Benefits of Collaboration on Capacity Investment and Allocation

Hyun Soo Ahn, Eren Çetinkaya, Izak Duenyas
Ross School of Business, University of Michigan, 701 Tappan Ave, Ann Arbor, Michigan 48109-1234,
Mengzhenyu Zhang*
School of Management, University College London, Level 38 and Level 50, One Canada Square, London E14 5AA
hsahn@umich.edu, erencet@umich.edu, duenyas@umich.edu, zhenyu.zhangmeng@ucl.ac.uk

This paper studies how capacity collaboration can benefit two competing firms. We consider a two-stage model where capacity decisions are made in the first stage when there are significant uncertainties about market conditions, and then production decisions are made in the second stage after most of these uncertainties are resolved. We vary the degree of collaboration between the two firms in their capacity and production decisions, examining multiple models and comparing the outcomes. We find that a firm can benefit from collaboration even with its competitor. Interestingly, the firms do not have to make production decisions jointly to realize the benefits of collaboration. Additionally, while collaborative capacity investment proves beneficial, collaborating on production with existing capacity can often yield greater benefits. We find that the advantages of collaboration are most pronounced when competition intensifies, demand fluctuates significantly, and investment costs are high.

Key words: competition; capacity management; collaboration; game theory; nash bargaining solution

1. Introduction

Most capacity decisions entail major resource commitments and can substantially change the firm's asset structure. Capacity decisions are often hard to reverse, and they are made under significant demand uncertainty. Several approaches are used to mitigate risks associated with capacity decisions, including flexible capacity (Bish and Wang 2004) and delayed differentiation (Aviv and Federgruen 2001). Another option that is becoming more common is for firms to jointly invest in capacity and/or coordinate the use of it. Interestingly, even competing firms coordinate. One approach is to establish a joint venture where firms contribute equity to build capacity, agree on how to utilize the capacity, and share the revenue. In fact, except for rare instances, such coordination and joint investment are approved by the Federal Trade Commission (FTC) and are not considered anti-trust violations. For instance, Toyota's Corolla crossover and Mazda's new CX-50 crossover are produced in the same plant in Huntsville, Alabama (Greimel 2021). This is a joint venture

* corresponding author

between Toyota and Mazda, whose similar models compete in the end-user market. While they use different lines for the two brands, the two lines share some common capacities, mainly stamping, painting, steel sourcing, and quality inspection. They are even considering mixing production of their similar car models in the same line in the future.

Of course, firms can collaborate and share capacity without a joint venture. In the automotive industry, Toyota and Fuji Heavy Industries agreed to share their manufacturing facilities (Toyota 2006). In the airline industry, code-sharing allows different carriers to share flight capacity (Wassmer et al. 2010, Chun et al. 2012). The two dominant newspapers in the Detroit market, *Detroit Free Press* and *Detroit News*, have an operating agreement to print in the same facility (Busterna and Picard 1993), although they put out separate newspapers every weekday. In 2012, Mazda used its own capacity to build a Toyota sub-compact vehicle based on a Mazda 2 platform at a plant in Salamanca, Mexico (Automotive Logistics 2012). It was the first time that Mazda built a vehicle for its rival. Despite the possibility that collaboration can be seen as an act of collusion, these joint ventures or collaborations between competing firms are not uncommon.

As a further example, in 2018, both U.S. and Korean governments approved a joint venture between Delta Air Lines and Korean Air for transpacific partnerships. This agreement combines the networks via fully reciprocal code-sharing between U.S. and Asia, and it implements joint sales and marketing initiatives.[1] Furthermore, Delta and Korean Air co-located to the new terminal at Incheon International Airport and plan to do one-roof warehousing.[2] There are other examples of sharing transportation and logistics capacities as well. Nestlé, the world's largest food manufacturer, and Pladis, the largest biscuit and snack food manufacturer in the UK, started to share truck capacity in 2009. This collaboration is estimated to reduce costs by £300,000 a year.[3] Another example can be found in retail industry. Shekar Natarajan, the chief supply chain officer at retailer American Eagle Outfitters, began building a logistics platform that other retailers including rivals can share in 2018.[4] The goal of the platform is to reduce shipping time and costs by consolidating packages. Over 100 partners have signed up to use the platform so far, including Kohl's, Steve Madden. Another example can be observed in the forest and paper products sector. Three companies, StoraEnso, Norske Skog, and UPM, consolidated the transportation of their inbound

[1] https://news.delta.com/delta-and-korean-air-launch-world-class-joint-venture-partnership

[2] http://asiacargobuzz.com/2018/07/17/delta-korean-air-press-their-transpacific-belly-cooperation-into-action/

[3] https://logistar-project.eu/wp-content/uploads/2020/02/LOGISTAR_D7.1_Definition_of_use_cases_and_validation_plan_v1.0.pdf

[4] https://www.usnews.com/news/best-states/pennsylvania/articles/2022-09-12/american-eagle-exec-works-to-modernize-the-supply-chain

materials from Sweden and Finland with a single, dedicated, short-sea vessel.[5] This collaboration allows them to reduce transportation and handling costs, and at the same time, to improve service through more frequent and reliable replenishments.

In this paper, we study capacity collaboration between two firms, including competitors. We consider a two-stage model where capacity decisions are made when there are significant uncertainties about market conditions (*first stage*), and production decisions are made after most of these uncertainties are resolved (*second stage*). To capture different collaboration scenarios, we consider several models that differ in the extent to which the firms collaborate in making capacity and/or production decisions.

Although many examples of capacity collaboration exist, it is unclear whether a firm benefits from such collaboration, especially when it collaborates with a competitor. Capacity collaboration allows firms to reduce investment costs. But doing so with a competing firm can be harmful because it gives the competing firm easy access to more capacity. Furthermore, as the previous examples show, collaboration scenarios vary. Firms build and operate capacity together, or firms build capacity together and operate autonomously, or firms just share the existing capacity. We are interested in how much each of these collaboration scenarios improves firms' profits and when simple collaboration scenarios (such as sharing existing capacities but not collaborating on building capacity) are as effective as more complicated ones.

Several research questions are central to this paper: (1) Can a firm benefit from collaborating with a competitor? (2) For a given collaboration scenario, what is the total capacity and how is it allocated? (3) How do firms gain from collaboration? Is most of the gain from deciding capacity together or from utilizing capacity together to fulfill demands? (4) How do the outcomes and gains from collaboration change in business parameters such as variability, cost, etc.?

We find that a firm can considerably benefit from collaboration even with a competitor. If the firms collaborate on both capacity and production decisions, we show that there is a mutually beneficial agreement under which the firms select the centrally optimal decisions. This supports the case for a joint venture. We also find that most of the benefits of collaboration can be captured even when the two firms compete in the production stage as long as they build the capacity together and trade their allocations after they observe the demand signals. However, efficiency is lost if the firms cannot collaborate in both stages. We find that if the firms can collaborate either only in the capacity investment stage or only in the production stage, collaboration during the production stage provides more benefits, except when demands are extremely predictable.

[5] https://www.supplychainbrain.com/articles/14531-time-and-money-advantages-of-logistics-clusters

We find that when the firms compete in the production stage, but can trade their capacity after they observe the demand signals, the total capacity is smaller than the capacity of a centralized firm. We also find that the total capacity when the firms compete in both stages might end up being smaller than the total capacity when the firms compete in capacity investment and collaborate in production. These are surprising results because competition typically leads to larger capacity than centralization in most existing literature, including Yang and Schrage (2009).

2. Literature Review

Several papers study capacity sharing decisions for two or more products from the perspective of single or multiple firms. However, the substitution effects (competition) among products are not considered. Wu et al. (2013) study the setting where the firm potentially shares capacity with a supplier not competing in the end-product market. Yu et al. (2015) analyze the scenario where multiple non-competing firms can invest in a shared facility that is modeled as a queuing system with finite service rates (first come, first served). They find that capacity sharing might not be beneficial when firms have heterogeneous work contents and service variabilities. With a view toward maximizing the service level, Jiang et al. (2022) explore how to allocate a shared capacity to fulfill customer demands with individual service levels. Khanjari et al. (2022) focus on the supplier's problems of whether to allow buyers to transfer unused capacity to other buyers and how much to charge for the transfer. They allow the demand faced by buyers to be dependent, but they do not model competition among buyers. Similarly, Van Mieghem (1998) and Roels and Tang (2017) model the capacity allocation problem between two firms with non-competing product lines. Roels and Tang (2017) find that ex-post transfer payment contracts might make one firm worse off. Yang et al. (2021) build a multilocation newsvendor model with multiple retail stores owned by a central planner. In one of the considered scenarios, each retail store decides the order quantity for their own store, whereas their inventories are pooled. The product substitution effects are not modeled. In a cooperative inventory transshipment setting, Anupindi et al. (2001) and Granot and Sosic (2003) consider a model where multiple retailers of a common product can transfer inventory after demand is realized. Van Mieghem (1999) studies a model with a manufacturer and a subcontractor. In this model, each firm separately decides on its capacity ex ante but has an option to trade capacity ex post. He shows that the firms can reach a centrally optimal solution only when the contract terms are contingent on the demand realizations. Our paper's key difference from this group of literature is that we consider the potential substitution effect (competition) between products of two firms seeking collaboration. The demand in our model is endogeneously determined and affected by the other player's decision.

Several papers use cooperative game or bargaining theory to study capacity sharing. Hu et al. (2013) use bargaining theory to study the outcome of negotiated proration rates between airlines for interline and code-share flights. Slikker et al. (2005) use cooperative games to study inventory centralization with coordinated ex-ante orders and ex-post allocation among retailers. Other papers studying inventory centralization include Hanany and Gerchak (2008), Ozen et al. (2008) and Chen and Zhang (2009). Plambeck and Taylor (2005) study a model of two original equipment manufacturers (OEMs) that collaborate on capacity and decide investment levels in demand-stimulating innovations. They characterize the effects of collaboration structures on equilibrium outcomes. Nishizaki et al. (2022) consider the setting where multiple manufacturers individually determine the production levels before demand realization. Each manufacturer faces independent demand. After the demands are realized, manufacturers jointly produce the products using pooled resources, and surplus products are transshipped to manufacturers with residual demands. All of these papers, however, assume that the demand of one product is independent of demand for the others. We consider a model where demands can be dependent and endogenous.

Another stream of literature studies capacity allocation with competing firms or substitutable products. None of the papers in the area, however, considers the problem with a cooperative solution in which the firms share the capacity to maximize the total profit, making the outcome closer to the centralized case. Qi et al. (2015) study the capacity investment decisions of two competing firms in the face of contractual restrictions that govern the capacity use. They model the problem as a Cournot quantity competition game, in which demand is endogeneously affected by the other firm's production quantity. In a multi-product competition setting, Caldieraro (2016) finds that strategic production outsourcing can occur between an entrant and an incumbent selling differentiated products. He shows that the firms might prefer high transfer prices to mitigate price competition. Guo and Wu (2018) study the capacity sharing problem between two firms that engage in price competition. Each firm has some fixed demand from loyal buyers and seeks to undercut its rival in competing for the non-loyal buyers. They consider a linear transfer price for capacity sharing and model the problem as a price-setting game. They show that capacity sharing softens price competition. Assuming independent and exogeneously given demand distributions of two firms, Kemahlıoğlu-Ziya (2015) considers the contract between two firms (selling the same product or substitutable products) with a manufacturer for capacity reservation and wholesale prices. After demand realization, the two firms can renegotiate their contract, agreeing to use either more or less than the reserved capacity. She finds that a firm's post-renegotiation profit can be either increasing or decreasing in its or its partner's demand variances. None of these papers considers a cooperative solution to the problem in which the firms share the capacity to maximize a total profit. In contrast, our paper not only considers the case where the two firms compete, but also analyzes how competition incentivizes (or discourages) capacity or production collaboration to maximize a joint profit. We propose a Nash bargaining solution for cooperative capacity and production planning decisions where two firms' demands are endogenously affected by each other.

We model the outcome of collaboration between two firms using a bargaining game. Bargaining has been extensively studied in economics literature and applied to model the outcomes of negotiations on wage settlement between unions and firms, price decisions between retailers and consumers, and terms of mergers and acquisitions (see Muthoo (1999) for an extensive review). To characterize the outcome of a bargaining game, we use the Nash bargaining solution (NBS). The NBS establishes that the equilibrium outcome maximizes the product of the firms' surpluses net of their disagreement payoffs (Nash 1950). Although the NBS does not directly specify the bargaining process, the outcomes of several bargaining processes (or situations) can be modeled as variants of the NBS, including alternating offers (Rubinstein 1982). Furthermore, a number of extensions of Rubinstein's model, such as the possibility of negotiation breakdown or presence of inside or outside options, lead to outcomes that are slight variations of the NBS outcome (Muthoo 1999). Significant experimental evidence indicates that the NBS is successful in predicting the outcomes of various bargaining situations (Roth 1995). A number of papers in OM literature use the NBS to model bargaining between two firms: Van Mieghem (1999), Chod and Rudi (2006), Plambeck and Taylor (2005), Nagarajan and Bassok (2008), Kostamis and Duenyas (2009), Kuo et al. (2011), Davis and Hyndman (2021), Melkonyan et al. (2017), Grennan (2014) etc. A comprehensive review of cooperative game theory in OM literature can be found in Nagarajan and Sosic (2008), Fiestras-Janeiro et al. (2011).

The remainder of this paper evolves as follows. In Section 2, we introduce the model, notation, and preliminaries. In Section 3, we present the analysis and results, starting with the production subgame, followed by the capacity investment decision. We carry out a computational study to gain further insights, which we present in Section 4. Section 5 provides future research directions and concluding remarks.

3. Model, Notation, and Preliminaries

We consider two firms, each producing a single product, engaging in competition and/or collaboration over two stages. In the first stage, firms build capacity before demand information is known. In the second stage, firms observe the demand signals and then determine the production quantities.

Scenario	Capacity Decision	Production Decision
Nn	no collaboration (N)	no collaboration (n)
Nc	no collaboration (N)	collaboration (c)
Cn	collaboration (C)	no collaboration (n)
Cc	collaboration (C)	collaboration (c)
Monopoly	centralized	centralized

Table 1 Capacity and production decisions under each scenario.

We assume that the two firms either compete or collaborate in either or both of the two stages. If they compete, each firm chooses its decisions (of capacity investment or production) to maximize its own payoff. If they collaborate, the firms make decisions jointly and negotiate over the division of the total payoff. Along with the benchmark scenario of a single centralized firm, four scenarios represent a varying degree of collaboration, as summarized in Table 1.

We will separately analyze each of the four scenarios, along with the centralized benchmark scenario. We aim to analyze the benefit that firms get from collaborating on joint capacity investments and/or using the capacity. Depending on the collaboration scenario, firms invest in capacity—together (C) or separately (N)—in the first stage. Let c be the capacity building cost per unit. The analysis for the case of different capacity costs in a centralized firm results in a degenerate outcome where the entire capacity is built where the cost is lowest. Hence, we do not consider this case. We denote K_i as the capacity endowment of firm i, for which firm i holds the ownership rights. If there is no collaboration in the subsequent production stage, K_i is the capacity level that firm i can use for production. If the firms collaborate in the production stage, they can negotiate over the use of capacity so that a firm can produce beyond its initial endowment.

In the second stage, the firms observe the demand signals. Let (Θ_1, Θ_2) be random variables that represent the demand information and θ_i be the realization of Θ_i, $i = 1, 2$. We assume that Θ_i has mean μ_i, standard deviation σ_i, marginal density function $f_i(\cdot)$, $i = 1, 2$, and joint density function $f(\cdot, \cdot)$. The observed demand information, (θ_1, θ_2), and the firms' capacity endowments, (K_1, K_2), define the production subgame $\omega := (K_1, K_2, \theta_1, \theta_2)$. Let Ω denote the set of all subgames with given capacity endowments: $\Omega := \{(K_1, K_2, \Theta_1, \Theta_2)\}$.

In a given production subgame $\omega \in \Omega$, firms decide the production quantities $\Big(q_1(\omega), q_2(\omega)\Big)$. If the firms do not collaborate in the production stage (scenarios Cn or Nn), each firm can only produce up to its initial endowment K_i. On the other hand, if the firms collaborate during the production stage (scenarios Cc and Nc), they jointly choose the production quantities based on the demand signals. We allow the two products to be (partially) substitutable. Hence, the price for product i is given as a function of quantities: $p_i\Big(q_1(\omega), q_2(\omega), \omega\Big) = \theta_i - b_i q_i(\omega) - \hat{b} q_j(\omega), i, j =$

$1,2, i \neq j$. We assume that $b_i \geq \hat{b} \geq 0$. In other words, a product's own quantity is more influential on its price than the other product's quantity. In addition, note from the inverse demand function that, *ceteris paribus*, the price for product i increases in θ_i (i.e., the demand signal becomes more favorable for firm i). This inverse demand function delivers a reasonable representation of reality, as it arises from a choice model wherein a consumer maximizes a quadratic and concave utility function (Singh and Vives 1984). Essentially, we are modeling firms competing in quantity. This demand model has been used in several papers, including Chod and Rudi (2005), Zhou and Zhu (2010), and Bish and Suwandechochai (2010). To avoid trivial outcomes, we assume that $b_i > 0$ and $\mu_i > \frac{\hat{b}}{2b_j}\mu_j + c$ for $i, j = 1, 2$ and $i \neq j$.

If the firms collaborate in the production stage, a transfer payment that allocates revenues in a mutually agreeable way can occur between the firms. Let $\Gamma(\omega)$ be the net transfer payment from firm 1 to firm 2 in a subgame ω: if $\Gamma(\omega) > 0$, firm 1 pays firm 2; if $\Gamma(\omega) < 0$, firm 2 pays firm 1.

For each of the four scenarios, we solve the problem using backward induction. We first determine the equilibrium production quantities and transfer payment for a production subgame ω. We use superscripts to denote the equilibrium decisions and outcomes. For instance, we let $\left(q_1^{\mathfrak{s}*}(\omega), q_2^{\mathfrak{s}*}(\omega)\right)$ be the equilibrium production quantities. Note that these quantities are chosen together if the firms collaborate in the production stage ($\mathfrak{s} = \mathrm{c}$ for the Cc and Nc scenarios), or separately if the firms compete in the production stage ($\mathfrak{s} = \mathrm{n}$ for the Cn and Nn scenarios). Similarly, we let $\Gamma^{\mathrm{c}*}(\omega)$ denote the equilibrium transfer payment in the Cc and Nc scenarios. If the firms do not collaborate on production (Cn and Nn scenarios), no transfer payment occurs: $\Gamma^{\mathrm{n}*}(\omega) = 0$. Combining these, the equilibrium revenue for firm i in a subgame $\omega = (K_1, K_2, \theta_1, \theta_2)$ is denoted by $R_i^{\mathfrak{s}*}(\omega)$ and is written as follows:

$$R_i^{\mathfrak{s}*}(\omega) = q_i^{\mathfrak{s}*}(\omega) p_i\Big(q_i^{\mathfrak{s}*}(\omega), q_j^{\mathfrak{s}*}(\omega), \omega\Big) + (-1)^i \Gamma^{\mathfrak{s}*}(\omega), \qquad i, j = 1, 2, \; i \neq j, \; \mathfrak{s} \in \{c, n\}. \tag{1}$$

Once we determine the equilibrium revenues for each setting, we calculate the expected profits by taking the expectation with respect to the demand signals and subtracting the capacity costs. For given capacity endowments (K_1, K_2), we let $\pi_i^{\mathfrak{s}*}(K_1, K_2)$ denote the expected profit of firm i under setting $\mathfrak{s}$:

$$\pi_i^{\mathfrak{s}*}(K_1, K_2) = \mathbb{E}\Big[R_i^{\mathfrak{s}*}(K_1, K_2, \Theta_1, \Theta_2)\Big] - cK_i, \qquad i = 1, 2, \mathfrak{s} \in \{c, n\}. \tag{2}$$

If the firms do not collaborate in the first-stage investment game, each firm chooses the capacity that maximizes only its expected profit (Equation (2)). On the other hand, if the firms collaborate in the investment stage, firms negotiate to jointly build the capacity and share the profit according

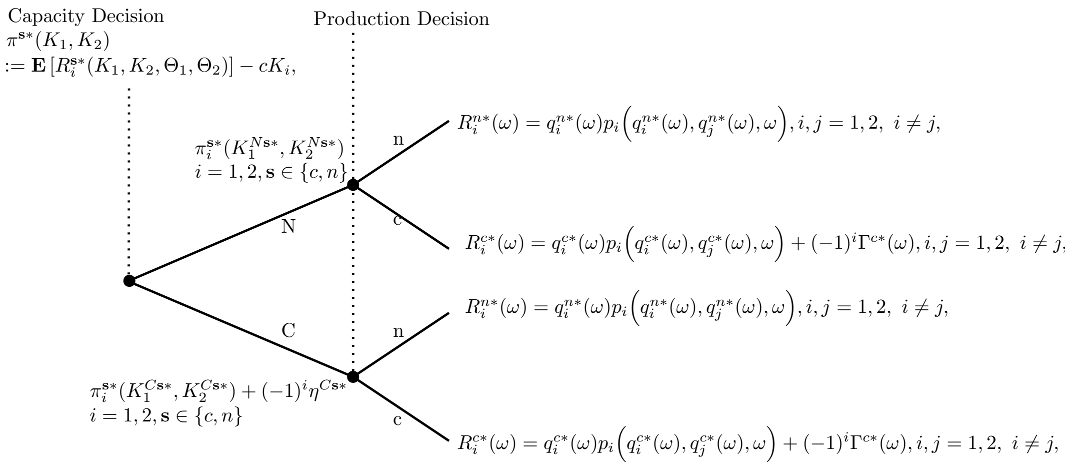


Figure 1 Decision tree with payoffs.

to the Nash bargaining solution. In the NBS, the capacity level that maximizes the sum of the profits, $\pi_1^{\mathfrak{s}*}(\cdot) + \pi_2^{\mathfrak{s}*}(\cdot)$ is chosen, and the profit is split in a manner that neither firm wants to deviate from the agreement (see Section 4.3). We note that for the decentralized cases (Nn, Nc, Cn, Cc), all our results apply to settings where capacity costs are different. We prove all our results using the general profit function $\pi_i^{\mathfrak{s}*}(K_1, K_2) = \mathbb{E}\Big[R_i^{\mathfrak{s}*}(K_1, K_2, \Theta_1, \Theta_2)\Big] - cK_i, i = 1, 2, \mathfrak{s} \in \{c, n\}$ for individual firm i, where the capacity investment cost c can be firm i-dependent (c_1, c_2 instead of c).

Once we obtain the profit functions, we solve for the equilibrium capacities under each scenario. To distinguish these, we use superscripts. For instance, $K_i^{\mathrm{Nn}*}$ is the equilibrium capacity of firm i in the Nn scenario, etc. To denote the outcomes and profit of a single centralized firm, we use a superscript "m" (representing *monopoly*). We finally conclude the model preliminaries by demonstrating the decisions to be made using a decision tree with payoffs in Figure 1.

Remark: Without loss of generality, we assume that the second-stage production cost is zero because any positive (and possibly asymmetric) cost can be accommodated in our model by shifting the demand variable Θ_i, $i = 1, 2$.

When a variable or a function represents a joint/total value, we use subscript "T". For instance, while K_1 denotes the capacity for firm 1, K_{T} denotes the joint/total capacity ($K_{\mathrm{T}} = K_1 + K_2$). We use $\mathbf{1}_{\{\}}$ to denote the indicator function. We also use ∇ as the differentiation operator: $\nabla g(x) = \frac{d}{dx} g(x)$, $\nabla^2 g(x) = \frac{d^2}{dx^2} g(x)$ for a function with single variable, and $\nabla_1 g(x_1, x_2) = \frac{\partial}{\partial x_1} g(x_1, x_2)$, $\nabla_2 g(x_1, x_2) = \frac{\partial}{\partial x_2} g(x_1, x_2)$ and $\nabla_{12}^2 g(x_1, x_2) = \frac{\partial^2}{\partial x_1 \partial x_2} f(x_1, x_2)$ for a function with multiple variables. Moreover, the differentiation operator has precedence over assignment. Hence, for instance, if $g(x) = x^2$, then $\nabla g(y^2) = 2y^2$ rather than $\nabla g(y^2) = 4y^3$.

3.1. Preliminaries—a Centralized Firm

We first consider a single firm that decides the capacity and production quantities for both products. For given capacity K_{T} and demand signals (θ_1,θ_2), the centralized firm solves the following problem to determine the optimal production quantities:

$$\begin{aligned}\max_{q_1,q_2\ge 0}\ & q_1\big(\theta_1-b_1q_1-\hat{b}q_2\big)+q_2\big(\theta_2-b_2q_2-\hat{b}q_1\big)\\ \text{s.t.}\quad & q_1+q_2\ \le\ K_{\mathrm{T}}.\end{aligned} \tag{3}$$

Let $\Big(q_1^{\mathrm{m}^*}(K_{\mathrm{T}},\theta_1,\theta_2),q_2^{\mathrm{m}^*}(K_{\mathrm{T}},\theta_1,\theta_2)\Big)$ denote the optimal production quantities. Then, the resulting revenue is

$$\begin{aligned}R^{\mathrm{m}^*}(K_{\mathrm{T}},\theta_1,\theta_2)\ &=q_1^{\mathrm{m}^*}(K_{\mathrm{T}},\theta_1,\theta_2)\Big(\theta_1-b_1q_1^{\mathrm{m}^*}(K_{\mathrm{T}},\theta_1,\theta_2)-\hat{b}q_2^{\mathrm{m}^*}(K_{\mathrm{T}},\theta_1,\theta_2)\Big)\\ &\quad+q_2^{\mathrm{m}^*}(K_{\mathrm{T}},\theta_1,\theta_2)\Big(\theta_2-b_2q_1^{\mathrm{m}^*}(K_{\mathrm{T}},\theta_1,\theta_2)-\hat{b}q_1^{\mathrm{m}^*}(K_{\mathrm{T}},\theta_1,\theta_2)\Big).\end{aligned} \tag{4}$$

Utilizing this, the expected profit is $\pi^{\mathrm{m}^*}(K_{\mathrm{T}})=\mathbb{E}\Big[R^{\mathrm{m}^*}(K_{\mathrm{T}},\Theta_1,\Theta_2)\Big]-cK_{\mathrm{T}}$. In the capacity building stage, the centralized firm selects the capacity level $K_{\mathrm{T}}^{\mathrm{m}^*}$ to maximize the expected profit:

$$K_{\mathrm{T}}^{\mathrm{m}^*}=\arg\max_{K_{\mathrm{T}}\ge 0}\ \pi^{\mathrm{m}^*}(K_T). \tag{5}$$

The following proposition characterizes the optimal production and capacity decisions.

Proposition 1. **[Centralized Firm]**

i. For given (K_T,θ_1,θ_2), define the two switching curves:

$$\tau_i^{m^*}(K_T,\theta_j)=\begin{cases}2b_iK_T, & \text{if } \theta_j\le 2\hat{b}K_T;\\ \frac{2(b_ib_j-\hat{b}^2)K_T-(b_i-\hat{b})\theta_j}{b_j-\hat{b}}, & \text{if } \theta_j>2\hat{b}K_T.\end{cases}\qquad i,j=1,2,\quad i\ne j. \tag{6}$$

Then, the optimal production quantities of a centralized firm, $q_1^{m^*}(K_T,\theta_1,\theta_2),q_2^{m^*}(K_T,\theta_1,\theta_2)$, are as follows:

$$\begin{cases}\left[\frac{\left(\theta_1b_2-\theta_2\hat{b}\mathbf{1}_{\{\theta_2b_1>\theta_1\hat{b}\}}\right)^+}{2\left(b_1b_2-\hat{b}^2\mathbf{1}_{\{\theta_2b_1>\theta_1\hat{b}\}}\right)},\frac{\left(\theta_2b_1-\theta_1\hat{b}\mathbf{1}_{\{\theta_1b_2>\theta_2\hat{b}\}}\right)^+}{2\left(b_1b_2-\hat{b}^2\mathbf{1}_{\{\theta_1b_2>\theta_2\hat{b}\}}\right)}\right] & \text{if } \theta_1\le\tau_1^{m^*}(K_T,\theta_2) \text{ and } \theta_2\le\tau_2^{m^*}(K_T,\theta_1);\\ \left[\frac{\left(2(b_2-\hat{b})K_T+\min\left((\theta_1-\theta_2),2(b_1-\hat{b})K_T\right)\right)^+}{2(b_1+b_2-2\hat{b})},\frac{\left(2(b_1-\hat{b})K_T+\min\left((\theta_2-\theta_1),2(b_2-\hat{b})K_T\right)\right)^+}{2(b_1+b_2-2\hat{b})}\right], & \text{otherwise.}\end{cases} \tag{7}$$

ii. There exists a unique optimal capacity $K_T^{m^*}$.

Figure 2 illustrates the optimal production policy characterized in Equation (7). In this figure, we can observe that it is optimal to fully utilize the capacity only when the firm gets favorable demand signals. (These areas are marked “binding” for binding capacity in Figure 2). The allocation of the capacity to production of products 1 and 2 depends on the relative values of the demand signals. (The gray areas in Figure 2 are areas where it is optimal to produce just one product, while in the white areas, it is optimal to produce both products.) Finally, Figure 2 also shows both thresholds τ_1,τ_2. The centralized case solution is interesting because it will serve as a benchmark against the decentralized cases with different levels of collaboration.

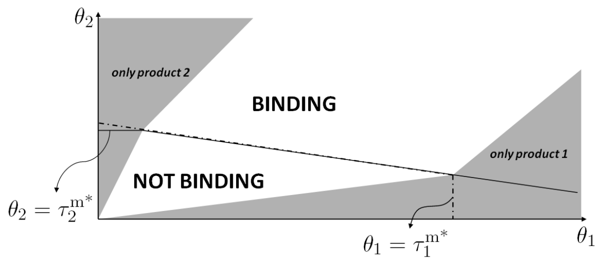


Figure 2 **Optimal production strategy for a centralized firm with respect to demand signals.**

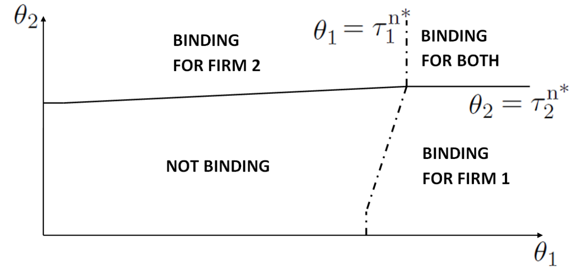


Figure 3 **Equilibrium production strategies when the firms compete in the production subgame, with respect to demand signals.**

4. Analysis of Decentralized Firms under Different Collaboration Scenarios

We now provide an analysis of the different scenarios of collaboration. We first start with the production subgames for each scenario and analyze the two different settings of the second stage production subgame—no collaboration (n) and collaboration (c)—for given capacity endowments and demand signals. We then roll back the outcome of the corresponding subgame to the first stage and determine the equilibrium strategies for each of the four different scenarios.

4.1. Noncollaborative Production

If the firms do not collaborate in the production stage (Nn and Cn scenarios), each firm individually chooses the quantity that maximizes its own revenue. Specifically, the equilibrium production quantities, for a given subgame $\omega = (K_1, K_2, \theta_1, \theta_2)$, must satisfy the following system of equations:

$$q_1^{n^*}(\omega) = \underset{q_1 \in [0, K_1]}{\arg\max}\ q_1 p_1\Big(q_1, q_2^{n^*}(\omega), \omega\Big) \qquad \text{and} \qquad q_2^{n^*}(\omega) = \underset{q_2 \in [0, K_2]}{\arg\max}\ q_2 p_2\Big(q_1^{n^*}(\omega), q_2, \omega\Big). \tag{8}$$

Solving this, we obtain the following equilibrium outcomes:

PROPOSITION 2. **[Subgame Equilibrium in the Nn and Cn Scenarios]** *In a subgame $\omega = (K_1, K_2, \theta_1, \theta_2)$, define the two switching curves $\tau_i^{n^*}(\omega)$:*

$$\tau_i^{n^*}(\omega) \;=\; \begin{cases} 2b_i K_i, & \text{if } \theta_j \le \hat{b} K_i; \\ \frac{(4b_i b_j - \hat{b}^2) K_i + \theta_j \hat{b}}{2b_j}, & \text{if } \hat{b} K_i < \theta_j \le 2b_j K_j + \hat{b} K_i; \\ 2b_j K_j + \hat{b} K_i, & \text{if } 2b_j K_j + \hat{b} K_i < \theta_j. \end{cases} \qquad i,j = 1,2 \quad i \ne j. \tag{9}$$

There exists a unique equilibrium production strategy $\Big(q_1^{n^*}(\omega), q_2^{n^*}(\omega)\Big)$ such that

$$\left(q_1^{n^*}(\omega), q_2^{n^*}(\omega)\right) = \begin{cases} \left(\frac{\left(2\theta_1 b_2 - \theta_2 \hat{b} \mathbf{1}_{\{2\theta_2 b_1 > \theta_1 \hat{b}\}}\right)^+}{4b_1 b_2 - \hat{b}^2 \mathbf{1}_{\{2\theta_2 b_1 > \theta_1 \hat{b}\}}}, \frac{\left(2\theta_2 b_1 - \theta_1 \hat{b} \mathbf{1}_{\{2\theta_1 b_2 > \theta_2 \hat{b}\}}\right)^+}{4b_1 b_2 - \hat{b}^2 \mathbf{1}_{\{2\theta_1 b_2 > \theta_2 \hat{b}\}}}\right), & \textit{if } \theta_1 \leq \tau_1^{n^*}(\omega) \textit{ and } \theta_2 \leq \tau_2^{n^*}(\omega); \\ \left(K_1, \frac{\left(\theta_2 - \hat{b} K_1\right)^+}{2b_2}\right), & \textit{if } \theta_1 > \tau_1^{n^*}(\omega) \textit{ and } \theta_2 \leq \tau_2^{n^*}(\omega); \\ \left(\frac{\left(\theta_1 - \hat{b} K_2\right)^+}{2b_1}, K_2\right), & \textit{if } \theta_1 \leq \tau_1^{n^*}(\omega) \textit{ and } \theta_2 > \tau_2^{n^*}(\omega); \\ (K_1, K_2), & \textit{if } \theta_1 > \tau_1^{n^*}(\omega) \textit{ and } \theta_2 > \tau_2^{n^*}(\omega). \end{cases} \tag{10}$$

Figure 3 illustrates the equilibrium quantities with respect to demand signals, θ_1 and θ_2. If both firms get poor demand signals, their production quantities are low and the initial capacity endowments do not play any role. If only one firm's demand signal is favorable, the firm with the favorable demand signal produces at its capacity. If both firms get favorable demand signals, they both produce at their capacities.

We let $R_i^{\mathrm{n}^*}(\omega)$ denote the revenue that firm i will earn in the sub-game ω such that

$$R_i^{\mathrm{n}^*}(\omega) = q_i^{\mathrm{n}^*}(\omega) p_i\left(q_1^{\mathrm{n}^*}(\omega), q_2^{\mathrm{n}^*}(\omega), \omega\right) \qquad i = 1, 2. \tag{11}$$

4.2. Collaborative Production

If the firms collaborate in the production stage (Cc and Nc scenarios), they jointly set the production quantities and share the total revenue obtained from both products. We assume that firms will decide on the optimal production quantities, and then use Nash bargaining to split the revenues.

To characterize the Nash bargaining solution (NBS), we first need to specify the disagreement payoff for each firm (that is, the payoff that each firm earns if there is no deal). Note that if the firms fail to reach an agreement, each firm chooses the quantity that maximizes its own revenue within its capacity endowment. Hence, the disagreement payoff for firm i is the equilibrium revenue in the no collaboration setting, $R_i^{\mathrm{n}^*}(\omega)$, for a subgame $\omega = (K_1, K_2, \theta_1, \theta_2)$.

The equilibrium quantities and transfer payment of the NBS solve the problem below, and the following proposition characterizes the equilibrium:

$$\max_{\Gamma, q_1, q_2} \quad \left(q_1 p_1(q_1, q_2, \omega) - \Gamma - R_1^{\mathrm{n}^*}(\omega)\right)\left(q_2 p_2(q_1, q_2, \omega) + \Gamma - R_2^{\mathrm{n}^*}(\omega)\right) \tag{12a}$$

$$\text{subsject to} \quad q_1 + q_2 \leq K_1 + K_2 \tag{12b}$$

$$q_1 p_1(q_1, q_2, \omega) - \Gamma \geq R_1^{\mathrm{n}^*}(\omega) \tag{12c}$$

$$q_2 p_2(q_1, q_2, \omega) + \Gamma \geq R_2^{\mathrm{n}^*}(\omega). \tag{12d}$$

PROPOSITION 3. **[Subgame Equilibrium in the Cc and Nc scenarios]** *Suppose that firms with capacity endowments (K_1, K_2) collaborate in the production stage. Then, for given $\omega = (K_1, K_2, \theta_1, \theta_2)$, there exists a unique equilibrium $\left(q_1^{c^*}(\omega), q_2^{c^*}(\omega), \Gamma^{c^*}(\omega)\right)$ such that* *i. the firms produce the same quantities as a centralized firm would:* $q_i^{c^*}(K_1, K_2, \theta_1, \theta_2) = q_i^{m^*}(K_1 + K_2, \theta_1, \theta_2)$, $i = 1, 2$;

ii. the transfer payment (from firm 1 to firm 2) is

$$\Gamma^{c^*}(\omega) \;=\; \frac{q_1^{c^*}(\omega) p_1\Big(q_1^{c^*}(\omega), q_2^{c^*}(\omega), \omega\Big) - R_1^{n^*}(\omega)}{2} - \frac{q_2^{c^*}(\omega) p_2\Big(q_1^{c^*}(\omega), q_2^{c^*}(\omega), \omega\Big) - R_2^{n^*}(\omega)}{2}. \tag{13}$$

Proposition 3 establishes that, in the NBS, the quantities produced by the two collaborating firms are equal to those of a single centralized firm. That is, for any demand signal, there exists a negotiation outcome where no efficiency is lost. In order to make the arrangement mutually beneficial for both firms (i.e., each firm's payoff is no less than its disagreement payoff), the transfer payment $\Gamma^{c^*}(\omega)$ is used to allocate the gains from collaboration, and capacity may have to be reallocated between the firms. Let $\chi^{c^*}(\omega)$ be the net capacity allocated from firm 2 to firm 1 in equilibrium, which is expressed as follows:

$$\chi^{c^*}(\omega) = \begin{cases} \big(q_1^{c^*}(\omega) - K_1\big), & \text{if } q_1^{c^*}(\omega) > K_1; \\ -\big(q_2^{c^*}(\omega) - K_2\big), & \text{if } q_2^{c^*}(\omega) > K_2; \\ 0, & \text{otherwise.} \end{cases} \tag{14}$$

Note that $\chi^{c^*}(\omega)$ is positive if firm 1 borrows capacity, and negative if firm 2 borrows capacity. Figure 4 illustrates how capacity is shared with respect to the demand signals, (θ_1, θ_2).

If both firms get poor demand signals, then each firm can serve its demand with its endowed capacity, hence no capacity reallocation needs to take place. Otherwise, the two firms readily trade the capacity to produce quantities that maximize the total revenue. Note that there are regions under which the entire capacity of one firm is reallocated to the other. This happens when one firm is better off by selling its entire capacity and receiving the transfer payment than by producing and selling its own product. One may argue that demand signals can be private information to the firms. Thus, we have also examined the setting where demand signals θ_i are private information in the noncooperative production game. In other words, the noncooperating production quantity only depends on the firm's own θ_i, not on the other firm's $\theta_j, j \neq i$. We find that with private demand information in the production stage, the firms still produce the same quantities as a centralized firm would (in other words, Proposition 3 still holds), with the only difference being the transfer payment (details can be found in Appendix A).

Note that for given total capacity, the equilibrium quantities depend on the demand signals but not on the individual capacity endowments of each firm. Consequently, for a given total capacity, the revenue a firm earns directly from sales (before the transfer payment) depends only on the demand signals, (θ_1, θ_2). However, the transfer payment (which depends not only on the demand

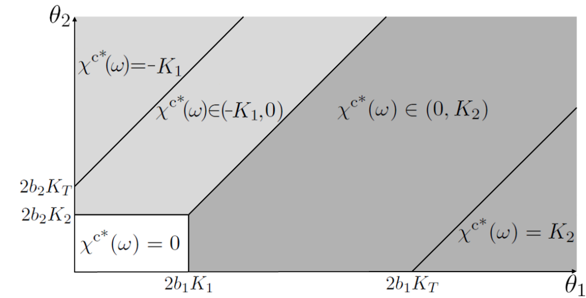


Figure 4 Equilibrium capacity trade with respect to demand signals.

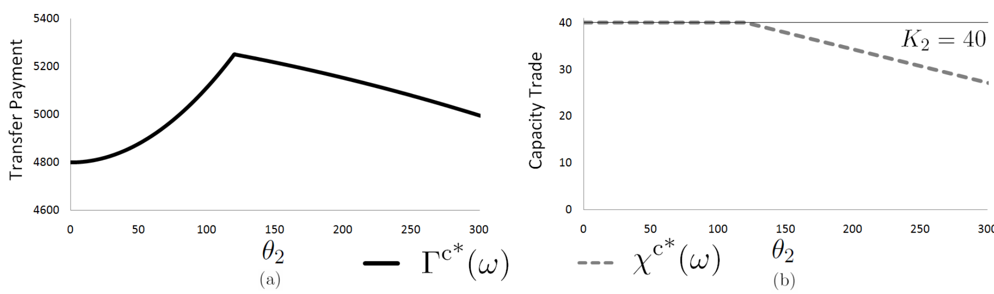


Figure 5 (a) Transfer payment from firm 1 to firm 2, $\Gamma^{c^*}(\omega)$, and, (b) amount of capacity traded from firm 2 to firm 1, $\chi^{c^*}(\omega)$, with respect to the demand signal, θ_2. ($b_1=3$, $b_2=4$, $K_1=K_2=40$, $\theta_1=400$)

signals, but also on the firms' individual capacity endowments K_1, K_2) balances the firms' payoffs according to their initial contributions to the total capacity.

One may expect the transfer payment to be monotone in demand signals or in capacity endowments because capacity becomes more valuable with higher demand. Figure 5 presents the transfer payment from firm 1 to firm 2, $\Gamma^{c^*}(\omega)$, and the amount of capacity that firm 1 acquires from firm 2, $\chi^{c^*}(\omega)$, with respect to the demand signal for firm 2. When the demand signal is poor (when θ_2 is low), firm 2 transfers its whole capacity to firm 1. In this range, as the demand signal for firm 2 becomes more favorable, the disagreement payoff of firm 2 (the opportunity cost for the capacity) increases. Consequently, although the amount of capacity traded remains the same, the transfer payment from firm 1 to firm 2 increases. However, when θ_2 becomes moderate or high, firm 2 finds it optimal to keep some of its capacity to satisfy its own demand. In this range, the amount of capacity that firm 2 trades to firm 1 decreases in θ_2. Consequently, firm 1 receives less capacity in trade and earns less with the traded capacity. Therefore, the transfer payment, $\Gamma^{c^*}(\omega)$, decreases.

To examine how much a unit capacity is worth when it is reallocated, we define the price per unit of reallocated capacity: $\gamma^{c^*}(\omega)=\Gamma^{c^*}(\omega)/\chi^{c^*}(\omega)$, for $\chi^{c^*}(\omega)\neq 0$. The next result shows that although the total transfer payment is not monotone, the unit price of capacity is monotone in demand signals and in capacity endowments when the two products are not substitutes.

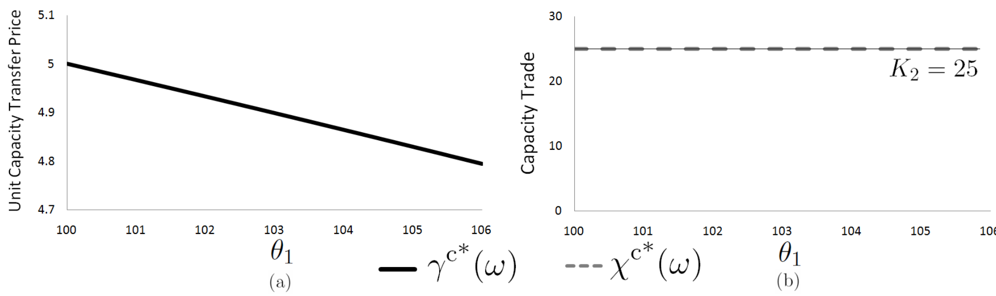


Figure 6 **(a) Unit capacity transfer price, $\gamma^{c^*}(\omega)$, and, (b) Capacity traded from firm 2 to firm 1, $\chi^{c^*}(\omega)$, with respect to the demand of firm 1, θ_1. ($b_1 = b_2 = 2$, $\hat{b} = 1$, $K_1 = K_2 = 25$ and $\theta_2 = 40$)**

PROPOSITION 4. [**Transfer Payment and Price of Capacity**] *Suppose that the two products are not substitutes (i.e., $\hat{b} = 0$). Then,*

i. the price of capacity (per unit), $\gamma^{c^*}(\omega)$, increases in θ_i, and decreases in K_i, $i = 1, 2$;

ii. the transfer payment is nonzero if and only if there is capacity trade: $\Gamma^{c^*}(\omega) \neq 0 \Leftrightarrow \chi^{c^*}(\omega) \neq 0$.

When a firm's demand signal becomes more favorable (i.e., θ_i increases), the market clearing price for its products will be higher. Thus, the firm's per-unit profit margin will increase. This increases the price that it is willing to pay for a unit of capacity. Likewise, when the demand signal for the firm that sells capacity becomes more favorable, the firm's opportunity cost of the capacity increases, which increases the per-unit capacity price it will charge to transfer capacity. Thus, the price of each unit capacity increases when either demand signal becomes more favorable. On the other hand, when either firm starts with large respective capacities, capacity trade becomes less valuable, and hence the transfer price of a unit capacity decreases. We also find that, when the products are not substitutes, a transfer payment is made if and only if there is nonzero capacity trade. This is intuitive, as one would exchange the payments to balance the revenue only when physical capacity is traded.

However, none of these intuitive results holds when the products are substitutes. Firms pay a non-zero transfer payment even when there is no capacity trade. In addition, even when capacity is traded, the unit capacity price is not necessarily increasing as either firm's demand increases. Figure 6 presents an example. In this example, the amount of capacity firm 1 buys from firm 2 is constant, but the price per unit of capacity decreases in the demand signal of firm 1, θ_1. In other words, firm 1 earns more but pays less per unit of capacity it acquires from firm 2. To understand why, first note that firm 2 sells its whole capacity to firm 1 in this case. As θ_1 increases, the price of product 1 increases. Hence, the production quantity of firm 1 also increases as a response to a higher market price. But, as firm 1 increases the quantity, the price of product 2 decreases.

Consequently, the opportunity cost for firm 2's capacity decreases, and, as a result, firm 2 gives up its capacity at a lower price. Note that when the products are not substitutes, this outcome will never happen, because the firm 2's opportunity cost will be independent from firm 1's demand signal.

4.3. Capacity Investment Stage

After solving the production subgame, we next study the capacity investment stage in two different cases, No collaboration (N) and Collaboration (C).

No collaboration

If the firms do not collaborate in the first stage (Nc and Nn scenarios), each firm strategically decides its capacity level to maximize its own expected profit. Therefore, the equilibrium capacity levels $(K_1^{\mathrm{N}\mathfrak{s}*}, K_2^{\mathrm{N}\mathfrak{s}*})$ must satisfy the following system of equations:

$$K_1^{\mathrm{N}\mathfrak{s}*} = \arg\max_{K_1 \geq 0} \; \pi_1^{\mathfrak{s}*}(K_1, K_2^{\mathrm{N}\mathfrak{s}*}) \quad \text{and} \quad K_2^{\mathrm{N}\mathfrak{s}*} = \arg\max_{K_2 \geq 0} \; \pi_2^{\mathfrak{s}*}(K_1^{\mathrm{N}\mathfrak{s}*}, K_2) \quad \text{for } \mathfrak{s} \in \{n, c\}. \tag{15}$$

Collaboration in capacity investment

If the firms collaborate in the first stage (Cc and Cn scenarios), they negotiate to build capacity jointly and share the capacity and its investment costs according to the Nash bargaining solution (NBS). We assume that each firm pays the capital to obtain its initial endowment, that is, firm i pays cK_i to have an endowment of K_i, which is a part of $\pi_k(\cdot)$, defined in Equation (2).

To determine the bargaining outcome, we first specify the disagreement payoff, i.e., what each firm earns if the negotiation fails. Let π_i^{d} denote the disagreement payoff for firm i. For both the Cc and Cn scenarios, if the firms fail to reach an agreement to collaborate in the first stage, then they never collaborate in the subsequent stages. Notice that we are modeling a situation in which firms agree to collaborate in a multi-stage partnership. If the agreement fails during the first stage, then we are assuming that this collaboration will end. We also separately analyze the case where firms agree to collaborate on capacity but not on production Cn and vice versa Nc.

Thus, under a multi-period agreement between parties in the Cc scenario, if the firms fail to reach an agreement at the first stage, each firm will decide its own capacity and production quantity separately to maximize its own profit. Therefore, each firm will then earn equilibrium profits in the Nn scenario, and we have $(\pi_1^{\mathrm{d}}, \pi_2^{\mathrm{d}}) = \Big(\pi_1^{\mathrm{n}*}(K_1^{\mathrm{Nn}*}, K_2^{\mathrm{Nn}*}), \pi_2^{\mathrm{n}*}(K_1^{\mathrm{Nn}*}, K_2^{\mathrm{Nn}*})\Big)$.

If there is a deal, the firms invest in the capacity and obtain the capacity endowments, K_1 and K_2. Then, demand signals are realized, and the firms play the production subgame and earn the revenue as illustrated in Subsections 3.1 and 3.2. Note, however, that the firms agree to a deal if their (ex-ante) profits are at least as large as their disagreement payoffs, $\pi_i^{\mathrm{n}*}(K_1^{\mathrm{Nn}*}, K_2^{\mathrm{Nn}*})$, $i = 1, 2$.

To guarantee this, one firm may need to make a transfer payment to the other firm so that it earns at least its disagreement payoff. Let η be the first-stage transfer payment firm 1 makes to firm 2 to induce an agreement (negative if the actual payment is from firm 2 to firm 1). According to the NBS, the equilibrium outcome, $(K_1^{\mathrm{C}\mathfrak{s}*}, K_2^{\mathrm{C}\mathfrak{s}*}, \eta^{\mathrm{C}\mathfrak{s}*})$, for $\mathfrak{s} \in \{n, c\}$, is the solution to the following optimization problem:

$$\begin{aligned} \max_{\substack{K_1,K_2\geq 0\\ \eta}} \quad & \left(\pi_1^{\mathfrak{s}*}(K_1,K_2) - \eta - \pi_1^{\mathrm{d}}\right)\left(\pi_2^{\mathfrak{s}*}(K_1,K_2) + \eta - \pi_2^{\mathrm{d}}\right) \\ \text{s.t.} \quad & \pi_1^{\mathfrak{s}*}(K_1,K_2) - \eta \geq \pi_1^{\mathrm{d}} \\ & \pi_2^{\mathfrak{s}*}(K_1,K_2) + \eta \geq \pi_2^{\mathrm{d}}. \end{aligned} \tag{16}$$

Notice that the transfer payment, η, plays a role of investment subsidy. Consequently, an equilibrium outcome in which $\eta = 0$ implies that each firm pays only for its own endowment because no firm gives or receives a payment in the first stage.

When the products are not substitutes ($\hat{b} = 0$), all of the results that follow will hold for any continuous demand distribution with positive support. This includes cases in which the demand signals are correlated. However, when the products are substitutes ($\hat{b} > 0$), obtaining analytical results is much more difficult. This is because the derivative of the firm's profit (following the subgame outcome) is discontinuous and makes verifying the second-order condition formidable. To overcome this, we make the following additional assumptions for the case of substitutable products: (i) Θ_1 and Θ_2 are independent, and (ii) Θ_i follows either an uniform or exponential distribution, $i = 1, 2$. Although our results are proven for the two distributions, our computational study shows that the results are still valid with other distributions as well.

Theorem 1. **[Equilibrium Outcome]**

i. In all four scenarios (Nn, Nc, Cn, and Cc), a pure strategy equilibrium exists. Moreover, the equilibrium is unique in the Nn and the Nc scenarios.

ii. When the firms collaborate on capacity investment (Cn or Cc scenarios), the following are true.

(a) *The difference between the firms' equilibrium profits is equal to the difference in their disagreement payoffs:*

$$\left(\pi_1^{\mathfrak{s}*}(K_1^{C\mathfrak{s}*}, K_2^{C\mathfrak{s}*}) - \eta^{C\mathfrak{s}*}\right) - \left(\pi_2^{\mathfrak{s}*}(K_1^{C\mathfrak{s}*}, K_2^{C\mathfrak{s}*}) + \eta^{C\mathfrak{s}*}\right) \;=\; \pi_1^d - \pi_2^d, \text{ for } \mathfrak{s} \in \{n, c\}. \tag{17}$$

(b) *In the Cc scenario, the total capacity in equilibrium is equal to the optimal capacity of a centralized firm:* $K_1^{Cc^*} + K_2^{Cc^*} = K_T^{m^*}$.

Theorem 1 implies that, while capacity collaboration makes both firms better off, the difference in profit remains the same as the difference in their disagreement payoffs. In other words, the negotiation outcome only increases the total surplus without changing the difference. Theorem 1 also establishes that, if the firms collaborate both in the capacity-building and the production stages (Cc scenario), the total capacity is the same as a centralized firm's capacity. In this scenario, the firms not only produce the centrally optimal quantities (Proposition 3) but also agree to build the optimal capacity of a centralized firm. Thus, no efficiency is lost in either stage. The result—that the Cc scenario achieves the centrally optimal solution—is consistent with the existing literature: cooperation usually leads to Pareto optimal solutions and improves profits for both parties. The primary reason lies in the dynamics of Cournot competition (when products are substitutes). In such a scenario, both firms will choose quantities that drive the price down, lowering revenues for both parties. However, in the Cc scenario, both firms can do better than the Cournot outcome even without a transfer payment (see Theorem 2 below) as the Nash bargaining solution guarantees both firms gain at least the profits in the Cournot competition. The benefit of cooperation increases as the substitutability (competition) increases.

An interesting question is how firms pay for the capacity they wish to purchase. Suppose the two firms agree to collaborate and build capacity K_1 and K_2. Do they each pay for the capacity they build or must that one of the firms provide a subsidy to the other? One might expect that each firm's contribution should be proportional to its endowment and that it is subsidy-free: in other words, $\eta^{C\mathfrak{s}*} = 0$. The next result characterizes the condition under which this occurs.

Theorem 2. **[When Do Two Firms Trade Capacity for Free?]** *For the Cc and Cn scenarios, the following are true.*

i. An investment equilibrium is subsidy-free (i.e., $\eta^{C\mathfrak{s}*} = 0$), if and only if the equilibrium endowments $(K_1^{C\mathfrak{s}*}, K_2^{C\mathfrak{s}*})$ satisfy the following:

$$\pi_1^{n^*}(K_1^{C\mathfrak{s}*}, K_2^{C\mathfrak{s}*}) - \pi_2^{n^*}(K_1^{C\mathfrak{s}*}, K_2^{C\mathfrak{s}*}) = \pi_1^d - \pi_2^d \qquad \text{for} \quad \mathfrak{s} \in \{n, c\}. \tag{18}$$

ii. If the products are not substitutes ($\hat{b} = 0$), then there always exists a subsidy-free investment equilibrium. If the products are substitutes ($\hat{b} > 0$), a subsidy-free investment equilibrium exists when the firms collaborate in the second stage (Cc scenario) and $K_T^{Nn^*} \geq K_T^{m^*}$. Otherwise, a subsidy-free investment equilibrium may not exist in general.

The condition in Equation (18) leads to a subsidy-free investment. Notice that the left-hand side is the difference between the profits when the firms compete in the second stage with the endowments $(K_1^{\mathrm{Cs}*}, K_2^{\mathrm{Cs}*})$. The right-hand side is the difference between the disagreement payoffs, same as in the condition in Equation (17). This implies that, under the subsidy-free equilibrium, the difference in profit must be the same regardless of whether they collaborate in the subsequent production stage.

Theorem 2 also implies that when the two products are not substitutes, a subsidy-free equilibrium exists regardless of whether the firms collaborate in the production stage or not (Cc and Cn, respectively). In the Cn scenario, as no capacity sharing occurs in the second stage, there is no gain from joint capacity investment. Hence, each firm agrees to build the endowment that maximizes its own profit, leading to no investment subsidy. On the other hand, in the Cc scenario, the firms share capacity in the second stage, and hence they gain from joint investment in capacity. The second-stage negotiation allocates these gains so that the firms do not need the investment subsidy to select the centrally optimal capacity in the first stage.

On the other hand, when the products are substitutes, a subsidy-free equilibrium exists only when the firms collaborate in both stages (Cc scenario) and the total capacity of a centralized firm is smaller than the total capacity in the Nn scenario (i.e., $K_{\mathrm{T}}^{\mathrm{m}^*} \leq K_{\mathrm{T}}^{\mathrm{Nn}^*}$). Note that when the products are substitutes and the firms collaborate in both stages, the gains of collaboration come not only from pooling capacities but also from pooling demands (i.e., forgoing competitive behavior). The negotiation in the second stage allocates the gains from pooling demands. If the total capacity of a centralized firm is smaller than the total capacity in the Nn scenario, by jointly building the centrally optimal capacity in the first stage, the firms achieve the investment cost savings. Consequently, in the first stage, the firms jointly build the centrally optimal capacity and select the endowments at which the savings from their capacity investment costs are split without an investment subsidy.

When the products are substitutes and the total capacity of a centralized firm is larger than the total capacity in the Nn scenario (i.e., $K_{\mathrm{T}}^{\mathrm{m}^*} > K_{\mathrm{T}}^{\mathrm{Nn}^*}$), a subsidy-free investment equilibrium does not necessarily exist even when the firms collaborate in both stages. Without the investment subsidy, the firms' total capacity might be less than that of a centralized firm. Thus, even if they collaborate in the second stage, the firms could earn less than what a centralized firm would earn for the same realization of demand signals. Therefore, the firms lose efficiency. Hence, one firm finds it beneficial to pay a subsidy to the other firm and induce it to agree on building a larger capacity to guarantee that the revenue in the second stage is the same as the revenue of a centralized firm.

When the products are substitutes and the firms do not collaborate in the second stage (Cn scenario), a subsidy-free investment equilibrium does not exist in general. Even without capacity sharing in the second stage, there still exists a gain from joint investment in capacity for substitutable products. However, because there is no ex-post recourse to resolve the inefficiencies (due to possible imbalance between endowments and demand signals), whether each firm realizes the gain or not depends on its initial endowment. Consequently, an up-front subsidy is generally needed so that the firms select the endowments that maximize the gains of collaboration.

When the firms collaborate in both stages, one might wonder how much will each firm invest in a subsidy-free equilibrium? First, note from Theorem 1 that the total capacity is equal to that of a centralized firm. This implies that $K_2^{\mathrm{Cc}*}$ can be replaced by $K_{\mathrm{T}}^{\mathrm{m}*} - K_1^{\mathrm{Cc}*}$ in Equation (18). Moreover, note that Equation (18) is defined by the disagreement payoffs and $\pi_i^{\mathrm{n}*}(\cdot,\cdot)$, $i = 1,2$. The disagreement payoffs do not depend on the endowments. In addition, $\pi_i^{\mathrm{n}*}(\cdot,\cdot)$ is a well-defined continuous function. Therefore, one can simply solve Equation (18) with a search in single variable in a bounded interval to determine the investment level under a subsidy-free equilibrium.

4.4. Collaboration in Capacity and Partial Collaboration in Production (Cp)

So far, we have analyzed the equilibrium outcomes for four scenarios and show that the firms gain the most if they can fully collaborate in both stages (Cc scenario). However, to achieve this outcome, the two firms must build capacity and set production quantities together. Furthermore, the two firms not only need to make decisions together but they also need to agree in detail on how to split the profit for each contingency. In the previous section, we show that if the two products are substitutes, the firms may exchange the transfer payment even when there is no physical exchange of the capacity. One alternative arrangement is that the firms collaborate on strategic decisions (e.g., building capacity), but each firm individually sets its production quantity, while they trade the capacity endowments if necessary. We call this arrangement the *Cp scenario* (where subscript *p* stands for *partial* collaboration in production).

In this scenario, the firms build joint capacity in the first stage. In the second stage, after the firms observe the demand signals, they trade capacity to establish new endowments. Then, each firm individually decides its production quantity within its new endowment. An example of such collaboration can be found in an arrangement between AMD and Fujitsu for producing flash memory chips (Devine 2003). Under this arrangement, the firms built a plant together (thus collaboratively choosing the total capacity), but each firm individually decided how much to purchase from the plant's output (Plambeck and Taylor 2005). Another such example is the limited joint operating agreement between two newspapers: the *Detroit Free Press* and *Detroit News*. Under this arrangement, the newspapers operate separately but are printed in the same, jointly built facility (Busterna and Picard 1993).

Once again, we solve for the equilibrium outcome using backward induction. Let $\hat{K}_i$, $i=1,2$, be the new endowment of firm i after the capacity trade, and let $\hat{\omega}$ be the vector that represents the new capacity endowments and demand signals, $\hat{\omega} := (\hat{K}_1, \hat{K}_2, \theta_1, \theta_2)$. For each realization of $\hat{\omega}$, the equilibrium production quantities must satisfy the following set of equations:

$$q_1^{\mathrm{Cp}^*}(\hat{\omega}) = \underset{q_1 \in [0,\hat{K}_1]}{\arg\max}\ q_1 p_1\Big(q_1, q_2^{\mathrm{Cp}^*}(\hat{\omega}), \hat{\omega}\Big) \quad \text{and} \quad q_2^{\mathrm{Cp}^*}(\hat{\omega}) = \underset{q_2 \in [0,\hat{K}_2]}{\arg\max}\ q_2 p_2\Big(q_1^{\mathrm{Cp}^*}(\hat{\omega}), q_2, \hat{\omega}\Big). \tag{19}$$

Note that the equilibrium outcome of this quantity-setting game is identical to the outcome of the Nn scenario (described in Equation (8)) except that $(\hat{K}_1, \hat{K}_2)$ replaces (K_1, K_2). Therefore, by Equation (11), firm i earns the revenue $R_i^{\mathrm{n}^*}(\hat{\omega})$ from sales. Given this, we can consider the preceding capacity trading game for given initial endowments (K_1, K_2) and demand signals (θ_1, θ_2). As before, let Γ be the transfer payment that firm 1 pays to firm 2 (negative if firm 2 pays to firm 1). Thus, if the firms agree on a deal, firm i earns $R_i^{\mathrm{n}^*}(\hat{K}_1, \hat{K}_2, \theta_1, \theta_2) + (-1)^i \Gamma, i = 1, 2$.

If the firms fail to reach a deal, each firm individually decides its production quantity using its initial endowment, K_i, as the capacity constraint. Hence, the disagreement payoff of firm i is $R_i^{\mathrm{n}^*}(K_1, K_2, \theta_1, \theta_2)$, $i = 1, 2$, and the NBS is a solution to the following problem:

$$\begin{aligned}
\max_{\Gamma, \hat{K}_1, \hat{K}_2}\ & \Big(R_1^{\mathrm{n}^*}(\hat{K}_1, \hat{K}_2, \theta_1, \theta_2) - \Gamma - R_1^{\mathrm{n}^*}(K_1, K_2, \theta_1, \theta_2)\Big)\Big(R_2^{\mathrm{n}^*}(\hat{K}_1, \hat{K}_2, \theta_1, \theta_2) + \Gamma - R_2^{\mathrm{n}^*}(K_1, K_2, \theta_1, \theta_2)\Big) \\
\text{s.t.}\ & \hat{K}_1 + \hat{K}_2 = K_1 + K_2 \\
& R_1^{\mathrm{n}^*}(\hat{K}_1, \hat{K}_2, \theta_1, \theta_2) - \Gamma \geq R_1^{\mathrm{n}^*}(K_1, K_2, \theta_1, \theta_2) \\
& R_2^{\mathrm{n}^*}(\hat{K}_1, \hat{K}_2, \theta_1, \theta_2) + \Gamma \geq R_2^{\mathrm{n}^*}(K_1, K_2, \theta_1, \theta_2) \\
& \hat{K}_1, \hat{K}_2 \geq 0.
\end{aligned}$$

The first constraint implies that the total capacity will remain the same before and after the trade. The remaining two constraints guarantee that both firms' payoffs must be improved after negotiation. We find that when the products are not substitutes, in the Cp scenario, the equilibrium total capacity is centrally optimal (equal to that of a centralized firm). We characterize the full equilibrium outcome and render the discussions of the Cp scenario in the Appendix (Section B).

One might also be interested in the case where firms with initial capacity endowments (K_1, K_2) trade capacity before the realization of demand signals (with transfer payment a linear function of the transferred capacity); and then, during the production stage, each firm maximizes their own expected revenue to decide the production quantity. To further distinguish this scenario from other settings we considered in the paper, we draw the timelines below in Figure 7. The two firms thus solve the following problem:

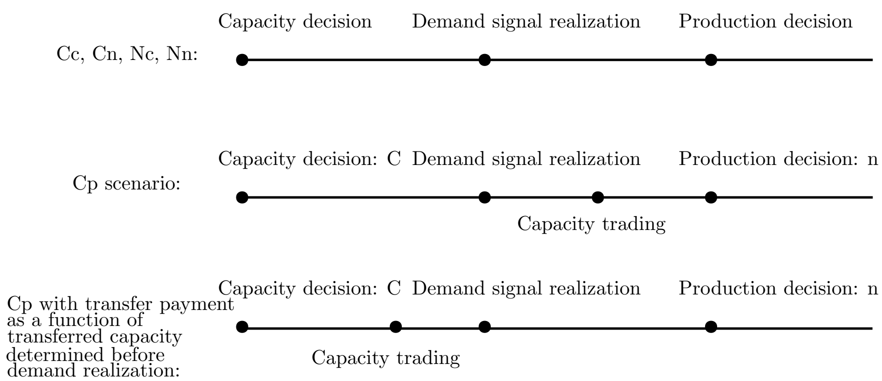


Figure 7 **Model timelines.**

$$
\begin{aligned}
\max_{\gamma,\hat{K}_1,\hat{K}_2} \quad & \left(\pi_1^{n*}(\hat{K}_1,\hat{K}_2)-\gamma(\hat{K}_1-K_1)-\pi_1^{\mathrm{d}}\right)\left(\pi_2^{n*}(\hat{K}_1,\hat{K}_2)+\gamma(K_2-\hat{K}_2)-\pi_2^{\mathrm{d}}\right) \\
\text{s.t.} \quad & \hat{K}_1+\hat{K}_2 = K_1+K_2 \\
& \pi_1^{n*}(\hat{K}_1,\hat{K}_2)-\gamma(\hat{K}_1-K_1)\geq \pi_1^{\mathrm{d}} \\
& \pi_2^{n*}(\hat{K}_1,\hat{K}_2)+\gamma(K_2-\hat{K}_2)\geq \pi_2^{\mathrm{d}} \\
& \hat{K}_1,\hat{K}_2\geq 0.
\end{aligned} \tag{20}
$$

The solution to the problem (20) is $\hat{K}_1^*=K_1^{Cn*}, \hat{K}_2^*=K_2^{Cn*}$ and $\gamma^*=\frac{\pi_1^{n*}(\hat{K}_1^*,\hat{K}_2^*)-\pi_1^{\mathrm{d}}}{2(\hat{K}_1^*-K_1)}-\frac{\pi_2^{n*}(\hat{K}_1^*,\hat{K}_2^*)-\pi_2^{\mathrm{d}}}{2(K_2-\hat{K}_2^*)}$. All the results related to the Cn scenario apply to this new setting.

5. Comparison of Equilibrium Capacities

In the previous section, we solved for the equilibrium capacity in five different scenarios. We show that in some cases, collaboration leads to a centrally optimal outcome. For example, if the two firms can fully collaborate in both stages (Cc scenario), the equilibrium joint capacity level is equal to the optimal capacity of a centralized firm. Similarly, if the two products are not substitutes (i.e., $\hat{b}=0$), the equilibrium joint capacity level in the Cp scenario is also equal to the optimal capacity of a centralized firm. However, in all other scenarios, some efficiency is lost and the equilibrium capacity level deviates from the capacity of a centralized firm. The next proposition compares the total equilibrium capacity levels under different scenarios.

Theorem 3. **[Comparison of Equilibrium Capacities]**

A. When the products are not substitutes (i.e., $\hat{b}=0$), the following results hold:

(i) $K_T^{Cn^*}=K_T^{Nn^*}$, *(ii)* $\min(K_T^{Cc^*},K_T^{Nn^*})\leq K_T^{Nc^*}\leq \max(K_T^{Cc^*},K_T^{Nn^*})$, *(iii)* $K_T^{Cp^*}=K_T^{Cc^*}$.

B. When the products are substitutes (i.e., $\hat{b}>0$), the following results hold:

(i) $K_T^{Cn^*}\leq K_T^{Nn^*}$, *(ii)* $\min(K_T^{Nn^*},K_T^{Cc^*})\leq K_T^{Nc^*}$, *(iii)* $K_T^{Cp^*}<K_T^{Cc^*}$.

Parts A(i) and B(i) of Theorem 3 compare the total capacity in the two scenarios—Nn and Cn. If the two products are not substitutes (part A(i)), the total capacity in the Cn scenario (i.e., collaborating in the capacity game, but not collaborating in the production subgame) is the same as that of the Nn scenario (i.e., not collaborating in both games). In other words, in terms of the total capacity, not collaborating in the production subgame is the same as not collaborating at all. On the other hand, if the products are substitutes, part B(i) implies that the Cn scenario yields smaller capacity than the Nn scenario, although the firms do not lend or borrow capacity from each other in the production subgame, $K_{\mathrm{T}}^{\mathrm{Cn}*} \le K_{\mathrm{T}}^{\mathrm{Nn}*}$. This is because, although no capacity is physically shared in the second stage, jointly deciding the total capacity together curbs the downstream competition by making capacity more scarce. Recall that in part B(i), products are substitutes whereas in part A(i), they are not, and it is interesting that collaborating in capacity investments without any production coordination and therefore, implicitly, pricing coordination results in decreasing competition when products are substitutes.

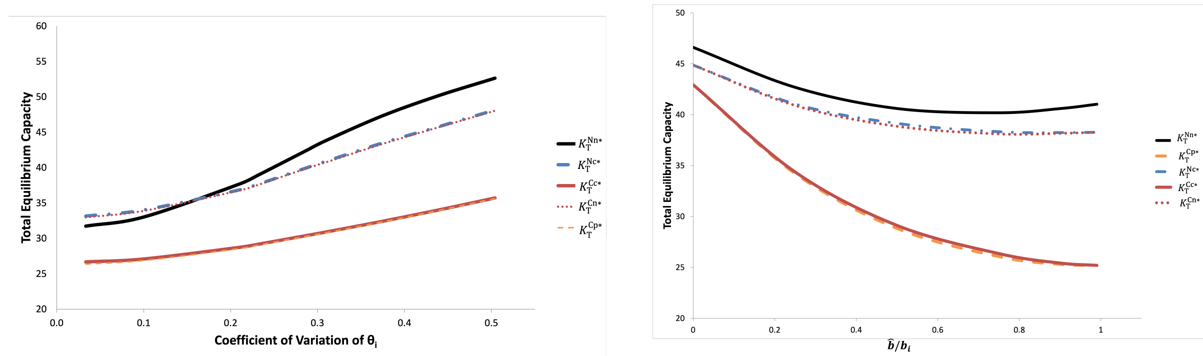
Parts A(ii) and B(ii) compare the total capacity in the Nc scenario with those in the Nn and Cc scenarios. If the products are not substitutes, the total capacity in the Nc scenario is always between $K_{\mathrm{T}}^{\mathrm{Cc}*}$ and $K_{\mathrm{T}}^{\mathrm{Nn}*}$. To see why, recall that firm i selects its capacity to maximize $\pi_i^{\mathrm{c}*}(K_1, K_2)$ in the Nc scenario. Note that from Proposition 3, we have

$$\pi_i^{\mathrm{c}*}(K_1, K_2) = \frac{1}{2}\Big(\pi^{\mathrm{m}*}(K_1 + K_2) + \pi_i^{\mathrm{n}*}(K_1, K_2) - \pi_j^{\mathrm{n}*}(K_1, K_2)\Big) \qquad i, j = 1, 2 \quad i \ne j.$$

Because the products are not substitutes, for given K_j, the disagreement payoff of firm j, $\pi_j^{\mathrm{n}*}(K_1, K_2)$, is independent of firm i's capacity, K_i. Thus, firm i chooses the capacity that maximizes $0.5\pi^{\mathrm{m}*}(\cdot) + 0.5\pi^{\mathrm{n}*}{}_i(\cdot)$. Notice that if each firm wants to maximize $\pi^{\mathrm{m}*}(\cdot)$, then this results in the total capacity of $K_{\mathrm{T}}^{\mathrm{Cc}*}$. On the other hand, each firm maximizing $\pi^{\mathrm{n}*}(\cdot)_i$ will result in the total of $K_{\mathrm{T}}^{\mathrm{Nn}*}$. Consequently, $K_{\mathrm{T}}^{\mathrm{Nc}*}$ falls between between $K_{\mathrm{T}}^{\mathrm{Cc}*}$ and $K_{\mathrm{T}}^{\mathrm{Nn}*}$.

However, this result is no longer true when the products are substitutes. Although we can establish $K_{\mathrm{T}}^{\mathrm{Nc}*} \ge \min(K_{\mathrm{T}}^{\mathrm{Cc}*}, K_{\mathrm{T}}^{\mathrm{Nn}*})$, $K_{\mathrm{T}}^{\mathrm{Nc}*}$ can exceed the maximum of $K_{\mathrm{T}}^{\mathrm{Cc}*}$ and $K_{\mathrm{T}}^{\mathrm{Nn}*}$. Figure 8a illustrates an example: $K_{\mathrm{T}}^{\mathrm{Nc}*} > K_{\mathrm{T}}^{\mathrm{Nn}*}$ when demand variability is low (the coefficient of variation is less than 0.35) and $K_{\mathrm{T}}^{\mathrm{Nc}*} < K_{\mathrm{T}}^{\mathrm{Nn}*}$ otherwise.

When the demand variability becomes smaller, the chances that one firm needs to borrow capacity from the other firm decrease. However, larger capacity can still be beneficial to the firms when products are substitutes. Note that the disagreement payoff of firm i's competitor, $\pi_j^{\mathrm{n}*}(K_1, K_2)$, decreases in K_i (see Lemma 2 in the appendix). Hence, increasing the firm's initial endowment can weaken the competitor's bargaining position and improve that of the firm. Consequently, both firms try to get an edge in the negotiation that will occur in the production subgame and end up building larger capacity in the capacity subgame than they would otherwise and this is why the



(a) Total capacity versus coefficient of variation.

(b) Total capacity versus $\hat{b}/b_i$ ratio.

Figure 8 Equilibrium capacity.

total capacity in the Nc scenario exceeds the total capacity in the Nn scenario. On the other hand, when demand becomes highly variable, the opposite effect prevails. When demands are highly variable, the value of pooling and sharing the capacity is high, resulting in smaller total capacity.

Finally, parts A(iii) and B(iii) compare the equilibrium capacity in the Cp and the Cc scenarios. One would expect the total capacity to be higher with increased competition, i.e., it may be reasonable to expect that $K_{\mathrm{T}}^{\mathrm{Cp}^*} \geq K_{\mathrm{T}}^{\mathrm{Cc}^*}$. However, surprisingly, Theorem 3 shows that the opposite is true. To understand why, first consider the case where each firm has ample capacity. Then, in the Cp scenario, for most demand realizations, each firm can satisfy its demand with its own capacity. In this case, it is unlikely that the two firms will trade capacity, thus they will be competing in the production subgame, deviating from producing the centrally optimal quantities. On the other hand, this adverse effect can be prevented if the firms do not build too much capacity in the first place. Therefore, in the first stage, the firms anticipate this outcome and build smaller joint capacity and reduce the intensity of competition.

We further examine the effects of demand uncertainty and substitutability on the equilibrium total capacities. The results are illustrated in Figures 8a and 8b ($\mu_1 = \mu_2 = 600$, $b_1 = b_2 \in \{5, \ldots, 35\}$, $\hat{b}/b_i \in \{0, \ldots, 0.99\}$, and $c \in \{20, \ldots, 160\}$). As the substitutability increases ($\hat{b}/b_i$ ratio increases), the total optimal centralized capacity K_T^{m*} (K_T^{Cc*} or K_T^{Cp*} in the figure) decreases, and the deviation of the total capacities of firms in situations in which they compete in either the capacity planning stage (N) or the production stage (n) from K_T^{m*} increases. This implies that the benefit of collaboration increases if the products are more similar to each other, which is also consistent with the profit improvement results illustrated in Figure 9 below. Keeping everything else the same, a higher level of substitutability (competition) makes it less desirable for firms to increase capacity to make production because the marginal benefit of increasing capacity decreases as $\hat{b}/b_i$

increases. As a result, it is optimal to build less capacity when the two firms collaborate. Similarly, in Figure 8a, we observe that the deviation from the centralized optimal capacity becomes larger as the demand variability increases. As the demand variability increases, firms must prepare more capacity to hedge against the variability. However, they must prepare even more capacity in the non-collaborating settings. This shows the potential benefit of collaborating in both stages as the demand variability increases.

6. Computational Study

We conduct a computational study to gain further managerial insights into the benefit of capacity collaboration. In particular, we aim to (1) measure the gains the firms can achieve via collaboration, (2) find out how the gains change in the level and the form of collaboration, and (3) assess the impacts of business parameters (such as costs, demand variability, etc.) on the gains from collaboration.

For this, we compare the firms' performances in five scenarios: Cc, Cp, Cn, Nc, and Nn. To examine the effects of the business parameters on the gains from collaboration, we systematically vary cost and demand parameters. Specifically, the following combinations of parameters (in 2,268 problem instances) are used in the computational study:

- $b_i \in \{5, 10, \ldots, 35\}$,
- $\hat{b}/b_i \in \{0, 0.25, 0.5, 0.75, 0.99\}$,
- $\mathrm{CV}_i = \sigma_i/\mu_i \in [0.03, 0.50]$,
- $c \in \{20, 40, \ldots, 120\}$.

Throughout the experiments, we use several distributions—uniform, truncated normal, triangular, etc.—with $E[\theta_i] = 600$, $i = 1, 2$. To measure the gains from collaboration in different scenarios, we compute the percentage improvement in total profit over the profit in the no-collaboration scenario:

$$\mathcal{I}^{\mathrm{Cc}} = \frac{\Pi_{\mathrm{T}}^{\mathrm{Cc}*} - \Pi_{\mathrm{T}}^{\mathrm{Nn}*}}{\Pi_{\mathrm{T}}^{\mathrm{Nn}*}} \times 100, \quad \mathcal{I}^{\mathrm{Cp}} = \frac{\Pi_{\mathrm{T}}^{\mathrm{Cp}*} - \Pi_{\mathrm{T}}^{\mathrm{Nn}*}}{\Pi_{\mathrm{T}}^{\mathrm{Nn}*}} \times 100, \quad \mathcal{I}^{\mathrm{Cn}} = \frac{\Pi_{\mathrm{T}}^{\mathrm{Cn}*} - \Pi_{\mathrm{T}}^{\mathrm{Nn}*}}{\Pi_{\mathrm{T}}^{\mathrm{Nn}*}} \times 100, \quad \mathcal{I}^{\mathrm{Nc}} = \frac{\Pi_{\mathrm{T}}^{\mathrm{Nc}*} - \Pi_{\mathrm{T}}^{\mathrm{Nn}*}}{\Pi_{\mathrm{T}}^{\mathrm{Nn}*}} \times 100.$$

The descriptive statistics are summarized in Table 2.

We observe that the overall gains from collaboration are significant except for the Cn scenario. In particular, the performance in the Cp scenario (where the firms invest in capacity together and decide jointly whether to trade capacity but independently on the production quantities) is very close to the performance in the Cc (full collaboration) scenario. We also observe that the gain is significant when the firms collaborate only in the production stage (Nc scenario): the total profit increases by 14.4% on average. On the other hand, if the firms cannot collaborate in the production stage, they do not gain much even when they build the capacity together (4.5% on

	$\mathcal{I}^{\mathrm{Cc}}$	$\mathcal{I}^{\mathrm{Cp}}$	$\mathcal{I}^{\mathrm{Cn}}$	$\mathcal{I}^{\mathrm{Nc}}$
Mean	20.45%	20.34%	4.50%	14.36%
Standard Deviation	17.78%	17.80%	5.35%	14.50%
Minimum	0.05%	0.05%	0.00%	0.02%
25^{th} Percentile	5.41%	5.24%	0.17%	2.87%
Median	15.37%	15.13%	2.24%	9.62%
75^{th} Percentile	31.31%	31.29%	7.47%	21.99%
Maximum	72.79%	72.78%	30.62%	62.60%

Table 2 Summary statistics for improvements in a dataset of 2,268 problem instances.

average). To understand why, notice that the investment decisions are made before observing the demand signals, but production decisions are made afterwards. Therefore, even if the firms build capacities close to the ideal level, they cannot capture most of the potential gain if they do not use shared capacity to respond to demand variability.

6.1. The Impact of Business Parameters

For each scenario, we examine the change in the performance with respect to the changes in the problem parameters. These results are presented in Figures 9a (substitutability), 9b (capacity cost) and 10 (demand variability). Figure 9(a) plots the percentage improvements with respect to $\hat{b}/b_i$. Note that b_i is the elasticity and $\hat{b}$ is the cross-elasticity of the inverse demand function. Therefore, the ratio of the two, $\hat{b}/b_i$, measures the degree of substitutability, or the intensity of competition between the two firms (Lus and Muriel 2009). We observe that the percentage improvement increases in the ratio $\hat{b}/b_i$, which implies that the gains from collaboration increase as the competition becomes more intense under all scenarios. On the other hand, we observe that the performance gaps between the scenarios where the firms collaborate only in one stage—the Cn and Nc scenarios—and the scenarios where the firms collaborate in both stages—the Cc and Cp scenarios—also increase. In the Cc scenario, the firms completely coordinate the decisions. In the Cp scenario, the firms compete in quantity, but they are able to trade the capacity. As the substitutability increases, the value of collaborating in both stages (Cc or Cp scenarios) increases. On the other hand, in the Cn and Nc scenarios, the firms collaborate in one stage but compete in the other stage. Thus, although the gains in the Cn and Nc scenarios increase in substitutability, the gain is much less than the gain in the Cc or Cp scenarios.

Figure 9b shows that the percentage gains increase in the unit capacity building cost. When capacity becomes more expensive, the firms build smaller capacity. When the firms collaborate in the production stage (Cc, Cp, and Nc scenarios), they utilize the limited capacities more efficiently. Therefore, in these scenarios, the gains from collaboration increase significantly when the unit

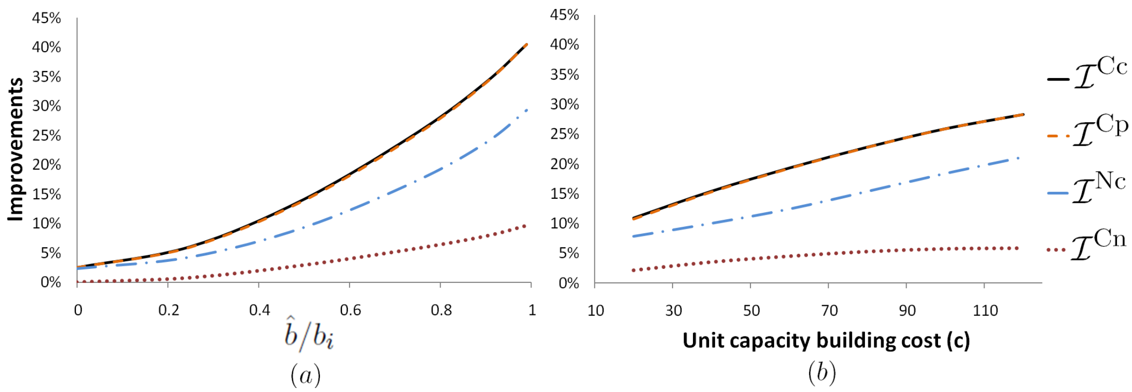


Figure 9 **Improvements versus (a) $\hat{b}/b_i$ ratio, and (b) unit capacity building cost c.**

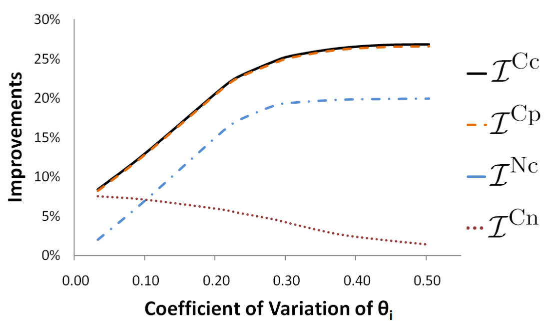


Figure 10 **Improvements versus the coefficient of variation of demand.**

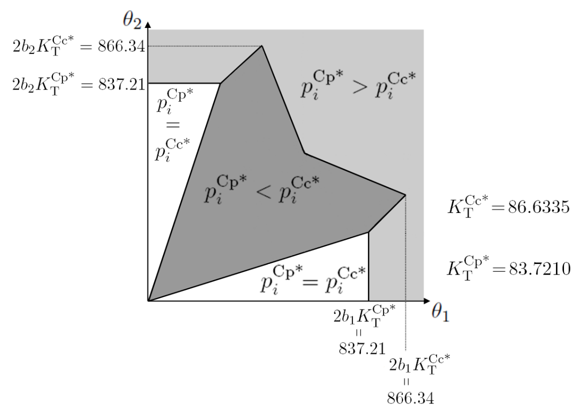


Figure 11 **Regions on $(\theta_1 \times \theta_2)$ space that determine whether the prices in the Cp scenario are larger than the prices in the Cc scenario in equilibrium.**

capacity building cost increases. On the other hand, in the Cn scenario, the gain slightly increases first, but the rate of increase diminishes as the cost increases. The firms are already building smaller capacities in the Nn scenario when the capacity cost is high, the gain in the Cn scenario is small and the growth diminishes with higher capacity cost.

Figure 10 illustrates how the gains change in demand variability. Overall, the gains increase in demand variability when the firms collaborate in the production stage (Cc, Cp, and Nc scenarios). If the firms do not collaborate in the production stage at all, however, the gain from collaboration on capacity alone (Cn scenario) decreases in demand variability. To see why, note that for given capacity endowments, as the demand variability increases, the probability that at least one firm is short of capacity increases as well. Thus, collaborating on production after observing the demand signals (reactive collaboration) becomes more valuable.

6.2. Full versus Partial Collaboration on Production and Implications on Price

Recall that in the Cp scenario, each firm sets its own production quantity and the firms exchange capacity once they observe demand signals. Therefore, one might expect that the increased competition compared to the full collaboration (Cc) scenario will force the firms to produce larger quantities resulting in lower prices. However, when we compare the equilibrium quantities and prices between the Cc and the Cp scenarios (see Figure 11, $b_1 = b_2 = 10$, $\hat{b} = 2.5$, $c = 20$, $\mu_1 = \mu_2 = 600$), we see that the exact opposite can happen. In fact, the possibility that coordinated production can result in a lower price for customers might explain why many joint investment and production coordination activities are permitted under anti-trust regulations. When demand signals for both products are unfavorable to moderately favorable, the price in the Cp scenario is strictly lower. When one firm observes a favorable demand signal and the other firm gets a poor one, the price in the Cp scenario is equal to the price in the Cc scenario. In other cases, however, the price in the Cp scenario is strictly higher than the price in the Cc scenario. To understand why, first note that, although the firms eventually engage in quantity competition in the Cp scenario, they trade the capacity when doing so is mutually beneficial. Thus, the firms engage in the full level of competition only when both firms have sufficient capacity (i.e., when they get demand signals that are unfavorable to moderately favorable). Otherwise, the capacity becomes binding. Recall that, firms build smaller joint capacity in the Cp scenario (Proposition 3). Therefore, when the capacity is binding under both the Cp and the Cc scenarios, the total output is smaller under the Cp scenario. As a result, the consumers pay higher prices.

7. Discussion and Conclusions

In this paper, we considered collaboration between two competing firms. Specifically, firms collaborate on both capacity building and/or production decisions. We considered several different collaboration scenarios and examined the resulting equilibrium outcomes.

We find that if the firms can fully collaborate on both capacity and production decisions (Cc scenario), they can achieve the centrally optimal outcome in equilibrium. In other words, no efficiency is lost. Moreover, compared to the scenario in which the firms do not collaborate at all, the gains from collaboration with a competitor can be substantial. This supports the joint venture agreements between competing firms, in which the firms jointly make decisions under a separate economic entity. However, for a joint venture agreement to be sustainable, the firms might need to agree on investment subsidies and on a detailed transfer payment schedule. Interestingly, we find that an investment scheme that is proportional to the capacity endowment structure (e.g., one firm owns 60% of capacity and the other owns 40% of capacity in a 60 - 40 joint venture) might not always be an equilibrium, and subsidies from one firm to the other could be necessary to achieve mutually beneficial collaboration.

We also study an arrangement where the firms jointly build capacity but still compete in production after trading their capacity endowments (Cp scenario). We find that most of the benefits from full collaboration can be captured in the Cp scenario. This coordination not only benefits firms but can also benefit consumers because prices will be lower than the centralized case and can even be lower than if no coordination took place at all. The sensitivity of the gains of collaboration with respect to different parameters implies that overall gains increase when (1) the products are more substitutable, (2) capacity is more costly to build, and (3) demand is more variable.

For future research, it would be interesting to consider collaboration structures other than the ones we consider in this paper. For instance, we assume that when the firms collaborate in the operational stage, they negotiate contract terms such as the capacity trade and the transfer payment. This causes the contract terms to be contingent on the demand signals (these contracts are labeled as *incomplete contracts* by Van Mieghem (1999)). One extension is to consider a simpler contract that can be agreed upon before demand signals are observed. It will be interesting to see how much of the benefit can be captured through such a simple mechanism.

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E-companion: Online Appendix

Appendix A: Private information

We first solve the production game in a noncooperative setting, of which the solution also serves as the disagreement value in the cooperative game (Nash bargaining setting). We let $K \triangleq (K_1, K_2)$ and μ_{θ_i} denote the expectation of $\theta_i, i = 1, 2$. Thus, we need to solve (revising (8))

$$q_1^{n^*}(K,\theta_1) = \arg \max_{q_1 \in [0,K_1]} q_1 \mathbf{E}_{\theta_2} \left(\theta_1 - b_1 q_1 - \hat{b} q_2^{n^*}(K,\theta_2) \right),$$

together with

$$q_2^{n^*}(K,\theta_2) = \arg \max_{q_2 \in [0,K_2]} q_2 \mathbf{E}_{\theta_1} \left(\theta_2 - b_2 q_2 - \hat{b} q_1^{n^*}(K,\theta_1) \right).$$

We further define the expected revenues to be

$$R_i^{n^*}(K,\theta_i) \triangleq q_i^{n^*}(K,\theta_i) \mathbf{E}_{\theta_j} \left(\theta_i - b_i q_i^{n^*}(K,\theta_i) - \hat{b} q_j^{n^*}(K,\theta_j) \right).$$

The solution satisfies

$$q_1^{n^*}(K,\theta_1) = \min \left\{ \frac{\theta_1 - \hat{b}\mathbf{E}_{\theta_2}(q_2^{n^*}(K,\theta_2))}{2b_1}, K_1 \right\},$$
$$q_2^{n^*}(K,\theta_2) = \min \left\{ \frac{\theta_2 - \hat{b}\mathbf{E}_{\theta_1}(q_1^{n^*}(K,\theta_1))}{2b_2}, K_2 \right\}.$$

For given $R_i^{n^*}(K,\theta_i)$, we solve the production Nash bargaining solution as follows

$$\begin{aligned}
&\max\nolimits_{\Gamma,q_1,q_2} \left(q_1 \left(\theta_1 - b_1 q_1 - \hat{b} q_2 \right) - \Gamma - R_1^{n^*}(K,\theta_1) \right) \left(q_2 \left(\theta_2 - b_2 q_2 - \hat{b} q_1 \right) + \Gamma - R_2^{n^*}(K,\theta_2) \right) \\
&\text{subject to } q_1 + q_2 \le K_1 + K_2, \\
&\qquad q_1 \left(\theta_1 - b_1 q_1 - \hat{b} q_2 \right) - \Gamma \ge R_1^{n^*}(K,\theta_1), \\
&\qquad q_2 \left(\theta_2 - b_2 q_2 - \hat{b} q_1 \right) + \Gamma \ge R_2^{n^*}(K,\theta_2).
\end{aligned} \tag{21}$$

With private demand information in the production stage, the firms still produce the same quantities as a centralized firm would, with the only difference in the transfer payment defined below:

$$\Gamma^{c^*}(\omega) = \frac{q_1^{c^*}(\omega) p_1 \left(q_1^{c^*}(\omega), q_2^{c^*}(\omega), \omega \right) - R_1^{n^*}(K,\theta_1)}{2} - \frac{q_2^{c^*}(\omega) p_2 \left(q_1^{c^*}(\omega), q_2^{c^*}(\omega), \omega \right) - R_2^{n^*}(K,\theta_2)}{2}. \tag{22}$$

We note that in Nash bargaining solutions, players always **cooperate** to jointly maximize an objective function knowing each other's parameters but the disagreement value can be defined with private information. Thus, in (21), the two firms decide each other's **cooperative** production quantities and the disagreement value is calculated based on maximizing the expected revenue given the other firm's demand signal to be unknown.

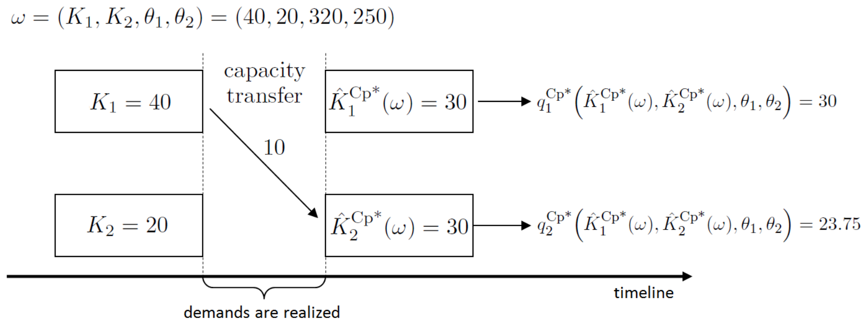

Figure 12 Equilibrium capacity trade and production under the Cp scenario. ($b_1 = b_2 = 4$, $\hat{b} = 2$)

Appendix B: Cp scenario

The next result characterizes the properties of the equilibrium outcome in the Cp scenario.

Proposition 5. [**Trading and Production Outcomes in the Cp Scenario**]

i. There exists an equilibrium in the capacity trading game. If $\hat{K}_i = K_i$, $i = 1, 2$ in equilibrium, then it must be $\Gamma^{Cp*}(\omega) = 0$ for any $\omega = (K_1, K_2, \theta_1, \theta_2)$.

ii. If the products are not substitutes (i.e., $\hat{b} = 0$), the equilibrium quantities in the subsequent production game are the same as those of a centralized firm with capacity $K_1 + K_2$.

Note that the equilibrium outcome described by Proposition 5 and equation (19) is different from the equilibrium outcome in a subgame under full collaboration (i.e., Cc scenario). For instance, unlike the Cc scenario, the firms exchange the transfer payment only if there is a physical trade of capacity. To see why, notice that, in the Cp scenario, each firm chooses the quantity that maximizes its own revenue. Therefore, if there is no capacity trade, each firm's revenue would be equal to its disagreement payoff, and hence there would be no transfer payment.

Another interesting outcome is about how the shared capacity is used. In the Cp scenario, one firm may buy capacity from the other firm and idle the purchased capacity: In Figure 12, we observe that firm 2 buys 10 units of capacity from firm 1, but uses only 3.75 units in production. Although such an outcome is counterintuitive at the first glance (why does it pay for capacity and waste some?), firm 2 actually gains more by reducing the intensity of competition by limiting firm 1's production. At the same time, firm 1 also benefits from selling a portion of capacity to firm 2 and limiting the competition.

After substituting the equilibrium outcome in the second stage, we write the equilibrium payoff of firm i, $R_i^{\mathrm{Cp}*}(\omega)$ for a given $\omega = (K_1, K_2, \theta_1, \theta_2)$ as follows:

$$R_i^{\mathrm{Cp}*}(\omega) = R_i^{\mathrm{n}*}\left(\hat{K}_1^{\mathrm{Cp}*}(\omega), \hat{K}_2^{\mathrm{Cp}*}(\omega), \theta_1, \theta_2\right) + (-1)^i \Gamma^{\mathrm{Cp}*}(\omega). \tag{23}$$

Now let us consider the first stage game. If two firms choose the initial endowments (K_1, K_2), the expected profit of firm i is $\pi_i^{\mathrm{Cp}*}(K_1, K_2) = \mathbb{E}\left[R_i^{\mathrm{Cp}*}(K_1, K_2, \Theta_1, \Theta_2)\right] - cK_i$. As in the analysis of the Cc scenario, an (upfront) investment subsidy might be needed for the firms to agree on a deal. Putting them altogether, the equilibrium capacity endowments, $(K_1^{\mathrm{Cp}^*}, K_2^{\mathrm{Cp}^*})$, and investment subsidy, η^{Cp^*}, are given by an optimization problem analogous to the one in (16). The next proposition characterizes the properties of the bargaining outcome.

PROPOSITION 6. **[Equilibirum Capacity under the Cp Scenario]**

i. There exists a pure strategy equilibrium in the capacity investment game such that the difference in the firms' equilibrium profits is equal to the difference between their disagreement payoffs.

ii. If the two products are not substitutes ($\hat{b}=0$), the equilibrium outcome in the Cp scenario is identical to the equilibrium outcome in the Cc scenario.

iii. A subsidy-free investment equilibrium exists (i.e., $\eta^{Cp^*}=0$), if and only if the equilibrium endowments $(K_1^{Cp^*}, K_2^{Cp^*})$ satisfy the following condition:

$$\pi_1^{n^*}(K_1^{Cp^*}, K_2^{Cp^*}) - \pi_2^{n^*}(K_1^{Cp^*}, K_2^{Cp^*}) = \pi_1^d - \pi_2^d \tag{24}$$

iv. Let $K_T^{Cp^*}$ be the total capacity in equilibrium under the Cp scenario. In addition to the conditions of Theorem 2(ii), assume that $K_T^{Cp^*} \geq K_i^{Nn^*}$, for $i=1,2$. Then, there exists a subsidy-free investment equilibrium in the Cp scenario.

Proposition 6 implies that, when the products are not substitutes, the equilibrium total capacity is centrally optimal (equal to that of a centralized firm). However, this is not necessarily true for the substitute case. When the products are substitutes, even with the same total capacity, the production quantities in the Cp scenario are different from the quantities of a centralized firm. For instance, when both firms get poor demand signals, there will be no capacity trade and the firms engage in quantity-setting game with their initial endowments. Because the second stage outcome is not always the same as the centrally optimal outcome, the firms' total expected profit is not the same as the profit of a centralized firm. As a result, in the Cp scenario, the firms build a joint capacity that is different from the capacity of a centralized firm.

Proposition 6 provides the condition that must be satisfied by the subsidy-free investment equilibrium. Note that this condition, given in equation (24), is analogous to (18), the equation that determines the subsidy-free investment equilibrium under the Cc scenario. Therefore, for the cases where the existence is guaranteed, similar to the Cc scenario, one can simply solve the condition in (24) with a search in single variable in a bounded interval to determine the investment level in a subsidy-free equilibrium.

Appendix C: Proofs of results

Proof of Proposition 1: The proof immediately follows from the fact that $r^{\mathrm{m}}(q_1, q_2, \theta_1, \theta_2)$ is concanve in (q_1, q_2) (part i) and the fact that $\pi^{\mathrm{m}^*}(K_{\mathrm{T}})$ is concave in K_{T}.

Proof of Proposition 2: Notice that, for a given subgame ω, the second-stage payoff of firm i, $q_i(\theta_i - b_i q_i - \hat{b} q_j)$, is concave in q_i. Also, the strategy space for (q_1, q_2) is compact, a pure strategy equilibrium exists

(Fudenberg and Tirole, 1991). To show uniqueness, for given q_j, the best response of firm i is $q_i^{\mathrm{n}}(q_j,\omega) = \min\left(\left(\frac{\theta_i - \hat{b}q_j}{2b_i}\right)^+, K_i\right)$. Taking its derivative with respect to q_j, we get:

$$\frac{d}{dq_j} q_i^{\mathrm{n}}(q_j,\omega) = \begin{cases} -\frac{\hat{b}}{2b_i}, & \text{if } 0 \le \frac{\theta_i - \hat{b}q_j}{2b_i} \le K_i; \\ 0, & \text{otherwise.} \end{cases}$$

Since $b_i > \hat{b}$, we have $-1 < \frac{d}{dq_j} q_i^{\mathrm{n}}(q_j,\omega) \le 0$ and the best response mapping is a contraction. Hence, the equilibrium (Fudenberg and Tirole, 1991) described in equation (10) is unique.

Proof of Proposition 3: To determine the NBS, we solve the optimization problem defined in (12). We first determine the optimal transfer payment, $\Gamma^{\mathrm{c}^*}(\omega)$, for given production quantities. Then, we solve for the optimal production quantities. For given (q_1,q_2), it can be shown that (12a) is strictly concave in Γ, and hence the optimal transfer payment $\Gamma^{\mathrm{c}^*}(\omega)$ is unique. To solve for $\Gamma^{\mathrm{c}^*}(\omega)$ (thereby proving part ii), we first write the KKT conditions. Let ν_1 and ν_2 be the Lagrangian multipliers. Then, the KKT conditions are as follows:

$$\Big(q_1p_1(q_1,q_2,\omega) - q_2p_2(q_1,q_2,\omega)\Big) - \Big(R_1^{\mathrm{n}^*}(\omega) - R_2^{\mathrm{n}^*}(\omega)\Big) - 2\Gamma - \nu_1 + \nu_2 = 0 \tag{25a}$$

$$\nu_1\Big(q_1p_1(q_1,q_2,\omega) - \Gamma - R_1^{\mathrm{n}^*}(\omega)\Big) = 0, \qquad \nu_1 \ge 0 \tag{25b}$$

$$\nu_2\Big(q_2p_2(q_1,q_2,\omega) + \Gamma - R_2^{\mathrm{n}^*}(\omega)\Big) = 0, \qquad \nu_2 \ge 0 \tag{25c}$$

From the KKT condition, we obtain

$$\Gamma^{\mathrm{c}^*}(\omega) = \frac{q_1p_1(q_1,q_2,\omega) - R_1^{\mathrm{n}^*}(\omega)}{2} - \frac{q_2p_2(q_1,q_2,\omega) - R_2^{\mathrm{n}^*}(\omega)}{2}, \tag{26}$$

To obtain the optimal production quantities, we rewrite (12) utlizing equation (26):

$$\left(\frac{q_1p_1(q_1,q_2,\omega) + q_2p_2(q_1,q_2,\omega)}{2} - \frac{R_1^{\mathrm{n}^*}(\omega) + R_2^{\mathrm{n}^*}(\omega)}{2}\right)^2 = \left(\frac{r^{\mathrm{m}}(q_1,q_2,\theta_1,\theta_2)}{2} - \frac{R_1^{\mathrm{n}^*}(\omega) + R_2^{\mathrm{n}^*}(\omega)}{2}\right)^2$$

Since the second part of the expression within the parentheses is independent of (q_1,q_2), the solution will maximize $r^{\mathrm{m}}(q_1,q_2,\theta_1,\theta_2)$ and this proves the result.

Proof of Proposition 4: i. We first determine the equilibrium transfer payment and capacity trade for given market signals. Then, we evaluate the unit price of capacity to establish the result. As illustrated in Figure 13, depending on the demand signals, the optimal capacity trade and the transfer payment can fall in one of 10 different regions. Table 3 presents the equilibrium capacity trade, $(\chi^{\mathrm{c}^*}(\omega))$, and unit price of capacity, $\gamma^{\mathrm{c}^*}(\omega)$, and condition for each of the 10 regions.

It can be shown that $\chi^{\mathrm{c}^*}(\omega)$ and $\Gamma^{\mathrm{c}^*}(\omega)$, are continuous functions in (θ_1,θ_2). Thus, $\gamma^{\mathrm{c}^*}(\omega)$ is also continuous in regions where $\chi^{\mathrm{c}^*}(\omega) \ne 0$. To show $\gamma^{\mathrm{c}^*}(\omega)$ is increasing in θ_i and decreasing in K_i, it suffices to show that $\gamma^{\mathrm{c}^*}(\omega)$ is monotone in each region. To illustrate this, we will show that $\gamma^{\mathrm{c}^*}(\omega)$ is increasing in θ_1 in Ψ_3^{c}. Other cases are similar, thus omitted. Taking the derivative of $\gamma^{\mathrm{c}^*}(\omega)$ in Ψ_3^{c} with respect to θ_1 and applying algebra, we get:

$$\frac{d}{d\theta_1}\gamma^{\mathrm{c}^*}(\omega) = \frac{\begin{bmatrix} (2b_2^2 - b_1b_2 - b_1^2)(\theta_1 - \theta_2 - 2b_1K_1 + 2b_2K_2)^2 - (b_1+b_2)^2(\theta_1 - 2b_1K_1)^2 \\ +2(b_1+b_2)^2(\theta_1 - 2b_1K_1)(\theta_1 - \theta_2 - 2b_1K_1 + 2b_2K_2) \end{bmatrix}}{4b_2(b_1+b_2)(\theta_1 - \theta_2 - 2b_1K_1 + 2b_2K_2)^2} > 0$$

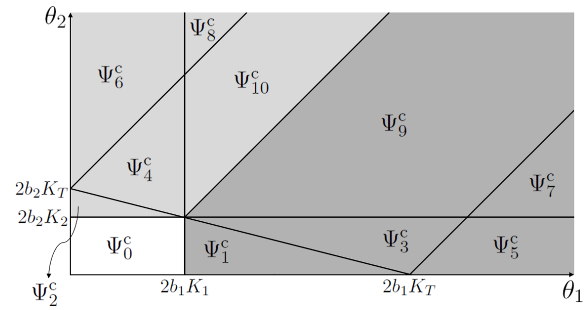

Figure 13 The capacity trade and transfer payment when firms collaborate in production.

Region	Definition	Capacity Trade, $\lvert\chi^{c^*}(\omega)\rvert$	Unit Price of Capacity, $\gamma^{c^*}(\omega)$
Ψ_0^c	$\theta_1 \le 2b_1K_1,\ \ \theta_2 \le 2b_2K_2$	0	Not defined
Ψ_1^c	$\theta_1 > 2b_1K_1,\ \ \theta_1b_2+\theta_2b_1 \le 2b_1b_2(K_1+K_2)$	$\dfrac{\theta_1}{2b_1}-K_1$	$\dfrac{\theta_1-2b_1K_1}{4}$
Ψ_2^c	$\theta_2 > 2b_2K_2,\ \ \theta_2b_1+\theta_1b_2 \le 2b_1b_2(K_1+K_2)$	$K_2-\dfrac{\theta_2}{2b_2}$	$\dfrac{\theta_2-2b_2K_2}{4}$
Ψ_3^c	$\theta_1b_2+\theta_2b_1 > 2b_1b_2(K_1+K_2)$, $\theta_2 \le 2b_2K_2, \theta_1-\theta_2 \le 2b_1(K_1+K_2)$	$\dfrac{\theta_1-\theta_2-2b_1K_1+2b_2K_2}{2(b_1+b_2)}$	$\dfrac{\begin{bmatrix} 4b_2(b_1+b_2)^2K_1(b_1K_1-\theta_1)+(b_1+b_2)^2\theta_2^2 \\ +b_2(2b_1(K_1+K_2)-\theta_1+\theta_2)\left[-2b_1\theta_2-b_2(\theta_1+\theta_2)\right] \\ +b_2(2b_2(K_1+K_2)+\theta_1-\theta_2)\left[2b_2\theta_1+b_1(\theta_1+\theta_2)\right] \\ +4b_1b_2^2(K_1+K_2)\left[(b_1-b_2)(K_1+K_2)-\theta_1+\theta_2\right]2 \end{bmatrix}}{4b_2(b_1+b_2)(\theta_1-\theta_2-2b_1K_1+2b_2K_2)}$
Ψ_4^c	$\theta_2b_1+\theta_1b_2 > 2b_1b_2(K_1+K_2)$, $\theta_1 \le 2b_1K_1, \theta_2-\theta_1 \le 2b_2(K_1+K_2)$	$\dfrac{\theta_1-\theta_2-2b_1K_1+2b_2K_2}{2(b_1+b_2)}$	$\dfrac{\begin{bmatrix} 4b_1(b_1+b_2)^2K_2(\theta_2-b_2K_2)-(b_1+b_2)^2\theta_1^2 \\ +b_1(2b_1(K_1+K_2)-\theta_1+\theta_2)\left[-2b_1\theta_2-b_2(\theta_1+\theta_2)\right] \\ +b_2(2b_2(K_1+K_2)+\theta_1-\theta_2)\left[2b_2\theta_1+b_1(\theta_1+\theta_2)\right] \\ +4b_1^2b_2(K_1+K_2)\left[(b_1-b_2)(K_1+K_2)-\theta_1+\theta_2\right] \end{bmatrix}}{4b_1(b_1+b_2)(\theta_1-\theta_2-2b_1K_1+2b_2K_2)}$
Ψ_5^c	$\theta_2 \le 2b_2K_2,\ \ \theta_1-\theta_2 > 2b_1(K_1+K_2)$	K_2	$\dfrac{\theta_2^2+4b_2K_2\left[\theta_1-b_1(2K_1+K_2)\right]}{8b_2K_2}$
Ψ_6^c	$\theta_1 \le 2b_1K_1,\ \ \theta_2-\theta_1 > 2b_2(K_1+K_2)$	$-K_1$	$\dfrac{\theta_1^2+4b_1K_1\left[\theta_2-b_2(2K_2+K_1)\right]}{8b_1K_1}$
Ψ_7^c	$\theta_2 > 2b_2K_2,\ \ \theta_1-\theta_2 > 2b_1(K_1+K_2)$	K_2	$\dfrac{\theta_1+\theta_2-2b_1K_1-(b_1+b_2)K_2}{2}$
Ψ_8^c	$\theta_1 > 2b_1K_1,\ \ \theta_2-\theta_1 > 2b_2(K_1+K_2)$	$-K_1$	$\dfrac{\theta_1+\theta_2-2b_2K_2-(b_1+b_2)K_1}{2}$
Ψ_9^c	$\theta_1-\theta_2 > 2b_1K_1-2b_2K_2$, $\theta_2 > 2b_2K_2,\ \ \theta_1-\theta_2 \le 2b_1(K_1+K_2)$	$\dfrac{\theta_1-\theta_2-2b_1K_1+2b_2K_2}{2(b_1+b_2)}$	$\dfrac{\begin{bmatrix} \theta_2(3b_1+b_2)+\theta_1(b_1+3b_2) \\ -2\left[b_1(b_1+3b_2)K_1+b_2(3b_1+b_2)K_2\right] \end{bmatrix}}{4(b_1+b_2)}$
Ψ_{10}^c	$\theta_2-\theta_1 > 2b_2K_2-2b_1K_1$, $\theta_1 > 2b_1K_1, \theta_2-\theta_1 \le 2b_2(K_1+K_2)$	$\dfrac{\theta_1-\theta_2-2b_1K_1+2b_2K_2}{2(b_1+b_2)}$	$\dfrac{\begin{bmatrix} \theta_2(3b_1+b_2)+\theta_1(b_1+3b_2) \\ -2\left[b_1(b_1+3b_2)K_1+b_2(3b_1+b_2)K_2\right] \end{bmatrix}}{4(b_1+b_2)}$

Table 3 The equilibrium capacity trade for the regions in Figure 13**.**

ii. The proof follows from Table 3, theorefore omitted.

Proof of Theorem 1: i. We present the proof when Θ_1 and Θ_2 are exponentially distributed with rates λ_1 and λ_2. The proof utilizes Lemma 1 which is stated and proved below.

LEMMA 1. *Suppose that Θ_1 and Θ_2 are independent exponential random variables with rates λ_1 and λ_2, respectively. Let $K_i^{N\mathfrak{s}}(K_j)$, $i \ne j$, be the firm i's best response when firm j sets its capacity to K_j of firm j under the scenario $N\mathfrak{s}$, $\mathfrak{s} \in \{n, c\}$. Then, we have the following:*

i. $\pi_i^{n^*}(K_1, K_2)$ *is concave in* K_i, $i = 1, 2$.

ii. $\nabla K_i^{Nn}(K_j) \in (-1,1)$, $i=1,2$, $i \neq j$.

iii. Define $\tilde{\pi}_i^{n^*}(K_1,K_2) = \pi_i^{n^*}(K_1,K_2) - \pi_j^{n^*}(K_1,K_2)$, $i \neq j$. *Then, we have* $\tilde{\pi}_i^{n^*}(K_1,K_2)$ *is concave in* K_i, $i=1,2$.

iv. $\nabla K_i^{Nc}(K_j) > -1$, $i=1,2$, $i \neq j$.

Proof of Lemma 1: We show that $\pi_1^{\mathrm{n}^*}(K_1,K_2)$ is concave in K_1. The proof for $\pi_2^{\mathrm{n}^*}(K_1,K_2)$ is symmetric. The second order derivative of $\pi_1^{\mathrm{n}^*}(K_1,K_2)$ with respect to K_1 is

$$\begin{aligned}\nabla_{11}^2 \pi_1^{\mathrm{n}^*}(K_1,K_2) \;=\; & \tfrac{\hat{b}^3 K_1}{2b_2}\left[\int_{2b_1K_1+\hat{b}K_2}^{\infty} f(t_1, 2b_2K_2+\hat{b}K_1)dt_1 - \int_{2b_1K_1}^{\infty} f(t_1,\hat{b}K_1)dt_1\right] \\ & - \tfrac{\hat{b}^2(4b_1b_2-\hat{b}^2)K_1}{4b_2^2}\int_{\hat{b}K_1}^{2b_2K_2+\hat{b}K_1} f\left(\tfrac{(4b_1b_2-\hat{b}^2)K_1+\hat{b}t_2}{2b_2}, t_2\right)dt_2 \\ & - 2b_1\left[\int_0^{\hat{b}K_1}\int_{2b_1K_1}^{\infty} f(t_1,t_2)dt_1dt_2 + \int_{2b_2K_2+\hat{b}K_1}^{\infty}\int_{2b_1K_1+\hat{b}K_2}^{\infty} f(t_1,t_2)dt_1dt_2\right] \\ & - \tfrac{2b_1b_2-\hat{b}^2}{b_2}\int_{\hat{b}K_1}^{2b_2K_2+\hat{b}K_1}\int_{\frac{(4b_1b_2-\hat{b}^2)K_1+\hat{b}t_2}{2b_2}}^{\infty} f(t_1,t_2)dt_1dt_2 \end{aligned} \tag{27}$$

Using the fact that Θ_i is exponentially distributed, the above equation can be simplified to

$$\begin{aligned}\nabla_{11}^2 \pi_1^{\mathrm{n}^*}(K_1,K_2) = \; & -2b_1 e^{-2b_1K_1\lambda_1} \\ & - \tfrac{\hat{b}(1-e^{\hat{b}K_2\lambda_1+2b_2K_2\lambda_2})(\hat{b}\lambda_2(2-\hat{b}K_1\lambda_2)+2b_1\lambda_1(1-\hat{b}K_1\lambda_2))}{\hat{b}\lambda_1+2b_2\lambda_2} e^{-(2b_1K_1+\hat{b}K_2)\lambda_1-(2b_2K_2+\hat{b}K_1)\lambda_2} \end{aligned} \tag{28}$$

Noting that $\hat{b}\lambda_1+2b_2\lambda_2>0$ and $e^{-(2b_1K_1+\hat{b}K_2)\lambda_1-(2b_2K_2+\hat{b}K_1)\lambda_2}>0$, $\nabla_{11}^2\pi_1^{\mathrm{n}^*}(K_1,K_2)$ must have the same sign as $\frac{\hat{b}\lambda_1+2b_2\lambda_2}{e^{-(2b_1K_1+\hat{b}K_2)\lambda_1-(2b_2K_2+\hat{b}K_1)\lambda_2}}\pi_1^{\mathrm{n}^*}(K_1,K_2)$. After some algebra, we have:

$$\begin{aligned} & \frac{\hat{b}\lambda_1+2b_2\lambda_2}{e^{-(2b_1K_1+\hat{b}K_2)\lambda_1-(2b_2K_2+\hat{b}K_1)\lambda_2}}\pi_1^{\mathrm{n}^*}(K_1,K_2) \\ & = -2b_1\left(e^{\hat{b}K_1\lambda_2}\right)\left(e^{\hat{b}K_2\lambda_1+2b_2K_2\lambda_2}\right)(\hat{b}\lambda_1+2b_2\lambda_2) - \hat{b}\left(1-e^{\hat{b}K_2\lambda_1+2b_2K_2\lambda_2}\right)\left(\hat{b}\lambda_2(2-\hat{b}K_1\lambda_2)+2b_1\lambda_1(1-\hat{b}K_1\lambda_2)\right) \\ & \leq -\lambda_2\left(2(2b_1b_2-\hat{b}^2)+\hat{b}^2(2b_1\lambda_1+\hat{b}\lambda_2)K_1\right)e^{\hat{b}K_2\lambda_1+2b_2K_2\lambda_2} - \hat{b}\left(\hat{b}\lambda_2(2-\hat{b}K_1\lambda_2)+2b_1\lambda_1(1-\hat{b}K_1\lambda_2)\right) \\ & \leq -2b_1\left(\hat{b}\lambda_1+2b_2\lambda_2\right) < 0. \end{aligned} \tag{29}$$

The first inequality comes from the fact that $-2b_1\left(e^{\hat{b}K_1\lambda_2}\right) \leq -2b_1$ and the second inequality comes from the fact $-\lambda_2 e^{\hat{b}K_2\lambda_1+2b_2K_2\lambda_2} \leq -\lambda_2$. The last inequality implies that $\nabla_{11}^2\pi^{\mathrm{n}^*}(K_1,K_2)<0$, hence $\pi_1^{\mathrm{n}^*}(K_1,K_2)$ is concave in K_1. The proofs for remaining parts use similar logic, thus omitted.

Proof of part (i): Nn Scenario: The best response of firm i to the firm j's action, K_j is uniquely determined (Lemma 1.i), and the mapping is a contraction (Lemma 1.ii). Note that $\lim_{K_i\to 0}\pi_i^{\mathrm{Nn}^*}(K_1,K_2)=0$ and $\lim_{K_i\to\infty}\pi_i^{\mathrm{Nn}^*}(K_1,K_2)\to-\infty$. Without loss of generality, it suffices to restrict the firm i's strategy space to be a compact interval, $[0,\bar{K}_i]$ for some $\bar{K}_i<\infty$. From Fudenberg and Tirole (1991), the equilibrium exists and it is unique.

Nc Scenario: Applying Proposition 3, the expected profit of firm i who has endowment K_i and will collaborate with firm j (with endowment K_j), in the second stage, $\pi_i^{\mathrm{c}^*}(K_1,K_2)$, is :

$$\pi_i^{\mathrm{c}^*}(K_1,K_2) = \frac{1}{2}\left(\pi^{\mathrm{m}^*}(K_1+K_2) + \pi_i^{\mathrm{n}^*}(K_1,K_2) - \pi_j^{\mathrm{n}^*}(K_1,K_2)\right)$$

Define $\tilde{\pi}_i^{\mathrm{n}^*}(K_1,K_2)=\pi_i^{\mathrm{n}^*}(K_1,K_2)-\pi_j^{\mathrm{n}^*}(K_1,K_2)$, $i\neq j$ and substitute this into the previous equation, we rewrite

$$\pi_i^{\mathrm{c}^*}(K_1,K_2)=\frac{1}{2}\Big(\pi^{\mathrm{m}^*}(K_1+K_2)+\tilde{\pi}_i^{\mathrm{n}^*}(K_1,K_2)\Big) \qquad i,j=1,2 \quad i\neq j. \tag{30}$$

It should be noted that $\pi_i^{\mathrm{c}^*}(K_1,K_2)$ is concave in K_i (this follows from the fact that $\pi^{\mathrm{m}^*}(K_1+K_2)$ and $\tilde{\pi}_i^{\mathrm{n}^*}(K_1,K_2)$ are concave: see Lemma 1.(iii). Similar to the Nn scenario, it suffices to restrict the firm i's strategy space to be a compact interval, which guarantees the existence (Fudenberg and Tirole, 1991).

For the uniqueness, first note that the best response mapping under the Nc scenario is not a contraction in general. We show the uniqueness by showing that the slopes of the best response functions are bounded in a way so that they will intersect exactly once. To start with, observe that the best response of firm i to the capacity K_j of firm j under the Nc scenario, $K_i^{\mathrm{Nc}}(K_j)$, is

$$K_i^{\mathrm{Nc}}(K_j)=\begin{cases}0, & \text{if } \nabla_i\pi_i^{\mathrm{c}^*}(0,K_j)\le 0;\\ \tilde{K}_i^{\mathrm{Nc}}(K_j), & \text{otherwise.}\end{cases} \qquad i,j=1,2 \quad i\neq j \tag{31}$$

where $\tilde{K}_i^{\mathrm{Nc}}(K_j)$ is the solution to the following first order condition for given K_j:

$$\nabla_i\pi_i^{\mathrm{c}^*}\big(\tilde{K}_i^{\mathrm{Nc}}(K_j),K_j\big)=\frac{1}{2}\Big\{\nabla\pi^{\mathrm{m}^*}(\tilde{K}_i^{\mathrm{Nc}}(K_j)+K_j)+\nabla_i\tilde{\pi}_i^{\mathrm{n}^*}(\tilde{K}_i^{\mathrm{Nc}}(K_j),K_j)\Big\}=0$$

Then, implicitly differentiating the first order condition, we get:

$$\nabla K_i^{\mathrm{Nc}}(K_j)=-1\cdot\frac{\nabla^2\pi^{\mathrm{m}^*}\big(K_i^{\mathrm{Nc}}(K_j)+K_j\big)+\nabla_{ij}^2\tilde{\pi}_i^{\mathrm{n}^*}\big(K_i^{\mathrm{Nc}}(K_j),K_j\big)}{2\nabla_{ii}^2\pi_i^{\mathrm{c}^*}\big(K_i^{\mathrm{Nc}}(K_j),K_j\big)}.$$

To obstain the bound on $\nabla K_i^{\mathrm{Nc}}(K_j)$, we first use Lemma 1(*iv*) and $\nabla K_i^{\mathrm{Nc}}(K_j)>-1$. Furthermore, since $\pi_i^{\mathrm{c}^*}(K_1,K_2)$ is strictly concave in K_i, the denominator has a negative sign. Hence, $\nabla K_i^{\mathrm{Nc}}(K_j)$ has the same sign as the numerator. Note that the first term in the numerator is negative since $\pi^{\mathrm{m}^*}(\cdot)$ is concave. Thus, in order for $\nabla K_i^{\mathrm{Nc}}(K_j)$ to be positive, the sign of the second term in the numerator must be postive. Subsutitute $q_i^{\mathrm{n}^*}(\omega)$ into $\tilde{\pi}_i^{\mathrm{n}^*}\big(K_1,K_2\big)$ and taking cross-partial derivative in K_i and K_j yield

$$\nabla_{ij}^2\tilde{\pi}_i^{\mathrm{n}^*}\big(K_i,K_j\big)=\hat{b}^2\Big\{K_i\int_{2b_iK_i+\hat{b}K_j}^{\infty}f(t_i,2b_jK_j+\hat{b}K_i)dt_i-K_j\int_{2b_jK_j+\hat{b}K_i}^{\infty}f(2b_iK_i+\hat{b}K_j,t_j)dt_j\Big\}. \tag{32}$$

If Θ_is are exponentially distributed, $\nabla_{ij}^2\tilde{\pi}_i^{\mathrm{n}^*}\big(K_i,K_j\big)$ becomes

$$\nabla_{ij}^2\tilde{\pi}_i^{\mathrm{n}^*}\big(K_i,K_j\big)=\hat{b}^2e^{-(2b_iK_i+\hat{b}K_j)\lambda_i-(2b_jK_j+\hat{b}K_i)\lambda_j}[K_i\lambda_j-K_j\lambda_i] \tag{33}$$

Note that this term is positive only if $K_i\lambda_j>K_j\lambda_i$. Therefore, $\nabla K_i^{\mathrm{Nc}}(K_j)$ can be positive only if $K_i\lambda_j>K_j\lambda_i$. This and the fact that $\nabla K_i^{\mathrm{Nc}}(K_j)>-1$ together implies that $\nabla K_i^{\mathrm{Nc}}(K_j)\in[-1,0]$ for $K_i\lambda_j<K_j\lambda_i$ and $\nabla K_i^{\mathrm{Nc}}(K_j)\ge -1$ for $K_i\lambda_j>K_j\lambda_i$: This is shwon in Figure 14(a). We now use this to show that the best response functions cannot cross multiple times. For this, suppose that the best responses cross twice or more and one of these intersections occurs at point A in Figure 14(b). Notice that from the fact that $\nabla K_2^{\mathrm{Nc}}(K_1)\in[-1,0]$ for $K_2\lambda_1<K_1\lambda_2$ and $\nabla K_2^{\mathrm{Nc}}(K_1)\ge -1$ elsewhere, the additional intersection point must be in the shaded region in Figure 14(b). However, if $K_1^{\mathrm{Nc}}(K_2)$ passes through point A, the other intersection (which is in the shaded region) must have $\nabla K_1^{\mathrm{Nc}}(K_2)<-1$, contradicts the fact that $\nabla K_1^{\mathrm{Nc}}(K_2)\ge -1$ (see Figure 14(a)). Thus, the response functions cannot cross more than once.

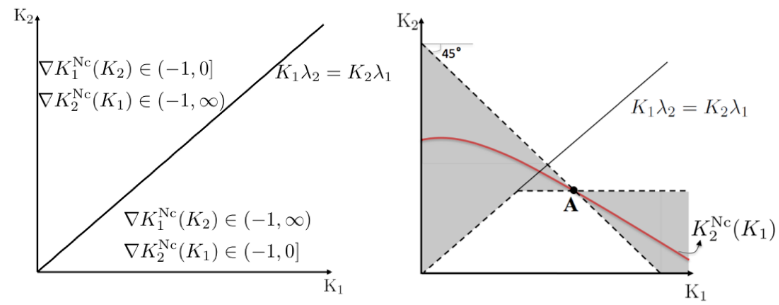

Figure 14 **(a) Signs of the best response functions' derivatives with respect to the regions on $K_1 \times K_2$ space, (b) Example for contradiction to prove that the intersection of the best response functions is unique.**

Cc and Cn Scenarios: The existence comes from the fact that $(K_1, K_2, \eta) = (K_1^{\mathrm{Nn}^*}, K_2^{\mathrm{Nn}^*}, 0)$ and the fact that the total profit is bounded above by that of a centralized firm.

The proof of part ii.(a) is algebraic, thus omitted. The proof for uniform distribution is similar.

For (b), observe that $(K_1^{\mathrm{C}\mathfrak{s}*}, K_2^{\mathrm{C}\mathfrak{s}*})$ maximizes $\pi_1^{\mathfrak{s}*}(K_1, K_2) + \pi_2^{\mathfrak{s}*}(K_1, K_2)$. In the Cc scenario, by Proposition 3, we have $\pi_1^{\mathrm{c}^*}(K_1, K_2) + \pi_2^{\mathrm{c}^*}(K_1, K_2) = \pi^{\mathrm{m}^*}(K_1 + K_2)$. Thus, $K_1^{\mathrm{Cc}^*} + K_2^{\mathrm{Cc}^*} = K_{\mathrm{T}}^{\mathrm{m}^*}$ holds. □

The proofs of Propositions 5, 7, and 8 use the following technical lemma.

Lemma 2. *We have the following:*

i. $\pi_i^{n^*}(K_i, K_j)$ *is decreasing in* K_j, $i = 1, 2$, $i \neq j$.

ii. The optimal capacity of a centralized firm is larger than an individual capacity of a firm under the Nn scenario: $K_T^{m^*} \geq K_i^{Nn^*}$, $i = 1, 2$.

iii. The joint capacity under the Cp scenario is smaller than the capacity of a centralized firm: $K_T^{Cp^*} < K_T^{m^*}$.

Proof of Lemma 2. We present the proof of Part **iii**. The proofs of parts (i) and (ii) are algebraic, therefore omitted. Under the Cp scenario, the firms negotiate to determine the reallocation of the total capacity in the second stage, before each firm individually sets its production quantity. Therefore, the total revenue, $R_{\mathrm{T}}^{\mathrm{Cp}^*}(K_{\mathrm{T}}, \theta_1, \theta_2)$, for given demand signals (θ_1, θ_2), is a function of the total capacity K_{T}. By the NBS, $K_{\mathrm{T}}^{\mathrm{Cp}^*}$ maximizes $E\left[R_{\mathrm{T}}^{\mathrm{Cp}^*}(K_{\mathrm{T}}, \theta_1, \theta_2)\right] - cK_{\mathrm{T}}$. We can show that $E\left[R_{\mathrm{T}}^{\mathrm{Cp}^*}(K_{\mathrm{T}}, \theta_1, \theta_2)\right] - cK_{\mathrm{T}}$ is increasing at $K_{\mathrm{T}} = 0$ and decreasing when $K_{\mathrm{T}} \to \infty$. Therefore, the first order condition is a necessary condition for $K_{\mathrm{T}}^{\mathrm{Cp}^*}$ to be optimal:

$$\tfrac{d}{dK_{\mathrm{T}}}\big|_{K_{\mathrm{T}} = K_{\mathrm{T}}^{\mathrm{Cp}^*}} \left\{\mathbb{E}\left[R_{\mathrm{T}}^{\mathrm{Cp}^*}(K_{\mathrm{T}}, \Theta_1, \Theta_2)\right] - cK_{\mathrm{T}}\right\} = 0$$

Note that for any incremental capacity, a centralized firm optimally allocates it to production while the efficiency is not guaranteed in the Cp scenario. This implies that, for any marginal increase in capacity, the increase in the total profit of a centralized firm is always larger than the increase in the total profit under the Cp scenario. Hence,

$$\tfrac{d}{dK_{\mathrm{T}}}\left\{\mathbb{E}\left[R_{\mathrm{T}}^{\mathrm{m}^*}(K_{\mathrm{T}}, \Theta_1, \Theta_2)\right] - cK_{\mathrm{T}}\right\} > \tfrac{d}{dK_{\mathrm{T}}}\left\{\mathbb{E}\left[R_{\mathrm{T}}^{\mathrm{Cp}^*}(K_{\mathrm{T}}, \Theta_1, \Theta_2)\right] - cK_{\mathrm{T}}\right\} \text{ for any } K_{\mathrm{T}}, \text{ and we have}$$

$$\frac{d}{dK_{\mathrm{T}}}|_{K_{\mathrm{T}}=K_{\mathrm{T}}^{\mathrm{Cp}^*}}\Big\{\mathbb{E}\big[R_{\mathrm{T}}^{\mathrm{m}^*}(K_{\mathrm{T}},\Theta_1,\Theta_2)\big]-cK_{\mathrm{T}}\Big\}>\frac{d}{dK_{\mathrm{T}}}|_{K_{\mathrm{T}}=K_{\mathrm{T}}^{\mathrm{Cp}^*}}\Big\{\mathbb{E}\big[R_{\mathrm{T}}^{\mathrm{Cp}^*}(K_{\mathrm{T}},\Theta_1,\Theta_2)\big]-cK_{\mathrm{T}}\Big\}=0$$

As $\mathbb{E}\big[R_{\mathrm{T}}^{\mathrm{m}^*}(K_{\mathrm{T}},\Theta_1,\Theta_2)\big]-cK_{\mathrm{T}}$ is concave in K_{T}, this implies that $K_{\mathrm{T}}^{\mathrm{Cp}^*}<K_{\mathrm{T}}^{\mathrm{m}^*}$.

Proof of Proposition 2: i. We first show that a subsidy-free equilibrium, $(K_1^{\mathrm{C}\mathfrak{s}*},K_2^{\mathrm{C}\mathfrak{s}*})$ satisfies equation (18). Note that applying $\eta^{\mathrm{C}\mathfrak{s}*}=0$, the equation in (17) is simplified as follows:

$$\pi_1^{\mathfrak{s}*}(K_1^{\mathrm{C}\mathfrak{s}*},K_2^{\mathrm{C}\mathfrak{s}*})-\pi_2^{\mathfrak{s}*}(K_1^{\mathrm{C}\mathfrak{s}*},K_2^{\mathrm{C}\mathfrak{s}*})=\pi_1^{\mathrm{d}}-\pi_2^{\mathrm{d}} \qquad \mathfrak{s}\in\{n,c\} \tag{34}$$

For the Cn scenario (i.e., $\mathfrak{s}=n$), this directly implies equation (18). For the Cc scenario (i.e., $\mathfrak{s}=c$), from (30), we have

$$\pi_i^{\mathrm{c}^*}(K_1,K_2)=\pi_i^{\mathrm{n}^*}(K_1,K_2)+\frac{\pi^{\mathrm{m}^*}(K_1+K_2)-\pi_i^{\mathrm{n}^*}(K_1,K_2)-\pi_j^{\mathrm{n}^*}(K_1,K_2)}{2} \qquad i,j=1,2 \quad i\neq j \tag{35}$$

Substituting $K_i=K_i^{\mathrm{Cc}^*}$, $i=1,2$ and applying in (34), we get (18).

We now show that any equilibrium $(K_1^{\mathrm{C}\mathfrak{s}*},K_2^{\mathrm{C}\mathfrak{s}*})$ that satisfies equation (18) is subsidy-free. The result $\eta^{\mathrm{Cn}^*}=0$ follows from algebra for the Cn scenario. For the Cc scenario, we first rewrite (18) using the expression of $\pi_i^{\mathrm{c}^*}(K_1,K_2)$ in equation (35). Then, we have $\pi_1^{\mathrm{c}^*}(K_1^{\mathrm{Cc}^*},K_2^{\mathrm{Cc}^*})-\pi_1^{\mathrm{c}^*}(K_1^{\mathrm{Cc}^*},K_2^{\mathrm{Cc}^*})=\pi_1^{\mathrm{d}}-\pi_2^{\mathrm{d}}$. Substituting this in (17), we obtain $2\eta^{\mathrm{Cc}^*}=0$, which completes the proof.

ii. From the fact that $(\pi_1^{\mathrm{d}},\pi_2^{\mathrm{d}})=\Big(\pi_1^{\mathrm{n}^*}(K_1^{\mathrm{Nn}^*},K_2^{\mathrm{Nn}^*}),\pi_2^{\mathrm{n}^*}(K_1^{\mathrm{Nn}^*},K_2^{\mathrm{Nn}^*})\Big)$, equation (18) can be expressed as:

$$\pi_1^{\mathrm{n}^*}(K_1^{\mathrm{C}\mathfrak{s}*},K_2^{\mathrm{C}\mathfrak{s}*})-\pi_2^{\mathrm{n}^*}(K_1^{\mathrm{C}\mathfrak{s}*},K_2^{\mathrm{C}\mathfrak{s}*})=\pi_1^{\mathrm{n}^*}(K_1^{\mathrm{Nn}^*},K_2^{\mathrm{Nn}^*})-\pi_2^{\mathrm{n}^*}(K_1^{\mathrm{Nn}^*},K_2^{\mathrm{Nn}^*}) \qquad \mathfrak{s}\in\{n,c\} \tag{36}$$

For $\hat{b}=0$, we will first consider the Cn scenario. Since $\hat{b}=0$, $q_i^{\mathrm{n}^*}(\omega)$ and the inverse demand function, $p_i(q_1,q_2,\omega)=\theta_i-b_iq_i$, are independent from K_j. Therefore, the profit $\pi_i^{\mathrm{n}^*}(K_i,K_j)$ is independent from K_j. Under the Cn scenario, the equilibrium, $(K_1^{\mathrm{Cn}^*},K_2^{\mathrm{Cn}^*})$ will maximize $\pi_1^{\mathrm{n}^*}(K_1,K_2)+\pi_2^{\mathrm{n}^*}(K_1,K_2)$. Since $\pi_i^{\mathrm{n}^*}(K_i,K_j)$ is independent from K_j, we have $K_i^{\mathrm{Cn}^*}=K_1^{\mathrm{Nn}^*}$. The proof for the Cc scenario is similar, thus omitted. Now, consider the Cc scenario when $\hat{b}>0$. First note that, by Theorem 1, $K_1^{\mathrm{Cc}^*}+K_2^{\mathrm{Cc}^*}=K_{\mathrm{T}}^{\mathrm{m}^*}$. For the proof, define the following function

$$h(K_1)=\Big(\pi_1^{\mathrm{n}^*}(K_1,K_{\mathrm{T}}^{\mathrm{m}^*}-K_1)-\pi_1^{\mathrm{n}^*}(K_1^{\mathrm{Nn}^*},K_2^{\mathrm{Nn}^*})\Big)-\Big(\pi_2^{\mathrm{n}^*}(K_1,K_{\mathrm{T}}^{\mathrm{m}^*}-K_1)-\pi_2^{\mathrm{n}^*}(K_1^{\mathrm{Nn}^*},K_2^{\mathrm{Nn}^*})\Big)$$

We next prove that there exists a K_1 such that $h(K_1)=0$. For this, observe that $h(\cdot)$ is continuous (since $\pi_1^{\mathrm{n}^*}(\cdot,\cdot)$ is continuous). In addition, we have:

$$h(K_1^{\mathrm{Nn}^*})=\Big(\pi_1^{\mathrm{n}^*}(K_1^{\mathrm{Nn}^*},K_{\mathrm{T}}^{\mathrm{m}^*}-K_1^{\mathrm{Nn}^*})-\pi_1^{\mathrm{n}^*}(K_1^{\mathrm{Nn}^*},K_2^{\mathrm{Nn}^*})\Big)-\Big(\pi_2^{\mathrm{n}^*}(K_1^{\mathrm{Nn}^*},K_{\mathrm{T}}^{\mathrm{m}^*}-K_1^{\mathrm{Nn}^*})-\pi_2^{\mathrm{n}^*}(K_1^{\mathrm{Nn}^*},K_2^{\mathrm{Nn}^*})\Big)$$

Since $K_2^{\mathrm{Nn}^*}\geq K_{\mathrm{T}}^{\mathrm{m}^*}-K_1^{\mathrm{Nn}^*}$ and $\pi_1^{\mathrm{n}^*}(K_1,K_2)$ is decreasing in K_2 (Lemma 2(i)), the terms in the first parenthesis is positive. Also, from the fact that $K_2^{\mathrm{Nn}^*}$ is the best response to $K_1^{\mathrm{Nn}^*}$, the terms in the second parantehesis is negative. Combining these, we have $h(K_1^{\mathrm{Nn}^*})\geq 0$.

A similar argument shows that $K_{\mathrm{T}}^{\mathrm{m}^*}-K_i^{\mathrm{Nn}^*}\geq 0$ (Lemma 2(ii)) and

$$h(K_{\mathrm{T}}^{\mathrm{m}^*}-K_2^{\mathrm{Nn}^*})=\Big(\pi_1^{\mathrm{n}^*}(K_{\mathrm{T}}^{\mathrm{m}^*}-K_2^{\mathrm{Nn}^*},K_2^{\mathrm{Nn}^*})-\pi_1^{\mathrm{n}^*}(K_1^{\mathrm{Nn}^*},K_2^{\mathrm{Nn}^*})\Big)-\Big(\pi_2^{\mathrm{n}^*}(K_{\mathrm{T}}^{\mathrm{m}^*}-K_2^{\mathrm{Nn}^*},K_2^{\mathrm{Nn}^*})-\pi_2^{\mathrm{n}^*}(K_1^{\mathrm{Nn}^*},K_2^{\mathrm{Nn}^*})\Big)\leq 0$$

These two inequalities imply that there exists K_1 between $\min(K_{\mathrm{T}}^{\mathrm{m}*} - K_2^{\mathrm{Nn}*}, K_1^{\mathrm{Nn}*})$ and $\max(K_{\mathrm{T}}^{\mathrm{m}*} - K_2^{\mathrm{Nn}*}, K_1^{\mathrm{Nn}*})$ that satisfies equation (36) (i.e., $h(K_1)=0$).

Proof of Proposition 5: i. The proof is similar to that for the existence of the NBS solution for the Cc senario, thus omitted. **ii.** Under the Cp scenario, the firms negotiate to trade capacity before they make quantity decisions. Notice that, if $\hat{b}=0$, the firm i's revenue is indepedent from the other firm's quantity. Hence, as long as the total endowment is the same, the quantities that firms will produce when they do not collaborate during the production are the same as the those chosen by a centralized firm.

Proof of Proposition 6: We omit the proof of part i. since it is similar to the proof of Theorem 1(i) (existence) and the proof of Theorem 1(ii)a (difference), respectively. For part **ii.**, from Proposition 5, observe that, the equilibrium production quantities are the same as the quanitites that a centralized firm with the same total capacity would produce. Since the firms fully collaborate in capacity investment in the first stage, the equilibrium outcome must coincide with the equilibrium outcome under the Cc scenario. The proof of part **iii.** is similar to the proof of Proposition 2(i), thus omitted. Finally, for part **iv.**, the case where $\hat{b}=0$ immediately follows from part (ii) of Proposition 6. Now, consider the case where $\hat{b}>0$. Note from Lemma 2(iii) that $K_{\mathrm{T}}^{\mathrm{m}*} > K_{\mathrm{T}}^{\mathrm{Cp}*}$. Furthermore, from the assumption, it must be $K_{\mathrm{T}}^{\mathrm{Cp}*} \geq K_i^{\mathrm{Nn}*}$ for $i=1,2$. Then, the result follows from a similar argument used in the proof of Proposition 2(ii) with $K_{\mathrm{T}}^{\mathrm{Cp}*}$ replacing $K_{\mathrm{T}}^{\mathrm{m}*}$.

Proof of Proposition 3.

(A) First note that from Proposition 2, $q_i^{\mathrm{n}*}(\omega)$ and $\pi_i^{\mathrm{n}*}(K_i, K_j)$ are independent from K_j when $\hat{b}=0$. Thus, we will simplify the notation and drop K_j from the arguments of $\pi_i^{\mathrm{n}*}(\cdot)$, $i,j=1,2$, $i\neq j$ in this proof.

(i.) Note that $(K_1^{\mathrm{Cn}*}, K_2^{\mathrm{Cn}*})$ maximizes $\pi_1^{\mathrm{n}*}(K_1) + \pi_2^{\mathrm{n}*}(K_2)$. Since $\pi_i^{\mathrm{n}*}(\cdot)$ is independent of K_j, $i,j=1,2$, $i\neq j$, $K_i^{\mathrm{Cn}*} = \arg\max \pi_i^{\mathrm{n}*}(K_i) = K_i^{\mathrm{Nn}*}$, $i=1,2$.

(ii.) From Theorem 1(ii)b, we have $K_{\mathrm{T}}^{\mathrm{Cc}*} = K_{\mathrm{T}}^{\mathrm{m}*}$. Therefore, it suffices to show that $\min(K_{\mathrm{T}}^{\mathrm{m}*}, K_{\mathrm{T}}^{\mathrm{Nn}*}) \leq K_{\mathrm{T}}^{\mathrm{Nc}*} \leq \max(K_{\mathrm{T}}^{\mathrm{m}*}, K_{\mathrm{T}}^{\mathrm{Nn}*})$. We provide the proof for the case where the equilibrium under the Nc scenario, $(K_1^{\mathrm{Nc}*}, K_2^{\mathrm{Nc}*})$ is an interior solution where both capacity endowments satisfy the first order conditions. The treatment for the boundary solution as the analysis is similar. From equation (30), the first order conditions that determine the equilibrium for the Nc scenario is expressed as follows:

$$\nabla_i \pi_i^{\mathrm{c}*}(K_i^{\mathrm{Nc}*}, K_j^{\mathrm{Nc}*}) = \nabla \pi^{\mathrm{m}*}(K_1^{\mathrm{Nc}*} + K_2^{\mathrm{Nc}*}) + \nabla \pi_i^{\mathrm{n}*}(K_i^{\mathrm{Nc}*}) = 0 \qquad i,j=1,2 \quad i\neq j \tag{37}$$

Thus, we must have $\nabla \pi_1^{\mathrm{n}*}(K_1^{\mathrm{Nc}*}) = \nabla \pi_2^{\mathrm{n}*}(K_2^{\mathrm{Nc}*})$.

Consider the case that $K_{\mathrm{T}}^{\mathrm{Nn}*} > K_{\mathrm{T}}^{\mathrm{m}*}$. Suppose that $K_1^{\mathrm{Nc}*} + K_2^{\mathrm{Nc}*} = K_{\mathrm{T}}^{\mathrm{Nc}*} < K_{\mathrm{T}}^{\mathrm{m}*}$. Since $\pi^{\mathrm{m}*}(\cdot)$ is concave, we have $\nabla \pi^{\mathrm{m}*}(K_{\mathrm{T}}^{\mathrm{Nc}*}) > 0$. Then, from equation (37), it must be $\nabla \pi_1^{\mathrm{n}*}(K_1^{\mathrm{Nc}*}) < 0$. Since, $\pi_i^{\mathrm{n}*}(\cdot)$ is also concave in K_i and it is independent of K_j, it must be the case that $K_i^{\mathrm{Nc}*} > K_i^{\mathrm{Nn}*}$, $i=1,2$. Hence, we have

$$K_{\mathrm{T}}^{\mathrm{m}*} \;>\; K_1^{\mathrm{Nc}*} + K_2^{\mathrm{Nc}*} \;>\; K_1^{\mathrm{Nn}*} + K_2^{\mathrm{Nn}*} = K_{\mathrm{T}}^{\mathrm{Nn}*}$$

which contradicts $K_{\mathrm{T}}^{\mathrm{Nn}*} > K_{\mathrm{T}}^{\mathrm{m}*}$. Hence, we must have $K_{\mathrm{T}}^{\mathrm{Nc}*} \geq K_{\mathrm{T}}^{\mathrm{m}*}$.

Now, to prove $K_{\mathrm{T}}^{\mathrm{Nc}*} \le K_{\mathrm{T}}^{\mathrm{Nn}*}$, suppose that $K_1^{\mathrm{Nc}*} > K_1^{\mathrm{Nn}*}$. Since $\pi_1^{\mathrm{n}^*}(\cdot)$ is concave, $\nabla\pi_1^{\mathrm{n}^*}(K_1^{\mathrm{Nc}*}) < 0$. Then, from equation (37), it must be the case that $\nabla\pi^{\mathrm{m}^*}(K_{\mathrm{T}}^{\mathrm{Nc}*}) > 0$. As $\pi^{\mathrm{m}^*}(\cdot)$ is concave, this implies $K_{\mathrm{T}}^{\mathrm{Nc}*} < K_{\mathrm{T}}^{\mathrm{m}*}$. Note that $\nabla\pi_1^{\mathrm{n}^*}(K_1^{\mathrm{Nc}*}) < 0$ also implies that $\nabla\pi_2^{\mathrm{n}^*}(K_2^{\mathrm{Nc}*}) < 0$, and hence $K_2^{\mathrm{Nc}*} > K_2^{\mathrm{Nn}*}$. Therefore:

$$K_{\mathrm{T}}^{\mathrm{m}*} \;>\; K_1^{\mathrm{Nc}*} + K_2^{\mathrm{Nc}*} \;>\; K_1^{\mathrm{Nn}*} + K_2^{\mathrm{Nn}*} = K_{\mathrm{T}}^{\mathrm{Nn}*}$$

which contradicts $K_{\mathrm{T}}^{\mathrm{Nn}*} > K_{\mathrm{T}}^{\mathrm{m}*}$. Hence, we must have $K_1^{\mathrm{Nc}*} \le K_1^{\mathrm{Nn}*}$ and hence $K_2^{\mathrm{Nc}*} \le K_2^{\mathrm{Nn}*}$, establishing $K_{\mathrm{T}}^{\mathrm{Nc}*} \le K_{\mathrm{T}}^{\mathrm{Nn}*}$. Therefore, we have $K_{\mathrm{T}}^{\mathrm{m}*} \le K_{\mathrm{T}}^{\mathrm{Nc}*} \le K_{\mathrm{T}}^{\mathrm{Nn}*}$. The proof when $K_{\mathrm{T}}^{\mathrm{Nn}*} \le K_{\mathrm{T}}^{\mathrm{m}*}$ is similar.

(iii.) Directly follows Proposition 6(*ii*).

(B) The proof is similar to part (A) but uses Lemma 2(*i*). Hence, we only provide the sketches and highlight difference. For part **(i)**, notice that, when the firms collaborate in the capacity building stage (as in Cn scenario), the NBS stipluates that the firms select the capacities to maximize the total profit. Therefore $(K_1^{\mathrm{Cn}*}, K_2^{\mathrm{Cn}*})$ satisfies the following conditions:

$$\nabla_1\pi_1^{\mathrm{n}^*}(K_1^{\mathrm{Cn}*}, K_2^{\mathrm{Cn}*}) + \nabla_1\pi_2^{\mathrm{n}^*}(K_1^{\mathrm{Cn}*}, K_2^{\mathrm{Cn}*}) = 0 \quad \text{and} \quad \nabla_2\pi_1^{\mathrm{n}^*}(K_1^{\mathrm{Cn}*}, K_2^{\mathrm{Cn}*}) + \nabla_2\pi_2^{\mathrm{n}^*}(K_1^{\mathrm{Cn}*}, K_2^{\mathrm{Cn}*}) = 0$$

Lemma 2(*i*) establishes that $\nabla_j\pi_i^{\mathrm{n}^*}(K_i, K_j) < 0$, for $i, j = 1, 2$, and $i \ne j$. Therefore, we have: $\nabla_1\pi_1^{\mathrm{n}^*}(K_1^{\mathrm{Cn}*}, K_2^{\mathrm{Cn}*}) > 0$ and $\nabla_2\pi_2^{\mathrm{n}^*}(K_1^{\mathrm{Cn}*}, K_2^{\mathrm{Cn}*}) > 0$. Recall that $K_i^{\mathrm{Nn}}(K_j)$ is the best response of firm i to the capacity K_j of firm j under the Nn scenario. Since $\pi_i^{\mathrm{n}^*}(K_i, K_j)$ is concave in K_i, we have $\nabla_i\pi_i^{\mathrm{n}^*}(K_i^{\mathrm{Nn}}(K_j), K_j) = 0$. Thus, $K_1^{\mathrm{Cn}*} < K_1^{\mathrm{Nn}}(K_2^{\mathrm{Cn}*})$ and $K_2^{\mathrm{Cn}*} < K_2^{\mathrm{Nn}}(K_1^{\mathrm{Cn}*})$. This is depicted in Figure 15, where $(K_1^{\mathrm{Cn}*}, K_2^{\mathrm{Cn}*})$ can only be in the lightly shaded region. In this figure, the dashed line represents the values where $K_1 + K_2 = K_{\mathrm{T}}^{\mathrm{Nn}*}$. The lightly shaded region is to the left of this line because $\nabla K_i^{\mathrm{Nn}}(\cdot) \in (-1, 1)$ by Lemma 1(*ii*). Therefore, $K_{\mathrm{T}}^{\mathrm{Cn}*} \le K_{\mathrm{T}}^{\mathrm{Nn}*}$.

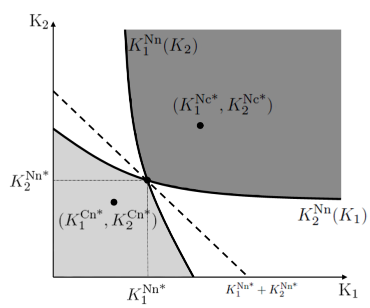

Figure 15 **The best response curves under the Nn scenario and the equilibrium capacity vectors under the Cn and Nc scenarios.**

(ii.) We use contradiction to prove the result that $\min(K_{\mathrm{T}}^{\mathrm{m}*}, K_{\mathrm{T}}^{\mathrm{Nn}*}) \le K_{\mathrm{T}}^{\mathrm{Nc}*}$. The argument is similar to the proof of part (A.ii) in Proposition 3, thus omitted.

(iii.) The result immediately follows Lemma 2.(*iii*).

References

Fudenberg, D. and J. Tirole (1991): *Game Theory*, MIT Press.