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Traversability in Spatial Configuration:

Some Theoretical and Practical Aspects

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ABSTRACT

Traversability, a relatively new concept in the field of space syntax study, was introduced by Hillier (2019) in his last paper, measuring the ease with which a layout can be traversed as a whole without crossing the same space twice. In his preliminary comparison of the traversability value of four structure types (a-. b-. c- and d-structures), Hillier argued that the understanding of traversability in graph structures, regarding its functional effects and social meanings, will contribute to the enrichment of space syntax theory. Taking Hillier's work as a starting point, this paper provides a more in-depth exploration of traversability in four dimensions. a. providing mathematical evaluations of the traversability value in a-, b-, c- and d-structure with the number of nodes ranging from four (the minimum number of nodes to form a d-structure) to n; b. conducting experiments with how changes in local spatial relationships may affect the global value of traversability in a configuration, thereby illustrating the architectural implications of the measure; c. classifying distinctions in social meanings between spatial configurations with high and low traversability values. This analytic is based on comparing traversability values with established classifications of layouts in space syntax research using the conceptual/analytical pairs of high/low integration and strong/weak programmes (Hillier 1996); d. measuring and comparing traversability values of real-world layouts using a python programme developed for this study. We *argue that these four dimensions of analysis highlight the significance of this new measure; we also explain what they add to existing applications of space syntax analysis. The paper concludes with directions for the future exploration of this new measure and possible applications in practical fields.*

KEYWORDS

Traversability, Hamiltonicity, spatial structure, space syntax.

1 INTRODUCTION

In his keynote paper for the twelfth Space Syntax Symposium, Hillier discussed the idea of traversability, as a new spatial measure that helps to determine and classify the type of graph structure underlying spatial configurations (Hillier 2019). Originally defined as Hamiltonicity in the graph theory, traversability quantifies the extent to which spatial nodes within a configuration can be traversed through a simple path without repetition or omission - a Hamiltonian path. Explained by Hillier (2019), the value of traversability will be maximised if there are no revisits required in the process of traverse, that is, '*when each step takes you to a new node*' (ibid., p.14) and minimised if frequent revisits are inevitable. This is a new concept brought to the space syntax field that differs from the well-discussed total depth/integration measure. While total depth accounts for the sum of topological distances across all spaces (Hillier and Hanson 1984), traversability considers not one-to-one depth but a consistent visit through all spaces at once, offering an alternative understanding of the potential functional effects of spatial configuration and its social-cultural significance.

However, this paper identifies two limitations in Hillier's study. The first is the absence of quantitative evaluations in his theoretical models of a-, b-, c- and d-structures. In the 2019 paper, Hillier's application of the traversability measure was limited to practical exercises and comparisons based on 7-node graph examples (Figure 1), in which he argued that c- and d-structures have high traversability values while a- and b-structures have comparatively lower values. It remains unclear whether the observed trend persists in graphs with a higher (or lower) number of nodes. The second deficit is a lack of a mathematical approach for calculating the traversability value of real-world spatial configurations. Hillier (2019) applied a series of manual calculations in his experiment on Tate Britain, an example that he suggested to be a d-structure type. Consequently, there is limited usefulness of traversability as a measure within the broader context of space syntax research if this new measure cannot be systematically quantified and globally compared within a standardized mathematical framework.

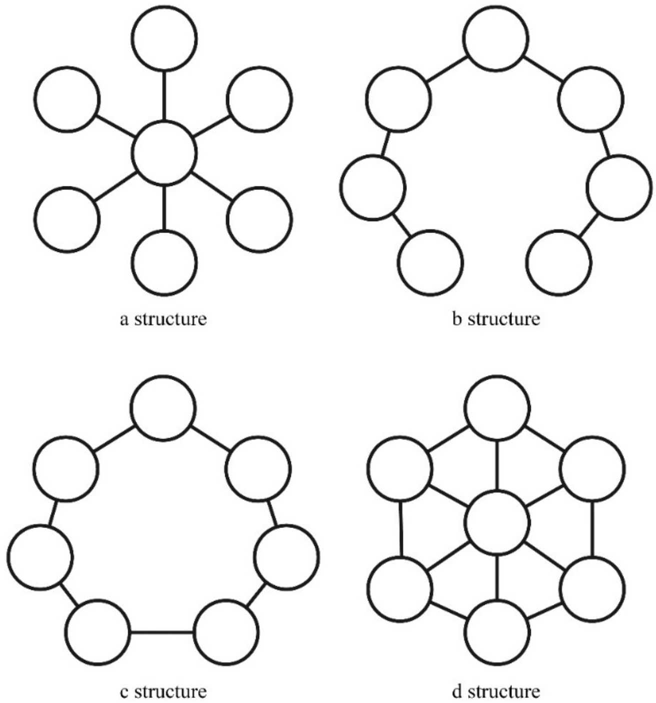

Figure 1: The a-, b-, c- and d-structure graphs with 7 nodes used as theoretical examples by Hillier in his experiments to calculate spatial properties (Hillier, 2019).

Shortly after the publication of Hillier's keynote paper, the global outbreak of COVID-19 posed a great challenge to public museums, prompting a need for spatial rearrangements to manage congestion flows and ensure social distancing protocols upon reopening. Addressing this context, Li and Psarra (2022) examined the reopening processes of five public museums located in London. Their study investigated spatial-curatorial adjustments implemented by these museums and explored the spatial resilience of the museum layouts in adapting to the reopening polices. One prevalent policy, required by the UK government, involved the establishment of a one-way navigation system for museum visitors to minimise face-to-face interactions. Li and Psarra contend that this one-way system requirement is closely related to the concept of traversability. Traversability, as introduced by Hillier, examines the extent to which a spatial configuration approximates Hamiltonicity. Notably, a Hamiltonian path or cycle represents a one-way route with no retracing steps - a characteristic that aligns with the mandated one-way navigation system implemented in response to the pandemic. Moreover, their study illustrates one of the architectural implications of traversability as a new measure. Museums with higher traversability values, such as the Tate Modern, demonstrate better resilience in adapting to one-way strategies. However, it was also noted that in certain cases, the influence of curatorial intensions may surpass that of spatial factors. As investigated by Li and Psarra, Tate Britain opted to close more exhibition rooms than strictly necessary to ensure its historical curatorial sequence.

Similar to the limitation observed in Hillier's keynote, Li and Psarra (2022) faced a similar constraint in their study as the calculations of traversability values were reliant on manual experiments. It is crucial to note that the traversability values derived in both Hillier's and Li and Psarra's studies did not constitute the total traverse steps from all nodes to all others, as initially suggested by Hillier. Instead, they employed a simplified methodology, identifying the longest possible Hamiltonian cycle or path in the configuration as a representative measure of traversability.

The aim of this paper is to extend the inquiry into the concept of traversability, building upon Hillier's foundational work. The intent is to establish a standardized framework for the evaluation, calculation, comparison, and discourse on traversability, thereby facilitating it as a critical spatial factor within evidence-based space syntax methodologies. The exploration will be conducted through three key disciplines:

1) Provide mathematical evaluations of traversability values in theoretical graph structures (a-, b-, c- and d-) with any given number of nodes.

2) Investigate how local changes in the spatial layout may impact the traversability value of the configuration, and how it informs architectural practices.

3) Empirically test the measurement in real-world layouts, discussing the socio-spatial meanings of traversability values at different levels.

2 LITERATURE REVIEW

2.1 Hamiltonicity and the Travelling Salesman Problem

Hamiltonicity, derived from the concept of Hamiltonian path/cycle problem in graph theory, refers to the existence of a path or a closed loop within a given graph that traverses each node (vertex) exactly once without repetition. It was originally raised by Sir William Rowan Hamilton in the 19th century and has since evolved into a critical tool in computational mathematics (Biggs, Lloyd and Wilson 1986). Closely related to the Hamiltonian problem, the Travelling Salesman Problem (TSP) poses an optimization challenge that asks for the shortest possible route a salesman can take to visit a set of cities (destinations) and return to the original start (Flood 1956). Unlike the Hamiltonian problem, which relies solely on the topological connections (edges), the TSP takes count of actual travel costs, such as metric distances between each destination.

Calculations for both Hamiltonicity and the TSP have been widely regarded as NP-hard (Non-deterministic Polynomial-time hard), especially in graphs with a large number of nodes. It has been argued by scholars like Michael Garey and David Johnson that it is theoretically impossible to establish an efficient algorithm to solve such problems (Garey and Johnson 1979, Garey 1997). In graph theory, there are two primary categories of solutions for the calculation. The first involves exhaustive methods, which require checking all possible permutations of vertices and propose the best combination through evaluations (Bellman 1962). This approach ensures the accuracy of results but sees a factorial increase in the calculation workload as the graph size increases. Hillier's manual experiments on traversability and the calculation approaches implied by this paper are based on the exhaustive methods. The second type of solutions comprises approximation and heuristic-based methods. For instance, Rosenkrantz, Stearns and Lewis (1977) applied the Nearest Neighbour algorithm in their study of the TSP, which picks a random start point and repeatedly visits the nearest unvisited destinations until all points have been covered. It offers a computationally less expensive approach for large graphs but is unable to guarantee a hundred percent of accuracy and does not consider the overall structure (Papadimitriou and Steiglitz 1998).

In the realm of smart cities and urban planning, the practical implications of Hamiltonicity and the TSP have been discussed in disciplines such as optimising public transportation efficiency (Batty and Longley 1994), strategic placement of urban utilities and amenities (Tero et al. 2010), and enhancing municipal waste collection routes to reduce operational costs (Laporte 1992). However, their application in the architectural field is less explored. Martin Werner's study on indoor navigation at the Munich airport, which compared exhaustive and approximation techniques for efficient route planning, had inconclusive results (Werner 2011). One of the reasons, discussed by Werner, was due to the complexity of the computation model, integrating multifarious variables like metric distances and utility availability. This paper suggests that Werner's research highlights a critical characteristic of space syntax theory, which considers space as an independent factor, thereby sidestepping these complexities as *a posteriori* variables. It is noticeable that the intension of this paper is not to find a mathematical solution for searching efficient traverse routes but to describe and justify levels of traversability values of spatial configurations and what it means in socio-cultural contexts.

2.2 Traversability and long/short models

Hillier (2019) proposed three fundamental structural properties crucial for defining the structural types of configurations: total depth, traversability and mean local choice. While total depth is a pre-existing spatial metric extensively examined in space syntax literature, traversability and mean local choice are posited as integral supplementary measures that, in conjunction with total depth, comprehensively describe spatial configurations and their associated social significances. In his analysis, Hillier calculated these properties for theoretical a-, b-, c-, and d-structures, each consisting of seven nodes (Figure 1). He suggested that variations in these structural properties are reflective of unique semantic implications inherent in each layout type (Figure 2 and 3). Consequently, '*each structure then has meaning, not in the sense of natural language, but in the syntactic sense of spatializing concepts and so reinforcing their expression*' (ibid., p.15). Essentially, long-model activity, which indicates restricted movement patterns under global constraints (Hillier and Hanson 1984; Hillier 1996), is correlated with a- and b-structures. On the other hand, c- and d-structures facilitate short-model activity, characterized by the potential for more free exploration.

This paper focuses on the concept of traversability, arguing that its quantitative value is instrumental in distinguishing between a-/b-structures and c-/d-structures. This distinction, in turn, makes traversability a critical indicator for differentiating between short- and long-model activities influenced by spatial structure. This argument is grounded on the observation that traversability is characteristically low in both a- and b-structures, yet notably high in c- and d-structures, as shown in Figure 2. A configuration with low traversability requires frequent re-entry into previously visited spaces, aligning with the concept of long-model activity. This indicates a spatial layout that inherently limits navigational freedom, enforcing a pattern of movement that involves repeated visits to certain areas. Conversely, high traversability in a configuration indicates a globally unrestricted navigation experience, enabling visitors to traverse the entire layout seamlessly in a single path without the necessity for backtracking. This characteristic is indicative of short-model activity, in which a more explorative movement pattern is grounded. Hence, the traversability value of a spatial configuration can potentially illuminate the types of social activities it facilitates. However, this raises two critical research questions: Firstly, what level of traversability values should be classified as high or low? Secondly, which spatial factors are likely to influence variations in traversability values?

	Total depth	Traversability	Mean local choice
a-structure	low	low	high
b-structure	high	low	low
c-structure	high	high	low
d-structure	low	high	high

Figure 2: Comparison of the three structural properties in a-, b-, c- and d-structures (Hillier, 2019).

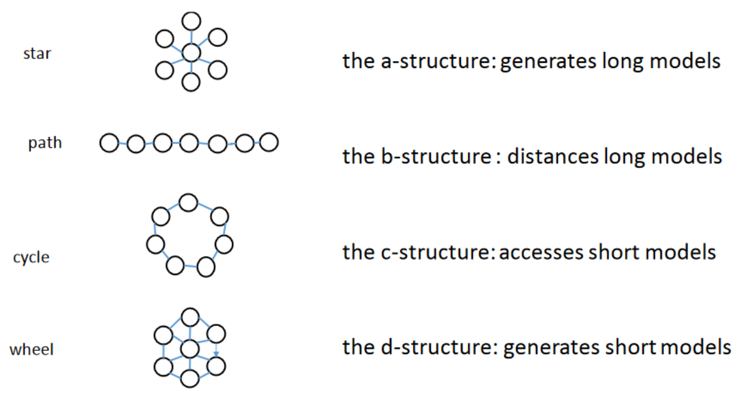

Figure 3: Hillier's suggestion on social concepts associated with each structure (2019).

3 DATASETS AND METHODS

3.1 Equations of the four spatial structures' traversability value

In Hillier's experiment of 7-node structural graphs, traversability was measured for each of the structure types by dividing the total number of steps, including revisits, required to go from each node to all others by the minimum possible for that number of nodes, essentially, when each step take you to a new node (Hillier 2019). The c- and d-structure graphs achieved a traversability value of ***1***, which means nodes in these structures could traverse all other nodes without repetition. Given a total of n nodes, each node in the c- and d-structures requires ***n-1*** steps to complete the traverse path, corresponding to the number of remaining nodes, as presented in Figures 4iv and 4v. Consequently, the aggregate traverse steps taken in c- and d-structures can be calculated as ***n(n-1)***, multiplying ***(n-1)*** by the total number of nodes n.

The a-structure represents the topological way of adding nodes to the 'star', with one central node and single-connected dead-end nodes attached to it (Figure 4i). The central node is one step away from all other nodes, necessitating a return to itself after visiting each edge node until the last one is reached. In an a-structure graph comprising n nodes, traversing each node from the central node, except the final one, requires two steps, totalling ***2(n-2)***. An additional step is needed to reach the last one, after which the journey is finished, culminating in ***2(n-2)+1***, which simplifies to ***2n-3***. For the edge nodes, the journey involved one step to the central nodes, then two steps for each subsequent node before the last, and finally one step to the last node, calculated as ***1+2(n-3)+1***, or ***2n-4***. The aggregate traverse steps in an a-structure are therefore ***(2n-3)+(n-1)(2n-4)***, which simplifies to $2n^2-4n+1$. The traversability value for the a-structure is derived by dividing ***n(n-1)*** (the minimum number of steps in an ideal layout) by $2n^2-4n+1$, the total steps calculated: $(n^2-n)/(2n^2-4n+1)$. Substituting ***n*** with *7*, as in Hillier's experiment, yields a traversability value of ***0.5915***, corroborating Hillier's result.

The calculation of b-structure's traversability, which it representative of a linear 'path' model, varies depending on whether the number of nodes n is odd or even. When ***n*** is odd, as shown in Figure 4ii, the central node first traverses to one end, taking ***(n-1)/2*** steps, and then proceeds to the other end, requiring an additional ***n-1*** steps. Thus, the total traverse for the central node amounts to ***(n-1)/2+(n-1)*** steps. The nodes at each end directly traverse to the opposite end without revisits, each taking ***n-1*** steps. For nodes adjacent to the ends, one step back is needed before completing the ***n-1*** step traversal to all other nodes. The cumulative step count, from the left end node to the right, is calculated as ***(n-1)+(n-1+1)+…+(n-1+(n-1)/2)+…+(n-1+1)+ (n-1)***, resulting in ***(5n-1)(n-1)/4***. In the case of an even ***n*** (Figure 4iii), the two central nodes each require ***(n-2)/2*** steps to reach the nearest end, followed by ***n-1*** steps to the opposite end. Th total step count is ***(n-1)+(n-1+1)+…+(n-1+(n-2)/2)+ (n-1+(n-2)/2)+…+(n-1+1)+ (n-1)***, which simplifies to ***n(5n-6)/4***. The traversability value for the b-structure is thus derived by dividing ***n(n-1)*** by the total step counts.

This results in a value of ***4n/(5n-1)*** for an odd ***n***, and ***(4n-4)/(5n-6)*** when ***n*** is even. Applying these equations to ***n*** equal to *7* yields a traversability value of ***0.8235***, which is consistent with Hillier's result (ibid.).

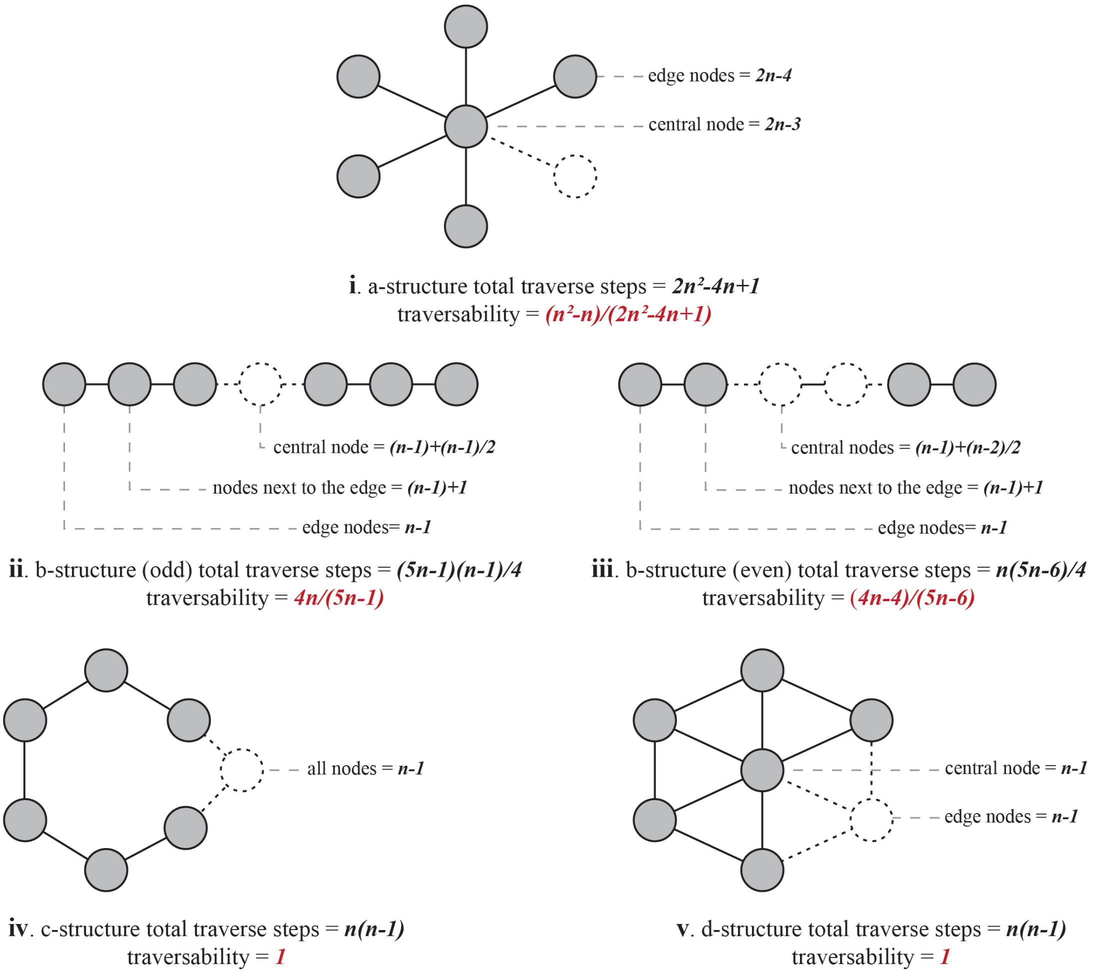

Figure 4: Evaluation of traversability value equations. (i) a-structure; (ii) b-structure with an odd number of nodes; (iii) b-structure with an odd number of nodes; (iv) c-structure; (v) d-structure.

In summary, given the number of nodes at n, the a-structure has traversability that equals ***(n²-n)/(2n²-4n+1)***. The b-structure has the value of ***4n/(5n-1)*** if n is odd and ***(4n-4)/(5n-6)*** if n is even. The c- and d-structures have the value of 1 regardless of the value of n. Figure 5 presents the curves of the traversability/node count of the four structural types. As n increases, the traversability for the a-structure asymptotically approaches 0.5, a theoretical minimum value signifying that for every step taken to a new node, an equivalent step back is required before proceeding to the next node. The b-structure's traversability, on the other hand, trends towards 0.8. According to Hillier's framework, which classifies the b-structure as having low traversability, any value at or below 0.8 can be theoretically considered as indicative of low traversability.

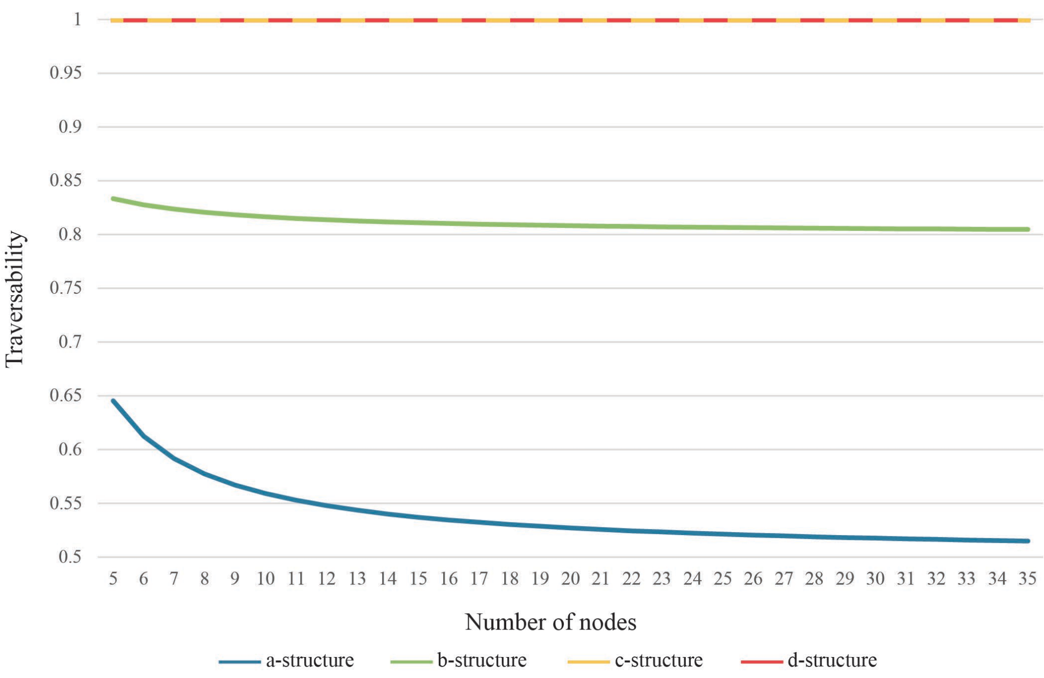

Figure 5: Curves of the traversability/node number of the four structural types.

3.2 The barring process on 18-node theoretical graphs

As stated in the previous section, this paper aims to investigate how local changes in the spatial layout may impact the overall traversability value of the configuration. This exploration is inspired by Hillier's barring experiment, detailed in the chapter '*Is architecture an ars combinatoria*?' of his book '*Space is the Machine*' (Hillier 1996), which uses 'bars', obstructing connections between spaces, and exploring how these locally placed bars affect the global property of total depth. Hillier articulated four key principles illustrating how these changes can lead to depth gain effects. For instance, the principle of centrality posits that bars positioned centrally in a layout induce a great increase in depth compared to those situated peripherally. Moreover, Hillier's experiment explored scenarios involving the addition of blocks and squares to the graph, simulating real-world spatial designs. His findings provide a quantitative understanding of the dynamic interplay between local alterations and the overall spatial configuration. These insights offer valuable reflections on design practices, underscoring the potential impacts of local design decisions on the global spatial characteristics.

Notably, studies in the field of space syntax have presented a wide range of experiments which make variations/ alterations to conceptual layouts to investigate changes in certain types of spatial properties (Shpuza and Peponis 2005; Conroy Dalton and Kirsan 2005; Koch and Miranda Carranza 2013). This paper particularly focuses on the examination of how local changes, such as placing bars, can affect the global traversability value of spatial configurations. To facilitate this investigation, a Python-based programme has been developed by the first author of this paper, calculating the total traverse steps necessary for each space within a graph to visit all others. The programme employs an exhaustive algorithm that identifies all potential traverse paths, thereby determining the minimum number of steps required for each space (Figure 6b). For the purpose of this study, a base graph comprising 18 nodes was created (Figure 6a). This size strikes a balance between computational manageability and realistic approximation of the spatial dimensions of public building layouts. Building upon this foundational graph, the study generated 60 alternative configurations by strategically placing bars in various arrangements (Figure 6c). These arrangements were guided by the logic of Hillier's four principles, aiming to examine the resultant effects on traversability. The objective is to discern whether patterns akin to Hillier's principles concerning depth gain emerge in the context of traverse step gains, which reduce the value of traversability.

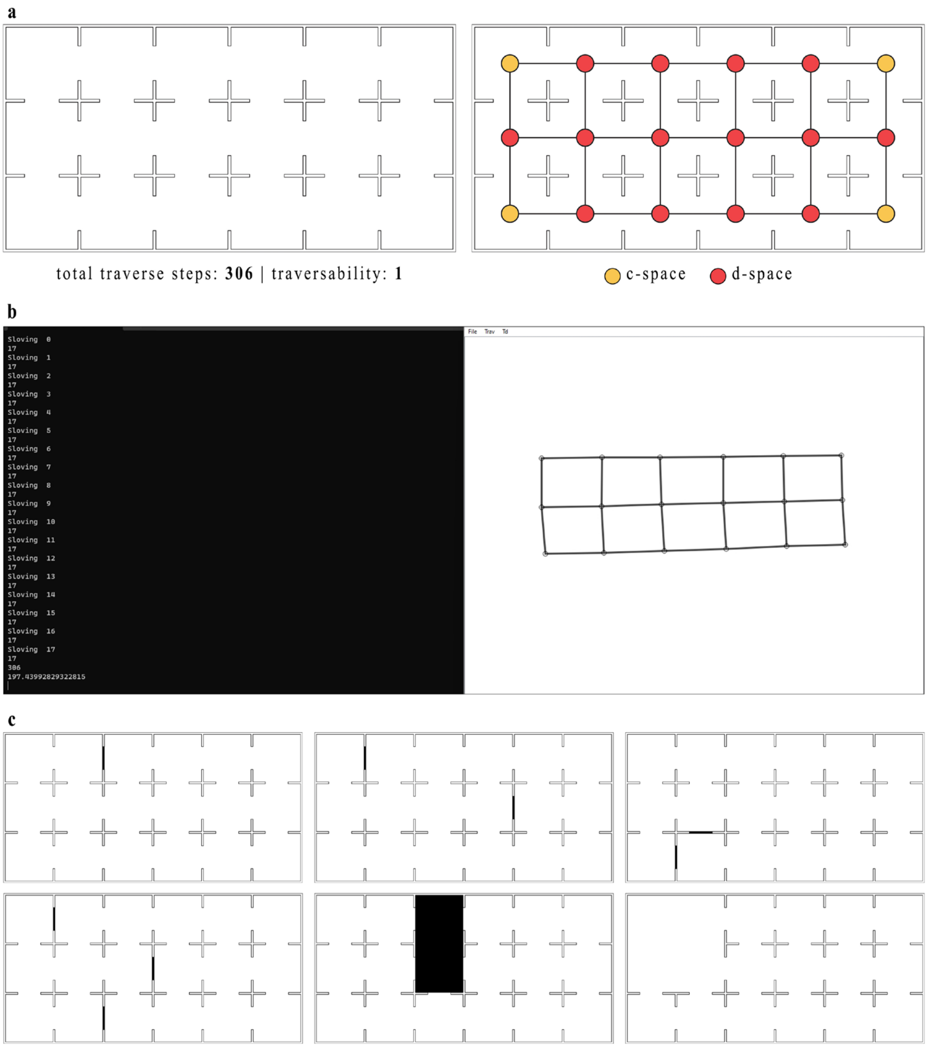

Figure 6a: Basic graph with 18 nodes, with a traversability value of 1, 12 d-spaces/ 4 c-spaces.
Figure 6b: Python programme to calculate the traverse steps for each space in the given graph.
Figure 6c: Examples of local spatial modifications on the basic graph.

4 RESULTS

4.1 Factors that lead to traverse step gain

Building on the defined concept of traversability, spaces within an 18-node graph require a minimum of 17 steps to traverse all others without repetition. Thus, a traverse step gain occurs when the shortest traverse route for a space exceeds 17 steps, necessitating inevitable revisits during navigation. Moreover, this study also examines changes in space types, investigating if specific changes or arrangements in space types correlate with consequent shifts in traversability values.

Putting a bar in the top-left corner of the graph transforms the c-space into an a-space (a dead-end space) and changes two adjacent d-spaces into c-spaces (Figure 7.1a). This modification results in an additional step required for 10 spaces in traversal routes. The remaining 8 spaces, including the new a-space, do not experience a traverse step gain (as described above, i.e. when the shortest traverse route exceeds *n-1* steps), indicating the presence of a Hamiltonian path encompassing all spaces, beginning or ending at the a-space. Shifting the bar from left to right, shown in Figures 7.1b-c, initially reduces the total traverse step gain to zero before increasing it to 10. This uncovers a noteworthy observation: although both graphs solely contain c- and d-spaces with equal ratios, one configuration leads to an increase in traverse steps, contrary to having Hamiltonian paths from all to all. This finding offers an alternative perspective to Hillier's theory, suggesting that even though c- and d-structures individually have a traversability value of 1, their combination might necessitate additional traverse steps. It also indicates a distinction between depth gain and traverse step gain; while the former follows Hillier's principle of centrality (Hillier, 1996), the latter appears in a different pattern.

Adding a second bar alongside the first on the same line of nodes generates another a-space and increases the total traverse steps from 10 to 21 (Figure 7.1d). Subsequent repositioning of the second bar from left to right then decreases the traverse step gain. Across all scenarios, as seen in Figures 7.1a-i, the emergence of a-spaces notably increases the traverse step gain. Particularly, when a-spaces are positioned in close proximity (Figure 7.1d), other spaces in the graph require more steps to complete their traverse. The conversion of d-spaces to c-spaces, which enlarges inner cycles rather than creating dead-ends, does not yield a clear principle in terms of how it affects gain in traverse steps. The logic behind these changes in traversability is not immediately discernible but relies on the computational analysis provided by the programme to understand how spatial disconnections may influence the total number of traverse steps.

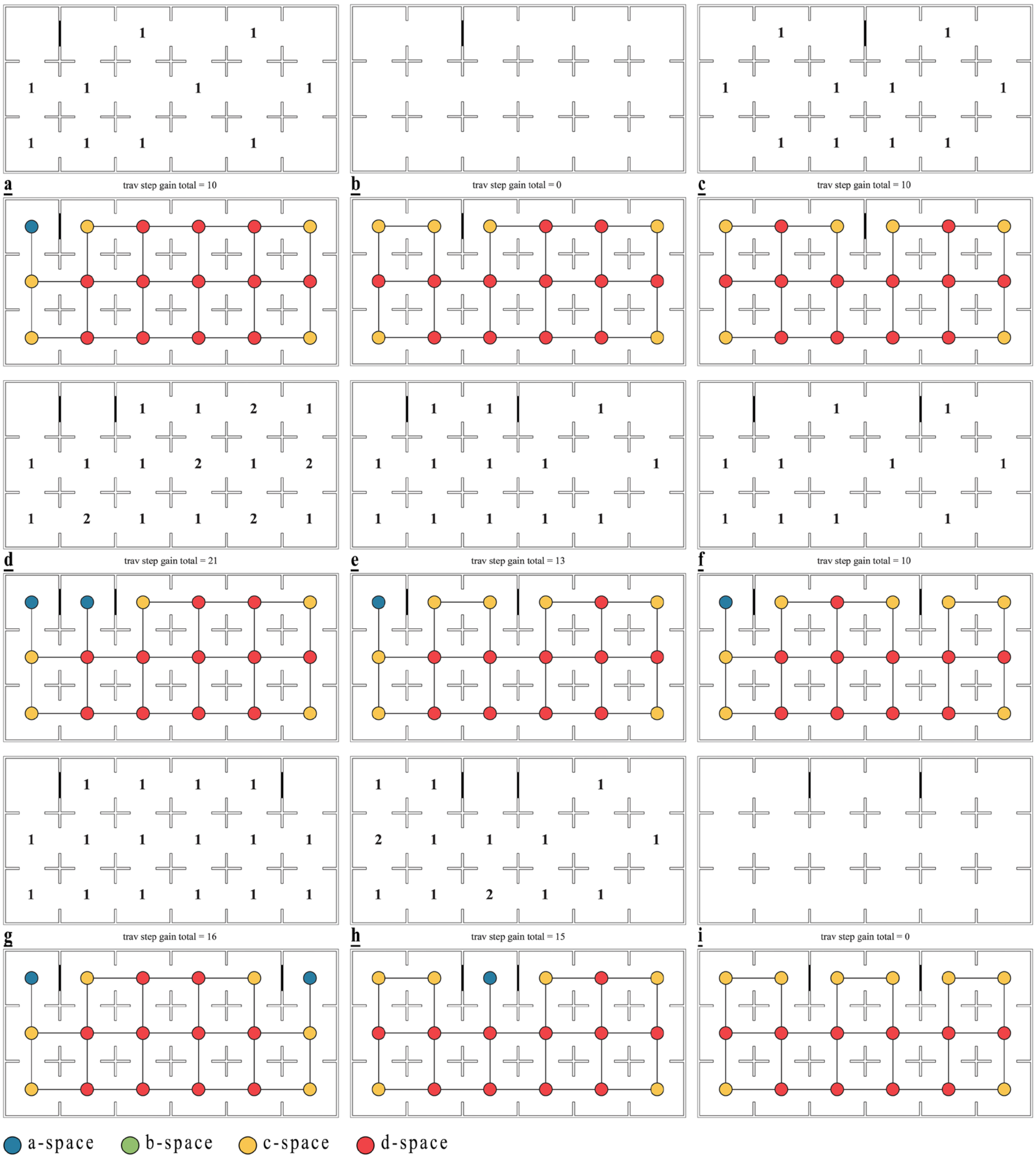

Figure 7.1: Putting bar(s) into the graph and its results in traverse step gain and space type change.

The addition of more bars along the top line of spaces, as demonstrated in Figure 7.2, results in the creation of additional a-spaces and a significant increase in the overall gain in traverse steps. This reinforces two key observations previously discussed: Firstly, the formation of a-spaces is directly correlated with an increase in the total traverse steps, as can be seen in the comparison of Figure 7.2a-d. Secondly, a-spaces situated in close proximity to each other contribute to a higher gain in traverse steps than those positioned more distantly, as illustrated by Figures 7.2e-f and 7.1d/g.

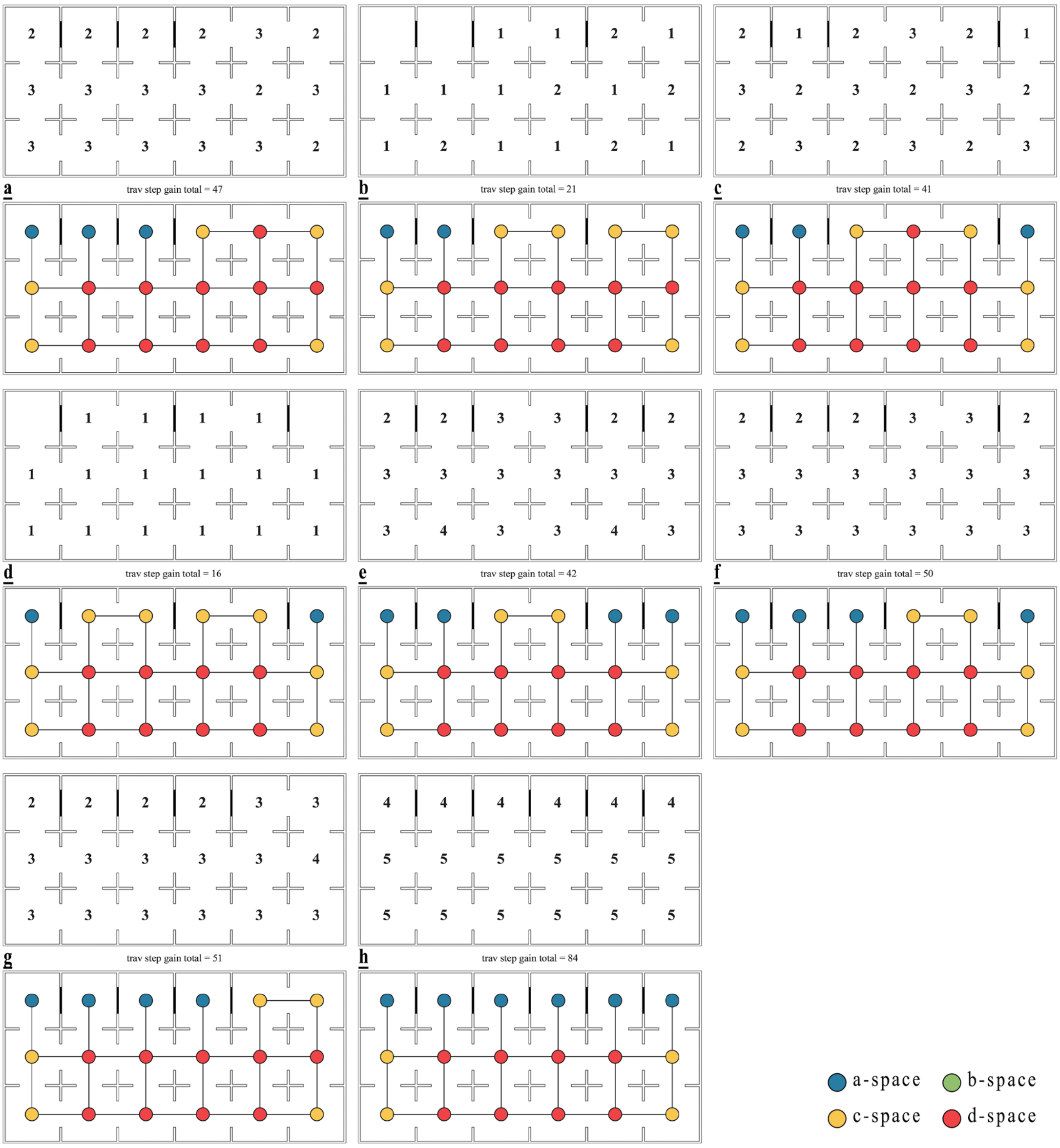

Figure 7.2: Putting bars into the graph and its results in traverse step gain and space type change.

When the second bar is positioned on an adjacent line instead of the same line, the resulting change in traverse step gain is not particularly significant, as illustrated in Figures 7.3a-h and 7.4a-b. One possible explanation for this could be that the non-contiguous arrangement of the bar essentially allows for at least one or several paths to navigate through the barred areas, thereby maintaining the shortest possible traversal route. In terms of changes to space types, in most instances, the placement of bars on an adjacent line does not alter the types of spaces but rather modifies two d-spaces from four-connected to three-connected. Additionally, bars that block an edge node, as shown in Figures 7.3a and e, result in the creation of a relatively large c-cycle which has negligible effect on changes in traversability.

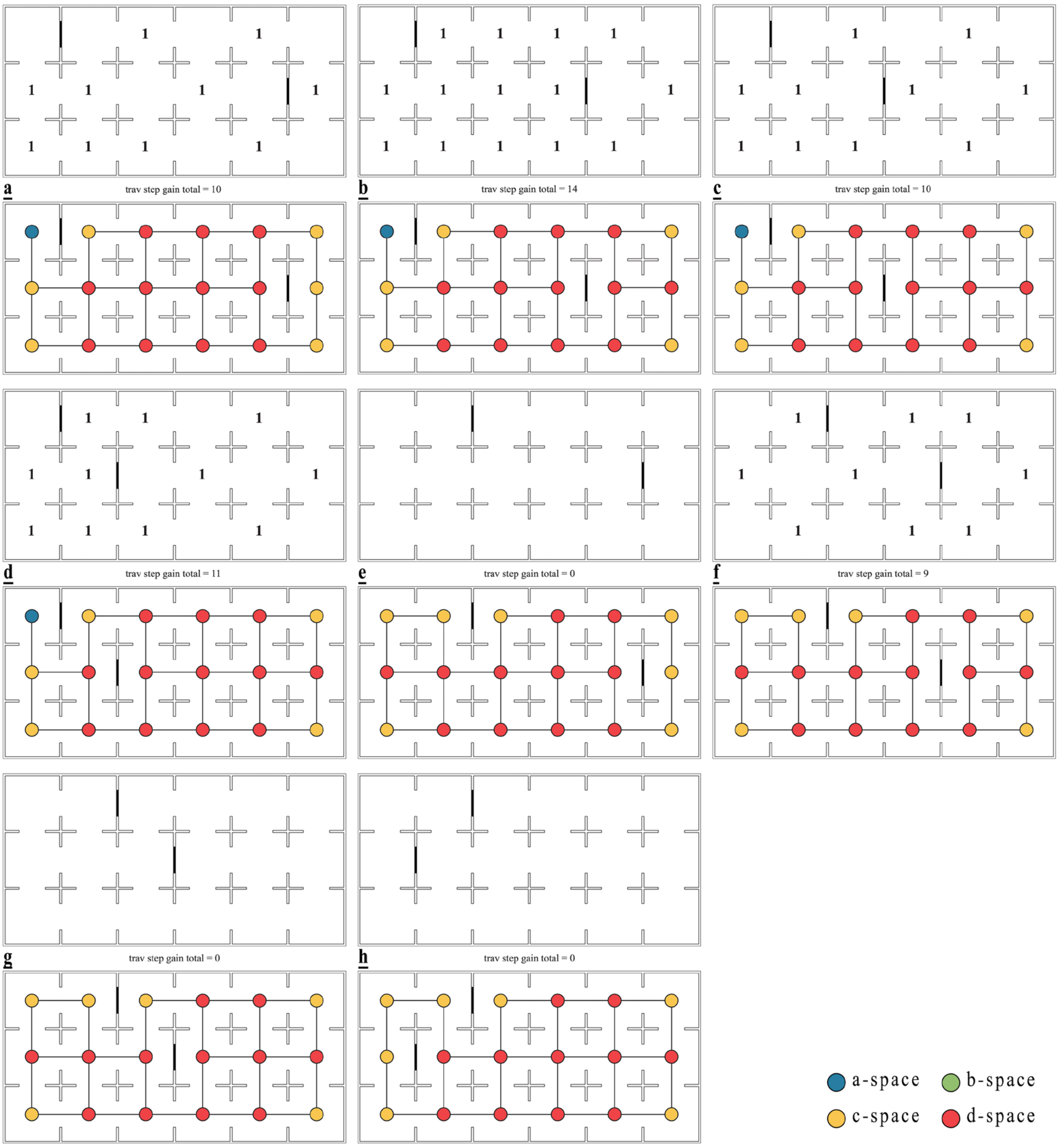

Figure 7.3: Putting bars into the graph and its results in traverse step gain and space type change.

A relatively large increase in traverse steps is observed when a second bar is placed to the left of the bottom axis (Figure 7.4c), resulting in the creation of a b-space attached with two a-spaces. In this configuration, all spaces are compelled to revisit two or three times during their traversal to other nodes. Specifically, it is required that one enters the b-space, reaches the dead-end, and then retraces one's steps to continue their journey. Another notable observation is that the majority of spaces experiencing an increase in traverse steps are d-spaces. This is primarily because d-spaces are less likely to be located at the ends of the traversal path, making them more susceptible to traverse step gains.

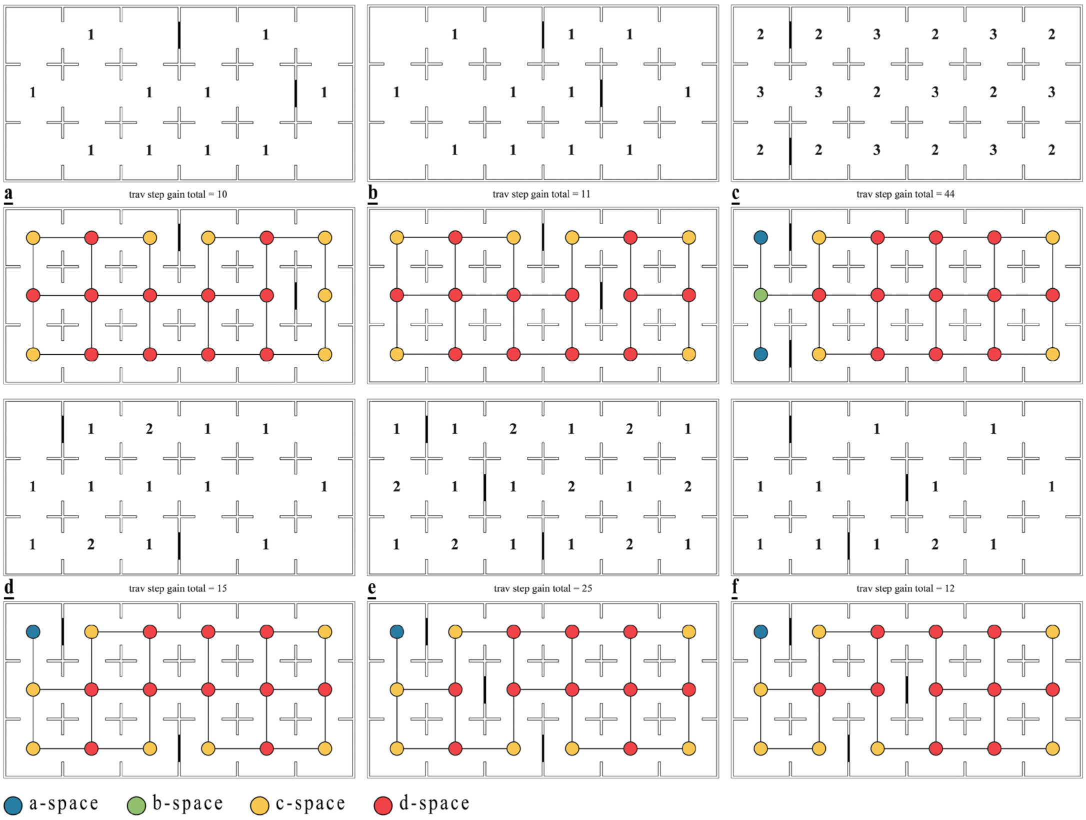

Figure 7.4: Putting bars into the graph and its results in traverse step gain and space type change.

The study next experiments with positioning the second bar orthogonally to the first, forming an 'L' shape (Figure 7.5). This arrangement results in an enlarged cycle within the graph, transforming d-spaces into c-spaces, which does not lead to substantial changes in traverse step gain. In contrast, the linearly contiguous bars arranged in the graph, as shown in Figure 7.6, create b-spaces leading to a dead-end. However, the emergence of b-structures, which Hillier (2019) associates with long model activities along with a-structures, does not result in an increase in traverse steps either. This observation partially aligns with Hillier's proposal that while the a-structure '*generates long models*', the b-structure '*distances long models*' (Hillier 2019, p.15). Essentially, the presence of a b-structure alone does not significantly impact traversability – the key indicator differentiating long and short models – but rather, it serves to distance the inevitable revisit routes diverted by the a-structure.

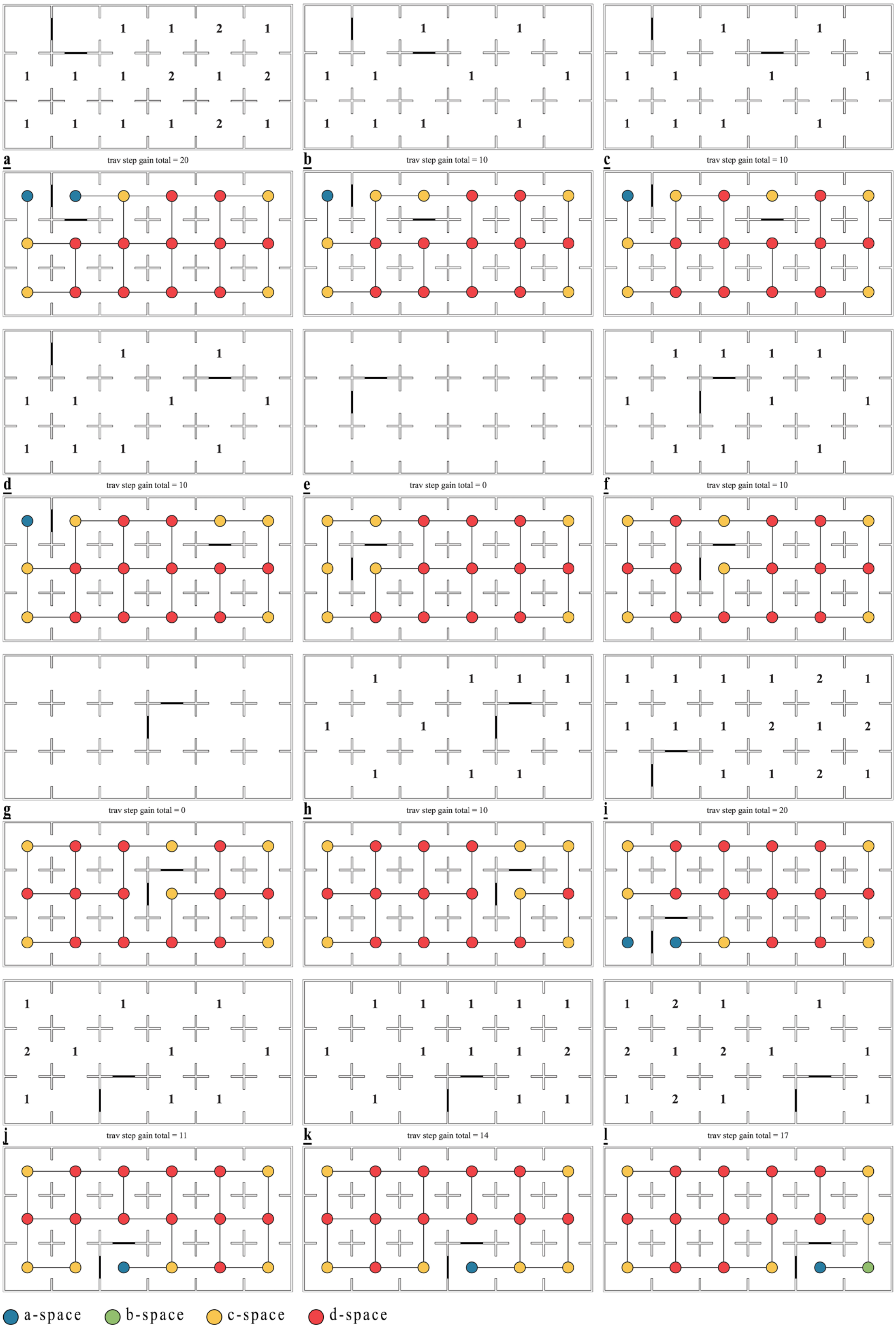

Figure 7.5: Putting bars into the graph and its results in traverse step gain and space type change.

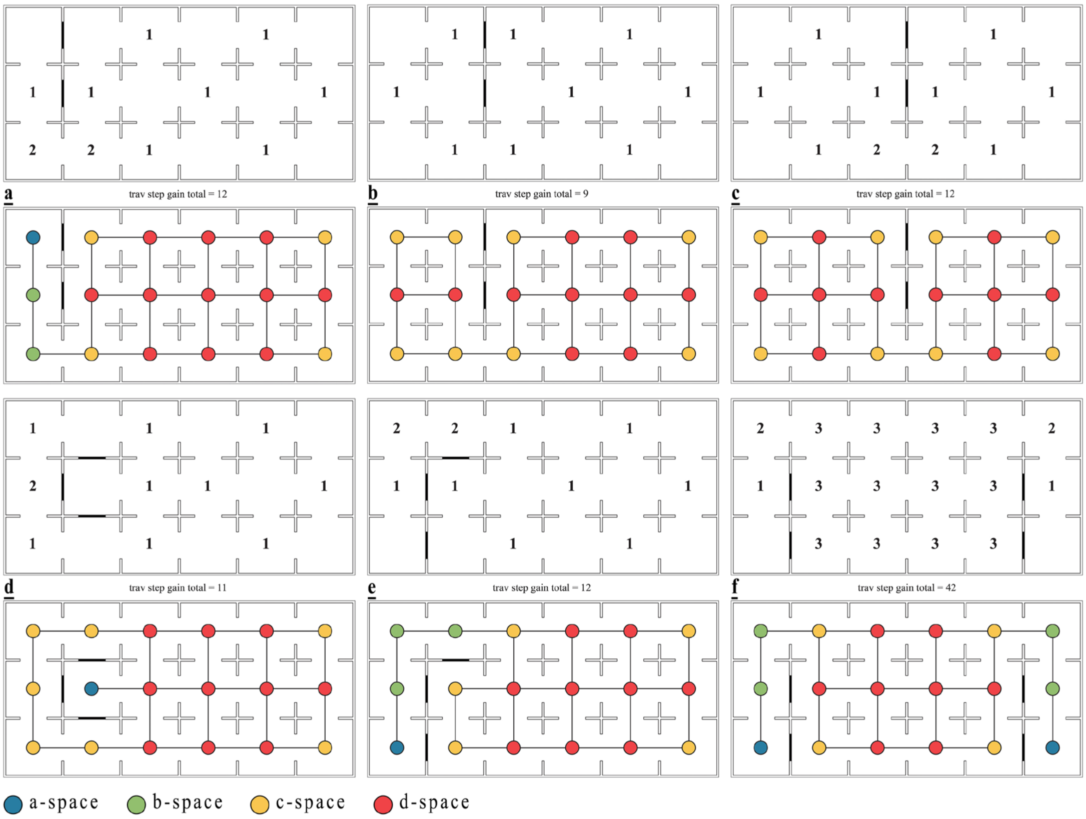

Figure 7.6: Putting bars into the graph and its results in traverse step gain and space type change.

Figures 7.7a-f present a series of blocks, or 'wells' as termed by Hillier (1996), situated within the graphs. These blocks are inaccessible from the configuration and thus are not considered part of the spatial structure. Similar to the L-shaped bars, blocks placed centrally within the graph generate an enlarged cycle, while those positioned at the edges transform spaces into a- and b-types. The gains in traverse steps observed in these graphs are not substantial. The arrangement of squares, as in Figures 7.7g-h, forms a highly connected d-space. Nonetheless, it does not alter the traversability value, given that the basic graph already possesses a value of 1 (as shown in Figure 6a). Consequently, the introduction of super connected d-spaces does not impact the existing maximum traversability in the spatial configuration.

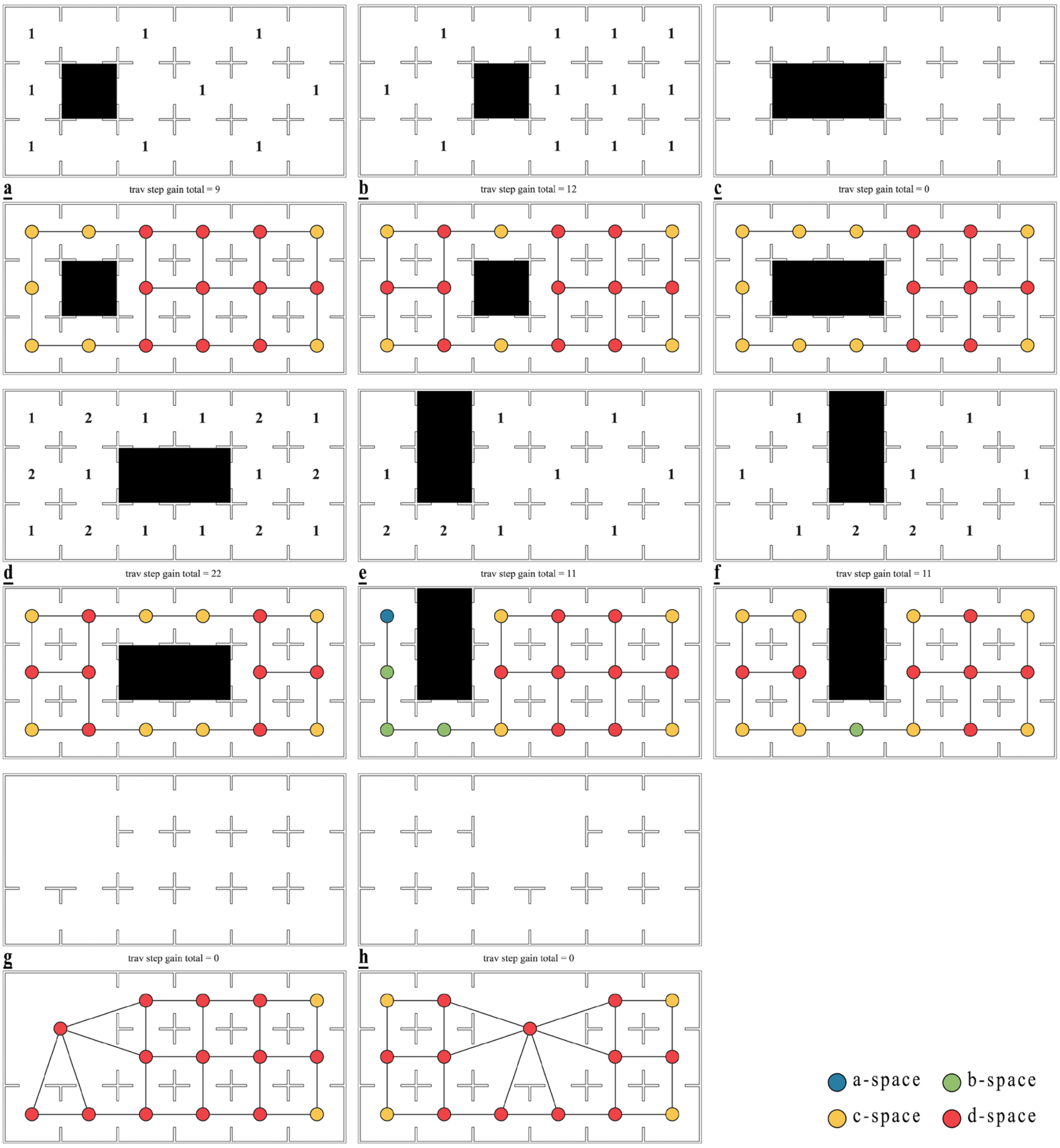

Figure 7.7: Arranging blocks/squares in the graph and its results in traverse step gain and space type change.

Building on these observations, this paper further explores extreme scenarios in which the arrangement of the maximum feasible number of bars in the graph either maximises or minimises the total gain in traverse steps, thus impacting the traversability value. Given that the graph has 18 nodes, the maximal number of bars that can be added without isolating any node is 10, maintaining 17 connections between the nodes. Since an increase in the number of a-spaces directly impacts the traverse step gain, Figure 7.8a illustrates an arrangement that maximises the conversion of spaces to a-spaces. This configuration results in a b-type path in the middle, connected with closely positioned a-spaces, leading to a total traverse step gain of 204. Consequently, the traversability value in this scenario decreases from 1 to 0.6. Conversely, to maintain efficient traversal, the optimal approach is to establish the longest possible Hamiltonian route. As shown in Figure 7.8b, strategically placing 10 bars within the graph forms a Hamiltonian path encompassing all 18 nodes, characterizing a typical b-structure. This results in an increased total of 72 traverse steps, significantly lower than that observed in the maximum scenario. It is noteworthy that connecting the two nodes at the bottom line of this configuration would transform the graph into a c-structure, a Hamiltonian cycle, which inherently possesses a traversability value of 1.

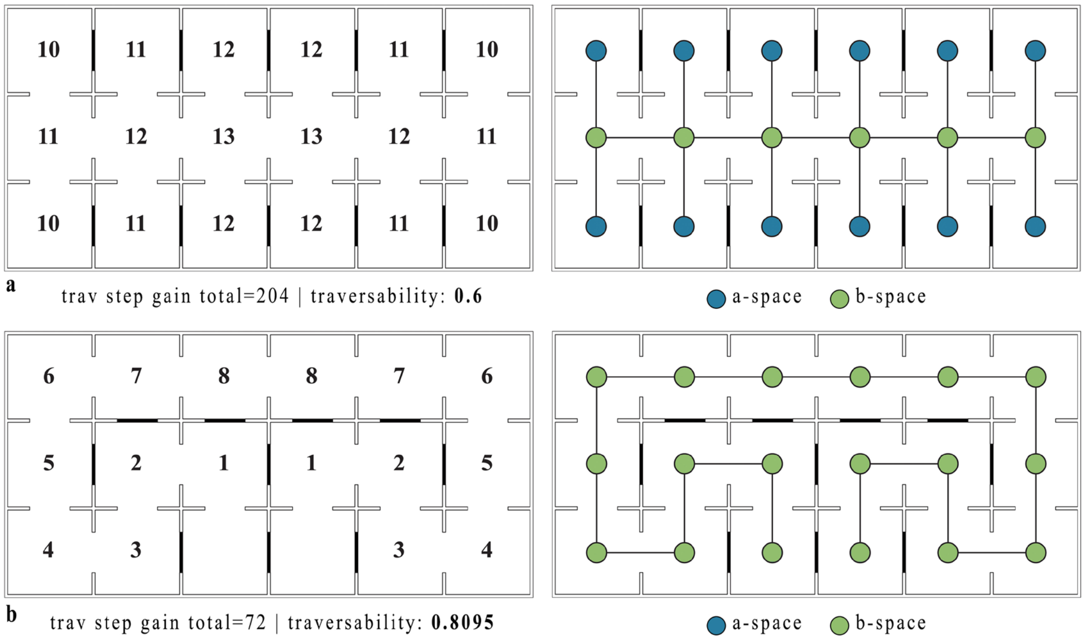

Figure 7.8a: Arrangement of bars that maximises the total traverse step gain.
Figure 7.8b: Arrangement of bars that minimises the total traverse step gain.

Figure 7.9 shows the statistical correlations between the total gain in traverse steps and the numbers of a-, b-, c-, and d-spaces in the experimental graphs. Consistent with earlier discussions, the number of a-spaces demonstrates a robust correlation with the increase in traverse steps, thereby reducing the traversability of the graph (R^2=0.853, p<0.0001). However, the correlation between traverse step gain and the number of b-spaces is not statistically significant (p=0.2537). Moreover, c- and d-spaces exhibit a negative correlation with traverse step gain, but this relationship is substantially weaker compared to that of a-spaces (R^2=0.1853, p<0.01 for c-spaces; R^2=0.1817, p<0.01 for d-spaces). This trend suggests that the pronounced effect of a-spaces on traverse step gain may overshadow the influence exerted by other types of spaces. An intriguing prospect for future research could involve exploring the traversability value when the impact of a-spaces is excluded, specifically focusing on how the combination of c- and d-spaces influences the overall spatial configuration. Such investigation could yield further insights into the spatial dynamics between local arrangements and global implications and their effect on traversability.

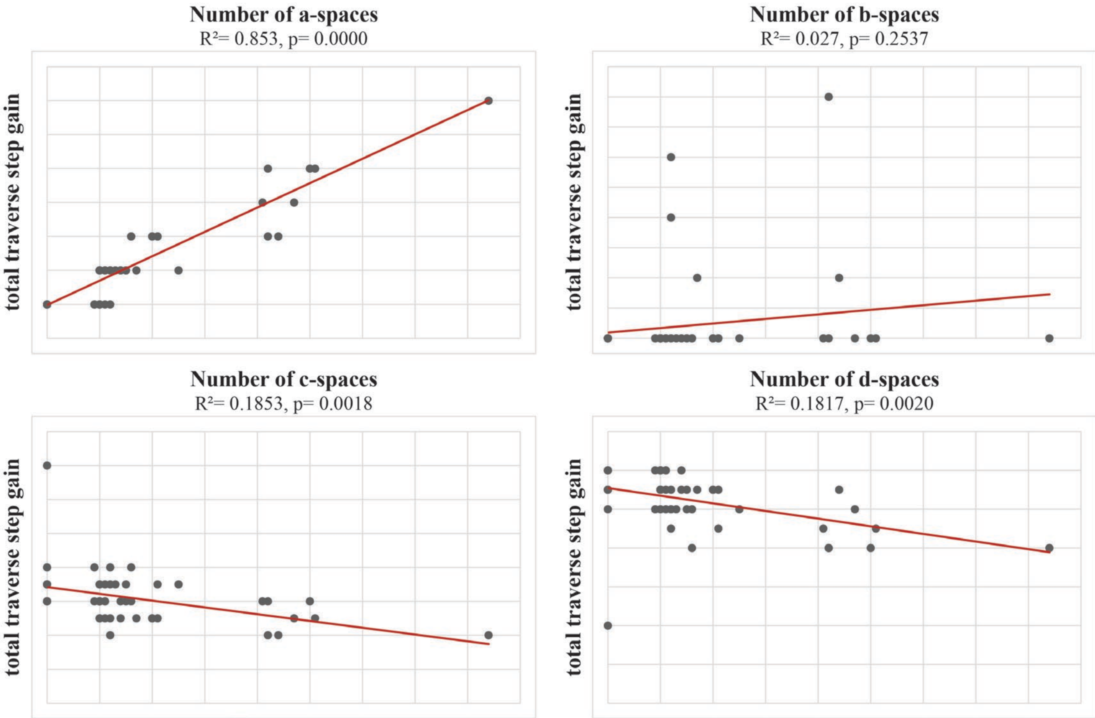

Figure 7.9: Correlation between total traverse step gain and the number of a-, b-, c- and d-spaces based on the analysed 60 alternative graphs.

4.2 Empirical attempts of traversability on the MoMA's 2019 extension

This study extends the theoretical evaluation of a-, b-, c-, and d-structures, as well as experiments on the dynamic interplay between local changes and global implications, to an empirical investigation of the Museum of Modern Art's (MoMA) 2019 expansion. This investigation seeks to comprehend how the new spatial expansion of the MoMA influences its traversability (Figure 8). The authors' previous research, presented at the 13th Space Syntax Symposium, investigated changes in spatial configurations in the 2019 extension of the MoMA in New York (Li and Psarra 2022). The 2022 study critically examined the evolution of the spatial layout of the museum from 2005 to the latest expansion, arguing that the changes not only complicated wayfinding and navigation but also diminished the museum's resilience in potentially implementing a one-way system, attributed to a reduction in traversability. However, the authors' approach to evaluating traversability was simplified due to the absence of a computational algorithm, which primarily focused on identifying the longest Hamiltonian cycle within the layout, rather than calculating the aggregate of traversal steps from all spaces to all others, as originally defined by Hillier (2019).

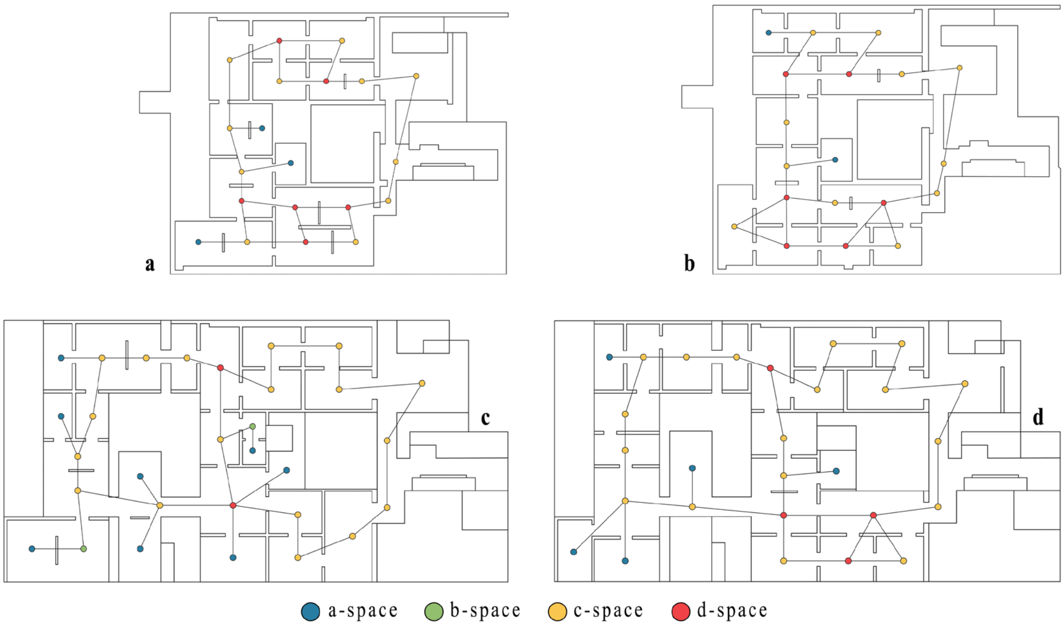

Figure 8: Unjustified graphs of the MoMA coloured by the spatial types. a) 2005 4th floor; b) 2005 5th floor; c) 2019 4th floor; d) 2019 5th floor.

Applying our newly proposed methodology, this paper conducts a configurational analysis of the MoMA's new extension in terms of its traversability, effectively building upon Li and Psarra's preliminary findings. The aim is to identify the specific spatial factors responsible for the changes in traversability values and to interpret the architectural significance of these changes. This involves comparing the findings to the theoretical traversability values of the a-, b-, c-, and d-structures evaluated through equations provided in the early section.

Figure 9a shows the justified graphs of MoMA's 4th and 5th floors, the primary exhibition areas, before and after the expansion. A notable change is the significant increase in a-spaces and a considerable decrease in d-spaces. In terms of traversability, calculated using the earlier mentioned Python programme (Figure 6b), the post-expansion MoMA exhibits a substantially lower traversability value, particularly on the 4th floor, which has a value of 0.767. This is lower than the theoretical b-structure value of 0.8, identified by Hillier as indicative of low traversability. In practical terms, this implies that visitors to the current MoMA face more challenges in traversing the entire layout, as more inevitable revisits are required to explore all spaces, thereby restricting free navigation.

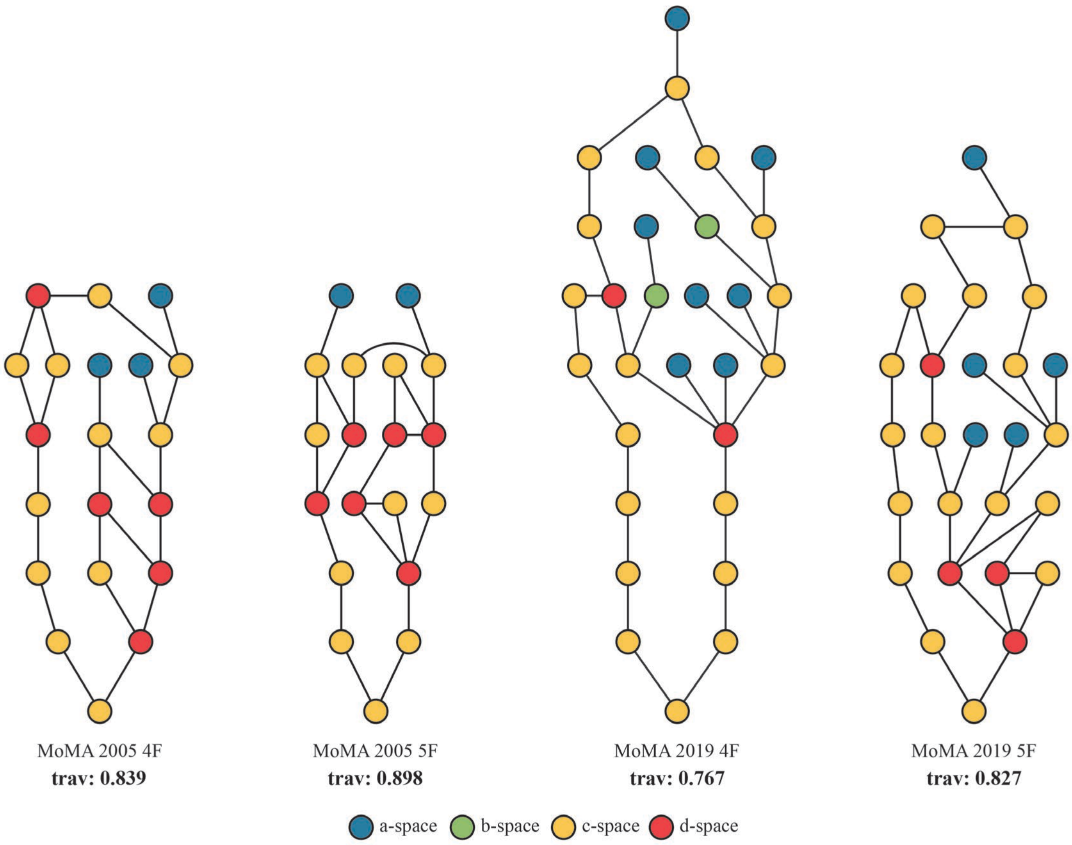

Figure 9a: Justified graphs of the MoMA coloured by the spatial types, and their traversability values.

The crucial question then arises: what factors contribute to this decrease in traversability? Our morphological experiments have indicated that the number and placement of a-spaces are key elements in increasing total traverse steps and thus lowering traversability. Figure 9b presents the calculated traversability values for MoMA's layout, excluding a- and b-spaces and considering only c- and d-spaces. The 2019 layouts show a marked increase in values, especially the 4th floor, which rises from 0.767 in 2005 to 0.941. However, does this imply that the elimination of a- and b-spaces in the MoMA's 2019 4th floor layout would enhance stronger short-model activities compared to 2005? Considering the presence of only two d-spaces, forming two large cycles with a single intersection, visitors in this setup would have fewer unnecessary revisits, but also limited route choices and difficulty accessing certain areas due to the absence of 'shortcuts' for transitioning between routes. It appears that a high traversability value merely establishes a precondition for short-model activities, ensuring traversal efficiency. Nonetheless, other spatial factors must be considered, including total depth (integration) and mean local choice, which, suggested by Hillier (2019), differentiate between c- and d-structures. The latter, as Hillier noted, is the structure that 'generates short models' (ibid., p.15).

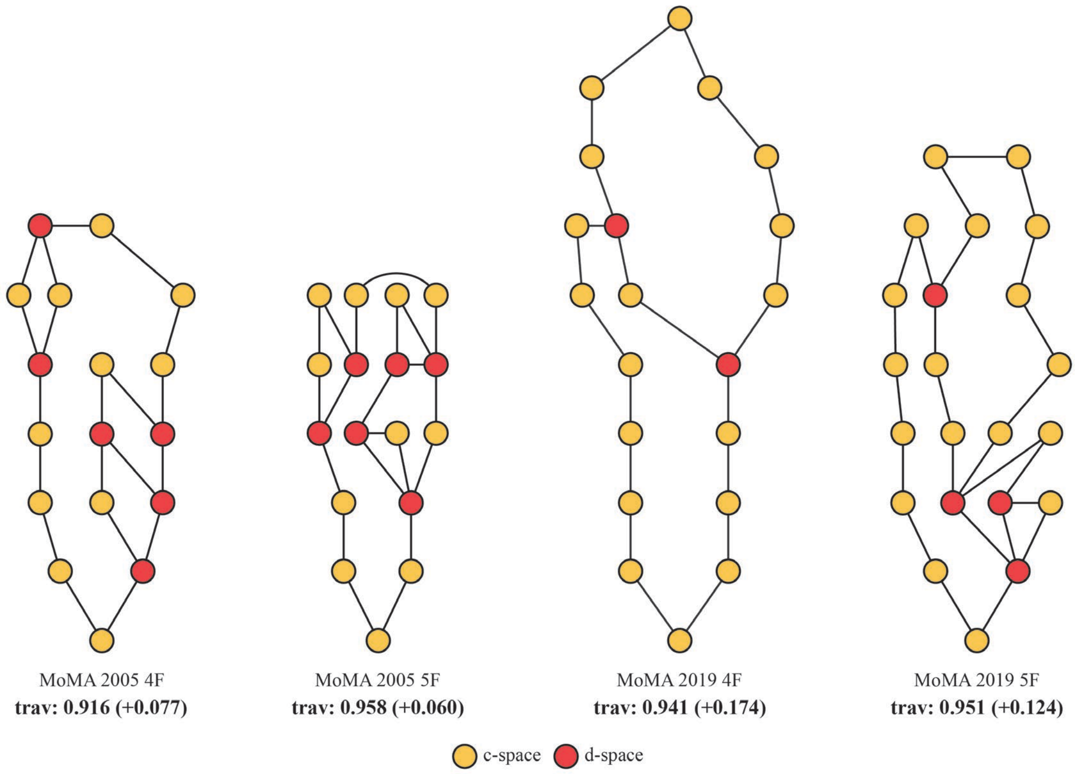

Figure 9b: Justified graphs and traversability values of the MoMA excluding a- and b-spaces.

5 CONCLUSIONS AND LIMITATIONS

This paper aimed to examine Hillier's concept of traversability, adding to the existing quantitative methods of spatial description in space syntax studies. It first calculated traversability values for theoretical a-, b-, c-, and d-structures. The study confirmed that the traversability value for the b-structure asymptotically approaches 0.8, indicative of a low traversability. The a-structure tends towards a value of 0.5 as the number of nodes increases, representing the theoretical minimum for traversability. Crucially, traversability serves as a critical indicator distinguishing between potential long- and short-model activities facilitated by spatial configurations. Building on this view, assessing the traversability value provides a mathematical means to quantify the potential for short-model activities within a given layout.

The examination of barring experiments highlighted the profound influence of a-spaces on traversability, supporting Hillier's proposal that a-structures are instrumental in generating long-model activities (Hillier 2019). It also revealed that combinations of c- and d-structures, despite being individually fully traversable, can result in relatively lower traversability values. This insight is particularly valuable for practical spatial design, suggesting that layouts without dead-ends do not automatically guarantee high global traversal efficiency. Traversability analysis in building layouts serves not only as an indicator for short-model activities but also has implications in other contexts, such as enhancing circulation efficiency in museums to prevent congestion (Thanou et al. 2019; 2020) or optimizing customer traversal in shopping malls (Li et al. 2022).

The exploration of traversability in spatial configurations is still in its early stages. This study has introduced initial theoretical concepts and practical approaches, laying the groundwork for further research. Future studies should include detailed investigations into the effects of c- and d-space arrangements on traversability. Empirical research on real-world public buildings, which rely heavily on efficient traversal system, is also essential. Moreover, future algorithmic development could incorporate heuristic methods to process configurations with numerous spatial nodes and potentially extend these studies to urban scales.

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