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Practice case study

Virtual Maths Circles: helping young people to think like researchers

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Abstract

The Edinburgh Maths Circles are free sessions for families, run by staff and students from the University of Edinburgh, UK. Their main aim is encouraging children to think and behave like mathematics researchers, as they explore open-ended questions, make their own conjectures, and explain their reasoning to others. Maths Circles started in 2016 as in-person events, with an online version created in response to the outbreak of Covid-19. To spread the initiative as widely as possible across Scotland, we have been running development workshops for over four hundred teachers and educators, and we have visited local schools and libraries. The ideas and methods of Maths Circles have moved beyond our disciplinary boundaries, as other departments at the university are planning activities based on this model. In this practice case study, we illustrate the aims and philosophy of Maths Circles, the challenges we faced, and how the initiative grew over time. We describe our work with local schools/communities, and how we adapted to Covid-19 restrictions. We also share personal experiences from researchers, university students and educators who contributed to the project. We aim to encourage colleagues to adopt a similar approach in other contexts and disciplines, and we are open to cross-disciplinary collaboration.

Keywords maths club; maths outreach; maths education; parental engagement; widening participation; problem solving; problem-based teaching; mathematics; education; NRICH

Key messages

- Mathematics often adopts an answer-based approach. An exploration-based approach, empowering young people to think like maths researchers is very challenging to put in place, but very much appreciated by participants and (arguably) very beneficial in the long term. Such an approach also lends itself to interdisciplinary collaborations.
- Creating a large-scale event may, initially, seem extremely intimidating. It is therefore important to proceed gradually, and to involve people who act as intermediaries between universities and the community. Moving online can massively increase the scale of such initiatives, but adapting a successful face-to-face event to a virtual setting is extremely challenging.
- Contributing to the organisation and delivery of public engagement activities is an excellent opportunity for researchers, and especially for students, starting from undergraduate level. They have the opportunity to develop necessary skills, as well as a different insight into, and renewed enthusiasm for, their discipline.

Introduction

Mathematics is often presented and perceived as a mechanical subject, often identified with numeracy, and with an emphasis on 'finding the right answer'. The authors, and probably all mathematicians, have personally experienced a change in their perception of the subject when progressing from school-level to university-level mathematics. At that point, we learnt that numeracy is only a part of mathematics; we discovered a new side of mathematics, where creativity, exploration and problem solving are emphasised, with many open questions still to be unpacked. We strongly feel the need to help people to familiarise themselves with this 'unknown' side of mathematics, and to get a glimpse of mathematical research from a young age. This is what we are striving to do through our Maths Circles.

In this practice case study, we first describe the aims and philosophy of Maths Circles. We then discuss how the initiative started in 2016, the challenges we faced, how it evolved over time, and how we responded to the Covid-19 pandemic. We go on to detail the associated challenges and feedback from participants, and to discuss our work with educators, schools and communities. Finally, we provide an insight into the experiences of the organisers and volunteers.

Maths Circles aims and philosophy: an investigative approach

The Edinburgh Maths Circles are free sessions for children aged 5–16 and their families, run by the School of Mathematics at the University of Edinburgh, UK. They aim to introduce children to mathematical research, and to involve parents in their children's learning. The latter has been shown to be crucial in raising attainment and engagement (Rasbash et al., 2010; Scottish Government, 2018).

Rather than engaging the audience with the content of current mathematical research, which is often difficult due to the abstract nature and the hierarchical structure of the discipline, we aim to equip them with the research mindset and processes. Ultimately, we want to raise awareness of what it means to be, and to think like, a mathematician. To reach our aim, we adopt a problem-based and exploration-based approach, rather than an answer-based approach.

Our activities prompt children to behave like little mathematicians, as they tackle open-ended questions, based on age-appropriate mathematical concepts. Each problem focuses on one or more aspects of mathematical thinking, such as thinking systematically, understanding the difference between a conjecture and a proof, developing a mathematical argument, or *generalising* (for example, the process of finding a general formula starting with limited data). Figure 1 shows an example of an activity which encourages generalisation and developing a mathematical argument.

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Figure 1. Example of an activity: seven flipped

We have seven mats which are red on one side and blue on the other. The mats all begin with the red side facing up, as shown above. The challenge is to flip the mats so that all seven mats end with the blue side facing up. However there are three rules we must follow:

1. We must flip exactly three mats in each move
2. The three mats we flip can be from anywhere in the line
3. A mat can be flipped over on one move and flipped over back again on a later move

The main question is: **What is the smallest number of moves we need to flip all the mats?**

It turns out this number is much smaller than we think.

Children are prompted to have a go, and to try different techniques to solve the problem. After solving the seven mats problem, children are prompted to explore what happens with a different number of mats, for example, five mats, or six mats, or ten mats, or nineteen mats. The ultimate aim is to figure out whether there is a general formula which, given the number of mats, determines the smallest number of moves we need to flip them all.

This problem appears to be very popular with all children, both because it does not immediately look mathematical, and because it provides a hands-on challenge that anyone can try (low threshold). On the other hand, the challenge is not trivial, and even mathematicians can get stuck.

After solving the first question, children are prompted to think deeper, explore patterns, make their own conjectures and, in the best-case scenario, come up with a general formula (high ceiling). This is a process which mathematicians call 'generalisation'.

In our experience, many children (and adults) tend to stop after solving the first challenge. Hence, some effort is required on our part to prompt them to go out of their comfort zone and explore further. If you are interested, you can read further about the problem, and find a solution at https://Nrich.Maths.org/4871.

Activities are 'low-threshold high-ceiling' (NRICH team, 2019), that is, they have a low barrier and are easy to get started on, but they contain deeper questions for those who want to delve further. In this way, no one feels intimidated, while participants get a chance to engage with research-like thinking, sometimes without even realising. Many of the questions are open-ended, or admit more than one right answer. We hope that this will debunk the perception of mathematics as a mechanical subject where everything is right or wrong, demonstrating the creativity and curiosity behind mathematical research (McClure, 2019). Unlike mathematical contests, we discourage competition, and no emphasis is placed on speed or avoiding mistakes. Maths Circles emphasise that getting stuck, making mistakes, and learning from them are important parts of mathematical thinking (Boaler, 2014; NRICH team, 2019). We hope that familiarising people with this idea from a young age will positively impact societal perception of mathematics.

We select activities from all areas of mathematics, from numeracy to geometry, from logic to statistics, hoping to raise awareness of the breadth of mathematics, and to attract people who do not like numbers.

How Maths Circles evolved over time: a challenging and rewarding journey

The structure of in-person Maths Circles

The idea of Maths Circles began in Cambridge with Dr Vicky Neale, and it was brought to Edinburgh in 2016 by Dr Zoe Wyatt (then a PhD student).

Until 2019, our Maths Circles were three-hour-long drop-in sessions for children aged 5 to 16 and their families. These sessions had a free structure, and were held in a large room with about 70 activities. Children were invited to choose a problem that interested them, explore it at their own pace, and spend

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as long as they liked on it, before moving on to another activity. The organisers, together with other staff and students from the University of Edinburgh, were on hand for guidance. We would encourage participants to explore the problems, make their own conjectures, and explain their reasoning to others. When we noticed a participant struggling, rather than providing solutions to the questions, we would give hints and point them in the right direction, while still allowing them to explore and apply critical thinking.

The free structure of the event was intentional, allowing children to learn some new mathematics without feeling intimidated by a rigid set-up. The activities were grouped by age, and placed on round tables, encouraging collaboration. We ensured that each table contained activities of different difficulty levels, to encourage children to try both 'easy' and 'hard' problems.

Holding the event in a large room, rather than splitting participants across different rooms according to age, was also intentional. This made things easier for families with children of different ages, and it removed any physical barriers for children to try activities which were nominally aimed at a lower or higher age.

Challenges at the time of setting up Maths Circles

Setting up the Edinburgh Maths Circles, and keeping them alive, was both very challenging and very rewarding. It required a lot of effort and thorough planning in terms of selecting appropriate activities, purchasing material, and analysing, improving and creating activities. Promoting a new event also required some work. Initially local schools were contacted by phone or email, and asked to share posters and flyers. Thankfully, the event has been very popular since day one, and it has mainly spread by word of mouth since then. A specific mailing list was then created, which has grown organically to over a thousand subscribers, and which now constitutes the main channel for promotion.

Training volunteers was another challenging aspect, especially due to the high turnover of students. Training was first delivered in the form of a short briefing on the day. In 2019, we started running additional bespoke sessions for the volunteers, outlining the aims of Maths Circles, and focusing on how to respond to several challenging scenarios.

Our efforts have been immensely rewarded, as we saw Maths Circles become increasingly popular, attracting an average of 250 visitors per event.

Moving online

The outbreak of Covid-19 presented us with the challenge of restructuring a successful face-to-face event to suit online delivery. We wanted to retain some of the key features of Maths Circles, such as the interactions between university students and staff and the public, and the free structure. Both features were very difficult to recreate online.

Our first online event, held on Zoom in August 2020, tried to recreate a similar environment to the in-person Maths Circles. We selected 12 problems, and emailed them to participants in advance of the session. During the live event, participants were able to move across four virtual rooms, based on the problems they wanted to tackle. Three of the virtual rooms were led by us, with volunteers leading discussion based on specific activities. One of the virtual rooms was led by the participants, who worked at their own pace on activities of their choice, with volunteers available to answer questions via the chat. Feedback showed that this set-up was confusing for participants, and it was difficult for us to handle. Moreover, participants seemed to prefer the semi-structured presentations over the self-paced activities. Such feedback led us to infer that an entirely free structure based on self-paced work did not work online.

We decided to restrict the age range, and to split the online Maths Circle into two one-hour-long slots, for ages 7–11 and 11–14, respectively, with each slot focusing on only four problems, with two breakout rooms available. All problems were emailed to participants in advance of the session. During the live event, children chose to explore two of the four problems, and to join the appropriate breakout

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rooms. Each of the problems were explored in depth by one of the demonstrators, who led a discussion, encouraging ideas and suggestions from participants. Feedback suggested that this revised structure worked much better.

While delivering our second online Maths Circle, we realised that while some participants enjoyed looking at the problems in advance, such expectation may create stress for others. Based on advice from experienced colleagues, we decided that for future events, we would send only half of the activities in advance, keeping the other half as a surprise.

Overall, by moving online, we compromised one of the features of the in-person Maths Circles, that is, the free and open structure. However, we still maintained an element of choice, and we were able to foster interaction between participants and our staff and students, as volunteers encouraged ideas and discussion. We also feel that we remained faithful to our ultimate goal of helping young people to develop research skills and mathematical thinking. Rather than presenting a solution straight away, we prompted participants to work together, to share their thoughts, to motivate their conjectures, and, in this way, to engage with mathematical thinking.

The different format was inevitably more challenging and demanding for our volunteers, especially our undergraduate students. Much more preparation and confidence was required of them to adapt to a new style of presenting: they needed to lead a large group discussion, constantly acting on participants' ideas. The organisers made a significant effort in training and encouraging the volunteers, both through bespoke sessions and through one-to-one guidance. This paid off immensely, as we saw our students growing in their passion and confidence.

Moving online allowed us to reach people from across Scotland, including remote areas, as well as other parts of the UK, and oversees. We recognise, however, that digital events may exclude other groups.

Our online Maths Circle took place almost every month between August 2020 and May 2022, with an average of 70 families attending each event, and many returning participants. We constantly sought and acted on feedback, while understanding issues of selection bias in optional surveys. Feedback was overall very positive, with participants consistently rating our virtual events as *good* or *very good*. Suggestions for improvements included comments on specific presentations, which were addressed by the presenter in question. Other inputs were practical, for example, having smaller age groups, smaller breakout rooms, longer sessions or extra sessions for those over 14; we were unfortunately unable to act on these due to limited capacity. A minority of participants asked for 'more difficult puzzles'. We found this comment tricky to act on, given the diverse background of participants.

Through the process of moving our Maths Circles online, we learnt that even something which seems ambitious and intimidating can be achieved; the key is making small steps at a time, and being flexible and ready to compromise and explore new ways of structuring activities. We also (re)learnt that it is important to keep things simple, and that, when it comes to technology, the audience does not expect everything to be perfect, but instead appreciates interaction with students and researchers as 'human beings'.

The approach and nature of the activities: helping children to think like mathematicians – challenges and reward

Our choice to adopt an exploration-based approach brought some challenges. It was not unusual to have children, or even adults, getting nervous, and asking for answers and solutions immediately. This attitude echoes literature which identifies some anxiety towards mathematics, not only in children, but also in adults and schoolteachers (Hunt and Sari, 2019; Wilson, 2018). To respond to this, we strived to create a supportive environment, and we always made it clear that mathematics is not about getting to the right answer quickly. We emphasised that exploring and 'getting stuck' is part of the learning experience, and that even professional mathematicians often struggle when trying to solve a problem.

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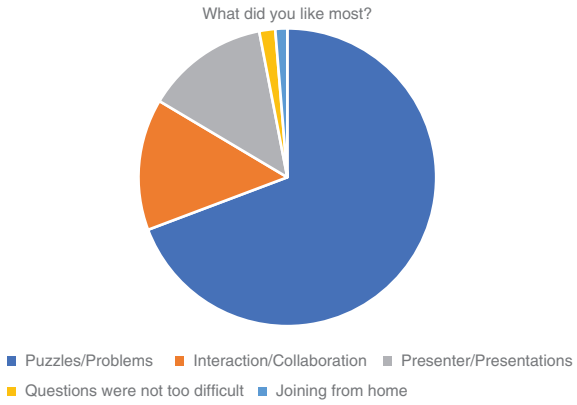
Figure 2. A summary of the answer to the question 'What did you like most about the event?', grouped according to key themes



There was a similar challenge with the low-threshold high-ceiling structure of the problems. Often participants would approach an activity, be content answering the most basic questions, and then move to the next activity. We would strive to encourage them to delve deeper, or to explore the open-ended questions.

We also found that it was best not to reveal the difficulty level of each problem to participants, to allow children to approach the activities without any preconceptions. In this way, less confident children would approach harder questions, and often do brilliantly.

Feedback seems to show that we have been successful in our goal of introducing the mindset behind mathematical research in a fun and approachable manner (and without calling it 'research'). Participants appreciated the style and nature of the activities, the problem-solving element, the interactive element and the discussion-based approach. All such aspects are at the heart of mathematical research.

This is evidenced by Figure 2, which is based on data collected from participants of each virtual Maths Circle between August 2020 and June 2021, and which covers 305 responses.

Most participants also stated that Maths Circles helped them to enjoy mathematics more, and that they found Maths Circles different from the way they learn mathematics at school. Such differences lie in the nature (and enjoyability) of the activities, the challenge, the space given to mathematical thinking or problem solving, and the interaction and discussion. A common theme was how participants encountered 'a different side of mathematics', which, arguably, is not prevalent in the school curriculum. These feelings are exemplified by quotations from participants:

> It is a fun way to view another side of math! (7-year-old)
>
> What I like most… is that it challenges me and makes me think about maths in a different way. (14-year-old)
>
> They are fun but challenging and make the kids think around the problem, exploring why and not just finding the answer but developing the solution. (parent)

Widening our reach: our work with schools and communities

Although Maths Circles were very popular, a significant proportion of the population may feel intimidated by the idea of visiting a university or joining an online mathematical event. Hence, we decided to involve schoolteachers and community workers, to act as intermediaries between the university and local communities, helping us spread the initiative more widely.

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School workshops

Since 2016, we have run Maths Circles in Scottish schools, both face-to-face, and virtually after 2020. Feedback showed that the investigative approach and the low-threshold high-ceiling activities engaged the whole class effectively. Most teachers commented that there were challenging elements for the high-achieving children, while even children who usually struggle were able to engage and learn something new. The level of enjoyment was very high, even when the class did not previously enjoy mathematics.

Workshops for teachers and educators

Despite the success and importance of school visits, we had limited resources and capacity to scale up.

In 2018, we decided to run workshops for schoolteachers and educators, where we shared some of our activities and resources, hoping to empower attendees to run Maths Circles in their own schools and communities. The workshops were advertised through local schools and specific social media groups for Scottish teachers.

In line with our mission, these workshops have a strong practical component. We first introduce the aims and philosophy of Maths Circles, and how we empower children to think like mathematicians. Then we give participants a chance to be in the children's shoes, as we explore challenging activities together. Throughout the workshop, we emphasise some of the pillars of our problem-based approach, discussed above. Between March 2018 and June 2021, we ran 11 workshops for Scottish schoolteachers – 4 in person, and 7 online after the start of the Covid-19 pandemic – as well as a workshop for library staff. The move to online workshops led to a massive increase in attendance and scale, and a much broader geographical reach. Over four hundred educators from across Scotland attended these workshops, and expressed extremely positive feedback (see Table 1).

This feedback mirrors that from Maths Circle participants. Like children and parents, educators seem to value our approach, where more space is given to 'research-like' attributes. Educators appreciate the style and variety of the resources, the problem-solving aspect and the investigative approach. They also enjoy the interaction with colleagues, researchers and students, and the practical element of the workshops. This is evidenced in Table 1 and Figure 3, which are based on feedback from attendees, collected over seven online teacher workshops between December 2020 and May 2021.

More than 75 per cent of those who completed the survey found our approach to the problems different from that in schools. They note that more space is given to exploration and discussion, that there is better focus on the process rather than the answer, and that the problems are open-ended. These threads are exemplified by these quotations from participants:

> [The workshop] showed how maths can be an investigation, not just a quest for the correct answer.

> Helpful to reinforce the idea of allowing pupils to get stuck and come up with solutions.

Table 1. The average score for some of the questions included in the surveys, over a total of 158 feedback forms

Question	Average score (out of 5)
How would you rate the workshop overall?	4.55
How useful did you find the workshop?	4.43
How enjoyable did you find the workshop?	4.45
How interesting did you find the material?	4.44

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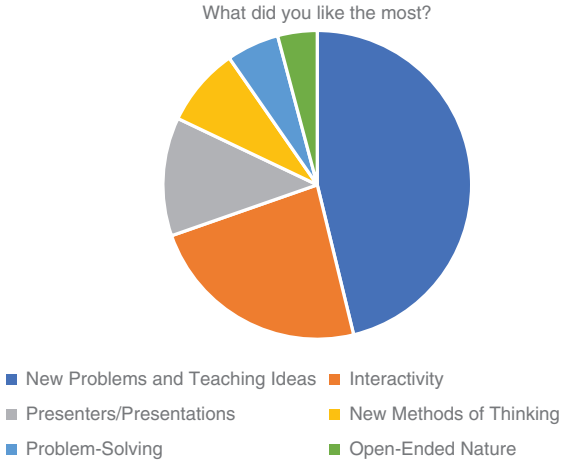
Figure 3. A summary of the answers to the question 'What did you like most about the workshop?', coded according to key themes, over a total of 141 responses



Most respondents also affirmed that they will use either specific resources that we shared, or our approach more generally:

> The idea of low-threshold high-ceiling is something I will integrate into my teaching.
>
> [As a result of the workshop, I will] get children talking about maths problems more than they possibly are at the moment.

Most respondents had no improvements to suggest. Some asked for more frequent or longer workshops, or for handouts/presentation slides to be emailed to them afterwards. We responded to the best of our capacity. Some participants asked to link the resources to the school curriculum, give practical suggestions on using them in the classroom, or focus on how to engage less confident students. We tried to address these requests by framing each activity within the curriculum, and by pointing out, for each resource, how people of different abilities could engage at different levels. Unfortunately, due to the large scale of the workshops, it was difficult to evaluate the long-term impact. Anecdotally, the organisers continuously receive positive written or verbal feedback from teachers who have previously attended our workshops, but no quantitative data are available.

Long-term collaborations

On a deeper level, our workshops sparked long-term collaborations with some organisations, as some of the educators decided to run Maths Circles on a regular basis in their community. An example is Edinburgh Libraries, who received very positive responses from participants, which surprised even library staff members. Another example is the Family Learning Team in North Ayrshire, who, after attending an educator workshop, asked us for follow-up training, and decided to use our model to create a family programme across their area.

As organisers, we hugely appreciate working with teachers and communities, and engaging in a powerful sharing of expertise: we shared our insight into mathematical research, while educators provided a unique perspective and many practical suggestions on how best to extend the initiative to underserved areas. Regarding the content, we learnt how educators and children really appreciate activities which link mathematics to games or to everyday life. From a practical point of view, we found that it is beneficial for educators to have hands-on resources ready to use. Hence, we produced a box with a selection of about 30 activities, which we are now loaning to schools/communities on demand. We also learnt that

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communities appreciate regular and constant dialogue. This has prompted us to think carefully about how to strike the right balance between large-scale events and small-scale but more regular contacts with specific communities.

Our experience and learning

This section is inspired by many personal quotations, through which organisers and volunteers have shared why Maths Circles have been such a great learning experience for them.

Organisers

The organising team has included people from different backgrounds: public engagement professionals, researchers, undergraduate and postgraduate students, all bringing experiences, skills and unique points of view to the project. We all learnt from each other and from the community, as Maths Circles kept fostering our passion for mathematics, and our enthusiasm for sharing this passion.

We found Maths Circles a very joyful experience, despite it being tiring and challenging at times. The children's contagious enthusiasm was inspiring, and at times we learnt from them as they approached some of the activities in unique ways. It was encouraging to see the children able to collaborate and arrive at solutions with only gentle guidance. It has been a privilege to be able to play a part in their mathematical journey.

It was really rewarding to see the initiative grow, as the programme spread to schools and communities across Scotland, to a scale we would have never anticipated.

We learnt not to be discouraged by ambitious projects, and that it is important to have very high aims, but to take small, achievable steps gradually. We also learnt about the pivotal importance of a constant and meaningful dialogue with schoolteachers and community workers, as they bring crucial insights and make us aware of their challenges.

Recruiting and training the volunteers (mostly university students) has also been very rewarding, as we could see how their enthusiasm, confidence and skills grew over time.

Researchers and university teachers

Maths Circles were an extremely valuable experience for the researchers and academics involved, who were able to share their passion for mathematics with young people.

As Zoe Wyatt suggests: 'The Maths Circles encourage two behaviours that are very important in mathematics. Firstly, the problems frequently build up a mathematical theory by using examples. … Secondly, the idea of mathematics being a collaborative endeavour.' A reinforcement of these behaviours benefits mathematicians at all stages of their careers, from young children to experienced researchers.

Academics were able to develop a better awareness of different learning styles and best practice in student-centred learning. University staff who participated were inspired to develop problem-driven learning across the curriculum. While the material differs significantly between Maths Circles and university-level, many of the approaches are directly transferable. The techniques for ensuring that students are motivated to try problems which may initially appear to be hard, working collaboratively, and setting up open-ended problems are valuable at all levels of teaching and learning.

University students

Feedback from student volunteers shows how rewarding they find Maths Circles, and how much they enjoyed the surprising enthusiasm, willingness to learn and creativity of the participants.

All volunteers commented on how the Maths Circles provide the perfect opportunity for them to practise exploring and presenting mathematics beyond their coursework, as well as listening to someone else's reasoning. Many of the volunteers have presented at several virtual Maths Circles, and have felt their

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confidence and ability in presenting increase each time. The volunteers also enjoyed hearing the different mathematical logic that participants used, which was often different from their own. This gave them good exposure to different viewpoints, and encouraged them to provide different styles of explanations, which will be extremely beneficial for their own studies.

Those students who became involved with the organisation improved their planning, management and leadership skills.

Moving beyond our disciplinary boundaries

Our belief that problem solving and critical thinking are the basis of many disciplines has been further evidenced by cross-curricular activities and collaborations with both science, technology, engineering and mathematics (STEM) and humanities researchers.

We are currently working with linguists on the production of resources at the interface between mathematics and linguistics, with an emphasis on problem solving and the study of patterns and structures. While still at an early stage, this project has already been beneficial to university staff and students, giving them an appreciation of unexpected links between disciplines. We ran successful workshops for mathematics and language teachers, and we are planning to collaborate with local schools and libraries. We hope that this initiative can shed a new light on the perception of both mathematics and linguistics, and, in general, on the links between STEM and the humanities.

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Declarations and conflicts of interest

Research ethics statement

Not applicable to this article.

Consent for publication statement

The authors declare that research participants' informed consent to publication of findings – including photos, videos and any personal or identifiable information – was secured prior to publication.

Conflicts of interest statement

The authors declare no conflicts of interest with this work. All efforts to sufficiently anonymise the authors during peer review of this article have been made. The authors declare no further conflicts with this article.

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