Asymmetric one-sided semi-device-independent steerability of quantum discordant states

Chellasamy Jebarathinam [1,*] Debarshi Das [2,†] and R. Srikanth [3]

[1] *Department of Physics and Center for Quantum Information Science, National Cheng Kung University, Tainan 701, Taiwan*
[2] *Department of Physics and Astronomy, University College London, Gower Street, WC1E 6BT London, England, United Kingdom*
[3] *Theoretical Sciences Division, Poornaprajna Institute of Scientific Research (PPISR), Bidalur post, Devanahalli, Bengaluru 562164, India*

(Received 21 July 2023; revised 21 September 2023; accepted 2 October 2023; published 17 October 2023)

Superlocality and superunsteerability provide operational characterization of quantum correlations in certain local and unsteerable states, respectively. Such quantum correlated states have a nonzero quantum discord. A two-way nonzero quantum discord is necessary for quantum correlations pointed out by superlocality. On the other hand, in this work, we demonstrate that a two-way nonzero quantum discord is not necessary to demonstrate superunsteerability. To this end, we demonstrate superunsteerability for one-way quantum discordant states. This in turn implies the existence of one-way superunsteerability and also the presence of superunsteerability without superlocality. Superunsteerability for nonzero quantum discord states implies the occurrence of steerability in a one-sided semi-device-independent way. Just like one-way steerability occurs for certain Bell-local states in a one-sided device-independent way, our result shows that one-way steerability can also occur for certain nonsuperlocal states but in a one-sided semi-device-independent way.

DOI: 10.1103/PhysRevA.108.042211

I. INTRODUCTION

Local quantum measurements on entangled states can be used to demonstrate quantum nonlocality, originating from an experimental situation proposed by Einstein, Podolsky and Rosen [1] and the Bohm-Aharonov version of it [2]. Bell proposed a framework to distinguish quantum nonlocality from a local realistic description of the measurement results by introducing an inequality, which is satisfied by any local hidden variable model for the observed correlations between spacelike separated observers [3]. Such an inequality is violated by certain quantum correlations and the phenomenon is referred as Bell nonlocality [4]. There exists another form of quantum nonlocality as pointed out by Schrödinger [5]. This form of quantum nonlocality is called quantum steering and its framework, analogous to the Bell's framework, was proposed by Wiseman, Jones, and Doherty (WJD) [6]. Apart from being fundamental aspects of quantum theory, both of the forms of quantum nonlocality find applications in quantum technologies (see Sec. IV in [4] and Sec. V in [7] for applications of Bell nonlocality and quantum steering, respectively). In contrast to Bell nonlocality of quantum correlations, quantum steering is an asymmetric form of quantum correlations, both from fundamental and application points of view. Quantum steering can exist with the one-way property, that is, certain entangled states have steerability from only one side [8] (also see Sec. III. D in [7]) and quantum steering can only provide one-sided device-independent applications [9] (also see Sec. V in [7]).

*jebarathinam@gmail.com
†dasdebarshi90@gmail.com

The quantification of quantum resources through appropriate quantifiers is an important aspect of quantum information science [10]. The quantification of quantum correlations beyond entanglement, called quantum discord, was proposed in [11,12]. This kind of quantum correlation has also emerged as a quantum resource for applications in quantum information science [13,14] (also see Sec. VI in [15]). From a quantum foundational perspective [16], quantum discord was proposed as Bohr's notion of nonmechanical disturbance [17]. Certain distinguishing features of quantum discord to quantum entanglement have been characterized, such as no death for discord, [18] and quantum discord may increase under certain decoherence conditions [19].

The simulation of certain local and unsteerable states using finite shared randomness has been shown to motivate the amount of shared randomness as a resource [20,21]. Superlocality [22] and superunsteerability [23] have been recently formalized to demonstrate a quantum advantage in simulating certain local and unsteerable correlations, respectively, in terms of a local Hilbert space dimension over the minimal amount of shared randomness required to simulate them. Such quantum advantage has been invoked to provide operational characterization of quantum correlations in certain local and unsteerable states having a nonzero quantum discord [24–26]. Superlocality or superunsteerability has also been found to be useful for certifying quantum discord in a measurement device-independent way [27] and also as a resource for measurement device-independent quantum key distribution protocols [28], quantum random access codes [29], and quantum random number generation [27].

Studying the precise relationships among quantum discord, i.e., superunsteerability and superlocality, could provide a better understanding of quantum correlations as well as their role as a resource in quantum information processing.

Superlocality is inequivalent to quantum discord [24]. This raises the question of whether superunsteerability is inequivalent to superlocality or quantum discord. Also, as superunsteerability is an asymmetric concept, a natural question that arises is whether superunsteerability can occur for nonzero quantum discord states with the one-way property, analogous to one-way quantum steering in the case of certain entangled states. In this work, we answer this question in the affirmative by demonstrating that quantum correlations in certain one-way quantum discordant states can be operationally captured by superunsteerability. For such states, superunsteerability cannot occur both ways because the state has zero quantum discord on one side. Thus, in this work, we demonstrate the existence of one-way superunsteerability. This in turn implies that superunsteerability is inequivalent to superlocality.

II. WJD'S FORM OF QUANTUM STEERING

Let us consider a steering scenario where two spatially separated parties, say Alice and Bob, share an unknown quantum system $\rho_{AB} \in \mathcal{B}(\mathcal{H}_A \otimes \mathcal{H}_B)$. Here, $\mathcal{B}(\mathcal{H}_A \otimes \mathcal{H}_B)$ stands for the set of all bounded linear operators acting on the Hilbert space $\mathcal{H}_A \otimes \mathcal{H}_B$. Alice performs a set of black-box measurements and the Hilbert-space dimension of Bob's subsystem is known. Such a scenario is called one-sided device independent since Alice's measurement operators $\{M_{a|x}\}_{a,x}$, which are positive operator-valued measures (POVM), are unknown. The steering scenario is completely characterized by the set of unnormalized conditional states on Bob's side, $\{\sigma_{a|x}\}_{a,x}$, which is called an unnormalized assemblage. Each element in the unnormalized assemblage is given by $\sigma_{a|x} = p(a|x)\rho_{a|x}$, where $p(a|x)$ is the conditional probability of getting the outcome a when Alice performs the measurement x; $\rho_{a|x}$ is the normalized conditional state on Bob's side. Quantum theory predicts that all valid assemblages should satisfy the following criteria:

$$\sigma_{a|x} = \mathrm{Tr}_A(M_{a|x} \otimes \mathbb{1}\rho_{AB}) \qquad \forall \sigma_{a|x} \in \{\sigma_{a|x}\}_{a,x}. \tag{1}$$

Definition 1. In the above scenario, Alice demonstrates Wiseman, Jones, and Doherty's (WJD) form of steerability to Bob [6] if the assemblage does not have a local hidden state (LHS) model, i.e., if for all a, x, there is no decomposition of $\sigma_{a|x}$ in the form

$$\sigma_{a|x} = \sum_{\lambda} p(\lambda)p(a|x,\lambda)\rho_{\lambda}, \tag{2}$$

where λ denotes the classical random variable which occurs with probability $p(\lambda)$. ρ_{λ} are called local hidden states which satisfy $\rho_{\lambda} \geqslant 0$ and $\mathrm{Tr}\rho_{\lambda} = 1$.

We can further define the detection of the above phenomenon from the no-signaling (NS) boxes (see Sec. I in the Supplemental Material [30]) as follows:

Definition 2. Suppose Bob performs a set of projective measurements $\{\Pi_{b|y}\}_{b,y}$ on $\{\sigma_{a|x}\}_{a,x}$. Then the scenario is characterized by the set of measurement correlations, or box between Alice and Bob, $P(ab|xy){:=}\{p(ab|xy)\}_{a,x,b,y}$, where $p(ab|xy) = \mathrm{Tr}(\Pi_{b|y}\sigma_{a|x})$. The box $P(ab|xy)$ detects WJD's steerability from Alice to Bob, iff it does not have a decomposition, as follows [6,31]:

$$p(ab|xy) = \sum_{\lambda} p(\lambda)p(a|x,\lambda)p(b|y,\rho_{\lambda}) \quad \forall a,x,b,y, \tag{3}$$

where $\sum_{\lambda} p(\lambda) = 1$, $p(a|x,\lambda)$ denotes an arbitrary probability distribution arising from the local hidden variable (LHV) λ [λ occurs with probability $p(\lambda)$], and $p(b|y,\rho_{\lambda})$ denotes the quantum probability of outcome b when measurement y is performed on local hidden state (LHS) ρ_{λ}.

III. QUANTUM DISCORD

In the following, we present the definition of quantum discord [11,12] from Alice to Bob, $D^{\rightarrow}(\rho_{AB})$. Quantum discord is defined as

$$D^{\rightarrow}(\rho_{AB}) = I(\rho_{AB}) - C^{\rightarrow}(\rho_{AB}) = S(\rho_{B|A}) - \tilde{S}(\rho_{B|A}). \tag{4}$$

Here, $I(\rho_{AB}) = S(\rho_B) - \tilde{S}(\rho_{B|A})$ is the quantum mutual information and can be interpreted as the total correlations in ρ_{AB}. $S(\sigma) = -\mathrm{tr}(\sigma \log_2 \sigma)$ is the von Neumann entropy of a density matrix σ. $\tilde{S}(\rho_{B|A}) = S(\rho_{AB}) - S(\rho_A)$ is the "unmeasured" quantum conditional entropy [32] (see, also, [33–35]). On the other hand, $C^{\rightarrow}(\rho_{AB}) = S(\rho_B) - S(\rho_{B|A})$ can be interpreted as the classical correlations in ρ_{AB}, where the quantum conditional entropy is defined as $S(\rho_{B|A}) = \min_{\{M_i^A\}} \sum_i p_i S(\rho_{B|i})$, with the minimization being over all POVMs, $\{M_i^A\}$, performed on subsystem A. Here, $p_i = \mathrm{tr}_{AB}(M_i^A \otimes \mathbb{I}_B \rho_{AB} M_i^A \otimes \mathbb{I}_B)$ is the probability of obtaining the outcome i, and the corresponding postmeasurement state for the subsystem B is $\rho_{B|i} = \frac{1}{p_i}\mathrm{tr}_A(M_i^A \otimes \mathbb{I}_B \rho M_i^A \otimes \mathbb{I}_B)$.

Quantum discord $D^{\rightarrow}(\rho_{AB})$ as defined above captures quantum correlation in the state from Alice to Bob. Similarly, quantum discord from Bob to Alice, $D^{\leftarrow}(\rho_{AB})$, can be defined. $D^{\rightarrow}(\rho_{AB})$ vanishes for a given ρ_{AB} if and only if it is a classical-quantum state of the form

$$\rho_{\mathtt{CQ}} = \sum_i |i\rangle\langle i|_A \otimes \rho_B^{(i)}, \tag{5}$$

where $\{|i\rangle\}$ forms an orthonormal basis on Alice's Hilbert space and $\rho_B^{(i)}$ are any quantum states on Bob's Hilbert space. On the other hand, $D^{\leftarrow}(\rho_{AB})$ vanishes for a given ρ_{AB} if and only if it is a quantum-classical state of the form

$$\rho_{\mathtt{QC}} = \sum_i \rho_A^{(i)} \otimes |i\rangle\langle i|_B, \tag{6}$$

where now $\{|i\rangle\}$ forms an orthonormal basis on Bob's Hilbert space and $\rho_A^{(i)}$ are any quantum states on Alice's Hilbert space.

IV. SUPERUNSTEERABILITY

We are going to present the formal definition of the notion *superunsteerability* for boxes having a LHV-LHS model [23]. Before that, we present the definition of *superlocality* for local correlations [22,24]. Consider a Bell scenario, where both parties perform black-box measurements. In this scenario, superlocality is defined as follows:

Definition 3. Suppose we have a quantum state in $\mathbb{C}^{d_A} \otimes \mathbb{C}^{d_B}$ and measurements which produce a local bipartite box $P(ab|xy) := \{p(ab|xy)\}_{a,x,b,y}$. Then, superlocality holds iff there is no decomposition of the box in the form

$$p(ab|xy) = \sum_{\lambda=0}^{d_\lambda-1} p(\lambda)p(a|x,\lambda)p(b|y,\lambda) \quad \forall a,x,b,y, \tag{7}$$

with dimension of the shared randomness or hidden variable $d_\lambda \leqslant \min(d_A, d_B)$. Here, $\sum_\lambda p(\lambda) = 1$, $p(a|x,\lambda)$, and $p(b|y,\lambda)$ denote arbitrary probability distributions arising from LHV λ [λ occurs with probability $p(\lambda)$].

In Ref. [24], an example of superlocality has been demonstrated with the noisy Clauser-Horne-Shimony-Holt (CHSH) local box given by

$$P(ab|xy) = \frac{2 + (-1)^{a\oplus b\oplus xy}\sqrt{2}V}{8}, \tag{8}$$

with $0 < V \leqslant 1/\sqrt{2}$. Such local correlations can be produced by a two-qubit pure entangled state or a two-qubit Werner state having entanglement or a nonzero quantum discord for appropriate local noncommuting measurements. On the other hand, it cannot be reproduced by a LHV model with $d_\lambda = 2$, as shown in Ref. [24].

Now, consider a different scenario where one of the parties (say, Alice) performs black-box measurements and another party (say, Bob) performs quantum measurements. In this steering scenario, the notion of *superunsteerability* has been defined as follows:

Definition 4. Suppose we have a quantum state in $\mathbb{C}^{d_A} \otimes \mathbb{C}^{d_B}$ and measurements which produce a unsteerable bipartite box $P(ab|xy) := \{p(ab|xy)\}_{a,x,b,y}$. Then, superunsteerability holds iff there is no decomposition of the box in the form

$$p(ab|xy) = \sum_{\lambda=0}^{d_\lambda-1} p(\lambda)p(a|x,\lambda)p(b|y,\rho_\lambda) \quad \forall a,x,b,y, \tag{9}$$

with dimension of the shared randomness or hidden variable $d_\lambda \leqslant d_A$. Here, $\sum_\lambda p(\lambda) = 1$, $p(a|x,\lambda)$ denotes an arbitrary probability distribution arising from LHV λ [λ occurs with probability $p(\lambda)$] and $p(b|y,\rho_\lambda)$ denotes the quantum probability of outcome b when measurement y is performed on LHS ρ_λ in $\mathbb{C}^{d_B}$.

In Ref. [23], two examples of superunsteerability have been demonstrated. In one of these examples, the unsteerable white noise BB84 family, given by

$$P(ab|xy) = \frac{1 + (-1)^{a\oplus b\oplus x.y}\delta_{x,y}V}{4}, \tag{10}$$

with $0 < V \leqslant 1/\sqrt{2}$, can be produced by a two-qubit Werner state having entanglement or a nonzero quantum discord for appropriate local noncommuting measurements. On the other hand, it cannot be simulated by a LHV-LHS model with $d_\lambda = 2$, as shown in Ref. [23]. These two examples demonstrate superunsteerability in both ways, as they are symmetrical with respect to interchanging Alice and Bob. In the following, we demonstrate an example of superunsteerability asymmetrically.

V. ONE-WAY SUPERUNSTEERABILITY

Consider that the two spatially separated parties (say, Alice and Bob) share the following separable two-qubit state:

$$\rho = \tfrac{1}{2}(|00\rangle\langle 00| + |+1\rangle\langle +1|), \tag{11}$$

where $|0\rangle$ and $|1\rangle$ are the eigenstates of the operator σ_z corresponding to the eigenvalue $+1$ and -1, respectively; $|+\rangle$ is the eigenstate of the operator σ_x corresponding to the eigenvalue $+1$. The above state has quantum discord $D^{\rightarrow}(\rho_{AB}) > 0$ and $D^{\leftarrow}(\rho_{AB}) = 0$ since it is not a classical-quantum state, but a quantum-classical state [11,12]. If Alice performs the projective measurements of observables corresponding to the operators $A_0 = \sigma_x$ and $A_1 = \sigma_z$, and Bob performs projective measurements of observables corresponding to the operators $B_0 = \sigma_x$ and $B_1 = \sigma_z$, then the following correlation is produced from the above quantum-classical state:

$P(ab|xy) =$

(x,y) \ (a,b)	(0, 0)	(0, 1)	(1, 0)	(1, 1)
(0, 0)	$\frac{1}{2}$	$\frac{1}{4}$	0	$\frac{1}{4}$
(0, 1)	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$	$\frac{1}{8}$
(1, 0)	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	0
(1, 1)	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$	$\frac{1}{8}$

(12)

Here, x, y denote the input variables on Alice's and Bob's sides, respectively, and a, b denote the outputs on Alice's and Bob's sides, respectively. In the following, we demonstrate that the box (12) detects superunsteerability of the quantum-classical state (11).

Let us now proceed to analyze simulating the correlation given by Eq. (12) with LHV at one side and LHS at another side. Before proceeding, let us define the following (for details, see the Supplemental Material [30]), which will be used throughout the article:

$$P_D^{\alpha\beta}(a|x) = \begin{cases} 1, & a = \alpha x \oplus \beta \\ 0 & \text{otherwise} \end{cases},$$

$$P_D^{\gamma\epsilon}(b|y) = \begin{cases} 1, & b = \gamma x \oplus \epsilon \\ 0 & \text{otherwise.} \end{cases}$$

The correlation given by Eq. (12) has the following LHV-LHS model:

$$P(ab|xy) = \sum_{\lambda=0}^{2} p(\lambda)P(a|x,\lambda)P(b|y,\rho_\lambda), \tag{13}$$

where $p(0) = \frac{1}{2}$, $p(1) = p(2) = \frac{1}{4}$; $P(a|x,0) = P_D^{00}(a|x)$, $P(a|x,1) = P_D^{10}(a|x)$, $P(a|x,2) = P_D^{11}(a|x)$, and

$P(b|y,\rho_0) =$

(y) \ (b)	(0)	(1)
(0)	$\frac{1}{2}$	$\frac{1}{2}$
(1)	$\frac{1}{2}$	$\frac{1}{2}$

$= \langle\psi_0'|\{\Pi_{b|y}\}_{b,y}|\psi_0'\rangle,$

$P(b|y,\rho_1) =$

(y) \ (b)	(0)	(1)
(0)	1	0
(1)	$\frac{1}{2}$	$\frac{1}{2}$

$= \langle\psi_1'|\{\Pi_{b|y}\}_{b,y}|\psi_1'\rangle,$

$P(b|y,\rho_2) =$

(y) \ (b)	(0)	(1)
(0)	0	1
(1)	$\frac{1}{2}$	$\frac{1}{2}$

$= \langle\psi_2'|\{\Pi_{b|y}\}_{b,y}|\psi_2'\rangle,$

where $\{\Pi_{b|y}\}_{b,y}$ corresponds to two arbitrary projective mutually unbiased measurements in the Hilbert space C^2 corresponding to the operators $B_0 = |\uparrow_0\rangle\langle\uparrow_0| - |\downarrow_0\rangle\langle\downarrow_0|$ and $B_1 = |\uparrow_1\rangle\langle\uparrow_1| - |\downarrow_1\rangle\langle\downarrow_1|$; here, $\{|\uparrow_0\rangle, |\downarrow_0\rangle\}$ is an arbitrary orthonormal basis in the Hilbert space C^2, and the orthonormal basis $\{|\uparrow_1\rangle, |\downarrow_1\rangle\}$ in the Hilbert space C^2 is such that the aforementioned two measurements define two arbitrary projective mutually unbiased measurements in the Hilbert space C^2. The $|\psi_\lambda'\rangle$s that produce the $p(b|y,\rho_\lambda)$s given above are given by $|\psi_0'\rangle = \frac{1}{\sqrt{2}}|\uparrow_0\rangle + i\frac{1}{\sqrt{2}}|\downarrow_0\rangle$, $|\psi_1'\rangle = |\uparrow_0\rangle$, and $|\psi_2'\rangle = |\downarrow_0\rangle$, which are all valid states in the Hilbert space C^2.

Hence, the LHV-LHS decomposition of the correlation given by Eq. (12) can be realized with a hidden variable having dimension 3 (with two arbitrary projective mutually unbiased measurements at the trusted party). This is also the minimal hidden variable dimension needed to simulate the correlation, as we show in the following lemma.

Lemma 1. The LHV-LHS decomposition of the correlation given by Eq. (12) cannot be realized with a hidden variable having dimension 2.

The proof of this lemma is given in Sec. II in the Supplemental Material [30]. We now show the following result:

Theorem 1. The correlation given by Eq. (12) demonstrates superunsteerability from Alice to Bob while having no superunsteerability from Bob to Alice.

Proof. We have shown that the unsteerable correlation given by Eq. (12) can have a LHV-LHS model, with the minimum dimension of the hidden variable being 3. On the other hand, we have seen that the unsteerable correlation given by Eq. (12) can be simulated by using a $2 \otimes 2$ quantum system (11). This is an instance of superunsteerability since the minimum dimension of shared randomness needed to simulate the LHV-LHS model of the correlation (12) is greater than the local Hilbert-space dimension of the shared quantum system (reproducing the given unsteerable correlation) at the untrusted party's side (who steers the other party, in the present case Bob).

On the other hand, the box (12) does not have superunsteerability from Bob to Alice. This is because it arises from a unsteerable state having zero discord from Bob to Alice, whereas nonzero discord from Bob to Alice is necessary for producing the superunsteerable (from Bob to Alice) correlation [23]. It can also be checked by providing a LHS-LHV model with $d_\lambda = 2$ as follows. From the decomposition of the box (12) in terms of the local deterministic boxes, we obtain the following LHS-LHV model:

$$P(ab|xy) = \sum_{\lambda=0}^{1} p(\lambda)P(a|x,\rho_\lambda)P(b|y,\lambda), \qquad (14)$$

where $p(0) = \frac{1}{2}$, $p(1) = \frac{1}{2}$;

$P(a|x,\rho_0) =$

(x) \ (a)	(0)	(1)
(0)	1	0
(1)	$\frac{1}{2}$	$\frac{1}{2}$

$= \langle\psi_0'|\{\Pi_{a|x}\}_{a,x}|\psi_0'\rangle,$

$P(a|x,\rho_1) =$

(x) \ (a)	(0)	(1)
(0)	0	1
(1)	$\frac{1}{2}$	$\frac{1}{2}$

$= \langle\psi_1'|\{\Pi_{a|x}\}_{a,x}|\psi_1'\rangle,$

where $\{\Pi_{a|x}\}_{a,x}$ corresponds to two arbitrary projective mutually unbiased measurements in the Hilbert space C^2 corresponding to the operators $A_0 = |\uparrow_0\rangle\langle\uparrow_0| - |\downarrow_0\rangle\langle\downarrow_0|$ and $A_1 = |\uparrow_1\rangle\langle\uparrow_1| - |\downarrow_1\rangle\langle\downarrow_1|$; here, $\{|\uparrow_0\rangle, |\downarrow_0\rangle\}$ and $\{|\uparrow_1\rangle, |\downarrow_1\rangle\}$ define two arbitrary projective mutually unbiased measurements in the Hilbert space C^2. The $|\psi_\lambda'\rangle$s that produce the $p(b|y,\rho_\lambda)$s given above are given by $|\psi_0'\rangle = |\uparrow_0\rangle$ and $|\psi_1'\rangle = |\downarrow_0\rangle$, which are all valid states in the Hilbert space C^2; and $P(b|y,0) = [P_D^{00}(b|y) + P_D^{10}(b|y)]/2$ and $P(b|y,1) = [P_D^{00}(b|y) + P_D^{11}(b|y)]/2$. ■

Note that Eq. (14) can be seen as a LHV-LHV decomposition for the correlation with hidden variable dimension $d_\lambda = 2$. Hence, the box (12) is not superlocal. In a similar way, it can be easily checked that any correlation, which is not superunsteerable at least in one direction, is not superlocal as well. Though the one-way discordant state (11) does not have quantum correlation pointed out by superlocality, the above result implies that it still have quantum correlation pointed out by superunsteerability asymmetrically. As an implication of this result, in Fig. 1, we depict a new hierarchy of quantum correlations in nonzero quantum discord states.

VI. A WEAK FORM OF QUANTUM STEERING

Here we address the connection of superunsteerability to a weak form of steering and its distinction from WJD's form of steerability at the level of certification of steerability. To this end, we note that in the given steering scenario, WJD's form of quantum steering implies the presence of steerability in a one-sided device-independent way, i.e., without making any assumption about the device used by the steering side (i.e.,

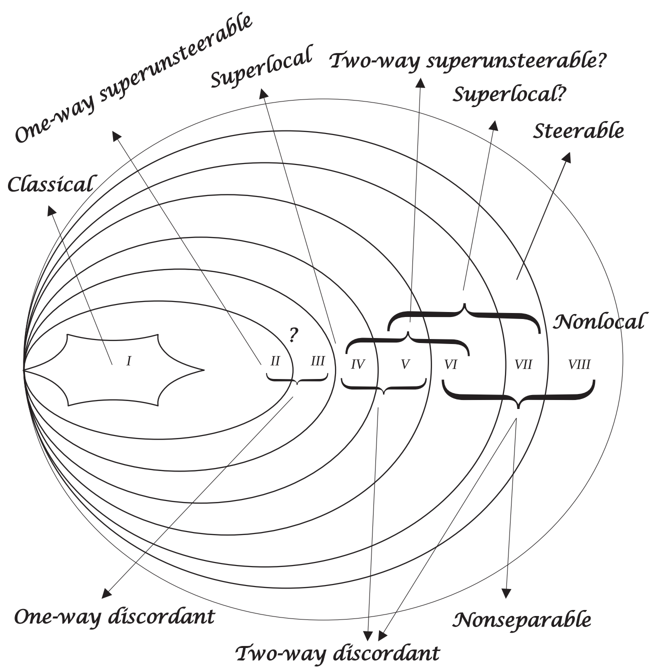

FIG. 1. Hierarchy of correlations in bipartite quantum states. The regions I, II, III, IV, and V represent the convex subset of correlations in separable states. The complement set, i.e., regions VI, VII, and $VIII$, represent nonseparable correlations in entangled states. Within the nonseparable correlations, the regions VII and $VIII$ represent one-sided device-independent steerable and Bell-nonlocal correlations, respectively, while the region VI represents the nonseparable correlations which are neither steerable nor Bell nonlocal. On the other side, within the correlations in separable states, correlations in one-way discordant states and two-way discordant states are depicted in regions II and III and regions IV and V, respectively, while the region I represents the classical correlations that do not require a discordant state to be produced. Within the discordant separable correlations, the regions II and IV represent the one-way superunsteerable and superlocal correlations that can be produced from one-way discordant and two-way discordant states, respectively. Whether one-way superunsteerable correlations form a strict subset of the correlations in one-way discordant states is an open question. Further, whether all correlations in regions IV, V, and VI are two-way superunsteerable and whether all correlations in regions V, VI, and VII are superlocal remain to be explored. Note that the noisy CHSH local box (8) exhibiting superlocality belongs to the regions V, VI, and VII; on the other hand, the unsteerable white noise BB84 family (10) exhibiting two-way superunsteerability and superlocality belongs to the regions IV, V, and VI.

untrusted side). On the other hand, in the following, we define another inequivalent form of steerability to imply the presence of steerability in a one-sided semi-device-independent way, i.e., by assuming only the dimension of the steering side.

Definition 5. In the steering scenario as described before for defining WJD's form of steering, Alice demonstrates steering to Bob in a one-sided semi-device-independent way if the assemblage does not have a LHS model with a restricted hidden variable dimension, i.e., if for all a, x, there is no decomposition of $\sigma_{a|x}$ in the form

$$\sigma_{a|x} = \sum_{\lambda=0}^{d_\lambda-1} p(\lambda)p(a|x,\lambda)\rho_\lambda, \tag{15}$$

with $d_\lambda \leqslant d_A$.

Note that for a given assemblage which has a LHS model with the hidden variable dimension d_λ as in Eq. (15), there exists a suitable choice of POVMs on Bob's side, $\{M_{b|y}\}$, such that it produces a LHV-LHS correlation $P(ab|xy)$ as in Eq. (9) with $p(b|y,\rho_\lambda) = \mathrm{Tr}(M_{b|y}\rho_\lambda)$ and the same d_λ. This implies that in the context of the above definition of steering, we have the following observation for the detection of it in a steering scenario where Bob performs particular measurements:

Observation 1. A bipartite box detects steerability in a one-sided semi-device-independent way if and only if there is no decomposition of the box in the form given by Eq. (9) with $d_\lambda \leqslant d_A$.

From the above discussions, it is clear that the correlation given by Eq. (12) detects one-way steerability in a one-sided semi-device-independent way. We have thus identified a different nonconvex subset of correlations in one-way discordant states, i.e., one-way superunsteerable correlations which do not exhibit superlocality as in Fig. 1, but exhibit one-sided semi-device-independent steerability asymmetrically. There are superlocal correlations which exhibit two-way superunsteerability as in the example given by Eq. (10) [23] and, hence, they exhibit two-way steerability in a one-sided semi-device-independent way.

VII. CONCLUSION

In this work, we have demonstrated the existence of a different asymmetric nature of quantum correlations in quantum discordant states. This asymmetric nature of quantumness arises due to one-way superunsteerability. For quantumness pointed out by superlocality, a two-way nonzero quantum discord is necessary. Whereas, superunsteerability being an asymmetric concept, to reveal quantumness pointed out by superunsteerability, a one-way nonzero discord suffices, as we demonstrated in this work. This result helps us to obtain a precise relationship among quantum discord, superunsteerability, and superlocality. We hope that this relationship stimulates the investigation of superunsteerability as a distinct resource rather than superlocality for quantum information processing.

Superunsteerability indicates a weak form of steerability, i.e., if an assemblage is produced in a one-sided semi-device-independent (SDI) steering scenario, then demonstrating superunsteerability is equivalent to steerability of the assemblage. This form of steerability is due to the quantum advantage in using a quantum system of lower local Hilbert-space dimension over the requirement of high dimensionality of the hidden variables. Since steering is truly a quantum phenomenon [5], superunsteerability captures a genuinely quantum effect, although it does not indicate steerability in the strongest way as the WJD's form of steerability occurs for certain entangled states [6]. Just like the WJD's form of steerability can be witnessed through a suitable inequality or criterion, in Ref. [27], we have shown a certification of quantum discord which provides sufficient criterion for determining superunsteerability of the two-qubit discordant states. While discord has been operationally understood as having classical correlations assisted by quantum coherence rather than quantum correlations [36], our results in this work and our other recent works on superunsteerability [23,27,29] thus provide insight that certain discordant states also have quantum correlations exhibiting quantum steering, just like certain entangled states have steerability, though in a weaker form.

ACKNOWLEDGMENTS

C.J. is grateful to Dr. Ashutosh Rai, Dr. Shin-Liang Chen, Dr. Manik Banik, and Professor Wei-Min Zhang for helpful discussions and would like to thank the National Science and Technology Council (formerly Ministry of Science and Technology), Taiwan (Grant No. MOST 111-2811-M-006-040-MY2). D.D. acknowledges the Royal Society (United Kingdom) for the support through the Newton International Fellowship (Grant No. NIF\R1\212007). R.S. acknowledges support from Interdisciplinary Cyber Physical Systems (ICPS) programme of the Department of Science and Technology (DST), India, Grant No. DST/ICPS/QuST/Theme-1/2019/6.

[1] A. Einstein, B. Podolsky, and N. Rosen, Can quantum-mechanical description of physical reality be considered complete? Phys. Rev. **47**, 777 (1935).

[2] D. Bohm and Y. Aharonov, Discussion of experimental proof for the paradox of Einstein, Rosen, and Podolsky, Phys. Rev. **108**, 1070 (1957).

[3] J. S. Bell, On the Einstein Podolsky Rosen paradox, Phys. Phys. Fiz. **1**, 195 (1964).

[4] N. Brunner, D. Cavalcanti, S. Pironio, V. Scarani, and S. Wehner, Bell nonlocality, Rev. Mod. Phys. **86**, 419 (2014).

[5] E. Schrödinger, Discussion of probability relations between separated systems, Math. Proc. Camb. Phil. Soc. **31**, 555 (1935).

[6] H. M. Wiseman, S. J. Jones, and A. C. Doherty, Steering, entanglement, nonlocality, and the Einstein-Podolsky-Rosen paradox, Phys. Rev. Lett. **98**, 140402 (2007).

[7] R. Uola, A. C. S. Costa, H. C. Nguyen, and O. Gühne, Quantum steering, Rev. Mod. Phys. **92**, 015001 (2020).

[8] J. Bowles, T. Vértesi, M. T. Quintino, and N. Brunner, One-way Einstein-Podolsky-Rosen steering, Phys. Rev. Lett. **112**, 200402 (2014).

[9] C. Branciard, E. G. Cavalcanti, S. P. Walborn, V. Scarani, and H. M. Wiseman, One-sided device-independent quantum key distribution: Security, feasibility, and the connection with steering, Phys. Rev. A **85**, 010301(R) (2012).

[10] E. Chitambar and G. Gour, Quantum resource theories, Rev. Mod. Phys. **91**, 025001 (2019).

[11] H. Ollivier and W. H. Zurek, Quantum discord: A measure of the quantumness of correlations, Phys. Rev. Lett. **88**, 017901 (2001).

[12] L. Henderson and V. Vedral, Classical, quantum and total correlations, J. Phys. A: Math. Gen. **34**, 6899 (2001).

[13] A. Datta, A. Shaji, and C. M. Caves, Quantum discord and the power of one qubit, Phys. Rev. Lett. **100**, 050502 (2008).

[14] S. Pirandola, Quantum discord as a resource for quantum cryptography, Sci. Rep. **4**, 6956 (2014).

[15] K. Modi, A. Brodutch, H. Cable, T. Paterek, and V. Vedral, The classical-quantum boundary for correlations: Discord and related measures, Rev. Mod. Phys. **84**, 1655 (2012).

[16] N. Bohr, Can quantum-mechanical description of physical reality be considered complete? Phys. Rev. **48**, 696 (1935).

[17] H. M. Wiseman, Quantum discord is Bohr's notion of non-mechanical disturbance introduced to counter the Einstein-Podolsky-Rosen argument, Ann. Phys. **338**, 361 (2013).

[18] A. Ferraro, L. Aolita, D. Cavalcanti, F. M. Cucchietti, and A. Acín, Almost all quantum states have nonclassical correlations, Phys. Rev. A **81**, 052318 (2010).

[19] A. Streltsov, H. Kampermann, and D. Bruß, Behavior of quantum correlations under local noise, Phys. Rev. Lett. **107**, 170502 (2011).

[20] J. Bowles, F. Hirsch, M. T. Quintino, and N. Brunner, Local hidden variable models for entangled quantum states using finite shared randomness, Phys. Rev. Lett. **114**, 120401 (2015).

[21] Y.-Y. Zhang and F.-L. Zhang, Local-hidden-state models for t-states using finite shared randomness, Europhys. Lett. **127**, 20007 (2019).

[22] J. M. Donohue and E. Wolfe, Identifying nonconvexity in the sets of limited-dimension quantum correlations, Phys. Rev. A **92**, 062120 (2015).

[23] D. Das, B. Bhattacharya, C. Datta, A. Roy, C. Jebaratnam, A. S. Majumdar, and R. Srikanth, Operational characterization of quantumness of unsteerable bipartite states, Phys. Rev. A **97**, 062335 (2018).

[24] C. Jebaratnam, S. Aravinda, and R. Srikanth, Nonclassicality of local bipartite correlations, Phys. Rev. A **95**, 032120 (2017).

[25] C. Jebaratnam, D. Das, S. Goswami, R. Srikanth, and A. S. Majumdar, Operational nonclassicality of local multipartite correlations in the limited-dimensional simulation scenario, J. Phys. A: Math. Theor. **51**, 365304 (2018).

[26] D. Das, C. Jebaratnam, B. Bhattacharya, A. Mukherjee, S. S. Bhattacharya, and A. Roy, Characterization of the quantumness of unsteerable tripartite correlations, Ann. Phys. **398**, 55 (2018).

[27] C. Jebarathinam and D. Das, Certifying quantumness beyond steering and nonlocality and its implications on quantum information processing, Quantum Inf. Comput. **23**, 379 (2023).

[28] K. T. Goh, J.-D. Bancal, and V. Scarani, Measurement-device-independent quantification of entanglement for given hilbert space dimension, New J. Phys. **18**, 045022 (2016).

[29] C. Jebarathinam, D. Das, S. Kanjilal, R. Srikanth, D. Sarkar, I. Chattopadhyay, and A. S. Majumdar, Superunsteerability as a quantifiable resource for random access codes assisted by Bell-diagonal states, Phys. Rev. A **100**, 012344 (2019).

[30] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevA.108.042211 for a detailed description of nosignaling boxes and the proof of Lemma 1.

[31] S. J. Jones, H. M. Wiseman, and A. C. Doherty, Entanglement, Einstein-Podolsky-Rosen correlations, Bell nonlocality, and steering, Phys. Rev. A **76**, 052116 (2007).

[32] S. M. Barnett and S. J. D. Phoenix, Entropy as a measure of quantum optical correlation, Phys. Rev. A **40**, 2404 (1989).

[33] N. J. Cerf and C. Adami, Negative entropy and information in quantum mechanics, Phys. Rev. Lett. **79**, 5194 (1997).

[34] B. Schumacher and M. A. Nielsen, Quantum data processing and error correction, Phys. Rev. A **54**, 2629 (1996).

[35] B. Groisman, S. Popescu, and A. Winter, Quantum, classical, and total amount of correlations in a quantum state, Phys. Rev. A **72**, 032317 (2005).

[36] V. Vedral, Foundations of quantum discord, in *Lectures on General Quantum Correlations and their Applications*, edited by F. F. Fanchini, D. d. O. Soares Pinto, and G. Adesso (Springer International, Cham, 2017), pp. 3–7.