Monetary Policy When the Phillips Curve Is Quite Flat

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This paper highlights how the presence of a monetary policy cost channel can offer new insights into the relation between monetary policy and inflation when the Phillips curve is quite flat. For instance, we highlight a key condition whereby lax monetary policy can push the economy to a low-inflation trap, and we discuss how, under the same condition, standard policy rules targeting inflation may need to be modified. In the empirical part of the paper, we explore the relevance of the condition that gives rise to these observations. The results support the key condition we emphasize. (JEL E24, E31, E43, E52)

Prior to the COVID crisis, the inflation rate has been below target for several years in many industrialized countries. At the same time, monetary policy has been sufficiently expansive to support unemployment rates that were close to historical lows. These low-inflation outcomes could have reflected a correlated reduction in the natural rate of unemployment across countries. However, such explanation appears unlikely given that only a few years ago the predominant puzzle was missing deflation with high unemployment. A more plausible candidate explanation for these outcomes is that the Phillips curve may be quite flat.1

The object of this paper is to explore the implications and empirical relevance of a relatively flat Phillips curve when a cost channel is present. The paper is divided into two main parts. In the first section, we highlight a set of theoretical implications of having a flat Phillips curve in the presence of a cost channel. As we shall show, this type of environment will offer a simple explanation for why inflation can get stuck below target, with low unemployment even if monetary policy appears quite aggressive. Such an outcome depends on parameters of the Phillips curve as well as on the sensitivity of aggregate demand to interest rates. Accordingly, in the second and third sections of the paper, we explore the empirical plausibility for the parameter configuration of interest. To this end, we present both (i) estimates derived from single-equation estimation of the Phillips curve and (ii) estimates based on structural estimation of a full model.

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1 By “slope of the Phillips curve,” we mean the partial relationship between inflation and a measure of market tightness such as either the output gap or the labor gap.
In terms of monetary policy, the findings of this paper have novel implications for how policy should be conducted to keep inflation close to an inflation target. In particular, our framework suggests that when trying to compensate for past departures from an inflation target, inflation-targeting central banks should not aim for quick redress by adopting slightly more aggressive nonstandard interest rate policy. Within our framework, this is precisely the type of strategy that can cause a persistent deviation of inflation from target. In such a situation, it is likely best to leave bygones be bygones and return quickly to a historical rule that has given good inflation results in the past.

The cost channel has been extensively studied in the literature. It was mentioned by Farmer (1984), then modeled by Blinder (1987); Fuerst (1992); Christiano and Eichenbaum (1992); and Barth and Ramey (2002) and discussed in the framework of the New Keynesian model by Christiano, Eichenbaum, and Evans (2005); Chowdhury, Hoffmann, and Schabert (2006); Ravenna and Walsh (2006); Rabanal (2007); Ravenna and Walsh (2008); Surico (2008); Tillmann (2008); Henzel et al. (2009); and Castelnuovo (2012). We contribute to this literature by both highlighting the implications of a cost channel when the Phillips curve is quite flat and by providing two sets of empirical results (single equation estimation of the Phillips curve and multiple equation structural estimation) that support the key parameter configuration that provides novel insight.

The remaining sections of the paper are structured as follows. In Section I, we derive some simple theoretical implications of having a relatively flat Phillips curve when a cost channel may also be operative. We show under which conditions this can change how an inflation targeting monetary authority should conduct policy to stabilize inflation. In this section we also discuss how an economy can get stuck in a low-inflation trap. In Section II, we begin by examining the plausibility of the parameter configuration of interest by presenting estimates of the Phillips curve when we allow for a cost channel. Given the challenges in estimating the slope of the Phillips curve with aggregate data, we also rely on the slope estimate provided in Hazell et al. (2020) that is identified by exploiting US regional variations, so as to focus on the relative strength of the cost channel. In Section III, we complement this partial equilibrium evidence by presenting estimates derived by an estimation of the full model. We also run some counterfactual simulations during the zero lower bound (ZLB) period. Finally, in Section IV, we offer some concluding comments.

I. Monetary Policy Implications of a Quite Flat Phillips Curve

The aim of this section is to highlight how the slope of the Phillips curve—or, more precisely, the sensitivity of the real marginal cost to market tightness—can

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2 It is worth noting that there are very few studies of the relevance of the cost channel using firm-level data. Exceptions are Gaiotti and Secchi (2006), who find robust evidence in favour of the presence of a cost channel using Italian firm data, and Suveg (2022), who uses Swedish firm data and finds that a 1 percentage unit increase in the policy rate leads to a 1 percent increase in the firm’s price via the working capital channel, with a pass-through that takes about 4 months.

3 See also Fitzgerald and Nicolini (2014) and McLeay and Tenreyro (2020) for the identification of the Phillips curve slope using regional variations.
affect the link between monetary policy and inflation stabilization in the presence of a cost channel. We will first present the model and highlight a condition—referred to as a “Patman condition”—under which restrictive monetary policy can potentially increase inflation. We will then examine the implications of this condition for monetary policy.  

A. A Simple New Keynesian Model with a Cost Channel

We modify the standard three-equation New Keynesian model to introduce a cost channel. The economy features a continuum of monopolists. Each monopolist produces a differentiated intermediate good using a basic input as the only factor of production and according to a one-to-one technology. The marginal cost of production will therefore be the price of that basic input. These monopolists will enter a Calvo lottery to draw price setting opportunities. Final output will be a CES aggregator of these differentiated goods and will be used for consumption and as intermediate input. Basic input $Q_t$ is produced by a representative firm with labor $L_t$ and intermediate input $M_t$ according to the following Leontief technology:

$$Q_t = \min \{a \Theta_t, b M_t\}.$$ 

We assume that the basic input representative firm must borrow at the risk-free nominal interest rate $i_t$ to pay for the input $M_t$. Assuming that the basic input firm discounts the future at rate $\beta$, this implies (see online Appendix C) that the real marginal cost will be given (in logs and omitting constants) by:

$$mc_t = \gamma_y y_t + \gamma_r (i_t - E_t \pi_{t+1}),$$

where $\gamma_y$ and $\gamma_r$ are functions of the model deep parameters. The key property of this modelling is that $\gamma_r / \gamma_y$ can take any positive value, depending on the deep parameters.

The household side is standard except that we allow for a discounted Euler equation specification. To do so, we assume that households must borrow in the morning to order the consumption goods they will consume in the afternoon, and we introduce some unobserved heterogeneity between households. Some households have access to commitment and always repay their debt, while other households cannot commit to repay. Type is not observable. Because of this, households will face an interest rate schedule that is increasing in the level of their consumption. This will translate (see online Appendix C) into a log-linearized discounted Euler equation.

From these assumptions, we can derive the two key equations of a canonical New Keynesian setup, where our microfoundations introduce only minor changes (explicit derivation is presented in online Appendix C). As is standard, all variables are expressed in deviations from the steady state. We are abstracting from capital...
accumulation, and we are assuming that technological progress follows a deterministic trend. Deviations of economic activity from its steady state therefore correspond to deviations of employment from its steady state. As a result, when talking about market tightness, we can refer interchangeably to the output gap or the labor gap.

\[ \pi_t = \beta E_t \pi_{t+1} + \kappa mc_t + \mu_t, \quad (2) \]
\[ y_t = \alpha_y E_t y_{t+1} - \alpha_r (i_t - E_t \pi_{t+1}) + d_t, \quad (3) \]

Equation (2) is the New Keynesian Phillips curve where inflation depends on expected inflation, the real marginal cost, and a markup shock \( \mu_t \). This equation is entirely conventional. Equation (3) is a Euler equation (the forward-looking IS curve) that is subject to preference shocks \( d_t \). As already mentioned, we allow for a discounted Euler equation specification by having \( 0 < \alpha_y < 1 \). Such a modification is not very substantial, as we allow \( \alpha_y \) to be arbitrarily close to one. However, it has the advantage of allowing us to consider a wider set of monetary policy rules without needing to worry about a unit root (induced when \( \alpha_y \) is exactly equal to one). In particular, we are able to consider environments where a central bank aims to influence real interest rates, which, in addition to being plausible, will be very convenient.

The main element we want to focus on is our specification of the real marginal cost as given by equation (1) above: we have the real interest rate included in the marginal cost. It is common to refer to this term \( \gamma_r \) as the cost channel of monetary policy even though the cost channel of monetary policy is most often associated with nominal interest rates affecting the real marginal cost. As our main results are not substantially modified by allowing for a nominal versus a real interest rate in the cost channel, we choose to maintain a real cost channel specification that is theoretically appealing, offers clearer results, and, most importantly, finds greater support in our later estimation.

B. The Patman Condition

The central message we want to convey in this section is that the way monetary policy influences inflation is closely tied to a particular condition involving \( \alpha_r, \gamma_y, \) and \( \gamma_r \). We will call the relevant condition the “Patman condition” after US Senator Wright Patman, who argued in the late 1970s that the Fed’s policy of increasing interest rates could be more of a contributor to inflation than a cure.\(^5\)

In an economy given by equations (2), (3), and (1), a marginal increase in the real interest rate \( i_t - E_t \pi_{t+1} \) has two effects on current inflation \( \pi_t \), holding expectations constant. A direct effect \( \gamma_r \) goes through the impact on the marginal cost in the Phillips curve. An indirect effect \( -\alpha_r \gamma_y \) runs through \( y_t \) via the Euler equation. When the direct effect dominates the indirect effect, then an increase in interest rates

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\(^5\)The view that tight monetary policy could be inflationary was discussed in Tobin (1980, 35): “More fundamentally, heretics from the populist Texas Congressman, Wright Patman, to John Kenneth Galbraith have disputed the orthodox view that tight money policies are anti-inflationary, claiming that borrowers mark up interest charges like other cost.” See also Driskill and Sheffrin (1985), who introduced interest costs into Taylor’s (1979) model of overlapping wage contracts.
tends to put upward pressure on inflation. We will refer to such a configuration as satisfying the temporary equilibrium (TE) Patman condition, defined as follows.

**DEFINITION 1:** *Temporary Equilibrium Patman Condition:* Current inflation will increase following a rise in real interest rate, holding expectations constant, if and only if the TE Patman condition $\gamma_r > \alpha_r \gamma_y$ is satisfied.

It is important to point out that many models with a cost channel have microfoundations that rule out the possibility that $\alpha_r \gamma_y \leq \gamma_r$ (see online Appendix B); that is, they rule out by assumption the possibility of this Patman condition being satisfied.\(^6\) However, this is not the case for our microfoundations. Also note that this condition is stated holding expectations constant, and for this reason we refer to it as a temporary equilibrium condition. As we shall see below, when we allow inflation expectations to adjust, the TE Patman condition will be only a necessary condition for an increase in interest rates to increase inflation. The more complete (general equilibrium) condition will also involve $\alpha_y$ and the persistence of the monetary shock.

**C. A First Look at Data through a Tight Lens of the Model**

The main idea behind the Patman condition is that monetary policy can have unconventional effects on inflation if increases in interest rates cause marginal cost to increase. In this subsection we want to have a first look at the possibility that monetary tightening may cause marginal cost to rise, taking the microfoundations presented in online Appendix C.2 seriously as a way to model marginal cost. As shown in this online Appendix, the model’s marginal cost can be expressed as

\[
mc_t = \frac{b - 1}{b} \times LabourShare_t + \frac{\beta}{b} E_t \left[ \frac{1 + i_{t+1}}{1 + \pi_{t+1}} \right],
\]

where $b$ is the inverse of the share of intermediate inputs in gross output and $\beta$ is the firms’ discount factor. To operationalize this measure of marginal cost, we set $\beta = 0.99$ and set $b = 2.28$ based on 2005 US data. We measure $P_t$ with the US Domestic Producer Prices Index for Manufacturing and the nominal interest rate as the Three-Month AA Financial Commercial Paper Rate to compute the time series of the real marginal cost implied by the model. Data are obtained from FRED (Federal Reserve Bank of St. Louis 2022). Since the resulting series has an important trend, we focus on the linearly detrended version as the measure of marginal cost. We then estimate the response of this marginal cost to a monetary contractionary shock using both smooth local projection\(^7\) and regular local projection. We use the Wieland and Yang’s (2020) extended series of Romer and Romer’s (2004) monetary shocks series as instrument for movements in the Fed funds rate. Figure 1 shows that the marginal cost responds positively to the monetary contractionary shock, pointing toward the

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\(^6\) However, this is not the case for some larger DSGE models. See, for instance, Rabanal (2007).

\(^7\) See Barnichon and Brownlees (2019). We have used the code they kindly provide.
dominance of the cost channel of monetary policy. Of course, this piece of evidence is very model dependent, and accordingly we will later explore more robust evidence by directly estimating the implied Phillips curves given by (1) without taking literally the restrictions on $\gamma_y$ and $\gamma_r$ implied by our specific microfoundations.

D. Implications for Monetary Policy

We first need to emphasize that as long as monetary policy is conducted in a way that maintains equilibrium determinacy, the Patman condition does not generally affect how the economy qualitatively responds to either demand or supply (markup) shocks. In particular, for a large class of monetary rules that includes standard Taylor rules, it is easy to show that demand shocks will always cause both activity and inflation to rise regardless of whether the Patman condition is met or not. Similarly, cost push shocks will lead to higher inflation and lower activity regardless of whether this condition is met. This observation is important as it implies that the relevance—or irrelevance—of the Patman condition can not be evaluated by simply examining the qualitative properties of how the economy reacts to such shocks. To see this, it is easiest to consider demand shocks and markup shocks sequentially. For demand shocks ($d$ shocks), if the response of monetary authorities is to increase real interest rates but to not overcompensate by causing a fall in activity (which is the case for a large set of policy rules including the form $i_t = E_t [\pi_{t+1}] + \phi_d d_t$, with $\phi_d < 1/\alpha_r$), then the demand shock will lead to an increase in both inflation and output regardless of whether the TE Patman condition is met or not. For markup shocks ($\mu$ shocks), if the response of monetary authorities is to increase real interest
rates but to not overcompensate by leading to a fall in inflation (which again is the case for a large set of rules), then the shock will lead to both an increase in inflation and a fall in activity regardless of whether the TE Patman condition is met or not.

We now turn to deriving implications of how different monetary stances affect the properties of the system given by equations (2), (3), and (1). First, it should be noted that when the monetary authorities follow a Taylor rule and when the economy is in the Patman regime (to be precise, if the general equilibrium Patman condition defined below holds for any persistence \( \rho < 1 \), then there is no need for a Taylor principle (of the type “monetary policy should be reacting aggressively enough to inflation”), but one should satisfy an “anti-Taylor” principle, in the sense that monetary policy should not be reacting too aggressively to inflation. Furthermore, being in the Patman regime is a sufficient condition for determinacy under a nominal or real interest rate peg policy.8 This being said, we want to emphasize here how traditionally prescribed anti-inflationary responses to shocks can have qualitatively different effects on inflation depending on whether the Patman condition is met or not. Starting from the steady state, we consider a period 0 demand or markup shock of arbitrary persistence. The policy response is to increase the real interest rate to the level \( r_0 \), with persistence \( \rho_r \), so that \( r_t = \rho_r r_{t-1} \). Combining equations (2) and (3) (for arbitrary processes for \( \mu_t \) and \( d_t \)), we can obtain the following equilibrium \((\pi_t, r)\) locus:

\[
\pi_t = \kappa \frac{\rho_r}{1 - \rho_r \beta} \left[ (\gamma_r - \alpha_r \gamma_y) - \frac{\rho_r}{1 - \rho_r \alpha_y \alpha_r \gamma_y} \right] r \\
+ \kappa \gamma_y \sum_{j=0}^{\infty} \beta^j \left( E_t \sum_{k=0}^{\infty} \alpha_y^k E_t d_{t+j+k} \right) + \sum_{j=0}^{\infty} \beta^j E_t \mu_{t+j}.
\]

Using equation (5), we can state Proposition 1.

**PROPOSITION 1:** If the TE Patman condition is not met, in response to either a positive supply shock or demand shock, engineering a rise in real interest rates will bring inflation closer to its target relative to keeping interest rates at their steady-state values. If the TE Patman condition is met, a not-too-persistent rise in the interest rate will push inflation further away from its target relative to keeping real rates at their steady-state values. By not too persistent, we mean \( \rho_r < \frac{(\gamma_r - \alpha_r \gamma_y)}{(\alpha_y \gamma_r)} \).

We will refer to this condition as the general equilibrium (GE) Patman condition.

**DEFINITION 2:** General Equilibrium Patman Condition: \( \rho_r < \frac{(\gamma_r - \alpha_r \gamma_y)}{(\alpha_y \gamma_r)} \).

The proof of Proposition 1 is in online Appendix A. Note that the GE Patman condition does not hold if the TE Patman condition fails. This proposition highlights that adding a cost channel does not alter how monetary policy can be used to help stabilize inflation if the Patman condition is not met. The first part of this

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8 See online Appendix D for a complete analysis.
proposition is illustrated in panel A of Figure 2, where in this figure we plot the \((\pi_t, r)\) relation that we derived earlier. The important property of the resulting relationship between \(\pi_t\) and \(r\) is that it is negatively sloped, as it is given by \((\gamma_r - \alpha_r \gamma_y) - \left[\rho_r / (1 - \rho_r \alpha_y)\right] \alpha_y \alpha_r \gamma_y\). Moreover, both demand and supply shocks shift this curve upwards. Therefore, in the absence of any move in interest rates, positive shocks to either demand or supply will increase inflation. Since the slope of the curve is negative, an increase in real interest rates will act to reduce inflation. The resulting increase in interest rates will help bring inflation closer to its target.

Panel B of Figure 2 corresponds to an economy in the Patman zone, for which the slope of the \((\pi_t, r)\) locus \((\gamma_r - \alpha_r \gamma_y) - \left[\rho_r / (1 - \rho_r \alpha_y)\right] \alpha_y \alpha_r \gamma_y\) is positive, which is true if the shock is not too persistent or, equivalently, if the GE Patman condition holds. The \((\pi_t, r)\) locus still shifts upward with demand or markup shocks. In that case, increasing the real interest following a positive demand or markup shock will move inflation further away from target. The intuition for why a standard anti-inflationary prescription might destabilize inflation instead of helping stabilize it is rather evident. Recall that the Patman condition relates to the property that the direct effect of an increase in interest rates is larger than the indirect effect. Hence, in the Patman zone, increases in interest rates have the opposite effect than what is traditionally predicted. The traditional assumption is that the indirect effect always dominates the direct effect, with the later often assumed to be zero. Note that the economy is more likely to be in the Patman zone when the Phillips curve is flat—i.e., when \(\gamma_y\) is small. The cost channel \(\gamma_r\) needs not to be large in absolute value; what matters is the relative size of the two components of the marginal cost—i.e., \(\gamma_r / \gamma_y\).

### E. Missing Deflation and Low-Inflation Trap

In this section we want to illustrate how a country can get trapped in a situation where, simultaneously, interest rates are at the effective lower bound, inflation is below target, and unemployment is below its steady-state value. In particular, we want to emphasize how this situation can arise when monetary authorities depart from their traditional rules after a period of effective lower bound constraint and low inflation—either to undo past inflation misses or simply to quickly bring inflation closer to its target.

To simplify exposition, we will assume that we are in a case where the Phillips curve is locally flat, so that \(\gamma_y\) is zero over a sufficiently wide interval around the steady state. This extreme assumption of a perfectly flat Phillips curve in a neighborhood of the steady state is not necessary for the point we want to make, but it simplifies our presentation substantially. We also assume that the only shock present is an i.i.d. demand shock, again for clarity of exposition. Note that in this case, all expected terms will be zero.

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9 Contrarily to the TE Patman condition that keeps expectations fixed, the GE Patman condition takes into account the endogeneity of expectations.

10 In the canonical New Keynesian model, there is not direct effect.
Under the i.i.d. assumptions, inflation is simply given by
\[ \pi_t = \beta E_t \pi_{t+1} + \gamma_r (i_t - E_t \pi_{t+1}) = \gamma_r i_t. \]

We call this economy an extreme Patman economy. By contrast, the inflation equation in a standard New Keynesian economy will be
\[ \pi_t = \beta E_t \pi_{t+1} + \gamma_y y_t = \gamma_y y_t. \]

In both economies, the Euler equation can be written as
\[ y_t = \alpha_y E_t y_{t+1} - \alpha_r (i_t - E_t \pi_{t+1}) + d_t = -\alpha_r i_t + d_t. \]

Finally, assume that the traditional monetary stance is to decrease real interest rates when demand falls, so that the desired policy rate would be to set
\[ (6) \quad i_t^d = E_t \pi_{t+1} + \psi_d d_t, \quad \psi_d > 0. \]

However, this policy is constrained by the effective lower bound, which we denote by \( \hat{i} \), so that the policy nominal rate is
\[ (7) \quad i_t = \max \{i_t^d, \hat{i}\}. \]

Since variables are expressed in deviation from their steady-state value, \( \hat{i} < 0 \).
**Missing Deflation:** Suppose the extreme Patman economy faces a temporary demand shock $-d < 0$ in period $t$ and the monetary authorities follow the policy in (6) and (7). There is a threshold $d = -\bar{i}/\psi_d$ such that the effective lower bound constraint will bind if and only if $d > \bar{d}$.

We consider an econometrician that estimates a (mispecified) Phillips curve $\pi_t = \hat{\gamma}_y y_t + \varepsilon_t$ using the demand shock $d_t$ as an instrument.

First, assume that the demand shock $d_t = -d < 0$ is not too severe—i.e., $d < \bar{d}$. The effective lower bound will not be binding, and monetary authorities will decrease the interest rate to the level $i_t = -\psi_d d$, so that $y_t = -(1 - \alpha_r \psi_d)d$ and $\pi_t = -\gamma_r \psi_d d$. Using demand shocks as an instrument, the IV estimated slope of the Phillips curve will be $\hat{\gamma}_y^N = \partial \pi_t / \partial y_t = (\gamma_r \psi_d) / (1 - \alpha_r \psi_d)$, where $N$ indicates that the effective lower bound is Not binding. Note that this estimated slope is a function of the monetary policy stance $\psi_d$.

Assume now that the demand shock is negative enough for the effective lower bound constraint to be binding—i.e., $d > \bar{d}$. Then, monetary policy will be $i_t = \bar{i}$, so that $\pi_t = \gamma_r \bar{i} = -\gamma_r \psi_d d$ and $y_t = \alpha_r \bar{i} - d = -(1 - \alpha_r \psi_d)d - (d - \bar{d})$. In this case, the IV estimated slope of the Phillips curve is $\hat{\gamma}_y = \partial \pi_t / \partial y_t = (\gamma_r \psi_d) / \left[1 - \alpha_r \psi_d + (d - \bar{d}) / \bar{d}\right] < \hat{\gamma}_y^N$, as $d > \bar{d}$. As we can see, using again the demand shock as an instrument, the IV estimated slope of the Phillips curve is flattening out when the effective lower bound binds. In contrast, in a standard New Keynesian model, the IV estimated slope of the Phillips curve will be constant and equal to $\gamma_y$.

In an extreme Patman economy, the period of mild deflation at the effective lower bound could easily be misinterpreted as an episode of missing deflation. In particular, if the monetary authority uses the past (linear) historical relationship between inflation and activity to predict how inflation should react in this episode, the fall in inflation when hitting the effective lower bound would be smaller than predicted. Therefore, when there is a cost channel to monetary policy, a period of perceived missing deflation at the effective lower bound is readily explained.

**Low-Inflation Trap:** Let us now consider a slightly different policy stance that can lead to poorer inflation outcomes despite looking more aggressive by design. Such a policy will be very much in line with the one suggested by Ben Bernanke in a 2017 blog post on the Brookings website:

To be more concrete on how the temporary price-level target would be communicated, suppose that, at some moment when the economy is away from the ZLB, the Fed were to make an announcement something like the following:

—The Federal Open Market Committee (FOMC) has determined that it will retain its symmetric inflation target of 2 percent. The FOMC will also continue to pursue its balanced approach to price stability and maximum employment. In particular, the speed at which the FOMC aims to return inflation to target will depend on the state of the labor market and the outlook for the economy.

—The FOMC recognizes that, at times, the zero lower bound on the federal funds rate may prevent it from reaching its inflation and employment goals, even with the use of unconventional monetary tools. The Committee
therefore agrees that, in future situations in which the funds rate is at or near zero, a necessary condition for raising the funds rate will be that average inflation since the date at which the federal funds rate first hit zero be at least 2 percent. Beyond this necessary condition, in deciding whether to raise the funds rate from zero, the Committee will consider the outlook for the labor market and whether the return of inflation to target appears sustainable. (Bernanke 2017, italics added by Bernanke)

We model such an idea in the following way. Consider the extreme Patman economy described in the previous paragraph. The desired policy stance is assumed to remain \( i_t^d = E_t \pi_{t+1} + \psi d_t \) in normal times. However, normal times are here defined in a slightly stricter way than previously. They correspond to either (i) when the interest rate was not at the effective lower bound last period or (ii) when the interest rate was at the effective lower bound last period but \( \pi_{t-1} \geq 0 \). In abnormal times, when both \( i_{t-1} = \bar{i} \) and \( \pi_{t-1} < 0 \), the rule is to set interest rates at the effective lower bound, \( i_t = \bar{i} \). Policy is then given by

\[
i_t = \begin{cases} 
\max\{\psi d_t, \bar{i}\}, & \text{if } (i_{t-1} > \bar{i}) \text{ or } (i_{t-1} = \bar{i} \text{ and } \pi_{t-1} \geq 0) \text{ in normal times;} \\
\bar{i}, & \text{if } (i_{t-1} = \bar{i} \text{ and } \pi_{t-1} < 0). 
\end{cases}
\]

This policy corresponds to keeping interest rates lower than standard policy when the economy has recently been at the effective lower bound and inflation has been below target. From a standard perspective, this approach may seem aggressive, as it is potentially correcting for low-inflation episodes by keeping interest rates at the effective lower bound even if the state of the economy would push the standard policy stance to increase interest rates. This type of policy can lead to situations where, in the absence of any new shocks, the policy rate gets stuck at the effective lower bound even after the negative demand shock that initiated the effective lower bound episode has dissipated. Such a situation is plotted in Figure 3, where we display responses of the nominal policy rate, the output gap, and inflation to a negative demand shock that occurs in period 1 and puts the economy at the effective lower bound. When the above-described aggressive policy is followed, inflation is stuck below target and unemployment is above its steady-state value. The economy could potentially remain stuck in such a low-inflation trap until a sufficiently big supply shock pushes inflation up and leads to a renormalization of policy. Note that the low-inflation trap is caused by the very specific price-level-targeting type of policy we have considered. In Section IIID, we find that with a more standard policy rule, the economy converges back to the steady state once shocks are back to zero.

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11 Here we exhibit an equilibrium in which agents expect the effective lower bound to be binding forever after the initial shock under the aggressive policy, which happens in equilibrium. We do not claim that this is the unique equilibrium.

12 One of the reasons monetary authorities may be tempted by this policy is the fear that inflation becomes unanchored after a period of low inflation at the effective lower bound. However, if the Patman condition is met, it is precisely following this policy that might trigger a deanchoring of inflation expectations.

13 Note that even if in this example we have a policy prescription somewhat similar to those associated with neo-Fisherian view, the mechanism is very different. In particular, in the current framework, the inflation trap can arise even if inflation expectations remain well anchored. The main mechanism is not through expectations but through the cost channel.
F. Monetary Shocks

Up to now, we have focused on the effects of the systematic part of monetary policy rules and we have not considered the effects of pure monetary shocks. To look at this issue, it is preferable to extend the model slightly to allow for some internal dynamics in order not to focus on knife-edge cases. The easiest way to do this is to allow for external habit in consumption. In this case, the Euler equation for consumption takes the form

\[ y_t = \alpha_y, f E_t y_{t+1} + \alpha_y, b y_{t-1} - \alpha_y (i_t - E_t \pi_{t+1}) + d_t. \]

Now consider a monetary shock that aims to increase real rates for a while. For instance, this would be the case of an interest rate rule of the form \( i_t = E_t \pi_{t+1} + \nu_t \), \( \nu_t = \rho_\nu \nu_{t-1} + \varepsilon_{\nu t} \), where \( \varepsilon_{\nu t} \) is the monetary shock. In this case, it is clear that a

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14To avoid a unit root associated with real interest rate rules, we are again assuming that \( 1 - \alpha_y, f - \alpha_y, b \) is greater than zero but can be arbitrarily close to zero.
tightening of monetary policy will lead to a persistent decline in economic activity as long as either $\rho_\nu$ or $\alpha_{y,b}$ are not equal to zero.

Inflation response to such a monetary shock will potentially cause the emergence of a price puzzle—i.e., inflation can rise on impact after a monetary contraction before declining below zero at later dates. The occurrence of a price puzzle following a monetary shock is not surprising in environments in the Patman regime. However, the more interesting observation is that the length of the price puzzle will vary depending on both the persistence of the shock $(\rho_\nu)$, the size of the shock and the extent of internal dynamics—i.e., the size of $\alpha_{y,b}$. In order to get a better sense of these forces, it is helpful to consider the effects of a temporary change in interest rates of size $r$ occurring at time 0. In this case, inflation for $t \geq 0$ is given by

$$\pi_0 = \left[ \gamma_r - \gamma_y \alpha_r \sum_{i=0}^{\infty} (\beta \lambda)^i \right] r,$$

$$\pi_t = -\gamma_y \alpha_r \lambda^t \left[ \sum_{i=0}^{\infty} (\beta \lambda)^i \right] r,$$

where $\lambda$ is the stable root of the polynomial $\alpha_{y,f} X^2 - X + \alpha_{y,b}$. Here there would be a price puzzle in period 0 if $\gamma_r - \gamma_y \alpha_r \sum_{i=0}^{\infty} (\beta \lambda)^i$ is greater than zero. The condition $\left[ \gamma_r - \gamma_y \alpha_r \sum_{i=0}^{\infty} (\beta \lambda)^i \right] > 0$ is the natural extension of the GE Patman condition for the case when there is external habit. Note that the basic TE Patman condition $\alpha_r \gamma_y < \gamma_r$ is a necessary condition for the price puzzle but is not sufficient. With a purely temporary increase in $r$, the price puzzle lasts one period in this case. After one period, inflation drops below steady-state inflation and then converges back to its steady-state value from below. In Figure 4, this response is displayed in dark gray. We also plot responses to a mildly persistent and very persistent shock. When persistence is mild, one observes several periods of “price puzzle,” while there are none in the case of a more persistent shock. Note that output gap decreases in all scenarios.

G. Nonlinear Model

It is beyond the scope of this paper to solve and estimate a nonlinear version of our model. However, it is worth noting that the Patman configuration—if satisfied—should be thought as a local phenomena, applicable only near the steady state. The slope of the Phillips curve $\gamma_y$ may be very close to zero (or equal to zero) when one is near the steady state of the system, and the Patman condition can be satisfied. However, when the economy deviates far from the steady state, it may be that the Phillips curve slope $\gamma_y$ increases, causing the parameterization to switch from Patman to a more regular case in which tight monetary policy decreases inflation. To illustrate this possibility, assume for simplicity that $\gamma_y = \tilde{\gamma}_y y_t^2$—as to represent

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15 We are aware that the price puzzle could be an artifact of poor controls for the Fed’s information set. Our reading of the literature is that the jury is still out on whether the price puzzle is a fact or an artifact.

16 A short-term price puzzle can also be obtained in a model with sticky prices and deep habits, as shown by Ravn et al. (2010).
that the effect of market tightness on wages may be more operative when far from the steady state—then the Phillips curve will take the form

\[ \pi_t = \beta E_t[\pi_{t+1}] + \gamma_y y_t^3 + \gamma_r \left( i - E_t[\pi_{t+1}] \right). \]

Now suppose that monetary policy was of the form \( i_t = E_t[\pi_{t+1}] + \phi_d d_t \), and for complete simplicity, assume that the demand shock \( d_t \) is an i.i.d. process and the only shock in the economy. In such a case, equilibrium inflation will be given by

\[ \pi_t = \gamma_\ell (1 - \alpha_r \phi_d)^3 d_t^3 + \gamma_r \phi_d d_t. \]

In such a model, a more activist monetary policy (higher \( \phi_d \)) destabilizes inflation in the Patman zone and stabilizes it outside the Patman zone. Hence, if the demand shock distribution has a large variance, then activist policy may help stabilize inflation, while if the shocks are not too large, it could destabilize inflation. Alternatively, in such a framework, one may want to choose a monetary policy that reacts very differently to small versus large shocks.

II. Estimating the Phillips Curve with Unrestricted Cost Channel

In this section, we explore properties of the New Keynesian Phillips curve when interest rates are allowed to directly affect real marginal costs. We do so by using the limited information—single equation approach initiated in the New Keynesian literature by Roberts (1995) and Galí and Gertler (1999).\footnote{See the surveys of Nason and Smith (2008) and Mavroeidis, Plagborg-Møller, and Stock (2014).}
body of literature that allows for a monetary policy cost channel, most papers impose parameter restrictions that rule out by assumption the Patman configuration that is of interest to us. Therefore, our objectives in this section are twofold. First, we want to examine, within the confines of the New Keynesian Phillips curve, whether interest rates have significant direct effects on inflation. Second, and most importantly, we want to look at whether the direct channel of monetary policy on inflation (i.e., the direct effect of interest rates) is large in comparison to the more standard indirect channel (i.e., working through market tightness).

A. Baseline Estimation

According to the first-order approximation of the model derived in Section I, the Phillips curve takes the form

$$\pi_t = \beta \pi_{t+1}^e + \gamma_x x_t + \gamma_r (i_t - \pi_{t+1}^e) + \mu_t,$$

where, as before, $\pi_{t+1}^e$ is expected inflation, $x_t$ is a measure of market tightness, $i_t$ represents the nominal interest rate, and $\mu_t$ is a markup shock. Note that all the variables are demeaned, so that there is no constant in the equation.

It is worth immediately noting that from an estimation point of view, the distinction between whether one should allow real interest rates or nominal interest rates in this equation is irrelevant. Both lead essentially to the same regression up to a recombination of terms. We will return to this point later when discussing the interpretation of coefficients.

The biggest challenge in estimating the Phillips curve (8) relates to the endogeneity of the regressors. In our case, the endogeneity problem is compounded by the fact that we allow for interest rates to have a direct effect on inflation, knowing very well that the setting of interest rates is likely responding to inflation. For this reason, in all our estimations we will treat output gap, inflation expectations, and interest rates as endogenous and follow Barnichon and Mesters (2020) in using identified monetary policy shocks as instruments. In particular, we will use six lags of the monetary policy shocks isolated in Romer and Romer (2004) and their squares as instruments.

There are many data choices associated with estimating equation (8). We will proceed in the following way. We use the US Congressional Budget Office (CBO) unemployment gap as our measure of market tightness. For our measure of the interest rate, we use the Federal Funds Rate. These two variables are obtained from FRED (Federal Reserve Bank of St. Louis 2022). For expected inflation we use the “Expected Change in Prices during the Next Year” of the Michigan Survey of Consumers.

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18 Notice that we normalize the coefficient attached to the marginal cost, $\kappa = 1$, as it is not separately identifiable from $\gamma_x$ and $\gamma_r$. However, this is not restrictive for our case, as the value of $\kappa$ is irrelevant when considering Patman condition, which is about the ratio $\gamma_x / \gamma_r$.

19 See online Appendix I for an estimation of a “hybrid” version of the Phillips curve.

Consumer Expectations (University of Michigan 2022), or we assume full information rational expectation (FIRE).22

First-Pass Estimations: For inflation, we use as headline CPI inflation as a first pass, and control for oil price in the estimation. The advantage of this choice is that we can use a long sample that starts in 1969. We first estimate the Phillips curve without including the cost channel of inflation (columns 1, 3, and 5 of Table 1). The slope of the Phillips curve \( \gamma_y \) is positive, significant at 1 percent with the survey measure of expectations, not at 10 percent otherwise. The clear and consistent results we obtain are that once we also include the interest rate, the cost channel \( \gamma_r \) is always positive and significant, while the slope \( \gamma_y \) becomes smaller and is always insignificant, with a point estimate that is not always positive, which points toward a Patman regime.

Table 2 presents a set of robustness check relative to the data choices made in Table 1. A larger set of robustness checks is available in Beaudry, Hou, and Portier (2020). In the first two columns of Table 2, we use the CBO output gap instead of the unemployment gap as our measure of economic slack. As can be seen, results are very similar to those in Table 1, with the interest rate effect continuing to enter our estimated Phillips curve significantly. In the next two columns, we replace headline CPI inflation by core CPI as our measure of inflation and no longer control for oil prices in estimation. Results are again robust to this modification and provide further support in the direction of the Patman condition.

Preferred Estimations Using Core R-CPI Inflation: Our preferred estimations use the BLS (US Bureau of Labor Statistics 2022) “Consumer Price Index retroactive series using current methods for all items less food and energy” (R-CPI-U-RS, or core R-CPI for short). We use this series instead of the CPI because before 1983, the shelter component of the CPI was computed using, among other small components, an index of house prices and an index of mortgage rates. Mortgage rates directly comove with the effective federal funds rate, and indirectly, through discount rates, house prices do too. This would mechanically make CPI inflation reacting to the federal fund rate. Since 1983, the BLS adjusted its methodology and changed the computation of the shelter component of the CPI in favor of a rental equivalence index, including an owner-occupied rental equivalence index. The BLS does not retroactively adjust the methodology in its price indexes. The advantage of the core R-CPI is that such

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22 In the Michigan Survey of Consumers, every month, a representative sample of consumers are asked the following question: “By about what percent do you expect prices to go (up/down) on the average, during the next 12 months?” The answer to this question is then the one-year-ahead inflation expectation \( E_t \pi_{t+12} \). To keep consistency with the quarter-to-quarter inflation we use in the estimation, we rescaled the one-year-ahead expected inflation assuming survey respondents believe that quarter-to-quarter inflation follows an AR(1) process with persistence \( \hat{\rho} \) that needs not to be equal to the actual persistence of inflation. See online Appendix G for details.

23 Note that the empirical literature on inflation expectations document prominent evidence on deviations from FIRE (see for example Coibion and Gorodnichenko 2015).
a retroactive adjustment is done. As this series is not seasonally adjusted, we use a year-to-year measure of inflation.\footnote{See online Appendix H for details.}

Finally, it is well known that the slope of the Phillips curve ($\gamma_y$) is difficult to estimate using aggregate data (Mavroeidis, Plagborg-Møller, and Stock 2014). The

| Table 1—First-Pass Estimation of the Phillips Curve Using Headline CPI Inflation |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
|                  | MSC             | FIRE            |
| $\pi^e$          | (1)             | (2)             | (3)             | (4)             |
| $\beta$          | 1.12            | 1.18            | 0.81            | 0.98            |
|                  | (0.079)         | (0.074)         | (0.098)         | (0.106)         |
| $\gamma_y$       | 0.12            | 0.06            | 0.08            | -0.07           |
|                  | (0.047)         | (0.053)         | (0.071)         | (0.076)         |
| $\gamma_r$       | 0.14            |                 | 0.21            |                 |
|                  | (0.041)         |                 | (0.062)         |                 |
| Observations     | 150             | 150             | 150             | 150             |
| $J$-test         | 7.607           | 8.515           | 5.538           | 5.919           |
| ($jp$)           | (0.815)         | (0.667)         | (0.938)         | (0.879)         |
| Weak ID test     | 3.387           | 3.091           | 1.804           | 1.643           |

Notes: All results are using IV-GMM procedure; Newey-West HAC standard errors with six lags are reported in parentheses. The constant term is omitted from the table. The measure of inflation is headline CPI inflation; the measure of market tightness is the US Congressional Budget Office unemployment gap. We use the Michigan Survey of Consumers to measure inflation expectations in the MSC columns and assume full-information rational expectations in the FIRE ones. Real oil price is added as a control in all the equations, and all regressors are instrumented using six lags of Romer and Romer’s (2004) shocks (as extended by Wieland and Yang 2020) and their squares as instruments. Sample is 1969:1–2007:IV.

| Table 2—First-Pass Estimation of the Phillips Curve, Robustness |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
|                  | Headline CPI    | Core CPI        |
| Gap              | $\gamma_{gap}$  | minus $ugap$    |
| $\pi^2$          | MSC             | FIRE            | MSC             | FIRE            |
|                  | (1)             | (2)             | (3)             | (4)             |
| $\beta$          | 1.10            | 0.74            | 0.97            | 0.74            |
|                  | (0.072)         | (0.096)         | (0.057)         | (0.053)         |
| $\gamma_y$       | 0.06            | 0.09            | -0.05           | 0.00            |
|                  | (0.025)         | (0.039)         | (0.053)         | (0.059)         |
| $\gamma_r$       | 0.17            | 0.21            | 0.19            | 0.47            |
|                  | (0.038)         | (0.058)         | (0.066)         | (0.065)         |
| Observations     | 150             | 150             | 150             | 150             |
| ($jp$)           | (0.647)         | (0.793)         | (0.588)         | (0.503)         |
| Weak ID test     | 5.015           | 2.629           | 2.865           | 2.734           |

Notes: All results are using IV-GMM procedure; Newey-West HAC standard errors with six lags are reported in parentheses. The constant term is omitted from the table. All regressors are instrumented using six lags of Romer and Romer’s (2004) shocks (as extended by Wieland and Yang 2020) and their squares as instruments. Sample is 1969:1–2007:IV.
most recent and credible estimates exploit cross-regional variations, as in Hazell et al. (2020). Therefore, to bypass controversies about $\gamma_y$, we repeat our estimation of the Phillips curve (8) imposing the slope parameter $\gamma_y = 0.0138$, as estimated by Hazell et al. (2020).25

Our preferred set of results is presented in Table 3. Our baseline estimation is column 1, where we use expectations surveys and estimate both $\gamma_y$ and $\gamma_r$: the slope of the Phillips curve $\gamma_y$ is small and negative, while the real interest rate coefficient $\gamma_r$ is positive and significant.26 When we set $\gamma_y$ to the estimated value of Hazell et al. (2020) (column 2), the cost channel parameter $\gamma_r$ is still highly significant and positive, and larger than when $\gamma_y$ is estimated. When we repeat the estimation assuming FIRE, we obtain similar results, although the size of the cost channel coefficient is smaller but still highly significant. This set of results indicates that the US economy may likely be operating in the Patman zone.

B. Iterating Forward the Phillips Curve

The previous results are of interest because we use only identified monetary policy shocks as instrumental variables and these monetary shocks are strong instruments. However, there are also drawbacks in such identification strategy. The standard formulation of the Phillips curve imposes strong restrictions on the timing of inflation variations. The identification through monetary policy shocks works like decomposing the responses of inflation, expectation, and real interest rate to these shocks. Empirically, these variables may not respond to the monetary shocks simultaneously, and in the Phillips curve relation, current inflation may also respond to economic slackness and real interest rate with lags. Looking at it this way, the timing restriction may make these estimates problematic. We address this by following Hazell et al. (2020) in iterating forward the Phillips curve. As explained in the previous paragraph, we choose not to estimate the slope parameter $\gamma_y$ and instead set it to the estimated value of Hazell et al. (2020) as to focus on the role of the cost channel.

First, we derive the long-run Phillips curve from equation (8):

$$\pi_t = \sum_{j=0}^{\infty} \beta^j (\gamma_y E_t x_{t+j} + \gamma_r E_t r_{t+j} + E_t \mu_{t+j}) + \lim_{j \to \infty} \beta^j E_t \pi_{t+j} = 0.$$  

25 In Hazell et al. (2020), the authors provide an implied aggregate slope of the Phillips curve, with R-CPI as the measure of inflation and negative unemployment gap as the measure of market slackness. From footnote 22 in Hazell et al. (2020), this aggregate slope of the Phillips curve is $0.58 \times 0.0062 + 0.42 \times 0.0243 = 0.0138$. Furthermore, although the authors didn’t estimate a Phillips curve with real interest rate, they did control for the time fixed effect in estimating the slope. If real interest rate is common across states, the impact of the direct cost channel is taken care of by the time fixed effect. For these reasons, it is appropriate to use their estimates in our analysis.

26 Estimates for $\gamma_r$ are smaller than first-pass estimates because mortgage rates are directly used to compute inflation is the later.
Assume that \( x_t \) and \( r_t \) have a long-run and a transitory component so that \( x_t = \tilde{x}_t + x_\infty \) and \( r_t = \tilde{r}_t + r_\infty \). We can then rewrite equation (9) as

\[
\pi_t = \gamma_y \sum_{j=0}^{\infty} \beta_j E_t \tilde{x}_{t+j} + \gamma_r \sum_{j=0}^{\infty} \beta_j E_t \tilde{r}_{t+j} + \frac{1}{1 - \beta} \left( \gamma_y E_t x_\infty + \gamma_r E_t r_\infty \right) \\
+ \sum_{j=0}^{\infty} \beta_j E_t \mu_{t+j} \\
= \gamma_y \sum_{j=0}^{\infty} \beta_j E_t \tilde{x}_{t+j} + \gamma_r \sum_{j=0}^{\infty} \beta_j E_t \tilde{r}_{t+j} + E_t \pi_\infty + \sum_{j=0}^{\infty} \beta_j E_t \mu_{t+j}.
\]

The last equation follows from equation (8), as when \( t \to \infty \), we have \( E_t \pi_\infty = \left[ 1 / (1 - \beta) \right] \left( \gamma_y E_t x_\infty + \gamma_r E_t r_\infty \right) \). We set the Phillips curve slope \( \gamma_y \) to the estimated value in Hazell et al. (2020) and use the year-to-year inflation rate. We truncate the infinite time horizon at \( T = 40 \), which is equivalent to ten years, and use the ten-year ahead CPI forecast from the Cleveland Fed as a measure of \( E_t \pi_\infty \). We then use ten-year moving average to compute the long-run component of real interest rate \( r_\infty \) and get \( \tilde{r}_t = r_t - r_\infty \). Then, following again Hazell et al. (2020), we use negative unemployment gap as \( \tilde{x}_t \). We can then estimate equation (10) by replacing \( \sum_{j=0}^{\infty} \beta_j E_t \tilde{x}_{t+j} \) and \( \sum_{j=0}^{\infty} \beta_j E_t \tilde{r}_{t+j} \) with \( \sum_{j=0}^{T} \beta^j \tilde{x}_{t+j} \) and \( \sum_{j=0}^{T} \beta^j \tilde{r}_{t+j} \) and instrument with monetary shocks prior to time \( t \). The sample we use here is 1982:I–2007:IV due to the availability of the ten-year-ahead CPI forecast. Results are presented in Table 4.

The key takeaway of Table 4 is that the estimate of \( \gamma_r \) is again positive, highly significant, and of the same magnitude as in Table 3.
C. Nominal versus Real Interest Rates?

So far, we have presented theory and data that emphasize the impact of the real interest rate on the marginal cost and thereby on inflation. However, in most of the literature on the cost channel of monetary policy, it is the nominal interest rate that is highlighted to affect inflation. As noted previously, in terms of Phillips curve estimation, this distinction does not matter for the estimation of $\gamma_r$, as both approaches lead to the same estimating equation. Indeed, the equation we have estimated is

$$\pi_t = \beta \pi_{t+1} + \gamma_y x_t + \gamma_r (i_t - \pi_{t+1}) + \mu_t,$$

and it can be rewritten as

$$\pi_t = (\beta - \gamma_r) \pi_{t+1} + \gamma_y x_t + \gamma_r i_t + \mu_t.$$

However, by focusing on estimated coefficients, especially the coefficient on expected inflation, one can get a sense of whether a real interest rate or a nominal interest rate interpretation is preferable. One of the implications of the New Keynesian Phillips curve literature is that the coefficient of expected inflation should be close to agents’ discount factor. Given the fact that we use quarterly data, this would suggest a coefficient of expected inflation close to 0.99. Taking our baseline estimation—looking at column 1 of Table 3—we must obtain a coefficient $\beta$ on expected inflation that is 0.91, which is already smaller than the hypothetical 0.99. However, if we were to adopt a nominal rate specification, then we would need to subtract the coefficient we found for real rates $\gamma_r$ (which is around 0.12) from the coefficient $\beta$ we estimated for expected inflation. This would imply a $\beta$ around 0.8 instead of around 0.91. This would be quite far from theoretical predictions, which points toward the real rate specification.

We can also directly compare the real and nominal rate specification if we set $\beta = 0.99$ and $\gamma_y = 0.0138$ and estimate the two following equations:

$$\pi_t = 0.99 \pi_{t+1} + 0.0138 x_t + \gamma_r (i_t - \pi_{t+1}) + \mu_t,$$

and

$$\pi_t = 0.99 \pi_{t+1} + 0.0138 x_t + \gamma_r i_t + \mu_t.$$
Results of these estimations are displayed in [Table 5]. The cost channel parameter $\gamma_r$ is significant in both specifications, but the $R^2$ is 0.25 for the real interest specification and only 0.02 for the nominal interest one; the real interest rate specification is unambiguously preferred by the data.

D. Some Further International Evidence

In a recent work, Coibion, Gorodnichenko, and Ulate (2019) estimate a New Keynesian Phillips curve by pooling across a range of countries that have consumer or firm surveys available. They assemble time series of inflation expectations for 18 countries/regions (Australia, Canada, Chile, Czechia, Denmark, Finland, France, Germany, Israel, Italy, Japan, New Zealand, South Korea, Sweden, Turkey, the United Kingdom, and the United States, as well as the entire eurozone) over different periods. They found “a robust and negative relationship between the inflation gap (the deviation of inflation from expected inflation) and the unemployment gap (the deviation of unemployment from the natural rate).” Although the panel regression does not use instrumental variables, its merit is to allow for estimation over more than 1,000 country-quarter observations. The estimated equation (omitting the constant) is

$$\pi_{it} - \pi_{it+1}^e = \gamma_y y_{it} + c_i + \varepsilon_{it},$$

where $i$ is a country index, $y_{it}$ is minus the unemployment gap, and $c_i$ are country fixed effects. We use the same data plus a measure of the real interest rate to estimate a Phillips curve augmented with a cost channel. The estimated equation is in this case

$$\pi_{it} - \pi_{it+1}^e = \gamma_y y_{it} + \gamma_r (i_{it} - \pi_{it+1}^e) + c_i + \varepsilon_{it}.$$

Estimation results are presented in Table 6.

Two results emerge. First, the real interest enters positively and significantly ($p$-value is 0.9 percent), and the coefficient $\gamma_r$ is pretty close to the one we obtain in our US estimates (0.2). Second, the slope of the Phillips curve $\gamma_y$ is reduced by almost half (0.19) and it loses significance ($p$-value is 17 percent). We find these results as an extra piece of evidence of the flatness of the Phillips curve and on the significance of the cost channel, and this points again toward the Patman zone.

III. Structural Estimation

The goal of this section is to estimate an extended three-equation New Keynesian model, where we do not a priori take any stance on whether parameters satisfy the Patman condition. Our objective is to see whether the Patman parametrization may offer a better fit to the data than more standard parametrizations implicit in most New Keynesian models.

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27 See Coibion, Gorodnichenko, and Ulate (2019) for a precise description of data and estimation method. We thank them for providing data and codes.

28 See online Appendix F for the choice of the nominal interest rate.
A. Estimated Model

We consider an extended version of our baseline model where we allow for internal propagation mechanisms in the three equations.\(^{29}\) To that effect, we follow the literature and introduce habit persistence, hybrid Phillips curve, and persistence in the policy rule. The derivation of the Euler equation and Phillips curve are presented in online Appendix C.

Private sector behavior is summarized by the two equations

\[
\text{(EE) } y_t = \alpha_y [\alpha_{y,f} E_t[y_{t+1}] + (1 - \alpha_{y,f}) y_{t-1}] - \alpha_r (i_t - E_t[\pi_{t+1}]) + d_t,
\]

\[
\text{(PC) } \pi_t = \beta [(1 - \beta_b) E_t[\pi_{t+1}] + \beta_b \pi_{t-1}]
\]

\[+ \kappa [\gamma_y y_t + \gamma_{y,b} y_{t-1} + \gamma_r (i_t - E_t[\pi_{t+1}])] + \mu_t,\]

\(^{29}\)See online Appendix J.1 for an estimation of the model without internal propagation mechanisms.
where $d_t$ and $\mu_t$ are assumed to be independent AR(1) processes. Here we are expressing market tightness by $y$, which should be interpreted as labor gap. Since in our simple framework the labor gap and the output gap are interchangeable, we chose to express market tightness by $y$ to remind the reader of this property.

With habit persistence, past output also enters in the marginal cost. In order to facilitate comparison with the Phillips curve estimations of Section II, we present estimates of the Phillips curve where we do not include lagged market tightness in the marginal cost—i.e., we use the Phillips curve equation of the form

$$(\text{PC}) \quad \pi_t = \beta \left[ (1 - \beta_b) E_t[\pi_{t+1}] + \beta_b \pi_{t-1} \right] + \kappa \left[ \gamma_y y_t + \gamma_r (i_t - E_t[\pi_{t+1}]) \right] + \mu_t.$$  

In online Appendix J.3, we show that results are unaffected when we estimate the model with (PC) rather than (PC').

We choose to close the model with the following class of policy rules:

$$(\text{Policy}) \quad i_t = E_t[\pi_{t+1}] + \phi_{r,b} (i_{t-1} - E_{t-1}[\pi_t]) + \phi_{\pi,b} \pi_{t-1} + \phi_{y,b} y_{t-1} + \phi_d d_t + \phi_\mu \mu_t + \nu_t.$$  

This class of policy rules is attractive as it minimizes difficulties associated with indeterminacy while simultaneously being very flexible, as it allows monetary policy to react to the state space of the system. With such a real rate rule, the equilibrium is determinate as long as $|\alpha_y| < 1$. In the baseline estimation, we assume quasi–no Euler discounting by setting $\alpha_y = 0.99$. Note that in this policy rule, $\nu_t$ will represent monetary shocks that we also assume to be AR(1). In online Appendix E, we prove that for any monetary rule that reacts to current endogenous variables and that guarantees determinacy of equilibrium—which includes the typical Taylor rule estimated in the literature—equilibrium allocations can be replicated with our class of policy rules. Estimating a model with our policy is therefore not restrictive and nests a Taylor rule specification.  

B. Estimation and Sample Period

As typical in the literature, a classical maximum likelihood method would become a nonlinear optimization problem that is quite unstable. We therefore perform a Bayesian estimation, as in this case the use of prior distributions over the structural parameters makes this optimization more stable. In online Appendix J.2, we present the choice of priors and show detailed results such as parameters priors and posterior distributions. As commonly done in the empirical macroeconomic literature, we calibrate some parameters. First, one cannot separately identify $\kappa$, $\gamma_y$, and $\gamma_r$. Instead, we can only get estimates of $\kappa \gamma_y$ and $\kappa \gamma_r$. Without loss of generality, we

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30 A somewhat intermediate modeling has been recently adopted by Bianchi and Melosi (2019) and Bianchi, Faccini, and Melosi (2022), as they model a situation in which the central bank reacts differently to different shocks.

31 In some related work (Beaudry, Preston, and Portier 2023), we study in more details the advantage of specifying such a general monetary policy rule.
therefore normalize $\kappa = 1$. We set $\beta$ to 0.99, which is in line with large parts of the literature. Our results are not sensitive to changing $\beta$ around this level. We set $\alpha_y$ to 0.99, so that although there is almost no discounting in the Euler equation, the model is always determinate. Data and sample are the same as in the baseline estimation of Section II. All estimations are performed using Dynare.32

C. Results

Table 7 presents the parameters estimates. Parameters are well identified and have the expected sign. Monetary policy is observed to increase interest rates in response to demand shocks ($\phi_d > 0$) and to decrease them in response to cost-push shocks ($\phi_\mu < 0$). The estimated value of $\alpha_y$ is low. There is recent micro evidence pointing toward such low values. Best et al. (2020) use quasi-experimental variation in interest rates on the UK mortgage market and find an average intertemporal elasticity around 0.1. There is also macro evidence. Following a DSGE approach, Kilponen, Vilmunen, and Vähämaa (2022) estimate a simple New Keynesian model with King, Plosser, and Rebelo’s (1988) preferences and show that the real interest rate elasticity of output is in the range of 0.05–0.20 in the United States. Regarding the Phillips curve, interestingly, at the mean of the posterior distribution, the slope of the Phillips curve $\gamma_y$ is $-0.03$ and is not significantly different from zero, while the cost channel $\gamma_\tau$ is significant and equal to 0.07. The parameters configuration is such that the TE Patman condition is met.33 As this is only a necessary condition in this model with persistent shocks, we also compute the GE Patman condition, which is given by the impact response of inflation to a 1 standard deviation monetary policy shock. As it can be checked in Table 7, that response is positive and significant at a 95 percent level. The model estimation confirms what we have found in the previous section: the Phillips curve appears to be very flat but has a positive and significant cost channel.

D. Accounting for the Zero Lower Bound Period

In this section, we use the model to evaluate the effect of monetary policy over the post-2007 ZLB period as a quantitative counterpart to our theoretical analysis of the low-inflation trap of Section IE. We set up a model with our previously estimated coefficients plus a ZLB constraint and use it to recover the three shocks series over the period 2007:I–2017:IV, solving a nonlinear smoothing problem because of the presence of the ZLB. With these smoothed shocks, the model does perfectly match the data by construction. We can now use the shocks and the model to run counterfactual exercises by shutting down some shocks or changing the monetary policy rule.34 Counterfactual simulations are displayed in online Appendix Figure J.2. Let us first focus on panel A of Figure J.2, which shows the evolution of the output gap.

32 See Adjemian et al. (2020).
33 In a previous version of this work (Beaudry, Hou, and Portier 2020), we show that the results we found here are robust to choices of samples and measures of inflation.
34 In online Appendix J.4, we repeat the exercise when reestimating the model over the post-Volcker period 1983:I–2007:IV and show that results are similar.
Note that all variables are in deviations from the model steady state that is zero. We start simulating the model in 2007:I with all the shocks. This is the thin dark line. By construction, it corresponds to the actual data. If we kill markup shocks from 2007:I and onward, we obtain the thick dark-gray line. It is essentially the same line as the actual data. Through the lens of the model, we see that the economy has been hit by a whole sequence of negative demand shocks. Markup shocks are irrelevant for output over that period. Now consider the totally nonreactive monetary policy that keeps the nominal interest rate at the 2007:I level. This produces an output path represented by the thick light-gray line. We see that the recession is slightly deeper—as expected, as monetary policy does not accommodate the negative demand shocks—and that the recovery is at the same speed. Monetary policy (the rule plus the shocks to the rule) has not been very effective in dampening the recession, because of the ZLB constraint. The evolution of the nominal interest rate in panel B of online Appendix Figure J.2 reveals that following the sequence of negative demand shocks, monetary policy has driven $i$ down to the ZLB (which is $-5$ percent in deviation from the steady state). Considering now inflation in panel C of online Appendix Figure J.2, we see that the actual (expansionary) monetary policy has contributed to low inflation (as the Patman effect would predict), and that with fixed $i$, inflation would have been much closer to its steady-state level.

We find the type of mechanism that was highlighted in Section IE: loose monetary policy did (slightly) reduce the output recession but, as a consequence, put inflation consistently below steady state. If the objective of the central bank is to balance output gap and inflation deviations, it is not obvious that a constant $i$ policy is doing worse, as it does not increase the recession much but keeps inflation much closer to the steady state. Whether it is better depends of course on the weights one puts on inflation and output gap in the welfare function. In that sense, it is indeed the reaction of the monetary authorities’ policy following the negative demand shocks that drove the economy to the ZLB.

### Table 7—Estimated Parameters, Extended New Keynesian Model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>95% Confidence Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_r$</td>
<td>0.02</td>
<td>[0.01, 0.03]</td>
</tr>
<tr>
<td>$\gamma_y$</td>
<td>-0.03</td>
<td>[-0.12, 0.05]</td>
</tr>
<tr>
<td>$\gamma_r$</td>
<td>0.07</td>
<td>[0.03, 0.12]</td>
</tr>
<tr>
<td>$\phi_d$</td>
<td>0.51</td>
<td>[0.31, 0.76]</td>
</tr>
<tr>
<td>$\phi_{\mu}$</td>
<td>0.73</td>
<td>[-0.88, -0.59]</td>
</tr>
<tr>
<td>$\sigma_d$</td>
<td>0.04</td>
<td>[0.03, 0.06]</td>
</tr>
<tr>
<td>$\sigma_\mu$</td>
<td>0.46</td>
<td>[0.34, 0.57]</td>
</tr>
<tr>
<td>$\sigma_\nu$</td>
<td>0.29</td>
<td>[0.16, 0.42]</td>
</tr>
<tr>
<td>$\rho_d$</td>
<td>0.85</td>
<td>[0.79, 0.90]</td>
</tr>
<tr>
<td>$\rho_\mu$</td>
<td>0.60</td>
<td>[0.49, 0.71]</td>
</tr>
<tr>
<td>$\rho_\nu$</td>
<td>0.04</td>
<td>[0.91, 0.96]</td>
</tr>
<tr>
<td>$\beta_b$</td>
<td>0.04</td>
<td>[0.01, 0.09]</td>
</tr>
<tr>
<td>$\phi_{\pi, b}$</td>
<td>0.03</td>
<td>[-0.10, 0.14]</td>
</tr>
<tr>
<td>$\phi_{\pi, f}$</td>
<td>0.14</td>
<td>[-0.01, 0.29]</td>
</tr>
<tr>
<td>$\sigma_\pi$</td>
<td>0.75</td>
<td>[0.67, 0.88]</td>
</tr>
<tr>
<td>$\phi_{\pi, b}$</td>
<td>0.08</td>
<td>[-0.30, 0.38]</td>
</tr>
</tbody>
</table>

Notes: This table shows the posterior mean estimates of the coefficients in equations (EE), (PC'), and (Policy) using unemployment gap, Core CPI Research Series, and the sample 1978:II–2007:IV. Parameters $\beta$ and $\alpha_y$ are not estimated and are set to 0.99 and 0.99. Parameter $\kappa$ is normalized to one. The posterior distribution is obtained using the Random Walk Metropolis algorithm with two chains of 1 million draws each and discarding the first 200,000 draws of each chain. The numbers between brackets represent the 95 percent confidence interval using the posterior distribution. TE Patman condition corresponds to $\gamma_r = \gamma_r \gamma_y$; GE Patman condition is the impact response of inflation $\pi$ to a 1 standard deviation monetary policy shock.
IV. Conclusion

During the two decades prior to the COVID-19 pandemic, the behavior of inflation has been puzzling in several countries. First, during 2008–2009 recession, inflation fell by less than anticipated given the depth of the recession. This became known as the missing deflation puzzle. After that, the puzzle reversed, with inflation generally remaining below target in many countries despite the experience of historically low rates of unemployment. This in turn became known as the missing inflation puzzle. Both these puzzles could reflect a relatively flat Phillips curve. This paper builds on this observation and goes a step further by exploring the monetary policy implications of a quite flat Phillips curve when a cost channel of monetary policy may also be present. We show how standard prescriptions for monetary policy may need to be modified in such an environment. In particular, to keep inflation close to its target in the face of positive demand and markup shocks, we argue that a
central bank may want to keep real interest rates unchanged or decrease them. One of the interesting features of this framework is that it offers a simple explanation of why and when a country may find itself trapped for a considerable amount of time at the effective lower bound with inflation below target and employment above its steady-state value. A large part of the paper has been devoted to showing that the condition under which these features arise is supported in US data.

REFERENCES


