Article

Urban Studies

Urban Studies
1–29

DOI: 10.1177/00420980241237410
journals.sagepub.com/home/usj


The role of analytical models and their circulation in urban studies and policy

Clémentine Cottineau
Delft University of Technology, The Netherlands

Michael Batty
University College London, UK

Itzhak Benenson
Tel Aviv University, Israel

Justin Delloye
Université Catholique de Louvain, Belgium

Erez Hatna
New York University, USA

Denise Pumain
Université Paris 1 Panthéon-Sorbonne, France

Somwrita Sarkar
University of Sydney, Australia

Cécile Tannier
CNRS-Université de Franche-Comté, France

Rūta Ubarevičienė
Delft University of Technology, The Netherlands
Lithuanian Centre for Social Sciences, Lithuania

Corresponding author:
Clémentine Cottineau, Department of Urbanism, Faculty of Architecture and the Built Environment, Delft University of Technology, Office BG West 700, Julianalaan 134, Delft 2628 BL, The Netherlands.
Email: c.cottineau@tudelft.nl

Abstract

Cities are so complex that we constantly build models to represent them, understand them and attempt to plan them. Models represent a middle ground between the singular configurations of cities and universal theories. This is what makes them valuable and prone to circulate (between places, institutions and languages) and evolve to adapt to new ideas, local conditions and/or other models. When it comes to analytical urban models (i.e. analytical representations of cities developed to study or simulate part of their structure or dynamics), there is a lack of academic understanding regarding how context and circulation affect their content, use and interpretation. What happens to analytical urban models and their reception during their circulation across geographical and disciplinary boundaries? How have different academic disciplines interacted with, contributed to and been influenced by analytical urban models? What are the consequences of urban models' mobility for our understanding of cities? In this article, we employ the policy mobilities framework to analyse the circulation of analytical urban models. We use six canonical models as case studies to determine how their assumptions came about and how these models have circulated across different domains of policy and application by using biographical information and model analysis. The first contribution of the article is to demonstrate by example that our hypothesis regarding the influence of context is consistent. We also show that highly transferable/mobile models share common characteristics relating to contingent factors such as their creators' biographies, institutional context and the traditional markers of power relations.

Keywords

biography, circulation, contingency, mobility, simulation, urban model

摘要

城市非常复杂，所以我们不断地给城市建模，去呈现、理解城市，甚至尝试规划城市。模型代表了单一城市配置和普遍理论之间的中间立场。这使得它们具有价值并且易于传播（在不同地方、机构和语言之间），并且能够不断发展以适应新的理念、当地条件和/或其他模型。当谈到分析性城市模型（即为研究或模拟城市部分结构或动态而开发的城市分析表示）时，关于背景和流通如何影响其内容、使用和解释，学术界缺乏相关的了解。分析性城市模型及其接受度在跨越地理和学科界限流通的过程中会发生什么变化？不同学科如何与分析性城市模型相互作用，如何影响分析性城市模型又受其影响呢？城市模型的流动性对我们理解城市有何影响？在本文中，我们采用政策流动性框架来分析分析性城市模型的流通。我们以六个典型模型作为案例，进行案例研究，通过传记信息和模型分析来确定它们的假设是如何产生的，以及这些模型如何在不同的政策和应用领域中传播。本文的第一个贡献是通过范例证明了我们关于背景影响的假设是不矛盾的。我们还表明，具有高度可转移性/流动性的模型，具有与一些偶然因素相关的共同特征，这些偶然因素包括创建者的人生经历、制度背景和权力关系的传统标志等。

关键词

传记、流通、偶然性、流动性、模拟、城市模型

Received March 2023; accepted January 2024

Introduction

Cities are complex human organisations composed of 'bodies and bricks' (Fox and Wolf, 2022) as well as multiple networks linking individuals, institutions and the built environment. To help us grasp how such organisations function, how they evolve with time and what cities could look like in the future, simplified representations in the form of urban models have been developed over the last 50 years or more (Pumain, 1998). These models are either generalised from specific exemplar cities or deductively devised from urban theories. Either way, models represent a middle ground and a mediator between the singular configurations of cities inscribed in a unique/contingent spatiotemporal frame on the one hand, and universal theories removed from a 'messy' urban reality on the other hand. 'Models are separate constructions, moving between, translating, arbitrating, and interceding in differences between the two worlds, and in so doing, making sense of them' (Barnes, 2008: 6). This is what makes urban models valuable. Generality makes their application possible and fruitful across many types of geographical space and organisation: from research departments to policy departments (and back again). During this circulation, the content and reception of urban models tend to change and adapt to local, epistemic and social conditions. Thus, urban models serve a dual bottom-up as well as top-down purpose. First, they contribute a significant and highly visible share of the current scientific approaches to cities (i.e. the 'new science of cities', see Batty, 2013), thereby allowing us to examine urbanism as a self-organised phenomenon with its own evolutionary trajectories and development. Second, they can enable testable policy interventions that allow us to examine urbanism as a process of engineering, invention and planning.

Although urban models can have multiple forms, functions and meanings, in this article we focus on the models that can be called analytical. Compared to more theoretical or normative urban models, analytical urban models aim to better understand and explain the processes behind certain urban features (i.e. centrality, urban form, population distributions, etc.) using a scientific and usually formal approach (e.g. equations, diagrams, statistics, simulations, etc.). Such models are described as analytical in that they proceed to decompose and simplify a complex matter to better understand it, in the tradition of spatial analysis. Unlike normative urban models (i.e. models of cities as blueprints of a desired future), the circulation of which is described and explained by the literature on (urban) *policy transfer* (Dolowitz and Marsh, 2000) and (urban) *policy mobilities* (McCann and Ward, 2012; Peck and Theodore, 2010), we lack a more detailed understanding of how individual analytical urban models travel and circulate in time and (social) space, and how this circulation can affect their content, use and interpretation, to build on the existing literature devoted to the role and history of analytical models in general science (see Dahan, 2010) and in geography (e.g. Barnes, 2004; Barnes and Abrahamsson, 2017; Orain, 2022).

What happens to analytical urban models and their reception during their circulation across geographical and disciplinary boundaries? How have different academic disciplines (e.g. economics, geography, physics and mathematics) interacted with, contributed to and been influenced by analytical urban models? What are the consequences of urban models' mobility on our understanding of cities? We ask all these questions to understand (1) the importance of contingent elements of context versus simplifications chosen on scientific grounds in the

design and circulation of analytical urban models and (2) the impacts that analytical urban model design has had both on urban studies and policymaking in the long run. We hypothesise that models, like ideas, are produced by actors embedded in contingent context settings that ultimately affect their circulation (Secord, 2004). We relied on reflexive methods from the fields of policy mobilities studies, the history of science and the sociology of knowledge production to study them. We selected six canonical analytical urban models as illustrative case studies to determine how and why their assumptions came about and how these models have circulated across different domains of policy and application by using biographical and model analysis. In particular, we show how some assumptions can become problematic once these models become appropriated by users who do not – or cannot – grasp the meaning and consequences of such assumptions for uses beyond their original purpose, such as forecasting and policy making. It was not our intention to specify the properties of these known models in relation to those of all existing or possible models in an exhaustive review, nor was it our aim to investigate the reasons for their success. Instead, we aimed to analyse how the ways in which analytical urban models circulate are comparable depending on the diversity of their objectives, the circumstances in which they were produced and the economic or political interests expressed by their users. Therefore, the first contribution of the article is to demonstrate by example that our hypothesis regarding the influence of context is corroborated. We also show that although highly transferable models share common characteristics relating to their content, a significant number of reasons for their trajectories seem to relate to contingent factors such as their designer's biography, the institutional context and the traditional markers of power relations (e.g. language, locus, gender, discipline and social capital). This information is key to future uses and adaptations that are aware of the contingency of their content and the limitations of their scope. It also corroborates previous findings in the sociology of knowledge (e.g. Collins and Evans, 2007; Secord, 2004) by highlighting how tacit knowledge and referred expertise facilitate the circulation of urban models from one discipline to the next, and from scientific institutions to policy spheres.

The first section of this article reviews the findings on urban model circulation gathered from the policy mobilities literature and discusses their value in studying analytical urban models. The second section presents the methodological strategy used to investigate the consequences of urban models' circulation on our understanding of cities. The third section presents descriptive results of the six analytical urban models' circulation and the fourth section discusses them comparatively to disentangle the factors of circulation into structural and contextual types (i.e. linked to the model components or its contingent position in a particular context). We conclude the article with a reflection on the contribution of this analysis to the future of urban modelling and simulation.

Adapting the policy mobilities framework to study analytical urban models

The circulation of normative planning practices or policies has recently become a topic of research (Peck and Theodore, 2015; Roy and Ong, 2011; Shore et al., 2011) and debate (Temenos and Baker, 2015) among geographers. The policy mobilities field of research emerged from critical policy studies to challenge:

> depictions of the smooth diffusion and adoption of best practices [...] that have been selected from a global marketplace of policy solution [...

> by being] attentive both to the constitutive role sociospatial context plays in the development and authorisation of 'successful' policy solutions (as well as those policies that are deemed to have failed) and to the relationality of policy-making sites (whether as sites of emulation, implementation, or contestation). [...] through the study of these aspects of policymaking, the policy mobilities approach seeks to be attentive to how decision-making is laden with power relations. (Theodore, 2019: 1)

This approach responded to certain policy-related questions (How can we slow down fast policies? How can we contest hegemonic policies?) as well as scientific-geographical questions (How can we explain the spread of some policy models? What are the social and political practices facilitating policy mobilities?). This represents a departure from the political science literature on policy transfer, represented by authors such as Dolowitz and Marsh (2000: 8), who focus on the international diffusion process of policies, and addresses questions such as 'What is transferred? What restricts or facilitates the policy transfer process? How is the process of policy transfer related to policy "success" or policy "failure"?'. In contrast, the policy mobilities literature is concerned with understanding the following areas: (1) the production of urban policies as socio-material assemblages rather than well-defined packages; (2) the production of cities through the global circulation of local urban policies; and (3) the role of political-economic institutions (particularly neoliberal ones) in shaping policy development and im/mobility (Baker and Temenos, 2015; McCann, 2011; McCann and Ward, 2012; Peck and Theodore, 2010; Prince, 2012). In particular, this literature argues for a '*peopling* of urban policy mobilities research' (Temenos and Baker, 2015: 843), which involves the development of a focus and methods to identify the actors involved in policy development and circulation, their role and their socio-institutional embeddedness (Larner and Laurie, 2010; McCann, 2011; Prince, 2012).

We argue that the literature on urban policy mobilities – despite being focused on different types of urban models (i.e. normative practice and public policies) – is relevant to the analysis of analytical urban model circulation for at least three reasons.

First, it acknowledges the fact that model circulation reflects the embeddedness of actors in power relations rather than rational choices. Even though analytical urban models are developed rationally by scientists and received critically by urban scholars and practitioners, 'science', much like policy, is part of the social production of knowledge. For instance, Peck and Theodore (2010: 170) asked the following questions:

> Which policies rise to the status of 'models', or objects of emulation? Do they simply rise to the top in some competitive marketplace for policies, or are they, instead, creatures of dominant interests, travelling from centres of authority along politically constructed and ideologically lubricated channels?

Their position was that urban models travel not only because of the quality of the 'supply side' but also because of their affinity with problems and needs for solutions of the demand side'.[1] This process can be partly explained by the actors' constraints:

> "Solutions-starved" actors, often under pressure to "deliver" successfully, quickly, and at low cost, "scan" globally looking for pre-tested policy models that have been anointed as "best" in one way or another, with the idea of "importing" them' (McCann and Ward, 2012: 45).

Here, a parallel exists with analytical urban models, where canonical models such as the gravity model or the monocentric city model are used as building blocks in larger models to represent typical urban patterns (e.g.

migration flows distribution and the centre–periphery organisation of urban activities respectively). Their availability as 'off-the-shelf' solutions presents the risk of reinforcing the dominance of these models outside of their application domain (e.g. the continued application of monocentric city models to real cities that have greatly diverged from that structure, or the continued application of Western models of cities to cities of the Global South). By contrast, some urban models have not been 'anointed' or have not reached the status of 'canons' despite the high quality of their content (e.g. the scholarship pertaining to ordinary cities; Robinson, 2006). This derives in part from the unequal access to sources of interactional expertise (in the sense of Collins and Evans; for instance, having direct access to policymakers and thus being able to learn the language of their needs to design a model that addresses them), communication resources (publishing in open-access journals or taking part in conferences, see Barnes and Roche, 2022) and mainstream academic legitimacy (positions in prestigious university and/or dominant disciplines). This is central to the policy mobilities literature:

> Not all policies, even ones that are locally successful, are mobilised or designated as best practices. Similarly, not all cities feature in the mental maps of policymakers or international institutions when they identify exemplars of best practice. […] Access to resources such as time, travel budgets, the media, translation services, and cultural capital would presumably make it easier for certain policy boosters to articulate their knowledge widely, and uneven access to those same resources would condition which urban actors are able to learn from global conversations about good policy. (McCann, 2011: 121–122)

Recognising this fact suggests a critical discussion of canonical urban models and their 'metadata' (i.e. the information about their creation, development and circulation). Indeed, by operating a selection of urban features, actors and scales to represent given aspects of cities, urban modellers create analytical urban models that can have a tremendous (but partly unjustified) influence on how cities are viewed and addressed by policymakers, depending on how their models circulate and are adopted and understood across time, space, languages and disciplines. Understanding the content of analytical urban models – as well as their hidden assumptions, purposes and contingent design choices – is thus crucial to guide the appropriate use of such models to avoid misleading conclusions and ultimately create better urban models that adapt to particular contexts. Additionally, this situation should entice urban modellers to broaden their search for urban model 'building blocks' beyond canonical urban models (through systematic literature reviews for instance; see Achter et al., 2024).

Second, the policy mobilities literature acknowledges the fact that representations of cities are situated and directly linked to particular contexts and individual agents. In the same manner, analytical urban models emerge from the work of individuals situated in space, time and social context, and circulate due to disciplinary translators and intermediaries. This similarity suggests that the recommended use of ethnographic techniques to analyse their significance and the importance of context in urban policy mobilities (see Peck and Theodore, 2010; Prince, 2012) could also be suited to the case of analytical urban studies. Who would be the equivalent of 'travelling technocrats' (Larner and Laurie, 2010) in analytical urban model circulations? We can think of *other* academic actors (those who read about new models, those who use models and adapt them, those who participate in the promotion of models through peer review or invited lectures, those who teach, those who oppose, those who summarise through overviews and literature

reviews) but also semi- or non-academic actors, such as consultants, researchers working for policymakers, as well as friends and romantic partners who bridge different scientific communities.[2] To be able to identify some of them, one must *follow* people and models around and analyse the historical *situations* in which they are embedded (McCann and Ward, 2012) – an 'externalist' approach being revived in contemporary geography and urban studies (Barnes, 2008; Ferretti, 2021; Rybski and Ciccone, 2023). We believe that researching the biography of analytical urban models' designers, intermediaries and translators can help us to better understand the significance and limitations of urban models' content and their potential for future use as building blocks of larger models. It is also an exercise of reflexivity because as urban modellers, academics and educators, we ourselves serve roles in the development, circulation and promotion of analytical urban models.

Third, the policy mobilities literature considers the materiality of urban policies as 'assemblages', which are not single fixed documents constantly delivering the essence of a policy applicable in the same way everywhere, but rather elements of discourse found in 'books, reports, documentaries, websites, blogs, press releases, newspaper reports, etc' (Prince, 2012: 193) assembled and interpreted in various ways by policymakers, consultants, politicians and academics, depending on the context.[3] In the case of analytical urban models, considering them as assemblages means distinguishing, for a given model, the set of equations described in the seminal publication making the model known to the modelling community from other versions of the model published in the past or subsequent documents, including simplified versions of the model in educational textbooks and the different implementations of the model in computer simulations (with all the hidden choices associated with the operationalisation of theories and concepts into analytical models; see Muelder and Filatova, 2018). The example of Thomas Shelling's models of segregation is representative in that respect. This example is referred to as either Schelling (1969) or Schelling (1971), although the model in the 1969 paper considers – only by implication – a one-dimensional space, while the 1971 model considers a two-dimensional space with very different resulting dynamics. Additionally, the default NetLogo implementation of Schelling's model differs from the original description of the model. Therefore, referring to *the* Schelling model of segregation is misleading and prone to misunderstanding since there are several variants implemented, which makes direct comparisons difficult to unravel. This is true of most models where more than a single application has been developed.

For all these reasons, we hypothesise that contingent elements of context (including social networks and power relations in which the modeller is involved, as well as the materiality of the model) can affect the content and circulation of urban models. We also hypothesise that ethnographic methods from policy mobilities studies (e.g. following actors and models and analysing the situations in which they are embedded) can help us to uncover these elements of context and their influence.

Methodological strategy to investigate urban models' circulation

We selected six important analytical urban models to demonstrate, by example, the role of contextual contingency on models' content and trajectories. Each model was analysed independently by a dedicated author following a common framework.[4]

The selection of the models and authors followed four principles:

Table 1. Overview of the models included in the analysis.

	Original language	Urban topic	Scale	Formalism	Author's expertise
Christaller's (1933) central place theory	German	Settlement distribution	Regional/national	Geometric	Interactional
Alonso's (1964) monocentric city	English	Centrality	Metropolitan	Algebraic	Contributory, interactional
Macro-economic base theory	English	Economic development	Metropolitan/regional	Algebraic	Interactional
Fractal urban models	English	Urban form/shapes	Nested scales	Geometric, statistical relationships	Contributory, interactional
	French/French				
Schelling's (1971) models of segregation	English	Residential segregation	Local/metropolitan	Rule-based simulation	Contributory, interactional
Forrester's (1969) Urban Dynamics	English	Growth	Metropolitan	Systems dynamics (nonlinear feedback)	Interactional

1. The final set of models should reflect the diversity of urban modelling. This means that they should focus on different aspects of cities (and not just transportation, for instance) and different urban scales (from the neighbourhood to the system of cities).
2. The final set of models should reflect a certain diversity of modelling formalisms (within algebraic thinking) and not be restricted to a single approach. This should ensure the presence of a plurality of paradigms and epistemologies, from methodological individualism to structuralism.
3. The models should have acquired a canonical status by having a strong influence on the discipline (rather than a measurable number of uses). This is acknowledged by the presence of said models being used in most urban studies and urban modelling curricula.[5]
4. The authors chosen to analyse the models should have a certain academic legitimacy in doing so. This implies a level of interactional expertise that can stem from more than one source. For example, they may have published books or articles discussing the model or taught the model (i.e. interactional expertise; contributing to its diffusion), or they may have contributed to its development by adapting it, extending it or applying it to new cases (contributory expertise). In distinguishing interactional from contributory expertise, we used Collins and Evans' (2007) conceptualisation of specialist expertise and its reliance on language.

As shown in Table 1, our final selection covers a diversity of urban dimensions: from urban morphology to economic activity and from spatial allocation to urban growth and dynamics. Models range from the local to the national scale, with the special case of

fractal models that cover multiple scales simultaneously by definition. Although many models are algebraic, geometric and rule-based formalisms are also represented.

We analysed these models along four lines:

1. The model overview, including its aim, components and mechanisms.
2. The biography of the model creator(s), intermediaries and the model itself, following the trail of the model's

Table 2. Model presentations.

Model	Issue/aim	Actors/features	Main finding resulting from model application
Christaller's central place theory	To explain the size, number and spatial distribution of cities that are defined as centres supplying services to the regional population.	Aggregate population, customer/supplier, transportation network, goods, centres (cities).	Few principles create a strong regularity in the distribution of cities and the functional organisation of centres into a hierarchy of nested hexagons.
Alonso's monocentric city	The formalisation of urban centrality and suburban growth both geometrically and functionally, as well as a utility maximisation-based optimisation framework.	Households, landlords, housing units, CBD, jobs, transportation costs, rents.	The trade-off between transportation costs and the consumption of other commodities creates a decreasing population density from city centres to their peripheries.
Macro-economic base theory	Explaining the urban development of cities based on their economic activities and the circulation of income between places.	Industries, cities, products, production, exports, residences, externalities.	The standard model (export base versus service) is suitable to understand how small areas develop due to the implantation of large infrastructure (e.g. industry complexes). The extended model (i.e. including circulation of other types of income, such as social and residential) shows the positive effects of the residential economy on the export and the private transfer basis.
Fractal urban models	A new kind of geometry that enables the analysis of complex, highly irregular shapes across different scales.	Geometrical sets of points that can represent buildings, transport networks, landscape features, etc.	The introduction of fractal geometric rules in the models enables geographers to better understand how space is organised through scales and how it is possible to modify this spatial organisation while considering the effects of scale dependence and the effects of distance.

(continued)

Table 2. Continued

Model	Issue/aim	Actors/features	Main finding resulting from model application
Schelling's models of segregation	The emergence of socio-spatial segregation with respect to local preferences for living adjacent to one's own kind.	Households of two mutually avoiding kinds, housing units, neighbourhood tolerance to neighbours of a different kind, relocation, segregation.	If agents leave a neighbourhood where less than a third of their neighbours are like them and then relocate to a neighbourhood where this fraction is above a third, then a city's population pattern can become totally segregated over time. The one-third threshold is essentially lower than the intuitively expected threshold of 50%, thus explaining how segregation emerges in a society of generally tolerant agents.
Forrester's Urban Dynamics	To simulate the growth of cities using positive (non-linear) and negative equilibrating feedback.	Capacity, population, jobs, stocks, flows.	A city, like a system, is dynamic. Since it is capacitated by limited resources, its exponential growth becomes logistic and a volatile equilibrium emerges that needs to be curbed by policy.

circulation across languages and disciplinary boundaries as much as possible.

3. The influence it has had on urban studies (i.e. on academic works, through replication, development of the model, adaptation/hybridisation, critique, results' discussion, etc.).
4. The influence it has had on urban policy (i.e. its application to existing cities, whether implicit or explicit).

For each of these four lines, we provide a table comparing the results for all models. Then, we provide our analysis of one or two models in further detail. For more details, we refer interested readers to the full presentation of each model in the recorded seminars[6] and edited podcasts.[7]

Analytical urban model circulation, some results

Model overviews. The model presentations correspond to the models' components and their main contributions to urban scholarship. Table 2 presents a comparative overview of the diversity of issues/aims addressed by the models selected, along with a description of the main actors and urban features modelled, as well as the main findings from the models. Some models work to answer a clear scientific question (e.g. Can segregation occur in a relatively tolerant society (Schelling)? What is the geographical impact of transportation cost on population density in monocentric cities (Alonso)?); therefore, their findings are easy to relate to the model's aim. In other cases, the model has the ambition of explaining the

Table 3. Model biographies.

Model	Known creator(s)	Creation context	Reception[a]	Circulation
Christaller's central place theory	Christaller (1933)	Christaller was a member of the German Social Democrat Party in 1922, produced his thesis in 1932, was an expert in planning institutions at Berlin and Freiburg (1935–1940), was a member of the Nazi party in 1940 and an expert at *Stabhauptamt für Planung und Boden* (1940–1944) and then held assumed Communist Party membership in 1945.	These theories were critically received in the 1930s among German geographers (as too theoretical), while being recognised by some at IGC (e.g. E. Kant). Enthusiastic and gained credence in the 1960s among actors of the new geography (quantitative turn) in that they were used to lay out plans for future cities. Critical in recent history of geography (Christaller's involvement with the Nazis).	First local, then international through IGU conference invitations. (Partial) English translations by Garrison (1941) and Baskin (1966).
Alonso's monocentric city	Alonso (1964), Mills (1967) and Muth (1969)	Alonso's *Location and Land Use* book, published in 1964, was the first thesis in Isard's new Regional Science Department. Muth's 1969 book added the income effect to the model and Mills' contribution in the 1970s added traffic congestion.	Wide in (urban) economics for very applied estimation cases. Criticised in geography within the context of the post-modernist rejection of quantitative and formal methods (see Berry, 1996; Rees, 1999).	As an integral part of new urban economics going back to von Thünen's model in the early 1800s.

(continued)

Table 3. Continued

Model	Known creator(s)	Creation context	Reception[a]	Circulation
Macro-economic base theory	Wiemer and Hoyt (1939) or Sombart (1902–1928)	These ideas emerged in the early 20th century when the increasing industrialisation and urbanisation of Europe combined with the globalisation of trade. This led to a renewed reflection on town-builders and town-fillers.	The notion that one could identify industries that were basic and that drove the development of services emerged during the 1930s, quickly being adopted by economics and policy advisors and linked with input–output analysis. There were many later criticisms (e.g. Andrews, 1953). Re-evaluations (on scale; see Lerousseau, 2020).	With multiple increments and regular empirical tests.
Fractal urban models	Arlinghaus (1985), Batty and Longley (1986, 1994), Frankhauser (1988, 1994)[b]	The introduction of fractal modelling for urban studies occurred a decade after Mandelbrot's (1967) seminal paper and his 1967 book about fractals (written in French; English version published in 1977). The creation of urban fractal models was concomitant and inextricably linked with the development of computer capacities. Batty and Longley's (1994) book was mainly predicated on using fractals to render numerical outputs of urban models as realistic pictures.	Wide due in part to their possible relations with Auerbach–Zipf's law and Christaller–Lösch central place hierarchy, but also the aesthetic qualities of fractal figures. Deeper engagement from parts of geography and planning linked to complex systems science (see Li Vigni, 2021).	Numerous applications in urban geography and statistical physics. Also a series of applications in ecology, architecture and environmental psychology.

(continued)

Table 3. Continued

Model	Known creator(s)	Creation context	Reception[a]	Circulation
Schelling's models of segregation	Sakoda (1971), Schelling (1969, 1971)	In the 1940s, after the experience of the Second World War and concentration camps for Japanese in the USA (Sakoda). In the 1960s, the context of post-Jim Crow segregation laws in the USA (Schelling).	Warm acceptance in communities that took a simulation turn within social science disciplines due to the model's conceptual simplicity and mathematical complexity. Scepticism in sociology.	First in academic circles, and then supported by Schelling's recognition as a US Cold War strategist (Hegselmann, 2017).
Forrester's Urban Dynamics	Forrester (1969)	After being instrumental in the development of the digital computer and feedback mechanisms, Forrester moved to the MIT Sloane School of Management, where he applied his ideas to industry, cities and the Earth, as reflected in his books *Industrial Dynamics, Urban Dynamics* and *World Dynamics*. Despite Boston being a hotbed of urban modelling at the time, Forrester's group was strangely disconnected from the mainstream movement.	Rapid acceptance in the transportation community for its freshness in suggesting that cities were intrinsically dynamic systems but faced criticism for its lack of inclusion of basic urban model features such as the growth of suburbs and the decline and recovery of the inner city.	Books, meetings, acquaintance with the mayor of Boston, influential position of Forrester in MIT then and in the Club of Rome.

Notes: [a]We use the plural form to indicate that all models were not received in a uniform manner in all communities, depending on their scientific traditions.
[b]The American geographer Arlinghaus also proposed fractal urban models early on (Arlinghaus, 1985), but her models have subsequently neither been applied nor developed by other researchers. Outside the field of urban studies, two American geographers, namely Goodchild and Mark, were pioneers in the application of fractal geometry to analyse geographical shapes (Goodchild, 1980; Goodchild and Mark, 1987).

larger structural or morphological organisation of urban areas under a descriptive representational framework (e.g. Christaller's aim of explaining the size, number and spatial distribution of cities) or a looser but more comprehensive focus (e.g. Forrester's Urban Dynamics model or fractal urban models). Interestingly, this sharpness of focus is not necessarily associated with the level of precision of the actors and urban features modelled. For example, Christaller's central place theory has a broad aim and a multiscale span with a small number of details on urban actors. Schelling's models of segregation have a focused aim and also a small number of features, whereas Forrester's Urban Dynamics is a large, entangled model of urban stocks and flows with a looser aim of simulating an entire city system, similar to fractal urban models, despite the simplicity of their generating principles.

An interesting aspect of model comparison here is the way the models can be summarised. For instance, the monocentric city model of Alonso works in a somewhat deductive manner. There are 13 assumptions on which the model relies (e.g. assumptions about the urban setting, the choices and behaviours of landowners and households, the fixation of rents, the bidding process for housing allocation, etc.). Understanding all the assumptions is central to the understanding of the model but also to assessing its contributions and limitations. Indeed, the main finding of the model (i.e. the concentric gradient of decreasing population density around the central business district) directly derives from framing the trade-off between location and movement as a formal utility maximisation optimisation problem. Solving the equilibrium of this set of assumptions is limited by the context created by these assumptions. In particular, the morphological centrality (the distribution of population density) is derived from the model, whereas the functional centrality (the fact that everyone works in the CBD) is assumed. Thus, the model is limited when it comes to explaining polycentric urban forms. Another crucial remark about this model is that when used for simulating the evolution of the social differences inside a city, it automatically locates the poorest income group close to the urban centre and the well-off populations in the suburban periphery, which is a typical configuration of north-American cities (Lemoy et al., 2010). As such, even a purely mathematical analytical model may be clearly dependent in its formulation on the geographic context of its elaboration (in this case, the United States).

In contrast, each fractal urban model can be summarised very simply by the reference fractal shape(s) or the fractal construction rules used for its development. These reference shapes or geometrical rules are used to assess how urban forms (which are not strictly self-similar but statistically self-similar) relate to these geometrical figures in order to analyse their self-similar properties – particularly their fractal dimensions – or generate fractal urban patterns (typically using randomness).

These different types of presentations reflect the highly heterogeneous nature of the models and their materiality (i.e. the set of documents that compose them). In the first case, the model corresponds to a mathematical deduction from a precise set of assumptions detailed in a unique document (Alonso's, 1964 book, adapted from his PhD dissertation), but is also part of a much wider set of ideas from urban economics that seek explain the monocentric city. In the second case, the model is an *assemblage* of ideas about geometry, protocols for measuring mathematical dimensions, generative processes, etc. The other models can be situated in between these two extremes. Furthermore, in the first case, the model is based on explaining behaviour in a system when

structure is given, whereas in the second case, explaining structure is the primary aim.

Biographies. The model biographies allow us to track the models back to their origin (whether unique or multiple), provide an account of the geographical, historical and institutional context of their creation and derive some hypotheses regarding the effects of this context on its reception and further circulation. Beyond the difference in contexts and immediate reception (see Table 3), a striking common element in these biographies is the position of the known model creators at the intersection of different disciplinary fields or between the academic and policy worlds. In a way, most of these canonical urban models were achieved using outside-the-box resources and referred expertise.[8] This is evident in the case of Forrester, who started his career as an electrical engineer and ended it by advising the Club of Rome on population sustainability. Schelling's fame as a political strategist during the Cold War also contrasts with the model for which he is remembered in sociology, geography and complexity science.[9] Moreover, Alonso was trained as an architect, not an economist.

Two models exhibit a particularly interesting biography and circulation history: Christaller's central place theory and Forrester's Urban Dynamics.

The history of Christaller's central place theory is embedded within the larger geopolitical context of the Second World War, denoting the importance of its reception for the potential adoption or rejection of the model by ideologically motivated scholars. Christaller's model reflects his own fascination with the issue of territorial division and his concern to rationalise spatial planning (Robic, 2001). This was also part of a 'new utopia of rational planning for social development' that had emerged in Germany at the end of the 19th century and was enshrined in Weimar legislation and then in that of the Third Reich, which was reflected in the constitutional law of the Federal Republic of Germany, with the continuity of institutions and careers on both sides of the Second World War (Rösslør, 1989). The model was effectively applied for the planning of territories conquered by the Nazis in Europe from 1939 to 1945 and also inspired the settlement locations of the IJsselmeerpolders in the Netherlands (Bosma, 1993, cited by Renes and Piastra, 2011; Sugiura, 2006). Additionally, it inspired the planning of the territories annexed by Israel after 1967 (Tilly, 2005). On the academic side, even if it needs to be contextualised and completed, central place theory is still a reference for describing settlement patterns anywhere or guiding archaeological explorations (Lapin, 2001), despite the political context of its first applications. Indeed, a former member of the Social Democrat Party, Christaller joined the Nazi party in 1940 to work as an expert at the Office for Planning – funded by Himmler – on the spatial structure of the Reich population, before moving on to the Communist Party in 1945 and back to the Social Democrats in 1959 (Hottes, 1981; Hottes et al., 1983; Rössler, 1987; Rösslør, 1989). This aspect of his personal and professional life did not motivate the earlier rejections of his model (which were rather based on his strong theoretical stance against nomothetism in geography), nor its later acceptance by the new generation of theoretical and quantitative geographers in the 1960s (e.g. Berry, 1967),[10] whose influential position in the new quantitative geography field at the time (i.e. on the transatlantic arc of quantitative geographers' migrations; see Johnston and Sidaway, 2004) provided a platform for Christaller (invited to give the opening plenary of the Lund IGU Symposium in 1960) and his model to diffuse (through the translation of Christaller's

Table 4. Model trajectories in urban studies.

Model	Main relays	Purpose evolution	Use evolution	Transfer/ mobility
Christaller's central place theory	Kant from Estonia to Sweden (Hägerstrand), Ullman and Garrison's group in Seattle (inc. Berry). English translators: Ullman's 1941 summary and Baskin's 1966 partial translation.	From urban planning to baseline models (in archaeology or the analysis of big data).	From regional planning to global networks.	Transfer
Alonso's monocentric city	Isard's Regional Science Department, Beckmann, Muth and Mills. School of New Urban Economics.	From full housing market models to the effects of transport on sprawl and explaining density profiles.	Parameter estimation in economics and scaling studies in geography (see Delloye et al., 2020).	Transfer
Economic base theory	Isard's Regional Science Department, Tiebout, Ponsard, Davezies and Blumenfeld.	From predicting the impacts of external demand on urban development to understanding why developing communities are not expanding and why exports are not sufficient to develop cities.	From export investment calibration and rural-to-urban migration predictions (Wiemer and Hoyt) to the production mix and spatial circulation of wealth analysis (Davezies).	Mobility
Fractal urban models	In urban studies: Benguigui, Marinov, Czamanski, Blumenfeld-Lieberthal (IL), Chen (CN), Pumain, Tannier (FR), Thomas (BE) and White (CA).	From improving and discussing methods to estimate and interpret urban fractal dimensions properly to introducing fractal dimensions or fractal generators into generative and/or explanatory models of urban growth and urban dynamics.	Three overlapping development stages (ongoing): - Description and comparison of the shape of numerous cities and their evolutions. - Search for explanations about the existence of urban fractal shapes (e.g. optimisation or aesthetic principles). - Introduction of fractal measures or fractal growth processes in urban growth models.	Transfer in the case of fractal analysis models Mobility in the case of fractal simulation models

(continued)

Table 4. Continued

Model	Main relays	Purpose evolution	Use evolution	Transfer/ mobility
Schelling's models of segregation	Benenson and Hatna (IL), Clark, Fossett, Vinkovic, Kirman (USA) and Banos (FR).	From the demonstration of basic counterintuitive behaviour to the explanation of model dynamics for a broad set of conditions and rules of interaction.	From abstract cities populated by agents of two kinds to realistic city patterns and agents differing in more characteristics (e.g. income, migration distance, complex interactions between agents and their neighbours).	Mobility
Forrester's Urban Dynamics	The LUTI community.	From the model itself to a demonstration that computer models can handle dynamics, and that cities should be modelled dynamically.	From applications of cases with data to the inclusion of dynamics into other models.	Transfer

book by Baskin in 1966 or its summary by Ullman in 1941; see Barnes and Roche, 2022).

Similarly, the history and mobility of the Urban Dynamics model are framed by the early years of urban development models in the USA (1965–1972) and by Forrester's own professional career. Knowledge of the urban development modelling context (in particular Lowry's land use transportation model and the later family of activity allocation models that build on the Lowry model, particularly in Britain; see Batty, 1976) reveals the extent to which Forrester did not make use of these at all, did not search academic connections with them and did not review, read or even absorb their work. Instead, Forrester produced a completely new model of urban growth[11] based on his background as an electrical engineer, his previous experience of computing during the war (see Barnes, 2008), his implementation of systems dynamics applied to industrial activity (Forrester, 1961) and, more anecdotally, his discussions about cities and urban problems with his acquaintance John F Collins, the mayor of Boston, MA, from 1960 to 1968 (who wrote the foreword to *Urban Dynamics*). Thus, the development of this model shows how external events (Forrester's position in the MIT Sloane School of Management in 1956 and his involvement with the influential Club of Rome in the 1970s) shaped the content, formalism and circulation of a model of cities removed from its contemporary counterparts in urban studies, which became the source of its appeal but also the cause for its insufficiency and subsequent abandonment.

Following these models, their creators and some intermediaries thus produced a deeper understanding of the choices behind certain elements of each model under consideration (e.g. the engineering formalism of Urban Dynamics), as well as the

contingency of some of their later trajectories of success and controversy (see Central Place Theory).

Influence on urban studies. We can summarise the influence of the models on urban studies in terms of how they made urban scholars evolve in their treatment of the urban features modelled, how they contributed to improving the model or how they changed its use (see Table 4).[12] In doing so, we have categorised the trajectory and influence of the models in urban studies into two options borrowed from the policy transfer and policy mobilities literature:

- A 'transfer' characterises the trajectory of a model used for replication across other case studies without being significantly transformed during the process. This seems to be the fate of Forrester's short-lived Urban Dynamics model in urban studies. The model was applied and calibrated to particular case studies, revealing some empirical invalidation (Gray et al., 1972); however, these did not produce hybrid forms of a model incorporating Forrester's initial formalism into land use transportation interaction (LUTI) models, for instance.[13] Instead, the main influence of Forrester on urban studies seems to have been his focus on the role of time and dynamics more generally in thinking about human systems. Most of the applications – particularly those associated with world dynamics – were based on plausible 'thought experiments' rather than focused empirical testing.
- In contrast, 'mobility' represents back-and-forth trajectories of influence, hybridisation and incremental development. This seems characteristic of the influence of the model(s) behind economic base theory, or of Schelling's models of segregation, with their multiple implementations and developments. These last two models provide a very interesting story of mobility in urban studies, which we focus on in the following paragraphs.

Economic base theory has an unknown origin (with influences traced back to Wiemer and Hoyt, Sombart and Perroux's growth poles as well as to Tiebout) but has well-documented trajectories of incremental developments that led the model from initially focusing on the impact of external demand on local development to explaining the maintenance and growth of residential local economies without an export base. This included the notable roles of Isard in changing the scale of analysis from one city to a regional focus including inter-city flows, Ponsard in measuring urban export activities at different spatial scales, Tiebout and Davezies in incorporating multiple income flows besides the export base (typically social incomes, national public expenditure and private transfers) and Blumenfeld in identifying the quality of relations between residential supply and export companies.

Schelling's models of residential segregation were influential in several theoretical and applied aspects. Unexpected self-organising minority–majority dynamics are observed in populations consisting of several groups of unequal size (Benenson and Hatna, 2011), with the further assumption that the agents' tolerance to strangers within the neighbouring community may be heterogeneous (Hatna and Benenson, 2012, 2015). Clark (2002) introduced a multi-ethnic setting and Benenson et al. (2009) and Spielman and Harrison (2014) introduced an economic component. With all these adaptations, the basic qualitative phenomenon remains the same: a mild level of individual intolerance is sufficient for the emergence of

Table 5. Model trajectories in urban policy.

Model	Policy recommendations	Applications	Evaluation	Recognition
Christaller's central place theory	Use theoretical regularity to optimise regional subdivision, rationalise transportation and estimate the number and size of new towns.	Estonia (1935), Germany and Poland (1940), The Netherlands (1950s), Israel (1960s), China (1990s).	Literal applications missed multi-purpose trips and online shopping.	Implicit
Alonso's monocentric city	Increase commuting costs to reduce urban sprawl.	Linked to LUTI.	Too simple but still relevant.	Implicit
Macro-economic base theory	Generate more exports (through subsidies or infrastructure) to increase migration and demand, thus creating more service jobs in supporting industries.	Multiplier effects of exports on derived employment.	The standard model is too simple and usually needs extensions to input–output analysis.	Explicit
Fractal urban models	Favour the creation of fractal forms because they are virtuous by nature (organicist argument) or because they are optimal with respect to diffusive processes, behaviours and practices. Some recommendations regarding the 'good' value of fractal dimension and the optimal urbanisation ratio.	Urban and regional development plans: Besançon and Paris (France), Vienna (Austria), North Indian Punjab.	Very few evaluations of the functional advantages of fractal versus non-fractal urban forms.	Explicit
Schelling's models of segregation	Impose policies to prevent segregation.	Singapore: building-level segregation threshold for each ethnic group. Rotterdam: restricted access of unemployed newcomers to deprived neighbourhoods.	Works in societies with strong law enforcement and low levels of ethnic conflicts (Singapore) but did not work in The Netherlands.	Implicit

(continued)

Table 5. Continued

Model	Policy recommendations	Applications	Evaluation	Recognition
Forrester's Urban Dynamics	Limit growth to prevent resource depletion, overpopulation and the collapse of capacitated systems.	Ad hoc applications to cities whose dynamics are stylised.	Demonstrations of how cities can embrace longer-term population dynamics.	Explicit

a strong spatially segregated residential pattern. Benenson et al. (2003) applied the Schelling model in the Israeli context, including Jewish–Arab residential segregation in Jaffa, Tel Aviv, while Flint et al. (2012) applied it to residential segregation in the ultraorthodox residential areas of Jerusalem. Grauwin et al. (2009) took up Schelling's spatial segregation model again and explicitly introduced a variable representing individuals' allowance for a collective reference. In the case of 'altruistic' individuals, the final spatial pattern maximises individuals' global utility. In the case of 'selfish' individuals (no consideration of a collective reference), space takes on a segregated spatial pattern in which individuals, for want of coordination, fail to maximise their utility.

In conclusion, the two aforementioned models have had a double influence on urban studies: they have imprinted scholars' qualitative intuition about urban dynamics and also generated a versatile quantitative tool to explore a broad variety of real-world scenarios. This material *assemblage* of theories and actionable tools most likely enhanced their academic circulation.

Influence on urban policy. We can summarise the influence of urban models on urban policy by looking at which policy recommendations can be drawn from the model, whether the model or these recommendations were ever implemented and, if so, whether the experiment directly credited the analytical urban model and how it was evaluated (Table 5).

For most models analysed in this article, the recognition of their role in policy is implicit, which means that the original model is not credited per se. Rather, the model informs the theoretical framework of the implemented policy (as in the case of central place theory) or is part of a more applied model (such as Alonso's model,

which is nested or embedded into LUTI and spatial interaction models).

Macroeconomic base theory is an exception here. Derived from policy initiatives and often geared towards applications, it was first applied in the post-war years very explicitly by US city planners – to the extent that as early as 1953, RB Andrews expressed the following warning:

> It seems fitting to point out that, in the case of urban base theory, we have operated far too long on a set of ideas which appear valid but which, despite substantial conceptual omissions and difficulties of application, seem to be accepted all too blithely as gospel by many researchers and active city planners. (Andrews, 1953: 167)

Fractal urban models are also used explicitly to derive planning principles and design recommendations. Although fractals have provided important (yet dispersed) theoretical insights in the field of planning, an obstacle to their policy application is the fact that notwithstanding what might appear to be top-down designs, fractal principles argue against urban designs and planning interventions that impose pre-defined and large-scale blueprints of forms on city morphologies (Jahanmiri and Parker, 2022). Thus, fractal principles contradict typical urban design practices based on zoning and favouring regular and smooth shapes (Tannier, forthcoming). Moreover, using measures of fractal dimensions, along with measures of density, is not common among urban planners and designers.[14]

In general, the influence of the economics-based models analysed on general urban policy is limited, and the more direct the applications of their recommendations, the less favourably their implementations tend to be evaluated. Although there are some very specific applications of models in domains such as urban transport planning, the general experience with such models in applied policymaking is mixed and the impact of simulations on the creation of new policy has been largely negative (Boyce and Williams, 2015).

Structural and contextual factors of circulation

Based on the above descriptive and biographical elements, we can distinguish between two sets of factors favouring the circulation of analytical urban models that have now become 'canons'. The first set of factors relates to the temporal/spatial/geopolitical context of model production and the personal attributes of the model creator(s) and intermediaries. This reflects the fact that academia, much like the sphere of policy, is a site of power struggles between individuals with various endowments. Having removed these factors, we can highlight a second set of factors facilitating the circulation of analytical urban models, which relate more directly to the form and content of the models themselves.

In terms of contextual factors, the main facilitator (or inhibitor) of model circulation in the models we have analysed seems to be that they originate from English-speaking places and times which were central to the development of the Anglo-American tradition of quantitative urban studies (see Table 1). In short, if the model is described in the English language and comes from scholars employed in American and Western European institutions (e.g. MIT, Harvard, U Penn), it has a better chance of spreading internationally. Analytical urban models presented in other languages will face important barriers to their circulation and adoption, even when they are translated. The case of central place theory is telling in that respect. Being originally written in German, it was initially discussed mostly by German-

speaking geographers in Germany, Estonia and later France and the USA. Its international recognition followed Christaller's presentation at the 1960 Symposium of Urban Geography organised by the IGU Commission in Lund, and its eventual translation into English by Baskin (Christaller and Baskin, 1966), thus producing a delay of three decades between the initial production and full reception of the model. Besides this temporal delay, the translation to English was only partial, leading to unjustified attacks from later readers. For instance, the third part of his thesis, where he discussed its limitations and offers predictions, was omitted in Baskin's translation, which means it barely existed for English readers (Robic, 2001). Similarly, the translation of Christaller's thesis into other languages has led to misconceptions, confusion in interpretation and appropriations by scientific and political currents. In French, for instance, Djament and Covindassamy (2005) describe a series of choices behind the translation of Christaller's main concepts, from his three location principles (*Versorgungsprinzip, Verkehrsprinzip* and *Absonderungsprinzip*) to distances (*Abstand* and *Entfernung and*). They were transparent and explicit about the options on offer to translate them: direct translations, the reuse of previously translated terms by other authors and conceptual equivalents in contemporary scientific language. In contrast, the 1998 translation to Chinese seems to have involved less transparency and a larger share of ideological interpretation of Christaller's theory, including a long preface by Zhang Dawei, who was not involved in the translation but situated the work of Christaller in the Chinese planning context (Radeff, 2019).

Another factor influencing the circulation of analytical urban models is the disciplines they originate from. Indeed, analytical models coming from 'hard' sciences and disciplines favouring formalisation (e.g. economics, physics, engineering) have higher chances of reaching a large audience and being granted legitimacy in bridging disciplines (e.g. urban geography) than models coming from social sciences (even from urban studies themselves). However, we also found that polymaths are particularly successful in creating and circulating such models, in part because they can play on different sources of legitimacy and expertise. They might also understand better the common problems faced by various disciplines.[15] All of the known/recognised creators of the models presented are male, which reflects the gender imbalance of academic power of the past decades (and sometimes also the appropriation of women's work by prominent male figures). Finally, although the policy mobilities research has shown the importance of institutional labelling/anointment in urban policy success (Béal et al., 2015; Peck and Theodore, 2010), its equivalent for analytical models is less obvious. Indeed, although the scientific practice of urban modelling is less dependent on institutional labels (however, it can be boosted by a creator or intermediaries affiliated with prestigious universities and inclusion in disciplinary textbooks), it may be more subject to data availability and path dependency in the way some urban features are modelled. For example, how Forrester's Urban Dynamics has circulated amongst different domains is primarily due to his novel way of considering time and evolution in urban development models at a time when most urban modellers were still using equilibrium static models out of habit and data restriction rather than out of holding the scientific conviction that it was the most suitable approach. Finally, we must acknowledge that the models that have found an audience in urban studies and the policy sphere reflect, match with and contribute to

the geopolitical *zeitgeist* of their time, which is exemplified by the capitalist assumptions and cost-benefit optimisation rationale of monocentric city models.[16]

In addition to these contextual elements, we have identified a series of circulation-enhancing factors more directly linked to the model's content. The first one is the model's simplicity and a clear focus on the basic phenomena. For instance, Schelling's models of segregation focus on the spatial clustering of similar individuals as a result of a simple mechanism of intolerance-induced moves. The simplicity and generality of the model thus derive from its limited representation of the process modelled in pursuit of a clear goal: showing that the simple individual-level mechanism (micro-motives) is *sufficient* to generate the population-level outcome of spatial segregation (macro-behaviour) (Schelling, 1978). A second factor increasing the mobility of analytical urban models is the transparency of the hypothesis modelled. Unlike urban policy and its material forms of political discourse and programmatic documents, the models we have presented most often include a mathematical or diagrammatic representation of the hypothesis (e.g. Alonso's monocentric city model) that is detailed and justified in the text to reduce ambiguity. This facilitates the process of scientific legitimation, replication and inclusion of such models as building blocks of subsequent larger models. A third favouring factor linked to the first two is that of transportability, which relates to their relevance and applicability to other case studies, sites, issues or disciplines from the original one, – this inter-disciplinary mobility being part of the circulation idea (see Temenos and Baker, 2015). Transportability can come from a common 'need' of modellers in various urban fields (i.e. explaining specialisation, centrality or polarisation), which can be adapted to transportation, economic planning or urbanisation models. In that case, being a polymath modeller is useful. Transportability can also come from the form in which the model is circulated. The beautiful aesthetics of fractals have played no small role in the scientific fascination surrounding them (Tannier, 2018). The presence of maps and graphics might also be the difference between the success of central place theory and its lesser known prior formulation by Reynaud in the 19th century, using text only (Robic, 1993). In the case of models represented in mathematical form, the formal language can work both ways: it can soothe differences and facilitate circulation among academic circles, but it can also create barriers in less mathematically literate worlds, such as the urban policy and planning spheres. Thus, the presentation of the model can hinder its adoption and circulation if its mathematics is beyond those who would be the key recipients of its practice, while too simple a presentation might suggest that the model is naïve.

Conclusion

In this article, we have addressed a gap in the literature on urban models' circulation by questioning the specificity of analytical urban models compared to urban policy models, using the theoretical framework and methodological tools of the policy mobilities debate. Starting from the hypothesis that contextual elements serve roles in the production and circulation of analytical urban models as they do in normative urban models, we have shown by example that such contingent elements of context can be identified (e.g. their institutional origin, geopolitical context, language, etc.) and distinguished from factors relating to models' content in their influence on the design and circulation of analytical urban models by following the models, their creators and intermediaries. We were able to uncover factors of unequal circulation common to urban policy's

unequal mobility, such as power relations in the institutional context of their assemblage and aspects of their materiality (the presence of graphics, equations, etc.), or the importance of travelling actors for knowledge in transit (Secord, 2004). In contrast, institutional anointment of the model itself seems less important to their circulation than path dependency and transportability for analytical urban models (although the institutional affiliation of its authors and intermediaries matter). We can thus conclude that the theoretical framework of the policy mobilities research is fruitful in analysing this type of urban model, with some modifications to account for differences in the nature, goals and behaviours of the actors involved in the circulation.

The circulation of (canonical) urban models remains ongoing. Most of the models presented are used as parts of other models (LUTI, ABMs, etc.), transformed and adapted to new uses. Notably, they have become incremental parts of the wider toolbox of urban modelling (Cottineau et al., 2019). To be useful and meaningful in the future, the canonical analytical urban models should remain transparent about where they come from and why they include a particular set of hypotheses, actors and features. Keeping this information attached to the models (as metadata) is crucial, and this work has attempted to contribute in this manner. This is especially important for canonical models because they tend to be chosen and used out of habit, ease and past dependency, even when new models, representations and ideas are needed (e.g. for urban policies; see McCann, 2011).

Acknowledgements

The authors wish to thank Ali Sobhani for his contribution to the early days of the Unforeseen Influence seminar series. They are grateful to all those who attended and contributed to the Unforeseen Influence seminars, where individual model circulations were presented, as well as the participants of external seminars, where earlier versions of this work were presented (CASA-UCL, LISER). They thank Nicolas Szende and two anonymous reviewers for their critical discussions and suggestions on the manuscript.

Declaration of conflicting interests

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Funding

The author(s) disclosed receipt of the following financial support for the research, authorship, and/or publication of this article: Clémentine Cottineau and Rūta Ubarevičienė are grateful to the Department of Urbanism at TU Delft for its funding of the seminar series and podcast production costs. Clémentine Cottineau acknowledges funding by the ERC Starting Grant 101039455 SEGUE.

ORCID iDs

Clémentine Cottineau https://orcid.org/0000-0002-2452-3901
Michael Batty https://orcid.org/0000-0002-9931-1305
Itzhak Benenson https://orcid.org/0000-0003-0704-0673
Justin Delloye https://orcid.org/0000-0003-3523-706X
Denise Pumain https://orcid.org/0000-0002-0730-4563
Somwrita Sarkar https://orcid.org/0000-0002-2037-0979
Cécile Tannier https://orcid.org/0000-0003-4158-6872
Rūta Ubarevičienė https://orcid.org/0000-0002-5894-7120

Notes

1. ‘A model can only become a model, needless to say, if it has followers, but it will not enrol followers unless it holds the promise

of extra-local salience. Models that travel therefore reveal at least as much about "demand-side" needs, imperatives, and anxieties as they do about supply-side inventiveness. The reason why many models achieve mobility in the first place is that they have, in some way or another, been ideologically anointed or sanctioned' (Peck and Theodore, 2010: 171).

2. Although we acknowledge the role of these 'others' frequently forgotten in the official histories of models, we remain attentive to the unequal diffusion power of these actors – or, using Peck and Theodore's (2015: 217) anatomical metaphor, 'networks as something more akin to sinews, sinews that are charged with varying degrees of energy and which are attached to different degrees (and forms) of policymaking "muscle"' at different times and under the influence of geopolitical contexts. For instance, models of urban production and management from Europe and the USA gained traction in the post-colonial context as expert tools to export and to sustain international dominance in the economic and political domains.
3. Two striking examples in the policy mobilities literature are conditional cash transfers and participatory budgets, which are policies 'associated with a cluster of material […] practices, from indicative schedules for neighbourhood assemblies to refined rules of deliberation and voting' (Peck and Theodore, 2015: 211) which favours their replication yet allows for a large variation in philosophies (see Bolsa Familia in Brazil versus PROGRESA / Oportunidades in Mexico, two canonical implementations of the conditional cash transfers policy).
4. Each model presentation/discussion has given way to an episode of the online seminar organised by Clémentine Cottineau and Rūta Ubarevičienė and been further edited into a podcast series call Unforeseen Influence: https://edu.nl/43b4g (accessed 10 December 2023). Which author has studied which model is not that important per se; however, since this introduces some unobserved bias in the analysis, we disclose it.
5. This criterion is the most subjective and contentious one. We are confident in the fact that most of the models chosen are 'canons', but not necessarily any more canonical than other models we have not included, such as the Auerbach-Lotka-Zipf law (see Rybski and Ciccone, 2023), Lowry's (1964) model of metropolis, MATSim (Axhausen et al., 2016), SLEUTH (Clarke et al., 1997) or UrbanSim (Waddell, 2002).
6. Urban Models Seminars series: https://edu.nl/pj3yt (accessed 10 December 2023).
7. Unforeseen Influence: https://edu.nl/43b4g (accessed 10 December 2023).
8. That is, 'the use of an expertise learned in one domain within another domain' (Collins and Evans, 2007: 14).
9. Despite having contributed to its dominance against Sakoda's earlier model (Hegselmann, 2017).
10. But rather the more recent positions against the model (e.g. Barnes, 2015).
11. Batty attributed the novelty of Forrester's model to his treatment of time and dynamics, which was unaccounted for in equilibrium models at the time due to the lack of available data and relevant theories of dynamics (see his presentation at the Urban Models seminar: https://edu.nl/bp8px (accessed 10 December 2023)).
12. Although the idea of a field of 'urban studies' is anachronical to the development of most of the models presented, we use the expression to encompass the influence that these models had on the connected disciplines and traditions interested in cities, such as urban geography, regional science, planning and transportation, etc.
13. With the exception of Sanders and Sanders (2004).
14. When the purpose is to use fractal analysis to characterise urban patterns, many ready-to-use software applications exist, such as the plugin FracLac for ImageJ, FracLab, Benoit and Fractalyse. When the purpose is the simulation of fractal scenarios in urban development, only two ready-to-use software applications exist, namely MUP-City (Tannier et al., 2010, 2012) and Fractalopolis (Frankhauser et al., 2018).

15. This is the case for the creators of fractal urban models, Batty, Longley and Frankhauser, with the latter having defended two doctoral theses: one in theoretical physics (University of Stuttgart) and one in geography (University of Paris).
16. 'The speed and reach of less disruptive models […] must not be tagged too narrowly to the internal characteristics or capacities of these models. They are travelling, after all, across neoliberalised terrains, the imperatives, constraints, and strategic orientations of which are reciprocally reinforced by the models themselves' (Peck and Theodore, 2015: e237).

References

Achter S, Borit M, Cottineau C, et al. (2024) How to conduct more systematic reviews of agent-based models and foster theory development – taking stock and looking ahead. *Environmental Modelling & Software* 173: 105867.

Alonso W (1964) *Location and Land Use*. Cambridge, MA: Harvard University Press.

Andrews RB (1953) Mechanics of the urban economic base: Historical development of the base concept. *Land Economics* 29(2): 161–167.

Arlinghaus SL (1985) Fractals take a central place. *Geografiska Annaler Series B Human Geography* 67(2): 83–88.

Axhausen WK, Horni A and Nagel K (2016) *The Multi-Agent Transport Simulation MATSim*. London: Ubiquity Press.

Baker T and Temenos C (2015) Urban policy mobilities research: Introduction to a debate. *International Journal of Urban and Regional Research* 39(4): 824–827.

Barnes T and Abrahamsson CC (2017) The imprecise wanderings of a precise idea: The travels of spatial analysis. In: Jöns H, Meusburger P and Heffernan M (eds) *Mobilities of Knowledge*. Cham: Springer, pp.105–121.

Barnes T and Roche M (2022) The international circulation and dissemination of geographical concepts and ideas. In: Schelhaas B, García-álvarez J, Heffernan M, et al. (eds) *A Geographical Century: Essays for the Centenary of the International Geographical Union*. Cham: Springer, pp.49–62.

Barnes TJ (2004) Placing ideas: Genius loci, heterotopia and geography's quantitative revolution. *Progress in Human Geography* 28(5): 565–595.

Barnes TJ (2008) Geography's underworld: The military–industrial complex, mathematical modelling and the quantitative revolution. *Geoforum* 39(1): 3–16.

Barnes TJ (2015) Geographies of knowledge and power. In: Meusburger P, Gregory D and Suarsana L (eds) "*Desk Killers": Walter Christaller, Central Place Theory, and the Nazis*. Dordrecht: Springer, pp.26–52.

Batty M (1976) *Urban Modelling. Algorithms, Calibrations, Predictions*. Cambridge: Cambridge University Press.

Batty M (2013) *The New Science of Cities*. Cambridge, MA: MIT Press.

Batty M and Longley PA (1986) The fractal simulation of urban structure. *Environment and Planning A* 18(9): 1143–1179.

Batty M and Longley PA (1994) *Fractal Cities: A Geometry of Form and Function*. London: Academic Press.

Béal V, Epstein R and Pinson G (2015) La circulation croisée. *Gouvernement et Action Publique* 4(3): 103–127.

Benenson I and Hatna E (2011) Minority–majority relations in the Schelling model of residential dynamics. 住宅动态迁移Schelling模型中的少数-多数关系. *Geographical Analysis* 43(3): 287–305.

Benenson I, Hatna E and Or E (2009) From Schelling to spatially explicit modeling of urban ethnic and economic residential dynamics. *Sociological Methods & Research* 37(4): 463–497.

Benenson I, Omer I and Hatna E (2003) Agent-based modeling of householders' migration behavior and its consequences. In: Bilary FC and Prskawetz A (eds) *Agent-Based Computational Demography: Using Simulation to Improve Our Understanding of Demographic Behavior*. Berlin and Heidelberg: Springer, pp.97–116.

Berry BJL (1967) *Geography of Market Centers and Retail Distribution*. Englewood Cliffs, NJ: Prentice-Hall.

Berry BJL (1996) Generational geographies: What next? *Urban Geography* 17(7): 569–571.

Bosma K (1993) *Ruimte voor Een Nieuwe Tijd: Vormgeving van de Nederlandse Regio, 1900–1945*. Rotterdam: NAi.

Boyce D and Williams H (2015) *Forecasting Urban Travel: Past, Present and Future*. Cheltenham: Edward Elgar Publishing.

Christaller W (1933) *Die Centralen Orte in Suddeutschland/The Central Places in the South of Germany*. Jena: Gustav Fischer.

Christaller W and Baskin CW (1966) *Die Zentralen Orte in Süddeutschland. Central Places in Southern Germany*. Translated by Baskin CV. Upper Saddle River, NJ: Prentice-Hall.

Clark WAV (2002) Ethnic preferences and ethnic perceptions in multi-ethnic settings. *Urban Geography* 23(3): 237–256.

Clarke KC, Hoppen S and Gaydos L (1997) A self-modifying cellular automaton model of historical urbanization in the San Francisco Bay area. *Environment and Planning B Planning and Design* 24(2): 247–261.

Collins H and Evans R (2007) *Rethinking Expertise*. Chicago, IL and London: University of Chicago Press.

Cottineau C, Chapron P, Le Texier M, et al. (2019) Incremental territorial modeling. In: Pumain D (ed.) *Geographical Modeling: Cities and Territories*, vol. 2. London: ISTE Ltd and Wiley, pp.95–123.

Dahan A (2010) Putting the Earth system in a numerical box? The evolution from climate modeling toward global change. *Studies in History and Philosophy of Science Part B: Studies in History and Philosophy of Modern Physics* 41(3): 282–292.

Delloye J, Lemoy R and Caruso G (2020) Alonso and the scaling of urban profiles. *Geographical Analysis* 52(2): 127–154.

Djament G and Covindassamy M (2005) Traduire Christaller en français. Textes seuils, réception, récit de découverte. *Cybergeo* 298. DOI: 10.4000/cybergeo.3144.

Dolowitz DP and Marsh D (2000) Learning from abroad: The role of policy transfer in contemporary policy-making. *Governance* 13(1): 5–23.

Ferretti F (2021) History and philosophy of geography II: Rediscovering individuals, fostering interdisciplinarity and renegotiating the 'margins'. *Progress in Human Geography* 45(4): 890–901.

Flint S, Benenson I and Alfasi N (2012) Between friends and strangers: Micro-segregation in a Haredi neighborhood in Jerusalem. *City and Community* 11(2): 171–197.

Forrester JW (1961) *Industrial Dynamics*. Cambridge, MA: MIT Press.

Forrester JW (1969) *Urban Dynamics*. Cambridge, MA: MIT Press.

Fox S and Wolf L (2022) What makes a place urban? *SocArXiv Papers*. DOI: 10.31235/osf.io/qfvry.

Frankhauser P (1988) Fractal aspects of urban systems. *Sonderforschungsbereich* 230: 67–76.

Frankhauser P (1994) *La Fractalité Des Structures Urbaines*. Paris: Anthropos.

Frankhauser P, Tannier C, Vuidel G, et al. (2018) An integrated multifractal modelling to urban and regional planning. *Computers, Environment and Urban Systems* 67: 132–146.

Goodchild MF (1980) Fractals and the accuracy of geographical measures. *Mathematical Geology* 12(2): 85–98.

Goodchild MF and Mark DM (1987) The fractal nature of geographic phenomena. *Annals of the Association of American Geographers* 77(2): 265–278.

Grauwin S, Bertin E, Lemoy R, et al. (2009) Competition between collective and individual dynamics. *Proceedings of the National Academy of Sciences* 106(49): 20,622–20,626.

Gray JN, Pessel D and Varaiya PP (1972) A critique of Forrester's model of an urban area. *IEEE Transactions on Systems Man and Cybernetics* SMC-2(2): 139–144.

Hatna E and Benenson I (2012) The Schelling model of ethnic residential dynamics: Beyond the integrated-segregated dichotomy of patterns. *Journal of Artificial Societies and Social Simulation* 15(1): 6.

Hatna E and Benenson I (2015) Combining segregation and integration: Schelling model dynamics for heterogeneous population. *Journal of Artificial Societies and Social Simulation* 18(4): 15.

Hegselmann R (2017) Thomas C. Schelling and James M. Sakoda: The intellectual, technical, and social history of a model. *Journal of Artificial Societies and Social Simulation* 20(3): 15.

Hottes K (1981) Walter Christaller – Ein überblick über Leben und Werke. In: Hlers E and Meynen E (eds) *Geographische Taschenbuch und Jahrweiser Für Landeskunde 1981/1982*. Wiesbaden: Franz Steiner Verlag, pp.59–69.

Hottes K, Hottes R and Schöller P (1983) Walter Christaller 1898–1969. In: Freeman TW (ed.) *Geographers: Biobibliographical Studies*, vol. 7. London: Bloomsbury Academic, pp.11–16.

Jahanmiri F and Parker DC (2022) An overview of fractal geometry applied to urban planning. *Land* 11(4): 475.

Johnston R and Sidaway JD (2004) The trans-Atlantic connection: 'Anglo-American' geography reconsidered. *GeoJournal* 59: 15–22.

Lapin H (2001) *Economy, Geography, and Provincial History in Later Roman Palestine*. Tübingen: Mohr Siebeck.

Larner W and Laurie N (2010) Travelling technocrats, embodied knowledges: Globalising privatisation in telecoms and water. *Geoforum* 41(2): 218–226.

Lemoy R, Raux C and Jensen P (2010) An agent-based model of residential patterns and social structure in urban areas. *Cybergeo* 512. DOI: 10.4000/cybergeo.23381.

Lerousseau J (2020) La question de l' échelle au centre d'un réexamen de la théorie de la base économique. *Revue d'Economie Regionale Urbaine* Octobre: 617–633.

Li Vigni F (2021) Complexity sciences: A scientific platform. *Science and Technology Studies* 34(4): 30–55.

Lowry I (1964) *A model of metropolis*. Report for the Rand Corporation, USA. Report RM-4035-RC, August.

McCann E (2011) Urban policy mobilities and global circuits of knowledge: Toward a research agenda. *Annals of the Association of American Geographers* 101(1): 107–130.

McCann E and Ward K (2012) Assembling urbanism: Following policies and 'studying through' the sites and situations of policy making. *Environment and Planning A* 44(1): 42–51.

Mandelbrot B (1967) How long is the coast of Britain? Statistical self-similarity and fractional dimension. *Science* 156(3775): 636–638.

Mills ES (1967) An aggregative model of resource allocation in a metropolitan area. *The American Economic Review, Papers and Proceedings* 57(2): 197–210.

Muelder H and Filatova T (2018) One theory – many formalizations: Testing different code implementations of the theory of planned behaviour in energy agent-based models. *Journal of Artificial Societies and Social Simulation* 21(4): 5.

Muth RF (1969) *Cities and Housing: The Spatial Pattern of Urban Residential Land Use. Third Series: Studies in Business and Society*. Chicago, IL: University of Chicago Press.

Orain O (2022) A social history of quantitative geography in France from the 1970s to the 1990s: An overview of the blossoming of a multifaceted tradition. In: Gyuris F, Michel B and Paulus K (eds) *Recalibrating the Quantitative Revolution in Geography: Travels, Networks, Translations*. London: Routledge, pp.102–117.

Peck J and Theodore N (2010) Mobilizing policy: Models, methods, and mutations. *Geoforum* 41(2): 169–174.

Peck J and Theodore N (2015) *Fast Policy: Experimental Statecraft at the Thresholds of Neoliberalism*. Minneapolis, MN: University of Minnesota Press.

Prince R (2012) Policy transfer, consultants and the geographies of governance. *Progress in Human Geography* 36(2): 188–203.

Pumain D (1998) Urban research and complexity. In: Bertuglia CS, Bianchi G and Mela A (eds) *The City and Its Sciences*. Heidelberg: Physica Verlag, pp.323–362.

Radeff A (2019) Walter Christaller traduit en chinois (1998–2010). *Cybergeo* 291. DOI: 10.4000/cybergeo.33486.

Rees J (1999) Regional science: From crisis to opportunity. *Papers in Regional Science* 78(1): 101–110.

Renes H and Piastra S (2011) Polders and politics: New agricultural landscapes in Italian and Dutch wetlands, 1920s to 1950s. *Landscapes* 12(1): 24–41.

Robic MC (1993) A hundred years before Christaller ... A central place theory. English translation [translator unknown]. *L'Espace Géographique, Espaces, modes d'emploi. Two Decades of l'Espace géographique, an Anthology* 1(1): 53–61.

Robic MC (2001) Walter Christaller et la théorie des "lieux centraux". Die zentralen Orte in Süddeutschland (1933). In: Lepetit B and Topalov C (eds) *La Ville Des Sciences Sociales*. Belin: Paris, pp.151–189.

Robinson J (2006) *Ordinary Cities: Between Modernity and Development*. London: Routledge.

Rössler M (1987) Die Institutionalisierung einer neuen "Wissenschaft" im Nationalsozialismus: Raumforschung und Raumordnung 1935–1945. *Geographische Zeitschrift* 75(3): 177–194.

Rösslør M (1989) Applied geography and area research in Nazi society; central place theory and planning, 1933 to 1945. *Environment and Planning D: Society & Space* 7(4): 419–431.

Roy A and Ong A (2011) *Worlding Cities: Asian Experiments and the Art of Being Global*. Malden, MA: Blackwell Publishing Limited.

Rybski D and Ciccone A (2023) Auerbach, Lotka, and Zipf: Pioneers of power-law city-size distributions. *Archive for History of Exact Sciences* 77(6): 601–613.

Sakoda JM (1971) The checkerboard model of social interaction. *Journal of Mathematical Sociology* 1(1): 119–132.

Sanders P and Sanders F (2004) Spatial urban dynamics. In: *Proceedings of the 2004 International System Dynamics Conference*, Oxford, UK, 25–29 July, pp.1–32. Oxford: Keble College.

Schelling T (1969) Models of segregation. *American Economic Review* 59(2): 488–493.

Schelling TC (1971) Dynamic models of segregation. *Journal of Mathematical Sociology* 1(2): 143–186.

Schelling TC (1978) *Micromotives and Macrobehavior*. New York, NY: Norton.

Secord JA (2004) Knowledge in transit. *Isis* 95(4): 654–672.

Shore C, Wright S and Però D (eds) (2011) *Policy Worlds: Anthropology and the Analysis of Contemporary Power*. New York, NY: Berghahn Books.

Sombart W (1927) *Der moderne Kapitalismus. Dritter Band: Das Wirtschaftsleben im Zeitalter des Hochkapitalismus*. 2nd edn. Berlin: Duncker & Humblot.

Spielman S and Harrison P (2014) The co-evolution of residential segregation and the built environment at the turn of the 20th century: A Schelling model. *Transactions in GIS* 18(1): 25–45.

Sugiura Y (2006) Planning on settlement location in the IJsselmeerpolders and Central Place, theory. *Geographical Review of Japan* 79(11): 566–587.

Tannier C (2018) About fractal models in urban geography and planning: Refuting the aesthetics and the universal norm. *Cybergeo Conversation*, 12 March. Available at: https://cybergeo.hypotheses.org/223 (accessed 24 November 2023).

Tannier C (forthcoming) Principles of fractal planning and urban design (Chapter 8). In: Tannier C (ed.) *Fractal Geometry in Human Geography and Planning*. ISTE-Wiley.

Tannier C, Vuidel G, Frankhauser P, et al. (2010) Simulation fractale d'urbanisation. MUP-city, un modèle multi-échelle pour localiser de nouvelles implantations résidentielles. *Revue internationale de géomatique* 20(3): 303–329.

Tannier C, Vuidel G, Houot H, et al. (2012) Spatial accessibility to amenities in fractal and nonfractal urban patterns. *Environment and Planning B: Planning and Design* 39(5): 801–819.

Temenos C and Baker T (2015) Enriching urban policy mobilities research. *International Journal of Urban and Regional Research* 39(4): 841–843.

Theodore N (2019) Policy mobilities. *Oxford Bibliographies*. Available at: https://www.oxfordbibliographies.com/display/document/obo-9780199874002/obo-9780199874002-0205.xml (accessed 28 November 2023).

Tilly C (2005) The colonial present: Afghanistan, Palestine, Iraq. *Political Science Quarterly* 120(1): 134–135.

Waddell P (2002) UrbanSim: Modeling urban development for land use, transportation, and environmental planning. *Journal of the American Planning Association* 68(3): 297–314.

Weimer AM and Hoyt H (1939) *Principles of Urban Real Estate*. 1st edn. New York: Ronald Press.