Giant Enhancement of SHG via Doubly Symmetry Protected Bound States in the Continuum of PhC Slabs

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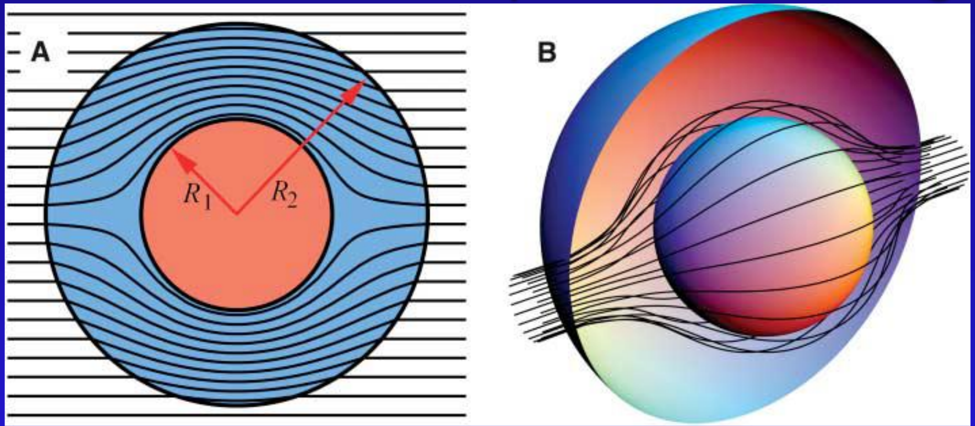
Outline

- ❑ Motivation
  - ✓ Synergy between topological photonics and nonlinear optics can lead to new physics
  - ✓ Impact at fundamental level and device applications
- ❑ Bound-states in the continuum (BICs) – a brief review
  - ✓ BIC properties
  - ✓ Relation to system topology and symmetry properties
- ❑ Optical structures with doubly-resonant BICs
- ❑ SHG in optical systems with doubly-resonant BICs
  - ✓ System design
  - ✓ SHG enhancement
- ❑ Conclusions

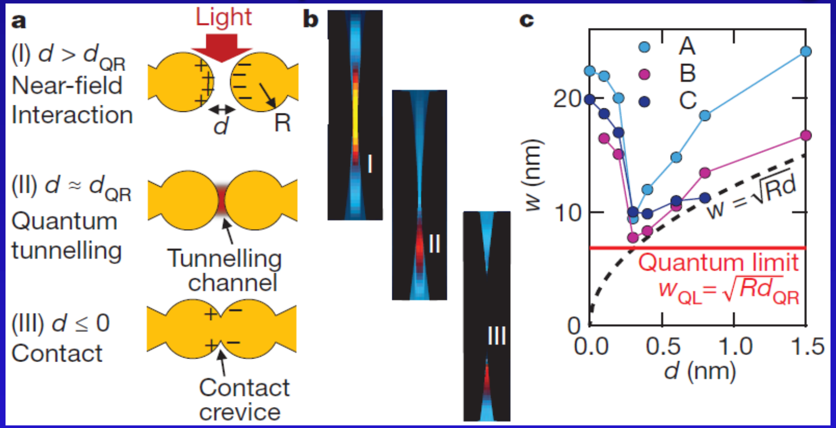
Light-matter interaction at the nanoscale

Transformation optics – Cloaking



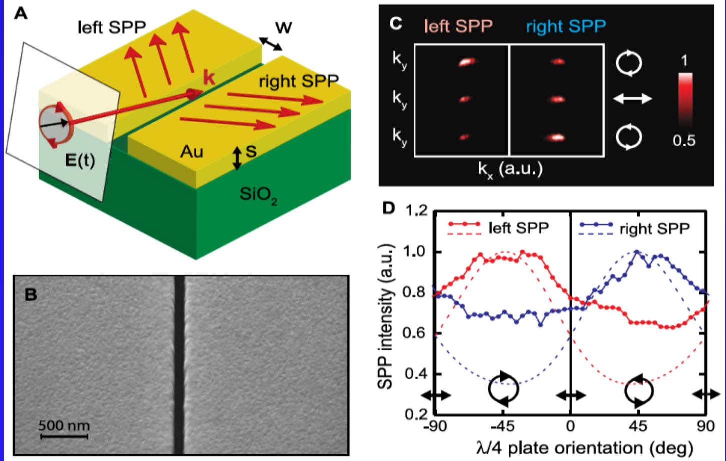
J.B. Pendy, et al., Science **312**, 1780 (2006).

Quantum plasmon tunnelling



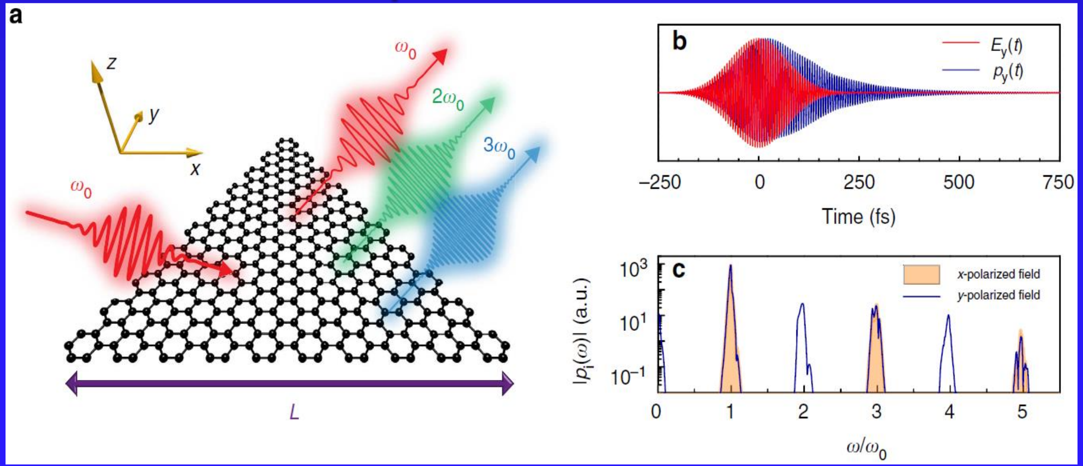
K.J. Savage et al., Nature **491**, 574 (2012).

Near-field manipulation



F. Rodriguez et al., Science **340**, 329 (2013).

Nonlinear optics in nanostructures



J. D. Cox & F. J. G. de Abajo, Nature Commun. **5**, 5725 (2014).

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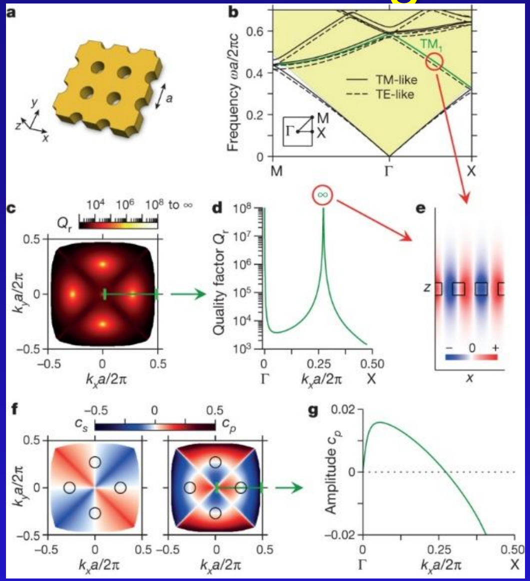
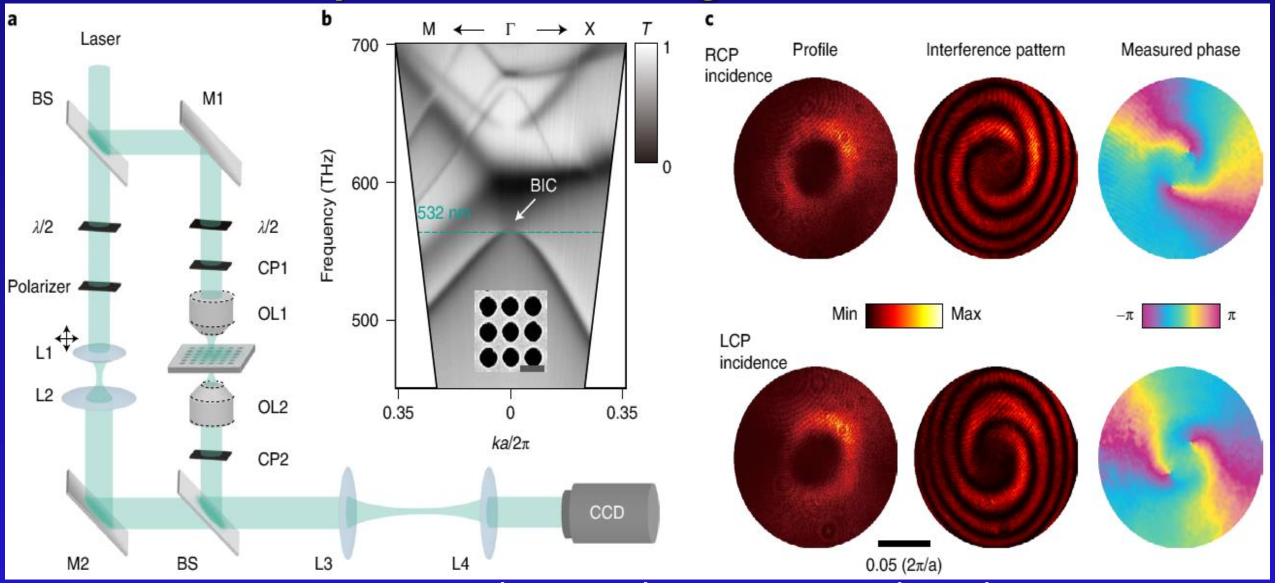
PhC slab waveguides



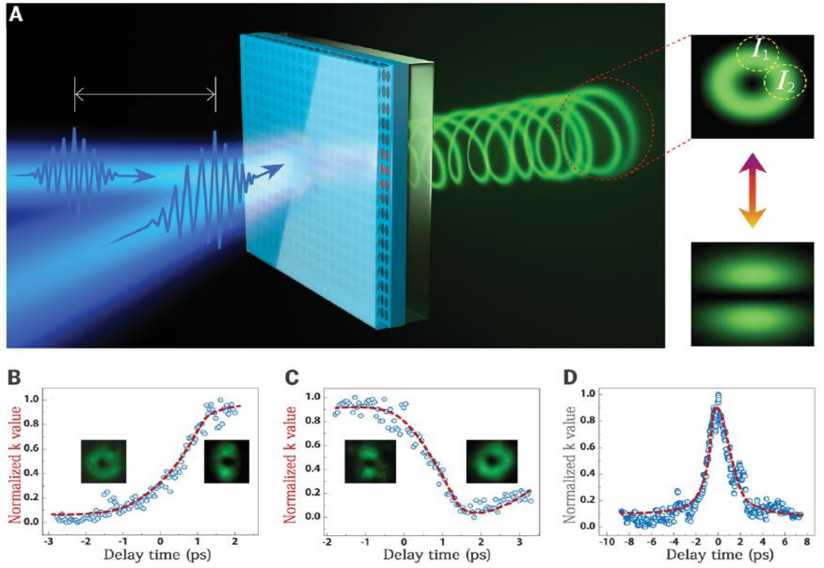
C.W. Hsu et al., Nature **499**, 188 (2013).

Optical vortex generation



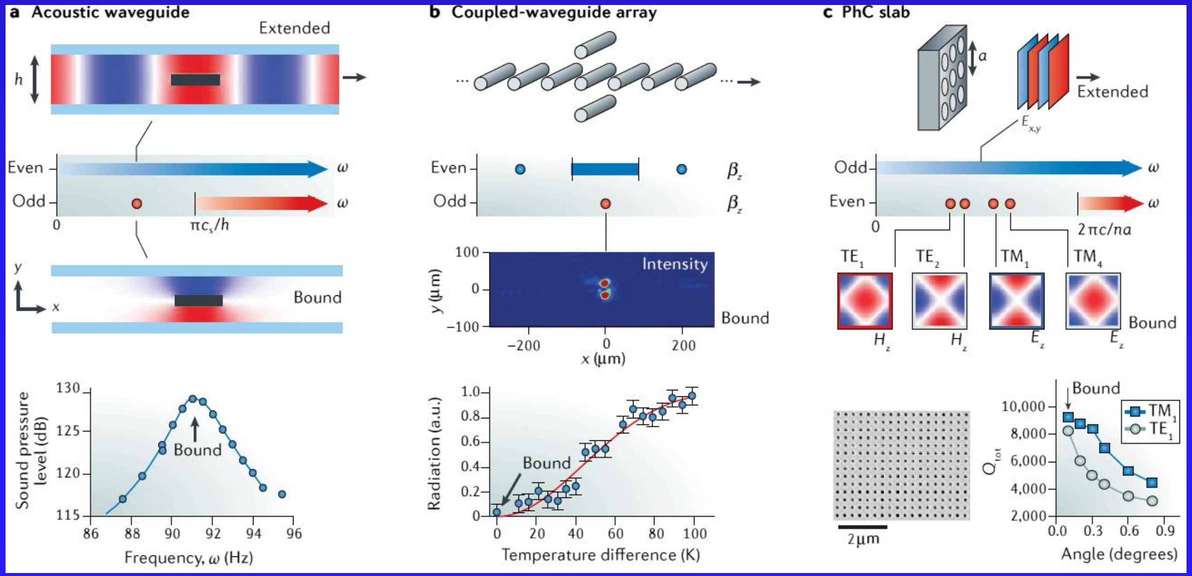
B. Wang, et al., Nat. Photon. **14**, 623 (2020).

Control of vortex microlasers



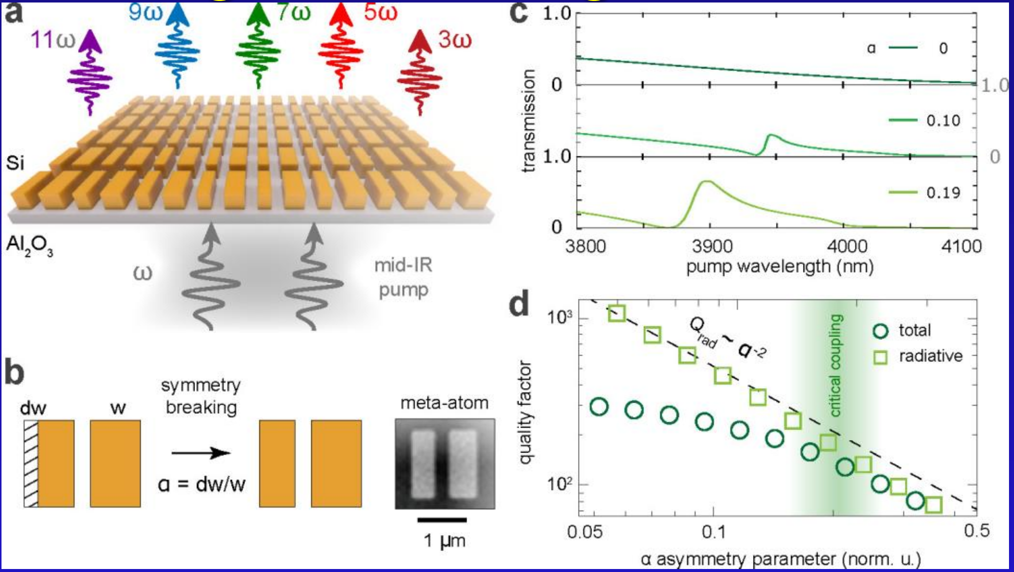
C. Huang, et al., Science **367**, 1018 (2020).

BICs – review



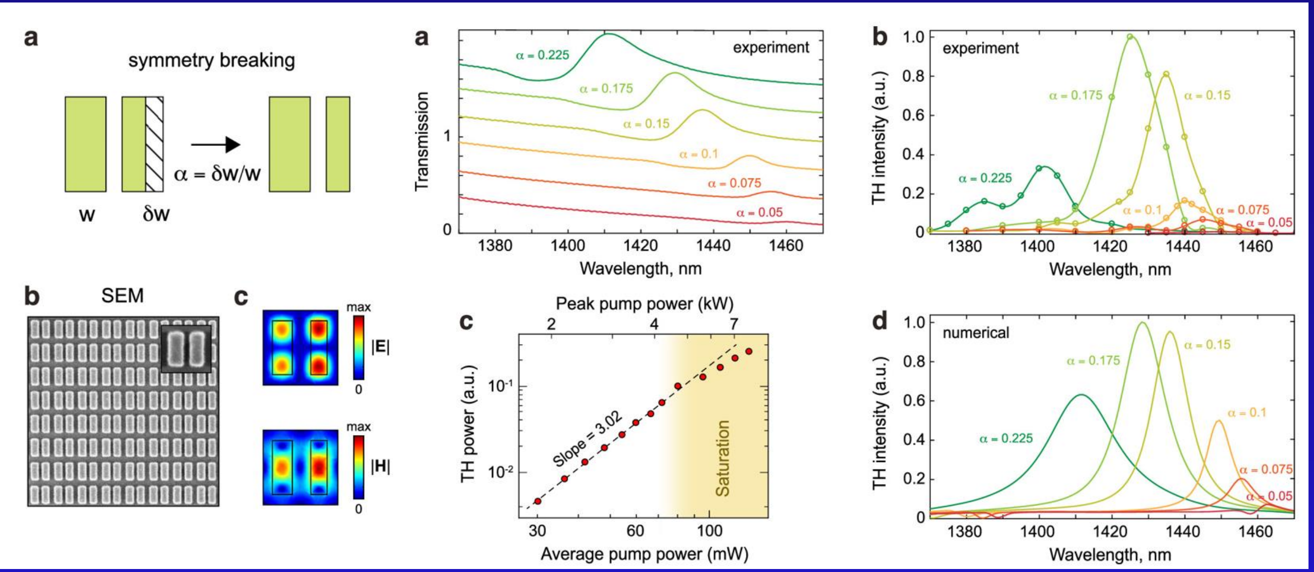
C. W. Hsu, et al., Nat. Rev. Mater. **1**, 16048 (2016).

High-harmonic generation



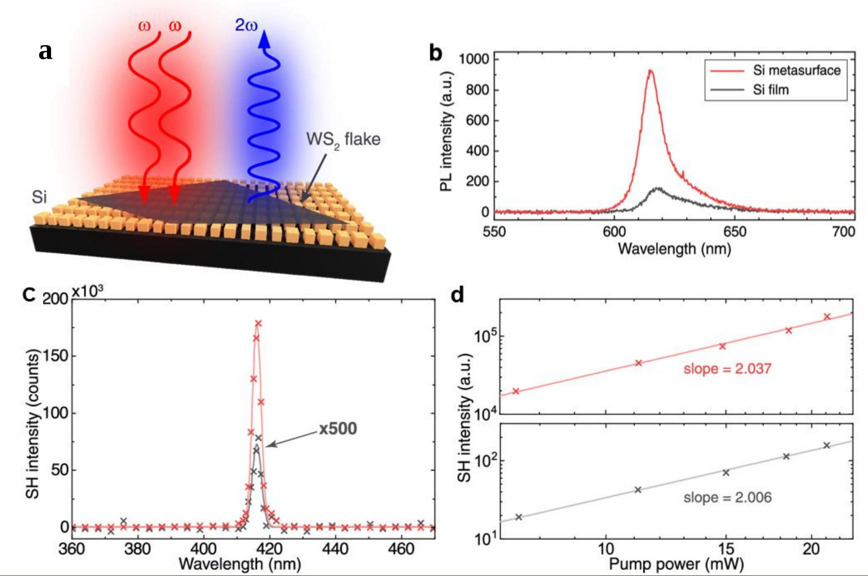
G. Zograf Hsu et al., ACS Photonics **9**, 567 (2022).

All-dielectric nonlinear metasurface



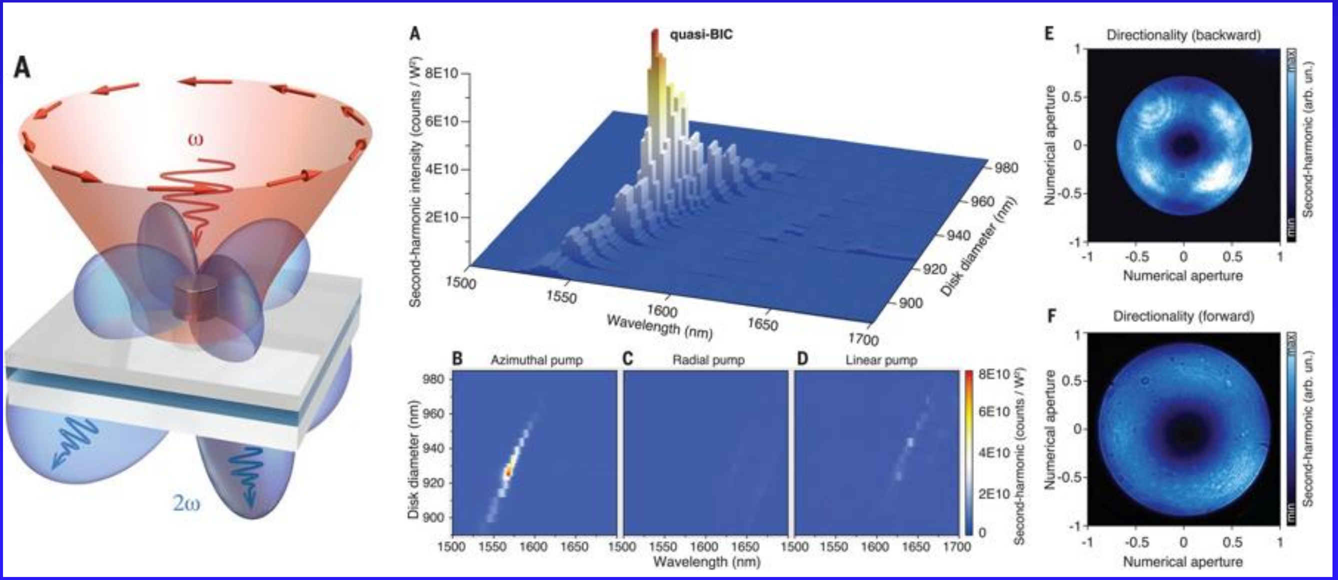
K. Koshelev, et al., ACS Photonics **6**, 1639 (2019).

Hybrid nonlinear photonic system



N. Bernhardt, et al., Nano Lett. **20**, 5309 (2020).

Nonlinear isolated dielectric resonators



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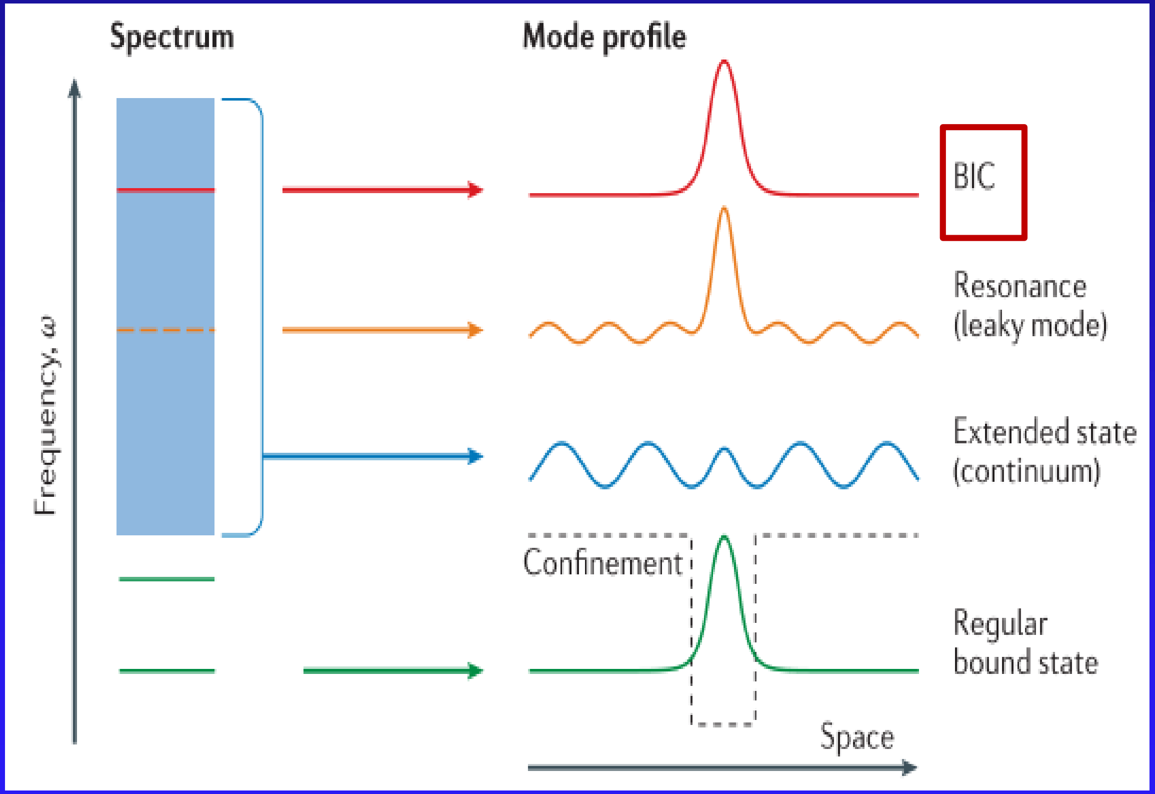
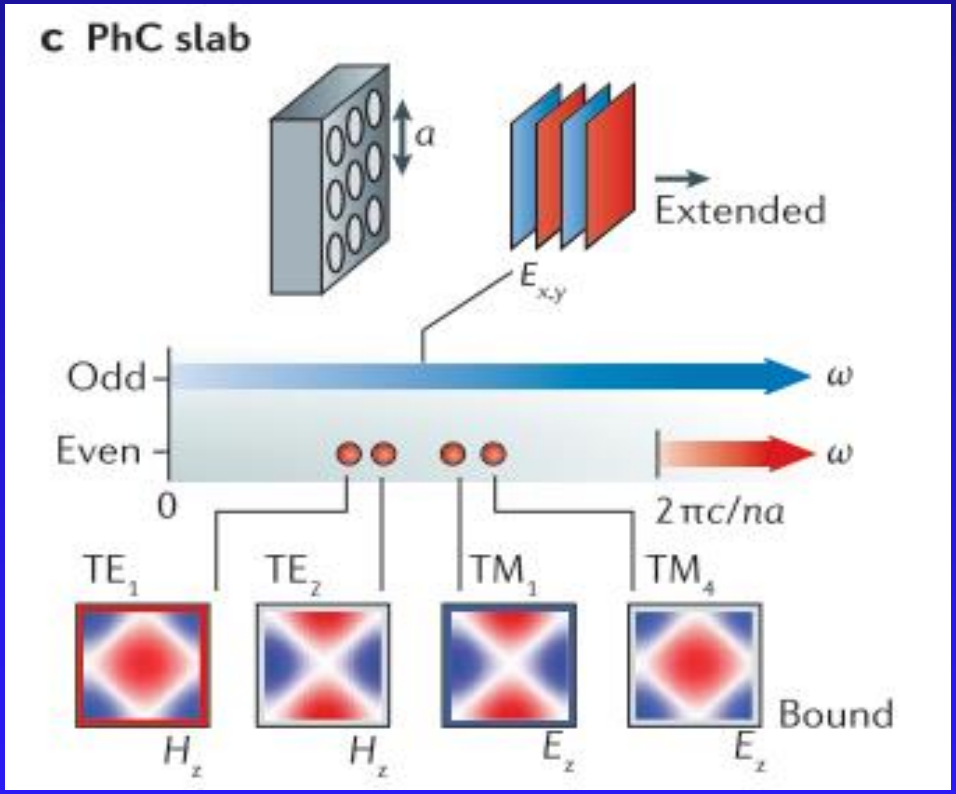
Background

Illustration of a BIC



Symmetry-protected BICs



C.W. Hsu, et al., *Nature Rev. Mater.* **1**, 16048 (2016).

- ✓ Symmetry-protected BICs
- ✓ Separable BICs
- ✓ Fabry-Pérot BICs

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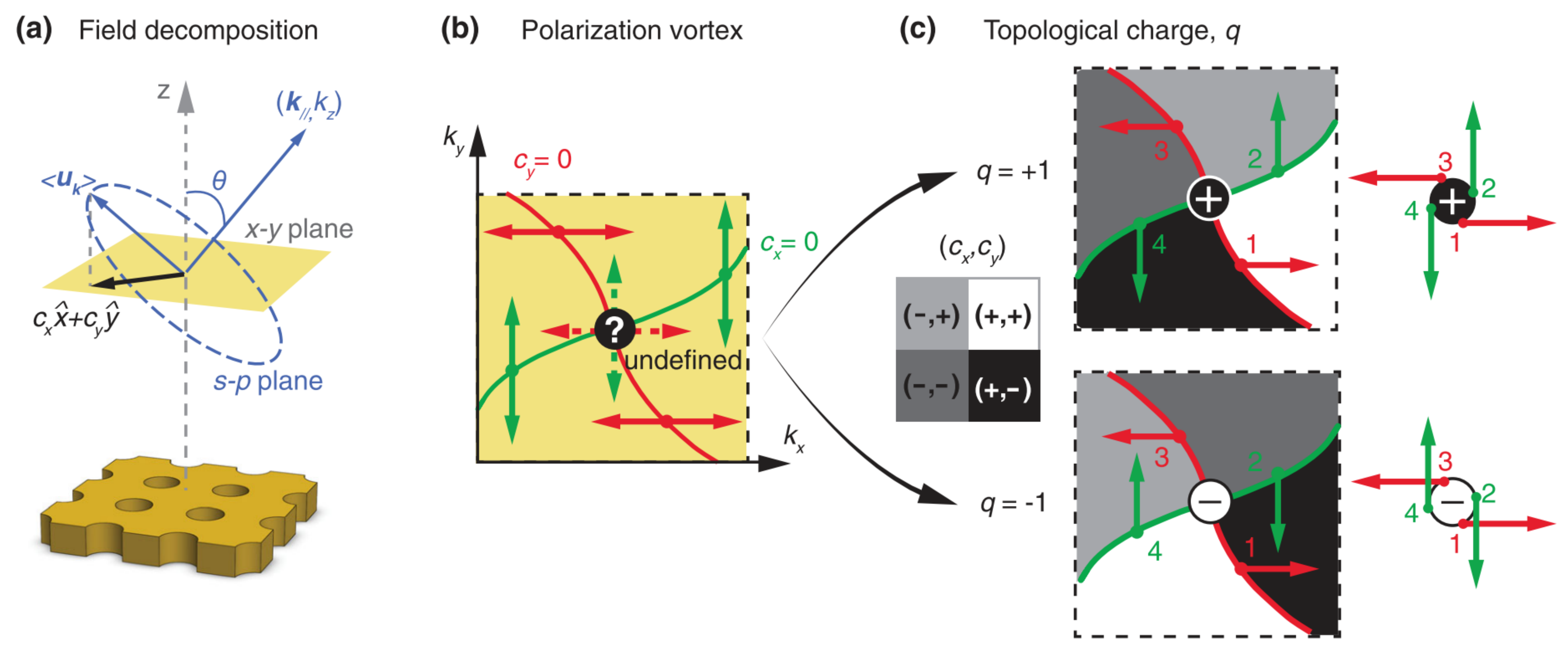
Topological nature of BICs



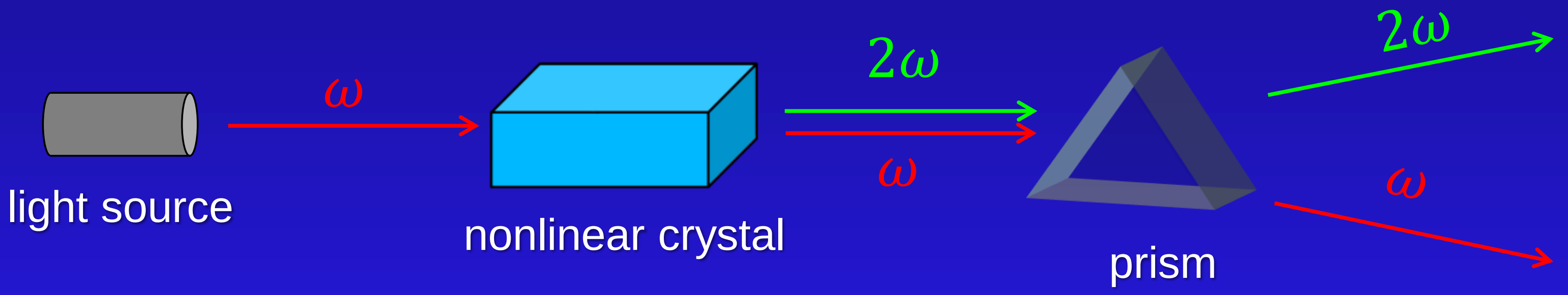
B. Zhen, et al., Phys. Rev. Lett. 113, 257401 (2014).

$$q = \frac{1}{2\pi}\oint d\mathbf{k}\cdot\nabla_{\mathbf{k}}\phi(\mathbf{k})$$

topological charge

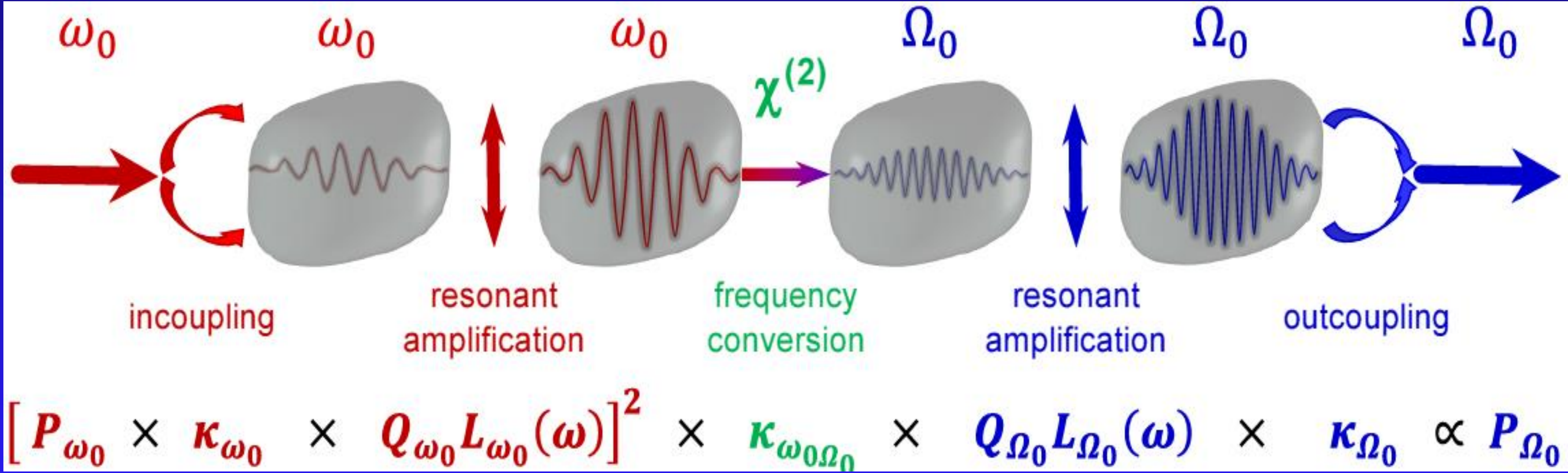
Nonlinear Optical Processes

- Nonlinear optics key for many applications: wavelength conversion, signal processing, optical microscopy



- Challenges:
  - ✓ Practical: nonlinear optical response is generally weak
    - ✓ need for local field enhancement
    - ✓ periodic structures are ideal for engineering optical near and far-fields
  - ✓ Theoretical: complex dependency between excitation and optical response
    - ✓ efficient numerical tools for nonlinear periodic structures essential

SHG enhancement via double-resonant states

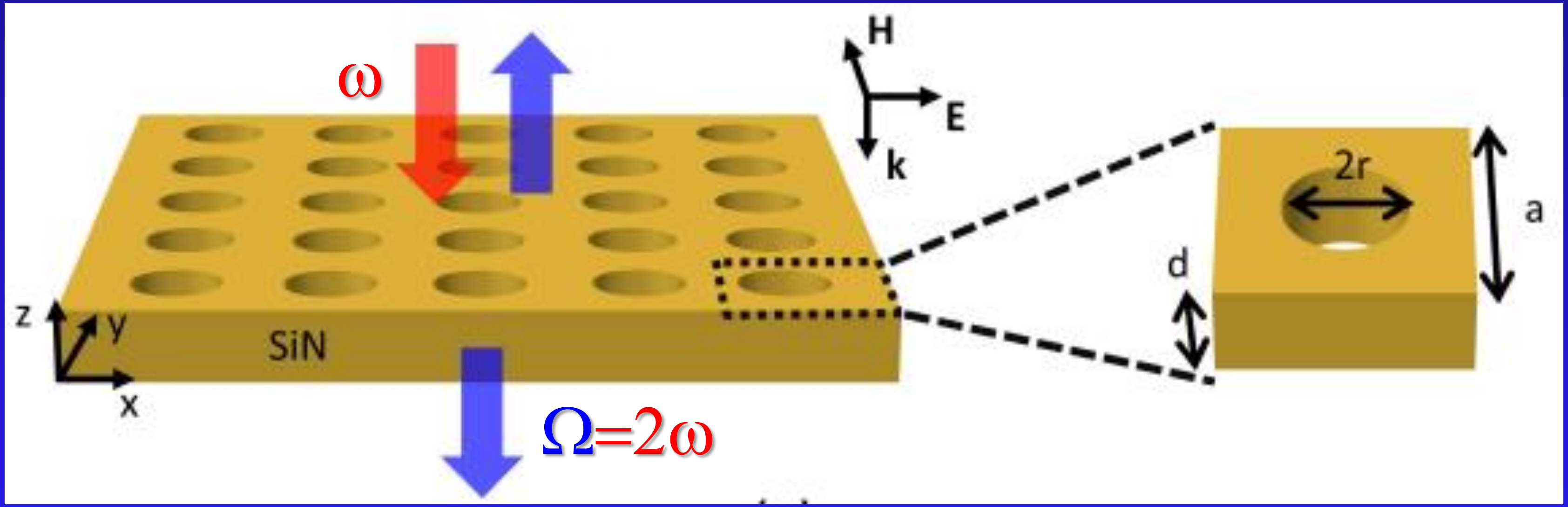


K. Koshelev et al., Science ***367****, 288 (2020).*

- ❑ Needed ingredients
  - ✓ Resonance at ω_0 and a resonance at $\Omega_0 = 2\omega_0$
  - ✓ Efficient coupling between modes at ω_0 and Ω_0

Optical System

PhC slab made of SiN



- ✓ PhC slab with square lattice of holes
- ✓ Period *a*, hole radius *r*, and thickness *t*
- ✓ Susceptibility: $\chi^{(2)}_{xxz} = \chi^{(2)}_{xzx} = \chi^{(2)}_{yyz} = \chi^{(2)}_{yzy} = 0.4$ pV/m

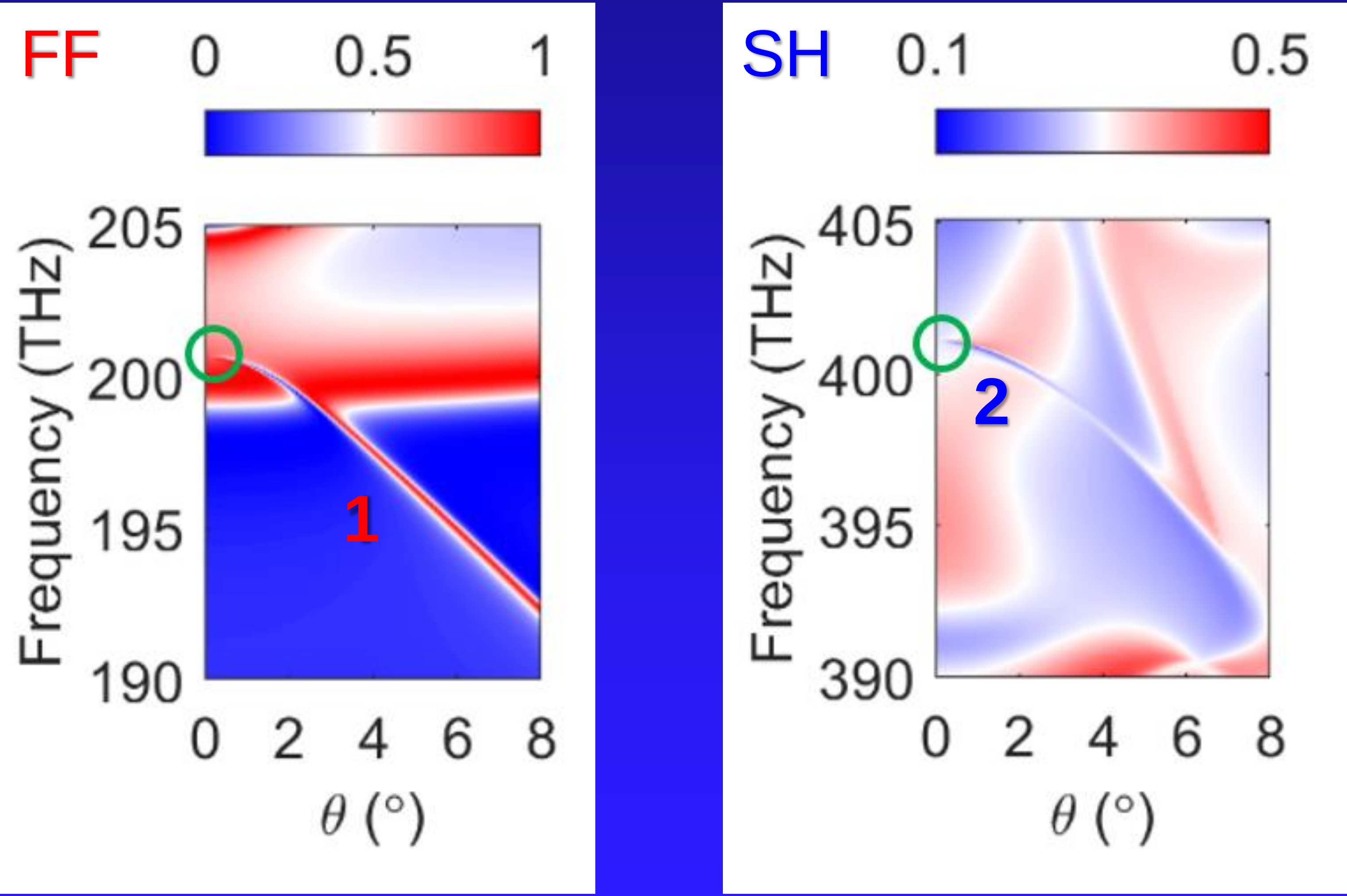
$$\chi^{(2)}_{zxx} = \chi^{(2)}_{xxz} = 0.34 \text{ pV/m}$$

$$\chi^{(2)}_{zzz} = 1.1 \text{ pV/m}$$

At–Γ BICs

Linear optical response (FF)

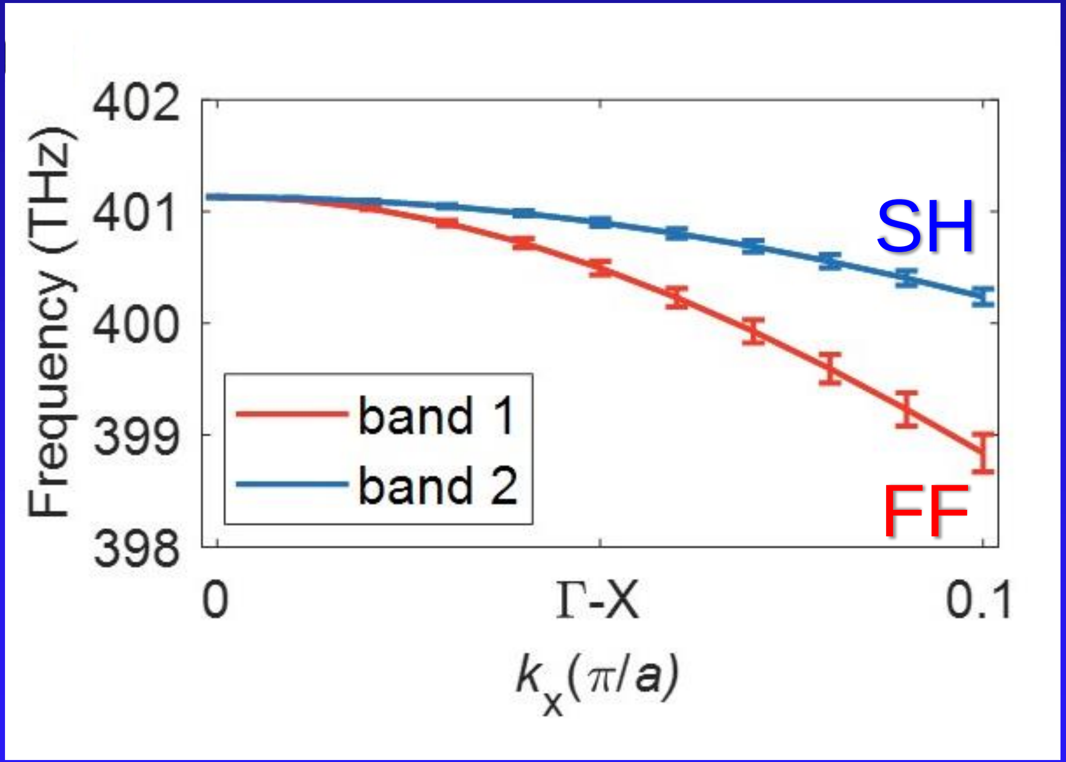
Reflectivity maps



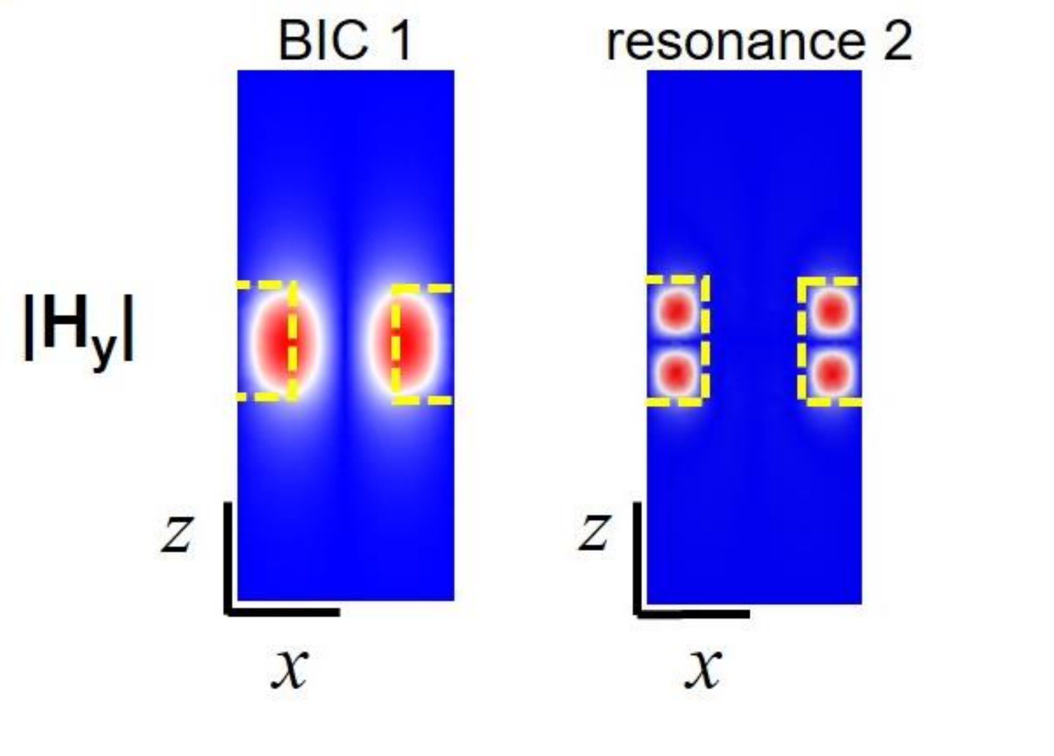
- ✓ BIC exists at FF
- ✓ BIC-like mode exists at SH
- ✓ At–Γ: double-resonance effect

Band engineering

Band diagram



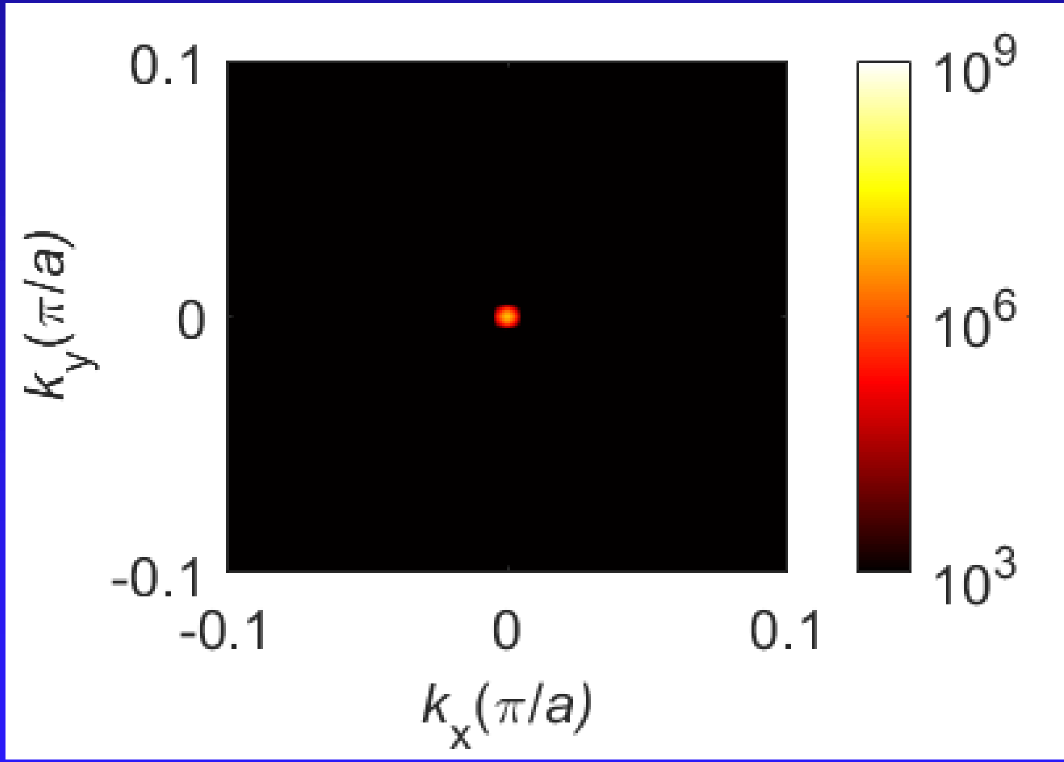
Field profiles



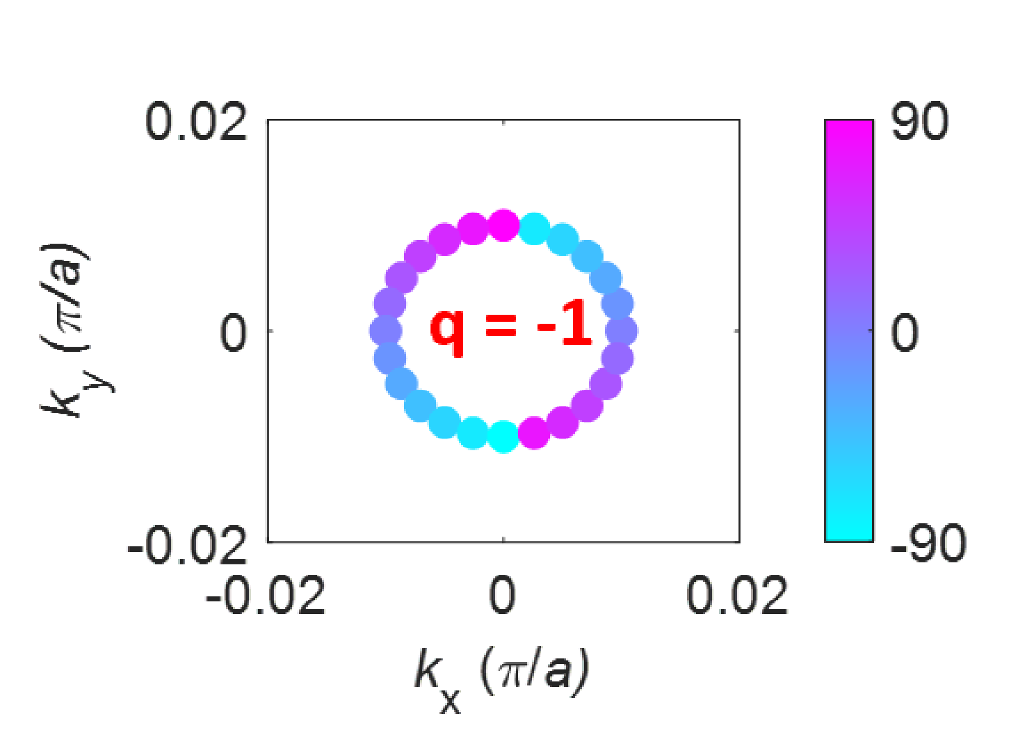
- ✓ Away from the Γ–point *Q*-factor decreases
- ✓ Effective overlap between the optical modes at the FF and SH

Topological properties of BIC 1 (FF)

Q map



Topological charge



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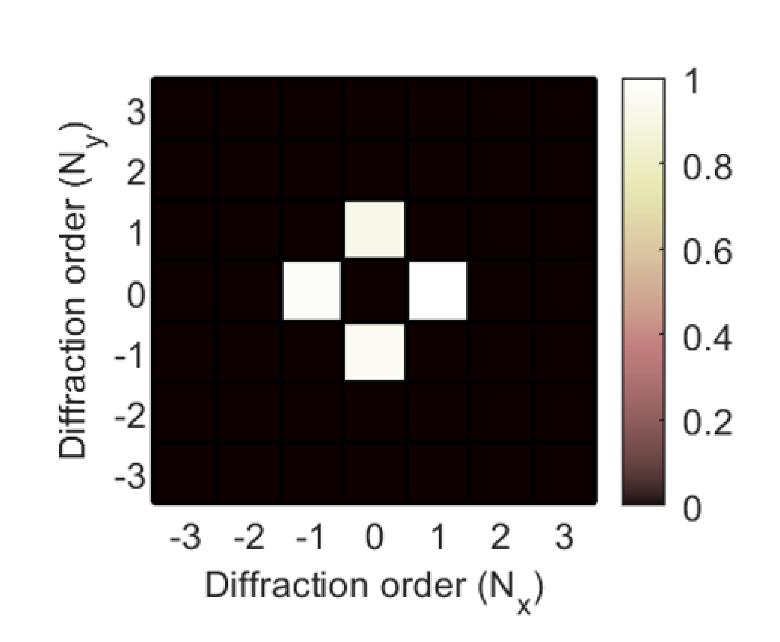
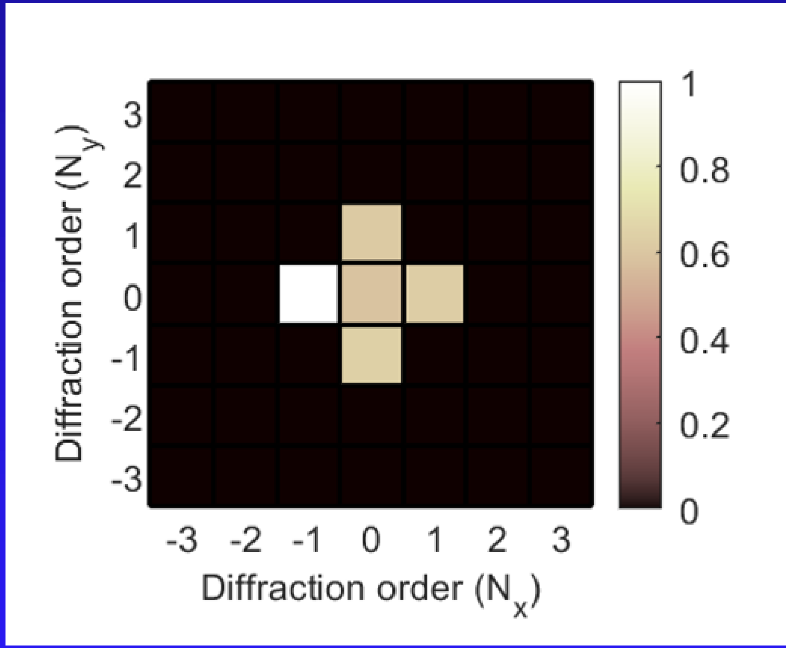
Fourier analysis of Resonance 2 (SH)

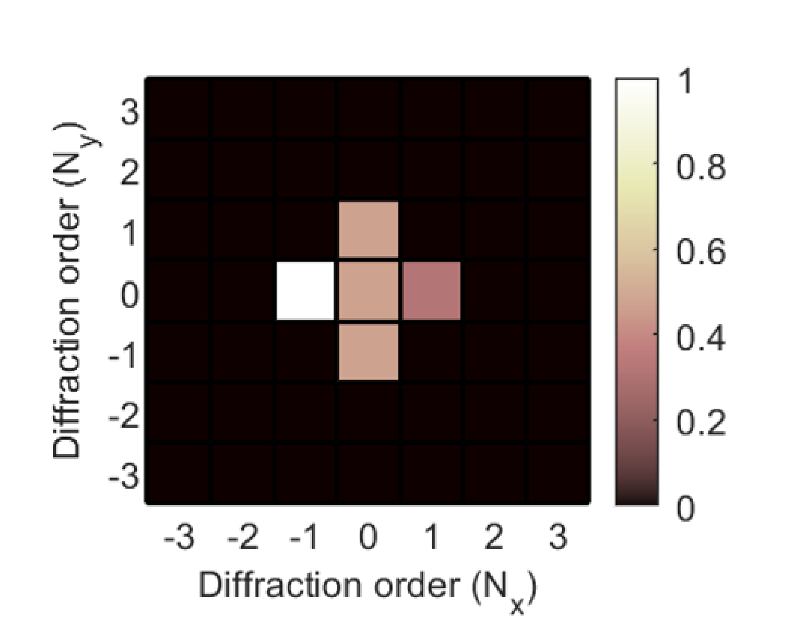
k_x=0, k_y=0



k_x=0.005 π/a, k_y=0



k_x=0.05 π/a, k_y=0



- ✓ Suppressed emission from (0,0) diffraction order (Γ–point)
- ✓ Four first-order diffraction channels exist above diffraction limit

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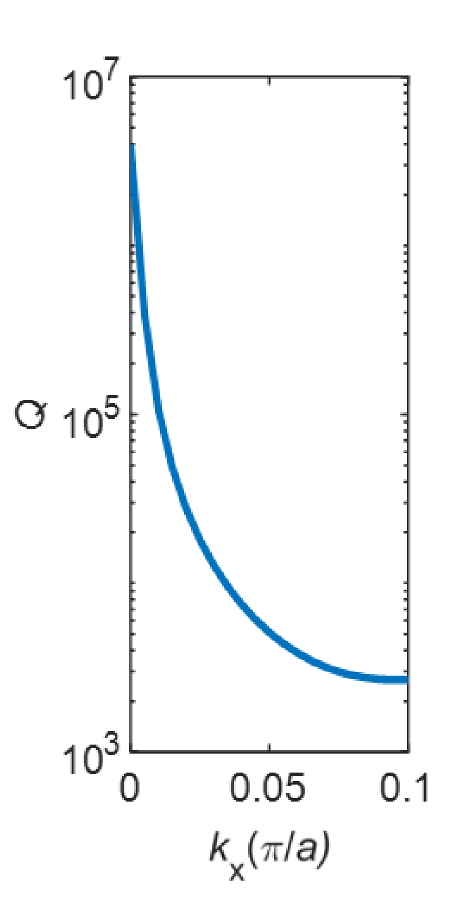
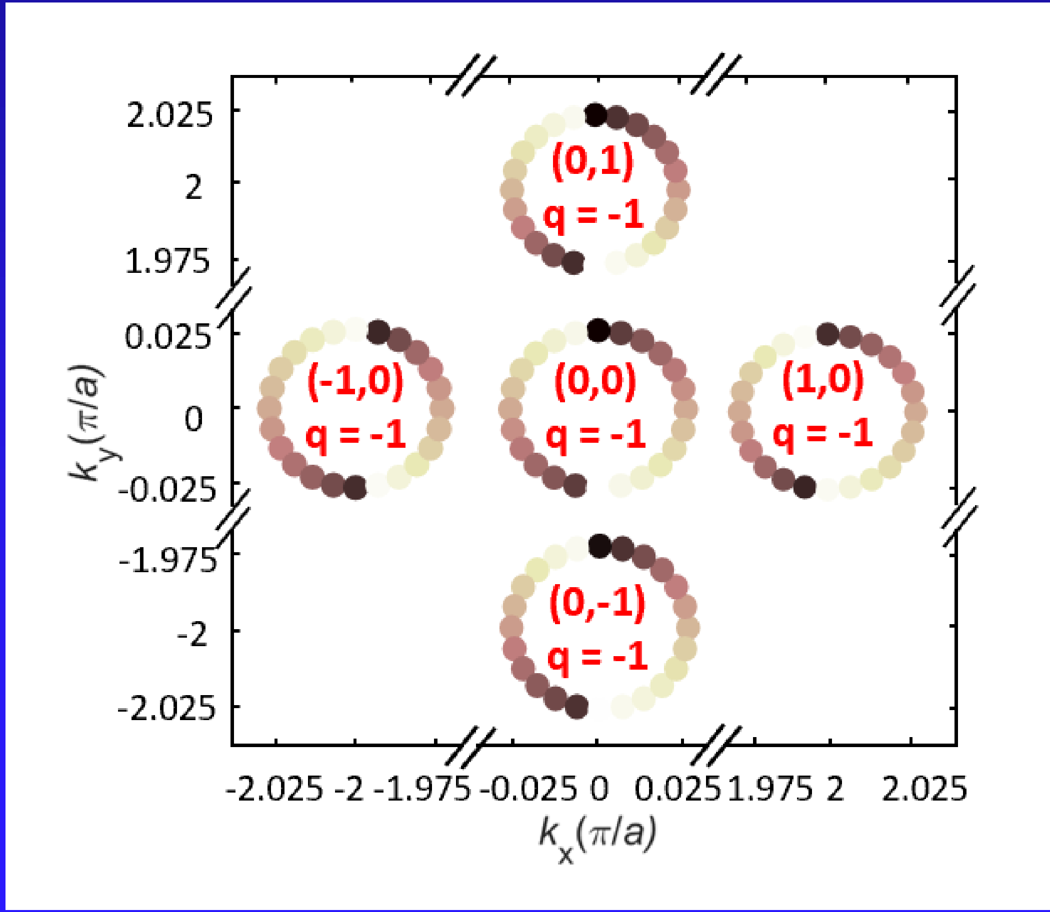
Topological properties of Resonance 2 (SH)

Q-factor



Topological charge



SHG: symmetry considerations

- Nonlinear polarization

$$\boldsymbol{P}^{(2)}(\Omega) = \epsilon_0 \chi^{(2)}(\boldsymbol{r}) \colon \boldsymbol{E}(\omega)\boldsymbol{E}(\omega)$$

$$\boldsymbol{E}(\omega) = \boldsymbol{E}_{in}(\omega) + \boldsymbol{E}_{BIC}(\omega) + \boldsymbol{E}_{bg}(\omega)$$

$$\boldsymbol{E}(\Omega) = \boldsymbol{E}_{BIC}(\Omega) + \boldsymbol{E}_{bg}(\Omega)$$

- Nonlinear mode coupling

$$\kappa = \int_V \boldsymbol{E}(\boldsymbol{r};\Omega) \cdot \boldsymbol{P}^{(2)}(\boldsymbol{r};\Omega) d\boldsymbol{r}$$

$$\epsilon_0 \int_V \boldsymbol{E}_{BIC}(\boldsymbol{r};\Omega) \cdot \chi^{(2)}(\boldsymbol{r}) \colon \boldsymbol{E}_{BIC}(\boldsymbol{r};\omega)\boldsymbol{E}_{BIC}(\boldsymbol{r};\omega) d\boldsymbol{r}$$

SHG: symmetry considerations (ctd)

Mode symmetry properties

	$\hat{\imath}$	$\hat{\sigma}_x$	$\hat{\sigma}_y$	$\hat{\sigma}_z$
BIC 1	–1	1	1	–1
BIC 3	1	–1	1	–1
Resonance 2	1	1	1	1
Resonance 4	–1	–1	–1	–1

Nonlinear polarization: symmetry properties

	$\hat{\imath}$	$\hat{\sigma}_x$	$\hat{\sigma}_y$	$\hat{\sigma}_z$
$\boldsymbol{P}^{(2)}(\Omega)$: BIC 1	–1	1	1	–1
$\boldsymbol{P}^{(2)}(\Omega)$: BIC 3	–1	1	1	–1
$\boldsymbol{P}^{(2)}(\Omega)$: BIC 3 (QPM)	1	1	–1	–1

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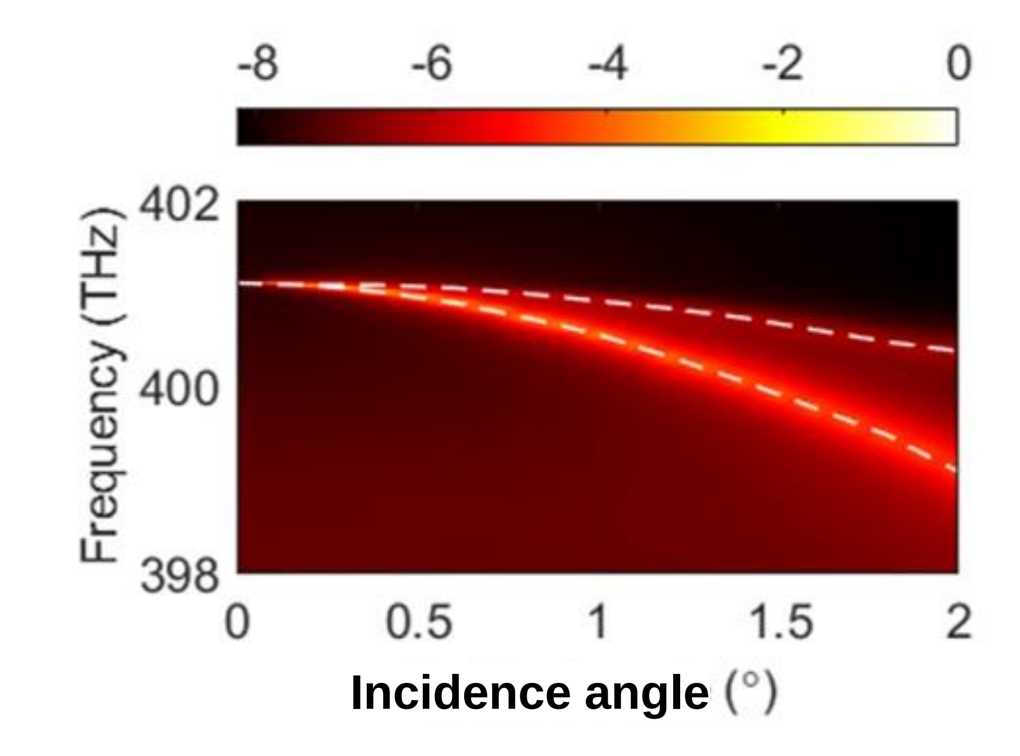
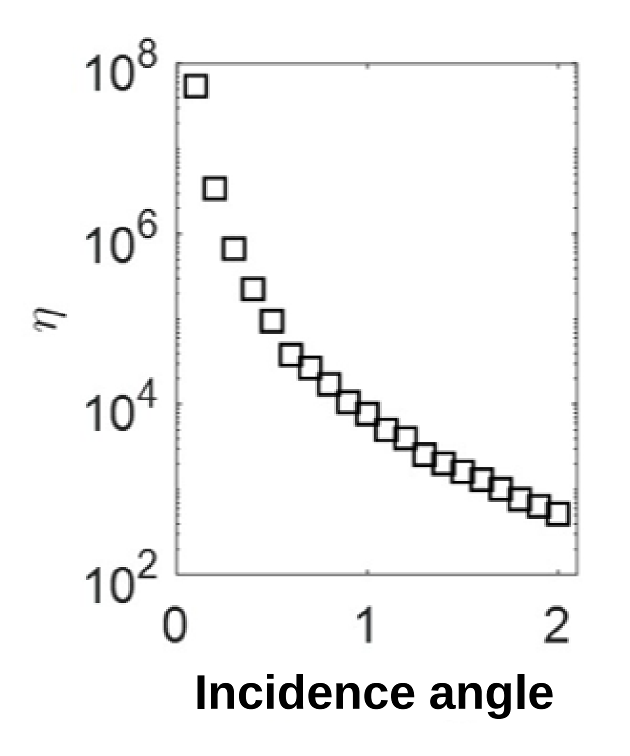
Nonlinear optical response (SH)

SHG simulation



SHG enhancement

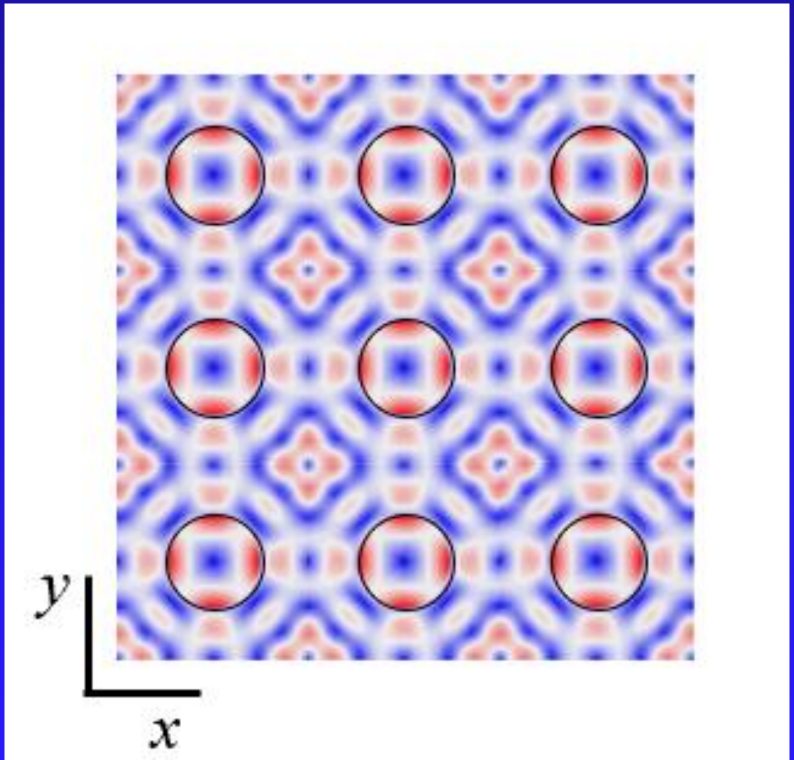


✓ Two peaks in the map of normalized SHG intensity

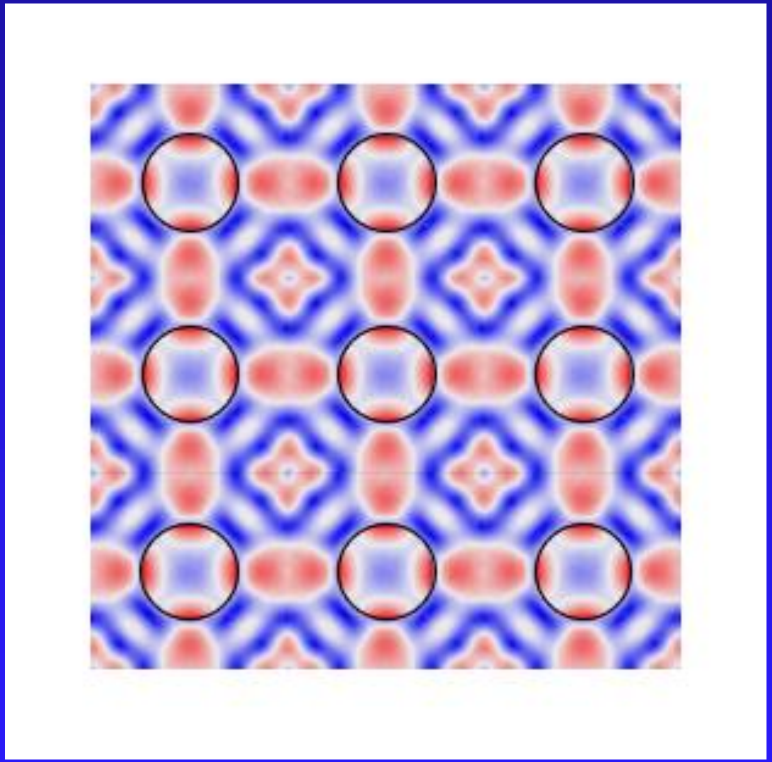
✓ Giant SHG enhancement of 10^8 due to double-resonance effect

Nonlinear optical response (SH)

|E|: Resonance 2

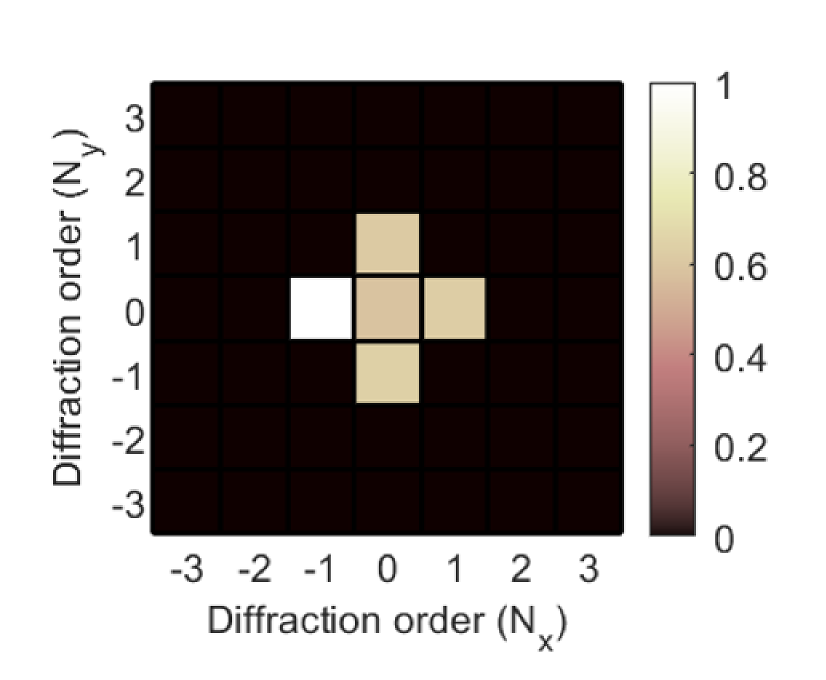


$|E_{SH}|$

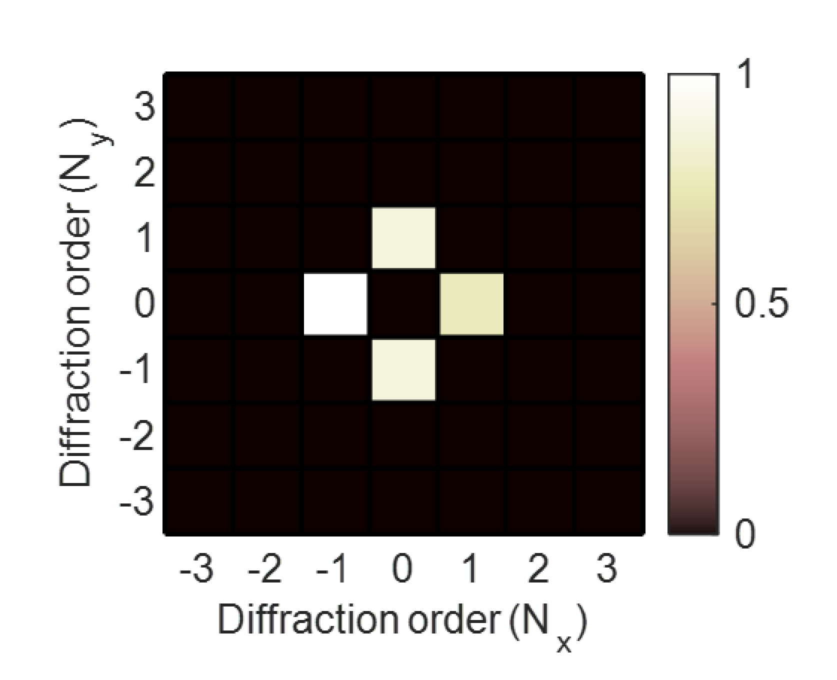


Diffraction analysis of SHG

k_x=0.005 π/a, k_y=0

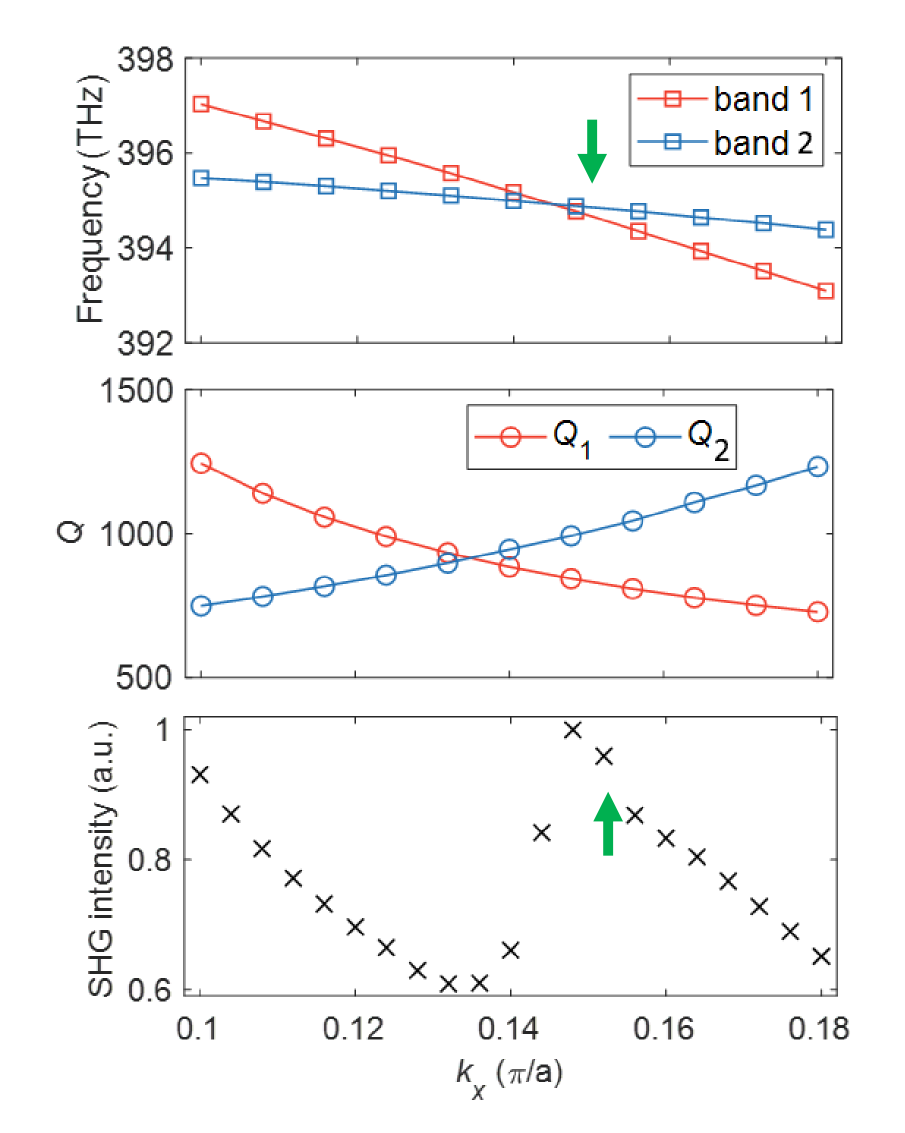


k_x=0.005 π/a, k_y=0



- ✓ Four diffraction channels exist in eigenmode analysis and nonlinear calculation
- ✓ Suppressed SH emission in (0,0) order due to structure symmetry

Coupling effect between BIC 1 and Resonance 2



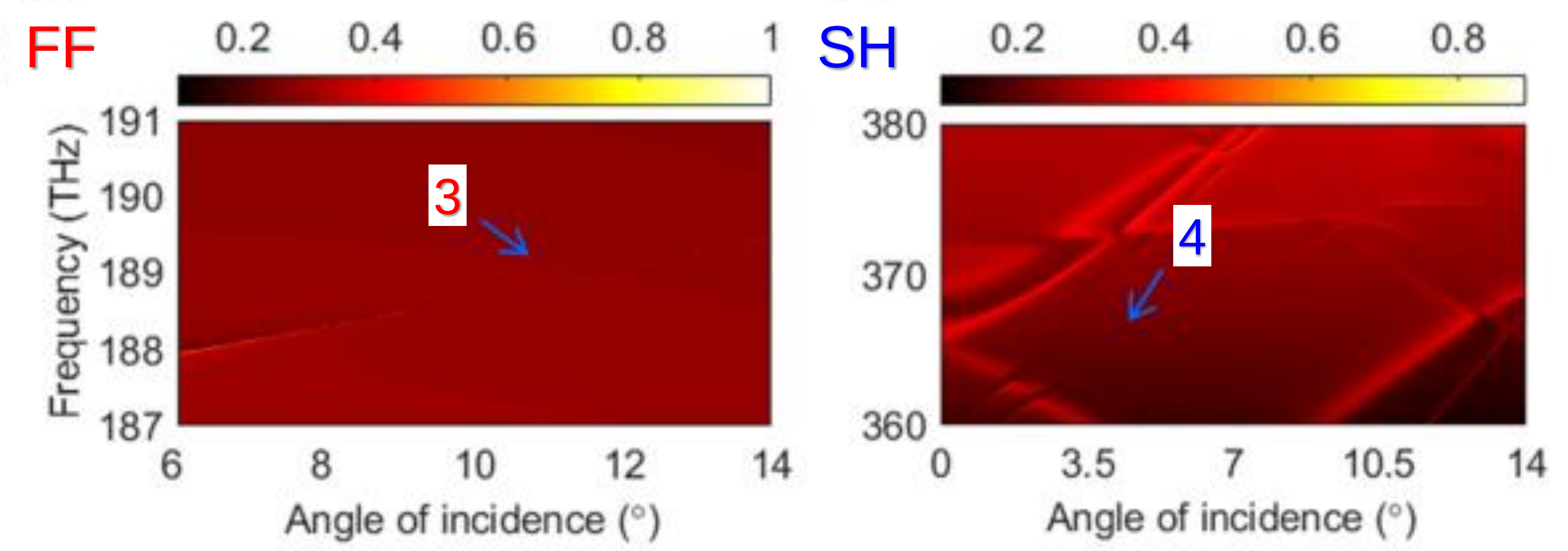
Tune radius of hole (to 220 nm)

- ✓ Double-resonance phenomenon also exists at off–Γ point
- ✓ $Q_1^2Q_2$ decreases with respect to k_x
- ✓ Local peak of SHG intensity arising from doubly resonant effect

Off–Γ BICs

Linear optical response (FF)

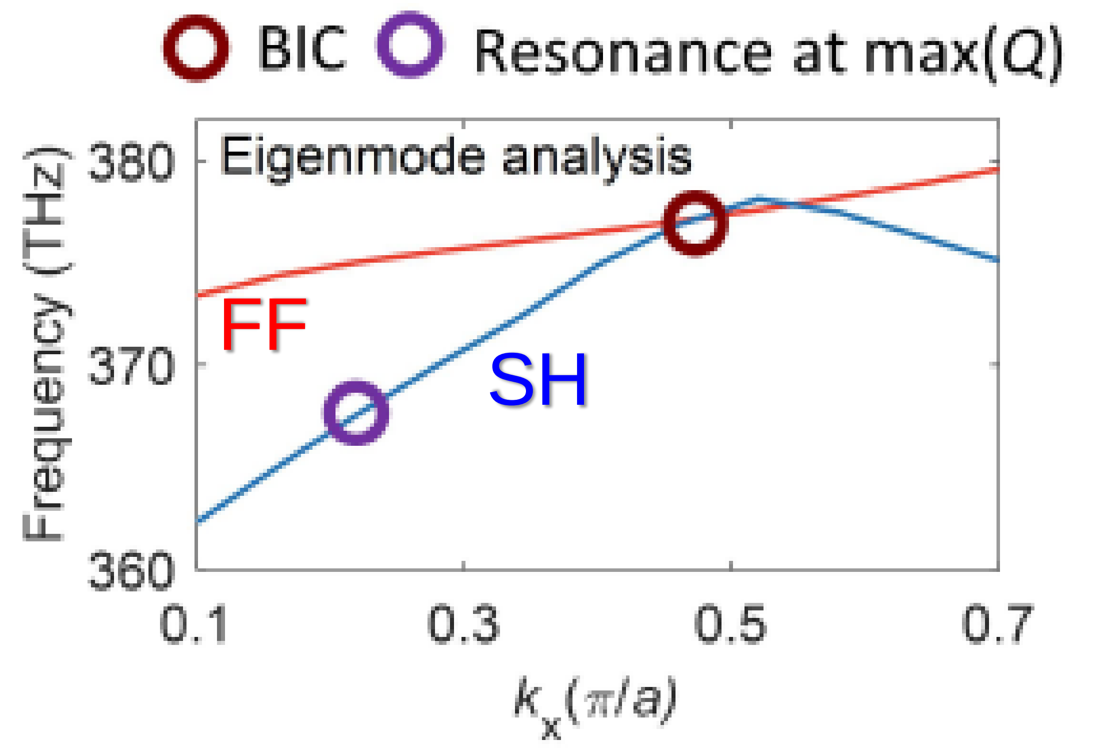
Reflectivity maps



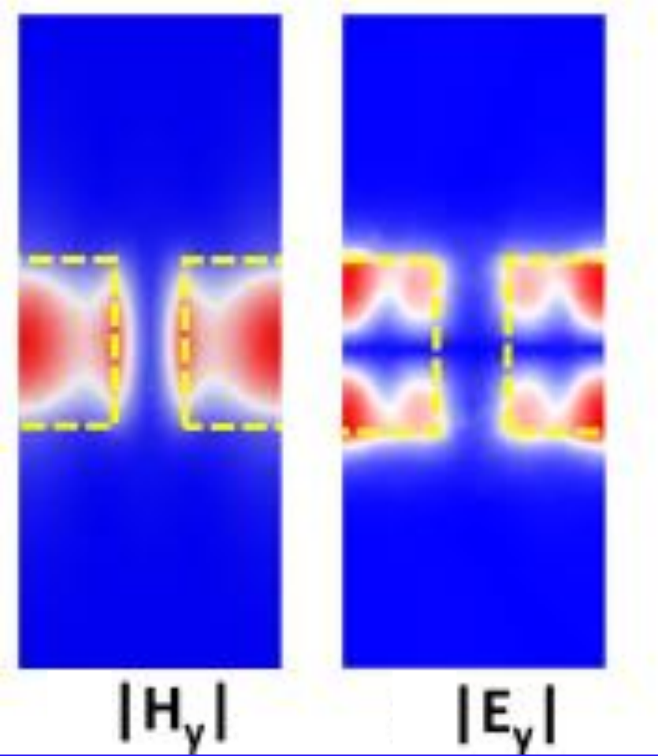
- ✓ BIC exists at FF
- ✓ BIC-like mode exists at SH
- ✓ Off–Γ: double-resonance effect

Band engineering

Band diagram



Field profiles

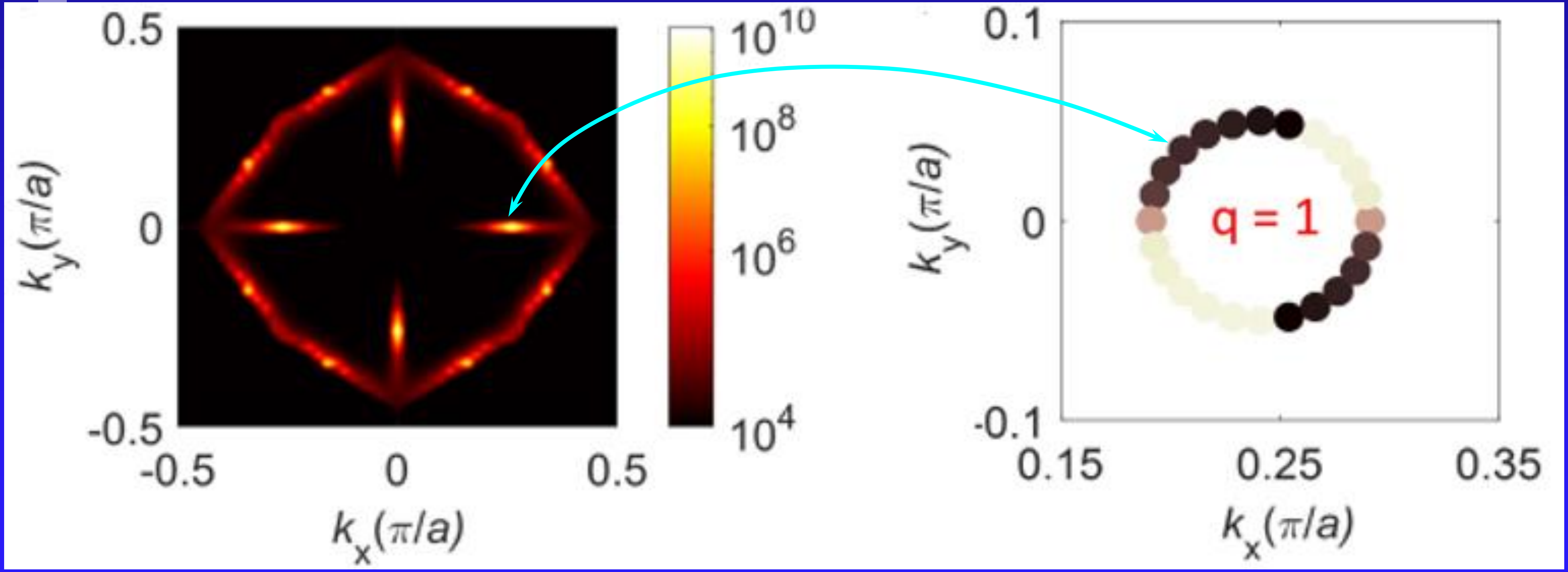


✓ FF and SH bands cross at the BICs

✓ Effective overlap between the optical modes at the FF and SH

Topological properties of BIC 3 (FF)

Q map Topological charge



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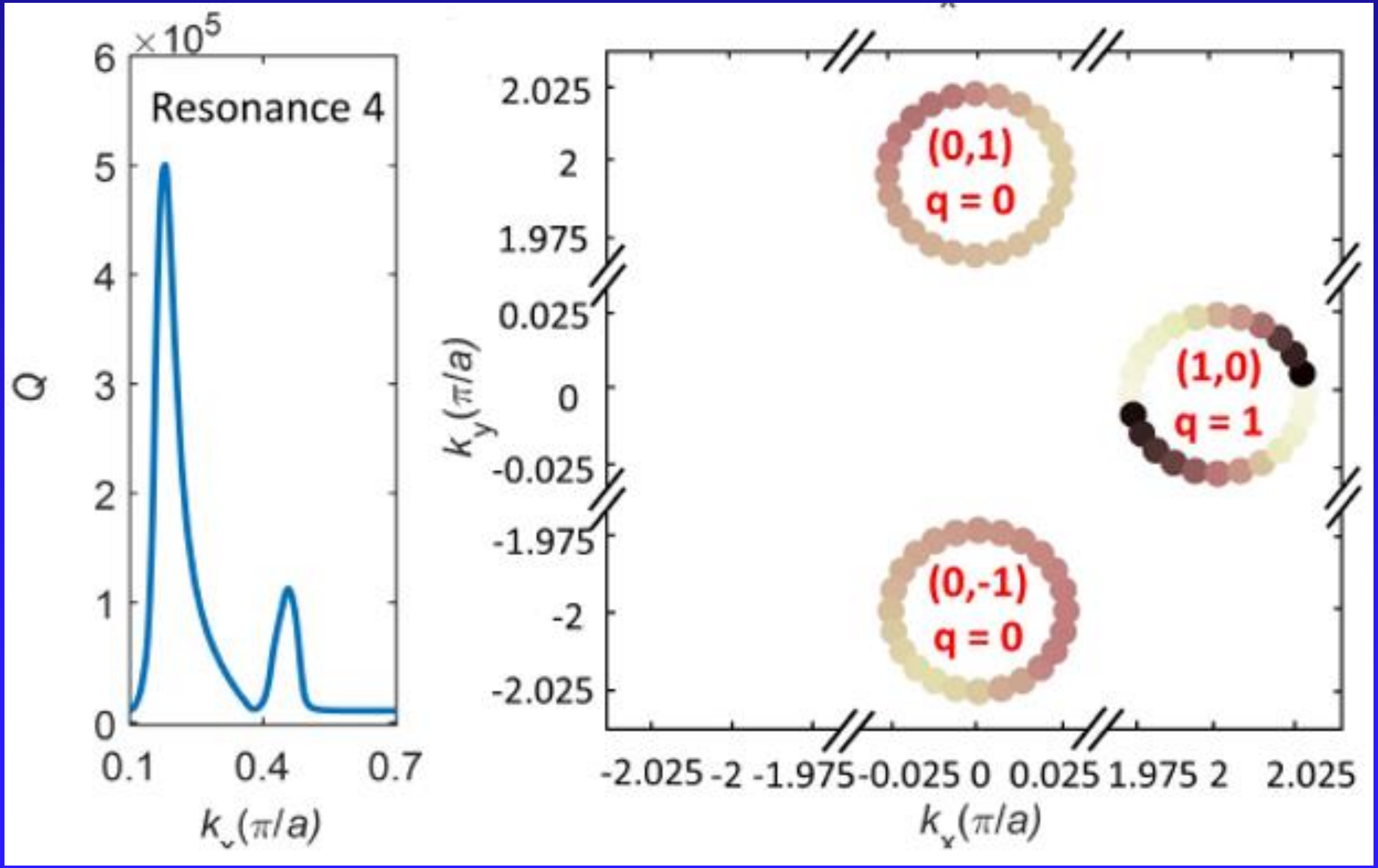
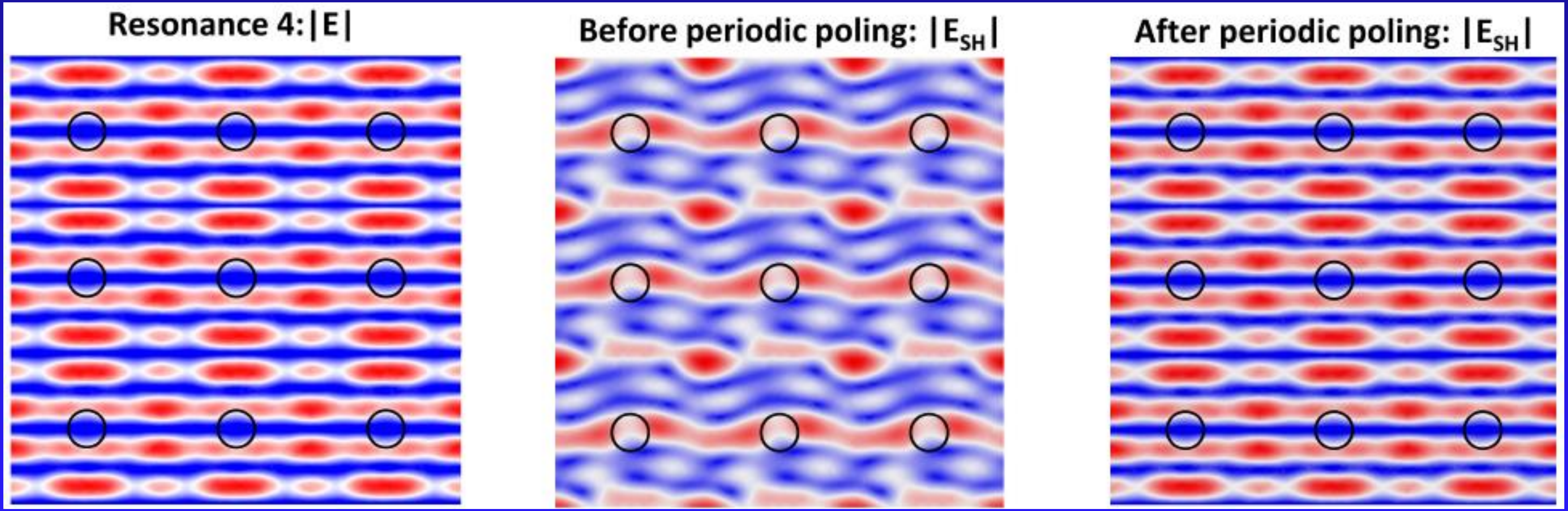
Topological properties of Resonance 4 (SH)

Q-factor

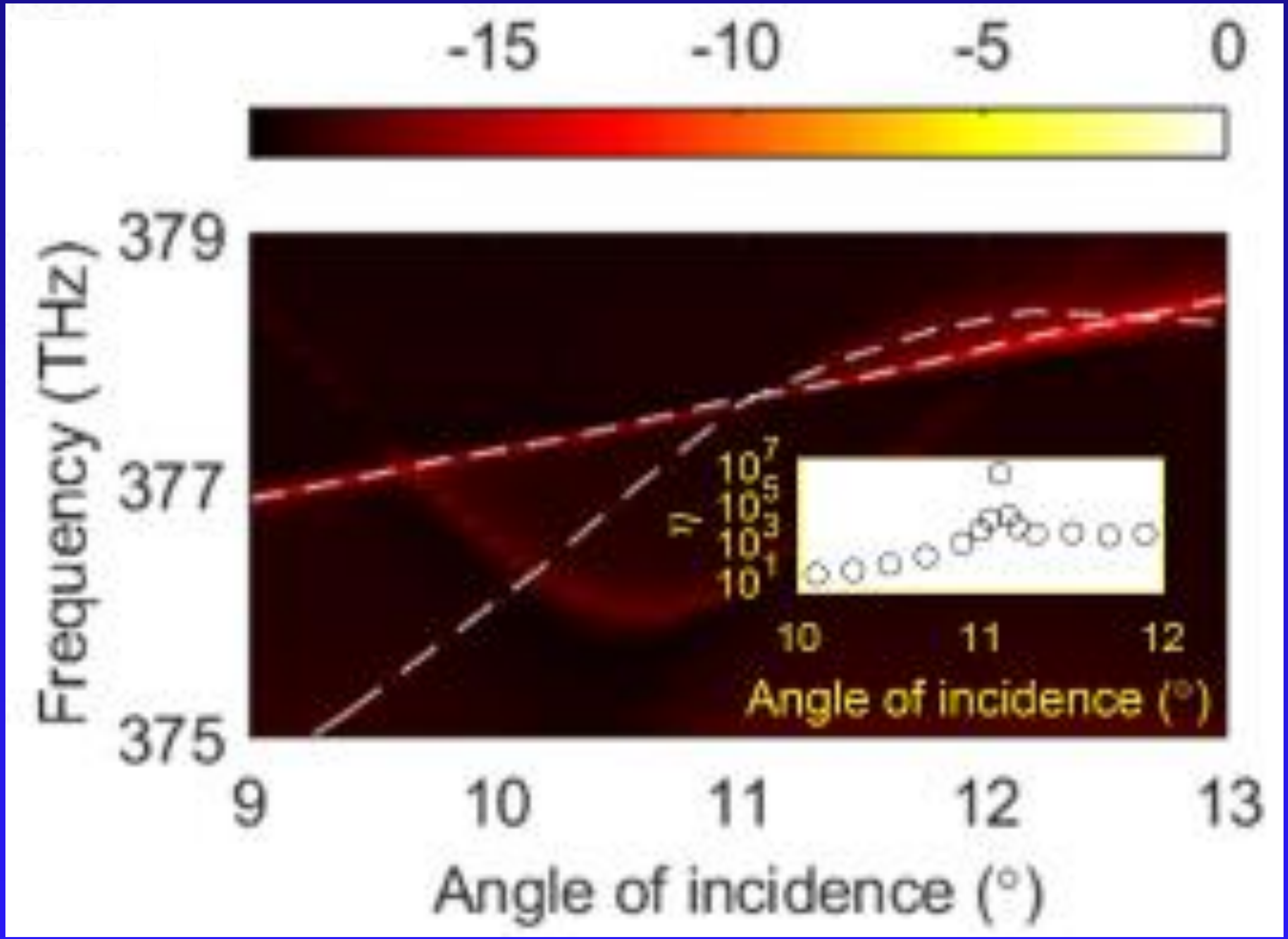
Topological charge



Nonlinear optical response (SH)



Coupling effect between BIC 3 and Resonance 4



- ✓ Double-resonance phenomenon also exists at off–Γ point
- ✓ Local peak of SHG intensity arising from maximum of $Q_1^2Q_2$

Conclusions

- We investigated SHG in systems that possess BICs at FF and SH.
- SHG enhancement depends strongly on the symmetry properties of the interacting BICs
- Strong SHG enhancement is achieved when the frequencies of the BICs are engineered to have the required ratio.
- Possible applications to new optical sources, quantum optics, and optical communications

Thank you!