Max-Min Fair Beamforing Design for a RIS-Assisted System with SWIPT

Yayun Wei, Ziyao Peng, Jie Tang, *Senior Member*, *IEEE*, Xiuyin Zhang, *Fellow*, *IEEE*, Kai-Kit Wong, *Fellow*, *IEEE*, and Jonathon Chambers, *Fellow*, *IEEE*

***Abstract*—This paper investigates a multiuser reconfigurable intelligent surface (RIS)-assisted simultaneous wireless information and power transfer (SWIPT) system, in which a RIS assists in establishing favorable wireless communication environment. Particularly, we study the max-min signal-to-interference-plus-noise ratio (SINR) problem of the system by joint optimizing the base station (BS) active beamforming vectors, the RIS passive beamforming vector, and the power splitting (PS) ratios while guaranteeing the BS maximum transmit power budget and the minimum harvested energy threshold. The considered max-min SINR problem is highly non-convex, which is arduous to directly solve. Hence, we present an efficient alternating optimization (AO) algorithm to decompose the max-min SINR problem into three tractable subproblems, which are solved in an alternating manner. Particularly, we apply semi-definite relaxation (SDR) technique and the bisection method to tackle the beamforming optimization subproblems, and then derive a closed-form solution to solve the PS ratios optimization subproblem. Numerical simulations verify the superiority of our proposed AO algorithm and demonstrate that deploying RIS can achieve significantly increased min-SINR value.**

***Index Terms*—signal-to-interference-plus-noise ratio (SINR), reconfigurable intelligent surface (RIS), simultaneous wireless information and power transfer (SWIPT), power splitting (PS).**

I. Introduction

WITH the proliferation of low-power Internet of Things (IoT) applications, frequent battery charging or replacement of massive IoT devices may result in significant maintenance costs. To overcome the limited battery life problem, simultaneous wireless information and power transfer (SWIPT) has aroused great research interests. Specifically, SWIPT has the ability of offering simultaneous energy and information access, and thus the IoT devices can continuously recharge themselves with energy from the surrounding wireless environment [1]. Meanwhile, reconfigurable intelligent surface (RIS) has been acknowledged as a disruptive technology to support cost-effective and energy-efficient communication for future wireless networks [2], [3]. A RIS equipped with an array of passive reflecting elements does not need any energy source. By intelligently reflecting incident signals to the desired direction with adjustable amplitude and phase shift, a RIS can flexibly configure the wireless signal propagation, thus enhancing the desirable signals and suppressing the interference.

Integrating RIS and SWIPT in wireless communication networks can efficiently inherent the advantages of these two technologies. Consequently, there are several significant works in the literature focusing on deploying the RIS in the SWIPT system [4]–[10]. The authors in [4] investigated a downlink

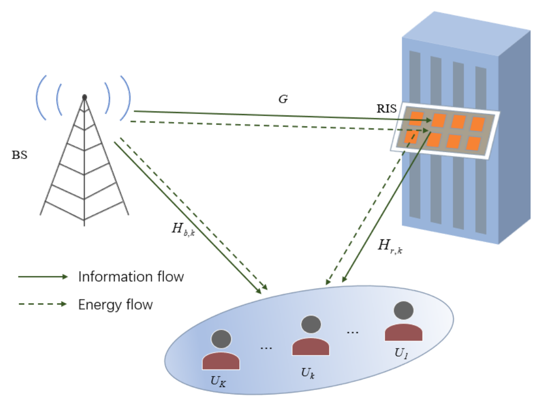

Fig. 1. A RIS-assisted system with SWIPT.

SWIPT multiple-input single-output (MISO) system with distributed RISs, where the energy efficiency (EE) performance was improved by reasonably designing the RIS control and the base station (BS) transmit beamforming. Furthermore, the authors in [5] explored the spectral efficiency maximization problem in a RIS-enhanced SWIPT multiple-input multiple-output (MIMO) system, while considering the imperfect primary user's channel state information. Additionally, the authors in [6] minimized the BS transmit power in a RIS-aided and power-splitting (PS)-based non-orthogonal multiple access (NOMA) network with SWIPT by utilizing an efficient two-stage algorithm. In addition, the authors in [7] jointly designed the RIS reflection coefficients, the UAV trajectory, and user scheduling in a RIS-unmanned aerial vehicle (UAV) network with SWIPT to address the max-min energy consumption optimization problem.

Apart from the aforementioned works, there have been other innovative studies for RIS-assisted SWIPT systems focusing on weighted sum power maximization [8], minimum EE maximization [9], and achievable secrecy rate maximization [8], minimum EE maximization [9], and achievable secrecy rate maximization [10]. In [11], the authors investigated the max-min signal-to-interference-plus-noise ratio (SINR) problem in the systems with two different user categories, i.e. information users and energy users. Nevertheless, to our best knowledge, the max-min SINR problem in a RIS-assisted SWIPT system based on the PS scheme has not been investigated yet. On the other hand, to ensure the desired level of fairness among users, the minimum SINR serves as an important performance indicator in SWIPT systems, which could provide better user fairness in multiuser MISO downlink systems. Furthermore, the rapid decrease in signal power with respect to the distance of SWIPT can be overcome by RIS. Therefore, we focus on a mathematical framework that investigates a RIS-assisted system with SWIPT to maximize the minimum SINR while meeting the maximum transmit power budget at the BS as well as the minimum harvested energy threshold. Furthermore, we consider a practical non-linear (NL) PS-based architecture, which enables energy harvesting (EH) and information decoding (ID) simultaneously for each user. Specifically, we propose an alternating optimization (AO) algorithm in which the BS active beamforming vectors, the RIS passive beamforming vectors, as well as the PS ratios are optimized alternatly. In particular, the active and passive beamforming vectors are optimized by the semi-definite relaxation (SDR) technique and the bisection method, and the PS ratios are optimized based on a closed-form solution. The performance advantages of the proposed AO algorithm as well as the benefit of deploying RIS are unveiled by numerical simulations.

II. System Description and Problem Formulation

As depicted in Fig. 1, we consider a RIS-assisted SWIPT system with K single-antenna users, in which the RIS and the BS are euipped with M reflecting elements and Nt transmit antennas, respectively. Moreover, the set of RIS reflecting elements and that of the users in the system are represented as $\mathcal{M} \triangleq \{1,\ldots,M\}$ and $\mathcal{K} \triangleq \{1,\ldots,K\}$. We assume that both the BS and RIS controller are capable of obtaining the perfect channel state information (CSI). [1] The channels of the BS-user k link, BS-RIS link, and RIS-user k link are specified as $\mathbf{H}_{b,k} \in {}^{N_t\times 1}$, $\mathbf{G} \in {}^{M\times N_t}$ and $\mathbf{H}_{r,k} \in {}^{M\times 1}$, respectively. In addition, the reflection-coefficients matrix is denoted by $\mathbf{\Theta} = \mathrm{diag}\left(\beta_1 \mathrm{e}^{\mathrm{j}\vartheta_1},\ldots,\beta_N \mathrm{e}^{\mathrm{j}\vartheta_M}\right)$, where $\beta_m \in [0,1]$ and $\vartheta_m \in [0,2\pi]$ denote the amplitude and the phase shift associated with the m-th RIS reflecting elements. Furthermore, we set the RIS passive beamforming vector as $\mathbf{v} = \left[\beta_1 e^{j\vartheta_1},\ldots,\beta_N e^{j\vartheta_M}\right]^H$, and thus we can express the equivalent reflective channel between the BS and user k as $\mathbf{H}_{r,k}^H \mathbf{\Theta} \mathbf{G} = \mathbf{v}^H \mathbf{\Psi}_k$, in which $\mathbf{\Psi}_k = \mathrm{diag}\left(\mathbf{H}_{r,k}^H\right)\mathbf{G}$. In addition, we consider all the users under the PS-based SWIPT setup consisting of a conventional ID circuit which is able to decode the information carried in the radio frequency (RF)-band signal and an EH circuit which is used for harvesting energy. Morever, we denote the PS ratio for user k as $p_k \in (0,1)$, $\forall k \in \mathcal{K}$. Accordingly, the received signal of user k is split into two separated streams, where the p_k part is for ID and the $1-p_k$ part is for EH.

[1]In practice, to acquire the CSI, the users/BS send pilots in the uplink/downlink, and thus the BS-user direct channels can be estimated. Meanwhile, RIS can estimate the CSI from the BS/users through the signals received by its sensors. Next, based on the beamforming vectors at the BS or the RIS controller and then sent to the other, the CSI is exchanged between the BS and the IRS [2]. To reduce the training overhead for the RIS-assisted system, there are a series of channel estimation schemes for RIS-assisted MISO systems based on signal processing techniques such as compressed sensing, deep learning, alternating least squares, and so on [12].

Under the above analysis, the signal from the BS to user k can be formulated as

$$y_k^{ID} = \sqrt{p_k}\sum_{i=1}^{K}(\mathbf{v}^H\mathbf{\Psi}_k + \mathbf{H}_{b,k}^H)\mathbf{w}_i d_i + n_k, \tag{1}$$

where $d_i \sim \mathcal{CN}(0,\, I_{N_t})$, and $\mathbf{w}_i \in {}^{N_t\times 1}$ are the transmit information symbol for user i, and the corresponding active beamforming vector, respectively. $n_k \sim \mathcal{CN}(0,\sigma_k^2)$ is the noise vector of user k.

Hence, we can express the SINR at user k as

$$\mathrm{SINR}_k = \frac{p_k\left|\left(\mathbf{v}^H\mathbf{\Psi}_k + \mathbf{H}_{b,k}^H\right)\mathbf{w}_k\right|^2}{p_k\sum\limits_{i=1,i\neq k}^{K}\left|\left(\mathbf{v}^H\mathbf{\Psi}_k + \mathbf{H}_{b,k}^H\right)\mathbf{w}_i\right|^2 + \sigma_k^2}. \tag{2}$$

From practical perspectives, we consider a non-linear EH model based on the sigmoidal function [13]. Accordingly, we can formulate the harvested energy at each user as

$$E_k^{NL} = \frac{\frac{Z_k}{1+\exp(-x_k(e_k-y_k))} - Z_k\Omega_k}{1-\Omega_k}, \forall k \in \mathcal{K}, \tag{3}$$

where $\Omega_k \triangleq \frac{1}{1+\exp(x_k y_k)}$, x_k and y_k are determined by the EH circuit characteristics, Z_k denotes the maximum power that user k can harvest, and $e_k = (1-p_k)\sum\limits_{i=1}^{K}\left|\left(\mathbf{v}^H\mathbf{\Psi}_k + \mathbf{H}_{b,k}^H\right)\mathbf{w}_i\right|^2$ represents the input power, i.e. the split power for EH at user k.

In this paper, our objective is to maximize the minimum SINR by optimizing the BS active beamforming vectors $\{\mathbf{w}_k\}$, the RIS passive beamforming vector $\mathbf{v}$, as well as the PS ratios $\{p_k\}$ in an alternating optimization manner. Accordingly, we can mathematically express the orginal max-min SINR optimization problem as

$$\text{(P1)} \quad \max_{\mathbf{v},\{\mathbf{w}_k\},\{p_k\}} \min_{k\in\mathcal{K}} \quad \mathrm{SINR}_k \tag{4}$$

$$\text{s.t.} \quad \sum_{k=1}^{K}\|\mathbf{w}_k\|^2 \le P_m, \tag{5}$$

$$E_k^{NL} \ge E_{\min}, \forall k \in \mathcal{K}, \tag{6}$$

$$|\mathbf{v}_m| \le 1, \forall m \in \mathcal{M}, \tag{7}$$

$$0 < p_k < 1, \forall k \in \mathcal{K}, \tag{8}$$

in which constraint (5) implies that the transmit power should satisfy the maximum budget P_m and constraint (6) guarantees the minimum harvested energy requirement E_{min} for each user. Furthermore, constraint (7) is the RIS reflection constraint, and constraint (8) limits the PS ratios of each user. Here we express constraint (6) in an equivalent form to tackle its non-convexity. The inverse function of (3) is given by

$$P_k(E_k^{NL}) = y_k - \frac{1}{x_k}\ln\left(\frac{Z_k}{(1-\Omega_k)E_k^{NL} + Z_k\Omega_k} - 1\right), \forall k \in \mathcal{K}. \tag{9}$$

Based on (9), we can transform constraint (6) into the convex form with respect to $\{p_k\}$ as follows,

$$(1-p_k)\sum_{i=1}^{K}\left|(\mathbf{v}^H\mathbf{\Psi}_k + \mathbf{H}_{b,k}^H)\mathbf{w}_i\right|^2 \ge P_k(E_{\min}), \forall k \in \mathcal{K}. \tag{10}$$

With the aim of facilitating the derivation, we reformulate problem (P1) by introducing an auxiliary variable γ as

$$\text{(P2)} \quad \max_{\mathbf{v},\{\mathbf{w}_k\},\{p_k\}} \quad \gamma \tag{11}$$

$$\text{s.t.} \quad \text{SINR}_k \geq \gamma, \forall k \in \mathcal{K}, \tag{12}$$

$$(5), (7), (8), \text{and } (10).$$

III. The Proposed AO Algorithm

Problem (P2) is arduous to directly solve, resulting from the presence of the coupled variables $\{\mathbf{w}_k\}$, $\mathbf{v}$, and $\{p_k\}$. To cope with this issue, we present an efficient AO algorithm in this section to deal with the max-min SINR problem by decomposing problem (P2) into three tractable subproblems. First, the active beamforming vectors $\{\mathbf{w}_k\}$ are optimized to design the optimal transmit beam pattern. Subsequently, we optimize the passive beamforming vector $\mathbf{v}$ with the acquisition of the active beamforming vectors. Finally, we derive a closed-form solution to optimize the PS ratios $\{p_k\}$ based on the achieved active and passive beamforming vectors.

A. Optimal BS Active Beamforming Vector

We set the combined channel between the BS and user k as $\mathbf{c}_k = \mathbf{\Psi}_k{}^H \mathbf{v} + \mathbf{H}_{b,k}$. Motivated by the SDR technique, we define $\mathbf{C}_k = \mathbf{c}_k\mathbf{c}_k^H$ and $\mathbf{W}_k = \mathbf{w}_k\mathbf{w}_k^H$, where $\mathbf{W}_k$ satisfies $\mathbf{W}_k \succeq \mathbf{0}$ and rank($\mathbf{W}_k$) ≤ 1. Thus, we can reformulate the SINR at user k as

$$\text{SINR}_k^{'} = \frac{p_k \sum_{i=1}^{K} \text{Tr}(\mathbf{C}_\text{k}\mathbf{W}_\text{i}) + \sigma_k^2}{p_k \sum_{i=1,i\neq k}^{K} \text{Tr}(\mathbf{C}_\text{k}\mathbf{W}_\text{i}) + \sigma_k^2}. \tag{13}$$

Based on the fixed $\mathbf{v}$ and $\{p_k\}$, we can formulate the BS active beamforming optimization problem as

$$\text{(P3)} \quad \max_{\{\mathbf{w}_k\}} \quad \gamma \tag{14}$$

$$\text{s.t.} \quad \text{SINR}_k^{'} \geq \gamma, \forall k \in \mathcal{K}, \tag{15}$$

$$\sum_{k=1}^{K} \text{Tr}(\mathbf{W}_k) \leq P_m, \tag{16}$$

$$(1-p_k) \sum_{i=1}^{K} \text{Tr}(\mathbf{C}_\text{k}\mathbf{W}_\text{i}) \geq P_k(E_{\min}), \forall k \in \mathcal{K}, \tag{17}$$

$$\mathbf{W}_k \succeq \mathbf{0}, \forall k \in \mathcal{K}, \tag{18}$$

$$\text{rank}(\mathbf{W}_k) \leq 1, \forall k \in \mathcal{K}. \tag{19}$$

To efficiently tackle the non-convex problem (P3), we fix γ and obtain the feasibility problem (P4) as follows,

$$\text{(P4)} \quad \text{find } \{\mathbf{W}_k\} \tag{20}$$

$$\text{s.t.} \quad \text{SINR}_k^{'} \geq \gamma, \forall k \in \mathcal{K}, \tag{21}$$

$$(16), (17), (18), \text{and } (19),$$

where γ is updated by the bisection method. specifically, we denote the optimal solution of γ to problem (P4) as γ^*. For a fixed γ, if problem (P4) is feasible, then we have $\gamma \leq \gamma^*$; otherwise we have $\gamma > \gamma^*$. Accordingly, we can perform the bisection search over $\gamma > 0$ while checking the feasibility of problem (P4) with any fixed γ to equivalently tackle problem (P3).

Note that problem (P4) is strictly concave in $\{\mathbf{W}_k\}, \forall k \in \mathcal{K}$ by relaxing the rank constraint (19), and thus the standard convex optimization methods [14] can be used to solve it. Moreover, the obtained solution is bound to satisfy $\text{rank}(\mathbf{W}_k) = 1, \forall k \in \mathcal{K}$, which can be mathematically proved [6]. Therefore, the eigenvalue decomposition can be applied to obtain the globally optimal BS active beamforming vectors.

B. Optimal RIS passive Beamforming Vectors

Similar to Section III-A, we optimize the RIS passive beamforming vector $\mathbf{v}$ by the bisection method and SDR technique while fixing $\{\mathbf{w}_k\}$ and $\{p_k\}$. Define $\mathbf{s}_{k,i} = \mathbf{\Psi}_k\mathbf{w}_i$, $t_{k,i} = \mathbf{h}_k^H\mathbf{w}_i, \forall k, i \in \mathcal{K}$, and we can obtain

$$\left|\left(\mathbf{v}^H\mathbf{\Psi}_k + \mathbf{H}_{b,k}^H\right)\mathbf{w}_i\right|^2 = \mathbf{v}^H\mathbf{S}_{k,i}\mathbf{v} + 2\text{Re}\left\{\mathbf{v}^H\mathbf{a}_{k,i}\right\} + |t_{k,i}|^2, \tag{22}$$

where $\mathbf{S}_{k,i} = \mathbf{s}_{k,i}\mathbf{s}_{k,i}^H$, $\mathbf{a}_{k,i} = \mathbf{s}_{k,i}t_{k,i}^H$. Furthermore, we define $\mathbf{F}_{k,i} = \begin{bmatrix} \mathbf{S}_{k,i} & \mathbf{a}_{k,i} \\ \mathbf{a}_{k,i}^H & 0 \end{bmatrix}$ and $\tilde{\mathbf{v}} = \begin{bmatrix} \mathbf{v} \\ 1 \end{bmatrix}$. Then we have

$$\left|\left(\mathbf{v}^H\mathbf{\Psi}_k + \mathbf{H}_{b,k}^H\right)\mathbf{w}_i\right|^2 = \tilde{\mathbf{v}}^H\mathbf{F}_{k,i}\tilde{\mathbf{v}} + |t_{k,i}|^2. \tag{23}$$

Accordingly, we can reformulate SINR_k as

$$\text{SINR}_k^{''} = \frac{p_k \sum_{i=1}^{K} \left(\text{Tr}\left(\mathbf{F}_{k,i}\mathbf{V}\right) + |t_{k,i}|^2\right) + \sigma_k^2}{p_k \sum_{i=1,i\neq k}^{K} \left(\text{Tr}\left(\mathbf{F}_{k,i}\mathbf{V}\right) + |t_{k,i}|^2\right) + \sigma_k^2}, \tag{24}$$

in which $\mathbf{V} = \bar{\boldsymbol{v}}\bar{\boldsymbol{v}}^H$.

Based on the above setups, the RIS passive beamforming optimization problem can be expressed as

$$\text{(P5)} \quad \max_{\mathbf{V}} \quad \gamma \tag{25}$$

$$\text{s.t.} \quad \text{SINR}_k^{''} \geq \gamma, \forall k \in \mathcal{K}, \tag{26}$$

$$(1-p_k) \sum_{i=1}^{K} \left(\text{Tr}\left(\mathbf{F}_{k,i}\mathbf{V}\right) + |t_{k,i}|^2\right) \geq P_k(E_{\min}), \forall k \in \mathcal{K}, \tag{27}$$

$$\mathbf{V} \succeq \mathbf{0}, \tag{28}$$

$$\text{rank}(\mathbf{V}) \leq 1, \tag{29}$$

$$\mathbf{V}_{m,m} \leq 1, \forall m \in \mathcal{M}, \tag{30}$$

$$\mathbf{V}_{M+1,M+1} = 1. \tag{31}$$

Similar to Section III-A, we fix γ and then relax the rank constraint (29), thus obtaining the feasibility problem (P6) as

$$\text{(P6)} \quad \text{find } \mathbf{V} \tag{32}$$

$$\text{s.t.} \quad \text{SINR}_k^{''} \geq \gamma, \forall k \in \mathcal{K}, \tag{33}$$

$$(27), (28), (30), \text{and } (31),$$

which is concave in $\mathbf{V}$. As such, the solution $\mathbf{V}^*$ of problem (P5) can be obtained by checking the feasibility of problem (P6), together with a bisection search over $\gamma > 0$. In particular, if rank $(\mathbf{V}^*) < 1$, we can obtain the optimal reflecting vector $\mathbf{v}^*$ by eigenvalue decomposition, otherwise we can apply Gaussian randomization procedure [15] to approximate $\mathbf{v}^*$.

C. Optimal PS ratios

Based on the variables $\{\mathbf{w}_k\}$ and $\mathbf{v}$ which have been optimized above, we finally formulate the problem of optimizing PS ratios as

$$\text{(P7)} \quad \max_{\{p_k\}} \quad \gamma \tag{34}$$

$$\text{s.t.} \quad \text{SINR}_k \geq \gamma, \forall k \in \mathcal{K}, \tag{35}$$

$$(1-p_k)\sum_{i=1}^{K}\left|\left(\mathbf{v}^H\mathbf{\Psi}_k+\mathbf{H}_{b,k}^H\right)\mathbf{w}_i\right|^2 \geq P_k(E_{\min}), \forall k \in \mathcal{K}, \tag{36}$$

$$0 < p_k < 1, \forall k \in \mathcal{K}. \tag{37}$$

We denote the derivative of SINR_k with respect to p_k as $\text{SINR}_k(p_k)$, which can be proved to satisfy $\text{SINR}_k(p_k) > 0$. On the other hand, constraints (36) and (37) can be equivalently expressed as

$$0 < p_k \leq 1 - \frac{P_k(E_{\min})}{\sum_{i=1}^{K}\left|\left(\mathbf{v}^H\mathbf{\Psi}_k+\mathbf{H}_{b,k}^H\right)\mathbf{w}_i\right|^2} < 1, \forall k \in \mathcal{K} \tag{38}$$

Accordingly, we can obtain the closed-form solution of problem (P7) as

$$p_k^* = 1 - \frac{P_k(E_{\min})}{\sum_{i=1}^{K}\left|\left(\mathbf{v}^H\mathbf{\Psi}_k+\mathbf{H}_{b,k}^H\right)\mathbf{w}_i\right|^2}, \forall k \in \mathcal{K} \tag{39}$$

Under the previous derivations and analyses, the summarized AO algorithm is shown in Algorithm 1. The computational complexity of the AO algorithm is $\mathcal{O}(I_{s1}\mathcal{O}_1 + I_{s2}\mathcal{O}_2 + \mathcal{O}(K))$, in which $\mathcal{O}_1 = \mathcal{O}(log(\frac{1}{\tau})(3K+1)({N_t}^{3.5}+3K{N_t}^{2.5}))$ and $\mathcal{O}_2 = \mathcal{O}(log(\frac{1}{\tau})(2K+1)(M^{3.5}+2KM^{2.5}))$, I_{s1} and I_{s1} are the iteration numbers of the bisection method for solving problem (P4) and (P6), respectively [9]. It is worth noting that the strict convergence of the overall AO algorithm can not be guaranteed due to the Gaussian randomization procedure. However, it has been mathematically and numerically proved that a good approximation of the optimal solution can be obtained by the SDR technique followed by Gaussian randomization (see [16] and the references therein). Therefore, the convergence can be improved by increasing the number of Gaussian randomizations.

Algorithm 1 The Proposed AO Algorithm

1: **INITIALIZE:** $\mathbf{v}^{(0)}, \{p_k\}^{(0)}$. Denote the accuracy threshold and iteration number as ε>0 and i = 0.
2: **REPEAT:**
3: $i = i + 1$
4: Based on given $\mathbf{v}^{(i-1)}$ and $\{p_k\}^{(i-1)}$, solve problem (P3) by the SDR technique and bisection method to obtain $\{\mathbf{w}_k\}^{(i)}$.
5: Based on given $\{\mathbf{w}_k\}^{(i)}$ and $\{p_k\}^{(i-1)}$, solve problem (P5) by the SDR technique and bisection method to obtain $\mathbf{v}^{(i)}$.
6: Based on given $\{\mathbf{w}_k\}^{(i)}$ and $\mathbf{v}^{(i)}$, calculate the closed-form solution (39) to obtain $\{p_k\}^{(i)}$.
7: Calculate the objective function in (P2) denoted as $\gamma^{(i)}$.
8: **UNTIL** convergence, i.e. $\left|\gamma^{(i)} - \gamma^{(i-1)}\right|^2 \leq \varepsilon$.

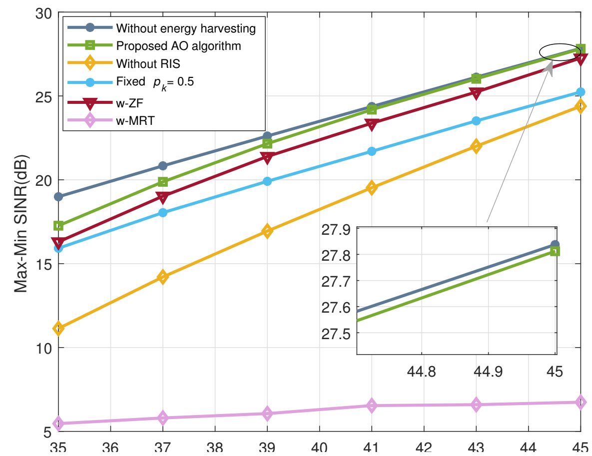

Fig. 2. Max-min SINR versus maximum transmit power at the BS.

IV. NUMERICAL SIMULATION

In this part of the paper, we offer numerical simulations to validate the performance superiority of our proposed AO algorithm. We consider a RIS-assisted MISO system with SWIPT, in which the 3 users ($K = 3$), one BS, and one RIS are positioned at (6m, 0), (5m, 1m), (5m, -1m), (0,0), and (5m, 0), respectively. Additionally, we formulate the distance-dependent path loss model as $D_L = D_0(\frac{q}{q_r})^{-\alpha}$ [17], where $D_0 = -30$dB is the path loss at $q_r = 1$m. Furthermore, for the BS-user, BS-RIS and RIS-user links, the path-loss exponents α are set to be 3.6, 2, and 2.5, respectively. Referring to [18], the RIS-user and BS-RIS links follow Rician fading, whereas the BS-user link follows Rayleigh fading. In addition, we set the Rician factor to 10. Furthermore, we suppose that the BS and RIS adopt a uniform linear array. These additional simulation parameters are set as: $N_t = 4, M = 10, \sigma_k^2 = -40$dBm, $P_m =$ 40dBm, $E_{min} = 1\mu$W, $P_M = 0.1$W, Z_k = 24 mW, $x_k = 150$, $y_k = 0.014$ [19].

To reflect the superiority of our model and algorithm, we have taken into account the following four benchmark schemes,

1) Scheme without RIS, marked as "Without RIS"
2) Scheme with fixed PS ratios, marked as "fixed p_k = 0.5"
3) Scheme applying Zero-Forcing (ZF) to optimize $\{\mathbf{w}_k\}$, marked as "w-ZF" [20]
4) Scheme applying Maximum Ratio Transmission (MRT) to optimize $\{\mathbf{w}_k\}$, marked as "w-MRT" [21].

First of all, we

we gradually increase the maximum transmit power P_m to study its effect on minimum SINR. As we can see in Fig. 2, the minimum SINR achieved by all the schemes increases monotonically in the P_m. This is due to the fact that with a higher power budget, more received power can be allocated to ID once the minimum harvested energy demand for each user is fulfilled, thus improving the data rate. Obviously, our proposed AO algorithm can achieve a higher minimum SINR as compared to both the "without RIS" and "fixed p_k = 0.5" schemes. The reason is that the energy and spectrum can be utilized more effectively with the auxiliary of RIS and the

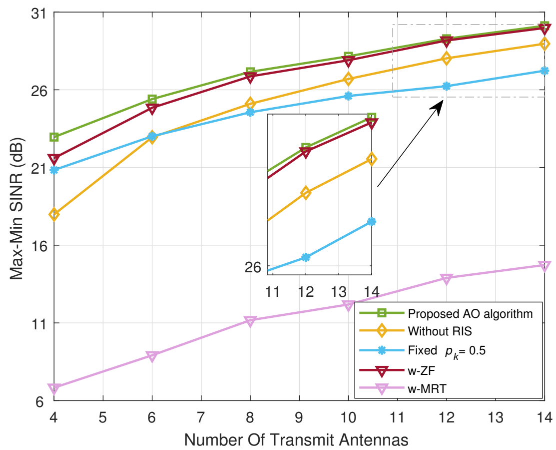

Fig. 3. Max-min SINR versus the number of transmit antennas.

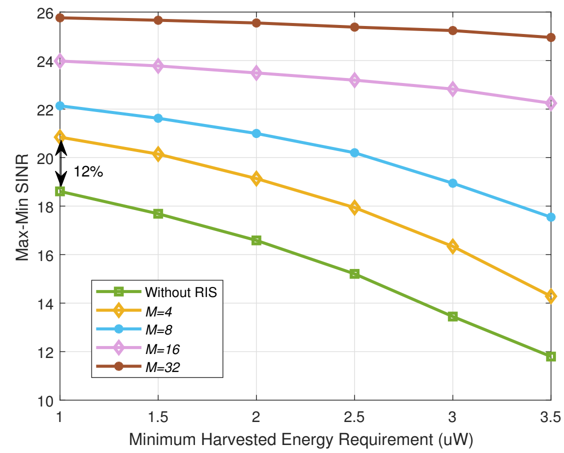

Fig. 4. Max-min SINR versus minimum harvested energy requirement.

reasonable PS ratios setting, thus enhancing the minimum SINR. In addition, our proposed algorithm can achieve a higher minimum SINR compared to the benchmark schemes applying the ZF and MRT approaches since higher transmit beamforming gain can be obtained by our proposed algorithm. In addition, we investigate the RIS-assisted system without energy harvesting. We can observe from Fig.2 that as P_m increases, the loss of the max-min SINR caused by adding the minimum harvested energy requirement gradually decreases. This is because with relatively large transmit power, there is still enough energy to increase SINR while meeting the minimum harvested energy requirement.

Next, we study the min-SINR performance versus the number of transmit antennas N_t. As we can see in Fig. 3 that by increasing N_t, the minimum SINR of all the schemes increases as well. The reason is that as N_t increases, higher transmit beamforming gain and spatial diversity gain can be obtained, thus yielding higher minimum SINR. Similarly, our proposed scheme can significantly enhance the minimum SINR compared with the four benchmark schemes, which indicates the superiority of the proposed AO algorithm. It should be noted that, as N_t becomes large, the min-SINR performance achieved by the ZF scheme approaches that of our proposed design. This is due to the fact that with a large number of transmit antennas, the ZF transmit beamforming can become asymptotically optimal, and thus obtaining the transmit beamforming gain similar to that in our proposed scheme.

Finally, we gradually increase the minimum harvested energy requirement E_{min} under different number of RIS reflecting elements M to investigate the effect of both E_{min} and M on the min-SINR performance. As we can see from Fig. 4, the minimum SINR decreases as E_{min} increases. This is explained by the fact that with a large value of E_{min}, more power needs to be used for EH to guarantee the minimum harvested energy threshold, resulting in a decrease in the achievable data rate. Furthermore, comparing the five curves, we can conclude that the minimum SINR increases monotonically with the increase of M. The reason is that stronger passive beamforming gain

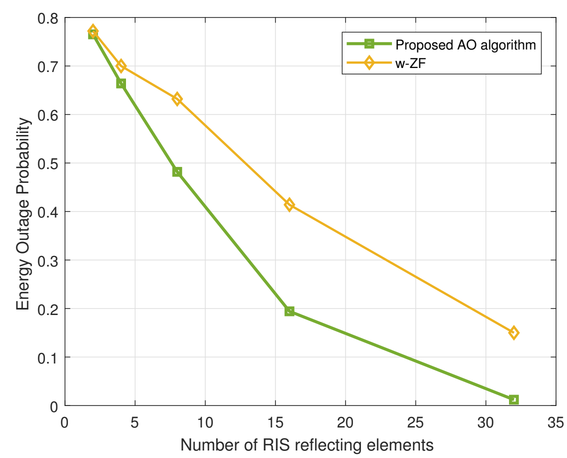

Fig. 5. Energy outage probability versus the number of RIS reflecting elements.

can be obtained with a larger M, thereby further enhancing the desired signals and mitigating the co-channel interference. Particularly, when $M = 4$, the minimum SINR of the RIS-assisted system is at least 12% larger than the system without RIS, which unveils the benefit of deploying RIS for improving the minimum SINR. In our final simulation, we define the energy outage probability as $\Pi_E(\tau) = \mathbb{P}\{\min \text{ SINR} < \tau\}, \tau = 28$. Fig 5. illustrates the energy outage probability in relation to the number of RIS reflecting elements *M*. As expected, the energy outage probability decreases with the increase of *M*. This indicates that the minimum energy constraint is more easily satisfied, which once again proves that the increase of *M* can improve the system performance.

V. Conclusion

This paper investigated the max-min SINR optimization problem for a RIS-assisted SWIPT system with NL EH model, while considering the minimum harvested energy threshold and the BS maximum transmit power budget. The original max-min SINR problem was extremely non-convex and complex due to the coupling of the variables, i.e., the BS active beamforming vectors, the RIS passive beamforming vector, and the PS ratios. To cope with the non-convex problem, we proposed an efficient AO algorithm to decompose the max-min SINR problem into three tractable subproblems. Specifically, the SDR technique as well as the bisection method were applied to tackle the beamforming optimization subproblems, and a closed-form solution was derived to solve the PS ratios optimization subproblem. Finally, numerical simulations unveiled the superiority of the proposed AO algorithm and demonstrated the auxiliary role of RIS is of great significance as compared to four benchmark schemes. In addition, this paper can be extended to a more realistic case by considering the imperfect CSI, which will be considered in our future work.

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