \frac{{}_kS_{d_1}}{{}_kS_{d_1} + {}_kS_{d_2}}Q$ and $G_{d_2} = \frac{{}_kS_{d_2}}{{}_kS_{d_1} + {}_kS_{d_2}}Q$.

The explained process, and its generalization presented in the pseudocode of the algorithm in Appendix A, guarantees the fulfillment of the FIFO rule within arcs.

We have implemented the ABSDTA algorithm in MATLAB; in section 5 we address different aspects related to the use of this computational tool in this matter.

5. Insights on the ABSDTA performance

To give an insight of how results of the computational implementation of our model behaves when opposed to existing literature and when different combinations of parameters are considered, we ran multiple analyses, supported by graphical results and indicators. To do this, we choose the assignment approach associated with the SDUE by Han (2003), discussed earlier in section 1 and further described in section 2. We consider this approach to be the most suitable match for comparison with our model. In fact, its dynamic nature comes from time-dependent demand rate functions; its stochasticity comes from the uncertainty in cost perceptions, represented through a logit rule in the choice model; and it applies the deterministic punctual queuing model as the traffic model. However, the fact that it is route-based, while ours is arc-based, sets the nature of the differences between the two approaches.

We consider the results presented in Han (2003) for its largest addressed scenario, which is the Sioux Falls Network with underlying digraph (N, A), represented in Figure 9 (LeBlanc 1975), with arc parameters shown in Table 2 and O-D pairs shown in Table 3. All of the O-D pairs are associated with the same demand rate profile shown in Figure 10, for a period of $T = 60$ min.

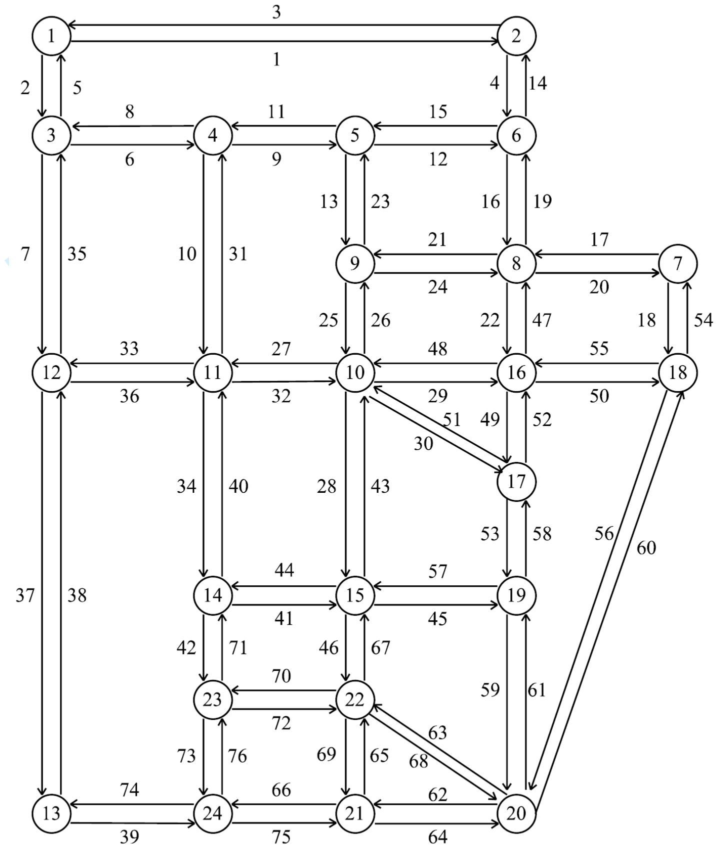

Figure 9. The Sioux Falls Network. Arc label next to each arc

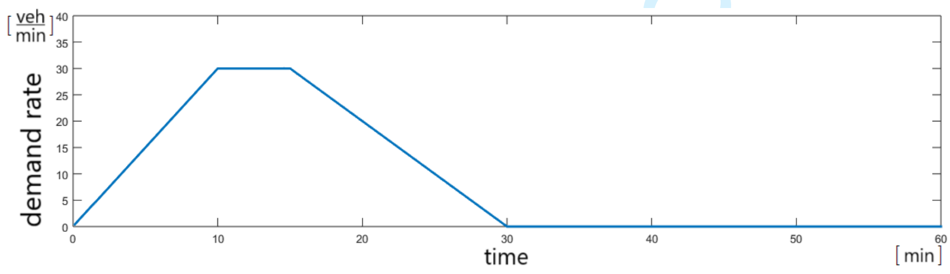

Figure 10. Demand [veh/min] for each O-D pair for the Sioux Falls network in Han (2003)

5.1. *General overview on the outputs evolution*

Before further analyses of the outputs, it is important first to highlight some aspects of the graphical representation of their evolution.

Table 2. Arc parameters for the Sioux Falls network in Han (2003).

a	ϕ_a $[min]$	Q_a $[\frac{veh}{min}]$	a	ϕ_a $[min]$	Q_a $[\frac{veh}{min}]$	a	ϕ_a $[min]$	Q_a $[\frac{veh}{min}]$
1	6	65	27	5	50	52	2	45
2	2	55	28	4	45	53	3	45
3	6	65	29	3	40	54	5	50
4	2	60	30	3	45	55	3	55
5	2	55	31	5	55	56	6	55
6	5	60	32	5	50	57	3	40
7	5	60	33	3	60	58	3	45
8	5	60	34	4	50	59	4	50
9	3	50	35	5	60	60	6	55
10	5	55	36	3	60	61	4	50
11	3	50	37	6	65	62	3	40
12	3	50	38	6	65	63	4	45
13	2	50	39	2	60	64	3	40
14	2	60	40	4	50	65	2	50
15	3	50	41	4	50	66	3	50
16	3	45	42	3	40	67	3	45
17	3	40	43	4	45	68	4	45
18	5	50	44	4	50	69	2	50
19	3	45	45	3	40	70	4	40
20	3	40	46	3	45	71	3	40
21	3	45	47	2	45	72	4	40
22	2	45	48	3	40	73	2	40
23	2	50	49	2	45	74	2	60
24	3	45	50	3	55	75	3	50
25	2	45	51	3	45	76	2	40
26	2	45						

Table 3. O-D pairs for the Sioux Falls network in Han (2003).

Origin node	Destination node
1	10
4	19
6	15
7	15
12	19
13	10
14	8
18	5
20	9
22	8
2	15
3	16

For a scenario with dispersion parameter $\theta = 0.2$ $[min^{-1}]$ and timestep size $\Delta t = 1$ $[min]$, we present the evolution of the outputs of arc 22, shown in Figure 11. These graphic results are typical of the different interactions in the evolution of the outputs of the arcs. Recall that the free flow travel time and the queue service capacity of arc 22 are $\phi_{22} = 2$ min and $Q_{22} = 45$ veh/min, respectively.

The following aspects highlight the output behaviour:

- As arc 22 is reasonable towards destination nodes 10, 15, 16 and 19 (but not to destination nodes 5, 8 and 9), there are four kinds of flows traversing the arc, one for each destination with positive assignment. Because they share arc 22, their aggregated values are affected by the queue service capacity Q_{22};
- Whenever the total inflow surpasses Q_{22}, after traversing the arc (after ϕ_{22}), not all traffic can leave the arc at the service capacity. Those that can exit, leave the arc as outflow rate, while those that are not able to exit, join the end of the queue;

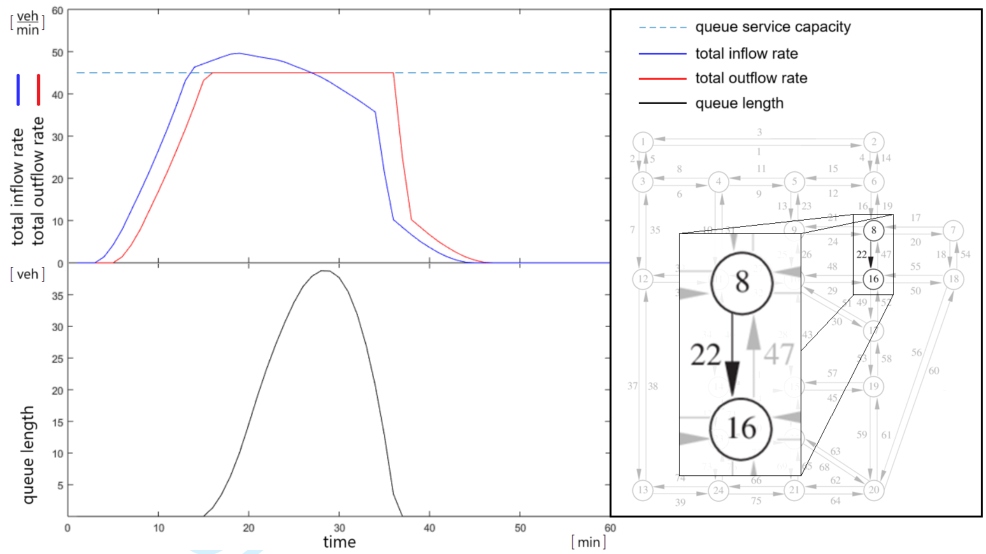

Figure 11. Evolution of outputs of arc 22.

- When the queue length is positive, the total outflow is equal to Q_{22}, as the discharge of the queue happens at maximum capacity;
- At a time increment k, the total outflow is equal to Q_{22} for either one of two reasons:
  - there is no queue but the total inflow that entered earlier $(k - \phi_{22})$ is at least Q_{22}; or
  - the queue already has positive length (residual form the previous time increment).

  In the latter case, the total outflow (equal to Q_{22}) is divided, according to the FIFO discipline, among those motorists that have been waiting longest to leave the arc to exit first.

We recall that only those arcs whose inflow rate overpasses its service capacity have positive queue lengths. In this scenario, we observe queues at some point in 8 out of the 76 arcs, as shown in Figure 12.

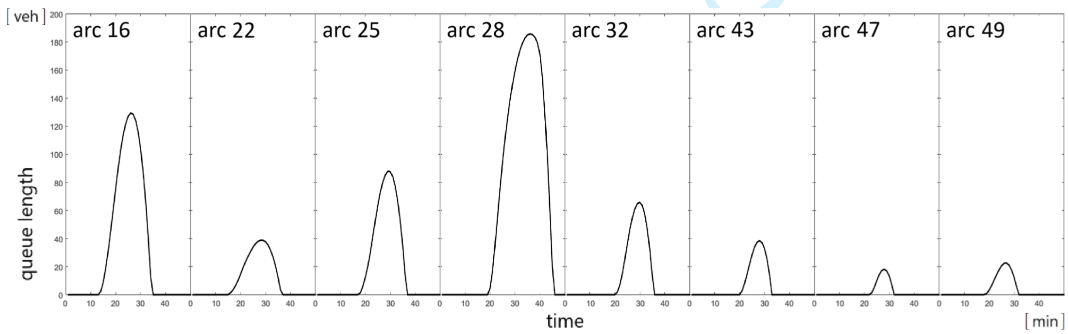

Figure 12. Queue length curves of arcs that have formed queues during the studied time period.

Considering the outputs of arc 22 again, particularly its queue length (bottom of Figure 11), Figure 13 presents how the aggregation of vehicles in the queue waiting to go to their respective destinations composes the total queue length.

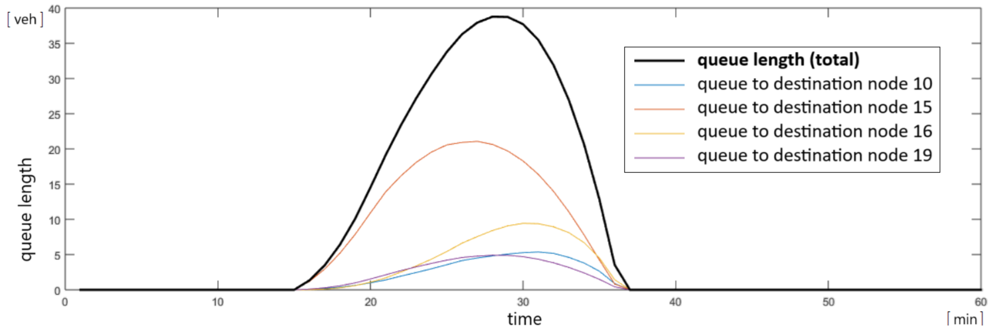

Figure 13. At the end of arc 22, a queue composed of vehicles going to destination nodes 10, 15, 16, and 19 has formed during the studied time period.

5.2. *Comparison opposed to existing contributions*

Before exploring the direct comparison between the results of the ABSDTA and the SDUE, we present comments on how the application of the reasonable arc concept drastically changes the structure of the outputs when it comes to arc use, and thus route use, from the outputs of exhaustive-search stochastic models.

5.2.1. The effect of considering reasonable arcs

So far, we have recalled the concept of reasonable arcs toward a destination multiple times for many analytical and formulation purposes. In this subsection, we present how application of this concept becomes a determinant feature of our approach while also supporting the remarks on the results of the following subsections.

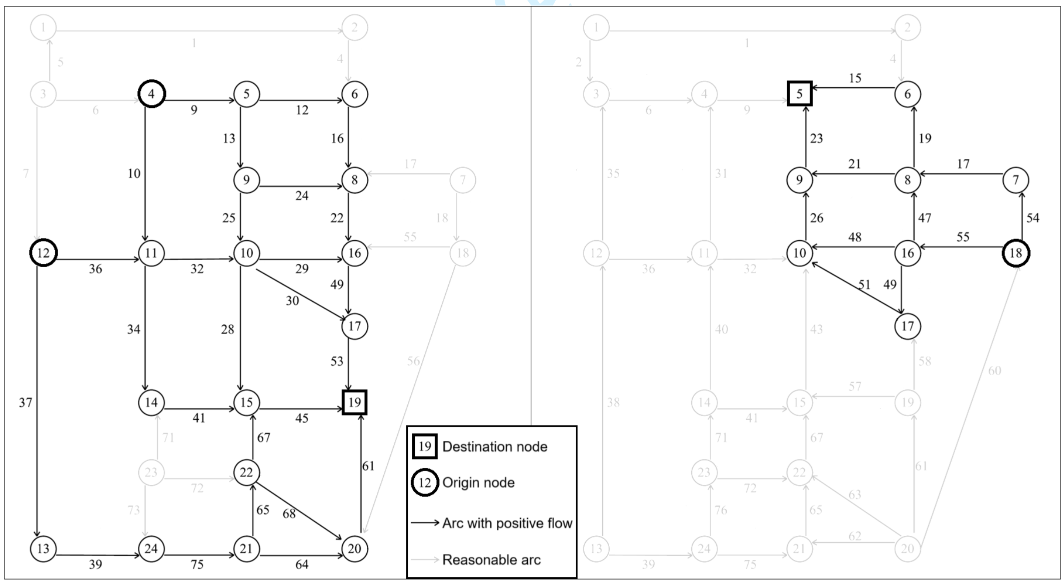

Figure 14. Reasonable arcs of the Sioux Falls network for destinations 19 and 5.

Recall that a reasonable arc is defined according to a destination. Figure 14 shows the subnetworks generated by the reasonable arcs towards destination nodes 19 (left) and 5 (right). In darker color we mark the arcs that are actually assigned with positive inflow. Notice that the concept of reasonable arc does not depend on the stochastic characteristics of our model; thus, regardless of the θ value being used, the set of reasonable arcs towards a fixed destination remains the same.

Given the application of the reasonability arc concept, our proposed ABSDTA approach results in sets of 36 arcs to be considered to travel to destination nodes 19 and 5, respectively, and actually assigns positive inflow to 26 and 12 of those arcs, respectively.

We consider these results worth being highlighted, as they intuitively show a more realistic scenario in which the whole network is not forced to move flow through all arcs and thus, through all possible routes. Instead, the ABSDTA just pushes flow through the subnetworks composed of arcs that are convenient for motorists to move forward to, in our case, through applying the reasonable arc concept.

5.2.2. *Graphic analysis: SDUE vs ABSDTA*

In this subsection, we compare graphically the inflow rates that results from the SDUE and the ABSDTA where, for the logit specifications, three values for the dispersion parameter θ are considered: 0.01 min^{-1}, 0.04 min^{-1} and 0.1 min^{-1}, while for the discretisation a timestep size of $\Delta t = 1\ min$ is considered.

Figures 15 and 16 depict the plots of inflow rate evolution for arcs 24 and 29 respectively (from Sioux Falls network in Figure 9), by presenting the results of the SDUE approach from Han (2003) and our proposed ABSDTA approach. We recall that the presented plots correspond to aggregated inflows of each arc, meaning that they represent the total flow that has been assigned to each arc, regardless of their specific destinations.

From both Figures, 15 and 16, there are two aspects that are worth to highlight:

- We first note that they show a similar behaviour before changes on the dispersion parameter θ. This is consistent with what can be expected from both approaches, as θ adjusts the level of disaggregation of the flows to be split among the available options. The greater the θ value is, the less dispersed the model behaves, meaning that the options that are perceived as more attractive are assigned with more inflow (and the opposite to those that are less attractive).
- Then, we note that the ABSDTA presents mainly higher levels of inflow rates associated with each arc, more noticeable in the case of arc 29 (Figure 16). This is an effect that comes from the fact that, under our approach, there are less options that are considered by motorists, given the assumption that they travel only through reasonable arcs, the concept that we introduced earlier in subsection 3.1. Those arcs that are considered, given that are reasonable, are assigned with larger flows. Recalling Figure 14, we note that arcs 24 and 29 are reasonable towards destination 19 but not to destination 5. In fact, our results show that both arcs are reasonable towards three of the destinations (nodes 8, 16 and 19).

This graphic analysis helps corroborate the consistency of the outputs of our model, as they behave accordingly when compared to those of the SDUE, and also the differences between them are consistent with the nature of the features of our approach.

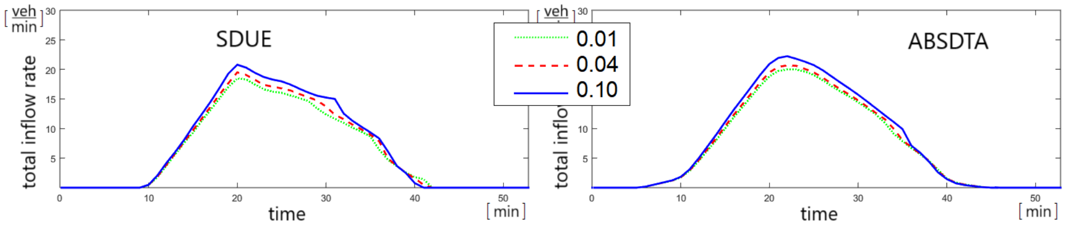

Figure 15. Inflows of arc 24 for the different θ $[min^{-1}]$ values. SDUE on the left, ABSDTA on the right

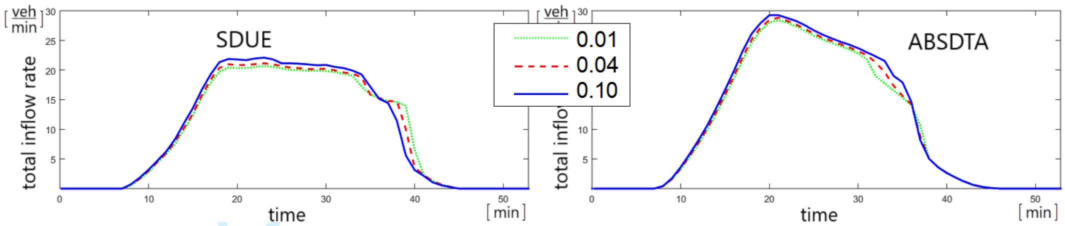

Figure 16. Inflows of arc 29 for the different θ $[min^{-1}]$ values. SDUE on the left, ABSDTA on the right

5.2.3. *Cost indicators: SDUE vs ABSDTA*

To compare in a global scale the results of both approaches, we use as indicators the *total travel cost* and the *total queuing delay*, analytically expressed as

$$TC = \int_t \sum_a E_a(t) c_a(t) dt, \tag{26}$$

$$TD = \int_t \sum_a E_a(t) \frac{L_a(t)}{Q_a} dt, \tag{27}$$

respectively (Han 2000). The first indicator represents the total time incurred because of the travels of all motorists, while the second one represents the total time spent in queues by all motorists. Next, in Table 4 we present the values of this indicator, of both approaches, for the three implemented values of the dispersion parameter θ.

Table 4. Comparison of costs indicators TC (Eq. 26) and TD (Eq. 27) for the different θ values.

θ $[min^{-1}]$	0.01		0.04		0.10	
model	SDUE	**ABSDTA**	SDUE	**ABSDTA**	SDUE	**ABSDTA**
TC $[min]$	126841	**99342**	125460	**98893**	123634	**98268**
TD $[min]$	14271	**5676**	13054	**5729**	11515	**6071**

From Table 4 we note that the ABSDTA model results in both of the indicators being less than those of the SDUE in each one of the three implemented cases. The TC indicator for the ABSDTA model is never greater than 80 % of the TC for the SDUE, while the TD indicator for the ABSDTA model is around 50 % of the TD for the SDUE.

Another noticeable result comes from the information in Table 5. There, we present the percentage of the total cost associated with the total time that all motorists spent in queues, for both the ABSDTA and the SDUE approaches and for the three θ values. In all cases, the ABSDTA results in lower percentages than those of the SDUE, meaning that, under our approach, motorists spend less of their time stuck in queues

Table 5. Comparison of percentage of delays because of queues over total costs for the different θ values.

θ [min^{-1}]	0.01		0.04		0.10	
model	SDUE	**ABSDTA**	SDUE	**ABSDTA**	SDUE	**ABSDTA**
TD/TC [%]	11.25	**5.71**	10.40	**5.79**	9.31	**6.18**

and more of the time actually moving through the transport network. This, added to the fact that motorists only travel through arcs that are reasonable and, thus, are moving closer to their destination, can be understood as that not only motorists spend more of their time effectively moving, but actually moving forward and closer to their destinations.

We remark that the implementation applied by Han (2000) corresponds to the DYNASTOCH algorithm (Ran & Boyce 1996) repeated for all O-D pairs in the network (as DYNASTOCH is originally for one O-D pair). A comparison of computation times between the two approaches would have been a natural experiment. However, we had no access to Han (2000) codes to execute both algorithms in similar computational environments. We believe that a comparison of execution times of our approach in the current computational platform with execution times in (Han 2000) will not be very illustrative.

Regarding worst-case time complexity, DYNASTOCH on the INITIALIZATION, BACKWARD and FORWARD steps executes their respective subroutines for all arcs and for all time increments, from where we have $\mathcal{O}(|A \times K|)$ operations. For general transport networks, DYNASTOCH has to be repeated for all O-D pairs, resulting in $\mathcal{O}(|OD \times A \times K|)$. On the other hand, our algorithm's INITIALIZATION executes its subroutine for all arcs, from where we have $\mathcal{O}(|A|)$, while BACKWARD and FORWARD apply their subroutines for all destinations, arcs, and time increments, resulting in $\mathcal{O}(|D \times A \times K|)$ operations. Thus, our algorithm has worst-case complexity $\mathcal{O}(|D \times A \times K|)$. For cases of dense networks with almost all pairs of nodes are OD pairs, the set of arcs has a size $\mathcal{O}(|N|^2)$, and then DYNASTOCH SDUE has complexity $\mathcal{O}(|N|^4|K|)$, while our algorithm has complexity $\mathcal{O}(|N|^3|K|)$.

We remark that our approach, given its arc-based construction, offers an algorithm that avoids the path enumeration process typically embedded in route-based traffic assignment models. Moreover, integrating the reasonability concept helps reduce the number of used arcs. Adding both effects improves the computation time of the implementation as no route has to be previously built nor stored to be later assigned with positive flow to its arcs, which have already been filtered to consider only those arcs fulfilling the reasonability criterion. When it comes to the application in real-life scenarios, every instance of the method where complexity can be reduced becomes determinant given the size of real transport networks associated with real cities.

5.3. *The dispersion parameter θ vs the timestep size Δt*

To dig deeper into the scopes of our computational implementation, we analyzed different combinations of the dispersion parameter θ and the timestep size Δt. We consider again the Sioux Falls network (Figure 9) with the same inputs as before but with multiple different values for θ and Δt.

We analyze the effects of the combination of 5 different θ (0.25, 0.5, 1, 2, and 4 min^{-1}) against nine corresponding Δt (9 values for each θ), chosen to obtain nine fixed values for the adimensional product $\theta\Delta t$ (0.25, 0.33, 0.5, 0.75, 1, 1.5, 2, 3, and 4). It is important to note that we present cases that result in a number of time increments in the range from 4 to 960, something that already gives an insight into the level of granularity that the implementation of our ABSDTA algorithm is able to reach, as we manage to discretise the time period up to 16 times the number of time increments used in the case presented in Han (2003).

As for the presented outputs, they correspond to the inflow rate going particularly to destination node 8 through arc 24. Note that the conclusions for the rest of the arcs are similar to those described here for arc 24.

We opted to analyze the behaviour of θ versus Δt because of the different nature of these parameters: the dispersion parameter θ is an element of the model that represents variability in travelers' route choice, whilst the time step parameter Δt is an element of the solution method for the problem, so should be chosen accordingly. Thus, to analyze any interaction between the two parameters, the first parameter to be set should be the dispersion parameter θ, as it is part of the model, and the parameter that should be defined consistently with a given θ is the timestep size Δt, as it determines the characteristic of the discretised solution method of the problem.

5.3.1. Highlights on graphic results

Figure 17 depicts, in a matrix display, the evolution of the inflow rates going to destination node 8 of arc 24. Rows represent the product $\theta\Delta t$ while columns represent the value of the θ parameter. The upper right corner of each plot shows its corresponding θ and Δt values. Plots associated with timestep size Δt equal to 1 min are highlighted with a wider frame to be considered for a comparison analysis with constant Δt. Also, plots associated with a Δt greater than the minimum free-flow travel time have a gray background for convenience regarding the following comments.

Considering the plots shown in Figure 17, the following results arise:

- Only plots associated with a timestep size Δt less or equal to the minimum free-flow travel time of the network (2 min for our case) have to be considered. When this condition is not met, as Δt adjusts how forward in time the algorithm performs the next computations, if the free-flow travel time of an arc is less than Δt, then motorists can enter and exit it within a single time increment, representing a cost of 0 and, thus, underestimating the cost of the arc. It also can be seen that, in all plots with gray background, as the timestep grows, the delay of the start of the curve also grows, resulting in unreliable plots;
- Given a fixed product $\theta\Delta t$, as the θ value increases (and, thus, Δt decreases), it can be observed how the curve of the inflow goes from a single peak to gradually two peaks. This, given that higher values of the dispersion parameter θ represent more sensitivity to cost changes (inversely proportional to the variance), resulting in motorists becoming gradually more willing to change to an alternative arc if the cost grows;
- Given a fixed dispersion parameter θ, as the Δt value increases, it can be observed how gradually the level of detail of the curve is lost. Note that small timesteps Δt are able to capture effects that cannot be observed with larger ones;
- For each θ value, the plots associated with Δt values less or equal to 1 min present no significant graphical differences between them. On the other hand, plots associated with Δt greater than 1 min tend to deviate from the plots of their respective column.

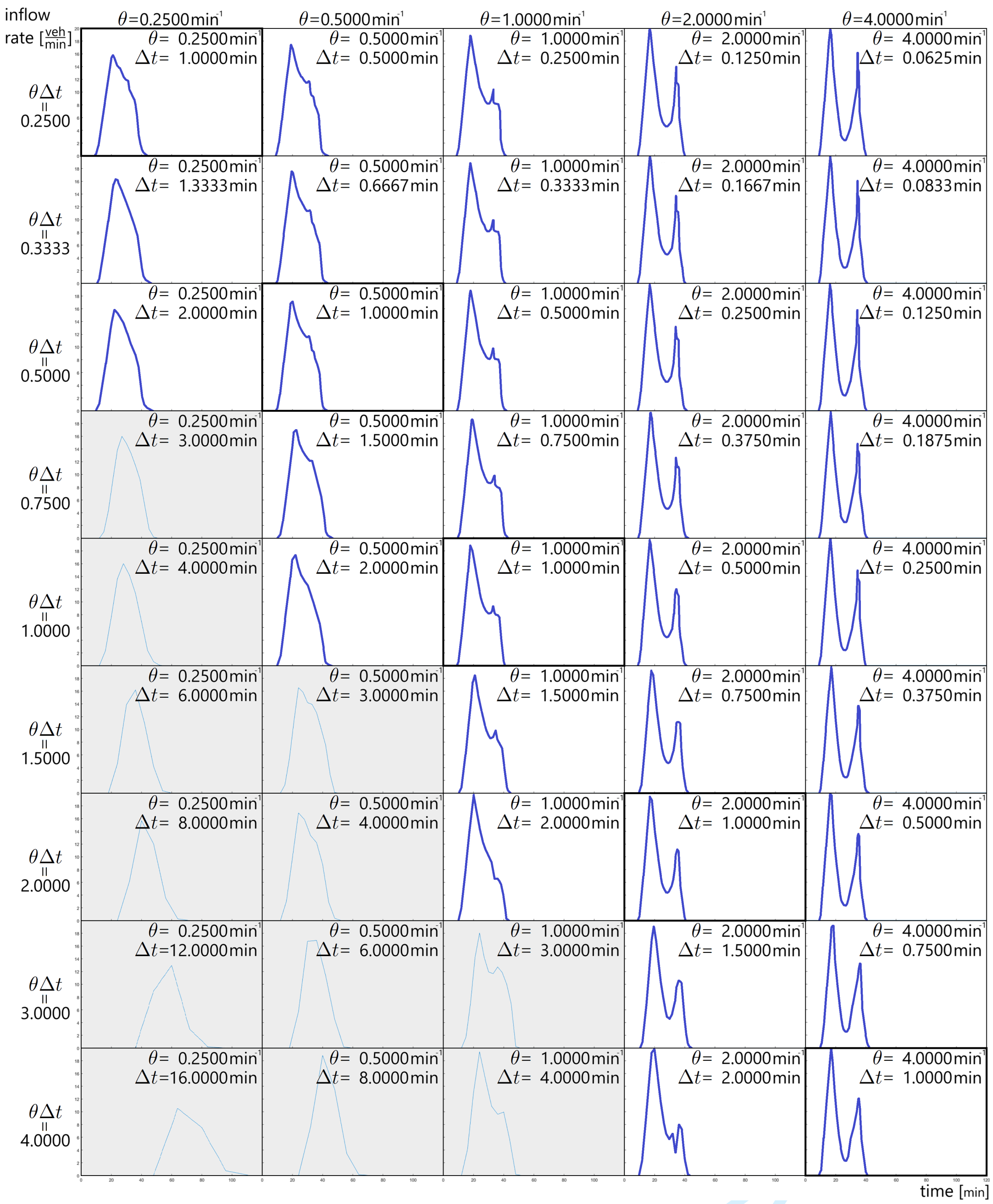

Figure 17. Evolution of the inflow rate going to node 8 of arc 24 under different combinations of dispersion parameter θ and timestep size Δt.

5.3.2. Highlights on cost indicators

Table 6 presents the total cost and the total delay indicators, along with the percentage of delay over the total cost, for all of the different combinations of θ and Δt values implemented for the example of this subsection. The display is similar to the one used for presenting the plots in the previous subsection, as rows represent the product $\theta\Delta t$ and columns represent the values for the dispersion parameter θ. Each cell for a given $\theta\Delta t$ and a given θ presents the timestep size Δt, the total cost indicator, the total delay indicator and the percentage of the latter with respect to the former. Again, similarly to the previous subsection, cells associated with timestep size Δt equal to 1 min are written in bold typing so that they can be considered for a comparison analysis with constant Δt. Finally, cells associated with a Δt greater than the minimum free-flow travel time are written in gray typing.

Table 6. Comparison of costs indicators TC (Eq. 26) and TD (Eq. 27) for the different combinations of θ and Δt values.

$\theta\Delta t$	θ $[min^{-1}]$	0.2500	0.5000	1.0000	2.0000	4.0000
0.2500	Δt $[min]$	**1.0000**	0.5000	0.2500	0.1250	0.0625
	TC $[min]$	**97646**	97588	98463	100990	102740
	TD $[min]$	**7581**	4820	3103	1980	1120
	TD/TC [%]	**7.76**	4.94	3.15	1.96	1.09
0.3333	Δt $[min]$	1.3333	0.6667	0.3333	0.1667	0.0833
	TC $[min]$	96481	97257	98439	100980	102730
	TD $[min]$	8656	6287	4153	2650	1490
	TD/TC [%]	8.97	6.46	4.22	2.62	1.45
0.5000	Δt $[min]$	2.0000	**1.0000**	0.5000	0.2500	0.1250
	TC $[min]$	96689	**97463**	98387	100950	102710
	TD $[min]$	14121	**9854**	6274	3990	2240
	TD/TC [%]	14.60	**10.11**	6.38	3.95	2.18
0.7500	Δt $[min]$	3.0000	1.5000	0.7500	0.3750	0.1875
	TC $[min]$	95907	96285	98079	100750	102650
	TD $[min]$	18634	12882	9179	5890	3350
	TD/TC [%]	19.43	13.38	9.36	5.85	3.26
1.0000	Δt $[min]$	4.0000	2.0000	**1.0000**	0.5000	0.2500
	TC $[min]$	94102	96451	**98229**	100860	102660
	TD $[min]$	20585	18945	**12811**	8070	4500
	TD/TC [%]	21.87	19.64	**13.04**	8.00	4.38
1.5000	Δt $[min]$	6.0000	3.0000	1.5000	0.7500	0.3750
	TC $[min]$	92461	96050	97231	100610	102440
	TD $[min]$	21003	26513	17482	11950	6650
	TD/TC [%]	22.72	27.60	17.98	11.88	6.50
2.0000	Δt $[min]$	8.0000	4.0000	2.0000	**1.0000**	0.5000
	TC $[min]$	91150	93841	97299	**100670**	102540
	TD $[min]$	17510	29812	25275	**16500**	9090
	TD/TC [%]	19.21	31.77	25.98	**16.35**	8.87
3.0000	Δt $[min]$	12.0000	6.0000	3.0000	1.5000	0.7500
	TC $[min]$	89396	92165	98062	99847	102280
	TD $[min]$	5150	34861	30110	23164	13510
	TD/TC [%]	5.76	37.82	40.90	23.20	13.20
4.0000	Δt $[min]$	16.0000	8.0000	4.0000	2.0000	**1.0000**
	TC $[min]$	88967	90684	94785	100480	**102300**
	TD $[min]$	0	34904	43229	34500	**18560**
	TD/TC [%]	0.00	38.49	45.61	34.34	**18.15**

From the information presented in Table 6, the following comments can be made:

- As in the results presented in the previous subsection regarding the plots, only cells associated with a timestep size Δt less or equal to the minimum free-flow travel time of the network have to be considered. The cells with gray font present erratic total delay indicators;
- Given a fixed product $\theta\Delta t$, as the value of θ increases, it can be observed that while the total cost indicators increase, the total delay indicators decrease. This

can be explained by the fact that higher θ values mean higher sensitivity to cost changes. Then, even though arcs chosen more frequently may experience congestion, those chosen less frequently may not reach a state of congestion and thus have no queue-associated delay;

- Given a fixed dispersion parameter θ, as the Δt value increases, it can be noticed how, while the total cost indicators remain alike, the total delay indicators gradually grow. With greater timestep sizes Δt, more flow rate is accumulated until the next assignment decision, which results in larger assignments of inflow at once and, thus, more opportunities where arcs reach states of congestion;
- Considering now the diagonal formed by the cells with constant timestep size $\Delta t = 1\ min$, it can be observed how higher θ values affect congestion. As choices become closer to those expected from a deterministic case, those arcs perceived as convenient by motorists gradually attract more flow rates, growing congestion and, thus, increasing the total delays.

In this subsection, we found that for values of the timestep size Δt greater than the minimum free-flow travel time or arcs of the network, the behaviour of outputs and indicators cannot be interpreted as realistic. We can also note that for Δt values less or equal to 1 min, the graphical results present no significant differences. We also corroborate the strong effect of increasing the dispersion parameter θ, as it can be noticed graphically how the decisions of motorists become more drastic and their effect on traffic congestion becomes stronger.

6. Final remarks and conclusions

The main contribution presented in this paper is the development of a novel model that allows facing a DTA problem under stochasticity in the motorists' decisions, namely, the ABSDTA model. The proposed approach comes from the integration of the contributions of two lines of work. The first is the traffic assignment that results from the arc-based decision model by Baillon & Cominetti (2008), initially applied for static cases. The second is the basis of the formulation presented by Addison & Heydecker (1996), who established that the DTA modelling process requires the development of a demand profile, a traffic model and a route-choice model. In the present formulation, the demand profile is exogenous, the traffic model is the deterministic point queue, and the route-choice model, as mentioned before, is arc-based as a dynamic extension of the choice model in the Baillon and Cominnetti's MTE.

Our main contribution is the ABSDTA model for general transport networks. The approach applies the notion that a motorist decides how to move forward considering the remaining part of their trip and does not decide according to his/her origin once they enter the transport network. To represent that, we introduce the *reasonable arc towards destinations* concept, which is an arc that, once traveled through, takes the motorist not farther from his/her destination if minimum cost routes are taken. Then, we assume that motorists travel through reasonable arcs only. The ABSDTA model has properties that are not usually found in DTA models from the literature, particularly in approaches that consider uncertainty. Given the arc-based approach rather than the usually assumed route-based approach, along with the within-arc interactions defined and formulated for the traffic model, the ABSDTA framework allows working with overlapping routes. This relevant aspect comes from the model of route choice as a recursive decision process over the arcs. From applying this reasoning, independence on the route costs is not assumed, as the formulations are constructed according to the arcs. Thus, the only aspect regarding routing behaviour, which is the computation of the expected minimum costs from a current node to the destination experienced by the motorist, is constructed through nested arc cost operators. Additionally, route enumeration, usually applied to analyze and compare motorists' options, is not required. In another aspect, even though the arc-choice model assigns the inflows according to the expected minimum costs through a logit rule, it is not limited only to this: given the model construction, there is the potential of using different models to perform the assignment. The same can be concluded for the cost functions, where other models, apart from the deterministic point queue model that we use in this paper, could be used.

Another relevant contribution of our approach is the ABSDTA algorithm. The method allows obtaining an assignment for a discretised version of the problem and thus an approximated solution for the original version (which considers continuous time). It works efficiently, considering that the computational effort in the algorithm's execution could become important since a dynamic and repeated computing of the flow assignment has to be performed. In addition, the construction of the ABSDTA algorithm allows initialization with non-empty transport networks. From this feature, we can study how a pre-loaded network empties over time if an ABSDTA approach is applied. Even though this property is not developed further here, we highlight it because it emphasizes the broader applicability of the ABSDTA model formulation and its solution method. Our proposed method is a remarkable accomplishment, as dynamic traffic assignment solution methods are already complex to deal with and the ABSDTA algorithm that we have developed is an efficient method that solves our proposed arc-based DTA approach through an elaborated dynamic programming algorithm that defines a routine that ensures the fulfillment of the FIFO rule, a defining achievement of this research.

From our comparison analyses, we have that the ABSDTA model behaves consistently opposed to Han's approach. Our results show that the ABSDTA leads to lower cost indicators than those of the SDUE while presenting an use of arcs close to what can be expected in real scenarios. Also, the fact that routes are recursively formed while traveling, can be understood as more instances of choice for motorists when compared to the route-based SDUE model.

Among the multiple instances we analyzed, we present the plots of different combinations of dispersion parameter θ and timestep size Δt values. We observe that higher values of θ result in more sensitivity of motorists to cost changes, resulting in more drastic choices and higher total delays. Also, in terms of graphic representation, plots for Δt values less or equal to 1 min show no significant differences. Also, regarding Δt, we corroborate that when it is greater than the minimum free-flow travel time of the network, the results can not be considered realistic. We, therefore, conclude the importance of appropriately choosing the range of dispersion parameters for the logit model, which would lead to feasible interpretations, along with the appropriate size of the timestep in the discretisation of the algorithm, that would lead to realistic implementations of the ABSDTA approach.

It is worth noting that there are other choice models in the literature that have been applied to route-based stochastic traffic assignment, such as Probit (Rosa & Maher 2002), (Gu *et al.* 2022), and Gammit models (Cantarella & Binetti 2002). Even though the application of these models can provide different representations of the choice behaviour of motorists, we do not address them at this stage of our work. Their adaptation from an arc-based dynamic approach is far from direct, particularly for those models that have not closed-form probability formulations, such as Probit and Gammit (Di Gangi & Polimeni 2022). Given the dynamic nature of the problem, the choice model needs to be solved at each time increment, and those would require solving through approximation methods, resulting in an integrated complex process nesting sets of complex subroutines.

Among the potential research opportunities and extensions of the ABSDTA approach that we propose, we are especially interested in the following aspects:

- Varying the concept of reasonability, particularly by extending it to the notion of an *expected set of reasonable arcs*. The intuition behind this approach is that, instead of being defined by the costs, the reasonability of an arc could depend on its expected costs. In this line of further research, we could explore the concept of choice-based prism recently proposed by (Oyama & Hato 2019) in the context of a static Markovian assignment;
- Use of different traffic models, such as Friesz' divided link model (Friesz *et al.* 1989) and its integration with queuing models, as in Mun (2007);
- Response to incidents and capacity reductions, not all known before departure from the origin and potential application to the study of infrequent situations, such as gridlocks. In this sense, a relevant article is Oyama & Hato (2017), where the authors address such situations by applying a sequential route-choice model according to expected utilities of the remaining part of the trip to the destinations.
- Integrate the model with public transportation ones with similar approaches, particularly on the recursive nature of the choices, as in Cortés *et al.* (2023), where the assignment is tackled from a static point of view.

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Appendix A. Detailed formulational description of the ABSDTA algorithm

The following routine represents the complete formulational proceedings of the ABSDTA algorithm (section 4):

- **Initial settings:** Parameters, sets, and initial values for the structures that change over every time increment are set.
  STEP 0: INITIALIZATION:
  For each arc $a \in A$ and at each $k = 1, ..., K$, set ${}_k c_a = \phi_a$.
  For each arc $a \in A$, set ${}_0 L_a = 0$.
  For each destination $d \in D$ and for each node $n \in N$, calculate the initial minimum cost M_n^d from node n to d.
  Set an order π_d of all nodes in increasing M_n^d.
  Identify the set of reasonable arcs towards d, given by

$$R^d = \left\{(n, m) \in A : M_n^d \geq M_m^d\right\}. \tag{A1}$$

  For each O-D pair $(o, d) \in OD$ and each time increment $k = 1, ..., K$, calculate the average demand as

$${}_k q^{(o,d)} = \frac{\int_{t=k\Delta t}^{(k+1)\Delta t} q^{(o,d)}(t)\, dt}{\Delta t}. \tag{A2}$$

- **Time increment update:**
  At each time increment $k = 1$ to $k = K$ until the stop condition is satisfied:

- **STEP 1: BACKWARD:**
  Calculate the expected minimum costs ${}_kW_n^d$ from each node $n \in N$ to each destination $d \in D$

$$ {}_kW_m^d = -\frac{1}{\theta} \ln \left(\sum_{b \in A_m^+} \exp\left(-\theta_k Z_b^d\right) \right). \tag{A3} $$

Calculate the expected minimum costs ${}_kZ_a^d$ using each reasonable link $a \in A \bigcap R_d$ to each destination $d \in D$:

$$ {}_kZ_a^d =_k c_a +_{k+\lfloor \tau_a(k\Delta t)/\Delta t \rfloor} W_m^d. \tag{A4} $$

- **STEP 2: COMPUTING OF ASSIGNMENT FACTORS:**
  For each destination $d \in D$ and for each arc $a \in A$, calculate the *assignment factor* as

$$ {}_kF_a^d = \begin{cases} \exp\left(-\theta_k Z_a^d\right) & \text{if } a \in R^d \ (a \text{ is reasonable towards } d) \\ 0 & \text{otherwise.} \end{cases} \tag{A5} $$

- **STEP 3: FORWARD:**
  For each node $n \in N$, check if there is any flow rate to be assigned from n, which happens if

$$ \sum_{d \in D} \left(\sum_{b \in A_n^-} {}_kG_b^d +_k q^{(n,d)} + \frac{\sum_{a \in A_n^+} {}_{k+\phi_a-1}L_a^d}{\Delta t} \right) > 0, \tag{A6} $$

where ${}_kq^{(n,d)} = 0$ if $(n,d) \notin OD$ and ${}_{k+\phi_a-1}L_a^d$ is the queue length of motorists towards destination d from the previous time increment, and check if the end of its outgoing arcs is reached during the time period,

$$ k + \max_{a \in A_n^+} \{\phi_a\} \le K. \tag{A7} $$

If the conditions are fulfilled, for each arc $a \in A_n^+$, calculate

$$ {}_kE_a^d = \begin{cases} \dfrac{{}_kF_a^d}{\sum_{a' \in A_n^+ \bigcap R^d} {}_kF_{a'}^d} \left(\sum_{bd \in A_n^-} {}_kG_b^d +_k q^{(n,d)} \right), & \text{if } a \in R^d, \\ 0, & \text{otherwise.} \end{cases} \tag{A8} $$

if $(n,d) \in OD$, otherwise

$$ {}_kE_a^d = \begin{cases} \dfrac{{}_kF_a^d}{\sum_{a' \in A_n^+ \bigcap R^d} {}_kF_{a'}^d} \sum_{b \in A_n^-} {}_kG_b^d, & \text{if } a \in R^d, \\ 0, & \text{otherwise.} \end{cases} \tag{A9} $$

Then, if the arc has not exceeded its queue service capacity Q_a, which happens if

$$\sum_{d\in D}\left(\frac{{}_{k+\phi_a-1}L_a^d}{\Delta t} + {}_k E_a^d\right) \leq Q_a. \tag{A10}$$

calculate the outflow rate as

$${}_{k+\phi_a}G_a^d = \frac{{}_{k+\phi_a-1}L_a^d}{\Delta t} + {}_k E_a^d, \tag{A11}$$

and set the queue length as

$${}_{k+\phi_a}L_a^d = 0. \tag{A12}$$

Otherwise, if (A10) is not met, set $Q \leftarrow Q_a$ and for all $d \in D$ set ${}_{k+\phi_a,k+\phi_a}S_a^d \leftarrow_k E_a^d$ and ${}_{k+\phi_a}G_a^d \leftarrow 0$, then from $l + \phi_a$ such that $\sum_{d'\in D}\, {}_{l+\phi_a,k+\phi_a}S_a^{d'} > 0$ and $\sum_{d'\in D}\, {}_{m+\phi_a,k+\phi_a}S_a^{d'} = 0$ for $m = 1, \dots, l-1$, perform the following subroutine:

– If $\sum_{d'\in D}\, {}_{l+\phi_a,k+\phi_a}S_a^{d'} \leq Q$, then update

$${}_{k+\phi_a}G_a^d \leftarrow {}_{k+\phi_a}G_a^d + {}_{l+\phi_a,k+\phi_a}S_a^d, \text{ for all } d \in D, \tag{A13}$$

$${}_{k+\phi_a,k+\phi_a}S_a^d \leftarrow 0, \text{ for all } d \in D, \tag{A14}$$

$$Q \leftarrow Q - \sum_{d'\in D} {}_{l+\phi_a,k+\phi_a}S_a^{d'}. \tag{A15}$$

Then, if $Q = 0$, end subroutine, otherwise, run it again.

– Otherwise, update

$${}_{l+\phi_a,k+\phi_a}S_a \leftarrow \sum_{d'\in D} {}_{l+\phi_a,k+\phi_a}S_a^{d'}, \tag{A16}$$

$${}_{k+\phi_a}G_a^d \leftarrow {}_{k+\phi_a}G_a^d + \frac{{}_{l+\phi_a,k+\phi_a}S_a^d}{{}_{l+\phi_a,k+\phi_a}S_a}Q, \text{ for all } d \in D, \tag{A17}$$

$${}_{k+\phi_a,k+\phi_a}S_a^d \leftarrow {}_{k+\phi_a,k+\phi_a}S_a^d - \frac{{}_{l+\phi_a,k+\phi_a}S_a^d}{{}_{l+\phi_a,k+\phi_a}S_a}Q, \text{ for all } d \in D, \tag{A18}$$

$$Q \leftarrow 0. \tag{A19}$$

and end subroutine.

Calculate queue lengths as

$${}_{k+\phi_a}L_a = \Delta t \sum_{d\in D}\sum_{l=1}^{k} {}_{l+\phi_a,k+\phi_a}S_a^{d'}. \tag{A20}$$

- **STEP 4: COST UPDATE:**
  For each arc $a \in A$ and each time increment $k \in K$, use the assigned flows

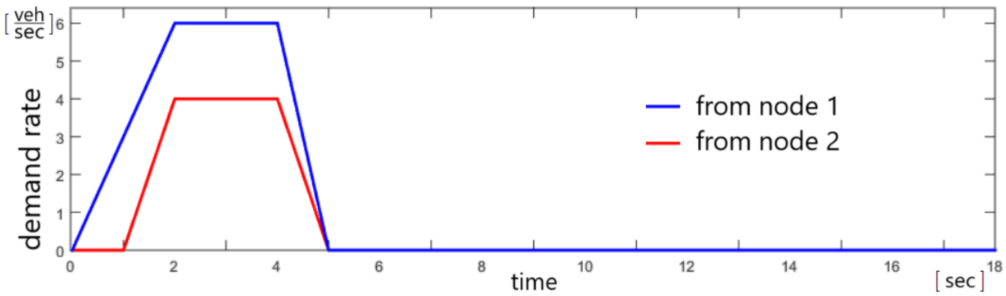
to update the queue lengths and link costs

$$ {}_{k+\phi_a}L_a = \max\left(0, {}_{(k+\phi_a-1)}L_a + ({}_kE_a - Q_a)\Delta t\right), \tag{A21} $$

$$ {}_kc_a = \phi_a + \frac{{}_{k+\phi_a}L_a}{Q_a}. \tag{A22} $$

- **STEP 5: STOP CRITERIA:**

$$ \text{If } k \le K \begin{cases} \text{If } {}_kE_a^d = 0 \ \forall a \in A, \forall d \in D & \begin{cases} \text{If } \sum_{a\in A} \sum_{l=k}^{l=K} {}_{l+\phi_a}L_a = 0, \text{ then Stop.} \\ \text{Otherwise, set } k = k+1 \text{ and return} \\ \text{to } \textbf{STEP 1}. \end{cases} \\ \text{Otherwise, set } k = k+1 \text{ and return to } \textbf{STEP 1}. \end{cases} \tag{A23} $$



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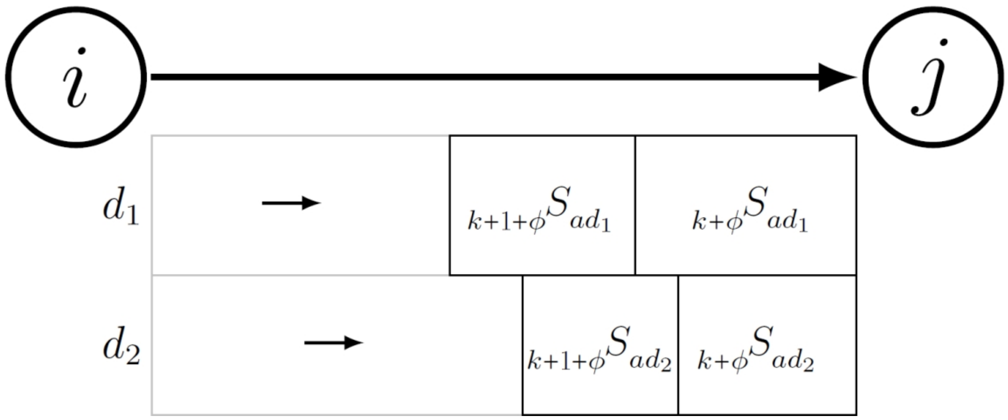

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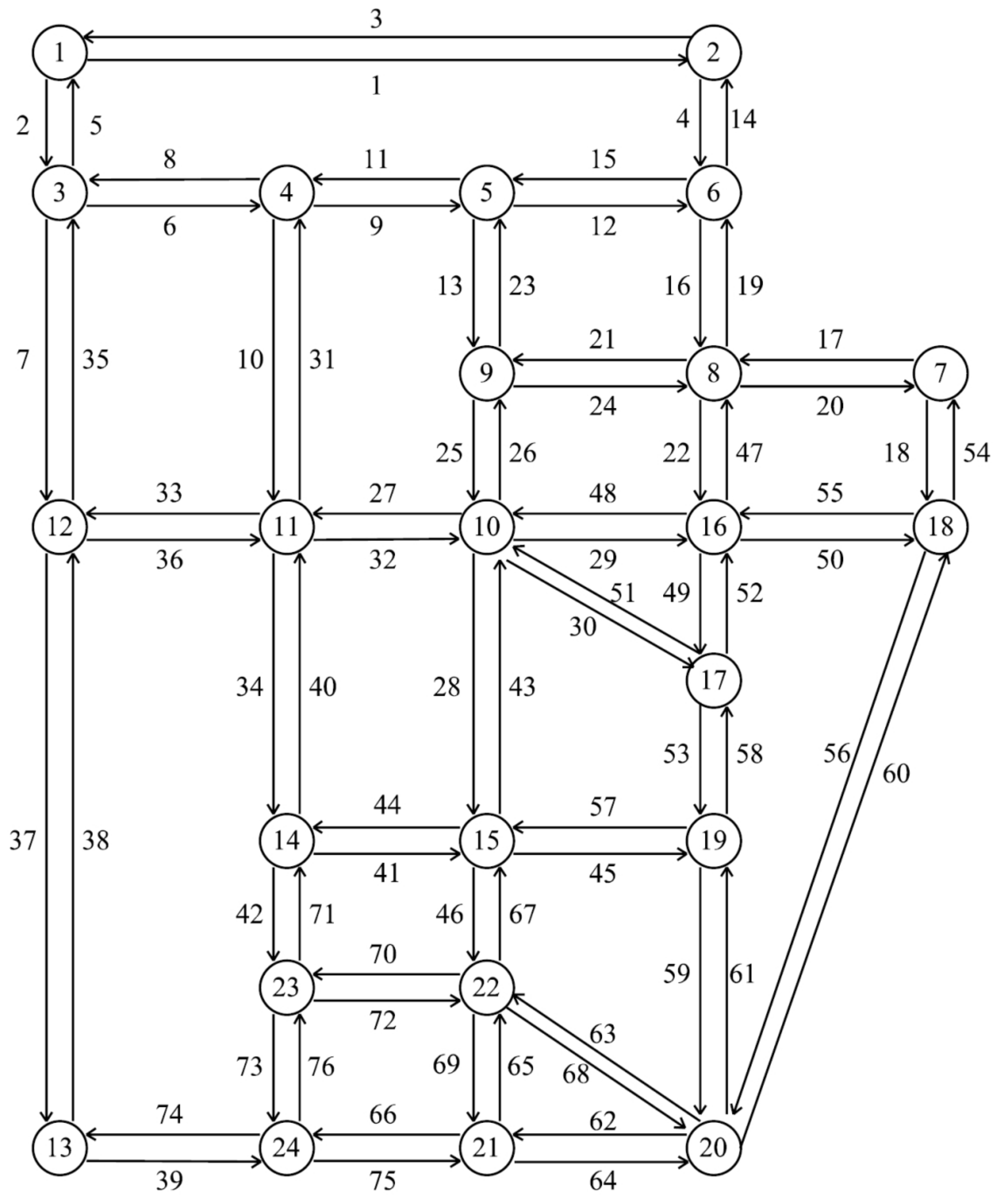

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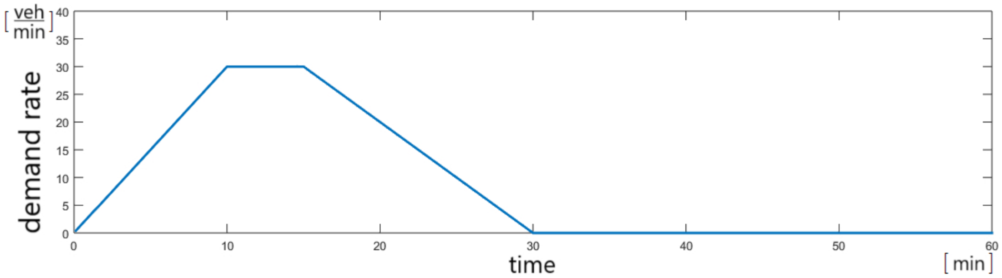

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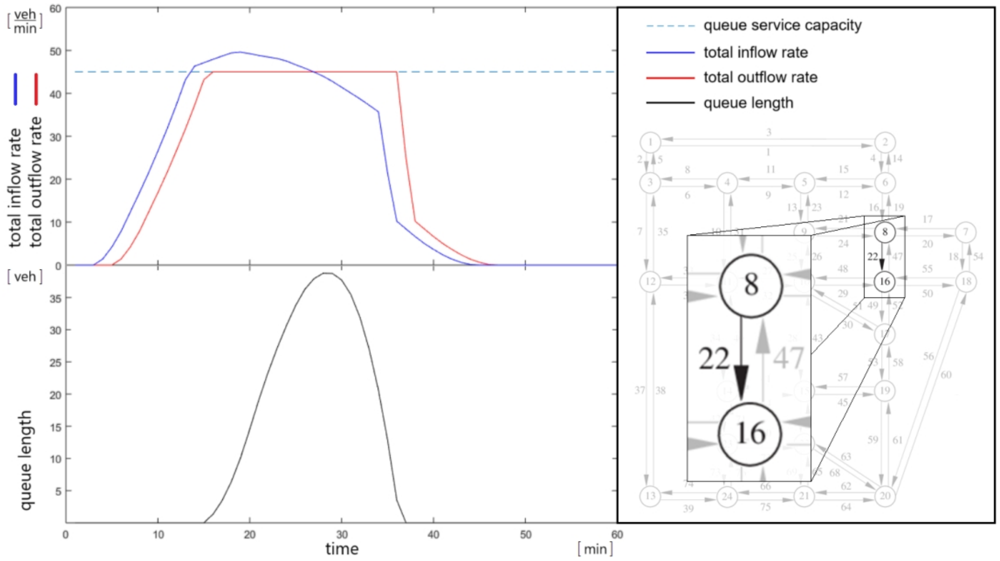

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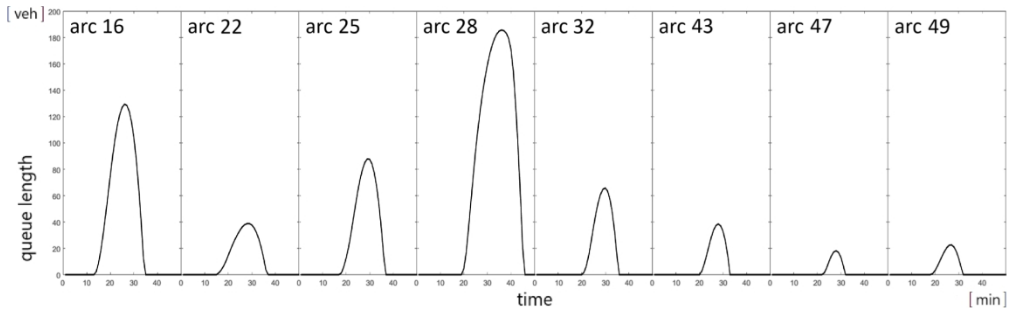

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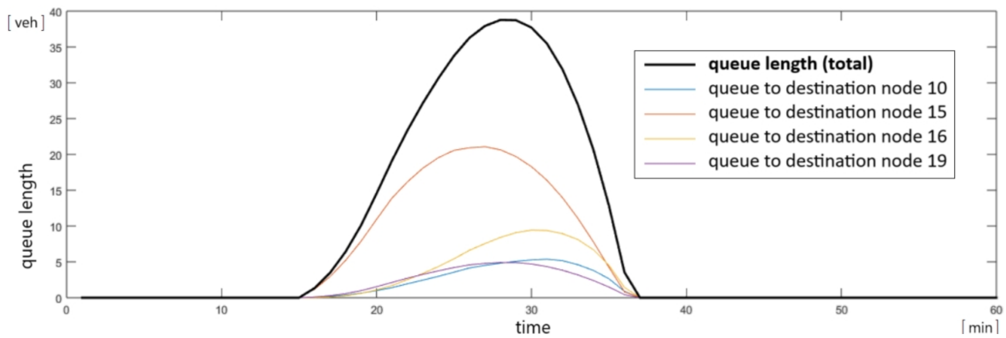

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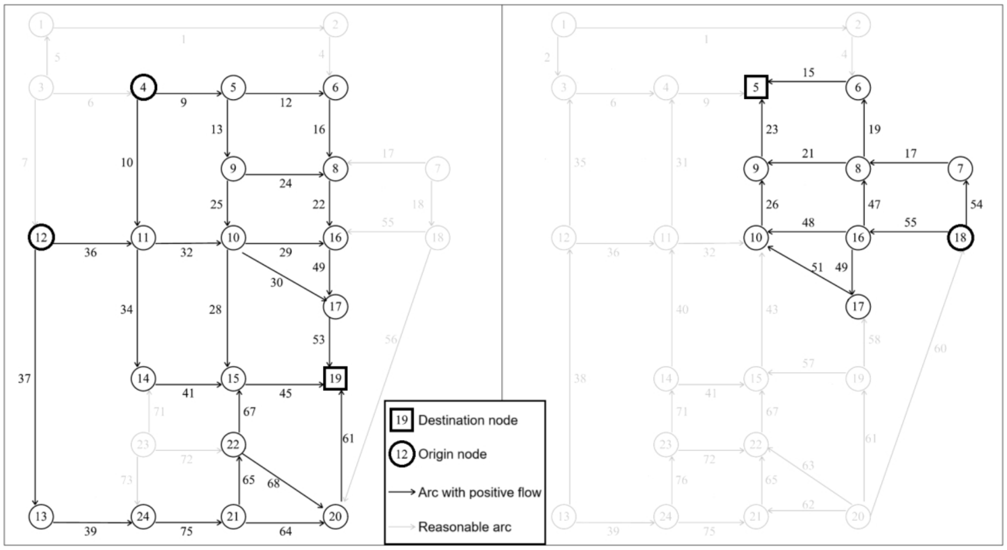

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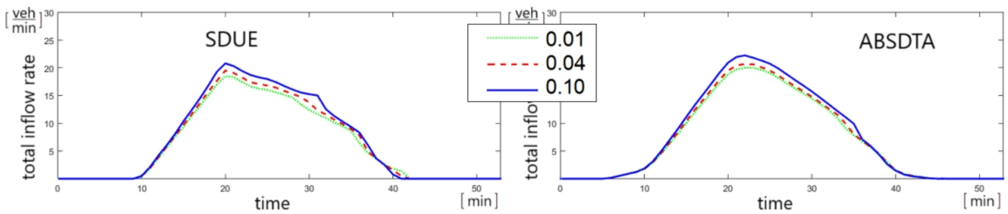

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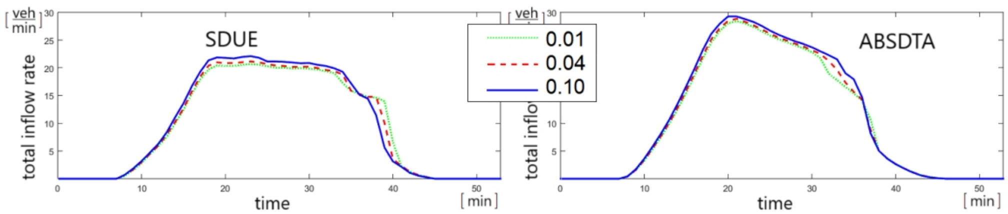

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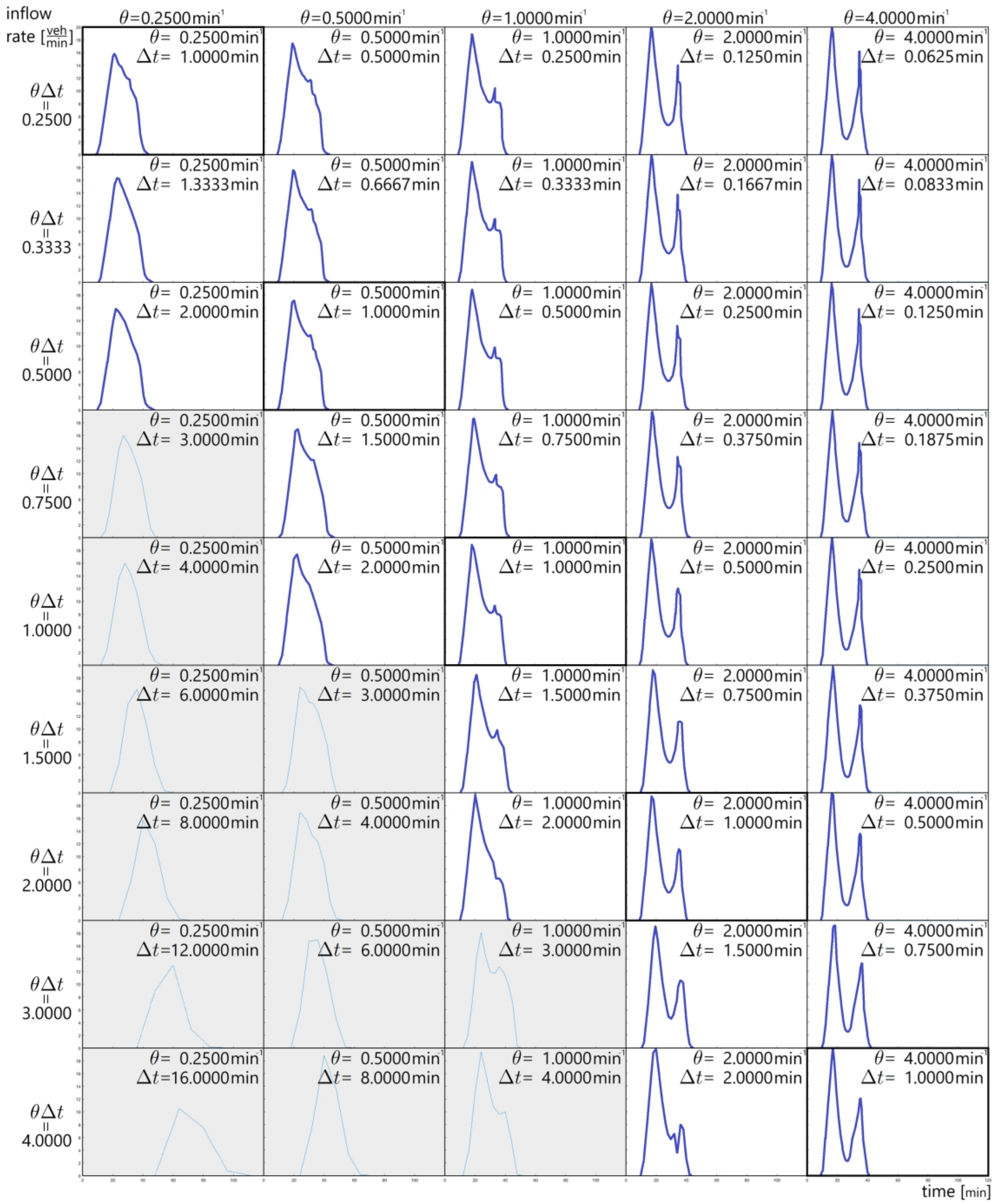

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