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Reliability-based Fracture Analysis for Shallow Shell Structure with the Dual Boundary Element Method

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Abstract

Reliability analysis has gained prominence as a stochastic approach to incorporate uncertainties in structural analysis. This study presents a novel methodology for evaluating the sensitivity of the crack tip stress intensity factor in a shallow shell structure. The work focuses on the reliability analysis of a shallow shell structure containing a crack originating from the corner of a fuselage window. The analysis incorporates uncertainties in both geometrical and loading parameters. The sensitivity of the crack tip stress intensity factors with respect to the considered uncertainties is determined. The Implicit Differentiation Method (IDM)-based First-Order Reliability Method (FORM) is utilized, and the results are compared to the results obtained from Monte Carlo Simulation (MCS), with a maximum difference of 2.99%. The reliability analysis aids in determining an appropriate inspection crack size that satisfies safety requirements, consequently facilitating the selection of the corresponding inspection technique.

Keywords: Shallow shell structure; First-Order Reliability Method (FORM); Dual Boundary Element Method (DBEM); Fracture.

1. Introduction

In engineering problems, uncertainties in design parameters are inherent in various problems. The conventional approach to addressing these uncertainties involves the application of safety factors which treats the design parameters deterministically. However, this simplistic approach often results in over-engineering of structures. To address this limitation, reliability analysis has emerged as an alternative technique for incorporating uncertainties in a stochastic manner. Reliability analysis offers insights into the influence of each uncertainty on the overall performance of structures.

Examples illustrating the application of reliability analysis to structural problems can be found in the literature Keshtegar (2021),Chowdhury (2013). Commonly used methodologies includes, the Monte Carlo Simulation (MCS), which involves extensive random sampling of design parameter values. To enhance computational efficiency, MCS is commonly combined with metamodelling techniques. However, for multi-dimensional problems, the computational time required becomes significant Li (2013). In contrast, the First-Order Reliability Method (FORM) and Second-Order Reliability Method (SORM) offer computationally efficient alternatives and have been widely adopted for a diverse

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range of engineering problems. FORM and SORM rely on approximating the performance function, which defines the boundary between success and failure, using first and second-order Taylor expansions. In both FORM and SORM, evaluating the sensitivities of the performance function with respect to each design parameter is necessary. Common methods for sensitivity evaluation include the Finite Difference Method (FDM) and the Implicit Differentiation Method (IDM). The FDM employs finite difference approximations to estimate sensitivity, but its accuracy is highly dependent on the chosen step size. Consequently, determining an optimal step size often requires a prior convergence test, resulting in increased computational time. To overcome this limitation, the IDM is employed in this work, enabling direct computation of derivatives from the underlying mathematical formulations.

In this study, the Dual Boundary Element Method (DBEM) is employed to model crack problem in a shallow shell structure. The DBEM has been successfully applied in various examples Morse (2019),Morse (2017), Huang (2015), demonstrating its effectiveness as an alternative to the Finite Element Method (FEM) for crack analysis and structural modelling. The Crack Surface Displacement Extrapolation (CSDE) techniques were adopted to accurately evaluate the crack tip stress intensity factor (SIF). An advantage of the DBEM lies in its requirement for meshing only the outer boundaries, with the crack surface treated as an external boundary. This substantially reduces the computational time needed for fracture analysis. A thorough introduction on the DBEM and the SIF evaluation techniques can be found in Aliabadi (2002), offering comprehensive details on the methodology employed in this work.

The application of shallow shell structures in engineering, such as aircraft fuselages and marine structures, is widespread due to their notable advantages of lightweight construction and high strength. Consequently, the design of shallow shell structures holds significant importance. The DBEM formulation for shallow shell structures was first proposed by Dirgantara (1999), Dirgantara (2001) based on the principles of shear deformable plate theory and plane stress elasticity. The fundamental solution was evaluated based on Reissner's plate theory Reissner (1952), and the Dual Reciprocal Method (DRM) was employed to transform domain integral equations into boundary integral equations Wen (1999). Notable examples of DBEM-modeled shallow shell structures can be found in Baiz (2007), Albuquerque (2010).

In the field of reliability analysis, research efforts have predominantly focused on coupling the FEM, with only a few notable works involving the application of DBEM Morse (2017)Huang (2015). However, to date, no investigations have been conducted on the reliability analysis of fractures in shallow shell structures using DBEM. Therefore, this study aims to investigate the reliability analysis of a shallow shell structure using DBEM, incorporating uncertainties in geometrical and loading parameters. The FORM is employed to evaluate the reliability index of the structure, while the IDM is employed to derive the sensitivity of the crack tip SIF with respect to design parameters. To assess accuracy, the results obtained from reliability analysis are compared with those derived from the MCS.

2. Methodology

2.1. IDM-based DBEM for shallow shell structure

In this study, the DBEM was employed to model the crack problem in a shallow shell structure. The formulation of the DBEM, along with the DRM, was initially proposed by Dirgantara (1999), Dirgantara (2001). A comprehensive introduction of the boundary integral equations, fundamental solutions, and the CSDE techniques utilized for evaluating the crack tip stress intensity factors can be found in Dirgantara (2001). For simplification, only a concise introduction to the DBEM formulation is provided and the derivatives of the boundary integral formulation are presented in this paper.

The DBEM incorporates the collocation of source points with field points to form a system of equations based on the boundary integral equations. This system of equations is represented as $\mathbf{Hu} = \mathbf{Gt}$, where $\mathbf{H}$ and $\mathbf{G}$ denote the coefficient matrices, and $\mathbf{u}$ and $\mathbf{t}$ represent the displacement and tractions, respectively. To simplify the equations, the equation can be rearranged such that the vector $\mathbf{X}$ contains all the unknowns, and the vector $\mathbf{F}$ contains all the known displacements and tractions. By solving the new system of equations in the form of $\mathbf{AX} = \mathbf{F}$, the unknowns can be determined.

To obtain the sensitivity of the displacement and traction with respect to the design parameters, the IDM-based DBEM formulation, leading to the system equation $\mathbf{H}_{,\mathbf{m}}\mathbf{u} + \mathbf{H}\mathbf{u}_{,\mathbf{m}} = \mathbf{G}_{,\mathbf{m}}\mathbf{t} + \mathbf{G}\mathbf{t}_{,\mathbf{m}}$ was solved. Where $\mathbf{H}_{,\mathbf{m}}$, $\mathbf{G}_{,\mathbf{m}}$, $\mathbf{u}_{,\mathbf{m}}$, and $\mathbf{t}_{,\mathbf{m}}$ represent the derivatives of the coefficient matrices, displacement, and tractions with respect to the design parameter

m. In this work, the design parameter m includes geometric parameters and curvature. Consequently, the system of equations can be rearranged as:

$$\mathbf{AX_{,m}} = \mathbf{F_{,m}} - \mathbf{A_{,m}X} \tag{1}$$

where $\mathbf{A_{,m}}$ represents the derivatives of the coefficient matrix with respect to the design parameters and $\mathbf{X_{,m}}$ contains the derivatives of the displacement and tractions.

2.2. Derivatives of the Stress Intensity Factors

The CSDE technique is used in this work for evaluating the crack tip stress intensity factors. The meshes near the crack tip by the discontinuous quadratic elements are represented in Fig.1.

The crack tip SIF can be linearly extrapolated by substituting the boundary displacement and rotation results of nodes

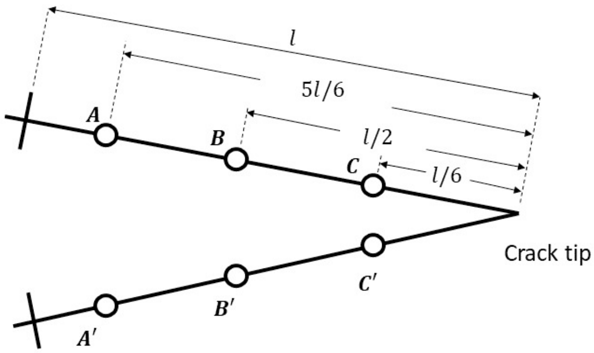

Fig. 1. Example of crack tip nodal element. Dirgantara (2001)

near the crack tip such that:

$$\{K\}^{tip} = \frac{r_{AA'}}{r_{AA'} - r_{BB'}}\left(\{K\}^{BB'} - \frac{r_{BB'}}{r_{AA'}}\{K\}^{AA'}\right) \tag{2}$$

where $\{K\}^{AA'} = \sqrt{\frac{6}{5l}}[\mathbf{C}]\left(\{w\}^{A} - \{w\}^{A'}\right)$ and $\{K\}^{BB'} = \sqrt{\frac{2}{l}}[\mathbf{C}]\left(\{w\}^{B} - \{w\}^{B'}\right)$. w consists of rotation in x and y direction ϕ_1, ϕ_2, the out-of-plane displacement w_3 and the in-plane displacements u_1, u_2 in x and y direction respectively.

The stress intensity factor at the nodes on the crack surfaces near the tip for three bending failure modes and two membrane failure modes can be found by:

$$\begin{aligned} &K_{1b} = \frac{Eh^3}{48\sqrt{r}}\sqrt{\frac{\pi}{2}}\Delta\phi_2 \quad K_{2b} = \frac{Eh^3}{48\sqrt{r}}\sqrt{\frac{\pi}{2}}\Delta\phi_1 \quad K_{3b} = \frac{5Eh}{24(1+\nu)\sqrt{r}}\sqrt{\frac{\pi}{2}}\Delta w_3 \\ &K_{1m} = \frac{Eh\sqrt{2\pi}}{8\sqrt{r}}\Delta u_2 \quad K_{2m} = \frac{Eh\sqrt{2\pi}}{8\sqrt{r}}\Delta u_1 \end{aligned} \tag{3}$$

where the subscript m, b represent membrane mode and bending mode respectively. by applying the extrapolation in equation 2, the sensitivities of the stress intensity factors are:

$$\{K\}^{tip}_{,m} = RRB_{,m}\{K\}^{BB'} + RRB\{K\}^{BB'}_{,m} - RRA_{,m}\{K\}^{AA'} - RRA\{K\}^{AA'}_{,m} \tag{4}$$

where

$$\begin{aligned} &RRA = \frac{r_{BB'}}{r_{AA'} - r_{BB'}} \qquad RRB = \frac{r_{AA'}}{r_{AA'} - r_{BB'}} \\ &RRA_{,m} = \frac{r_{BB',m}}{r_{AA'} - r_{BB'}} - \frac{r_{BB'}(r_{AA',m} - r_{BB',m})}{(r_{AA'} - r_{BB'})^2} \\ &RRB_{,m} = \frac{r_{AA',m}}{r_{AA'} - r_{BB'}} - \frac{r_{AA'}(r_{AA',m} - r_{BB',m})}{(r_{AA'} - r_{BB'})^2} \end{aligned} \tag{5}$$

The sensitivity of the stress intensity factors with respect to some geometrical variable or curvature can be derived from the above equation. The derivatives of $\{K\}$ can be written as:

$$K_{1b,m} = \frac{Eh^3}{48\sqrt{r}}\sqrt{\frac{\pi}{2}}\left\{\Delta\phi_{2,m} - \frac{r_{,m}}{2r}\Delta\phi_2\right\} \quad K_{2b,m} = \frac{Eh^3}{48\sqrt{r}}\sqrt{\frac{\pi}{2}}\left\{\Delta\phi_{1,m} - \frac{r_{,m}}{2r}\Delta\phi_1\right\}$$
$$K_{3b,m} = \frac{5Eh}{24(1+\nu)\sqrt{r}}\sqrt{\frac{\pi}{2}}\left\{\Delta w_{3,m} - \frac{r_{,m}}{2r}\Delta w_3\right\} \quad K_{1m,m} = \frac{Eh\sqrt{2\pi}}{8\sqrt{r}}\left\{\Delta u_{2,m} - \frac{r_{,m}}{2r}\Delta u_2\right\} \tag{6}$$
$$K_{2m,m} = \frac{Eh\sqrt{2\pi}}{8\sqrt{r}}\left\{\Delta u_{1,m} - \frac{r_{,m}}{2r}\Delta u_1\right\}$$

The sensitivities of the maximum stress intensity factors were considered, and the maximum SIF through the thickness of the shell is:

$$[1 + \frac{h}{4}(\frac{1}{R_1} + \frac{1}{R_2})]K_I^{max} = \tfrac{1}{h}K_{1m} + \tfrac{6}{h^2}K_{1b}$$
$$[1 + \frac{h}{4}(\frac{1}{R_1} + \frac{1}{R_2})]K_{II}^{max} = \tfrac{1}{h}K_{2m} + \tfrac{6}{h^2}K_{2b} \tag{7}$$
$$[1 + \frac{h}{4}(\frac{1}{R_1} + \frac{1}{R_2})]K_{III}^{max} = \tfrac{3}{2h}K_{3b}$$

The derivatives of the maximum SIF can be found by:

$$[1 + \frac{h}{4}(\frac{1}{R_1} + \frac{1}{R_2})]K_{I,g}^{max} = \frac{1}{h}K_{1m,g} + \frac{6}{h^2}K_{1b,g}$$
$$[1 + \frac{h}{4}(\frac{1}{R_1} + \frac{1}{R_2})]K_{II,g}^{max} = \frac{1}{h}K_{2m,g} + \frac{6}{h^2}K_{2b,g} \tag{8}$$
$$[1 + \frac{h}{4}(\frac{1}{R_1} + \frac{1}{R_2})]K_{III,g}^{max} = \frac{3}{2h}K_{3b,g}$$

$$K_{I,\rho}^{max} = \left(\frac{1}{h}K_{1m,\rho} + \frac{6}{h^2}K_{1b,\rho}\right)T_\kappa + \left(\frac{1}{h}K_{1m} + \frac{6}{h^2}K_{1b}\right)T_{\kappa,\rho}$$
$$K_{II,\rho}^{max} = \left(\frac{1}{h}K_{2m,\rho} + \frac{6}{h^2}K_{2b,\rho}\right)T_\kappa + \left(\frac{1}{h}K_{2m} + \frac{6}{h^2}K_{2b}\right)T_{\kappa,\rho} \tag{9}$$
$$K_{III,\rho}^{max} = \left(\frac{3}{2h}K_{3b,\rho}\right)T_\kappa + \left(\frac{3}{2h}K_{3b}\right)T_{\kappa,\rho}$$

$$K_{I,h}^{max} = \left(\frac{1}{h}K_{1m,h} + \frac{6}{h^2}K_{1b,h} - \frac{1}{h^2}K_{1m} - \frac{12}{h^3}K_{1b}\right)\cdot T_\kappa + \left(\frac{1}{h}K_{1m} + \frac{6}{h^2}K_{1b}\right)\cdot T_{\kappa,h}$$
$$K_{II,h}^{max} = \left(\frac{1}{h}K_{2m,h} + \frac{6}{h^2}K_{2b,h} - \frac{1}{h^2}K_{2m} - \frac{12}{h^3}K_{2b}\right)\cdot T_\kappa + \left(\frac{1}{h}K_{2m} + \frac{6}{h^2}K_{2b}\right)\cdot T_{\kappa,h} \tag{10}$$
$$K_{III,h}^{max} = \left(\frac{3}{2h}K_{3b,h} - \frac{3}{2h^2}K_{3b}\right)\cdot T_\kappa + \left(\frac{3}{2h}K_{3b}\right)\cdot T_{\kappa,h}$$

where terms $()_{,g}$, $()_{,\rho}$ and $()_{,h}$ represent the derivatives with respect to geometrical variables, curvatures and thickness respectively. Denote that $T_\kappa = [1 + \frac{h}{4}(\frac{1}{R_1} + \frac{1}{R_2})]^{-1}$, the corresponding derivatives are $T_{\kappa,\rho} = -\left[\frac{h}{4}(\kappa_{11} + \kappa_{22})\right]^{-2}\frac{h}{4}\left(\kappa_{11,\rho} + \kappa_{22,\rho}\right)$ with respect to the curvature and $T_{\kappa,h} = \frac{\partial T_\kappa}{\partial h} = -\left[\frac{h}{4}(\kappa_{11} + \kappa_{22})\right]^{-2}\frac{1}{4}(\kappa_{11} + \kappa_{22})$ with respect to the thickness.

2.3. First-order reliability analysis (FORM)

The main purpose of the FORM is to evaluate the reliability index which can be thought of as a measurement of the ability that a structure remains its safety during the operation. The reliability index is usually associated with the probability of failure which quantifies the risk of structural failure.

In the field of reliability, the region confined by the random variables **Z** can be separated into safe and failure regions such that $g(\mathbf{Z}) \leq 0$ in the safety region and $g(\mathbf{Z}) > 0$ in the failure region, where $g(\mathbf{Z})$ is a performance function act as a boundary between the safe and failure zone. The performance function is usually defined in terms of the structural resistance $R(\mathbf{Z})$ and the structure demand $G(\mathbf{Z})$. Some common structural resistance includes yield strength maximum allowable stress or deflection. The performance function can be written as:

$$g(\mathbf{Z}) = R(\mathbf{Z}) - G(\mathbf{Z}) \tag{11}$$

The probability of failure is defined as the probability that the performance function $g(\mathbf{Z}) \leq 0$ such that:

$$P_F = P\{g(\mathbf{Z}) \leq 0\} = \int_{g(\mathbf{Z})\leq 0} f_Z(\mathbf{Z})dZ \tag{12}$$

where P_F is the failure probability and P_R is the reliability which can be calculated as $1 - P_F$. In FORM, the random variables are required to transform into a standardized coordinate system U-space assuming that the CDFs of the random variables are unchanged. In the case when a variable follows a normal distribution $Z_i \sim (\mu_i, \sigma_i)$. The U-space relates to the Z-space by $Z_i = \mu_i + \sigma_i U_i$. Equ. 12 is therefore transferred to the $\mathbf{U}$-space as:

$$P_F = P\{g(\mathbf{U}) \leq 0\} = \int_{g(\mathbf{U})\leq 0} f_U(\mathbf{U})dU \tag{13}$$

The reliability index is found by the shortest distance from the performance function to the origin of the U-space. The point with minimum distance to the origin is called the Most-probable point (MPP) where the distance is denoted as β. The details of the MPP search algorithm can be found in Jia (2009). Once found, the probability of failure and reliability can be evaluated as:

$$P_R = 1 - P_F = 1 - \Phi(-\beta) = \Phi(\beta) \tag{14}$$

where Φ is the standardized cumulative distribution function.

3. Numerical Example

A numerical investigation was conducted on a fuselage window structure featuring a crack subjected to membrane, bending, and uniform pressure loads. The structure is composed of Aluminium 6061-T6 and is modelled using 40 quadratic elements for the outer boundaries and 48 quadratic elements for the inner boundaries. The initial crack is represented by 4 elements, and a total of 78 DRM points, uniformly distributed within the domain, were employed. To model crack propagation, 2 quadratic elements are incrementally added to the tip of the propagating crack on both the upper and lower surfaces.

Figure 2 illustrates the geometry of the structure, highlighting region A as the location of the crack, initiated at the corner of the window frame, which typically experiences the highest stress concentration. The length of the initial crack was set as with a length of $0.02m$. The mesh configuration of the structure is presented in Figure 3(a), while a detailed view of the mesh around the crack is presented in Figure 3(b). The mesh density near the crack area is increased to ensure that the mesh size around the corner is comparable to that on the crack surface.

In the context of reliability analysis, it is necessary to define the limit state function, which relates the structural resistance to a specific loading condition, as discussed in Section 2.3. The LSF is subsequently assessed by applying the FORM to the limit state function. In this study, the limit state function is formulated in terms of the crack tip SIF and fracture toughness. Specifically, the limit state function is defined to ensure that the crack tip SIF remains below the fracture toughness value. It is expected that as the crack size increases, the reliability diminishes.

A suitable LSF in terms of the failure criteria used in this work is:

$$g(\mathbf{Z}) = K_{IC} - K_{eff}(\mathbf{X}) \tag{15}$$

where K_{IC} is the fracture toughness and K_{eff} is the effective stress intensity factor. The vector $\mathbf{Z}$ contains the design variables that can influence the value of g where $\mathbf{Z} = (W_2, L_2, R_2, h, \kappa, N, M, P, K_{IC})$ and $\mathbf{X}$ consist of the design variables in $\mathbf{Z}$ without the fracture toughness $\mathbf{X} = (W_2, L_2, R_2, h, \kappa, N, M, P)$. The distribution of the above design variables is given in Table. 1.

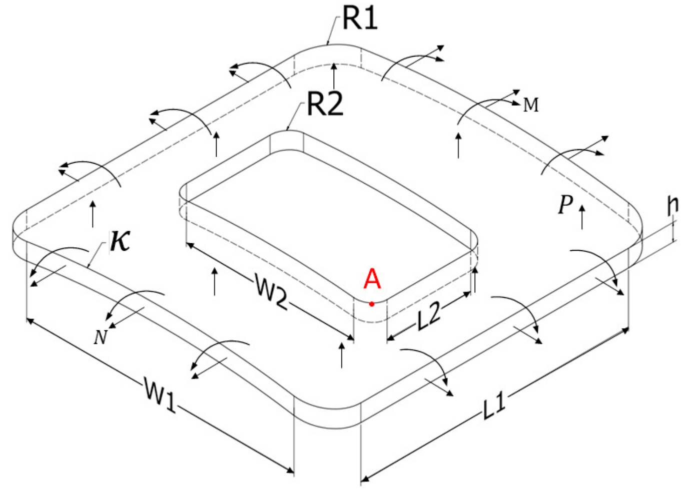

Fig. 2. The geometry of the shallow shell fuselage window structure. The window structure consists of inner boundaries and outer boundaries with a crack initiated at point A. The loading consists of bending moment, tension and domain pressure.

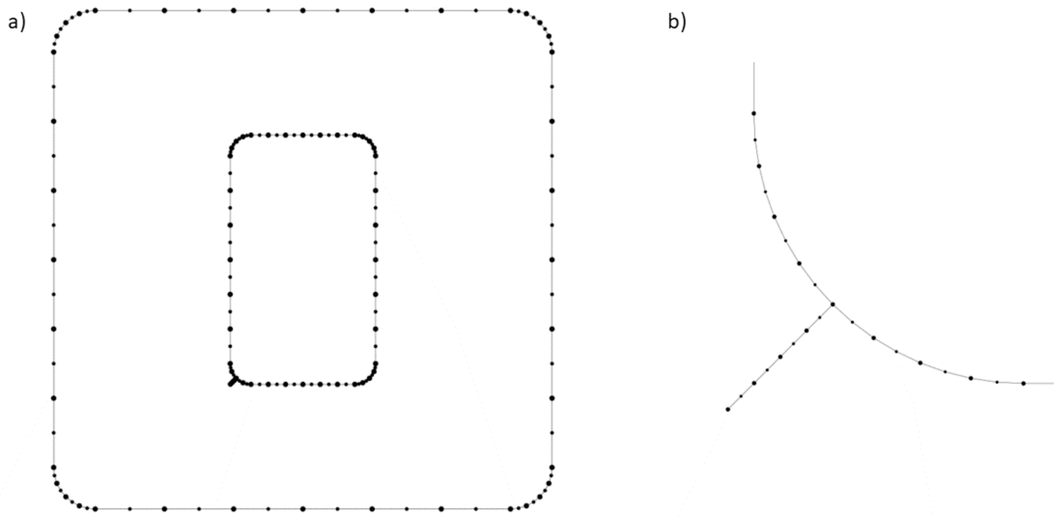

Fig. 3. a): The meshed of the shell using quadratic elements. b): The meshes around the crack area. Denser meshes were using around the corner of the window so that the element size on the corner is of similar size to the element size on the crack.

In FORM, the sensitivity of $g(\mathbf{Z})$ with respect to design parameter Z_i is needed. The derivatives of the nodal coordinate with respect to the Z_i can be evaluated analytically in the DBEM-IDM. Variables $Z_1 - Z_3, Z5$ are geometrical variables and the sensitivities were evaluated analytically using the DBEM-IDM. While the sensitivities with respect to the non-geometrical variables $Z_4, Z_6 - Z_8$ were evaluated using the DBEM-FDM with a step size of $\Delta Z'_g = 1 \times 10^{-2}$. The reliability analysis was conducted over a range of crack lengths. It is expected that the longer the crack propagates,

Table 1. Details of the shell structure parameters and random variables used in the reliability analysis.

Z_i	U_i	Parameter	Description	Distribution	Mean	COV
Z_1	U_1	W_2	Inner width	Lognormal	$0.5m$	0.01
Z_2	U_2	L_2	Inner length	Lognormal	$0.25m$	0.01
Z_3	U_3	R_2	Inner radius	Lognormal	$0.05m$	0.01
Z_4	U_4	h	Thickness	Lognormal	$0.05m$	0.01
Z_5	U_5	κ	Curvature	Lognormal	$0.1m^{-1}$	0.01
Z_6	U_6	N	Boundary traction	Lognormal	$0.8\ MNm^{-1}$	0.1
Z_7	U_7	M	Boundary moment	Lognormal	$0.01\ MN$	0.1
Z_8	U_8	P	Domain pressure	Lognormal	$0.01\ MNPa$	0.1
Z_9	U_9	K_{IC}	Fracture toughness	Deterministic	$29\ MNPa$	
Z_{10}	U_{10}	a_0	Initial crack length at A	Deterministic	$0.02\ m$	

the structure will have lower reliability.
The effective SIF was calculated vis the effective energy release rate:

$$\begin{aligned} K_{eff} &= \sqrt{EG_{eff}} \\ G_{eff} &= G_1 + \alpha(G_2 + G_3 + G_4 + G_5) \\ \alpha &= \sqrt{\frac{|\Delta K_{Ib}|}{|\Delta K_{Ib}|+|\Delta K_{Im}|}} \end{aligned} \tag{16}$$

and the corresponding derivatives can be evaluated as:

$$K_{eff,g} = \frac{E}{2K_{eff}} G_{eff,g} \tag{17}$$

where the derivatives of the energy release rate components were obtained from:

$$\begin{aligned} &G_{1,m} = \tfrac{2}{Eh^2} K_{1m} K_{1m,m} \quad G_{2,m} = \tfrac{24\pi}{Eh^4} K_{1b} K_{1b,m} \quad G_{3,m} = \tfrac{72\pi}{Eh^4} K_{2b} K_{2b,m} \\ &G_{4,m} = \tfrac{36(1+\nu)\pi}{5Eh^2} K_{3b} K_{3b,m} \quad G_{5,m} = \tfrac{2}{Eh^2} K_{2m} K_{2m,m} \end{aligned} \tag{18}$$

The results of the reliability index against the length of the crack are given in Fig.4. The reliability index decreases as the crack length extended which is within the expectation as the structure tends to be more unstable when a larger crack exists. The MPP search algorithm converged successfully in three iterations, and the computational time for evaluating one reliability index was 2475.2 s. As a validation, the results obtained from the IDM-FORM were compared with the results obtained from 40,000 MCS. The computational time required for the MCS was $3.0788 \times 10^7\ s$. The results are in good agreement with a maximum difference of 2.99% found in a crack length of $0.0725m$.
These outcomes highlight the reliability and accuracy of the proposed IDM-FORM approach, providing an efficient alternative to MCS for evaluating reliability indices in shallow shell structures with cracks.

3.1. Conclusion

In this research, we propose a novel methodology that employs the Dual Boundary Element Method (DBEM) to derive the sensitivity of the crack tip stress intensity factor in a shallow shell structure. The reliability analysis of a shallow shell structure with a crack initiated from the fuselage window corner was conducted. The fracture reliability of the structure was assessed, accounting for uncertainties in geometrical and loading parameters. By considering these uncertainties, the sensitivities of the crack tip stress intensity factors were derived. The reliability index was obtained by applying the Implicit Differentiation Method (IDM)-based First-Order Reliability Method (FORM). The results were compared with those obtained from Monte Carlo Simulation (MCS). As expected, the reliability of the structure decreases as the crack propagates, indicating an increased probability of failure, aligning with our expectations. The maximum difference between the results obtained from FORM and MCS was found to be 2.99% at a crack length of 0.0725 m, which demonstrates the reliability and accuracy of the proposed methodology.
One of the significant advantages of using FORM is the substantial reduction in computational effort. The computational time required to evaluate one reliability index using FORM is merely 2475.2 seconds, whereas performing

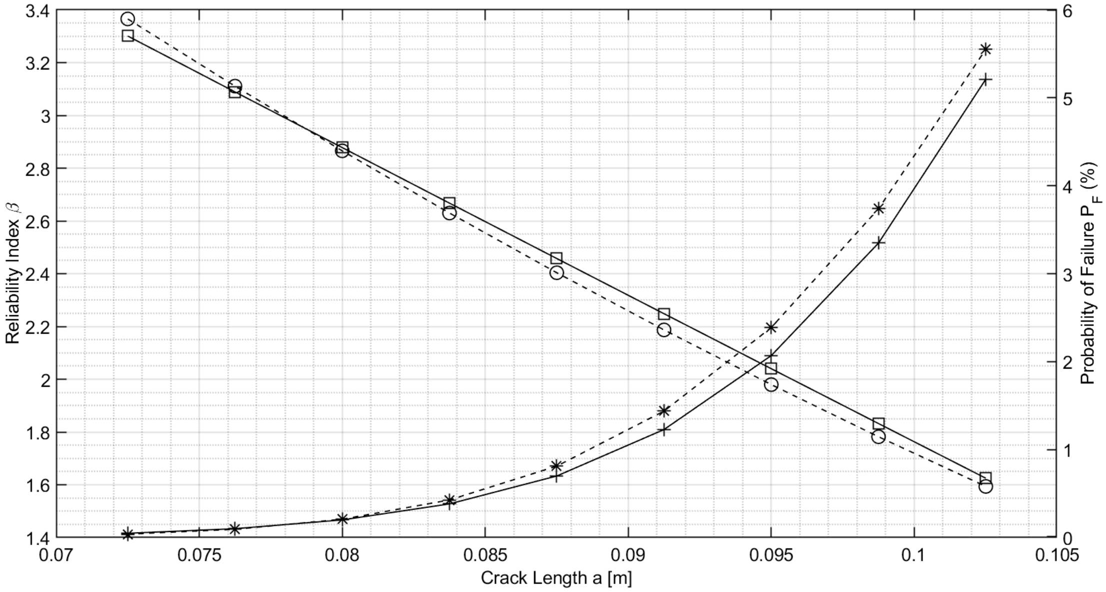

Fig. 4. Reliability indices and the corresponding probabilities of failure evaluated at each crack length via the IDM-based FORM.

40,000 MCS demands a significantly higher computational time of 3.0788×10^7 seconds. The practical applicability of reliability analysis is evident, as it enables engineers to determine a suitable inspection crack size while meeting safety requirements. Once the optimal inspection crack size is identified, the corresponding inspection technique can be deployed, ensuring the structural integrity of the shallow shell system.

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