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**THE FINANCIAL IMPACT OF CARBON EMISSIONS ON  
POWER UTILITIES UNDER CLIMATE SCENARIOS**

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2 *F. Krach, A. Macrina, A. Kanter, E. Hampwaye, S. Hlalukana & N. T. Rateele*

Power utilities, especially those that generate electricity by burning fossil fuels, produce significant amounts of carbon emissions. Mitigation of CO<sub>2</sub>e-emissions can be achieved by replacing power plants with renewable power installations and by adopting carbon-sequestration technologies. Physical upgrades are expensive, but carbon taxes, or the purchase of certificates and allowances on a voluntary carbon market, can be costly, too. Carbon costs may increasingly become a threatening liability for power utilities, eating into profits and undermining the financial viability of emission-intensive electricity generation. Thus, we consider an asset-and-liability, structural firm model to investigate the creditworthiness of a generic power utility. The utility's assets dynamics are driven by the financial returns generated from the sold electricity for a set tariff which is modelled by a simple stochastic process. The liabilities not only depend on fuel, running, and depreciation costs, but also on the costs of CO<sub>2</sub>e-emissions. As a case study, we consider Eskom, the South African power utility. We show the evolution of Eskom's default probability under various fuel mix plans and technologies (as per SA's Integrated Resources Plan (IRP) 2019), and under the Network for Greening the Financial System (NGFS) carbon price scenarios. The obtained results and insights present a trying path ahead, especially for carbon-intensive power utilities.

*Keywords:* Generation of electricity; power utility; fossil fuels; carbon emissions; climate change; emission reduction policies; carbon price scenarios; asset & liability; default probability; Eskom.

## 1. Introduction

Consider a power utility firm that generates electricity. For simplicity, we may assume that electricity generation is the firm's sole line of business. To generate electricity, the power utility relies on a fuel mix that includes fossil (chiefly coal and gas) and nuclear fuels, and renewable sources like aeolian, solar, and hydro power. In the process of generating electricity, the power utility releases greenhouse gases, of which amounts are measured in CO<sub>2</sub>-equivalent (CO<sub>2</sub>e) quantities. The more carbon-intensive and the more of such fuels are used, the more CO<sub>2</sub>e greenhouse gases are emitted. In jurisdictions where CO<sub>2</sub>e-emissions are penalized, by, e.g., taxation, carbon certificates and allowances, generating electricity by using CO<sub>2</sub>e-intensive energy sources can become expensive. Depending on the severity of the imposed penalization and how carbon-heavy the electricity generation is, the resulting carbon costs can lead to a firm becoming financially distressed, in the sense that the firm's default risk is untenable for financial investors or underwriters. In the case where an existing power utility no longer has the ability to access financial markets, or no longer can be underwritten by a third-party (e.g., a government), the firm will have to close and its assets become stranded. One can imagine a scenario in which CO<sub>2</sub>e-intensive electricity generation is no longer financially viable and a whole sector (e.g., energy providers) becomes stranded, unless fundamental physical and financial redesign are implemented. Kenyon *et al.* (2024, 2023b,a) propose the carbon equivalence principle (CEP) and show how accounting for carbon flows in all financial instruments (e.g., loans, credit lines, etc.) impacts the valuation of assets and consequently investment decision-making. The shift in the assessment of what constitutes a financially viable investment is exposed by considering project finance, and in particular the financing of power plants of various types. Kenyon

*et al.* (2023b) conclude that under the Network for Greening the Financial System (NGFS) scenarios (Network for Greening the Financial System (NGFS) 2022) leading to meaningful emissions reductions, all electricity generation that relies on fossil-fuel combustion is already financially untenable. That is, new fossil-fuel power plants are already stranded assets, from the start. So, associated questions emerge: What strategy should be pursued in the case of an ageing network of fossil-fuel power plants? Should these (a) be refurbished, (b) refurbished and retrofitted with emissions-reducing technology, or (c) decommissioned and replaced with “green” power plants? These questions become increasingly topical as networks of power plants age, and maintenance and depreciation costs grow. Here, an important example is Eskom, South Africa’s monopoly power utility, generating most electricity by burning coal. Eskom relies on old power plants, and is often unable to satisfy the electricity demand in South Africa, so much so that power cut schedules must be implemented across the country to safeguard the electricity grid. While, in a coal-rich country as South Africa, refurbishing the existing coal-fired power plants and building new ones seems to be an obvious solution to guarantee current and future electricity demand, the picture changes dramatically when the costs of carbon emissions are accounted for. This is especially the case, if CO<sub>2</sub>e-costs are projected into the future to 2050 based on emission-abating carbon price scenarios as provided by, e.g., the NGFS. Then, questions pertaining a power utility’s future fuel mix plans, the refurbishment of power plants and adoption of emissions-reducing technologies (e.g., carbon capture and storage), and outright replacement of fossil-fuel-fired power plants with renewable electricity-generating installations gather importance and pace. Though our case study focuses on South Africa, the challenge faced by the energy sector in tackling climate change mitigation and adaptation is not limited to a specific country, see, e.g., Mi & Sun (2021) and Lau *et al.* (2023). In the context of the European power sector, we refer to Cormack *et al.* (2020).

In this paper, we focus on the financial impact of carbon emissions on power utilities under climate scenarios. In particular, we investigate how the default probability of a power utility that has issued debt, is affected by the costs arising from CO<sub>2</sub>e-emissions. We take a structural firm approach, see Merton (1974), to price the equity (assets minus liabilities) of a stylized power utility and to derive the probability of default used to price a zero-coupon bond issued by the firm. In this approach, a firm defaults when the liabilities exceed the assets. In the case of a power utility, whose core business is to generate and sell electricity, the main source of assets growth is derived from the sold electricity. On the other hand, the firm’s liabilities are driven by fuel, running, maintenance and depreciation costs, in addition to the CO<sub>2</sub>e-emissions costs. For all of these costs, along with the assets, we produce simple dynamical models with the necessary parameters to capture enough of the power plants features and mode of operation. We then go on to calibrate the model’s degrees of freedom to available power plant specifications and market data. Once a specific power utility is chosen, its calibrated default proba-

4 *F. Krach, A. Macrina, A. Kanter, E. Hampwaye, S. Hlalukana & N. T. Rateele*

bility can be obtained and compared with the market-implied one. The calibrated model can then be used to create scenarios for the default probability of a power utility given the carbon price scenarios provided by, e.g., the NGFS. This in turn allows for an analysis of a power utility's default risk depending on, for example, its fuel mix plans, adoption of emission-abating technologies, and imposed regulatory emissions-reducing policies, all of which impact the carbon emission liability (and the other cost sources), and thus the credit-worthiness of a power utility. As a case study, we apply our modelling setting to Eskom, South Africa's monopoly power utility (Eskom 2022), to shed light on Eskom's probability of default on its issued debt, once Eskom's carbon footprint is accounted for. We find that, given Eskom's heavy reliance on fossil fuels (mainly coal) to generate electricity, it has substantial exposure to carbon price risk. In most carbon price scenarios out to 2050, bar those with little to no impact on emissions reduction, Eskom's credit-worthiness declines inexorably. With current policies in place, our findings show that Eskom is already in a financially non-viable state when its carbon (price) risk exposure is taken into account.

This paper is organised as follows: In the next section, we introduce the asset-and-liability structural firm model for pricing defaultable discount bonds, and thus we also obtain the expression for the probability of default in this setting. Then we go on to model the assets and liabilities of a generic power utility with electricity generation as their core business, before offering brief suggestions about how to use our model setting. In Section 3, we focus on Eskom as a case study. Here, we gather enough data to produce a stylized version of South Africa's actual power utility and calibrate our asset-and-liability model. We obtain an excellent fit to Eskom's market-implied default probability even though we work with a rather simplistic stochastic process for the electricity sell price. We provide graphs showing the resulting term structure and time-evolution of Eskom's default probability for different fuel mix plans and under various carbon price scenarios. We also derive the minimum electricity price for Eskom to remain solvent given its asset and liability profile under different carbon cost scenarios until 2050. In the last section we offer a few concluding remarks. Details regarding the optimization method employed to calibrate our asset-and-liability model can be found in the appendix.

## 2. Power Utility Firm Model

We base our analysis of a power utility firm on a simple asset-and-liability model that we design to reflect the main income and cost sources of such a company. This model allows us to study the influences of different carbon price scenarios as well as investment scenarios on the firm, after fitting it to current market data available to us.

In the following, we describe the general modelling framework with the possible options of how to use it. The implementation of the model is available in the GitHub repository (Krach *et al.* 2023).

The focus of this work is to quantify carbon cost liabilities under different carbon price scenarios and the profitability of a power utility firm in conjunction with these liabilities. In particular, we focus on the Scope 1 carbon emissions<sup>a</sup> of a power utility firm, i.e., those emissions which the company can influence directly<sup>b</sup>. Moreover, we focus on their main CO<sub>2</sub>e-emissions, which are generated by burning fuels to produce electricity, while we ignore the comparably small emissions caused by the transportation of the fuels.

### 2.1. *Asset-and-Liability Model*

We use a Merton structural firm value model, see Wang (2009). At time  $t \geq 0$ , we define the equity  $E(t)$  of a firm as the difference between its assets  $A(t)$  and liabilities (or debts)  $L(t)$ , i.e.,

$$E(t) = A(t) - L(t). \quad (2.1)$$

However, instead of assuming that  $A(t)$  follows a Black–Scholes model and that the debt is only due at maturity, we instead suggest the use of data-driven modelling of  $A(t)$  and  $L(t)$  as the main income and cost sources of a power utility firm. In particular, we model these quantities on a time grid  $t_0 < t_1 < \dots < t_m < \dots < t_N$  suitable for the application at hand (e.g., corresponding to months or years) and survey whether the company defaulted at time step  $t_m$ . For simplicity, we assume that the time grid has equidistant steps  $\Delta t$ . One could adopt a continuous-time modelling approach; however, for practical purposes, a discrete-time model is sufficient. To keep notation light, we use subscripts  $m \in \{0, \dots, N\}$  whenever quantities correspond to time  $t_m$ , e.g., we write  $A_m$  for the assets value  $A(t_m)$  at time  $t_m$ . We assume that  $t_0$  is the reference (e.g., current) time, at which the initial equity  $E_0 = A_0 - L_0$  is known. Then, throughout time, assets and liabilities are accumulated as described in Sections 2.4 and 2.5 below, until the maturity date.

### 2.2. *Discounting and Inflation*

Discounting is used to value future income streams and costs in today's terms. We introduce a bank account process  $B_m$  in terms of which we define a discounting factor by

$$D_{mn} := \frac{B_m}{B_n}, \quad (2.2)$$

over the time interval  $[t_m, t_n]$ , where  $m < n \in \{0, \dots, N\}$ . Moreover, depending on the currency denomination, the valuation of equity may be subject to inflation for

<sup>a</sup>Scope 1 emissions are the direct emissions of a company, while scope 2 and 3 emissions are indirect emissions, which the company causes by the energy it uses (scope 2) and all emissions caused by suppliers to the company and customers of the company (scope 3).

<sup>b</sup>We note that for a power utility company this usually also includes their scope 2 emissions, since they produce their needed energy themselves.

6 *F. Krach, A. Macrina, A. Kanter, E. Hampwaye, S. Hlalukana & N. T. Rateele*

which we introduce an inflation factor ( $I_{mn}$ ). This factor is defined as the quotient of a nominal (zero-coupon) bond price process  $Z_{mn}$  and its associated inflation-linked (zero-coupon) bond price process  $Z_{mn}^I$  with maturity  $t_n$ . At bond maturity, i.e.,  $m = n$ , the inflation factor  $I_{nn}$  equals the (consumer, retail) price index value prevailing at time  $t_n$ .

### 2.3. Probability of Default and Zero-Coupon Bond Price

The asset and liability price processes  $(A_m)_{m=0,\dots,N}$  and  $(L_m)_{m=0,\dots,N}$  are defined on a filtered probability space  $(\Omega, \mathcal{F}, \mathbb{F} = (\mathcal{F}_m)_{m=0,\dots,N}, \mathbb{Q})$ , where  $\mathbb{Q}$  is the market's pricing measure, and the price processes are  $(\mathcal{F}_m)$ -adapted. In the following, all expectations are with respect to this probability measure.

The probability of default  $P_{mn}^d$  conditional on  $t_m < t_n \leq t_N$  (implicitly assuming that no default occurred before or at  $t_m$ ) is defined for any time  $t_n \in \{t_m, \dots, t_N\}$  by

$$P_{mn}^d := P^d(t_m, t_n) := \mathbb{Q}(\exists m \leq k \leq n : A_k \leq L_k \mid \mathcal{F}_m) := \mathbb{E} [\mathbb{I}_{\{\exists m \leq k \leq n : A_k \leq L_k\}} \mid \mathcal{F}_m], \quad (2.3)$$

and its term structure is given by  $t_n \mapsto P^d(t_m, t_n)$ , for  $t_n \geq t_m$ . The corresponding defaultable zero-coupon bond (ZCB) price process  $(Z_{mn}^d)$  with recovery rate  $R = 0$  and maturity  $t_n \leq t_N$ , is given by

$$Z_{mn}^d := Z^d(t_m, t_n) := \mathbb{E} [D_{mn} \mathbb{I}_{\{\forall s \leq n : A_s > L_s\}} \mid \mathcal{F}_m]. \quad (2.4)$$

The  $t_n$ -parametrized term structure of the defaultable ZCB is given by the function  $t_n \mapsto Z^d(t_m, t_n)$ , for  $t_n \geq t_m$ .

**More realistic assumptions.** Our definition of default, as the event occurring when a firm's equity becomes negative, i.e.,  $E(t) \leq 0$ , is rather elementary. It implies that the equity owners (e.g., shareholders) do not intervene unless the firm's assets are less than the liabilities, in which case they would liquidate all assets to pay back the liabilities—or as much as possible if  $E(t) < 0$ . In such a situation, the equity owners lose their entire capital. More realistically, equity owners usually declare bankruptcy when  $E(t) \leq G(t)$  for some positive quantity  $G(t) \geq 0^c$ , a covenant defined as, e.g., a fraction of current assets  $G(t) = p \cdot A(t)$  for some  $p \in [0, 1]$ . This way, liquidating all the assets to pay back the liabilities leaves the equity owners with a capital remainder that amounts to  $G(t)$ . Furthermore, it is rather common in the event of a default—independently of the choice of  $G(t)$ —that debt is restructured (according to a rescue plan) instead of a drastic liquidation of the firm's assets (Schönbucher 1998). In this case, bond holders agree on foregoing a fraction  $q \in [0, 1]$  of their (original) claims to provide rescue capital that is invested in the defaulted firm. Furthermore, the recovered fraction  $R = 1 - q$  is not paid out in “cash”, but in new defaultable bonds with the same maturity. Since these bonds can also default, Schönbucher (1998) defines stopping times  $\tau_1 < \tau_2 < \dots$

<sup>c</sup>By setting  $G(t) = 0$  we get back our previous definition.

representing the default times of the company. At each default time  $\tau_i$ , a loss quota  $q_i \in [0, 1]$ , implying a recovery rate  $R_i := 1 - q_i$ , is agreed on such that the defaultable ZCB with maturity  $T$  pays out  $Q(T) = \prod_{\tau_i \leq T} R_i$ , i.e., what is left after all recoveries. Therefore, the price  $Z_{mT}^d$  at time  $t_m \leq T$  of the defaultable ZCB with maturity  $T$  and recovery is

$$Z_{rec}^d(t_m, T) := \mathbb{E}[D(t_m, T)Q(T) | \mathcal{F}_m].$$

To link this to our asset-and-liability model, we define  $\tau_0 = 0$  and  $R_0 = 1$  and

$$\tau_i := \inf\{t > \tau_{i-1} | E(t) \leq G(t)\}. \quad (2.5)$$

The corresponding recovery rates  $R_i$  can either be fixed in advance (market standards are  $R_i \in [0.4, 0.8]$ ) or defined as  $\mathcal{F}_i$ -measurable random variables. Moreover, at the default time  $\tau_i$ , the liabilities change, which is expressed by a jump in the liabilities value  $L(\tau_i) = R_i \cdot L(\tau_i-)$  at the default time<sup>d</sup>.

In this work, we are mainly interested in the probabilities of default, because they are quoted in the market, hence, allow for calibration to market data without the need of modelling ZCB with recovery. Nevertheless, we will also show the term structure of the corresponding ZCB prices with zero-recovery, where an extension to bond prices with recovery is straightforward following the discussion above. Since the default probabilities  $P_{mn}^d$  are increasing as a function of the maturity  $t_n$ , the term structure  $t_n \mapsto P_{mn}^d$  does not allow to investigate a firm's future financial viability without taking into account the entire time period. An alternative may be to consider different starting dates  $t_m$  along with their corresponding term structures. However, this would require one to define the state of the model at  $t_m$  (i.e., fix a scenario in  $\mathcal{F}_m \setminus \mathcal{F}_{m-1}$ ). Instead, we propose to use the instantaneous probability of default at time  $t_m$ , given by

$$P_m^d \equiv P^d(t_m) = \mathbb{Q}(E_m \leq G_m), \quad (2.6)$$

$0 \leq m \leq N$ , which is well-defined in our discrete-time setting. The function  $t_m \mapsto P^d(t_m)$  of this probability is not (necessarily) increasing, and thus provides one with a measure of how financially viable a firm is at each instant  $t_m \in \{t_0, t_1, \dots, t_N\}$ .

#### 2.4. Modelling of Assets

Typically, power utility companies sell two major goods, these are electricity and gas. Generating electricity causes substantial CO<sub>2</sub>e-emissions world-wide. Electricity generation was responsible for 43% of carbon emissions in 2020, see Climate Watch (2020), and CO<sub>2</sub> emissions from energy combustion and industrial process

<sup>d</sup>This definition of the jump of the liabilities  $L$  at default implies that the default times defined in (2.5) are well-defined in the sense that  $\tau_i > \tau_{i-1}$ , as long as  $R_i$  is chosen such that  $E(\tau_i) = A(\tau_i) - R_i \cdot L(\tau_i-) > G(\tau_i)$ . Assuming that  $G(\tau_i) = G(\tau_i-)$  and  $A(\tau_i) = A(\tau_i-)$ , this is equivalent to  $R_i < \frac{A(\tau_i-) - G(\tau_i-)}{L(\tau_i-)}$ .

8 *F. Krach, A. Macrina, A. Kanter, E. Hampwaye, S. Hlalukana & N. T. Rateele*

accounted for 89% of energy-related greenhouse gas emissions in 2022, see International Energy Agency (IEA) (2023). Gas sales do not cause scope 1 emissions, if emissions arising from extraction and transportation are ignored. Therefore, we only consider modelling the electricity-producing branch of a power utility firm in what follows. Furthermore, for simplicity, we do not consider income from financial investments or subsidiary companies of the power utility firm. Thus, the main income source for such a firm is the sale of electricity, which is calculated as the sold energy times the prevailing electricity price  $\pi_m^{\text{elect}}$  and, where necessary, discounted to the time at which the income is computed. The amount of sold energy is calculated as the power output capacity  $\kappa_m^{\text{run}}$ , at which all power plants run on average in the period  $(t_{m-1}, t_m]$ , multiplied by the length of the time period  $\Delta t_m = t_m - t_{m-1}$  and a factor  $\psi_i \in [0, 1]$  that determines how much of the produced energy is sold (transmission losses, e.g., influence this factor). Hence, for a given initial asset value  $A_0$  the asset process  $(A_m)_{m=0, \dots, N}$  is defined by

$$A_m = \frac{A_0}{D_{0m}} + \sum_{i=1}^m \frac{\kappa_i^{\text{run}} \cdot \psi_i \cdot \pi_i^{\text{elect}}}{D_{im}} \Delta t_i, \quad (2.7)$$

where, for  $i \leq m = 1, \dots, N$ , the output capacity ( $\kappa_m^{\text{run}}$ ), the electricity price ( $\pi_m^{\text{elect}}$ ), and the discount factor ( $D_{im}$ ) are stochastic processes.

## 2.5. Modelling of Liabilities

In line with the simplifying assumptions for the modelling of the power utility's income stream, we do not consider possible debt or interest payments like the pay-back of bonds or their coupons. Hence, we model the liabilities as the main costs that arise from generating electricity.

**Fuel mix.** We first introduce the notion of a fuel mix (sometimes referred to as “energy mix”) for a power utility and all related quantities. This is the firm's composition of different power plant types which amount to the entire energy-generating portfolio. Power plant types include coal, gas/diesel, nuclear, hydro-power, wind, and solar<sup>e</sup>. Let  $\mathcal{J}$  be the set of indices for the different types of electricity-generating plants. We use the standard convention that for each energy type  $j \in \mathcal{J}$  the maximal generation capacity  $\kappa_m^{j \text{max}}$  (i.e., its maximal power output) of all power plants of type  $j$  during the period  $(t_{m-1}, t_m]$  is known. However, the maximal capacity is not achievable, since, for example, a photo-voltaic power plant can only produce as much electricity as there is convertible sunshine. The capacity realization constant  $\gamma_{\text{crf}}^j \in (0, 1)$  is an average that adjusts the maximal capacity to generate electricity with all power plants of type  $j$  at time  $t_m$  to its realizable capacity (or power output) given by

$$\kappa_m^{j \text{rlz}} = \gamma_{\text{crf}}^j \cdot \kappa_m^{j \text{max}}.$$

<sup>e</sup>In this work, we consider prototype power plants for each energy type with their specifics reported in Table 1. In particular, the different energy types used in this work are listed in the table.



Hence, if the power plants of energy type  $j$  run at full realizable capacity, then they produce  $\kappa_m^{j\text{rlz}} \cdot \Delta t_m$  of electricity in the period  $(t_{m-1}, t_m]$ . However, usually only a fraction, defined by the power production factor  $\gamma_m^{j\text{PPf}} \in (0, 1)$ , is actually produced by the power utility, such that its electricity production meets the demand. In particular, we assume that the power production factor  $\gamma_m^{j\text{PPf}}$  is determined such that the produced electricity  $\gamma_m^{j\text{PPf}} \cdot \kappa_m^{j\text{rlz}} \cdot \Delta t_m$  equals the actually produced electricity amount in period  $(t_{m-1}, t_m]$ . In other words,  $\gamma_m^{j\text{PPf}}$  is the fraction of the realizable power output at which the power plants run on average in the period  $(t_{m-1}, t_m]$ . So, we define the average running power output capacity,

$$\kappa_m^{j\text{run}} = \gamma_m^{j\text{PPf}} \cdot \kappa_m^{j\text{rlz}}, \quad (2.8)$$

such that we may write the electricity amount (matching the actually produced amount) generated by plant type  $j$  in period  $(t_{m-1}, t_m]$  as  $\kappa_m^{j\text{run}} \cdot \Delta t_m$ . The total maximal generation capacity (power output) is

$$\kappa_m^{\text{max}} = \sum_{j \in \mathcal{J}} \kappa_m^{j\text{max}}. \quad (2.9)$$

Similar expressions can be obtained for  $\kappa_m^{\text{rlz}}$  and  $\kappa_m^{\text{run}}$ . Given a fuel mix, the proportion of the maximum capacity in the period  $(t_{m-1}, t_m]$  for generation type  $j$  is given by  $(\kappa_m^{j\text{max}} / \kappa_m^{\text{max}})_{j \in \mathcal{J}}$ . Similar expressions hold for  $\kappa_m^{\text{rlz}}$  and  $\kappa_m^{\text{run}}$ . We refer to these as the maximal realizable and running proportional fuel mixes, respectively.

**Costs for generating electricity.** In what follows, we list the costs which are considered in our setting. The list is non-exhaustive, so we focus on the main cost sources. Costs that are not included in the list below fall under “additional costs”.

- (1) We introduce the average price  $\pi_m^{j\text{fuel}}$  per unit of fuel type  $j$ , averaged over  $\Delta t_m$ , for each plant type  $j \in \mathcal{J}$ . Moreover, for each  $j \in \mathcal{J}$ , let  $\varphi^j$  be the fuel consumption rate per generated unit of electricity. Then, the total fuel costs in the period  $(t_{m-1}, t_m]$  amounts to

$$C_m^{\text{fuel}} = \sum_{j \in \mathcal{J}} \varphi^j \cdot \kappa_m^{j\text{run}} \cdot \pi_m^{j\text{fuel}} \cdot \Delta t_m.$$

- (2) The running costs  $C_m^{\text{run}}$  over the period  $\Delta t_m$  are a combination of the labour and maintenance costs  $C_m^{\text{labour}}$  and  $C_m^{\text{maint}}$ , respectively. In principle, these costs could be linked to the fuel/plant type  $j$  and its generated electricity and associated emissions cost. However, the corresponding cost factors are hard to determine. Therefore, we shall model these costs as incremental running costs, which can be inferred from a power utility’s balance sheet. The incremental quantity is adjusted for inflation over the interval  $(t_{m-1}, t_m]$ , and is proportional to the change in the realizable power output over the same time period. Hence, the running costs are given by

$$C_m^{\text{run}} = \frac{\kappa_m^{\text{rlz}}}{\kappa_{m-1}^{\text{rlz}}} \cdot C_{m-1}^{\text{run}} \cdot I_{m-1m} \cdot \frac{\Delta t_m}{\Delta t_{m-1}}.$$

For clarity, we emphasize that a discrete-time quantity indexed by  $m$  applies over the interval  $(t_{m-1}, t_m]$  and one indexed by  $m - 1$  is applicable over  $(t_{m-2}, t_{m-1}]$ . The labour and maintenance costs might change depending on the energy mix, which is not reflected in the expression above. This implies that the running costs are the same for each realizable unit of power output, independently of the fuel type. Therefore, a better definition would link the running costs to the fuel types and their maximal, realizable (or average) running power output. Although this is achievable in our setting, we keep the simplification above. In absence of the needed cost factors, the obtained results hold, at least as a good proxy if the energy mix does not change significantly over time.

- (3) For power plant type  $j \in \mathcal{J}$ , we define the depreciation factor  $d_m^j$  during  $(t_{m-1}, t_m]$  per unit of maximal capacity. This is the capital amount per unit of maximal capacity by which the value of a power plant generating electricity on fuel type  $j$  depreciates. For power plant type  $j$ , let  $\pi_{\text{build}}^j$  be the capital cost at  $t_0$  for the construction of a power plant<sup>f</sup> with maximal capacity  $K_{\text{max}}^j$  and a lifespan  $l^j$ . We emphasize that  $K_{\text{max}}^j$  is the maximal capacity of one electricity-generating plant of type  $j$ , as opposed to the aggregate maximal capacity  $\kappa_m^{j, \text{max}}$  of all plants of type  $j$  in a network. Then, the depreciation factor  $d_m^j$  over  $\Delta t_m$  is given by

$$d_m^j = \frac{\pi_{\text{build}}^j}{K_{\text{max}}^j \cdot l^j} \Delta t_m.$$

We use the simplifying convention that an investment into a new power plant does not directly change the overall asset-and-liability balance of the firm, since, at the time of funding the new power plant<sup>g</sup>, it is an exchange from cash to the equal value now presented by the new power plant. However, the power plant loses value over time (since it is only operational for its lifespan  $l^j$ ), where we assume equal value reduction in each time period until it becomes worthless at the end of its lifespan. Since we compute the depreciation of prototype power plants (these might differ from those brought online by a specific power utility firm), we introduce a depreciation adjustment  $\gamma_{\text{df}}$  that is useful for the calibration of the model to market data. To compute the depreciation cost at a future time  $t_m \geq t_0$ , we adjust the value  $d_m^j$  by inflation. Since the power plant's value increases with inflation, future depreciation costs also increase with inflation. Equivalently, building the same power plant at a future time  $t_m$  costs more

<sup>f</sup>For simplicity, we consider typical prototype power plants, with their specifics reported in Table 1. Moreover, we assume that these prototype power plants can be scaled linearly to any wanted size such that we only need to consider the proportional costs per capacity unit of these prototype plants for each energy type.

<sup>g</sup>For further simplification, we assume here that the power utility firm pays for a new power plant at the moment it becomes operational.

than at an earlier time  $t_0$  by a factor of  $I_{0m}$ . We have

$$C_m^{\text{dep}} := \gamma_{\text{df}} \cdot I_{0m} \sum_{j \in \mathcal{J}} d_m^j \cdot \kappa_m^{j \text{max}},$$

where  $t_m > t_0$ . By this definition of the depreciation cost, one can increase the capacities of the different energy types over time, without the need to account for the investments into new power plants, since these investments are paid over the future time periods through the depreciation. On the other hand, we assume that reductions of the maximal capacities are made at the end of the lifespan of a power plant, or are compensated by selling the respective power plant exactly for the future depreciation costs (such that no additional costs arise). Otherwise, the power production factor of the respective energy type  $\gamma^j \text{PP}^{\text{f}}$  can be adjusted so to model the reduced amount of electricity that is produced for a certain plant type while the maximal capacity does not change.

- (4) The CO<sub>2</sub>e emission costs  $C_m^{\text{emiss}}$  at time  $t_m$  play a central role. Their impact on the financial viability of electricity-generating power plants is considered under different future scenarios for the CO<sub>2</sub>e price  $\pi_m^{\text{CO}_2\text{e}}$ . For each energy type  $j \in \mathcal{J}$ , let  $e^j$  be the amount of CO<sub>2</sub>e-emissions per unit of generated electricity. Recalling the average power output capacity  $\kappa_m^{j \text{run}}$  of a power plant of type  $j$  over the period  $(t_{m-1}, t_m]$ , we compute the total CO<sub>2</sub>e-emissions produced over the period  $(t_{m-1}, t_m]$  by all power plant types in the network by

$$\mathcal{E}_m = \sum_{j \in \mathcal{J}} e^j \cdot \kappa_m^{j \text{run}} \cdot \Delta t_m.$$

Since we compute the emissions of prototype power plants, i.e., emissions of the actually installed plants might differ, we introduce an adjustment factor  $\gamma_{\text{ef}}$  that is useful for calibrating the emissions model to market data. Then, the financial costs due to CO<sub>2</sub>e-emissions during  $(t_{m-1}, t_m]$  are

$$C_m^{\text{emiss}} = \gamma_{\text{ef}} \cdot \mathcal{E}_m \cdot \pi_m^{\text{CO}_2\text{e}}.$$

In Section 3.1, we shall consider carbon price scenarios, in particular the NGFS scenarios, and their usage for building liability scenarios potentially faced by a power utility due to their CO<sub>2</sub>e-emissions and future carbon price evolution.

- (5) Finally, the additional costs  $C_m^{\text{add}}$  can be used to model any other costs not included in the costs defined above. For example, additional investment costs when building a new power plant, which are not covered by the depreciation costs, could be included in  $C_m^{\text{add}}$ .

We can now determine the financial liability process  $(L_m)_{m=0, \dots, N}$  of a power utility firm, generating electricity based on a certain fuel mix profile, that accounts for the costs listed above. We have,

$$L_m = \frac{L_0}{D_{0m}} + \sum_{k=1}^m \frac{1}{D_{km}} \left( C_k^{\text{fuel}} + C_k^{\text{run}} + C_k^{\text{dep}} + C_k^{\text{emiss}} + C_k^{\text{add}} \right), \quad (2.10)$$

where  $L_0$  is the initial liability at time  $t_0$ .

12 *F. Krach, A. Macrina, A. Kanter, E. Hampwaye, S. Hlalukana & N. T. Rateele*

## 2.6. Calibration and Suggested Usage of the Model

We work with a data-driven approach to fit the model-implied default probabilities to the market-implied ones. In the case study we focus on in Section 3, we inform as many of the model ingredients as possible with market data (or make assumptions about how to extrapolate them deterministically) and model the remaining quantities with parameterised stochastic processes. Then, for any fixed set of parameters, the model-implied default probabilities can be approximated by Monte Carlo by using i.i.d. sampling from the stochastic processes and by approximating (2.3) by the corresponding empirical mean. Hence, the model can be calibrated to the market default probabilities by solving the inverse problem of finding the parameters such that the model-implied default probabilities match the market default probabilities as well as possible—a goodness-of-fit measured by the Euclidean norm. The calibrated model, equipped with the fitted parameters, can be used to investigate the impact of various carbon price scenarios. Here we take the view that financial markets (at least significant portions of it) have not yet priced future carbon price developments (steered by climate change policies) in the firm’s financial viability. In particular, we study how a power utility firm’s default probabilities change under different (i) carbon price scenarios, and (ii) fuel mix development strategies, modelled by different scenarios for the construction of new and decommissioning of old power plants, which influences the fuel mix of the generated electricity.

## 3. Eskom: South Africa’s State-Owned Power Utility Firm

In this section, we apply the model developed in Section 2 to South Africa’s state-owned power utility firm, Eskom. It has a monopoly position in the electricity market generating approximately 90% of South Africa’s electricity supply (Eskom 2023a). First, we describe the market data and our considered scenarios in Section 3.1 followed by the stochastic modelling approach for the electricity price in Section 3.3. Based on those, we calibrate our model in Section 3.4 and investigate the impact of different carbon price and fuel mix development scenarios on Eskom’s financial viability in Sections 3.5 and 3.6.

### 3.1. Market Data and Scenarios

We describe the market data for the variables introduced in Section 2. We focus on describing the data qualitatively and give their source. When not stated, the exact values can be found in the data configuration of our implementation (Krach *et al.* 2023).

Eskom’s last financial report was published dated 31 March 2022. So, we use 1 April 2022, which is the beginning of Eskom’s 2022/2023 fiscal year, as starting date  $t_0$ . Yearly time steps are used (the corresponding time step is  $\Delta t = 8760$  hours) such that  $t_m$  corresponds to 1 April 2022 +  $m$ , for  $m \in \{0, \dots, N\}$ , where  $N = 28$  correspond to 28 yearly steps, i.e., an end date of 1 April 2050. This time horizon

is chosen because many climate-related targets and strategies are determined with an end date of 2050, especially the 2015 Paris Agreement (UNFCCC 2018).

**Inflation and discounting factors.** The South African discount and inflation factors  $D_m$  and  $I_m$  are derived from the South African nominal sovereign (SAGBs) and inflation-linked bonds, respectively. These can be retrieved from Bloomberg with base date  $t_0$  for all available maturities. The inflation factor is obtained as the quotient of the nominal and inflation-linked bond for each available maturity (in the case that the term structure is needed). Linear interpolation is then used to determine the discount and inflation factors for each year. Similarly, the US inflation and discount factors,  $I_m^{\text{US}}$  and  $D_m^{\text{US}}$ , respectively, are computed from the corresponding US bonds.

**Market default probabilities and credit ratings.** The market survival (non-default) probabilities of Eskom are retrieved from the Eskom bonds quoted on Bloomberg with base date  $t_0$  for all available maturities. The market default probabilities are computed by subtracting the survival probabilities from one. Linear interpolation is used to obtain the default probabilities at any step  $t_m$  on the time grid. To better understand what the computed default probabilities signify in a financial context, we link the S&P Global<sup>h</sup> long-term issuer credit ratings (ranging from “AAA” to “D”) to the default probabilities obtained for Eskom. We retrieve from Bloomberg, the 5Y, 10Y and 20Y value of the yield curves corresponding to the ratings between “AAA” and “B-”, where “AAA” is the most creditworthy (lowest default probability) rating. For each credit rating, its respective default intensity  $\lambda_T$  with maturity  $T \in \{5, 10, 20\}$  is computed as the credit spread between this yield curve value and the corresponding US government bond’s yield curve value (which is used as a reference). These default intensities are extrapolated to any maturity  $T$  by

$$\lambda_T = \begin{cases} \lambda_5, & 0 \leq T < 10, \\ \lambda_{10}, & 10 \leq T < 20, \\ \lambda_{20}, & 20 \leq T, \end{cases}$$

as is standard practice. Then the probability of default  $P^d(T)$  corresponding to each credit rating with maturity  $T \geq 0$  is given by

$$P^d(T) = \frac{1 - e^{-\lambda_T T}}{(1 - R)}, \quad (3.1)$$

where the 40% recovery rate market standard is used, i.e.,  $R = 0.4$ .

**Power plant (fuel) types and fuel prices.** We consider the power plant types (which we also refer to as *fuel types*) listed in Table 1. Since the combined-cycle single shaft turbines can be powered by diesel and by gas, their power plant specifics are the same, but their emission and fuel specifics differ. The values for  $K_{\text{max}}^j$ ,  $\pi_{\text{build}}^j$

<sup>h</sup><https://www.spglobal.com/ratings/en/index>

and  $\tau^j$  are taken from (Kenyon *et al.* 2023b, Table 1). The values for the CO<sub>2</sub>e-emissions are taken from (US Energy Information Administration 2022a). As stated in the documentation by the National Energy Regulator of South Africa (NERSA) (2023b) (Paragraphs 8.7.14 to 8.7.17, Table 72), Eskom pays a special coal price of 672.9 ZAR/ton in 2023, fixed by the National Energy Regulator of South Africa (NERSA). This is approximately 39.74% of the market coal price. The current fuel price  $\pi_m^{j\text{fuel}}$  for diesel is taken as the average of Eskom’s normal contract prices for diesel, as stated in (Business Tech 2023). The fuel amounts  $\varphi^j$  for coal is taken from (National Energy Regulator of South Africa (NERSA) 2023b, Paragraph 8.7.16), and for diesel from (US Energy Information Administration 2022b). The fuel price  $\pi_m^{j\text{fuel}}$  at time  $t_m$  for LPG-gas (the type which is used by Eskom) is computed as the mean of the prices stated in (South African Department of Mineral Resources and Energy 2023), and its fuel amount  $\varphi^j$  is taken from (ELGAS 2022). For nuclear energy, the fuel price  $\pi_m^{j\text{fuel}}$  and fuel amount  $\varphi^j$  are taken from (World Nuclear Association 2022). The price stated in USD as of September 2021 is converted (spot) to South African Rand (ZAR), using first the US inflation calculator (US Bureau of Labor Statistics 2023) to adjust for inflation to  $t_0$ . The currency is converted at the ( $t_0$ ) exchange rate of 14.5954 ZAR/USD. For future fuel prices we use a simple extrapolation of the current fuel prices instead of a stochastic model, so to keep our model simple. In particular, we scale the current fuel prices (except for coal) with the US inflation factor  $I^{\text{US}}$  and discount them with the US discount factor  $D^{\text{US}}$ . We use US factors instead of South African ones because the commodity market is based on US dollars. Hence, these prices are subject to US inflation. Moreover, discounting by the US factor, instead of the South African one, adjusts directly for the different future foreign exchange rates. For coal, which is mined in South Africa and where the price is set by NERSA, we scale and discount with the South African inflation factor  $I$  and discount factor  $D$ .

**CO<sub>2</sub>e emission price scenarios and fuel cost efficiency.** The CO<sub>2</sub>e price data for the six most common scenarios (Net Zero 2050, Below 2°C, Divergent Net Zero, Delayed Transition, Nationally Determined Contributions (NDC) and Current Policies) are obtained from the Network for Greening the Financial System (NGFS) portal Network for Greening the Financial System (NGFS) (2023). These prices are given in 2010 USD, and thus are US inflation-adjusted. We adjust them by inflation to  $t_0$  and convert them to ZAR, using the same procedure as above. In our model, we discount future carbon prices by the US discounting factor  $D^{\text{US}}$  (as for the fuel prices) to  $t_m$ . We add the No-Cost scenario, where CO<sub>2</sub>e trades at the constant price of zero, and the scenario of the South African Carbon Tax (SACT) (South African Revenue Service (SARS) 2023) that is computed according to its definition by an annual increase of the current rate by South Africa’s inflation factor  $I$  and discounted by the South African factor  $D$  to  $t_m$ . The CO<sub>2</sub>e price dynamics in these eight scenarios are plotted in Figure 1. One sees that the SACT approximately coincides with the Current Policies scenario. By using the data of the different

Table 1. Power plant types with technology characteristics, capital costs and life span. NB: Carbon capture and storage (CCS), (B)ZAR = (billion) South African rand, MW = Megawatt, KWh = Kilowatt-hour.

$j$	Abbre- viation	Technology	Size (MW): $K_{\max}^j$	Capital cost (BZAR): $\pi_{\text{build}}^j$	Lifespan (Years): $l^j$	Capacity realiza- tion factor: $\gamma_{\text{eff}}^j$	CO <sub>2</sub> e- emissions (kg/KWh): $\epsilon^j$	Fuel amount per KWh: $\varphi^j$	Fuel price at $t_0$ (ZAR per amount unit): $\pi_0^{j \text{ fuel}}$
1	Coal	Ultra-supercritical coal (USC)	650	37.247	40	0.85	0.328	0.610 kg	0.673
2	Coal (CCS90)	USC with 90% CCS	650	59.535	40	0.85	0.033	0.610 kg	0.673
3	Diesel	Combined-cycle single shaft (diesel fired)	418	7.064	40	0.87	0.253	0.303 l	21.093
4	Gas	Combined-cycle single shaft (gas fired)	418	7.064	40	0.87	0.181	0.074 kg	31.118
5	Gas (CCS90)	Combined-cycle with 90% CCS (gas fired)	377	14.581	40	0.87	0.018	0.074 kg	31.118
6	Nuclear	Nuclear-small modular reactor	600	57.900	40	0.9	0	$2.78 \cdot 10^{-6}$ kg	25485.758
7	Hydro	Conventional hydropower	100	8.290	50	0.5	0	0	0
8	Wind (onshore)	Wind onshore	200	3.941	25	0.38	0	0	0
9	Wind (offshore)	Wind offshore	400	27.279	25	0.39	0	0	0
10	Solar	Solar photovoltaic (PV) with tracking	150	3.065	30	0.158	0	0	0

fuel types (Table 1), the fuel prices and the CO<sub>2</sub>e-emissions, we compare the cost efficiency of all fuel types in different CO<sub>2</sub>e price scenarios, as presented in Figure 2.

At the time of writing, Eskom is in a transition phase, whereby it is moving from a previous environmental levy, which amounts approximately to 0.035 ZAR/kg, to the new SACT, which is currently 0.159 ZAR/kg. According to current information, Eskom will pay the environmental levy until end of 2025 and the SACT starting from 2026 (Eskom 2023c, p. 116). We assume that the market uses this combined CO<sub>2</sub>e price scenario (of environmental levy for the first four years and SACT thereafter) for pricing Eskom’s assets and liabilities. Therefore, we calibrate our model to market data assuming this scenario. While it is certain that Eskom will be paying the environmental levy until 2026, it is possible that the levy will not be replaced by the SACT, but by a different, more expensive CO<sub>2</sub>e price scenario, see Eskom (2023c, p. 116). In Sections 3.5 and 3.6, we therefore use the combined scenarios of environmental levy until 2026 followed by the different NGFS or the SACT CO<sub>2</sub>e price scenarios to illustrate their impact on the viability of Eskom. Only the No-Cost scenario is not combined with the environmental levy, and it is used as baseline for comparison.

**Eskom-specific data.** We consider the South African power utility firm Eskom SOC Ltd (Eskom), which is a monopoly holding the responsibility of supplying

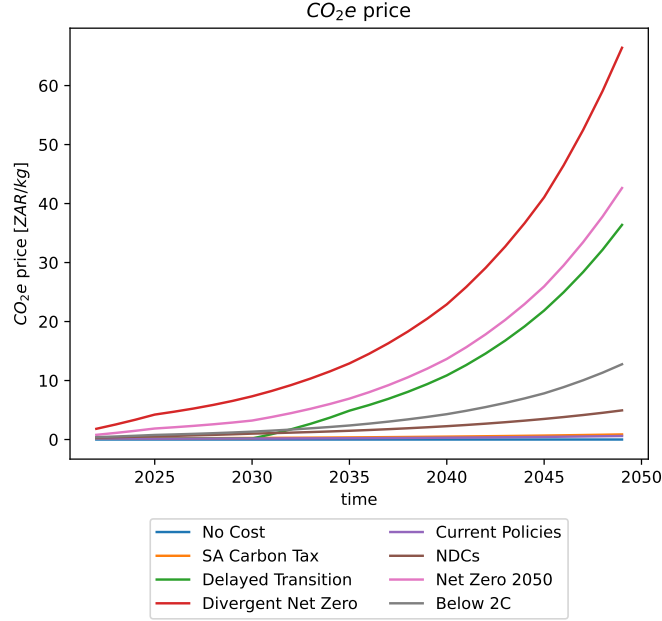


Fig. 1. NGFS, South African Carbon Tax, and No-Cost CO<sub>2</sub>e price scenarios. No discounting is applied, i.e., prices are not divided by  $D$  to obtain non-discounted prices in ZAR.

the majority of electricity in South Africa. It is entirely owned by the South African government, and it is accountable for generating and distributing electricity, serving industries and municipalities across the country. Furthermore, Eskom buys electricity from independent power producers (IPP) and international sources based in southern Africa (Eskom 2022). Eskom’s latest balance sheet (Eskom 2022) and the corresponding integrated report (Eskom 2023c) refer to the end of the fiscal year on 31 March 2022, and therefore represent the starting values at  $t_0$ . The relevant data for our purpose is shown in Table 2. Moreover, Eskom’s current fuel mix (Eskom 2023a,d) is illustrated in Table 3, where we treat pumped storage hydro plants as standard hydro plants. The actually produced capacities  $\kappa_0^{j\text{run}}$  are derived from Eskom (2023c, p. 77) by dividing the stated produced energy amounts by the hours per year (8760 hours). The power production factors  $\gamma^{j\text{PPF}}$  can then be derived via (2.8). All of Eskom’s fuel type capacities are listed. We first note that Eskom’s theoretically realizable annual energy production is 376.575 TWh. Comparing this to the produced electricity, Eskom’s overall power production factor amounts to  $\gamma_0^{\text{PPF}} = 58.87\%$ . Moreover, the produced and imported electricity, when compared to the sold electricity, implies an energy sales factor of  $\psi_0 = 86.14\%$ . Based on these calibrated parameters, the model-implied CO<sub>2</sub>e-emissions are  $e_0 = 37.358$  megatons (MT), which is much smaller than Eskom’s reported value. According to Eskom’s Carbon Footprint Report (Eskom 2022), these emissions stem from burning fuels, mainly coal, only. Therefore, we suspect that the higher level of emissions



Inflation Adjusted Costs (depreciation + fuel + CO<sub>2</sub>e-emission) per KWh

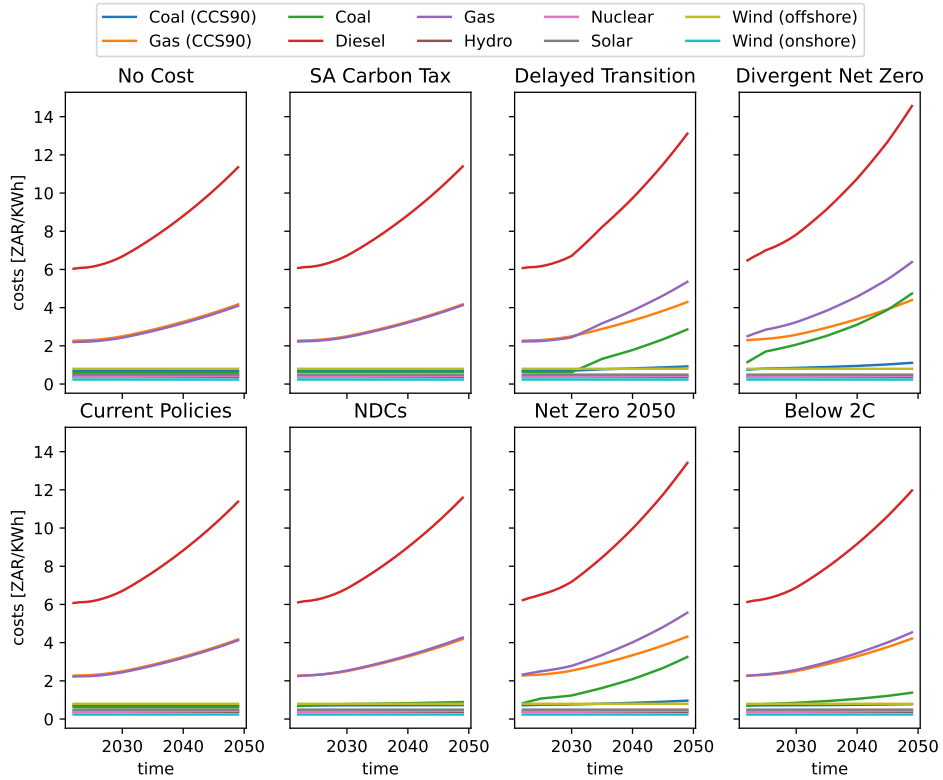


Fig. 2. Cost comparison of all different energy types in different scenarios for the emission price.

Table 2. Eskom’s balance sheet data as of 31 March 2022. NB: BZAR = billion ZAR, MT = million tons, TWh = terawatt hours.

Equity	235.314 BZAR
Annual depreciation costs	32.009 BZAR
Annual maintenance costs	24.113 BZAR
Annual labour costs	32.985 BZAR
Annual sold electricity	198.3 TWh
Annual produced electricity	221.7 TWh
Annual electricity imports	8.5 TWh
Annual CO <sub>2</sub> e emission	207.230 MT

are due to older technology used in Eskom’s coal power plants than assumed in our study. For the model calibration of the emissions level to the available data, we set  $\gamma_{ef} = 554.71\%$ , where Eskom’s coal, gas and diesel power plants are not equipped

18 *F. Krach, A. Macrina, A. Kanter, E. Hampwaye, S. Hlalukana & N. T. Rateele*

Table 3. Eskom’s energy mix with the maximal capacities  $\kappa_0^{j \max}$ , the realizable capacities  $\kappa_0^{j \text{rlz}}$  (computed with the respective  $\gamma_{\text{crf}}^j$  factors), and the actually produced capacities  $\kappa_0^{j \text{run}}$  stated in megawatt (MW), together with their respective ratios  $\kappa_0^j/\kappa_0$  and the implied power production factors  $\gamma_0^{j \text{PPf}}$

j	Source	$\kappa_0^{j \max}$	Ratio	$\kappa_0^{j \text{rlz}}$	Ratio	$\kappa_0^{j \text{run}}$	Ratio	$\gamma_0^{j \text{PPf}}$
1	Coal	44013.0	84.88%	37411.0	87.027%	21860.7	86.38%	58.43%
3	Diesel	2078.3	4.01%	1808.1	4.21%	267.1	1.06%	14.77%
4	Gas	342.0	0.66%	297.5	0.69%	44.0	0.17%	14.77%
6	Nuclear	1934.0	3.73%	1740.6	4.05%	1415.5	5.59%	81.32%
7	Hydro	3385.4	6.53%	1692.7	3.94%	1682.9	6.65%	99.42%
8	Wind Onshore	100.0	0.19%	38.0	0.09%	37.8	0.15%	99.42%
	Total	51852.7	100%	42988.0	100%	25308.0	100%	58.87%

with carbon capture and storage (CCS) technology. Similarly, the model-implied depreciation cost is  $C_0^{\text{dep}} = 81.382$  billion ZAR (BZAR), which is much larger than the market value. This is most likely due to lower cost bases in South Africa compared to the prototype costs listed in Table 1. To calibrate the model depreciation costs to market data, we set  $\gamma_{\text{df}} = 42.93\%$ .

**Future fuel mix till 2030.** South Africa’s Integrated Resource Plan (IRP) of 2019 (South African Department of Mineral Resources and Energy 2019) states the planned addition and decommissioning of power plants that will be implemented between 2023 and the end of 2030. In Table 4 we show the planned changes relevant to the present study, i.e., excluding storage projects and the unspecified listings in the category of “other”. Considering the large model-implied/calibrated emission factor  $\gamma_{\text{ef}} = 554.71\%$ , we assume that newly added coal, gas and diesel power plants will operate with CCS technology. Together with  $\gamma_{\text{ef}}$  this leads to CO<sub>2</sub>e-emissions of 55.4% of the emissions of these plants without CCS technology, which is a reasonable compromise for our modelling approach. In particular, it is not clear whether (and to what extent) Eskom invests in CCS technology in their new power plants. However, it is reasonable to assume that the new technology is more CO<sub>2</sub>e-emission efficient than the old coal plants that are still in use. Since Eskom is also aware of its environmental impact, the assumption that newly built plants have average CO<sub>2</sub>e-emissions between the current prototype plants with and without CCS technology seems justified. For gas-fired power plants with CCS technology, we assume that the power production factor  $\gamma_0^{j \text{PPf}}$  is the same as without CCS technology, since the limiting factor here is the cost of production. In contrast to this, the limiting factor for coal-fired plants is the old facilities’ poor conditions, which often need to be shut down involuntarily due to technical problems. Hence, it is plausible that newly built coal-fired plants will be operated with a high power production factor, since burning coal is comparably cheap for Eskom. The same reasoning holds true for nuclear

plants, therefore it seems reasonable to use the same factor for “clean coal”. To additionally take into account that decommissioned coal-fired plants will be those, which already now do not have a satisfactory output capacity (due to technical problems), we increase the factor for the newly built “clean coal” plants further to  $\gamma_0^{j\text{PPF}} = 90\%$ <sup>i</sup>. South Africa’s wind farms are onshore, so we shall assume that added wind farms will also be onshore. This is further justified considering that South Africa has enough “unused” land (compared for example to countries in Europe) such that they are able to build the more cost-efficient onshore wind farms. Moreover, the IRP 2019 includes using gas, instead of diesel, to power electricity generation given that emission costs are higher when burning diesel. Thus, we assume that any newly introduced “gas & diesel” power plant will be gas-fired. At the same time, we keep the currently diesel-fired plants in this category and do not assume that these are switched to gas-fired plants. In absence of more precise information, the latter two assumptions together seem to be a reasonable compromise between the usage of (the more efficient) gas- and (the currently more used) diesel-fired plants for our modelling approach. Hence, we define the future maximal capacity  $\kappa_m^{j\text{max}}$  starting from the current maximal capacities reported in Table 3 and following the changes listed in Table 4. In particular, we make the conservative assumption that capacity changes listed for year  $m$  are implemented as of the start of the following fiscal year  $m + 1$  (e.g., the changes listed for 2023 are in place from  $t_2 = 1$  April 2024). This defines  $\kappa_m^{j\text{max}}$  for  $1 \leq m \leq 10$ .

**Fuel mix scenarios after 2030.** We consider the following three fuel mix scenarios after 2030, all based on the IRP 2019 (South African Department of Mineral Resources and Energy 2019).

- **Base case** In our base case scenario for the future capacities, we assume that the capacities do not change between 2031 and 2050. This is the scenario used to calibrate our model to market data, i.e., the scenario we believe the market currently prices.
- **Green continuation** The second scenario is a green continuation of the IRP 2019, where we assume that for coal, solar and wind the average change of the last three years (2028 to 2030) in Table 4 is continued. Each year, from 2031 to 2050, coal is reduced by 1073 MW, solar is increased by 1000 MW and on-

<sup>i</sup>A different possibility to account for this assumption would be to increase the power production factor for coal whenever old coal-fired plants are decommissioned. Since determining the amount of those increases would need additional assumptions, we simply use a higher factor for “clean coal”, instead. For orientation, assuming that all decommissioned coal plants already have zero output now and that the overall output of coal-fired plants (not including newly added “clean coal”) stays constant over time, the power production factor for coal would increase to  $\gamma_0^{j\text{PPF}} = 67.37\%$  by 2031. However, this is a strong assumption, and given the development in past years, one is lead to assume that additional coal-fired plants will start to have technical issues over this time period. It thus seems most reasonable to keep the power production factor for coal-fired plants constant over time, while using a higher factor for newly built “clean coal” facilities, which should not be constrained by technical problems in the near future.

Table 4. Decommissioning (Dec.) of existing and addition (Add.) of new capacities between 2023 and 2030 (South African Department of Mineral Resources and Energy 2019, Table 5) stated in megawatt (MW).

Year	Coal			Nuclear	Hydro	Solar	Wind	Gas & Diesel
	Add.	Dec.	Change	Add.	Add.	Add.	Add.	Add.
2023	750	-555	195	0	0	1000	1600	0
2024	0	0	0	1860	0	0	1600	1000
2025	0	0	0	0	0	1000	1600	0
2026	0	-1219	-1219	0	0	0	1600	0
2027	750	-847	-97	0	0	0	1600	2000
2028	0	-475	-475	0	0	1000	1600	0
2029	0	-1694	-1694	0	0	1000	1600	0
2030	0	-1050	-1050	0	2500	1000	1600	0

shore wind is increased by 1600 MW. Moreover, once every ten years, nuclear and hydro power generation is increased by 1860 MW and 2500 MW, respectively. For nuclear power this happens in the years 2034 and 2044, while for hydro-generated electricity this happens in 2037 and 2047. The gas and diesel capacities are not increased further, hence the terminology “green continuation”.

- Green continuation & CCS technology** In the third scenario, we assume a switch to carbon capture and storage (CCS) technology for fossil-fuelled power plants starting from  $t_1 = 1$  April 2023<sup>j</sup>, in addition to the adaptations in the green continuation scenario. In particular, we assume that diesel and gas power plants are all replaced with or upgraded to gas (CCS90) plants in equal fractions over four years starting from  $t_1$ . Additionally, gas plants which are newly built according to the IRP 2019 4 are assumed to deploy CCS90 technology. For the coal plants we assume that every year, from 2023 until 2029, 5500 MW of coal plants are upgraded to coal CCS90 power plants and 500 MW worth of coal-fired electricity generation is decommissioned. The latter adaptation measure applies to power plants that are too old to be upgraded. We note here that the costs for these upgrades are priced in through the different depreciation costs of the power plants with CCS technology.

The realizable capacities  $\kappa_m^{j\text{rlz}}$  and the energy mixes in the three scenarios are plotted in Figure 3. In all scenarios, the total realizable capacity grows approximately by the same amount. Along with these management scenarios, we assume that the power production factors  $\gamma^{j\text{PPF}}$ , the emission and depreciation factors  $\gamma_{\text{ef}}$  and  $\gamma_{\text{df}}$

<sup>j</sup>Eskom is considering using CCS technology (Wendell Roelf 2021), therefore this scenario is not too unrealistic, even though the quick change we consider here might be implausible.

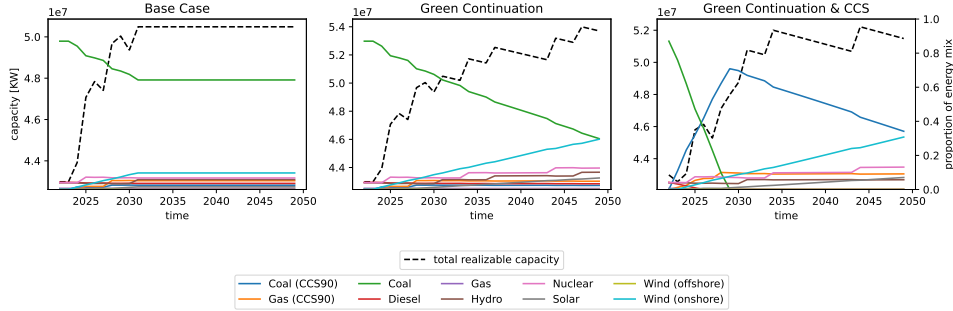


Fig. 3. Realizable capacity  $\kappa_m^{rlz}$  and proportional energy mix  $\kappa_m^j / \kappa_m^{rlz}$  for different fuel types according to the scenarios based on South Africa’s IRP 2019.

are constant throughout the years. In the third scenario, we assume that the CCS power plants are run with the same power production factors  $\gamma^{j\text{PPF}}$  as their more polluting alternatives. Further, we assume that Eskom will be able to sell all the produced electricity, also in the case that their capacity increases. This assumption is justified by the fact that South Africa has been forced to apply “loadshedding” (planned power cuts) since 2007, because Eskom has not been able to produce enough electricity to cover demand. The deficit between supply and demand is so significant and the necessary financial commitment for sufficient upgrade so high that it is reasonable to expect that for the foreseeable future any level of plausible increase in electricity supply will be met by demand in South Africa.

### 3.2. Minimal Electricity Prices to Cover the Costs

Given our asset and liability firm model for a power utility along with the market data defined in Section 3.1, we can now calculate the minimal electricity price  $\pi_m^{\text{min elect}}$  that is necessary to cover all costs in every period  $(t_{m-1}, t_m]$ , by solving  $A_m - A_{m-1} = L_m - L_{m-1}$  for  $\pi_m^{\text{min elect}}$ . In Figure 4, the minimal electricity prices are plotted for the different scenarios of the emission price  $\pi^{\text{CO}_2\text{e}}$  and for the three fuel mix scenarios we introduced above. In the “green continuation” scenario,  $\pi_m^{\text{min elect}}$  increases much more moderately than in the base case scenario. Moreover, in the scenario “green continuation with CCS technology”, the (inflation-adjusted) price even decreases until 2030 to then keep relatively constant.

### 3.3. Electricity Price Model

South Africa’s electricity price is set by the National Energy Regulator of South Africa (NERSA) once per year. In particular, Eskom applies for a certain percentage price increase (so to stay viable) for the following year and NERSA decides whether

<sup>k</sup>Here, we use the standard CO<sub>2</sub>e price scenarios without environmental levy during the first four years.

22 *F. Krach, A. Macrina, A. Kanter, E. Hampwaye, S. Hlalukana & N. T. Rateele*

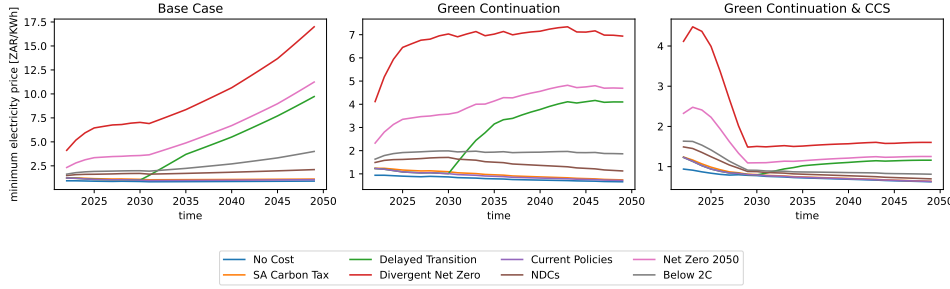


Fig. 4. Inflation adjusted minimal electricity prices  $\pi_n^{\min \text{ elect}}$  to cover all costs under the different emission price scenarios in our base case (left), the green continuation case (middle) and the green continuation & CCS technology case (right).

Table 5. Eskom’s electricity selling prices from 2012 to 2023 with relative changes.

Year	Tariff (ZAR/KWh)	Relative change
2012	0.5027	-
2013	0.5849	16.35%
2014	0.6281	7.39%
2015	0.6763	7.67%
2016	0.7538	11.46%
2017	0.8177	8.48%
2018	0.8249	0.88%
2019	0.8512	3.18%
2020	0.9595	12.73%
2021	1.0562	10.08%
2022	1.2275	16.21%
2023	1.3455	9.61%

to grant this or only a fraction of it—this is partly also a political decision. Therefore, electricity prices are not determined by demand and supply, in the usual sense of a market. In Table 5, we report the historical average price<sup>1</sup> for which Eskom sold the generated electricity, as reported in (Eskom 2023b). The electricity price for 2023 was computed based on the percentage price increase of 9.61% reported in (National Energy Regulator of South Africa (NERSA) 2023a). These prices always apply for one year overlapping with Eskom’s fiscal year where e.g. the price stated for 2022 applies from 1 April 2021 to 31 March 2022, i.e.,  $\pi_0^{\text{elect}} = 1.2275$  ZAR/KWh. Under the real-world probability measure  $\mathbb{P}$ , the mean of the price adjustment is 9.46%

<sup>1</sup>Eskom provides different electricity tariffs for local authorities, residential, commercial, industrial, mining, agriculture, traction, and international costumers. We report the price for which Eskom sold their electricity on average, i.e., the total revenue divided by the total sales volume.

compared to which its standard deviation of 4.79% is very large. Moreover, we note that the set prices always increase. Due to the unpredictability of the level at which prices are set and the high variance of the yearly percentage price increases, we propose to use a simple stochastic process to model the future electricity prices. In particular, we model the electricity price dynamics by

$$\pi_m^{\text{elect}} = \pi_0^{\text{elect}} \cdot \alpha_m \cdot \prod_{i=1}^m (1 + X_i), \quad m > 0, \quad (3.2)$$

where  $\alpha_m > 0$  are scaling factors and  $X_m$  are i.i.d. random variables drawn from an exponential distribution, i.e.,  $X_m \sim \text{Exp}(\lambda^{-1})$ . The parameter  $\lambda$  represents the mean percentage price jumps of the electricity price (e.g.,  $\lambda = 0.2$  corresponds to a 20% average increase). The calibration of the mean percentage price jump parameter  $\lambda$  and the scaling factors  $(\alpha_m)_{m \geq 2}$  to market data is discussed in Section 3.4. The paths of this calibrated electricity price model are paths with distribution under the (market) pricing measure  $\mathbb{Q}$ , so not the real-world probability measure  $\mathbb{P}$ .

### 3.4. Calibration of the Power Utility Model to Market Data

We use the market data described in Section 3.1 for the corresponding model parameters. For the emission prices, we use the South African Carbon Tax scenario and for the energy mix we use the Base Case scenario, since we assume that these are the scenarios currently used for pricing by the market. (We note that the following calibration procedure could similarly be performed with any other policy choice.) For any fixed set of parameters  $\lambda$  and  $(\alpha_m)_{m \geq 2}$ , the model-implied probabilities of default can be approximated via Monte Carlo by (i) sampling i.i.d. paths from the stochastic electricity price model and (ii) approximating (2.3) by the corresponding empirical mean. The goal is to find the parameters  $\lambda$  and  $(\alpha_m)_{m \geq 2}$  such that the model-implied term structure of the probability of default with starting date  $t_0$ , i.e.,  $T \mapsto P^d(t_0, T)$  for  $t_0 \leq T \leq t_N$  (where we use the simplest formulation with  $G = 0$ ), best matches the corresponding term structure observed in the market. More formally, let  $P_{\text{market}}^d(t_0, t_m)_{0 \leq m \leq N}$  be the probabilities of default observed in the market<sup>m</sup> and let us write  $\widehat{P}_{\lambda, (\alpha_m)_{m=2}^N}^d \approx P^d$  for the model-implied probabilities computed by Monte Carlo for a given set of parameters. Then, we set out to solve the  $\ell_2$ -minimization problem

$$\min_{\substack{\lambda > 0 \\ \alpha_2 > 0, \dots, \alpha_N > 0}} \left\{ \sum_{k=0}^N \left( \widehat{P}_{\lambda, (\alpha_m)_{m=2}^N}^d(t_0, t_k) - P_{\text{market}}^d(t_0, t_k) \right)^2 \right\}. \quad (3.3)$$

We compute (an approximation to) the solution of this minimization problem numerically using the Nelder-Mead algorithm (Gao & Han 2012). For more details, see the appendix.

<sup>m</sup>We note that these values, observed in the market, correspond to the (market) pricing measure  $\mathbb{Q}$  and not the real-world measure  $\mathbb{P}$ .

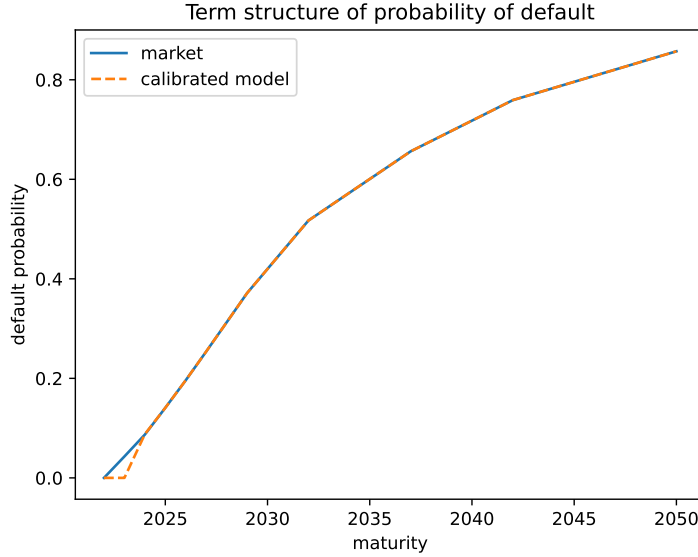


Fig. 5. Term structure of the market default probabilities,  $T \mapsto P_{\text{market}}^d(t_0, T)$ , and of the calibrated model,  $T \mapsto P_{\lambda, (\alpha_m)_{m=2}}^d(t_0, T)$ .

The term structure of the (rather simple) calibrated market model versus the market observations is shown in Figure 5. We see that our model fits the market data accurately, except at  $t_1$  (corresponding to the year 2023) where the model-implied default probability is 0. The reason for this is the large initial equity amount  $E_0$ , which is not (i.e., in none of the sampled paths) exhausted by the amount of liability before  $t_2$ . One could argue that the model-implied probabilities of default are reasonable until  $t_1$ , since Eskom’s (real world) probability of default under  $\mathbb{P}$  within the next year should be zero (implying that this is also the case under the pricing measure  $\mathbb{Q}$ , by equivalence of the real-world measure and the pricing measure,  $\mathbb{P} \sim \mathbb{Q}$ ). This argument becomes more plausible if one recalls that Eskom is for all practical purposes underwritten by the South African state, at least for the foreseeable future of a year.

On the other hand, since we use a simplistic toy model that focuses on Eskom’s main income and cost streams, our model does not capture all of Eskom’s cash flows (e.g., large debt and interest payments, transportation costs for coal and other fuels, costs of running the electricity grid, etc.). An effective and convenient way to alleviate this shortcoming is to decrease the initial equity amount. In particular, using only  $E_0/3$  for Eskom’s initial equity at  $t_0$ , we can account for the missing costs in our model. Re-running the same calibration as above with a third of the initial equity amount yields a nearly perfect fit of the model, see Figure 6. In the following we shall therefore use the adjusted model, with only 1/3 of Eskom’s initial equity. The mean percentage price jump parameter  $\lambda$  is 0.033, and the scaling



Table 6. Fitted values for  $\{\alpha_m\}_{m=1,\dots,28}$  corresponding to the years 2023 to 2050.

$m$	1	2	3	4	5	6	7	8	9	10
$\alpha_m$	0.50	0.89	0.89	0.88	1.07	1.05	1.07	1.06	1.04	1.01
$m$	11	12	13	14	15	16	17	18	19	20
$\alpha_m$	1.06	1.07	1.07	1.08	1.08	1.15	1.16	1.14	1.15	1.14
$m$	21	22	23	24	25	26	27	28		
$\alpha_m$	1.27	1.31	1.29	1.31	1.31	1.31	1.35	1.29		

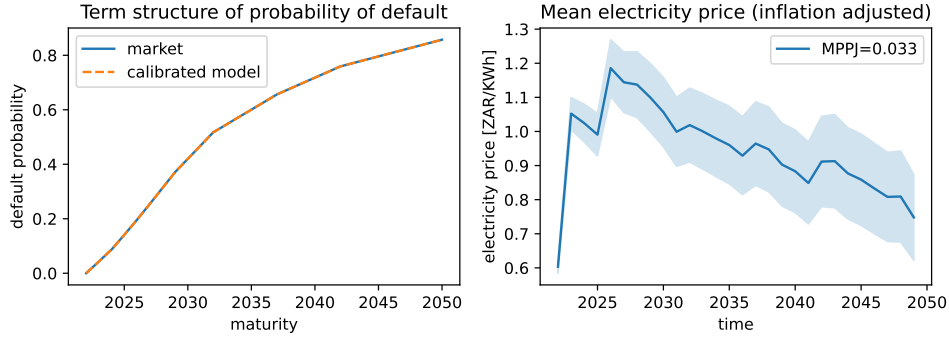


Fig. 6. Left: term structure of probabilities of default in the market,  $T \mapsto P_{\text{market}}^d(t_0, T)$ , and of the calibrated model,  $T \mapsto P_{\lambda, (\alpha_n)_{n=2}}^d(t_0, T)$ , when using  $1/3$  of the initial equity  $E_0$ . Right: electricity price evolution (mean  $\pm$  standard deviation) under the calibrated model.

parameters  $\{\alpha_m\}_{m=1,\dots,N}$  are reported in Table 6. In Figure 6, we include a plot for the mean and standard deviation of the (inflation adjusted) electricity price paths  $(\pi_m^{\text{elect}})_{m=1,\dots,N}$ , obtained from the calibrated model. The adjustment of  $E_0/3$  can be further understood by recalling that in our theoretical model, we define default as the event where  $E(t) = A(t) - L(t) \leq G(t)$ . So, rescaling  $E_0$  amounts to introducing a non-zero covenant level  $G(0) > 0$ .

### 3.5. Impact of NGFS Scenarios on the Calibrated Model

In Figure 7, we show the term structure of the probability of default (2.3),  $T \mapsto P^d(t_0, T)$ , in different CO<sub>2</sub>e price scenarios (starting from 2026 until such time when the environmental levy is in place), which are also included. We note that the instantaneous probabilities of default (2.6),  $T \mapsto P^d(T)$ , are nearly the same, therefore we do not show the corresponding plot, here. For comparison, we also plot the probabilities of default corresponding to the different levels of Standard & Poor’s credit ratings. Under the South African Carbon Tax, Eskom is mainly rated between BBB and BB. In all scenarios with higher CO<sub>2</sub>e costs than the South African Carbon Tax, i.e., all scenarios except “No-Cost” and “Current Policies”,

26 *F. Krach, A. Macrina, A. Kanter, E. Hampwaye, S. Hlulukana & N. T. Rateele*

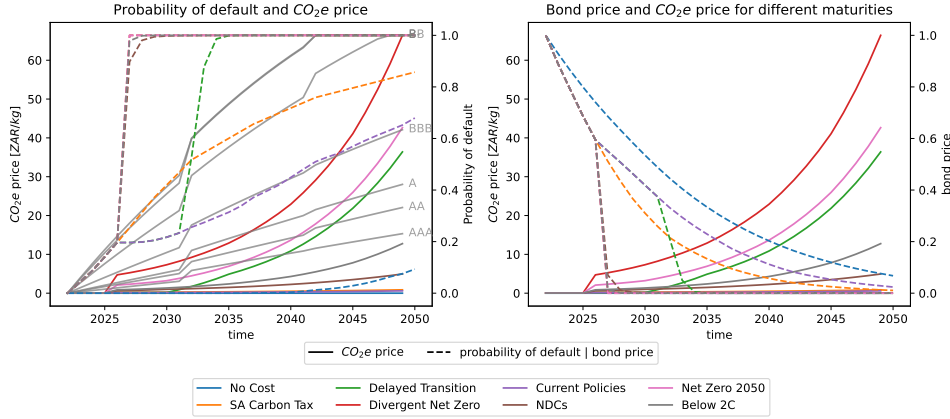


Fig. 7. Left: Term structure of the default probability  $T \mapsto P^D(t_0, T)$  under different CO<sub>2</sub>e price scenarios. Default probabilities corresponding to different S&P credit ratings are plotted in gray. Right: Term structure of the bond prices  $T \mapsto Z^d(t_0, T)$  under different CO<sub>2</sub>e price scenarios.

the probability of default increases rapidly. In the “Delayed Transitions” scenario the default probability reaches  $P^d = 1$  by 2035. This result indicates that Eskom’s operations become financially non-viable if the CO<sub>2</sub>e emission costs are accounted for in a scenario with an expedite and sustained CO<sub>2</sub>e-emissions reduction policy. We also plot the corresponding bond price term structure (2.4),  $T \mapsto Z^d(t_0, T)$ , assuming zero recovery ( $R = 0$ ), in Figure 7.

### 3.6. Impact of Different Energy Mix Scenarios

The impact of the fuel mix scenarios (i) “green continuation” and (ii) “green continuation with CCS”, which we have produced following the IRP 2019, is shown in Figure 8. There, we present the impact on (i) the term structures of the default probability, (ii) the instantaneous default probability, and (iii) the impact on the term structure of bond prices. In the green continuation scenario the probability of default under the SA Carbon Tax only grows to 0.6 compared to its growth to slightly below 0.9 in the base case. However, in all more expensive CO<sub>2</sub>e scenarios, we do not see a difference to the base case. The instantaneous probability of default is similar to the probability of default in all CO<sub>2</sub>e price scenarios. In the “green continuation with CCS technology” scenario the situation looks different. In particular, only in the “divergent net zero” and the “net zero 2050” scenario, the probability of default increases (nearly) to one, immediately after replacing the environmental levy. In the “Delayed Transition” scenario it increases to about 0.6 until 2050, while it only increases to 0.35 in the “Below 2C” scenario. In all other, cheaper, CO<sub>2</sub>e price scenarios, it stays below 0.1 and is therefore classified as AAA in the long run. The instantaneous probability of default in the “Net Zero 2050” and “Below 2C” scenarios has a (steep) initial increase from 2026 to 2028, then decreases again as soon as the switch to the CCS technology becomes substantial

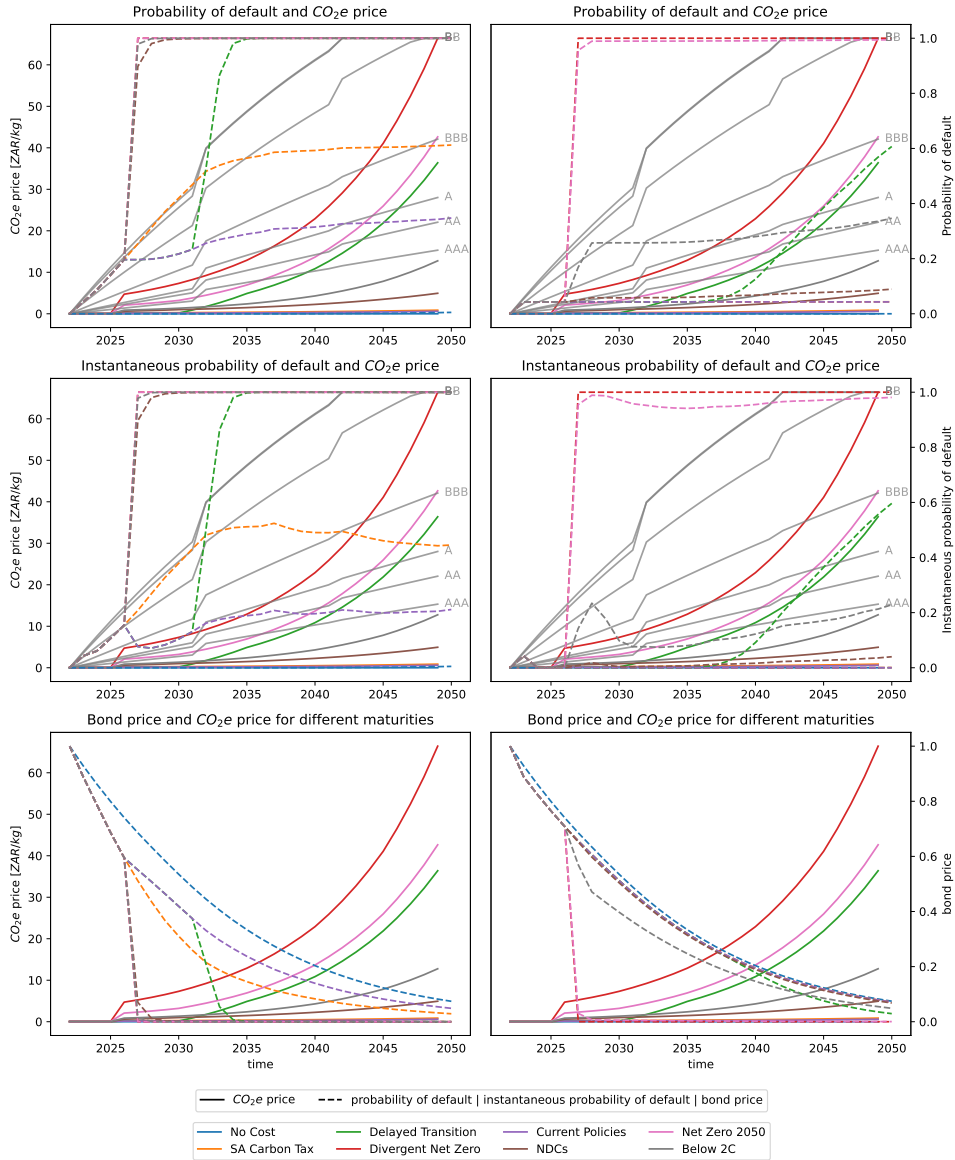


Fig. 8. Left: green continuation scenario. Right: green continuation & CCS technology scenario. Top: term structure of the probability of default  $T \mapsto P^d(t_0, T)$ . Middle: term structure of the instantaneous probability of default  $T \mapsto P^d(T)$ . Bottom: term structure of the bond prices  $T \mapsto Z^d(t_0, T)$ . All plotted for the different CO<sub>2</sub>e price scenarios.

enough, though thereafter it increases once more.

28 *F. Krach, A. Macrina, A. Kanter, E. Hampwaye, S. Hlalukana & N. T. Rateele*

#### 4. Conclusions

In this work we introduced a simple, yet effective, modelling framework for electricity-generating companies that captures their main income and cost streams. We apply this approach to the case study of South Africa’s power utility Eskom. In particular, we first calibrate the model to the current market data and then use it to study the impact of (i) different emission cost scenarios, see (Network for Greening the Financial System (NGFS) 2022), and (ii) different fuel mix scenarios. We observe, for example, that under the “Delayed Transition” scenario for emission costs, Eskom’s operations become non-viable by 2035 if one assumes the fuel mix base case. However, this can be prevented by a determined transition to less carbon-intensive energy sources as suggested in the “green continuation and CCS technology” scenarios. The obtained results suggest that large investments in greener electricity production would pay off relatively quickly, by keeping Eskom’s business viable in an emission cost scenario that corresponds to an expedite and sustained CO<sub>2</sub>e-emission reduction policy. Moreover, in the long run, these investments could in addition lead to lower electricity prices for end users, as suggested by our calculated minimal electricity price to cover all costs. Our model implementation is available at (Krach *et al.* 2023), so that our approach can in principle be used to model other setups for electricity-generating firms and to study the effects of customised scenarios (for emission costs, fuel mixes etc.) on the firm’s viability.

#### Appendix A. Details on model calibration

**Joint calibration.** We use the Nelder-Mead optimization algorithm (Gao & Han 2012) implemented in Python’s SciPy package (Virtanen *et al.* 2020) to find a numerical approximation of the solution to the calibration problem (3.3). The quality of the solution of this simplex algorithm depends on the initial guess for the parameters  $\lambda$  and  $(\alpha_n)_n$ . Therefore, we use an iterative, half-automated calibration scheme, that can be summarized as follows.

- (1) Make an initial guess for  $\lambda$  and  $(\alpha_n)_n$ . Our initial guess is  $\lambda = 0.094$  (the mean percentage jump from the historical electricity price data in Table 5) and  $\alpha_n = 1$ .
- (2) Run the Nelder-Mead algorithm with the initial guess.
- (3) If the calibrated model fit is not good enough, come up with a (handcrafted) new initial guess based on the fitted parameters from Step 2. Use this as new input for Step 2 and iterate.

As a general “engineering rule”, the scaling factor  $\alpha_m$  needs to be increased whenever the probability of default of the calibrated model  $P_{\lambda, (\alpha_m)_{m=2}}^d(t_0, t_m)$  is larger than the respective market value  $P_{\text{market}}^d(t_0, t_m)$  and vice versa. Indeed, increasing  $\alpha_m$  makes the electricity price larger at this time step and therefore leads to larger equity, which means a smaller default probability. Moreover, we note that restarting

the Nelder-Mead algorithm with its last output as a new initial guess can further improve the model fit (due to the stopping conditions of the algorithm).

The calibrated models reported in Figure 5 and Figure 6 were achieved after two and three iterations of the above calibration scheme (i.e. after using Nelder-Mead twice or thrice), respectively. The exact initial values used are available in the data configuration of our implementation (Krach *et al.* 2023).

**Sequential calibration of  $(\alpha_n)_n$ .** Once a good enough joint fit of the parameters  $\lambda$  and  $(\alpha_n)_n$  is achieved, or after choosing a reasonable candidate for  $\lambda$ , the scaling factors  $(\alpha_n)_n$  can (additionally) be optimized sequentially to minimize the objective function. In particular, since  $\alpha_n$  only influences  $\hat{P}_{\lambda, (\alpha_m)_{m=2}^N}^d(t_0, t_k)$  for  $k \geq n$ , looping over  $n \in \{1, \dots, N\}$  to optimize

$$\alpha_n^* = \arg \min_{\alpha_n > 0} \left\{ \left( \hat{P}_{\lambda, (\alpha_2^*, \dots, \alpha_{n-1}^*, \alpha_n, \dots, \alpha_N)}^d(t_0, t_n) - P_{\text{market}}^d(t_0, t_n) \right)^2 \right\}, \quad (\text{A.1})$$

yields improved scaling factors  $(\alpha_m^*)_{m=2}^N$ , which should lead to the best possible fit for  $n \geq 1$  based on the fixed parameter  $\lambda$ .

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30 *F. Krach, A. Macrina, A. Kanter, E. Hampwaye, S. Hlalukana & N. T. Rateele*

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32 *F. Krach, A. Macrina, A. Kanter, E. Hampwaye, S. Hlalukana & N. T. Rateele*

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