Inconsistency of the capital asset pricing model in a multi-currency environment

Khalifa Al-Thani[*], Domenico Mignacca[†]

Gianluca Fusai[‡], Fabio Caccioli[§], Guido Germano[¶]

May 31, 2023

Abstract

The capital asset pricing model (CAPM) is a widely adopted model in asset pricing theory and portfolio construction because of its intuitive nature. One of its main conclusions is that there exists a global market portfolio that each rational investor should hold in proportion with the risk-free asset. In this paper we demonstrate theoretically and through an example that the CAPM cannot hold in a multi-currency environment. This is because it produces different market risk premia depending on the investor's base currency unless each exchange rate is uncorrelated with the asset prices in the portfolio. This finding has significant implications, including questioning the starting point of the Black & Litterman (1992) model, which is widely used in asset allocation and assumes that the CAPM equilibrium provides a neutral starting point for estimating expected risk premia. However, this assumption may not hold in a multi-currency environment, potentially rendering the model less effective.

[*]**Corresponding Author**, Investment Strategy, Qatar Investment Authority, P.O. box 23224, Doha, Qatar, kkalthani@qia.qa

[†]Investment Risk, Qatar Investment Authority, P.O. Box: 23224, Doha, Qatar, dmignacca@qia.qa

[‡]Faculty of Finance, Bayes Business School (formerly Cass), City, University of London, 106 Bunhill Row, London EC1Y 8TZ UK, gianluca.fusai.1@city.ac.uk, and Dipartimento SEI, Università del Piemonte Orientale, Via Perrone 18, 28100 Novara, Italy

[§]Department of Computer Science, University College London, 66–72 Gower Street, London WC1E 6EA, United Kingdom, f.caccioli@ucl.ac.uk, and Systemic Risk Centre, London School of Economics and Political Science, Houghton Street, London WC2A 2AE, United Kingdom

[¶]Department of Computer Science, University College London, 66–72 Gower Street, London WC1E 6EA, United Kingdom, g.germano@ucl.ac.uk, and Systemic Risk Centre, London School of Economics and Political Science, Houghton Street, London WC2A 2AE, United Kingdom

I Introduction

The capital asset pricing model (CAPM) (Sharpe 1964, Lintner 1965) is an easily understandable and straightforward model that explains the relationship between risk and expected return in an efficient market and is widely regarded as the initial and most commonly used asset pricing model. Together with the Markowitz portfolio selection model (Markowitz 1952), it is at the foundation of modern financial theory. Despite numerous proposed advancements over the past five decades, both the original CAPM and Markowitz models remain the primary tools used by scholars and investors for asset pricing and allocation (Rubinstein 2002).

The Markowitz (1952) portfolio selection model proposes that constructing portfolios with minimum variance given an expected return constraint can generate an efficient frontier, where each portfolio on the frontier provides either the highest expected return for a given level of risk or the lowest risk for a target return. The CAPM utilizes this result by demonstrating that, under specific assumptions, in equilibrium, the market portfolio, calculated by dividing each asset's market capitalization by the total market capitalization of all assets, lies on the efficient frontier. Consequently, a linear relationship between the asset's risk premium and the market portfolio risk premium can be established. This relationship is captured by the asset beta, which represents the covariance of the market return with the asset returns, standardized by the variance of the market portfolio.

The international capital asset pricing model (ICAPM) has been proposed as an extension of the traditional CAPM to address the challenge of a multi-currency environment (Solnik 1974*a*, Stulz 1981, Adler & Dumas 1983). The ICAPM recognizes that investors are concerned with consumption in their respective local currencies, and therefore evaluate portfolio risk, which includes both market and currency risks, differently based on their base currency. While the literature on the ICAPM focuses on taking into consideration currencies as another factor in estimating risk premia, our paper shows that including currencies that are correlated to the assets will produce different risk premia depending on the base currency.

The aim of the present study is to contribute to the theoretical discussion surrounding the CAPM by examining its applicability in a multi-currency environment. The question posed here is, if an investor calculates their implied returns in a base currency, for example USD, and converts these returns to another currency, for example EUR, will they retrieve the same implied returns if they convert the asset in EUR and calculate their implied risk premia? In other words, are the implied returns in USD converted to EUR the same as the implied returns calculated in EUR? As we shall see, these two values are not equal, which makes the CAPM inconsistent in a multi-currency environment: investors holding different base currencies may imply different equilibrium risk premia. We prove this inconsistency and illustrate it with a simple example.

Our findings have significant empirical and practical implications for the asset management industry.

Empirically, the relationship of the CAPM may be untestable in an international setting, as different risk premia are obtained depending on the investor's base currency. From an asset management perspective, the Black-Litterman (Black & Litterman 1992) model extends the CAPM to estimate the implied expected returns of assets and adjust them based on the investor's views. However, our results suggest that the Black-Litterman model's CAPM cannot provide a neutral starting point for estimating expected risk premia for investors with different base currencies.

In their paper, (Black & Litterman 1992) argue that incorporating the global capital asset pricing model (CAPM) equilibrium can enhance the efficacy of investment models. Specifically, they assert that "consideration of the global CAPM equilibrium can significantly improve the usefulness of these models. In particular, equilibrium returns for equities, bonds and currencies provide a neutral starting point for estimating the set of expected excess returns needed to drive the portfolio optimization process. This set of neutral weights can then be tilted in accordance with the investor's view." While the authors acknowledge that their results are presented from a U.S. dollar perspective, they also suggest that the use of other currencies would yield similar findings (Black & Litterman 1992, page 30). However, our results indicate that the market risk premia utilized in the Black-Litterman model cannot be uniquely determined, and the determination of market implied views necessitates reconsideration if correlations between assets and currencies are not zero. Therefore, the neutral starting point for asset allocation in the Black-Litterman model is contingent upon the investor's base currency, and further study is required to discern its application in an international context.

The remainder of this paper is organized as follows. Section II introduces the international CAPM and shows that the CAPM cannot hold in a multi-currency environment unless the correlations of the assets and exchange rates are zero. Section III provides an example showing that the CAPM does not hold in a multi-currency environment. Finally, Section IV concludes the paper and discusses opportunities for future research.

II Inconsistency of the CAPM in a multi-currency environment

To illustrate the inconsistency of the CAPM in a multi-currency framework, let us consider an investor with base currency k, for example $k = \$, €, \ldots$, and let us assume that there is no segmentation of the international capital market, that is, national capital markets are perfectly integrated. The single-factor ICAPM, also referred to as the "global CAPM" (GCAPM), states that, if markets are in equilibrium, then the risk premia in currency k are

$$\mathbf{r}_k = \frac{R_k}{\sigma_k} \mathbf{\Sigma}_k \mathbf{w}, \tag{1}$$

where $\mathbf{r}_k$ is the vector of risk premia in currency k, whose elements $r_{i/k}$ are the risk premia of asset i, $i = 1, \ldots, n$, in currency k; $\mathbf{\Sigma}_k$ is the covariance matrix in currency k whose elements $\sigma_{i/k,j/k}$ are the covariance between assets i and j in currency k; σ_k is the volatility of the world market portfolio in currency k, and $\mathbf{w}$ is the vector of the asset weights of the world market portfolio whose elements w_j are the weights of asset j, $j = 1, \ldots, n$. The Sharpe ratio in currency k,

$$R_k = \frac{r_k}{\sigma_k}, \tag{2}$$

is the ratio of the risk premium r_k of the world portfolio in currency k and its volatility σ_k. The risk premium is the difference $r_k = \mu_k - r_{\mathrm{f}/k}$ between the expected return μ_k and the risk-free interest rate $r_{\mathrm{f}/k}$, both in currency k. The Sharpe ratio divided by the volatility of the market portfolio is commonly interpreted as the market price of risk.

Eq. (1) implies that to calculate the vector of the implied risk premia, an assumption should be made about the market portfolio's Sharpe ratio, and the CAPM assumption that all investors have the same market portfolio regardless of their base currency should hold. However, as we shall see, this assumption does not hold unless the returns of the assets in the portfolio and the exchange rates are uncorrelated.

If an investor wants to calculate the implied risk premium of asset i in the base currency k, they would use Eq. (1) and get

$$r_{i/k} = \frac{R_k}{\sigma_k} \sum_{j=1}^{n} \sigma_{i/k,j/k} w_j = \beta_{i/k} r_k, \tag{3}$$

where $\beta_{i/k}$ is the beta of asset i in currency k and is computed as

$$\beta_{i/k} = \frac{1}{\sigma_k^2} \sum_{j=1}^{n} \sigma_{i/k,j/k} w_j. \tag{4}$$

For instance, an investor with the US Dollar (USD) as their base currency would use

$$r_{i/\$} = \frac{R_\$}{\sigma_\$} \sum_{j=1}^{n} \sigma_{i/\$,j/\$} w_j = \beta_{i/\$} r_\$. \tag{5}$$

The price $S_{i/€}(t)$ of an asset i in EUR at time t is obtained from its price $S_{i/\$}(t)$ in USD multiplying the latter by the exchange rate of 1 USD to $X_{\$/€}(t)$ EUR,

$$S_{i/€}(t) = S_{i/\$}(t) X_{\$/€}(t). \tag{6}$$

Assuming that both assets and the FX rate follow a log-normal distribution, the asset price in currency k

after a time interval $\Delta t = t_1 - t_0$ (assumed to be the time horizon of market participants, or the duration between portfolio readjustments) is

$$S_{i/k}(t_1) = S_{i/k}(t_0) \exp\left(\left(r_{i/k} + r_{\mathrm{f}/k} - q_{i/k} - \frac{1}{2}\sigma^2_{i/k}\right)\Delta t + \sigma_{i/k}\varepsilon_{i/k}\sqrt{\Delta t}\right), \quad k = \$, €, \tag{7}$$

where $q_{i/k}$ is the convenience (e.g., dividend) yield of asset i in currency k that for simplicity in the following we will set to 0, $\sigma_{i/k}$ is the volatility of asset i in currency k, and $\varepsilon_{i/k}$ is a standard normal random variable. In the same manner, the FX rate after a time interval $\Delta t = t_1 - t_0$ is

$$X_{\$/€}(t_1) = X_{\$/€}(t_0) \exp\left(\left(r_{\$/€} + r_{\mathrm{f}/€} - r_{\mathrm{f}/\$} - \frac{1}{2}\sigma^2_{\$/€}\right)\Delta t + \sigma_{\$/€}\varepsilon_{\$/€}\sqrt{\Delta t}\right), \tag{8}$$

where $r_{\$/€}$ is the risk premium of the FX rate, $\sigma_{\$/€}$ is the volatility of the FX rate, and $\varepsilon_{\$/€}$ is a standard normal random variable. From Eqs. (6)–(8) we get

$$\begin{aligned} &S_{i/\$}(t_1)X_{\$/€}(t_1) \\ &\quad = S_{i/\$}(t_0)X_{\$/€}(t_0) \exp\left(\left(r_{i/\$} + r_{\mathrm{f}/€} - \frac{1}{2}\sigma^2_{i/\$} + r_{\$/€} - \frac{1}{2}\sigma^2_{\$/€}\right)\Delta t + \left(\sigma_{i/\$}\varepsilon_{i/\$} + \sigma_{\$/€}\varepsilon_{\$/€}\right)\sqrt{\Delta t}\right), \end{aligned} \tag{9}$$

which must be equal to

$$S_{i/€}(t_1) = S_{i/€}(t_0) \exp\left(\left(r_{i/€} + r_{\mathrm{f}/€} - \frac{1}{2}\sigma^2_{i/€}\right)\Delta t + \sigma_{i/€}\varepsilon_{i/€}\sqrt{\Delta t}\right). \tag{10}$$

In the following, we set without loss of generality $\Delta t = 1$ units of time. Therefore, we must have

$$r_{i/€} - \frac{1}{2}\sigma^2_{i/€} = r_{i/\$} - \frac{1}{2}\sigma^2_{i/\$} + r_{\$/€} - \frac{1}{2}\sigma^2_{\$/€} \tag{11}$$

and

$$\sigma_{i/€}\varepsilon_{i/€} = \sigma_{i/\$}\varepsilon_{i/\$} + \sigma_{\$/€}\varepsilon_{\$/€}. \tag{12}$$

This is possible if and only if

$$\sigma^2_{i/€} = \sigma^2_{i/\$} + \sigma^2_{\$/€} + 2\rho_{i/\$,\$/€}\sigma_{i/\$}\sigma_{\$/€}, \tag{13}$$

where $\rho_{i/\$,\$/€}$ is the correlation of the price of asset i in USD and the FX rate $X_{\$/€}$. Replacing this condition in Eq. (11), we have

$$r_{i/€} = r_{i/\$} + r_{\$/€} + \rho_{i/\$,\$/€}\sigma_{i/\$}\sigma_{\$/€}. \tag{14}$$

All quantities in Eq. (14) are known, except the FX risk premium $r_{\$/€}$. Although we know that $r_{\$/€}$ should be the same across all assets i for a given currency pair, we now show that this condition can only hold if $\rho_{i/\$,\$/€} = 0$ for all i.

The determination of the implied risk premia in EUR requires the corresponding covariance matrix in Euro. This can be obtained by using equation Eq. (9) with reference to assets i and j and then by computing the covariance between the log-returns of the two assets. It turns out (Fusai et al. 2023)

$$\sigma_{i/€,j/€} = \sigma_{i/\$,j/\$} + \sigma_{i/\$,\$/€} + \sigma_{j/\$,\$/€} + \sigma^2_{\$/€}, \tag{15}$$

where $\sigma_{i/\$,\$/€}$ is the covariance of asset i in USD with the FX currency rate. In the Appendix we also illustrate how to implement the transformation of the covariance from one currency to another via a simple matrix multiplication as illustrated in Fusai et al. (2023).

If we knew the value of the FX risk premimum $r_{\$/€}$, then, using Eq. (14), we could calculate the Sharpe ratio of the EUR portfolio,[1]

$$R_€ = \frac{1}{\sigma_€}\left(r_{\$/€} + \sum_{j=1}^{n} w_j r_{j/\$} + \sum_{j=1}^{n} w_j \sigma_{j/\$,\$/€}\right), \tag{16}$$

where $\sum_{j=1}^{n} w_j \sigma_{j/\$,\$/€}$ is the covariance of the portfolio in USD and the FX return. Substituting Eq. (16) into Eq. (1) with $k = €$ gives the risk premium in EUR of asset i,

$$r_{i/€} = \beta_{i/€}\left(r_{\$/€} + \sum_{j=1}^{n} w_j r_{j/\$} + \sum_{j=1}^{n} w_j \sigma_{j/\$,\$/€}\right). \tag{17}$$

Combining Eqs. (14) and (17) we can solve for the FX risk premium and we obtain

$$r_{\$/€} = \frac{1}{\beta_{i/€} - 1}\left[r_{i/\$} + \rho_{i/\$,\$/€}\sigma_{i/\$}\sigma_{\$/€} - \beta_{i/€}\left(r_\$ + \sum_{j=1}^{n} w_j \sigma_{j/\$,\$/€}\right)\right]. \tag{18}$$

We prove now that the CAPM holds only if $\rho_{i/\$,\$/€} = 0,\ i = 1, \ldots, n$. Indeed, with this assumption Eq. (18) becomes

$$r_{\$/€} = \frac{r_{i/\$} - r_\$ \beta_{i/€}}{\beta_{i/€} - 1}. \tag{19}$$

Moreover, using the zero-correlation assumption and converting the covariances from EUR to USD using

[1] Eq. (16) is valid if we assume that the portfolio is continuously rebalanced, see (Merton 1990, p. 127), i.e. if the time interval Δt shrinks to zero.

Eq. (15), we have

$$\beta_{i/€} = \frac{1}{\sigma_€^2} \sum_{j=1}^{n} w_j \sigma_{i/€,j/€} = \frac{1}{\sigma_€^2} \sum_{j=1}^{n} w_j \sigma_{i/\$,j/\$} + \frac{\sigma_{\$/€}^2}{\sigma_€^2} = \beta_{i/\$} \frac{\sigma_\$^2}{\sigma_€^2} + \frac{\sigma_{\$/€}^2}{\sigma_€^2}. \tag{20}$$

Under the assumption of zero correlation we have $\sigma_{i/€}^2 = \sigma_{i/\$}^2 + \sigma_{\$/€}^2$, and then at portfolio level it must hold also $\sigma_€^2 = \sigma_\$^2 + \sigma_{\$/€}^2$. In this way, we rewrite Eq. (20) as

$$\beta_{i/€} - 1 = (\beta_{i/\$} - 1) \frac{\sigma_\$^2}{\sigma_€^2}, \tag{21}$$

and a simple formula to transform the asset beta from one currency to another, which is valid only assuming a zero correlation with the exchange rates. Inserting Eqs. (3), (14) and (21) into Eq. (19), we get

$$r_{\$/€} = r_\$ \frac{\beta_{i/\$} - 1 - (\beta_{i/\$} - 1) \frac{\sigma_\$^2}{\sigma_€^2}}{(\beta_{i/\$} - 1) \frac{\sigma_\$^2}{\sigma_€^2}}, \tag{22}$$

and finally

$$r_{\$/€} = r_\$ \left(\frac{\sigma_€^2}{\sigma_\$^2} - 1 \right), \tag{23}$$

that is, when the correlation between the price of asset i and an exchange rate is zero, the FX risk premium is the same for all assets. However, if the zero correlation assumption is not satisfied, we illustrate in the next section with a numerical example that distinct FX equilibrium risk premia exist for each asset. This finding renders the CAPM inconsistent. An additional remark could be raised regarding Eq.(23); even if the assets are uncorrelated with the currency rates, the FX risk premium can be positive or negative depending on the ratio between the variances of the market portfolio in the two currencies. Therefore the implied forward currency rate is a biased predictor of the future spot currency rate, providing therefore a theoretical basis to the empirical result in (Sarno et al. 2012).

III A numerical example illustrating the inconsistency of the international CAPM

To demonstrate the inadequacy of the CAPM in a multi-currency environment, we present an example that highlights the divergence in equilibrium risk premia obtained when moving from one currency to another. Without loss of generality, we assume that the risk-free rates in the three currencies are identical in this example.

Let us consider a scenario where an investor holds a portfolio consisting of three assets: Apple (AAPL), Volkswagen (VOW), and Unilever (ULVR), each denominated in a different currency (USD, EUR, and British Pound (GBP)). To analyze this portfolio, we use the monthly time series in USD of the three assets, spanning from 29 January 2010 to 30 September 2022. We also incorporate the time series for the EUR/USD and GBP/USD exchange rates over the same period to obtain the covariance matrix presented in Table I. For instance, the covariance between the log-returns of AAPL and VOW is 2.31. We use this covariance matrix $\mathbf{\Sigma_\$}$ as a starting point to derive the implied equilibrium risk premia in EUR.

With $\mathbf{\Sigma_\$}$ as a starting point, we assume a Sharpe ratio of $R_\$ = 0.5$ for the US market portfolio to derive the equilibrium risk premia in USD, as shown in Eq. (5). We also assume that the world market portfolio is equally weighted, as indicated in column 2 of Table II. The volatility of the market portfolio in USD, $\sigma_\$ = \sqrt{\mathbf{w}'\mathbf{\Sigma}_\$\mathbf{w}}$, is equal to 5.585%. The covariances of each asset with the market portfolio are computed as $\mathbf{\Sigma}_\$\mathbf{w}$ and are equal to 3.11 (AAPL), 4.68 (VOW), and 1.56 (ULVR). The resulting USD risk premia are calculated by applying Eq. (1) and are presented in column 3 of Table II. For instance, the risk premium in USD for AAPL is 2.79%.

To convert the implied equilibrium risk premia from USD to EUR, we first need to determine the risk premium of the USD/EUR exchange rate. Assuming that the covariances between the assets denominated in USD and the USD/EUR exchange rate are as shown in Table I, we use Eq. (18) to determine the USD/EUR risk premium required to make the converted implied returns from USD to EUR equivalent to the implied returns calculated in EUR. For Apple, this risk premium is found to be -2.52%. Using this value and Eq. (14), we obtain the fourth column of Table II, which presents the risk premia in EUR for VOW and ULVR. For instance, the risk premia in Euros for VOW and ULVR are calculated to be 1.573% and -1.170%, respectively. We can now compute the risk premium in EUR of the market portfolio using the average of the EUR risk premia in column 4 of Table II, which results in a value of 0.214%. We also need to determine the covariances in EUR of each asset with the market portfolio. To accomplish this, we convert the USD covariance matrix to EUR using Eq. (15).

The resulting EUR covariance matrix $\mathbf{\Sigma_€}$ can be found in Table AI in the Appendix. This matrix is used to calculate the betas in EUR for each asset, which are presented in column 5 of Table II. Using Eq. (1) with $k = €$, we compute the implied risk premia in EUR and report them in column 6 of the same table.

However, our calculations reveal that the implied risk premia in EUR differ from the equilibrium risk premia in USD converted to EUR using Eq. (14). These two sets of risk premia are only equivalent, by construction, for AAPL. For the CAPM to be valid, they should be equivalent for all assets. In fact, as we proved, the risk premia presented in columns 4 and 6 of Table II are only consistent when the covariances between the asset prices and the EUR/USD exchange rate are zero.

Let us assume a zero-covariance between the assets and the currency rates. We then use Eq. (18) to determine the equilibrium FX risk premium, which is found to be $r_{\$/€} = 0.56\%$ for all assets. We can now apply Eq. (14) to convert the USD risk premia to EUR risk premia, which are reported in column 4 of Table III and give a risk premium of the EUR portfolio equal to 3.349%. We can also convert the USD covariance matrix $\mathbf{\Sigma}_{\$}$ of Table AII to EUR. The EUR covariance matrix $\mathbf{\Sigma}_{€}$ is given in Table AIII in the Appendix and can be used to calculate the betas in EUR for each asset. They are reported in column 5 of Table III. With the EUR portfolio risk premium and the EUR betas, we use Eq. (1) to calculate the implied risk premium in EUR for each asset. The final result is presented in the last column of Table III, and we obtain the same risk premia as in column 4 of the same table. This confirms that the implied EUR risk premia are equal only when the correlation between the assets and the USD/EUR exchange rate is zero.

Table I. Covariance matrix in USD

	AAPL/USD	VOW/USD	ULVR/USD	USD/EUR
AAPL/USD	6.04	2.31	0.98	-0.25
VOW/USD	2.31	10.39	1.34	-0.97
ULVR/USD	0.98	1.34	2.37	-0.48
EUR/USD	-0.25	-0.97	-0.48	0.62

Table I: Covariance matrix of assets in USD and USD/EUR exchange rate with values multiplied by 1000.

Table II. Comparison of risk premia in EUR

Asset	Weights w_i	USD implied risk premia ($r_{i/\$}$)	EUR risk premia from Eq. (14)	EUR beta $\beta_{i/€}$	EUR implied risk premia, Eq. (1)
AAPL/USD	0.333	2.79%	0.239%	1.118	0.239%
VOW/USD	0.333	4.19%	1.573%	1.444	0.309%
ULVR/USD	0.333	1.40%	-1.170%	0.439	0.094%

Table II: Calculation of EUR asset risk premia assuming non-zero correlation among assets and currency rates. The table allows the comparison between the EUR risk premia (column 4) derived from the implied USD risk premia (column 3) and the implied EUR risk premia (column 6). To illustrate, the Apple Euro risk premium $r_{\mathrm{AAPL}/€}$ of 0.239% in column 4 is calculated using Eq. (14): $r_{\mathrm{AAPL}/€} = r_{\mathrm{AAPL}/\$} + r_{\$,€} + \sigma_{\mathrm{AAPL}/\$,\$/€} = 2.79\% - 2.52\% - 0.025\% = 0.239\%$. Similarly, for VOW, we obtain $r_{\mathrm{VOW}/€} = 4.19\% - 2.52\% - 0.097\% = 1.573\%$. The beta in EUR of each asset (column 5) is obtained by converting the USD covariance matrix ($\mathbf{\Sigma}_{\$}$) to EUR ($\mathbf{\Sigma}_{€}$), as shown in Table AI in the Appendix, and then computing $\mathbf{\Sigma}_{€}\mathbf{w}/\mathbf{w}'\mathbf{\Sigma}_{€}\mathbf{w}$. With these betas and the portfolio risk premium $((0.239\% + 1.573\% - 1.170\%)/3 = 0.214\%)$ in EUR, we can use Eq. (1) to derive the implied risk premia of each asset that are reported in column 6.

Table III. Comparing risk premia in EUR when correlation equals zero					
Asset	$r_{i/\$}$	$r_{\$,€}$	$r_{i/€}$ from Eq. (17)	$\beta_{i/€}$	$r_{i/€}$ from Eq. (1)
AAPL	2.79%	0.56%	3.343%	0.998	3.343%
VOW	4.19%	0.56%	4.748%	1.417	4.748%
ULVR	1.40%	0.56%	1.956%	0.584	1.956%

Table III: Calculation of EUR asset risk premia assuming zero correlation among assets and currency rates. Column 2 contains the USD risk premia ($r_{i/\$}$) from column 3 in Table II; column 3 gives the FX risk premium $r_{\$,€}$ computed according to Eq. (18) where we set $\rho_{i/\$,\$/€} = \sigma_{j/\$,\$/€} = 0$, or equivalently Eq. (23); column 4 gives the EUR risk premia computed from USD risk premia applying Eq. (14); column 5 gives the beta of asset i in EUR ($\beta_{i/€}$) computed by converting the USD covariance matrix to EUR in Table AIII in the Appendix and then computing $\mathbf{\Sigma_€ w}/\mathbf{w'\Sigma_€ w}$; the last column reports the implied EUR risk premia from Eq. (1), using the EUR betas and the EUR portfolio risk premium of 3.349% = (3.343%+4.748%+1.956%)/3.

IV Conclusion

The CAPM is an intuitive model and a useful starting point in the process of asset allocation and portfolio construction. However, as we have shown in this study, it not only fails to hold empirically in a single currency world, it also provides inconsistent results in a multi-currency world. We expect that in any portfolio construction exercise, investors will find that all currencies will be correlated with asset-classes in one way or another. Thus, each base currency will imply different asset risk premia and therefore lead to different optimal allocations. This is inconsistent with the traditional ICAPM result that the asset risk premia are equal regardless of the base currency.

This has a significant implication on investors and asset managers, as they rely heavily on the Black-Litterman model (1992) on asset allocation decisions. The model uses a global market portfolio as a starting point, where it is assumed all investors should hold the same portfolio regardless of their base currency. We have shown in this paper that this not the case, as investors with different base currencies will estimate different risk premia.

The raised inconsistency of the ICAPM opens the door for future research on asset pricing and allocation, specifically on how to estimate the risk premia in a multi-currency portfolio and how to use them in portfolio construction, see for example (Lustig et al. 2011, Corte et al. 2016).

References

Adler, M. & Dumas, B. (1983), 'International portfolio choice and corporation finance: A synthesis', *Journal of Finance* **38**(3), 925–984.

Black, F. & Litterman, R. (1992), 'Global portfolio optimization', *Financial Analysts Journal* **48**(5), 28–43.

Corte, P. D., Riddiough, S. J. & Sarno, L. (2016), 'Currency premia and global imbalances', *Review of Financial Studies* **29**(8), 2161–2193.

Fusai, G., Al-Thani, K. & Mignacca, D. (2023), 'Converting a covariance matrix from local currencies to a common currency'. SSRN 4454299.

Lintner, J. (1965), 'The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets', *Review of Economics and Statistics* **47**(1), 13–37.

Lintner, J. (1969), 'The aggregation of investor's diverse judgments and preferences in purely competitive security markets', *Journal of Financial and Quantitative Analysis* **4**(4), 347–400.

Lustig, H., Roussanov, N. & Verdelhan, A. (2011), 'Common risk factors in currency markets', *Review of Financial Studies* **24**(11), 3731–3777.

Markowitz, H. (1952), 'Portfolio selection', *Journal of Finance* **7**(1), 77–91.

Merton, R. C. (1973), 'An intertemporal capital asset pricing model', *Econometrica* **41**(5), 867–887.

Merton, R. C. (1990), *Continuous-Time Finance*, Blackwell, Cambridge, Massachusetts.

Reinganum, M. R. (1981), 'A new empirical perspective on the capm', *Journal of Financial and Quantitative Analysis* **16**(4), 439–462.

Rubinstein, M. (2002), 'Markowitz's "portfolio selection": A fifty-year retrospective', *Journal of Finance* **57**(3), 1041–1045.

Sarno, L., Schneider, P. & Wagner, C. (2012), 'Properties of foreign exchange risk premiums', *Journal of Financial Economics* **105**(2), 279–310.

Sharpe, W. F. (1964), 'Capital asset prices: A theory of market equilibrium under conditions of risk', *Journal of Finance* **19**(3), 425–442.

Solnik, B. H. (1974*a*), 'An equilibrium model of the international capital market', *Journal of Economic Theory* **8**(4), 500–524.

Solnik, B. H. (1974*b*), 'The international pricing of risk: An empirical investigation of the world capital market structure', *Journal of Finance* **29**(2), 365–378.

Stulz, R. M. (1981), 'A model of international asset pricing', *Journal of Financial Economics* **9**(4), 383–406.

Converting a covariance matrix among currencies

In order to convert a covariance matrix for assets whose prices are all in the same currency, e.g. \$, to another currency, e.g. €, we perform the calculation, see (Fusai et al. 2023)

$$\mathbf{\Sigma}_{€} = \mathbf{B}'\mathbf{\Sigma}_{\$}\mathbf{B}. \tag{24}$$

In our example, involving three assets and the currency rate €/\$, the matrix $\mathbf{B}$ is given by

$$\mathbf{B} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

whilst $\mathbf{\Sigma}_{\$}$ is in Table I. By performing the above product, we obtain the covariance matrix in EUR $\mathbf{\Sigma}_{€}$ in Table AI. This matrix is then used to compute the beta of each asset in €, by computing $\mathbf{\Sigma}_{€}\mathbf{w}/\mathbf{w}'\mathbf{\Sigma}_{€}\mathbf{w}$.

	AAPL/EUR	VOW/EUR	ULVR/EUR
AAPL/EUR	6.17	1.72	0.88
VOW/EUR	1.72	9.08	0.52
ULVR/EUR	0.88	0.52	2.04

Table AI: Covariance matrix in EUR converted from the USD covariance matrix in Table I.

The betas expressed in Euro for each asset are reported in the first column of Table III. Let us now assume that the covariance between the different assets in USD and the currency rate USD/EUR is zero: to do so, we modify the last column and last row of the covariance matrix in Table I as in Table AII.

	AAPL/USD	VOW/USD	ULVR/USD	USD/EUR
AAPL/USD	6.04	2.31	0.98	0.00
VOW/USD	2.31	10.39	1.34	0.00
ULVR/USD	0.98	1.34	2.37	0.00
USD/EUR	0.00	0.00	0.00	0.62

Table AII: USD covariance matrix assuming zero covariance between asset and FX returns.

The resulting covariance matrix $\boldsymbol{\Sigma}_{€}$ in Euro is finally presented in Table AIII.

	AAPL/EUR	VOW/EUR	ULVR/EUR
AAPL/EUR	6.66	2.93	1.61
VOW/EUR	2.93	11.01	1.96
ULVR/EUR	1.61	1.96	2.99

Table AIII: Covariance matrix in EUR converted from the USD covariance matrix assuming zero correlation among assets and FX returns.