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The mathematics pipeline in England: inclusion and the excellence stream

Christopher Brignell[†,*], Andrew Noyes[‡] and Laurie Jacques[§]
[†] *School of Mathematical Sciences, University of Nottingham, University Park, Nottingham, NG7 2RD, UK*
[‡] *School of Education, University of Nottingham, Jubilee Campus, Wollaton Road, Nottingham, NG8 1BB, UK*
[§] *UCL Institute of Education, 20 Bedford Way, London, WC1H 0AL, UK*

[*] *Corresponding author. Email:* chris.brignell@nottingham.ac.uk

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In England, there is currently heightened political interest in increasing mathematics attainment and maximizing post-16 participation. The latter is not merely an issue for upper secondary but requires a long-view of students' mathematical progress. This paper reports such a system-level overview of the mathematics education pipeline in England based on analysis of linked data from the National Pupil Database (NPD) and Higher Education Statistics Agency (HESA). A sequence of mixed-effect logistic regression models at each phase of the education pipeline are developed to predict attainment in General Certificate of Secondary Education (GCSE), A level and undergraduate mathematics and participation in post-16 and university mathematics. In particular, we focus on the impact of prior attainment and student demographics: gender, ethnicity and socio-economic status. The analysis identifies four important findings. Firstly, female participation and attainment in A level Mathematics and Further Mathematics is lower than that of male students, which impacts their representation in undergraduate mathematics, despite them marginally outperforming male students at both GCSE and degree level. Secondly, the attainment and participation of students of Asian ethnicity in GCSE and A level Mathematics is striking in its positive divergence from the trends of other ethnic groups, whereas White students have higher attainment than other ethnic groups at university. Thirdly, many students from poorer backgrounds fall behind their peers during years 7 to 11, and their participation and attainment in A level Mathematics is also reduced compared to their more affluent peers. Lastly, students who study A level Further Mathematics are more likely to study undergraduate mathematics, but they are no more likely to achieve a first-class degree, compared to those who haven't studied Further Mathematics. The findings should be informative to policymakers and other stakeholders who can put in place enhancements and interventions to improve the outcomes for underrepresented groups.

1. Introduction

Evidence for the personal, social and economic benefits of mathematics has generated broad and sustained interest in improving mathematical education in advanced economies around the world. This is no different in England, where the importance of mathematics is evidenced in a plethora of reports on everything from everyday numeracy (National Numeracy, 2019) to the contributions of mathematics research (Deloitte, 2013; Bond, 2018). Early in 2023, Prime Minister Rishi Sunak put mathematics education centre stage in education policy, in particular focusing on higher attainment and mathematics to 18[1]. The mathematics-to-18 agenda has long been a focus of education policy (Smith, 2004; Gove, 2011; Smith, 2017) and research (Matthews & Pepper, 2007; Noyes, 2009; Hodgen *et al.*, 2010), but to understand the post-16 problem properly, one must carefully examine students' progression up to 16—the pipeline—and consider factors that might explain socially differentiated patterns in outcomes and participation.

The metaphoric language of *pipeline* (see Noyes & Adkins, 2016 for a critique) to describe students' progress through the mathematics education system is associated with flow volumes and rates, and with leaks (Van Den Hurk *et al.*, 2019). Within the *pipeline*, the group of students who have potential to progress to undergraduate and postgraduate study and possibly thereafter to research in mathematics, we term as the *excellence stream*. To understand post-16 mathematics participation, it might be tempting to focus attention on the points where the mathematics pipeline narrows or 'leaks' most dramatically (i.e. at the end of compulsory education following General Certificate of Secondary Education (GCSE) exams at age 16, and at the transition to university following A level exams at age 18). These are critically important points for action and others have already explored these contractions in the pipeline (see above). Yet a focus on leaks is limiting as the evidence is clear that many young people have effectively drifted out of, or been filtered from, the *excellence stream* well before the age of 16, such that their progress leaves them unqualified for, or unenthusiastic about, progression to advanced mathematics.

Successive governments and multiple organizations have tried to address such disparities in student outcomes from the mathematics pipeline. For example, the government-funded Advanced Mathematics Support Programme (AMSP) targets support to areas of low social mobility while charities, such as Maths4Girls and the LMS levelling up scheme, seek to increase participation from underrepresented groups. These interventions sit alongside developments in national policy such as changes to the national curriculum in 2014, the creation of mathematics hubs to support teaching excellence and reforms to GCSEs in 2017 and A levels in 2019. Meanwhile the Office for Students, created in 2018, has placed greater emphasis on widening participation in universities. Nevertheless, despite these initiatives and many others, important variations in mathematical outcomes associated with socio-economic status, ethnicity and gender have persisted over decades.

Of the general attainment gap between disadvantaged students and those from more affluent backgrounds that has formed by age 16, 40% can already be accounted for during the pre-school years. A further 20% of the attainment gap has been shown to develop during the primary phase and another 40% during the secondary phase of education, culminating in a cumulative gap of up to 19 months by age 16 (Hutchinson *et al.*, 2016). Evidence from national assessment data in England and in multiple large-scale research studies shows that both ethnicity and gender predict inequalities along the mathematics pipeline (Sammons *et al.*, 2012; Shaw *et al.*, 2017; Elliot Major & Parsons, 2022). Furthermore, geographical analysis of student outcomes for mathematics indicates that a larger proportion of the

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[1] https://www.gov.uk/government/news/prime-minister-outlines-his-vision-for-mathematics-to-18

highest GCSE grades are achieved in the more affluent south of England[2]. Such geographical inequalities (see also Smith, 2017) impact on employability and earning potential, the opportunity to pursue further mathematical study and to contribute as an engaged citizen.

Prior attainment at GCSE remains the strongest predictor of participation at A level (Noyes, 2009; Noyes & Adkins, 2016). Previous studies have suggested that, once GCSE attainment is included in the predictive models, the effects of socio-economic status all but disappear. In effect, social class differences appear to have already been embedded in GCSE outcomes by the time students choose A levels (Boylan *et al.*, 2016). In addition, these and other studies (Mendick, 2005; Brown *et al.*, 2008) show notable differences in attainment and post-16 engagement by gender and ethnicity. This is important because it means social patterns of underachievement, formed at earlier points in the pipeline, contribute to students avoiding mathematical study beyond age 16 (Codiroli Mcmaster, 2017).

These patterns generally continue after the transition to higher education. Despite efforts by all universities to widen participation for underrepresented groups (BIS, 2016), the literatures show that in the higher education phase of the mathematics pipeline there are considerably fewer students with low socio-economic status compared to the population, or indeed, the higher education sector (Sutton Trust, 2021). Those who do enter higher education are also less likely to complete their course and more likely to leave with a lower degree class, with the gaps persisting into postgraduate study and employment (Crawford, 2014). Similarly, there are well-established, and intersectional, associations between ethnicity, gender and degree outcomes (Richardson *et al.*, 2020).

2. The mathematics pipeline project

This paper arises from a recently completed national study exploring students' progress in England at key points in mathematics education, from the start of reception (age 4) through Key Stages 1 (ages 5–7), 2 (ages 7–11), 3 (ages 11–14), 4 (ages 14–16) and thereafter into sixth forms & colleges (Key Stage 5, ages 16–18) and universities (age 18+); the Mathematics Pipeline in England (Noyes *et al.*, 2023). Note that students usually attend primary school for reception, KS1 and KS2; and secondary school for KS3 and KS4. Important qualifications are compulsory GCSEs taken at age 16 (end of KS4) and around 14% of students take A level Mathematics at age 18 (end of KS5), sometimes continuing in the same school at age 16 and sometimes moving to a different college.

This Pipeline Project makes an important contribution to our systemic understanding of mathematics education by taking a holistic, longitudinal view and examining the diversity of the *excellence stream* from the earliest phases of mathematics education to 18 and beyond. This is done by paying particular attention to underrepresented groups of students (as defined by gender, social-economic status and ethnicity) at assessment points along the pipeline to identify phases between these critical milestones where particular groups of students appear to have been 'filtered out' of the *excellence stream*.

The Pipeline Project report (Noyes *et al.*, 2023) provided summary statistics of participation and attainment for underrepresented groups. However, it did not consider the relationship between variables, such as the dependency between socio-economic status and ethnicity. This paper extends the findings of the original study to examine in greater detail the diversification of the *excellence stream*. In particular, we use statistical modelling to identify the statistically significant effects whilst controlling for other variables including, importantly, which school or university the student attended. Therefore, disparities noted in the results of this paper are less likely to be artefacts of confounding variables but differences that

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[2] https://analytics.ofqual.gov.uk/apps/GCSE/County/

remain after accounting for many other potential sources of variability. We use this analysis to suggest phases of the pipeline where students who get filtered from the *excellence stream* might be prevented from doing so, with the intention of retaining a greater number of students in this *excellence stream* overall, and in particular those from underrepresented groups.

2.1 *Methods*

The study used two sets of linked cohort data from the Department for Education's National Pupil Database (NPD) and Higher Education Statistics Agency (HESA):

- Cohort 1 comprises a full, national cohort of students in the NPD who took GCSEs in the academic year 2016–2017, linked to their academic records back to the end of Key Stage 1 (in 2007–2008) and forward to A levels (in 2018–2019). This cohort was chosen to capture the most recent cohort to have typically completed A levels prior to the impact of the Covid-19 pandemic.
- Cohort 2 comprises a base HESA cohort of first-year undergraduate students in 2015–2016. These are linked back to their A level and GCSE outcomes and forward to undergraduate outcomes and postgraduate degree choices; they might well have come from different A level cohorts, and they may end up in different graduating cohorts. This cohort was selected as the most recent cohort to typically complete their degree programme pre-pandemic.

Cohort 1 was the first to experience both the new grading system at GCSE (9–1) and the reformed A level Mathematics qualification, both of which have an increased focus on problem-solving and modelling and with greater focus on terminal assessment, whereas the earlier Cohort 2 experienced modular A levels, which allowed for some choice between the study of mechanics, statistics and 'decision' mathematics.

We fitted statistical models that predict the probability of success at different stages of the pipeline based on prior attainment and demographics. Models are mixed-effect logistic regression models (Bates, 2010) where institution (e.g. school) is a random effect and student-level variables (e.g. prior attainment and demographic information) are fixed effects nested within institution. Random effects are applied to account for the nested structure of the data. For example, in our first model (see Table 1) we use the specific primary school attended as a random effect, combined with student-level attainment at the end of primary school to provide a baseline, and then use secondary school type and demographic variables to predict whether the student achieves GCSE grade 7 or above at age 16. We use the Income Deprivation Affecting Children Index (IDACI) as a measure of socio-economic deprivation. The index is based on Lower layer Super Output Areas (neighbourhoods of around 1500 people) in England. Raw scores for each student are converted into quintiles with quintile 5 being the least deprived and quintile 1 being the most deprived.

Quality of model performance is assessed by measuring the area under the curve (AUC) of the receiver operating characteristic (ROC) curve. The ROC plots the specificity (true-negative rate) against sensitivity (true-positive rate) for all thresholds of a binary classification model. A model that perfectly predicts binary outcomes has an AUC value of 1, reducing to 0.5 for an uninformative classifier. The effect of variables and interaction terms is determined by testing whether the relevant parameter in the linear predictor of the logistic model is statistically significantly different from zero on the log-odds scale.

Models 1, 3, 5 and 7 predict attaining high grades at GCSE, A level Mathematics, Further Mathematics and undergraduate levels, respectively (see Table 1 for grade criteria), for students who attempt the qualification, whereas models 2, 4, 6 and 8 predict continued participation in the pipeline in A level

TABLE 1. *Summary of regression models. The data column indicates the number of students included in the analysis. Full definitions of the variables are included in the appendix*

Model	Data	Response: did they . . .	Random effect (Institution)	Fixed effects: gender, ethnicity, IDACI quintile, FSM, SEN, first language, plus:
1	Cohort 1 n = 515,695	achieve GCSE levels 7 to 9?	KS2 School	KS2 Mathematics, KS2 English, KS4 School type
2	Cohort 1 n = 515,725	enter A level Mathematics?	KS4 School	KS4 Mathematics, KS4 English
3	Cohort 1 n = 71,890	achieve A level Mathematics grades A or A*?	KS4 School	KS4 Mathematics, KS4 English, KS5 (A level) Institution type
4	Cohort 1 n = 506,755	enter A level Further Mathematics?	KS4 School	KS4 Mathematics, KS4 English
5	Cohort 1 n = 9985	achieve A level FM grades A or A*?	KS4 School	KS4 Mathematics, KS4 English, KS5 (A level) Institution type
6	Cohort 2 n = 70,625	enter a Mathematics degree?	KS5 (A level) Institution	A level Mathematics, A level Further Mathematics, UCAS A level points, parental education, region of nationality, socio-economic class
7	Cohort 2 n = 9985	achieve 1^{st} class Mathematics degree?	First degree HEI	A level Mathematics, A level Further Mathematics, UCAS A level points, HEI type, Mode of study, Degree level, Degree subject, Parental education, Region of nationality, Socio-economic class
8	Cohort 2 n = 7375	enter postgraduate Mathematics degree?	First degree HEI	First degree classification, first degree level, first degree subject, parental education, region of nationality, socio-economic class

Mathematics, Further Mathematics, undergraduate and postgraduate levels, respectively, for students who completed the previous phase. Note the data only allows us to measure participation at A level by whether a student is assigned a grade, including failed grades, at completion so we are unable to measure students who started an A level but did not complete it.

In each case we take our reference level for each fixed-effect variable to be a student with the most advantage (e.g. IDACI quintile 5) where there is a clear ordinal scale or the most commonly represented (e.g. male, white, no FSM, no SEN, English first language, did not study A level Further Mathematics). In particular, our reference-level student has the highest attainment at each stage (i.e. level 6 at KS2, grade 9 at GCSE, grade A* in A level Mathematics) and obtained first-class honours in a full-time single-honours BSc Mathematics degree at a university outside the Russell Group. All reference student levels are shown in bold in the Appendix. Note that by defining ‘success’ to be attaining the highest grades and continuing to study mathematics, we do not wish to imply that lower grades or studying other subjects are not worthwhile achievements. Our aim is to highlight inequalities in the later stages of the pipeline and, by implication, draw attention to the factors that correspond to students making slower progress or leaving the pipeline.

3. Results

The mixed-effect logistic regression models allow us to investigate the effect of variables while controlling for others and to investigate interactions where the effect of one variable depends on another. This gives more nuanced results compared to just looking at summary statistics for each variable, as was done in our original report (Noyes *et al.*, 2023). However, care must be taken when interpreting the results, as we can only display results for particular combinations of the fixed-effect variables. To plot the results, we calculate the fitted probability of ‘success’, with a 95% confidence interval, from each model for the reference-level student and then vary one or two other variables of interest to see their effect. Note that the probability range on the vertical axis varies between graphs to improve readability. While the observed patterns will be true for other students, any quoted probability will be different for alternative choices of the reference levels. Full details of the fitted models are shown in the supplementary material. In particular, this shows the coefficients for each term in the models, the standard error of the estimate and the p-value denoting its significance. Note that negative coefficients imply a negative effect and positive coefficients imply a positive effect. Coefficients are on the log-odds scale and the inverse logit function is applied to convert linear combinations of coefficients to fitted probabilities. Models 1, 2, and 4 have an AUC value above 0.9; Models 3, 5 and 6 have an AUC value above 0.8; and Models 7 and 8 have AUC values above 0.7; which may be interpreted as excellent, good and fair model performance, respectively. In the remainder of this section, we take a chronological view of the mathematics pipeline, outlining the social patterns and characteristics of student attainment at GCSE, participation and attainment at A level and undergraduate study and concluding with participation for postgraduate study.

3.1 GSCE attainment

Model 1 predicts student attainment in GCSE Mathematics at the end of KS4 (age 16). Prior attainment at the end of primary school (end of KS2) is the strongest indicator of attainment at GCSE. As shown in Fig. 1(a), our reference-level student who achieved national curriculum level 6 at KS2 has a 97% chance of achieving grades 7–9 at GCSE, compared to 84% and 55% for a similar student who was at the upper end (5U) or lower end (5L), respectively, of level 5. A similar student who achieved level 4 or lower (4−) at KS2 has only a 13% chance of achieving grades 7–9 at GCSE. To some extent this means student outcomes are already highly predictable but placing responsibility solely on previous education phases ignores the fact that some students progress at different rates and in different phases. For example, Fig. 1(a) shows that those whose first language is not English make greater progress than those whose first language is English. We should also note that prior attainment in KS2 English also impacts GCSE Mathematics performance, although not to the same extent as KS2 Mathematics attainment. For example, our reference-level student who achieves level 6 in KS2 English and Mathematics has a 97% chance of achieving grades 7–9 in GCSE Mathematics but this reduces to 86% if they achieve level 4 or below in KS2 English.

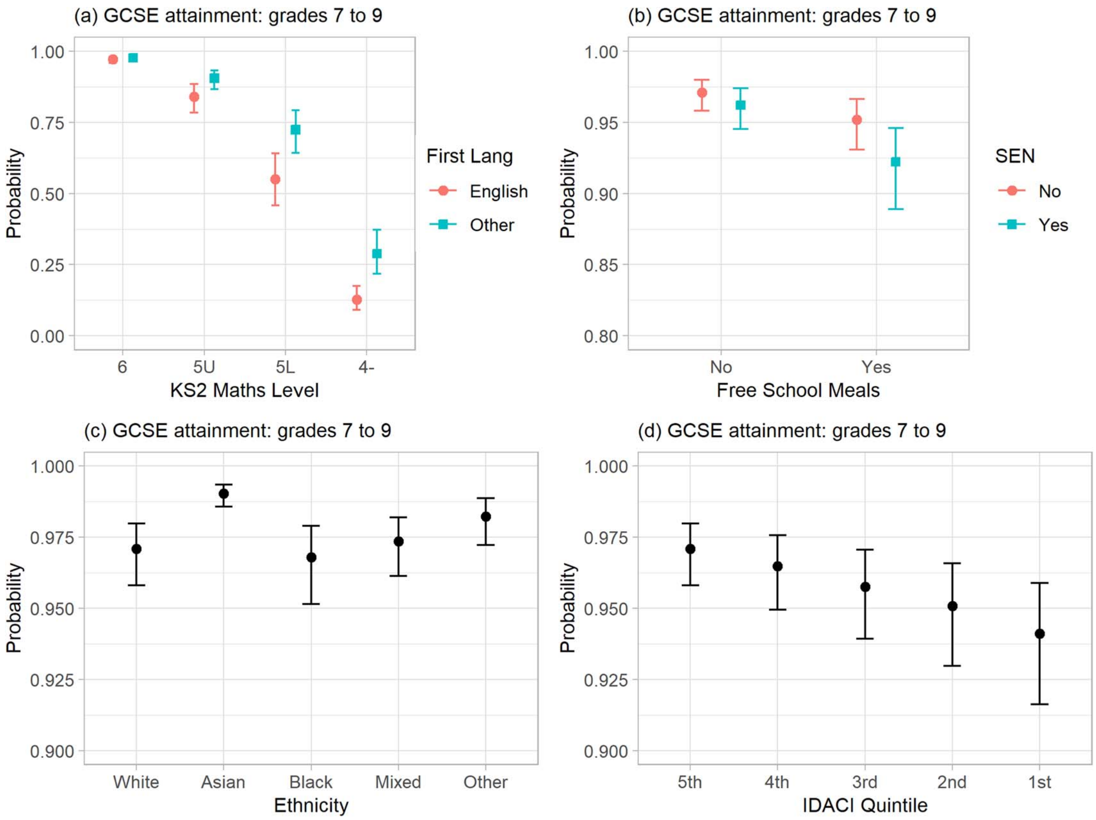

Fig. 1. The probability that the reference-level student achieves grades 7–9 in GCSE Mathematics, with 95% confidence intervals. For each plot, all fixed-effect variables are held at their reference levels unless otherwise stated. See the Appendix for full definitions of variables and levels.

Students with special educational needs (SEN) are significantly less likely to achieve grades 7–9 at GCSE than those without. Figure 1(b) shows that this attainment gap widens if the student receives Free School Meals. This might mean that measures to support SEN students are less effective for lower income students and/or that lower income families are less able to supplement SEN support in school. Either way, it suggests that SEN support for lower income students needs to be increased.

Overall, male students are more likely to achieve grades 7–9 in GCSE Mathematics than female students (20.5% of male students vs. 19.1% of female students). However, when controlling for prior attainment, the effect is reversed. For example, 92.1% of females with KS2 Mathematics level 6 achieve grades 7–9 at GCSE compared to 90.3% for males. (The female advantage is 6.7 percentage points for those at the upper end of level 5, and 5.8 percentage points for the lower end of level 5.)

Meanwhile model 1 predicts students of Asian and Other heritage are more likely to achieve grades 7–9 than White students, while there is no significant difference in the model between those of White and Black or mixed ethnicity (Fig. 1(c)). Furthermore, Fig. 1(d) shows the effect of income deprivation: our White IDACI quintile 5 reference-level student is 3.0 percentage points more likely to achieve grades 7–9 than the same student from quintile 1. This effect is more pronounced among students of White and mixed ethnicity compared to Asian, Black and other ethnicities. For example, the attainment gap is only 0.7 percentage points if they were of Asian ethnicity.

3.2 A level participation

Model 2 shows that GCSE performance is the strongest indicator of A level Mathematics participation. Male students are more likely to take A level Mathematics than female students and there is an interaction effect between prior attainment and gender. Figure 2(a) shows that while our male grade 9 reference-level student is 9.2 percentage points more likely to participate in A level Mathematics than their female counterpart, this gap widens to 22.8 percentage points for the equivalent students with grade 8. The gender gap is also significantly wider for students in higher income areas compared to lower income areas. For example, grade 9 male students have a 5.4 percentage point advantage for IDACI quintile 1 students compared to 9.2 percentage points for IDACI quintile 5.

A level Mathematics participation is also higher amongst students of Asian and other heritage. This is true even after controlling for GCSE attainment where Asian, Black and other ethnicities outperformed all other ethnicities. However, Fig. 2(b) shows the variation between ethnicities is significantly reduced if the student's first language is not English. The ethnicity gaps widen for lower-attaining students. For example, our White grade 9 reference-level student is 8.9 percentage points less likely to participate in A level Mathematics than their counterpart of Asian heritage, but the gap widens to 19.7 percentage points for equivalent students with grade 8.

Our IDACI quintile 5 reference-level student has an 83% chance of participating in A level Mathematics compared to 77% for a similar student from IDACI quintile 1. Furthermore, Fig. 2(c) shows this participation gap widens for students with SEN. As we noted with GCSE attainment, the negative effects of both income deprivation and SEN are compounded when both factors are present. Free School Meals is also a negative indicator of A level participation. Figure 2(d) shows that the probability of our reference-level student participating in A level Mathematics reduces to 80% if they receive Free School Meals. Of course, A level participation is partly down to a student's choice as well as opportunity. Figure 2(d) also shows that students who obtain grade 9 in GCSE English are less likely to participate in A level Mathematics compared to students who obtain grade 5. We speculate that a similar pattern will be observed for other (non-mathematics) GCSE subjects and might be explained by students attaining highly as having a wider range of potential options to pursue post-16.

Model 4 investigated participation in A level Further Mathematics. Figure 2(e) shows the gender gap is wider for Further Mathematics, with a 24.1 percentage point difference between our male grade 9 reference-level student and his female equivalent (compared with 9.2 percentage points for A level Mathematics in Fig. 2(a)). Amongst students whose first language is English, Fig. 2(f) shows the differences in Further Mathematics participation are less significant than for Mathematics (see Fig. 2(b)). For those without English as a first language, participation in Further Mathematics is higher amongst students of White and mixed ethnicities, but lower among Black students. We might speculate that students perceive mathematics as being less demanding on English language skills than other subjects and therefore attracts disproportionally more students without English as a first language, but this isn't universally true amongst the different ethnicities for either Mathematics or Further Mathematics.

3.3 A level attainment

Although female students with the same prior attainment outperform male students at GCSE, this gender gap reverses alarmingly during the A level phase. Our top-performing (grade 9) reference-level student

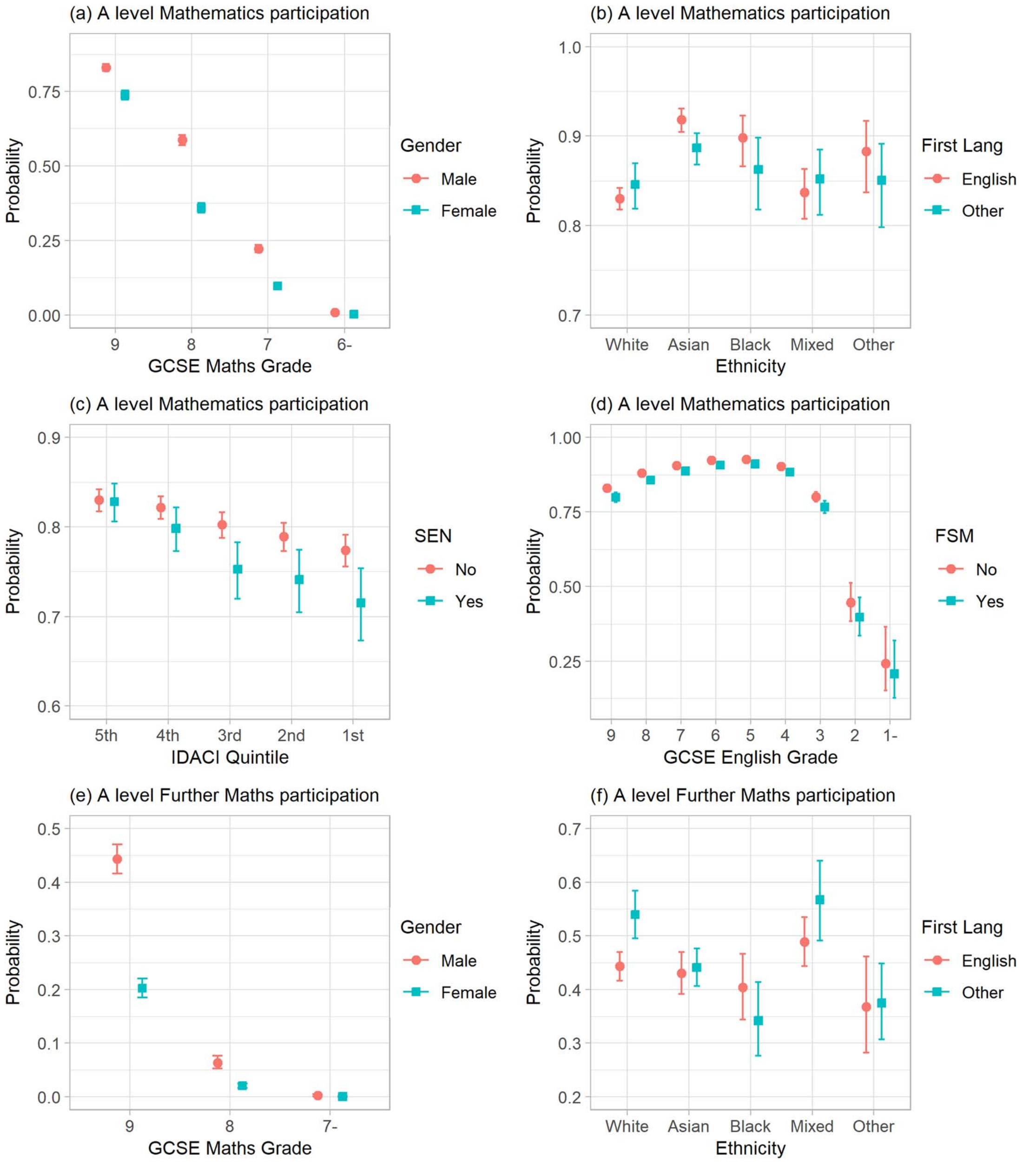

FIG. 2. The probability that the reference-level student participates in A level Mathematics (a–d) or A level Further Mathematics (e–f), with 95% confidence intervals. For each plot, all fixed-effect variables are held at their reference levels unless otherwise stated.

is 5.3 and 12.8 percentage points more likely than his female counterpart to achieve a grade A or A* in A level Mathematics and Further Mathematics, respectively. Interestingly, Fig. 3(a) shows this gender gap

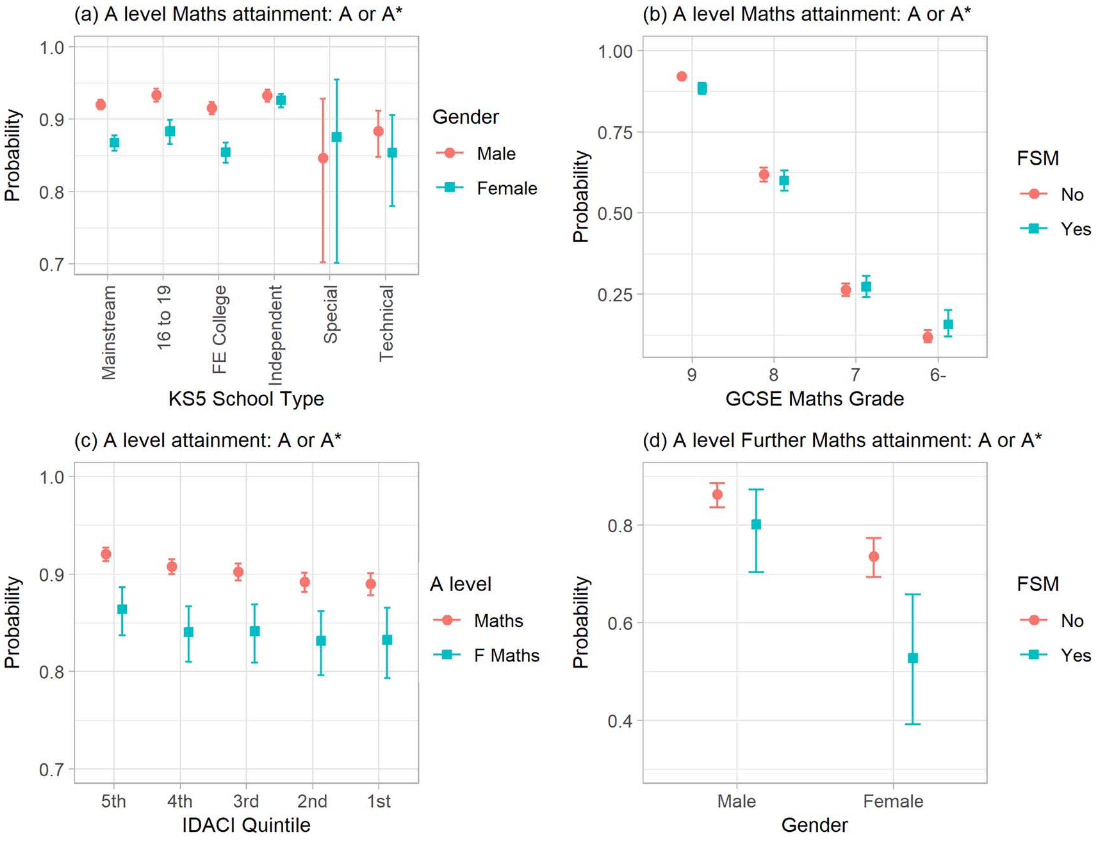

FIG. 3. The probability with 95% confidence interval that the reference level student achieves A level grades A or A∗. For each plot, all fixed-effect variables are held at their reference levels unless otherwise stated.

is mostly neutralized for A level Mathematics in Independent schools, which might be due, in part, to the greater prevalence of single-sex classes. The same figure also shows students in Independent and 16–19 schools outperform those in 11–18 maintained schools and Further Education colleges for Mathematics (and Model 5 shows the same is true for Further Mathematics).

While GCSE attainment is the strongest predictor of A level attainment and students from lower incomes have reduced attainment at GCSE, Fig. 3(b) shows that students on Free School Meals catch up by a small but significant amount during the A level phase, although much of the disadvantage still remains. For example, students in lower income areas (IDACI quintile 1) are significantly less likely to achieve A or A∗ compared to their more affluent IDACI quintile 5 peers for both A level Mathematics and Further Mathematics (Fig. 3(c)). Furthermore, for Further Mathematics, Fig. 3(d) shows the gender gap widens for students in receipt of Free School Meals. Those of mixed and other ethnicities make more progress through A level Mathematics than those of White, Asian or mixed ethnicities. For example, Model 3 suggests the White reference-level student has a 92.0% chance of obtaining grade A or A* in A level Mathematics, rising to 93.2% and 93.5% for students of mixed and other ethnicities, respectively.

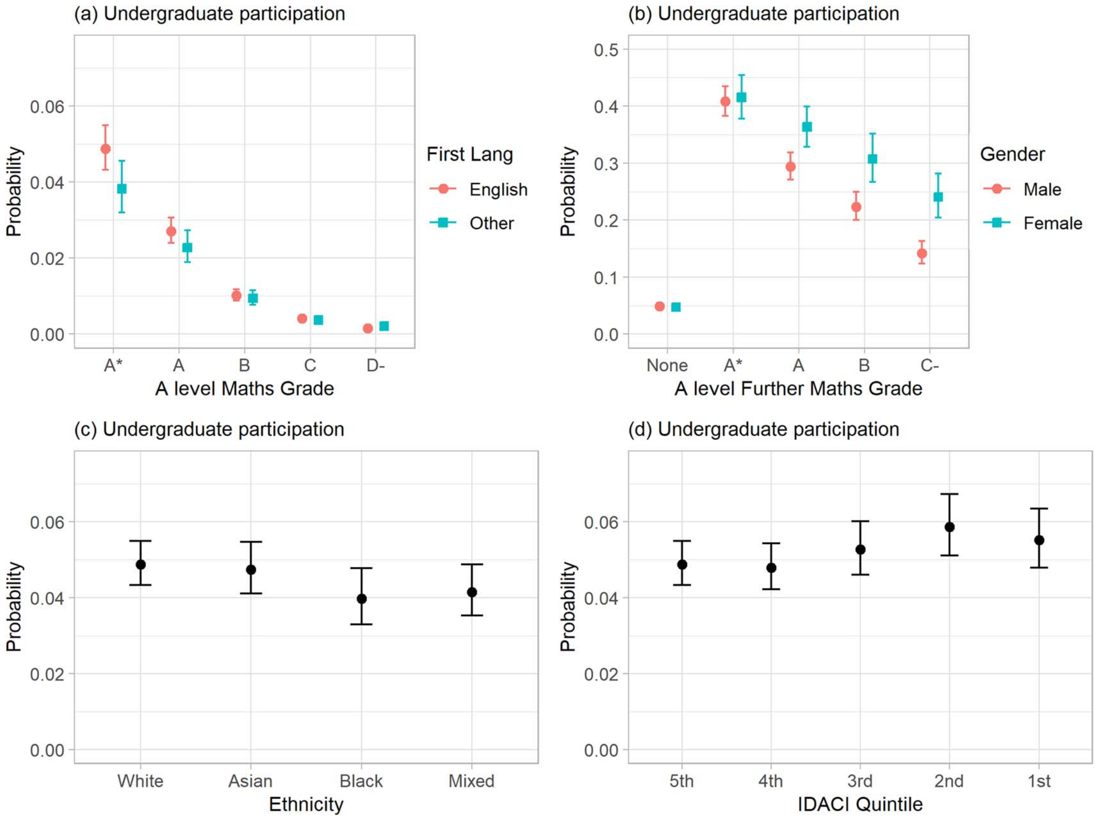

FIG. 4. The probability with 95% confidence interval that the reference-level student participates in undergraduate mathematics. For each plot, all fixed-effect variables are held at their reference levels unless otherwise stated.

3.4 Undergraduate participation

We define undergraduate participation to be enrolling in a degree where Mathematics comprises at least 30% of the curriculum. The number of students entering Mathematics (single and joint honours) degrees is comparatively small given the popularity of A level Mathematics. Even our reference-level student with grade A* in A level Mathematics (but did not take A level Further Mathematics) has just a 5% probability of entering an undergraduate Mathematics degree and, as Fig. 4(a) shows, that probability is even smaller if English is not their first language, or if they were to achieve a lower A level grade. This is despite some universities that accept a grade B in their entry requirements recruiting below their target and suggests students are not participating out of choice or through a misconception that they are not good enough.

Figure 4(b) shows that students who achieve any pass grade in A level Further Mathematics are much more likely to enter an undergraduate mathematics degree than those who have no A level Further Mathematics grade (shown as ‘none’ in Fig. 4(b)). Overall, there is no significant gender effect in this undergraduate transition phase but, as Fig. 4(b) also shows, female A level Further Mathematics students who have not achieved the top grade are more likely to enter an undergraduate mathematics degree compared to male students with the same grade. However, as reported above, the A level cohort is male-dominated and female students are less likely to achieve the highest A level grades, so the undergraduate cohort ends up being more male-dominated than the A level cohort. This suggests that if the gender gaps in A level participation and attainment are closed then this might feed through to the undergraduate cohort.

Regarding ethnicity, Fig. 4(c) shows those of Black and mixed ethnicities are less likely to enter an undergraduate mathematics degree compared to those of White and Asian ethnicities. However, after controlling for the other variables, those with greater income deprivation are more likely to enter undergraduate mathematics degrees (Fig. 4(d)). This difference may be the result of wealthier students having greater cultural capital that makes it easier to pursue a wider range of options such as medicine and law. While the difference between IDACI quintiles partly closes the participation gap observed at A level, students from low-income areas are still underrepresented in university mathematics compared to the wider population. For example, 24.8% of the undergraduate cohort are from IDACI quintile 5 compared to 15.8% from IDACI quintile 1.

3.5 *Undergraduate attainment*

The effect of prior attainment seems less dramatic at undergraduate degree level (Fig. 5(a)) compared to GCSE (Fig. 1(a)) and A level (Fig. 3(b)). Figure 5(a) also shows female students outperform male students with the same prior attainment and this effect is stronger for those entering with A level grade B or lower. This helps to offset the gender attainment gap created at A level and suggests female students may adapt better to the independent learning and continuous assessment experienced at university. However, Fig. 5(b) shows all minority ethnicities make less progress than White students, especially for female students of Black heritage, suggesting that White students may be advantaged by having greater socio-cultural capital.

Figure 5(c) shows that students who have not taken A level Further Mathematics are not disadvantaged compared to those with the additional qualification and anything less than an A* in A level Further Mathematics is a negative indicator. This is particularly true for male students, whereas for female students, who are disadvantaged during the A level phase, they are more likely to overcome a lower grade in Further Mathematics. Another potential admissions criterion, overall UCAS points score (a translation of pre-university qualifications and grades into a numerical value), is also shown to be a good predictor of undergraduate attainment (Fig. 5(d)). Of course, the criteria for a first-class degree vary by course and institution as, unlike for GCSE and A level, these are not nationally set examinations. Model 7 controls for this and we also note from Fig. 5(d) that first-class BSc degrees appear to be slightly easier to obtain at universities outside the research-intensive Russell Group. Model 7 also shows that a student who studies full-time is much more likely to achieve a first-class Mathematics degree than an equivalent part-time student. Similarly, a student without SEN is more likely to achieve a first-class Mathematics degree than a counterpart with SEN, although the effect is much smaller than the effect of part-time study. There was no significant difference in the chance of receiving a first-class outcome between BSc and MMath degrees.

3.6 *Postgraduate participation*

The number of students taking postgraduate mathematics degrees (taught and research) is comparatively small so there are fewer significant findings. Our reference-level student with a first-class single-honours BSc degree has an 11% probability of taking a postgraduate degree in mathematics, compared to 4% for the same student with a 2:1 degree class. Aside from prior attainment, Fig. 6(a) shows students are more

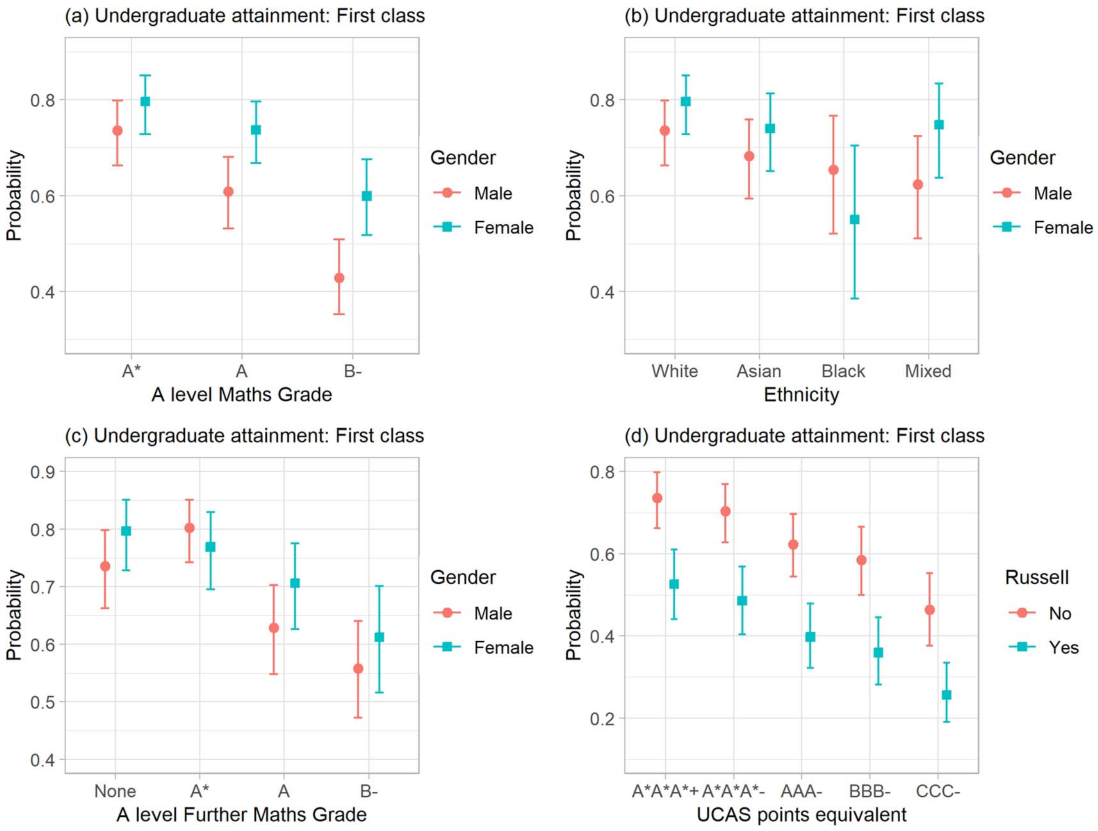

FIG. 5. The probability with 95% confidence interval that the reference-level student achieves a first-class degree in undergraduate mathematics. For each plot, all fixed-effect variables are held at their reference levels unless otherwise stated.

likely to take a postgraduate mathematics qualification if they have taken a single-honours (compared to joint honours) Mathematics degree and, to a lesser extent, an MMath degree (compared to BSc degree).

As was noted for A level and undergraduate transitions, Fig. 6(b) shows that participation rates for female students falls again in the postgraduate phase but only for those of British nationality. Students of Asian nationality are more likely to take a postgraduate mathematics qualification than British students (Note that HESA data records the nationality of each student by continental region. Students from the Asia region should not be confused with students of Asian ethnicity (heritage) of British or other nationality referred to elsewhere.).

4. Discussion

The Mathematics Pipeline Project (Noyes *et al.*, 2023), together with the extended analysis herein, set out to understand the flows of students in the higher-attaining *excellence stream*, i.e. those who might, at any particular stage of the education system, be considered more likely to progress to Advanced and Higher study of mathematics. The rationale for this systems-level overview was to provide stakeholders

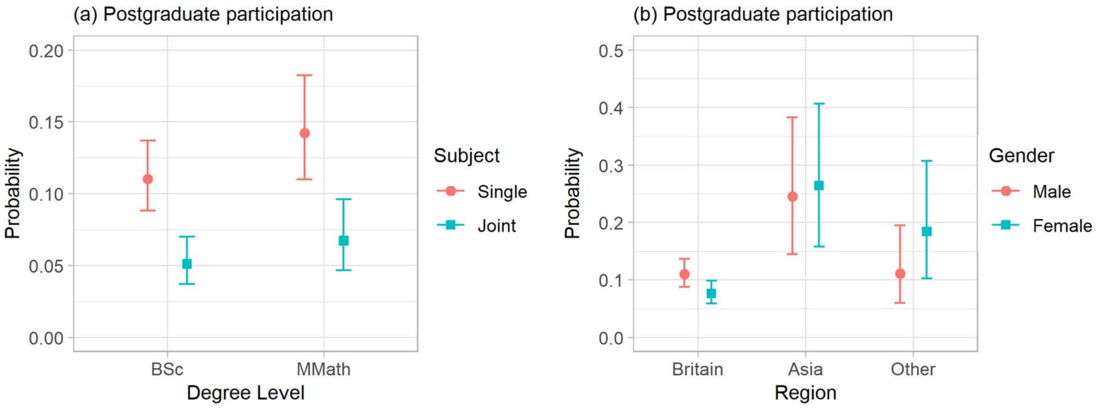

FIG. 6. The probability with 95% confidence interval that the reference-level student participates in postgraduate mathematics. For each plot, all fixed-effect variables are held at their reference levels unless otherwise stated.

who are working to improve the flow in the *excellence stream* with insights to inform their actions, whether through policy or other interventions. Our aim was to highlight patterns and inequalities in the later stages of the pipeline and, by implication, draw attention to the factors that correspond to students making differential progress or leaving the pipeline. An important feature of the analysis herein was to take a holistic view of the pipeline from the end of KS2 to the end of compulsory education and thereafter into higher education.

Our analysis adds to previous studies that show how prior mathematics attainment predicts attainment at each stage of the pipeline (e.g. Sylva *et al.*, 2014; Hodgen *et al.*, 2020) and participation in mathematical study post-16 (Noyes & Adkins, 2017). Policy initiatives and interventions, such as those proposed by EEF (2017), aimed at raising attainment in mathematics (and, to a lesser extent, other subjects) at earlier stages of the pipeline are therefore critical and are likely to have compound long-term impact. However, by identifying a reference student at each assessment point along the *excellence stream* of the mathematics pipeline we have illuminated details of the ebbs and flows of students with particular characteristics as they move along the pipeline. This analysis suggests that whilst prior attainment is the strongest predictor of attainment this does not account for different students making progress at different rates and at different stages. A summary of these ebbs and flows, and the possible implications for various stakeholders, are discussed below.

The gender gap varies at different points along the *excellence stream.* For instance, at GCSE, female students who achieve KS2 Mathematics level 6 are more likely to achieve grades 7–9 compared with male students and this advantage remains the case for those who achieved level 5 at the end of KS2. However, this gap reverses at A level both in attainment and participation. Female students, particularly those with grades 8 or 7 at GCSE, are considerably less likely to take A level Mathematics or A level Further Mathematics, and then perform worse than their male counterparts. This might be explained by the five factors affecting students' choice of mathematics at A level identified by Smith (2014) and observed by Golding *et al.* (2021). Their respective findings suggest prior attainment, enjoyment of the subject, interest in the subject, perceived utility of the subject and perceived competence in mathematics explain the differential preferences for girls' or boys' choices to study mathematics at A level. Additionally, this may well be on account of the relationship between their mathematics attainment and that in other subjects (see, for example Noyes, 2009). Although female students are underrepresented in the undergraduate mathematics cohort, when controlling for prior attainment and other variables, there is no significant difference in the probability of them starting a mathematics degree compared to male students. Therefore, interventions aimed at female students should target the secondary phase with the aim of enhancing participation and attainment in the A level phase of the pipeline, as this should then feed through to greater parity at undergraduate level. Despite female students outperforming male students once enrolled on undergraduate courses, they are then less likely to choose postgraduate degrees in mathematics. A robust evaluation of the mentoring, STEM clubs, events and social media content run by Stemettes might reveal potential interventions to address gender inequalities.

Students with higher income deprivation, as measured by IDACI scores or Free School Meals eligibility, are less likely to achieve top grades in the GCSE and A level phases of the pipeline compared to their more affluent peers. The differentiation in A level participation by IDACI quintile contrasts with an earlier study (Noyes, 2009; Noyes & Adkins, 2016), which found no effect. In the intervening period there has been a notable increase in students opting to take A level mathematics, but our data suggests this growth has not been balanced across the IDACI quintiles. Changes in education policy, for example increasing university tuition fees and promoting mathematics as a means of increasing earning potential, may be a contributing factor. Furthermore, when other student characteristics interact with high levels of income deprivation, such as ethnicity or SEN, the likelihood of progressing along the *excellence stream* is further reduced. For instance, students from low-income backgrounds are less likely to take A level Mathematics, and this effect is stronger if they are also White, male or have English as their first language (which, incidently, are the largest demographics of student in the A level cohort). If students with low-income backgrounds can overcome the disadvantage to obtain high A level grades, they are then more likely to enrol on a mathematics degree than their wealthier peers and, once controlling for prior attainment and other variables, there is no significant difference in their attainment at degree level. However, students who study part-time, which may be a necessity for students from low-income backgrounds with work commitments, are less likely to achieve first-class degrees than full-time students. Sector-wide approaches such as IntoUniversity's long-term support and Sutton Trust's short-term summer schools are both interventions seeking to inspire young people from economically disadvantaged backgrounds.

At GCSE, Asian ethnicity students are likely to outperform those of White, Black and mixed ethnicities, and Asian and Black ethnicity students with less than the highest grade 9 are more likely to participate in A level Mathematics or Further Mathematics than their White or mixed ethnicity counterparts. These differences might be explained by differing cultural attitudes to mathematics or parental expectations for their children. After controlling for prior attainment and other variables, there is no significant difference between the ethnicities in A level performance. Despite having among the lowest GCSE attainment and A level participation, White ethnicity students are most likely to enrol on an undergraduate mathematics degree (along with Asian ethnicity students) and most likely to achieve a first-class degree compared to students of Asian, Black and mixed heritage, and particularly Black females. Interventions aimed at reducing ethnic disparities through raising aspirations and attainment would therefore do well to target non-Asian ethnicities at GCSE and A level transition phases and non-White ethnicities in the Higher Education phase. The LMS Levelling Up: Maths for Black Heritage Students programme, which started in 2023, is an intervention taking this approach working in partnership with five UK university mathematics departments.

In addition to these ebbs and flows for these students there are several other patterns that surface from our analysis. Firstly, the apparent value of Further Mathematics as an indicator for undergraduate study may be a red herring. Somewhat counterintuitively, though performance in A level Further Mathematics is associated with increased likelihood of undertaking a degree in mathematics, undergraduates without Further Mathematics are no less likely to achieve a first-class degree outcome once we have controlled for institution and other variables. For universities where Further Mathematics is an optional part of the entry criteria, this suggests admissions tutors should not place too much emphasis on it, while for other students it remains an essential component of accessing courses at the minority of elite institutions where knowledge of Further Mathematics is assumed in the year 1 curriculum. Students of Further Mathematics still feel unprepared for more abstract and proof-based university mathematics (Lyakhova & Neate, 2019) and reforming the qualification to address this point may result in Further Mathematics being a more positive predictor of degree outcomes in our models. Incidentally, while all models fitted the data well, we note that the quality of the model fit is lower for the undergraduate attainment and postgraduate participation models. This is partly due to the smaller cohort size at these phases of the pipeline but might also indicate that ‘success’ in these phases is also due to variables not included in the data. These unknown variables might be associated with the maturity and motivation of the learner as they transition away from parental and school support to independent living and studying. Secondly, we observe that students with SEN often progress at different rates compared to their peers, sometimes faster (e.g. A level) and sometimes slower (e.g. GCSE). Since SEN encompasses a wide range of disabilities and learning differences, further research is needed to fully understand these patterns and address the inequalities. Thirdly, we note there are institutional differences. After controlling for prior attainment and other variables, both independent and 16–19 schools outperform mainstream schools and further education colleges in the A level phase. In the Higher Education phase each institution sets its own degree standards so care should be taken when making comparisons, but it appears that first-class degrees are less likely to be awarded by Russell Group universities when controlling for prior attainment and student demographics (Fig. 5(d)).

Our analysis provides a snapshot of two cohorts, Cohort 1 negotiating GCSEs in 2017 and A-levels in 2019 and Cohort 2 negotiating university starting in September 2015. As noted earlier, Cohort 1 was the first to experience the numerical GCSE grading system and revised A level Mathematics qualification and the last to complete their A levels prior to the Covid-19 pandemic. Analysis of subsequent cohorts may observe, for example, the ‘Sawtooth Effect’ as new qualifications bed in (Ofqual, 2016) and an increased impact of economic disadvantage in the teacher-assessed grades of 2020 and 2021 (Cullinane *et al.*, 2023). We have not sought to measure the effect of other school-level variables, such as streaming of students into higher and foundation GCSE papers, pedagogical practise and choice of curriculum and resources, which are not recorded in the National Pupil Database. Regular further analysis is needed to confirm how the patterns of participation and attainment noted in our analysis evolve in response to changes in qualifications, policies, practise and global events.

5. Conclusion

Our analysis highlights the eddying currents and complex flows of learners through mathematics education from the start of school to postgraduate study. Some of the findings, such as gender and income-deprivation gaps, were expected and these gaps have been systematically tracked as they widen and narrow through the different education phases. Other findings, such as the apparent lack of benefit of A level Further Mathematics for some university courses, are more unexpected. Overall, the complexity of the results suggests there is no magic bullet that can resolve the challenge of increasing engagement, outcomes and participation in mathematics to 18 and beyond. While some interventions, such as the AMSP (Boylan *et al.*, 2016), have made a positive impact, consideration also needs to be given to the wider landscape in which they operate (Noyes & Adkins, 2016). In truth, there remains inequalities at all stages of the pipeline and further action is required by government and the mathematics education community. In the Pipeline Report (Noyes *et al.*, 2023) we made some suggestions about potential interventions that might be developed, or phases where efforts might be best focused, in order to get some purchase on the mathematics problem. However, there is always a risk that one ends up with a version of policy or intervention whack-a-mole; as progress is made on one issue, several others pop up to limit or even counteract that progress. Perhaps, therefore, the key idea we should take from this analysis is the need for more systems thinking that produces well-coordinated or orchestrated approaches to understanding and improving mathematics education for all, and thereby increasing the *excellence stream*. As explained by Mason (2016):

> . . . because we can never know well enough the combination and salience of factors that are causing the school's or the system's failure, or exactly what it is that will turn things around, our best chance of success lies in addressing the problem from as many angles, levels and perspectives as possible.

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Chris Brignell is Associate Professor of Statistics at the University of Nottingham where he is Head of Mathematics Education and Scholarship in the School of Mathematical Sciences. His research interests focus on pedagogical approaches and student engagement in higher education mathematics, as well as interdisciplinary science education and education for sustainable development. He is an accredited ONS researcher and conducted the analysis of the National Pupil Database and HESA data on this project.

Andy Noyes is Professor of Education at the University of Nottingham where he has been Head of the School of Education and APVC for Research in the Faculty of Social Sciences. Andy is Chair of the Joint Mathematical Council of the UK and a member of the Royal Society's Advisory Committee on Mathematics Education. His research interests centre on post-16 education, on change in complex systems and on educational policy. He has advised DfE on post-16 mathematics and worked with regulators in England and Scotland on qualifications reform.

Laurie Jacques studied undergraduate mathematics and has an MA and PhD in Mathematics Education. Formerly a primary teacher (1998–2009), she has been a member of the Advisory Committee on Mathematics Education (2005–2010), panel member of the Williams Review (2008) and Director for Policy and Quality (and later for Primary) for the NCETM (2009–2013). She now works as a teacher educator and researcher in mathematics education with research interests in policy implementation from a classroom perspective and teaching mathematics for equitable classrooms.