Innovation in Decentralized Markets: Technology vs. Synthetic Products*

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Abstract

Advances in market-clearing technology for multiple assets and synthetic products present alternative ways to leverage complementarities and substitutabilites in asset returns. This paper compares their equilibrium and welfare effects. In competitive markets, either instrument can mimic the efficient design. When traders have price impact, however, synthetic products and market-clearing technology provide separate instruments for impacting markets' performance and can generate synergies or tradeoffs. Neither instrument can generally reproduce the other's payoffs state by state. Moreover, innovation in market clearing renders additional synthetic products nonredundant. Our analysis points to the advantages of each type of innovation while also exposing potential risks.

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1 Introduction

The success of synthetic financial products like derivatives has transformed the functioning of markets. Synthetic products now play an important role in portfolio strategy and risk management. Such products are sought by market participants who wish to improve liquidity and

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enhance traders' inference about underlying asset prices.[1] As technology advances, the design of multi-asset market-clearing algorithms offers an alternative for impacting market performance. Electronic trading platforms for financial assets increasingly allow traders to express their demands for one asset contingent on the prices of other assets.[2] By allowing sophisticated preference expression over many assets, thereby permitting more flexible diversification of cross-asset risk, technology has been assuming the traditional role of securities. In this paper, we ask whether innovation in multi-asset market-clearing algorithms can substitute for synthetic products in their impact on efficiency and liquidity.

Regulators have scrutinized the effects of each type of innovation.[3] Changes to synthetic financial product availability or changes to the market-clearing rules across assets are often motivated by the objective to provide market participants with information about other assets, facilitate risk diversification, or reduce participation fees. For large market participants, however, exogenous transaction fees are only a small part of their trading costs; price impact continues to be a dominant cost component.[4] Our analysis suggests that accounting for price impact is essential when determining whether changes to synthetic products or market-clearing rules are more effective for impacting diversification and welfare.

We build on the model of Rostek and Yoon (2021a,b) based on the standard uniform-price double auction for $K > 1$ assets that can be cleared jointly or independently and $I < \infty$ strategic traders privately informed about their asset holdings.[5] In the baseline market, traders submit *uncontingent* (net) demand schedules for each asset: their quantities are contingent only on the price of that asset. With uncontingent demands, assets can clear independently; with demands that are *contingent* on the prices of all assets, all assets must clear jointly.[6] The

[1] Over \$13 trillion notional amounts were outstanding in the global equity-linked derivatives market in 2020; `https://stats.bis.org/statx/toc/DER.html`. The daily average turnover of ETF futures and options was \$6.7 trillion in 2020; `https://www.bis.org/statistics/extderiv.htm`. In the United States, nearly 2500 ETFs were listed in the first quarter of 2021, and the average daily value of US ETF transactions was \$151.47 billion in total cash flow; `https://www.nyse.com/etf/exchange-traded-funds-quarterly-report`.

[2] Several types of cross-asset conditioning are available in futures and options markets (e.g., multi-leg orders) and electronic trading platforms, including Active Trader Pro, Etrade, Street Smart, Tradehawk.

[3] In fact, concerns about these and related effects motivated many sections of Dodd-Frank and MiFID I/II; see also Focault (2012). To mention a couple of recently revised rules, the SEC relaxed restrictions on ETFs to allow issuers to bring more flexible custom-designed baskets to market (SEC 2019). On the other hand, the SEC's Reg NMS intends for trades submitted to one trading venue to be cleared at prices displayed by other venues. Updates to these rules aimed to improve traders' access to information (Final Rule: Regulation NMS, 2020). However, many transactions involving joint trading of multiple securities (e.g., equities and related derivatives) are exempt from such price linkages.

[4] See, e.g., Bollen and Whaley (2004), Garleanu, Pedersen, and Poteshman (2009), Bali, Beckmeyer, Moerke, and Weigert (2022), Zhang (2022) and references there.

[5] Analysis of multi-asset markets with multi-unit demands in this framework—competitive and imperfectly competitive—has focused on markets in which traders' demands for each asset are *contingent* on the prices of all assets (e.g., Wilson (1979), Klemperer and Meyer (1989), Kyle (1989), Vives (2011)). Such schedules, however, are not yet common in practice. A recently introduced version of this model relaxes the contingent-demands assumption (Chen and Duffie (2021), Wittwer (2021), and Rostek and Yoon (2021a,b)).

[6] One might ask whether independence in market clearing across securities matters, given that markets are

setting is quadratic-Gaussian.

In markets with K underlying assets that clear independently, we compare the equilibrium and welfare effects of two types of innovation: We consider the introduction of synthetic products, whose returns are defined as bundles of the underlying assets' returns (derivatives), which trade along with the underlying assets and clear independently. We also allow changes to the market-clearing technology of the *securities* (underlying assets and synthetic products), which allow multiple securities to clear jointly. Changes to market-clearing technology encompass the introduction of new exchanges for underlying assets, exchange mergers, and listings of underlying assets in exchanges where they were not traded before. Neither innovation alters the underlying asset span, traders' initial asset holdings, or asset supply.

These two innovations affect the game in different ways: Synthetic products change the joint distribution of *securities' returns*, while technology innovations change *traders' strategies*. We show that in markets with arbitrary derivatives and exchanges that clear jointly any subset of underlying assets and synthetic products they list, the welfare effects of the two types of innovations can be compared by analyzing how they change the price impact and, equivalently, the cross-security inference (Theorem 1 and Lemma 1). We report four main results.

First, our results underscore the difference that price impact makes in evaluating the effectiveness of these instruments. In fact, in competitive markets, either instrument can reproduce the efficient design, which with price-taking traders corresponds to the fully contingent design: Rostek and Yoon (2021a,b) showed that the outcome of the fully contingent design (where additional synthetic products or exchanges would be neutral) can be implemented by a market structure with either sufficiently many derivatives or sufficiently many multi-asset exchanges that clear independently. In practice, introducing derivatives may be less costly than developing and implementing multi-asset market-clearing technology, mergers of privately run exchanges, or asset listings. Nevertheless, in this paper, we show that with imperfect competition, the effects of these instruments are generally no longer equivalent and need not "substitute" in the efficient design.

We examine when, in imperfectly competitive markets, synthetic products can reproduce the equilibrium effects of innovation in market-clearing technology for the corresponding assets state by state (i.e., for all realizations of traders' asset holdings), or vice versa. We show that this is generally not possible, even with multiple derivatives for the same assets or additional derivatives for other assets (Proposition 1). Both innovations allow traders' total demands

dynamic and a nontrivial fraction of assets can be traded continuously. By conditioning on past outcomes, demands with any contingent variables allow information from past shocks to be at least partially incorporated. Conditioning on contingent variables for current-round outcomes affects the way current-round shocks impact behavior. These effects continue to impact outcomes with dynamic trading even when trading is continuous, provided that the relative frequency of shocks to information or liquidity renewing the gains from trade (whose realization requires multiple rounds) is not too low relative to trading frequency. See also Section 5.2 and Lyu, Rostek, and Yoon (2021a).

for underlying assets to condition on additional information (i.e., other securities' prices in addition to own prices), thus permitting more flexible cross-asset diversification. Yet, they induce different inference about the prices of the underlying assets in traders' demands and, consequently, imply different equilibrium price impact. Derivatives offer an additional degree of freedom in the weights that bundle asset returns; however, with derivatives, the underlying assets must be traded in fixed proportions.

We provide a necessary and sufficient condition for the equivalence: Traders' cross-asset price impacts must be symmetric (i.e., $\lambda^i_{k\ell} = \lambda^i_{\ell k}$ for all k and $\ell \neq k$; Proposition 1, Examples 4 and 5). Two results underly this condition: First, innovation in market-clearing technology generally leads to asymmetric price impact, while in markets for securities that clear independently the cross-asset price impact is always zero and hence symmetric (Example 4). Second, when the price impact is symmetric—but not otherwise—adjustments in derivative trade to a price change of a correlated asset can mimic the adjustment for a corresponding underlying asset with joint clearing (Proposition 1, Example 3).[7]

The asymmetry of the cross-asset price impacts arises because traders' demands for those assets depend on their inference on *other* asset prices.[8] Cross-asset price impact is asymmetric when either the asset returns or market structure are asymmetric across assets. In that case, price changes in the new exchange's assets induce different inference about those other assets' prices (Corollary 4).[9] Consequently, innovations that let total demands for an underlying asset be contingent on prices of the *same* underlying assets need not be equivalent (Proposition 1).

Second, either instrument can dominate in welfare terms, depending on market characteristics (asset covariances and traders' asset holdings; Example 7). For instance, when the technology that clears some assets jointly increases welfare, the introduction of derivatives for the same assets may not be beneficial. Specifically, asymmetric trading costs can be beneficial when the underlying asset covariances or traders' desired positions are heterogeneous across

[7]If a trader could choose which type of demands to submit for those assets, individual optimization would imply the choice of contingent schedules, as they allow conditioning on the actual realizations of the assets' prices (and hence trades). Yet, the types of schedules the traders can submit are determined by the providers. As technology advances, exchanges allow traders to express their demands for an asset as a function of the prices of other assets. Nevertheless, such cross-asset conditioning is still limited: it applies to only a small number of assets. Our analysis suggests that a provider generally does not have incentives to allow traders to submit fully contingent demands. Specifically, the provider's incentives are aligned with efficiency: the efficiency gains from suitably designed innovation come from a liquidity improvement and more trading volume.

[8]When traders cannot condition their demand for an asset on the price realizations of all other assets, they condition demands on the best estimate of these prices (expected trades).

[9]In terms of market characteristics, the equilibrium price impact is indeed symmetric in markets with fully contingent demands (all securities clear jointly) and markets with uncontingent demands (all securities clear independently). However, in more general market structures, the price impact symmetry requires joint symmetry assumptions on both the asset returns (i.e., the covariances must be the same for all assets; Example 4) and the market structure (i.e., a condition that captures how demands for different assets are linked through market clearing or synthetic products; Example 5). Any asymmetries in market characteristics render cross-asset inference and hence price impact asymmetric.

assets. On the other hand, derivatives offer more flexibility for designing security returns (Example 7), absent derivative weight regulations (e.g., ETF Rule 2019). While always beneficial for a trader's individual optimization problem, synthetic products or exchanges can increase or lower traders' equilibrium welfare by changing their endogenous transaction costs.

Third, when traders have price impact, changes to market-clearing can make new synthetic products nonredundant, but these products would be neutral in markets where securities clear independently (Proposition 2, Example 6). Intuitively, when the cross-asset price impact is asymmetric, even if the inference error is zero among some assets, a derivative defined on these assets' returns is generally nonredundant as it alters the inference about the prices of assets not listed in the exchange where it is introduced (Corollary 1). Conversely, a non-zero inference error among some assets does not imply that a derivative defined on these assets' returns is nonredundant. When derivatives are cleared independently, inference errors among their underlying assets remain non-zero, yet additional derivatives on those assets are redundant given symmetric price impact (Proposition 2).

These results demonstrate that new synthetic products and new trading protocols constitute distinct instruments in financial market design and that they should be regulated jointly. The results also clarify whether regulation of market-clearing technology can be partially bypassed when issuing arbitrary derivatives is allowed. Namely, bypassing is feasible when the underlying assets and the market structure are symmetric. If trading platforms can be creative about synthetic products, rules prohibiting mergers, asset listings, or the addition of new trading protocols may be ineffective.

Fourth, the choice of efficient design may depend on what data on traders' asset holdings is available to the regulator. Holding fixed the traders' initial holdings, derivatives can give strictly higher welfare relative to the maximal welfare feasible by designing market-clearing protocols (Example 7), because of the choice of the derivative weights they allow. Likewise, derivatives can lead to a strictly lower welfare relative to the minimal welfare feasible with a market-clearing design. Hence, regulation that cannot rely on information about traders' portfolios may favor innovation in market-clearing technology. It is a less risky option that offers a greater reduction of the maximal welfare loss from innovations to the uncontingent market for the underlying assets alone. In turn, with information about traders' asset holdings, derivatives have an advantage in design. Increased reporting requirements for asset holdings can make a difference for determining effective innovation.

Related literature. Our paper belongs to the growing body of literature on financial market design with large traders.[10] Our comparative analysis of multi-asset designs and the identifica-

[10]e.g., Allen and Wittwer (2021), Du and Zhu (2017a,b) Baisa and Burkett (2018), Antill and Duffie (2021), Kyle, Obizhaeva, and Wang (2017), Kyle and Lee (2018), Duffie (2018), Zhu (2018a,b), Rostek and Yoon (2019, 2021a,b), Chen and Zhang (2020), Zhang (2022), Babus and Hachem (2021), Cespa and Vives (2021), Chen and Duffie (2021), Somogyi (2021), Wittwer (2021).

tion of their joint effects contributes to the study of markets that dispense with the assumption that demands are contingent (Chen and Duffie (2021), Rostek and Yoon (2021a,b), and Wittwer (2021)).[11] There is no scope for evaluating these innovations in the standard model based on fully contingent demands. Dispensing with the assumption that demands are fully contingent opens up a rich language for designing demand conditioning (and hence market-clearing technology) and synthetic products. The substantive observation is that when demands are not fully contingent, spanning does not hold. Consequently, various types of innovations that are neutral to the underlying assets' span are not neutral to welfare, even when all traders participate in all exchanges and trade all securities. Existing work has examined the equilibrium effects of each innovation separately (Rostek and Yoon (2021a,b)). Our paper shows that there can be trade-offs in employing either instrument and synergies from employing both.

Additionally, our analysis highlights the welfare and design implications of price inference across assets. In particular, the effects we report do not rely on the inference about asset value across traders. Relatively little is known about inference across assets because models for multiple assets commonly apply the assumption that demands are fully contingent (e.g., the survey by Rostek and Yoon (2021c)).[12] We explore how information about traders' asset holdings carried by the asset prices differs when the design allows synthetic products versus demand conditioning among the underlying assets, and how it changes when both instruments are allowed (Corollary 1 and Example 3).

The market structures we analyze are decentralized: not all assets clear jointly. There is a growing interest in how market fragmentation influences security design (e.g., Allen and Carletti (2006), Rahi and Zigrand (2009), Zawadowski (2013), Babus and Hachem (2020, 2021), Biais, Hombert, and Weill (2021); see also Cabrales, Gale, and Gottardi (2015)). Our results draw attention to the role that market fragmentation in the sense of limited demand conditioning rather than limited trader participation plays in financial innovation. We explore how imperfectly competitive fragmented markets motivate innovation in market-clearing technology vis-a-vis or jointly with security design.

Our model introduces a new class of package auction designs with a language that allows two ways of expressing preferences for portfolios with packages (bundles) or combinatorial (contingent) bids. The existing literature on combinatorial allocation problems has largely focused on indivisible-goods auctions in which bidders have unit demands (e.g., Cramton, Shoham, and

[11] Chen and Duffie (2021) examine one-asset markets with noise traders, Wittwer (2021) studies two-asset markets with supply shocks, Rostek and Yoon (2021a) analyze multi-asset markets with exchanges for many assets, and Rostek and Yoon (2021b) examine uncontingent multi-asset markets with derivatives.

[12] Cespa (2004) and Chen and Duffie (2021) examine how uncontingent demands affect the inference across traders. In a competitive market with uncontingent demands, Cespa (2004) shows that the prices can provide more information about the fundamental values than with fully contingent demands, depending on the determinants of asset price covariances, such as fundamental values, informed traders' private information, or noise. Chen and Duffie (2021) show that the prices of the same asset traded in multiple exchanges are each less informative relative to a single exchange, but jointly, the prices are more informative.

Steinberg (2006), Vohra (2011)). Our model encompasses competitive and imperfectly competitive markets, in which traders have multi-unit demands and heterogeneous values about which they are privately informed, as well as arbitrary substitutabilities and complementarities among the asset returns. Our analysis indicates that different types of package bids are generally not equivalent for efficiency or revenue.

2 Model

Our model is based on the uniform-price double auction cast in the quadratic-Gaussian setting (Definition 1). Unlike the standard multi-asset version of that model, where *all* assets clear jointly, we consider markets with K assets that clear independently. In such markets with K underlying assets, we consider two types of innovation. One innovation is the introduction of synthetic products (derivatives) whose returns are linear combinations of the underlying assets' returns, which trade along with the underlying assets and, like the underlying assets, clear independently. We also consider innovation in market clearing for *securities* (i.e., underlying assets and derivatives) that allows demands for some securities to clear jointly rather than independently. Neither innovation affects the underlying asset span, traders' initial asset holdings, or asset supply.

Traders and assets.[13] Consider a market with $I \geq 3$ strategic traders who trade K risky assets whose returns are jointly normally distributed $\mathbf{r} = (r_k)_k \sim \mathcal{N}(\boldsymbol{\delta}, \boldsymbol{\Sigma})$ with a vector of expected returns $\boldsymbol{\delta} = (\delta_k)_k \in \mathbb{R}^K$ and a positive definite covariance matrix $\boldsymbol{\Sigma} = (\sigma_{k\ell})_{k,\ell} \in \mathbb{R}^{K \times K}$. There is also a riskless asset (a numéraire).

Each trader i has a quadratic in the quantity of risky assets (mean-variance) utility:

$$u^i(\mathbf{q}^i) = \boldsymbol{\delta} \cdot (\mathbf{q}^i + \mathbf{q}_0^i) - \frac{\alpha}{2}(\mathbf{q}^i + \mathbf{q}_0^i) \cdot \boldsymbol{\Sigma}(\mathbf{q}^i + \mathbf{q}_0^i), \tag{1}$$

where $\mathbf{q}^i = (q_k^i)_k \in \mathbb{R}^K$ is trade, $\mathbf{q}_0^i = (q_{0,k}^i)_k \in \mathbb{R}^K$ is trader i's initial holding of risky assets, and $\alpha \in \mathbb{R}_+$ is traders' risk aversion. Gains from trade come from risk sharing and diversification: asset holdings are heterogeneous. Asset holdings $\{\mathbf{q}_0^i\}_i$ are traders' private information and are independent of asset returns $\mathbf{r}$. To ensure that the per capita aggregate asset holdings (equivalently, price) is random in the limit large market ($I \to \infty$), we allow for the common value component in traders' asset holdings (see Eq. (13) in Appendix A.1).

Innovation in synthetic products. We let $D \geq 0$ derivatives be traded in addition to the K assets. Traders' asset holdings for the derivatives are zero, $q_{0,d}^i = 0$ for all i and d. The return of derivative d is a linear combination of asset returns $r_d = \mathbf{w}_d'\mathbf{r}$ for some weight vector $\mathbf{w}_d = (w_{dk})_k \in \mathbb{R}^K$, $w_{dk} \in \mathbb{R}$ for any $k \in K$ and $d \in D$. A derivative thus allows

[13] This section closely follows Rostek and Yoon (2021a).

long and short positions of the underlying assets.[14] Let the weight matrix of all derivatives be $\mathbf{W}_d \equiv (\mathbf{w}_1, \cdots \mathbf{w}_D) \in \mathbb{R}^{K\times D}$ and the d^{th} column $\mathbf{w}_d$ correspond to the d^{th} derivative. Given the distribution of K assets and derivatives' weights $\mathbf{W}_d$, the returns of $K + D$ securities are jointly normally distributed according to $\mathcal{N}(\boldsymbol{\delta}^+, \boldsymbol{\Sigma}^+)$, where the moments are

$$\boldsymbol{\delta}^+ \equiv \begin{bmatrix} \boldsymbol{\delta} \\ \mathbf{W}_d'\boldsymbol{\delta} \end{bmatrix} \in \mathbb{R}^{K+D} \quad \text{and} \quad \boldsymbol{\Sigma}^+ \equiv \begin{bmatrix} \boldsymbol{\Sigma} & \boldsymbol{\Sigma}\mathbf{W}_d \\ \mathbf{W}_d'\boldsymbol{\Sigma} & \mathbf{W}_d'\boldsymbol{\Sigma}\mathbf{W}_d \end{bmatrix} \in \mathbb{R}^{(K+D)\times(K+D)}. \tag{2}$$

The utility of trader i depends on his total trades $q_k^i + \sum_d w_{dk} q_d^i$ of each underlying asset k. Given the vector of trades of the K assets, $\mathbf{q}_a^i \equiv (q_k^i)_k \in \mathbb{R}^K$ and the vector of trades of D derivatives, $\mathbf{q}_d^i \equiv (q_d^i)_d \in \mathbb{R}^D$, the vector of total trades is $\mathbf{q}_a^i + \mathbf{W}_d\mathbf{q}_d^i$, and the utility of trader i in (1) is $u^i(\mathbf{q}_a^i + \mathbf{W}_d\mathbf{q}_d^i)$ (Eq. (1)). By the definition of $\boldsymbol{\delta}^+$ and $\boldsymbol{\Sigma}^+$ in Eq. (2), we can equivalently represent utility $u^i(\mathbf{q}_a^i + \mathbf{W}_d\mathbf{q}_d^i)$ by treating the derivatives as distinct assets:

$$u^i(\mathbf{q}^i) = \boldsymbol{\delta}^+ \cdot (\mathbf{q}^i + \mathbf{q}_0^{i,+}) - \frac{\alpha}{2}(\mathbf{q}^i + \mathbf{q}_0^{i,+}) \cdot \boldsymbol{\Sigma}^+(\mathbf{q}^i + \mathbf{q}_0^{i,+}), \tag{3}$$

where $\mathbf{q}^i = (\mathbf{q}_a^i, \mathbf{q}_d^i) \in \mathbb{R}^{K+D}$ is the vector of trades for all $K+D$ securities, and $\mathbf{q}_0^{i,+} = (\mathbf{q}_0^i, \mathbf{0}) \in \mathbb{R}^{K+D}$ is the asset holdings vector whose elements corresponding to derivatives are zeros.

Innovation in market-clearing technology. Along with the synthetic products, we examine innovation in market-clearing technology. Today's financial markets offer many trading protocols for the same or distinct assets. Venues for financial securities are cleared independently. While the assets traded in each venue are typically cleared independently as well, in some markets, including electronic trading platforms for financial assets, traders can express their demands for one asset contingent on the prices of other assets. Advances in technology have sparked interest in innovation in market-clearing technology across multiple exchanges.[15] When available, such contingent orders allow cross-asset conditioning among only a limited number of assets.

Accordingly, our model allows the $K + D$ securities to trade in *exchanges*, each defined by the securities traded (*listed*) there: there can be multiple securities per exchange, and securities

[14] Futures, ETFs, and ETPs are examples of derivatives whose returns are in the linear span of the underlying assets. Although options and exotic derivatives whose returns are non-linear functions of the underlying asset returns are not modeled in this paper, the qualitative effects underlying the welfare trade-off will carry over. The first-order conditions for any of the designs we analyze can be written for general utility functions analogous to those in Wittwer (2021, Appendix F) for the uncontingent model. For derivatives traded over the counter, transactions tend to occur through protocols that resemble bargaining rather than a uniform-price mechanism. The effects studied in this paper are not specific to price mechanisms such as the uniform-price auction: The essence of our results is the inefficiency present with two-sided (buyer and seller) private information among strategic traders in any (budget-balanced) mechanism, and how changes to the available financial products or technology affect that inefficiency.

[15] Electronic trading platforms (e.g., Active Trader Pro, Etrade, Street Smart, Tradehawk) experiment with and innovate such orders.

can be traded in multiple exchanges. All traders participate in all exchanges. Each exchange is organized as the uniform-price double auction (e.g., Kyle (1989), Vives (2011)) in which traders submit (net) demand schedules. Assets clear jointly within an exchange and independently across exchanges.

Definition 1 (Double Auction with Multiple Exchanges) *Consider a market with $M = K + D$ securities. An* exchange *n is defined by the subset of securities traded $K(n) \subseteq M$. A* market structure *$N = \{K(n)\}_n$ is described by N exchanges.*

A trader's demand for security $m \in K(n)$ is contingent on the prices of the securities $K(n)$ traded in exchange n, $\mathbf{p}_{K(n)} = (p_\ell)_{\ell \in K(n)} \in \mathbb{R}^{K(n)}$, $q^i_{m,n}(\cdot) : \mathbb{R}^{K(n)} \to \mathbb{R}$ for $m \in K(n)$. For $q^i_{m,n} > 0$, trader i is a buyer of security m; for $q^i_{m,n} < 0$, the trader is a seller.

Exchanges clear independently. The market-clearing price vector $\mathbf{p}_{K(n)}$ in exchange n is determined by $\sum_j q^j_{m,n}(\mathbf{p}_{K(n)}) = 0$ jointly for all securities $m \in K(n)$ traded in this exchange.[16] *Trader i trades $\{q^i_{m,n}\}_{m,n}$, pays $\sum_{m,n} p_{m,n} q^i_{m,n}$, and receives a payoff of $u^i(\mathbf{q}^i) - \mathbf{p} \cdot \mathbf{q}^i$.*

Example 1 illustrates that demand conditioning determines how the market clears.

Example 1 (Uncontingent and Contingent Markets) Consider a market with $M = K+D$ securities.

(a) *(Contingent market: A single exchange for all securities $N = \{M\}$)* The standard multi-asset model is based on *contingent schedules.* Each trader i submits M demand functions $\mathbf{q}^{i,c}(\cdot) \equiv (q^{i,c}_1(\mathbf{p}), \ldots, q^{i,c}_M(\mathbf{p}))$, with each $q^{i,c}_m(\cdot) : \mathbb{R}^M \to \mathbb{R}$ specifying the quantity of security m demanded for any realization of price vector $\mathbf{p} = (p_1, \ldots, p_M)$. With contingent demands, securities must clear *jointly*: the zero aggregate net demand in all exchanges determines the equilibrium price vector, $\sum_i \mathbf{q}^{i,c}(p_1, \cdots, p_M) = \mathbf{0} \in \mathbb{R}^M$.

(b) *(Uncontingent market: M exchanges, each for one security, $N = \{\{m\}\}_m$)* Each trader i submits M uncontingent demand schedules $\mathbf{q}^i(\cdot) \equiv (q^i_1(p_1), \ldots, q^i_M(p_M))$, with each $q^i_m(\cdot) : \mathbb{R} \to \mathbb{R}$ specifying the quantity of security m demanded for any realization of price p_m. The market clears independently across securities: the zero aggregate net demand in exchange m determines the equilibrium price p_m, $\sum_i q^i_m(p_m) = 0$. □

Example 2 illustrates the two types of innovation we consider: derivatives and market-clearing technology.

Example 2 (Derivatives vs. Innovation in Market Clearing) In the market structure $N = \{\{1\}, \{2\}, \{3\}\}$, consider two types of innovations. First, suppose that a non-replicating derivative that bundles the returns of assets 1 and 2 is introduced to be traded in a separate

[16] If prices $\mathbf{p}_{K(n)}$ such that $\sum_j q^j_{m,n}(\mathbf{p}_{K(n)}) = 0$ do not exist or are not unique for some m and n, the market ends with no trade for all securities.

exchange; the market structure is $N' = \{\{1\},\{2\},\{3\},\{d\}\}$, $r_d = w_{d1}r_1 + w_{d2}r_2$. Each trader i submits demands $q^i_k(p_k) : \mathbb{R} \to \mathbb{R}$ for the underlying assets $k = 1,2,3$ and $q^i_d(p_d) : \mathbb{R} \to \mathbb{R}$ for the derivative. Traders' demands for each security clear independently.

Second, suppose instead that assets 1 and 2 are traded in one exchange. In the market structure $N'' = \{\{1,2\},\{3\}\}$, each trader i submits demands $q^i_1(p_1,p_2) : \mathbb{R}^2 \to \mathbb{R}, q^i_2(p_1,p_2) : \mathbb{R}^2 \to \mathbb{R}$, and $q^i_3(p_3) : \mathbb{R} \to \mathbb{R}$. Traders' demands for asset 3 clear independently of those for assets 1 and 2, which clear jointly. □

These two innovations affect the game in different ways: Security innovations change the joint distribution of *securities' returns*, while technology innovations changes *traders' strategies*.

Equilibrium. We study the Bayesian Nash Equilibrium in linear demand schedules (hereafter, *equilibrium*); i.e., schedules have the functional form of $\mathbf{q}^i(\cdot) = \boldsymbol{a}^i - \mathbf{B}^i\mathbf{q}^i_0 - \mathbf{C}^i\mathbf{p}$.

Definition 2 (Equilibrium) *Consider a market for $M = K+D$ securities with $N = \{K(n)\}_n$ exchanges, with securities $K(n) \subseteq M$ listed in exchange n. A profile of (net) demand schedules $\{\{q^i_{m,n}(\cdot)\}_{m,n}\}_i$ is a linear Bayesian Nash equilibrium if for each i, $\{q^i_{m,n}(\cdot)\}_{m,n}$ maximizes the expected payoff:*

$$\max_{\{q^i_{m,n}(\cdot)\}_{m,n}} E[\boldsymbol{\delta}^+ \cdot (\mathbf{q}^i + \mathbf{q}^{i,+}_0) - \frac{\alpha}{2}(\mathbf{q}^i + \mathbf{q}^{i,+}_0) \cdot \boldsymbol{\Sigma}^+ (\mathbf{q}^i + \mathbf{q}^{i,+}_0) - \mathbf{p} \cdot \mathbf{q}^i | \mathbf{q}^i_0], \tag{4}$$

given the schedules of other traders $\{\{q^j_{m,n}(\cdot)\}_{m,n}\}_{j \neq i}$ and market clearing $\sum_j q^j_{m,n}(\cdot) = 0$ for all m and n.

We will evaluate the effects of innovation with imperfectly competitive traders against the competitive-market benchmark.

Definition 3 (Competitive Market, Competitive Equilibrium) *Consider a market with $I < \infty$ traders. The* competitive market *is the limit game as $I \to \infty$, holding fixed all other primitives. Letting $\{\mathbf{q}^{i,I}(\cdot)\}_i$ be the equilibrium in the market with $I < \infty$ traders, the* competitive equilibrium *$\{\mathbf{q}^i(\cdot)\}_i$ is the limit of equilibria $\{\mathbf{q}^{i,I}(\cdot)\}_i$ as $I \to \infty$:*

$$\mathbf{q}^i(\cdot) = \lim_{I \to \infty} \mathbf{q}^{i,I}(\cdot) \qquad \forall i.$$

3 Equilibrium

In this section, we characterize equilibrium with M securities (Definition 1), which include K assets and D derivatives traded in market structure $N = \{K(n)\}_n$, where $K(n) \subseteq M$ for each n (Theorem 1 and Corollary 2 in Appendix A.1).

Optimization problem. Consider the optimization problem (4). The well-known equivalence between individual trader optimization in (net) demand functions contingent on price realizations (i.e., the optimization problem (4)) and *pointwise* optimization with respect to the realizations of $\mathbf{p} \in \mathbb{R}^M$ in the contingent model also holds in market structure $N = \{K(n)\}_n$ (Definition 1) with respect to the realizations of the relevant contingent variables, $\mathbf{p}_{K(n)} \in \mathbb{R}^{K(n)}$: for each trader i, for each security m in exchange n,

$$\max_{q^i_{m,n} \in \mathbb{R}} E[\boldsymbol{\delta}^+ \cdot (\mathbf{q}^i + \mathbf{q}_0^{i,+}) - \frac{\alpha}{2}(\mathbf{q}^i + \mathbf{q}_0^{i,+}) \cdot \boldsymbol{\Sigma}^+ (\mathbf{q}^i + \mathbf{q}_0^{i,+}) - \mathbf{p} \cdot \mathbf{q}^i | \mathbf{p}_{K(n)}, \mathbf{q}_0^i] \qquad \forall \mathbf{p}_{K(n)} \in \mathbb{R}^{K(n)}, \quad (5)$$

given a profile of his residual supply functions $\{S^{-i}_{\ell,n'}(\cdot) \equiv -\sum_{j \neq i} q^j_{\ell,n'}(\cdot) : \mathbb{R}^{K(n')} \to \mathbb{R}\}_{\ell,n'}$ and his demands for other securities $\{q^i_{\ell,n}(\cdot)\}_{\ell \neq m}$ and $\{q^i_{\ell,n'}(\cdot)\}_{\ell,n' \neq n}$. A profile of a trader's residual supply functions is the sufficient statistic of his residual market $\{\{q^j_{m,n}(\cdot)\}_{m,n}\}_{j \neq i}$.[17]

The security by security pointwise optimization (5) implies that the first-order conditions of trader i equalize, for each security $m \in K(n)$ in exchange n, his *expected* marginal utility with *expected* marginal payment for all $\mathbf{p}_{K(n)} \in \mathbb{R}^{K(n)}$:

$$\delta_m - \alpha \boldsymbol{\Sigma}^+_m E[\mathbf{q}^i + \mathbf{q}_0^{i,+} | \mathbf{p}_{K(n)}, \mathbf{q}_0^i] = p_m + (\boldsymbol{\Lambda}^i_{K(n)})_m \mathbf{q}^i_{K(n)} \qquad \forall \mathbf{p}_{K(n)} \in \mathbb{R}^{K(n)}, \quad (6)$$

where $\boldsymbol{\Lambda}^i_{K(n)} \equiv (\frac{dp_m}{dq^i_\ell})_{\ell,m} \in \mathbb{R}^{K(n) \times K(n)}$ is the *price impact* of trader i (i.e., "Kyle's lambda") in exchange n, and the m^{th} row of $\boldsymbol{\Lambda}^i_{K(n)}$ is denoted by $(\boldsymbol{\Lambda}^i_{K(n)})_m$.

Price impact. For each trader i, price impact $\boldsymbol{\Lambda}^i_{K(n)} \equiv (\frac{d\mathbf{p}_{K(n)}}{d\mathbf{q}^i_{K(n)}})'$ in every exchange n is characterized as the transpose of the Jacobian matrix of the inverse residual supply (Eq. (19)):

$$\boldsymbol{\Lambda}^i_{K(n)} = -\big(\big(\sum_{j \neq i} \frac{\partial \mathbf{q}^j_{K(n)}(\cdot)}{\partial \mathbf{p}_{K(n)}}\big)^{-1}\big)'. \quad (7)$$

Since the market clears independently across exchanges, traders' cross-exchange price impacts $\lambda^i_{m\ell} \equiv \frac{dp_\ell}{dq^i_m}$ are zero for $m \in K(n)$, $\ell \in K(n')$, and $n \neq n'$. Hence, the traders' price impacts are block-diagonal matrices: $\boldsymbol{\Lambda}^i \equiv diag(\boldsymbol{\Lambda}^i_{K(n)})_n \in \mathbb{R}^{(\sum_n K(n)) \times (\sum_n K(n))}$ for all i. Nevertheless, the equilibrium outcome is not independent across exchanges due to cross-exchange inference. We next explore how the cross-exchange inference affects the price impact in each exchange.

Even though each innovation affects the equilibrium price impact differently (Eq. (7))—technology innovations change the restrictions on which element of the price impact matrix is zero, and security innovations change the dimension and the basis of the space where the price impact matrix lies—we can construct a sufficient statistic for comparing welfare across market structures with either or both innovations, the per-unit price impact, defined for a

[17] The idea of considering a trader's pointwise optimization problem, taking as given the trader's residual market, goes back to Klemperer and Meyer (1989) and Kyle (1989).

counterfactual for a single exchange for K assets. See Definition 4 and Lemma 1 below.

Cross-asset inference. When demands are not contingent, traders cannot express the quantity demanded of a security as a function of the price realizations of other securities. Thus, they rely on expected prices (which are one-to-one with expected trades in Eq. (6)). Consequently, unlike in the contingent design, traders' equilibrium price impacts depend on cross-asset inference, as their demands (i.e., the demand Jacobians $\frac{\partial \mathbf{q}^i(\cdot)}{\partial \mathbf{p}}$) do. Innovation influences how traders' private information $\{\mathbf{q}_0^i\}_i$ is aggregated in the outcome, which impacts diversification across securities and risk sharing.

To see the cross-exchange *inference effect* in price impact, consider the counterfactual in which trader i increases his demand for security $m \in K(n)$. Other traders who assume the optimal behavior of all others interpret the higher price of security m as the result of a lower average initial holding for security m and update their posteriors about prices in other exchanges $\mathbf{p}_{K(n')}$ accordingly (cf. the term $\frac{\partial E[\mathbf{p}_{K(n')}|\mathbf{p}_{K(n)},\mathbf{q}_0^j]}{\partial \mathbf{p}_{K(n)}}$ in Eq. (8) below). This updating by other traders gives rise to an *inference effect* in trader i's price impact $\mathbf{\Lambda}^i_{K(n)}$:[18]

$$\mathbf{\Lambda}^i_{K(n)} = -\Big(\underbrace{\sum_{j\neq i} \frac{\partial \mathbf{q}^j_{K(n)}}{\partial \mathbf{p}_{K(n)}}}_{\text{Direct effect}} + \underbrace{\sum_{j\neq i}\sum_{n'\neq n} \frac{\partial \mathbf{q}^j_{K(n)}}{\partial \mathbf{p}_{K(n')}} \frac{\partial E[\mathbf{p}_{K(n')}|\mathbf{p}_{K(n)}, \mathbf{q}_0^j]}{\partial \mathbf{p}_{K(n)}}}_{\text{Inference effect}} \Big)^{-1}, \tag{8}$$

Eq. (8) shows that the price impact in each exchange n depends on the joint return distribution of *all* securities within and across exchanges. Inference effects in the price impact change the effect of innovation (Proposition 1 and Example 4) and their welfare implications (Section 4.3).

Although the cross-exchange inference and price impact create a complex fixed-point problem, we show that price impact is a sufficient statistic for equilibrium (Theorem 1, stated in Appendix A.1).[19] In particular, when characterized as a fixed point in price impacts $\{\mathbf{\Lambda}^i\}_i$, the equilibrium fixed point endogenizes traders' conditional inference $\{E[q^i_\ell|\mathbf{p}_{K(n)}, \mathbf{q}^i_0]\}_{\ell\notin K(n),n}$ for all i as functions of price impacts $\{\mathbf{\Lambda}^i\}_i$.

Remarks.

1. Traders' initial asset holdings do not change with the introduction of a derivative or an exchange. Equilibrium is neutral to an arbitrary split of initial asset holdings for duplicated assets (Eqs. (16)-(17) of Theorem 1 in Appendix A.1).

[18]Direct and inference effects in price impact (Eq. (8)) are characterized by differentiating the first-order condition (6) of trader $j \neq i$ in exchange n with respect to $\boldsymbol{p}_{K(n)}$ and substituting $\frac{\partial \boldsymbol{q}^j_{K(n)}(\cdot)}{\partial \boldsymbol{p}_{K(n)}}$ into Eq. (7). As a result, $\frac{\partial \mathbf{q}^j_{K(n)}}{\partial \mathbf{p}_{K(n)}} \equiv -(\alpha \mathbf{\Sigma}^+_{K(n),K(n)} + \mathbf{\Lambda}^j_{K(n)})$ and $\frac{\partial \mathbf{q}^j_{K(n)}}{\partial \mathbf{p}_{K(n')}} \equiv -(\alpha \mathbf{\Sigma}^+_{K(n),K(n)} + \mathbf{\Lambda}^j_{K(n)})\alpha \mathbf{\Sigma}^+_{K(n),K(n')} \frac{\partial \mathbf{q}^j_{K(n')}(\cdot)}{\partial \mathbf{p}_{K(n')}}$ for each $n' \neq n$.

[19]Theorem 1 in Appendix A.1 extends the equilibrium characterization for uncontingent markets with securities (Rostek and Yoon (2021b)) and for exchanges with multiple assets (Rostek and Yoon (2021a)) to market structures that allow both instruments.

2. Analyzing price impact directly will help identify the role of imperfect competition. As $I \to \infty$, then $\mathbf{\Lambda} \to \mathbf{0}$ (Lemma 2 in Appendix A.1). With positive price impact $\mathbf{\Lambda} > \mathbf{0}$, trader i demands (or sells) less relative to the schedules he would submit if the market were competitive.

4 Derivatives vs. Market-Clearing Technology

In this section, we address the main question of the paper: How do the equilibrium and welfare effects of new market-clearing protocols and synthetic products for the corresponding underlying assets differ?

Relative to opening multi-asset exchanges, derivatives allow the design of the traded securities' covariance, i.e., the weights with which asset returns are bundled. Given the additional degree of freedom in the weights, could innovation of synthetic products substitute for innovation in market-clearing technology state by state (i.e., for all realizations of asset holdings)? We show that this is generally not true (Proposition 1), because derivatives give rise to symmetric price impact while the price impact is generally asymmetric with technology innovations (Example 3). Conversely, exchanges cannot reproduce the effects of derivatives either (Proposition 1, Example 4 (b)). Taking as an objective the traders' *ex ante* total welfare, $\sum_i E[u^i(\mathbf{q}^i) - \mathbf{p} \cdot \mathbf{q}^i]$ (Eq. (25)), we present three results.

First, the market primitives matter for the payoff equivalence between synthetic products and technology innovations. Any asymmetries in market characteristics—the fundamental asset covariances or the market structure, which captures how assets are linked through exchanges—render cross-asset inference and hence price impact asymmetric (Corollary 4 in Appendix A.2, Examples 4 (a) and 5). Second, innovation in market-clearing technology renders additional synthetic products nonredundant (Proposition 2, Example 6). Third, either type of innovation can dominate in welfare terms, given the traders and assets; however, derivatives create a riskier distribution of welfare across realizations of asset holdings (Example 7).

When comparing the effects of new synthetic products and market-clearing protocols in this section, we focus on the impact of new securities traded along with underlying assets. In practice, traders often trade synthetic products because their access to the underlying assets is restricted. Our results (Propositions 1 and 2) apply to security innovation in markets in which some or all of the underlying assets are not traded.

Sufficient statistic for equilibrium and welfare ranking. When markets clear independently, innovation in market-clearing technology or securities—which are redundant if well-defined in markets with fully contingent demands—are generally not neutral, as they affect cross-exchange inference and price impact (Rostek and Yoon (2021a,b)). To compare the equilibrium and welfare effects of these innovations in subsequent sections, price impact matrices $\mathbf{\Lambda}$ cannot be ranked directly; not only do their dimensions and the corresponding securities differ across the arbitrary market structures we allow, but the two innovations also change the game in different ways: Technology innovations change *traders'* strategies, while security innovations change the joint distribution of *securities'* returns (covariances).

One can still relate equilibrium outcomes across markets with arbitrary market structures and arbitrary securities through the *per-unit price impact* $\widehat{\mathbf{\Lambda}} \in \mathbb{R}^{K\times K}$ (Definition 4 and Lemma 1), as $\mathbf{W}$ is defined for a weight matrix or an indicator matrix.

Notation 1. The weight matrix $\mathbf{W} \equiv (\mathbf{W}_1, \cdots, \mathbf{W}_N) \equiv ((\mathbf{w}_{m,n})_{m\in K(n)})_n \in \mathbb{R}^{K\times(\sum_n K(n))}$ represents the weight vector $\mathbf{w}_{m,n} \in \mathbb{R}^K$ of each security m in each exchange n for the underlying assets in each exchange n, given the market structure $N = \{K(n)\}_n$.

Definition 4 (Per-Unit Price Impact) *Suppose* $\widehat{\mathbf{q}}^i \equiv (\widehat{q}^i_k)_k = \mathbf{W}\mathbf{q}^i \in \mathbb{R}^K$ *is trader i's total equilibrium trade in market* $N = \{K(n)\}_n$ *for securities M. The* per-unit price impact $\widehat{\mathbf{\Lambda}} \in \mathbb{R}^{K\times K}$ *is a positive semi-definite matrix, such that*

$$E[\widehat{\mathbf{q}}^i] \equiv E[\mathbf{W}\mathbf{q}^i] = (\alpha\mathbf{\Sigma} + \widehat{\mathbf{\Lambda}})^{-1}\alpha\mathbf{\Sigma}(E[\overline{\mathbf{q}}_0] - E[\mathbf{q}^i_0]) \qquad \forall i \quad \forall\{E[\mathbf{q}^i_0]\}_i \in \mathbb{R}^{IK}. \tag{9}$$

The per-unit price impact maps traders' price impacts of any dimension defined by a market structure with arbitrary exchanges for any subset of underlying assets and synthetic products onto the smaller space of the underlying assets.[20] It matches the moments of each trader's total equilibrium trade $\widehat{\mathbf{q}}^i \equiv (\widehat{q}^i_k)_k = \mathbf{W}\mathbf{q}^i \in \mathbb{R}^K$, as if the market consisted of a single exchange for the K underlying assets. Then, a change in this *single-exchange* counterfactual $\widehat{\mathbf{\Lambda}}$ identifies nonredundant innovation (Lemma 1).

Lemma 1 (Nonredundancy of Innovation) $I < \infty$ and consider two markets: $N = \{K(n)\}_n$ with $M \geq 1$ securities and $N' = \{K(n')\}_{n'}$ with $M' \geq 1$ securities. The innovation is redundant, i.e.,

$$u^{i,N}(\mathbf{q}^{i,N}) - \mathbf{p}^N \cdot \mathbf{q}^{i,N} = u^{i,N'}(\mathbf{q}^{i,N'}) - \mathbf{p}^{N'} \cdot \mathbf{q}^{i,N'} \qquad \forall i \qquad \forall\{\mathbf{q}^i_0\}_i \in \mathbb{R}^{IK},$$

if and only if the per-unit price impacts coincide: $\widehat{\mathbf{\Lambda}}^N = \widehat{\mathbf{\Lambda}}^{N'}$.

Remark.

3. In the standard model, based on fully contingent demands, all innovations—new exchanges or new securities—are redundant. Equilibrium is *ex post*, and price impact is independent of inference. Indeed, applied to the contingent market for arbitrary M securities, Theorem

[20] Although the per-unit price impact is not by itself a sufficient statistic for equilibrium (see Eq. (25)), it allows a welfare comparison through the counterfactual price impact of dimension $K \times K$ for the underlying assets themselves.

1 in Appendix A.1 gives the price impact of[21]

$$\mathbf{\Lambda}^c = \frac{\alpha}{I-2}\mathbf{\Sigma}^+. \tag{10}$$

Substituting for the price impact in Eq. (10) into Eq. (23) in Lemma 1 shows that the per-unit price impact $\widehat{\mathbf{\Lambda}}^c = \frac{\alpha}{I-2}\mathbf{\Sigma}$ is invariant to the change in $\mathbf{W}$. Hence, innovation (in synthetic products or market-clearing) does not change equilibrium outcomes—the underlying asset prices $\mathbf{p}_a^c = \boldsymbol{\delta} - \alpha\mathbf{\Sigma}\overline{\mathbf{q}}_0$ and their *total trade* $\mathbf{q}_a^{i,c} + \mathbf{W}_d\mathbf{q}_d^{i,c} = \frac{I-2}{I-1}(\overline{\mathbf{q}}_0 - \mathbf{q}_0^i)$—and traders' payoffs in the contingent market.

4.1 Derivatives and Technology Innovations Are Not Equivalent

Example 3 illustrates when, given the traders and the underlying assets, an equilibrium in a market structure with exchanges that clear multiple assets jointly can be mimicked by a market structure with derivatives whose returns bundle the corresponding assets traded in each exchange and which clears securities independently.

Example 3 (When Can Derivatives Implement the Contingent Market Outcome?) Consider a market with $K = 2$ underlying assets. Holding fixed other traders' strategies, a trader wishes to mimic his fully contingent demands in $N = \{\{1, 2\}\}$. When a derivative whose returns are defined over the two assets is introduced instead, i.e., $N' = \{\{1\}, \{2\}, \{d\}\}$, a trader absorbs some units of both assets, thus hedging their correlated risk, which he cannot do with just the uncontingent demands for the underlying assets. We will show that the symmetry of price impact—which always holds in two asset markets N and N'—is crucial for whether the trader's total demands for each underlying asset can match the contingent demands.

(a) A trader prefers fully contingent demands, for any residual market. Let trader i's price impact matrix be an arbitrary positive definite matrix $\mathbf{\Lambda}^i$. Absent restrictions on demand conditioning, optimization implies that trader i would choose the fully contingent demands for the two assets $\boldsymbol{q}^{i,c}(p_1, p_2) = (\alpha\mathbf{\Sigma} + \mathbf{\Lambda}^i)^{-1}(\boldsymbol{\delta} - \boldsymbol{p} - \alpha\mathbf{\Sigma}\boldsymbol{q}_0^i)$, irrespective of the trader's price impact and its symmetry. Suppose that these fully contingent demands are

$$\begin{aligned} q_1^{i,c}(p_1, p_2) &= const. - c_{11}^c p_1 - c_{12}^c p_2 \\ q_2^{i,c}(p_1, p_2) &= const. - c_{21}^c p_1 - c_{22}^c p_2. \end{aligned}$$

On the other hand, in an uncontingent market with a derivative $r_d = w_1 r_1 + w_2 r_2$, trader i

[21] Theorem 1 in Appendix A.1 applies to fully contingent demands by replacing the matrix operator $[\mathbf{M}]_N$ (which captures that demand for each security is contingent only on the prices listed in the same exchange) with $\mathbf{M}$ (which captures that demand for each security is contingent on all securities' prices) for any matrix $\mathbf{M} \in \mathbb{R}^{M\times M}$.

splits his total demand for asset 1 between the underlying asset 1 and the derivative:

$$\begin{aligned} q_1^i(p_1) + w_1 q_d^i(p_d) &= const. - c_{11}^c p_1 + \frac{c_{12}^c}{w_2} w_1 p_1 - \frac{c_{12}^c}{w_2} \underbrace{(w_1 p_1 + w_2 p_2)}_{=p_d} \\ &= \underbrace{const. - (c_{11}^c - \frac{c_{12}^c}{w_2} w_1) p_1}_{=q_1^i(p_1)} + w_1 \underbrace{- \frac{c_{12}^c}{w_1 w_2} p_d}_{=q_d^i(p_d)}. \end{aligned}$$

This split allows the trader to mimic the fully contingent demand for asset 1: trader i hedges the risks in asset 1 (correlated and asset-specific) perfectly.

(b) The symmetry of price impact $\mathbf{\Lambda}^i$ is necessary for two market structures to be equivalent. By an analogous argument, the trader splits his total demand for asset 2 into $\widetilde{q}_2^i(p_2) = const. - (c_{22}^c - \frac{c_{21}^c}{w_1} w_2) p_2$ and $\widetilde{q}_d^i(p_d) = -\frac{c_{21}^c}{w_1 w_2} p_d$. For the constructed demands $(q_1^i(p_1), q_d^i(p_d))$ that mimic the fully contingent demand for asset 1 and $(\widetilde{q}_2^i(p_2), \widetilde{q}_d^i(p_d))$ that mimic the fully contingent demand for asset 2 to constitute equilibrium, $q_d^i(p_d) = \widetilde{q}_d^i(p_d)$ must hold. Hence, the key step for two innovations to be substitutable is whether $c_{12}^c = c_{21}^c$, equivalently, $\lambda_{12}^c = \lambda_{21}^c$ holds.

When $\lambda_{12}^c = \lambda_{21}^c$, the equivalence result is invariant to the choice of weights (w_1, w_2) of a non-replicating derivative: The total demands for each underlying asset are the same and mimic the fully contingent demands for the underlying assets. However, if $\lambda_{12}^c \neq \lambda_{21}^c$,[22] the total demand in N' differs from the contingent one and thus cannot be optimal pointwise with respect to all realizations of both asset prices, i.e., cannot hedge the trader's risks in assets 1 and 2. Equivalently, the inference errors between the total trades for assets 1 and 2 are non-zero (Eq. (8)). Consequently, the derivative cannot mimic equilibrium with contingent demands. □

Example 3 naturally extends to markets with more than two assets: For any $K \geq 2$, derivatives can substitute for merging exchanges state by state (i.e., for all realizations of asset holdings) if all exchanges merge, thus implementing the contingent market for all assets K. This raises the question whether an analogous result holds for innovations that involve joint market clearing for some but not all underlying assets. As indicated by Example 3 (b), Proposition 1 shows that this is not possible unless the per-unit price impact is symmetric. In fact, its proof shows that equilibrium with exchanges cannot be reproduced even with multiple derivatives for these corresponding assets or additional derivatives that include other assets.

Proposition 1 (Derivatives and Technology Innovations Are Generally Not Equivalent) Let $I < \infty$ and $N = \{K(n)\}_n$. Suppose that a new exchange n' is introduced in market

[22] When the cross-asset price impact is symmetric—e.g., it always is when $K = 2$—the cross-asset demand substitution is symmetric $c_{12}^c = c_{21}^c$ and linear pricing holds $p_d = w_1 p_1 + w_2 p_2$; thus, additional derivatives are redundant. However, in markets with $K > 2$ and an asymmetric price impact (e.g., in the market structure $\{1, 2\}, \{3\}$), $c_{12}^c \neq c_{21}^c$ and $p_d \neq w_1 p_1 + w_2 p_2$. Thus, trading additional derivatives cannot allow traders to mimic the demands in $\{\{1, 2\}, \{3\}\}$.

N, i.e., $N' = N \cup \{n'\}$. There exists a set of D derivatives such that traders' *ex ante* equilibrium payoffs in market structure $N'' = N \cup \{\{d\} : d \in D\}$ are the same as those in market structure N' for all distributions of asset holdings $F(\boldsymbol{q}_0^i)_i$ if and only if one of the following conditions holds:

(i) The price impact in exchange n' is symmetric, i.e., $\boldsymbol{\Lambda}_{K(n')}^{N'} = (\boldsymbol{\Lambda}_{K(n')}^{N'})'$.

(ii) The new exchange n' is redundant, i.e., $\widehat{\boldsymbol{\Lambda}}^N = \widehat{\boldsymbol{\Lambda}}^{N'}$.

Remarks.

4. Proposition 1 extends the existence and uniqueness of equilibrium in Rostek and Yoon (2021b, Proposition 7); see Corollary 3 for a class of symmetric markets in Appendix A.2.

5. The proof of Proposition 1 allows many new exchanges. The proof also shows that exchanges generally cannot mimic derivatives state by state, given the exchanges' asymmetric price impact (as in Proposition 1(i)) and the flexibility of derivatives' weights on all assets (see Example 4(b)).

6. Corollary 4 in Appendix A.2 provides a sufficient condition on the primitives for derivatives to substitute for innovations in market-clearing technology: When the market structure and asset covariances are symmetric for all asset pairs in new exchange n' (Definition 5 below), condition (i) in Proposition 1 holds.

Proposition 1 provides a necessary and sufficient condition for derivatives to mimic innovation in market-clearing technology state by state: The equilibrium cross-asset price impacts must be symmetric,

$$\lambda_{k\ell}^{N'} = \lambda_{\ell k}^{N'} \quad \forall k, \ell \in K(n');$$

hence the per-unit price impacts must be symmetric. Then, demands for derivatives can reproduce the same per-unit price impacts: When asset k's price changes, a trader's uncontingent demand for the correlated assets $\ell \neq k$ cannot adjust but the demand for a derivative that weighs assets k and ℓ can. However, when the demand substitution is not symmetric between assets, i.e., $\frac{\partial V^i(\cdot;\boldsymbol{\Lambda})}{\partial q_k^i \partial q_\ell^i} \neq \frac{\partial V^i(\cdot;\boldsymbol{\Lambda})}{\partial q_\ell^i \partial q_k^i}$, where $V^i(\cdot;\boldsymbol{\Lambda})$ is trader i's expected payoff, equivalently, when price impact $\boldsymbol{\Lambda}_{K(n')}^{N'}$ is not symmetric, no derivative—no function of p_d alone—can mimic the asymmetric cross-asset adjustments to asset k's price changes and those of asset ℓ's price changes simultaneously (Example 3).

In which markets can we expect price impacts to be asymmetric? We can recast the condition on the endogenous price impacts (condition (i) in Proposition 1) in terms of the market primitives (Corollary 4 in Appendix A.2). When all securities clear jointly, the asymmetry does not arise: In the contingent market, the cross-asset inference is perfect and therefore symmetric. Asymmetry also does not arise when all securities clear independently: In the uncontingent market, the cross-asset price impact is zero. In more general market structures, however, the equilibrium price impact is symmetric only under the condition of joint symmetry among all the underlying asset covariances (i.e., $\sigma_{kk} = \sigma$ for all $k \in K(n)$ and $\sigma_{k\ell} = \sigma\rho$ for all $k \in K(n)$ and $\ell \notin K(n)$ for all n) and the market structure *across underlying assets* (Definition 5). Intuitively, a market structure is symmetric if each asset is linked with others—via exchanges or derivative weights—in the same way.

Definition 5 (Symmetric Market Structure) *There are K underlying assets. A market structure $N = \{K(n)\}_n$ for M securities is* symmetric for assets k and $\ell \neq k$ *if the following conditions both hold:*

(i) (Symmetric exchanges) For any exchange n such that $k \in K(n)$, either $\ell \in K(n)$ or there exists an exchange n' such that $K(n') \setminus \{\ell\} = K(n) \setminus \{k\}$; and

(ii) (Symmetric derivatives) For any derivative d with return $r_d = \sum_{k' \in K} w_{dk'} r_{k'}$, $w_{dk} \neq 0$, there exists a derivative $d' \in M$ whose weights satisfy $w_{dk} = w_{d'\ell}$, $w_{d\ell} = w_{d'k}$, and $w_{dk'} = w_{d'k'}$ for all $k' \neq k, \ell$.

A market structure $N = \{K(n)\}_n$ for M securities is symmetric *if conditions (i) and (ii) hold for all asset pairs k and $\ell \neq k$.*

Example 5 illustrates the role that market asymmetries play in redundancy. Example 4 explains why price impact is asymmetric and expounds the advantages of each type of innovation.

Example 4 (Derivatives vs. Technology Innovation) Consider the introduction of a derivative in market structure N' and the exchange merger in N'' for assets 1 and 2 in Example 2. These innovations do not generally yield the same outcomes state-by-state, because they induce different cross-asset inference and therefore different cross-asset per-unit price impact (i.e., the conditions of Proposition 1 are violated).

(a) *(Derivatives cannot mimic cross-asset demand conditioning)* In the symmetric market structure N'' (Definition 5), assets 1 and 2 clear jointly, and thus involve no mutual inference effects. When these assets are asymmetrically correlated with asset 3 (i.e., condition (i) in Corollary 4 in Appendix A.2 is violated), the inference effects between each of these assets and asset 3 differ while the direct effects are the same. Therefore, the cross-asset price impacts differ as well.

For example, if asset 3 is correlated with asset 2 but not with 1 (i.e., $\sigma_{13} = 0, \sigma_{23} \neq 0$), a price change of asset 2 has no inference effect on the demands for asset 1 (i.e., $\alpha\sigma_{13} E[q_3^i(p_3)|p_1, p_2, \mathbf{q}_0^i] = 0$ in Eq. (6) for asset 1), whereas a price change of asset 1 has a non-zero inference effect on other traders' demands for asset 2 due to the imperfect inference about asset 3's price:

$$\frac{\partial q_2^j(\cdot)}{\partial p_1} = \underbrace{\frac{\partial q_2^j}{\partial p_1}}_{\text{Direct effect}} + \underbrace{\frac{\partial q_2^j}{\partial p_3} \frac{\partial E[p_3|p_1, p_2, \mathbf{q}_0^i]}{\partial p_1}}_{\text{Inference effect}\neq 0}. \tag{11}$$

Notably, the prices of assets 1 and 3 are not independent, and hence $\frac{\partial E[p_3|p_1,p_2,\mathbf{q}_0^i]}{\partial p_1} \neq 0$, even when $\sigma_{13} = 0$. With fully contingent demands, the price of asset k incorporates aggregate information $\overline{q}_{0,k}$ for *correlated* assets alone (Eq. (10) in Remark 4). However, with a restriction of demand conditioning, the asset price depends on the aggregate information for *all* assets (Eq. (20) in Appendix A.1). Consequently, the price covariance between two assets depends not only on the asset covariance of that pair, but also all other asset covariances. The per-unit price impact in $\{\{1,2\},\{3\}\}$ is *symmetric*: $\mathbf{\Lambda}^{N''}_{\{1,2\}} = (\mathbf{\Lambda}^{N''}_{\{1,2\}})'$ (hence, $\widehat{\mathbf{\Lambda}}^{N''} = (\widehat{\mathbf{\Lambda}}^{N''})'$) if and only if the asset covariances are symmetric between assets 1 and 2 (i.e., $\sigma_{11} = \sigma_{22}$ and $\sigma_{13} = \sigma_{23}$).

With the derivative instead, the cross-asset price impacts of $\mathbf{\Lambda}^{N'}$ are zero. The per-unit price impact matrix $\widehat{\mathbf{\Lambda}}^{N'} = (\mathbf{W}(\mathbf{\Lambda}^{N'})^{-1}\mathbf{W}')^{-1})^{-1} = ((\mathbf{\Lambda}^{N'}_a)^{-1} + \mathbf{W}_d(\mathbf{\Lambda}^{N'}_d)^{-1}\mathbf{W}'_d)^{-1}$ is symmetric by the symmetry of $\mathbf{\Lambda}^{N'}_a = diag(\lambda^{N'}_k)_k$ and $\mathbf{\Lambda}^{N'}_d = diag(\lambda^{N'}_d)_d$. It follows from Lemma 1 that the introduction of an exchange and a derivative for the same underlying assets leads to distinct equilibrium price impacts and outcomes.

(b) *(Cross-asset demand conditioning cannot mimic derivatives)* Derivatives can induce cross-asset per-unit price impact that exchanges cannot. In the market structure of Example 2, $\{\{1\},\{2\},\{3\}\}$, suppose that a derivative d whose return bundles *all* assets is introduced: $r_d = \sum_k w_k r_k$ for $w_k \neq 0$ for all k. Denoting the price impact in this market by $\Lambda = diag((\lambda_k)_k, \lambda_d)$, the per-unit price impact is

$$\widehat{\mathbf{\Lambda}} = \begin{bmatrix} \frac{\lambda_2\lambda_3\lambda_d^2\overline{\lambda}}{(\lambda_d+w_2^2\lambda_2)(\lambda_d+w_3^2\lambda_3)} & -\frac{w_1w_2\overline{\lambda}}{\lambda_d\lambda_3} & -\frac{w_1w_3\overline{\lambda}}{\lambda_d\lambda_2} \\ -\frac{w_1w_2\overline{\lambda}}{\lambda_d\lambda_3} & \frac{\lambda_1\lambda_3\lambda_d^2\overline{\lambda}}{(\lambda_d+w_1^2\lambda_1)(\lambda_d+w_3^2\lambda_3)} & -\frac{w_2w_3\overline{\lambda}}{\lambda_d\lambda_1} \\ -\frac{w_1w_3\overline{\lambda}}{\lambda_d\lambda_2} & -\frac{w_2w_3\overline{\lambda}}{\lambda_d\lambda_1} & \frac{\lambda_1\lambda_2\lambda_d^2\overline{\lambda}}{(\lambda_d+w_1^2\lambda_1)(\lambda_d+w_2^2\lambda_2)} \end{bmatrix}, \tag{12}$$

where $\overline{\lambda} \equiv 1/det(\widehat{\mathbf{\Lambda}}^{-1})$. The per-unit price impact $\widehat{\mathbf{\Lambda}}$ (Eq. (12)) has *non-zero off-diagonal elements* for all k and $\ell \neq k$, which generally differ from those in the contingent market, where $(\mathbf{\Lambda}^c = \frac{\alpha}{I-2}\mathbf{\Sigma})$.[23] This is because when demands are not contingent, the cross-asset price impact for any asset pair k and $\ell \neq k$ depends on *all* asset covariances and not merely $\sigma_{k,\ell}$. Notably, one derivative cannot generally make equilibrium *ex post*.[24]

Consider now designs with exchanges. All cross-asset demand slopes $\frac{\partial \overline{q}^i_\ell}{\partial p_k} = \frac{1}{I-1}((\widehat{\mathbf{\Lambda}}^{N''})^{-1})'_{k\ell}$ can be non-zero if and only if each pair of assets $k, \ell \neq k$ clears *jointly* in some exchange. However, with exchanges, this would imply that either equilibrium outcome coincides with that in a single exchange for all assets or the equilibrium price impact matrix must be asymmetric.[25] Neither case can mimic the per-unit price impact in the market $\{\{1\},\{2\},\{3\},\{d\}\}$.

Market $\{\{1\},\{2\},\{3\},\{d\}\}$ is not locally contingent for assets $1, 2,$ and 3 (i.e., condition

[23]This can be shown using the proof of Corollary 3 in Rostek and Yoon (2021b).

[24]The exception occurs in markets where asset covariances and derivative weights are symmetric among all assets (i.e., $\sigma_{kk} = \sigma$ for all k, $\sigma_{k\ell} = \sigma\rho$ for all k, $\ell \neq k$, and $w_k = w_\ell$ for all $k, \ell \neq k$).

[25]This can be shown using Corollary 2 in Rostek and Yoon (2021a).

Proposition 2 (i) does not hold) while technology innovation cannot link all pairs of these assets without introducing the locally contingent exchange $\{1,2,3\}$. Thus, derivatives allow more flexible designs than technology innovation even with symmetric market primitives (i.e., symmetric price impact). □

Thus, joint market clearing of some assets can (i) create asymmetries in cross-asset price impacts (Example 4(a)). $\widehat{\mathbf{\Lambda}}$ is symmetric if and only if $\mathbf{\Lambda}$ is symmetric, which holds only when $\mathbf{\Sigma}$ and the market structure are both symmetric (Definition 5). However, in uncontingent markets, derivatives do not induce such asymmetric cross-asset price impacts, irrespectively of the heterogeneity in asset covariance $\mathbf{\Sigma}$ and derivative weights $\mathbf{W}$. As we will show in the next section, price impact asymmetry further implies that exchanges, but not derivatives, (ii) make additional synthetic products nonredundant. Derivatives, on the other hand, can induce welfare levels that are not feasible with exchanges. Essentially, this is possible because apart from (iii) offering flexibility in weighing the returns, (iv) with a derivative whose return weighs $L \subseteq K$ assets' returns, the number of (linearly independent) contingent variables in the total demand for each underlying asset can be less than L, whereas with exchanges for the same listed assets, that number is, by design, at least L. In Sections 4.2 and 4.3, we explore the implications of these effects for nonredundancy and welfare.

Example 5 underscores the role of asymmetry in market structure (Definition 5).

Example 5 (Market Structure and Redundancy) Consider $N = \{\{1\},\{2,3\},\{4\}\}$ (as in Proposition 2 (i)) and suppose $\mathbf{\Lambda}^{N}_{\{2,3\}}$ is symmetric (as in (Proposition 2 (ii), which requires symmetric covariances of assets 2 and 3 (Corollary 4 (i) in Appendix A.2). Introducing a derivative that weighs assets 2 and 3 is redundant (Proposition 2 below). Such a derivative mimics the equilibrium effects of an exchange $\{2,3\}$ (Proposition 1).

Suppose now that asset 2 is listed in the venue for asset 1 so the market structure becomes $N' = \{\{1,2\},\{2,3\},\{4\}\}$ (condition (ii) in Corollary 4, Appendix A.2, is violated). Now, $\mathbf{\Lambda}^{N'}_{\{2,3\}}$ is generally not symmetric, even if all pairwise asset covariances are the same, because the market structure is not symmetric for assets 2 and 3. Consequently, introducing a derivative that weighs assets 2 and 3 in the market structure N' is not redundant. □

4.2 Innovation in Market Clearing Makes Additional Derivatives Nonredundant

Per Proposition 1, because exchanges can induce asymmetric cross-asset price impacts, derivatives cannot mimic such price impacts. In this section, Proposition 2 points to other implications of the asymmetry: innovation in market clearing (i.e., exchanges) can make additional derivatives nonredundant. Corollary 1 provides the reasoning behind this result by showing that the underlying asset prices cannot price those derivatives linearly.

With contingent trading, any security—traded or new—whose return lies in the span of the traded assets can be priced via a linear combination of the traded asset prices, i.e., $p_d^+ = p_d = \mathbf{w}_d' \mathbf{p}_a$ holds for a derivative whose return is created synthetically as $r_d = \mathbf{w}_d' \mathbf{r}$. Rostek and Yoon (2021b, Corollary 4) observed that with uncontingent demands, linear pricing does not generally hold for new securities *to be introduced*, i.e., $p_d^+ \neq \mathbf{w}_d' \mathbf{p}_a$, yet it holds for traded securities, i.e., $p_d = \mathbf{w}_d' \mathbf{p}_a$ (Corollary 2 in Appendix A.1 and Proposition 2). This result holds in competitive and imperfectly competitive markets regardless of whether price impact is symmetric or asymmetric, because the introduction of the new securities changes the underlying assets' equilibrium prices, i.e., $\mathbf{p}_a^+ \neq \mathbf{p}_a$.

Corollary 1 shows a stronger result in markets where technology allows for exchanges that clear multiple assets: linear pricing does not hold, even for *traded* securities whose return lies in the span of the traded assets.

Corollary 1 (Linear Pricing Does Not Hold) Let $I < \infty$. Consider a market structure $N = \{K(n)\}_n$ with K traded assets and D securities. For the traded securities D with returns $r_d = \mathbf{w}_d' \mathbf{r}$ for each $d \in D$, linear pricing $p_d = \mathbf{w}_d' \mathbf{p}_a$ holds if and only if the equilibrium price impact $\mathbf{\Lambda}$ is symmetric.

The fact that when technology allows exchanges that clear multiple assets traded securities cannot be priced linearly comes from the securities' ability to change traders' inference among the traded assets across exchanges (Lemma 1). When an asset is traded jointly with heterogeneously correlated assets, the inference about the prices of assets from *other* venues differs across the exchanges. Hence, the prices of the same asset traded in different venues carry different information (Eq. (20)).

Unlike the case of nontraded securities, the failure of linear pricing for traded derivatives is an imperfectly competitive phenomenon: In the competitive market, linear pricing always holds for traded assets, regardless of the market structure or security returns. In imperfectly competitive markets, decentralized trading motivates new types of financial innovation that are based on spanning as well as new types of innovation that are not based on spanning. Yet, there are no arbitrage opportunities from the failure of the law of one price because trades induce price impact.

Our key example, Example 6, illustrates the implications of the failure of linear pricing. It shows that innovations that let total demands for an underlying asset be contingent on prices of the *same* underlying assets need not be equivalent.

Example 6 (Innovation in Market Clearing Makes New Derivatives Nonredundant) Let $N = \{\{1\}, \{2\}, \{3\}\}$. Suppose a derivative d that weighs the underlying assets 1 and 2 is introduced, i.e., $N' = N \cup \{\{d\}\}$. Then, another derivative d' whose returns weigh assets $1, 2$ is redundant in N'.

However, in market structure $N'' = \{\{1,2\},\{3\}\}$, the derivative d' is generally not redundant. Unless $\mathbf{\Lambda}^{N''}_{\{1,2\}}$ is symmetric (i.e., assets 1 and 2 are symmetrically correlated with asset 3; Corollary 4 (i) in Appendix A.2), the price of the derivative $p_{d'}$ becomes a new contingent variable in traders' demands for the underlying assets $1, 2$ that is linearly independent of those asset prices p_1, p_2 (Corollary 1):

$$p_{d'} \neq w_{d'1} p_1 + w_{d'2} p_2.$$

The new conditioning variable $p_{d'}$ changes the cross-asset inference in the total demands for all assets, and thus, price impact.

In contrast, in the market structure N', the inference effects about the price of asset 3 are the same in traders' uncontingent demands. Consequently, derivative d' is linearly priced by the underlying asset prices (Corollary 1), and the cross-asset inference and price impact does not change the equilibrium total trades and payoffs. □

Example 6 indicates more general observations about the nonneutrality of new securities or exchanges. First, allowing some assets to clear jointly can make nonredundant additional synthetic products defined for some of these assets, which were redundant in uncontingent markets (Proposition 2), because it can alter cross-asset inference about the prices of assets that are *not* part of the innovation. Since the prices of the assets underlying the derivative carry different information about the prices of *other* assets, the security cannot be priced linearly.

Second, zero inference error among some assets does not imply that synthetic products whose returns weigh those assets are redundant. Market structure N'' in Example 6 provides a counterexample. Even though the demands for assets 1 and 2 are contingent in N''—thus, there is no inference error among them—synthetic products on these assets are generally not redundant, because they change the price impact, as seen in the example. At most one such synthetic product can be nonredundant (Proposition 2 (i) below).

Third, conversely, a non-zero inference error among some assets does not imply that a derivative or an exchange defined on these assets' returns is nonredundant. In N', derivative d' on assets 1 and 2 is redundant, even though there is an inference error in demands for each assets 1 and 2, and derivative d, due to imperfect inference about the price of the *other* asset 3. Similarly, due to imperfect inference about the price of asset 3, the introduction of exchange $\{1,2\}$ in N' is *not* redundant unless $\mathbf{\Lambda}^{N'}_{\{1,2\}}$ is symmetric.To reiterate, what matters for nonredundancy of innovation is whether it *alters inferences about the prices of other* assets, which derivative d' does not alter in N' but does in N''.[26]

Fourth, in markets in which some but not all assets clear jointly, there are two types of nonre-

[26]When the price inference in demands for assets 1 and 2 in market structures N' and N'' is symmetric with respect to asset 3 (so that $\widehat{\mathbf{\Lambda}}^{N'}_{\{1,2\}}$ and $\mathbf{\Lambda}^{N''}_{\{1,2\}}$ are symmetric), innovation on assets 1 and 2 does not affect cross-asset inference, and hence price impact in either market structure. Proposition 1 then gives an equivalence (i.e., symmetric market; Corollary 4 in Appendix A.2).

dundant derivatives: those that weigh the returns of multiple assets and those that replicate the returns of one of the underlying assets. For example, in market structure $N'' = \{\{1,2\},\{3\}\}$, introducing a replicating security (e.g., $\{1\}$) and a derivative with underlying assets $1,2$ are both nonredundant except when $\mathbf{\Lambda}^{N'''}_{\{1,2\}}$ is symmetric.

All the observations due to price impact asymmetries are imperfectly competitive phenomena: if the market were competitive (i..e, if the price impact were zero), none would hold. Proposition 2 characterizes when a new exchange is neutral.

Proposition 2 (Nonredundancy of Innovations) Let $I < \infty$ and $N = \{K(n)\}_n$. Suppose that equilibrium in the market structure N is not *ex post*. Introducing an exchange n' with $L \subset K$ assets that does not replicate an existing exchange or a derivative d with underlying assets L is redundant if and only if the following conditions hold:

(i) (*Locally contingent market*) There exists a set of exchanges $N'' \subset N$ such that, for each pair of assets in L, these assets or a derivative that bundles their returns is traded in an exchange in N''; there must be a separate such derivative, but not an exchange, per pair of assets in L.[27]

(ii) (*Symmetric price impact*) The per-unit price impact submatrix corresponding to exchanges N'' in condition (i), $\widehat{\mathbf{\Lambda}}_{K(N'')} \equiv (\mathbf{W}_{K(N'')} diag(\mathbf{\Lambda}^{-1}_{K(n'')})_{n'' \in N''} \mathbf{W}'_{K(N'')})^{-1}$, and the price impact of the new exchange $\mathbf{\Lambda}_{K(n')}$ are symmetric.

As Example 6 illustrates, the relevant asymmetry of the price impact matrix for identifying nonredundant innovation in condition (ii) is of the submatrix of the assets over which innovation is defined. This asymmetry is due to the inference effects about other assets (see Eq. (8) and Example 4).

Locally contingent market. In some market settings, it is of interest to ensure that the inference error across some assets $L \subset K$ is minimal (e.g., in benchmarking). Does the elimination of inference error among some assets require the schedules for these assets to be contingent? We highlight Proposition 1's implications.

In markets with assets and derivatives that all clear independently, the introduction of $\frac{L(L-1)}{2}$ linearly independent derivatives for $L < K$ assets implements a *locally contingent market*, i.e. condition (i) of Proposition 2. A locally contingent market does *not* imply zero inference error: as long as the prices of assets L and $K \setminus L$ are correlated, the inference about the prices of assets $K \setminus L$ is imperfect. Inference errors are zero in the locally contingent market only when

[27] For example, market $\{\{1,2,3\},\{4\}\}$ is locally contingent for assets $1,2$, and 3. In market $\{\{1\},\{2\},\{3\},\{4\}\}$, the introduction of a single derivative for assets $1,2,3$ (e.g., Example 4 (b)) would not induce a locally contingent market for assets $1,2$, and 3; instead, three linearly independent derivatives for assets $1,2,3$ would induce a locally contingent market. However, introducing a single exchange $\{1,2,3\}$ makes the market locally contingent for assets $1,2$, and 3.

an exchange for all L assets is introduced (i.e., $N'' = \{L\}$ in condition (i) of Proposition 2) or the per-unit price impact in the locally contingent market is symmetric (condition (ii) of Proposition 2). As we discussed below Example 6, the elimination of inference error among some assets is neither necessary nor sufficient for the redundancy of additional derivatives on those assets.[28] By Proposition 2, a locally contingent market is not sufficient for the redundancy of additional innovations.[29]

Notably, with either type of innovation, the per-unit price impact for the L assets still differs from that in the contingent market. For example, with either a derivative $\{d\}$ or exchange $\{1, 2\}$ introduced in N, the price impact submatrix for assets 1 and 2 is different than it would be in the contingent market. This follows from the non-separability of price impact across asset pairs when demands are not contingent, as we explained in Example 4. The inference effects about the prices of assets $K \setminus L$ affect traders' demands for assets L.

Role of imperfect competition. One takeaway from our analysis is that imperfect competition is crucial for comparing the effects of new securities and technology. When some assets are allowed to clear jointly in markets with large traders who have price impact, derivatives that would be neutral in competitive markets become nonredundant (Corollary 5 in Appendix A.2).

Consider market structure N'' in Example 6. If the market was competitive, the inclusion of any derivatives on assets 1 and 2 would be redundant *even with asymmetric inference* about the price of asset 3.[30] In the competitive market, price impact is a zero matrix, and thus, it is symmetric irrespective of the symmetry in asset covariances $\boldsymbol{\Sigma}$ and market structure N''. Essentially, asymmetric cross-asset inference leads to nonredundancy only insofar it impacts prices; with price taking traders, it does not.

If the derivative is introduced for assets traded in a locally contingent market (e.g., the introduction of an exchange $\{1\}$ in $\{\{1, 2\}, \{3\}\}$), the traders would split their total demands

[28] The lack of equivalence between zero inference error and the redundancy of additional innovation gives rise to a difference in the maximal number of nonredundant securities versus exchanges: In markets with uncontingent demands, there exist at most $\frac{K(K-1)}{2}$ nonredundant derivatives. On the other hand, technology innovation for the underlying assets K allows the introduction of $\max\{3 \cdot 2^{K-2} - K - 1, \frac{K(K-1)}{2}\}$ nonredundant exchanges. The corresponding bound in markets with derivatives and exchanges is $(3 \cdot 2^{K-2} - K - 1) + (\frac{K(K-1)}{2} - 1)$. In the competitive market, the maximal number is $\frac{K(K-1)}{2}$.

[29] A locally contingent market does not imply innovation redundancy: The introduction of $\frac{L(L-1)}{2}$ exchanges for pairs of the L assets, which implements a locally contingent market, makes additional exchanges on these L assets redundant when the market is competitive but *not* when traders have price impact, except when the per-unit price impact submatrix for the L assets is symmetric (e.g., in N'' in Example 6, $K = 3$ and $L = 3$). On the other hand, the introduction of $\frac{L(L-1)}{2}$ derivatives for pairs of the L assets, which also implements a locally contingent market, makes additional derivatives on these L assets redundant in both competitive and imperfectly competitive markets, even though the inference error is generally not zero among assets L.

[30] An exchange that does not increase the number of contingent variables in any asset's demand would be redundant as well (e.g., the introduction of an exchange $\{1\}$ in $\{\{1, 2\}, \{3\}\}$). Adding exchange $\{1, 2\}$ to market structure $\{\{1\}, \{2\}, \{3\}, \{d\}\}$ where the derivative has return $r_d = w_1 r_1 + w_2 r_2$ would still be nonredundant with asymmetric demand slope $\widehat{\mathbf{C}} = \mathbf{W}\mathbf{C}\mathbf{W}'$, as it would increase the number of contingent variables in assets' demand (condition (iii) in Corollary 5 in Appendix A.2), and change cross-asset inference.

for an asset traded among the exchanges while preserving their total demands. Then, the price of the same asset would carry the same information about the random variables in the competitive market (but not in the imperfectly competitive market; Corollary 1).[31]

Our results revealed three implications regarding the irrelevance of asymmetric cross-asset inference in the competitive market: (1) innovation in market clearing does not make additional derivatives nonredundant, (2) innovation is redundant if there is no inference error among its underlying assets, and (3) derivatives that replicate the returns of a listed security are redundant. None of these results are applicable in markets with large traders.

Table 1 summarizes the results in this section.

A. Imperfectly competitive market ($I < \infty$)

Innovation	Market structure with Exchanges	Market structure with Derivatives
Exchanges	✓*	✓
Derivatives	✓*	×

B. Competitive market ($I \to \infty$)

Innovation	Market structure with Exchanges	Market structure with Derivatives
Exchanges	×	✓
Derivatives	×	×

Table 1: This table shows when innovations (exchanges or derivatives) for assets $L \subsetneq K$ that satisfy condition (i) of Proposition 2 (i.e., define a locally contingent market) are nonredundant. Each row ("Exchanges" or "Derivatives") represents a type of innovation; each column corresponds to the market structure where innovation is introduced; "×"=redundant, "✓"=nonredundant, *= nonredundant if price impact is asymmetric.

4.3 Welfare: New Securities vs. New Market-Clearing Technology

As Example 7 illustrates, no particular derivative or exchange design dominates in welfare terms for all realizations of asset holdings.[32] The payoff distribution induced by synthetic products is weakly riskier. In fact, Example 7 shows that derivatives can give strictly higher welfare relative to the maximal welfare feasible with exchange design. This holds because of the derivatives' degree of freedom in weighing returns. Likewise, derivatives can give a strictly lower welfare relative to the minimal welfare feasible with exchange design. Thus, whereas derivatives have an advantage in markets where the designer has information about the traders' asset holdings, a design that cannot rely on that information may favor innovation in market-clearing technology.

Example 7 (Derivatives vs. Innovation in Market-Clearing Technology: Welfare) Consider a market with three assets, two of which (assets 2 and 3) have symmetric asset

[31] In imperfectly competitive markets with asymmetric market structures, the splitting of total demands for asset 1 distorts the cross-asset price impact in exchange $\{1,2\}$: When p_2 changes, a trader's total demand changes with an adjustment of $q^j_{1,\{1,2\}}(p_1,p_2)$ but not $q^j_{1,\{1\}}(p_1)$. Consequently, the introduction of exchange $\{1\}$ changes price impact $\lambda_{12,\{1,2\}}$ (Eq. (7)) and hence the per-unit price impact $\widehat{\lambda}_{12}$ (Eq. (23)). See ft. 38.

[32] Clearly, given covariance $\mathbf{\Sigma}$ and a *realization* of $(\mathbf{q}^i_0)_i$, the level of *ex post* welfare in a market with exchanges for multiple assets can be reproduced by a market with derivatives, even though the per-unit price impact cannot.

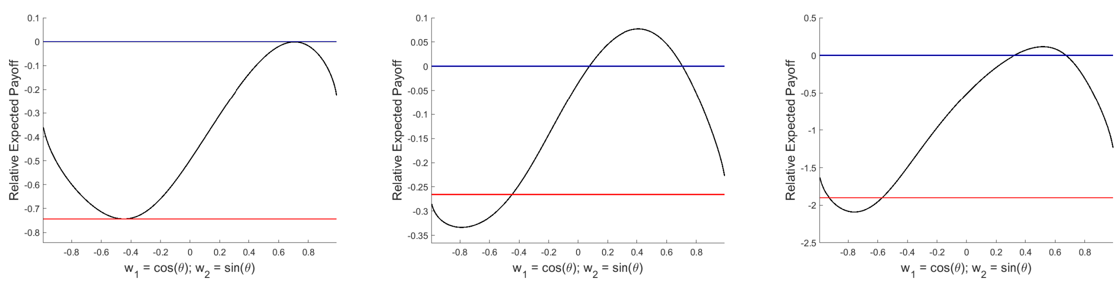
covariances and symmetric *ex ante* trading needs for all traders (i.e., $E[\overline{\mathbf{q}}_0] - E[\mathbf{q}_0^i] = x^j E[\overline{\mathbf{q}}_0] - E[\mathbf{q}_0^j]$ for some $x^j \in \mathbb{R}$ for any i, j):

$$\mathbf{\Sigma} = \begin{bmatrix} 1 & \rho_k & \rho_k \\ \rho_k & 1 & \rho_\ell \\ \rho_k & \rho_\ell & 1 \end{bmatrix} \quad \text{and} \quad E[\overline{\mathbf{q}}_0] - E[\mathbf{q}_0^i] = \begin{bmatrix} E[\overline{q}_{0,k}] - E[q_{0,k}] \\ E[\overline{q}_{0,\ell}] - E[q_{0,\ell}] \\ E[\overline{q}_{0,\ell}] - E[q_{0,\ell}] \end{bmatrix}.$$

With $K = 3$ underlying assets, there are thirteen possible designs with exchanges $\{K(n)\}_n$, including the uncontingent market $\{\{1\}, \{2\}, \{3\}\}$ and the contingent market $\{\{1, 2, 3\}\}$.

Consider the uncontingent market and suppose that a derivative with return $r_d = w_k r_1 + w_\ell r_2 + w_\ell r_3$ is introduced: $N = \{\{1\}, \{2\}, \{3\}, \{d\}\}$. We compare the *ex ante* welfare in the market structure N for any weight vector (w_k, w_ℓ, w_ℓ) relative to that with arbitrary exchange design. This market structure is not locally contingent (i.e., condition (i) in Proposition 2 is violated) and cannot be equivalent to any market structure with technology innovation state by state (Proposition 1 and Example 4 (b)).

Figure 1: Welfare in an Uncontingent Market with Derivative $(w_k, w_\ell, w_\ell) \in \mathbb{R}^3$



(*A*) (*B*) (*C*)

Notes. In Panel (A), asset covariances and *ex ante* trading needs are symmetric ($\rho_k = \rho_\ell = -0.1$ and $E[\overline{q}_{0,k}] - E[q_{0,k}^i] = E[\overline{q}_{0,\ell}] - E[q_{0,\ell}^i] = 10$). In Panel (B), asset covariances are heterogeneous. In Panel (C), *ex ante* trading needs are heterogeneous. Trading venues for multiple underlying assets create bounds on the *ex ante* welfare: the blue line represents the most efficient design with multiple venues, and the red line represents the least efficient one. The *ex ante* welfare with the derivative varies depending on the weights w_k (x-axis) and w_ℓ parameterized by $w_k = cos(\theta)$ and $w_\ell = sin(\theta)$ for $\theta \in [0, \pi]$ to make the variance of the derivative return constant.

(i) *Symmetric covariance and trading needs.* In markets with symmetric asset covariances and trading needs for all assets (i.e., $\rho_k = \rho_\ell$ and $E[\overline{q}_{0,k}] - E[q_{0,k}^i] = E[\overline{q}_{0,\ell}] - E[q_{0,\ell}^i]$; Fig. 1A), the *ex ante* welfare with the welfare-maximizing derivative is the same as the *ex ante* welfare that can be attained with the welfare-maximizing exchange design.

Moreover, the *ex ante* welfare for any derivatives and any exchange design is bounded between the market for securities equivalent to the contingent and uncontingent markets for the underlying assets (Fig. 1A).

(ii) *Heterogeneous covariance or trading needs.* When either covariances or trading needs are heterogeneous *across assets* (i.e., $\rho_\ell/\rho_k \neq 1$ or $|E[\overline{q}_{0,k}] - E[q_{0,k}]| \neq |E[\overline{q}_{0,\ell}] - E[q_{0,\ell}]|$), there exists a derivative (i.e., $N = \{\{1\}, \{2\}, \{3\}, \{d\}\}$) with which the *ex ante* welfare in N is higher than the maximal welfare feasible with exchange design (thus also with the contingent and uncontingent markets) and a derivative with which the *ex ante* welfare is lower than the minimal welfare feasible with exchange design (Figs. 1B and 1C).

Depending on market characteristics, the same derivative can be welfare-maximizing or welfare-minimizing, and the welfare induced by the derivative is more volatile than with exchange design. □

Either derivatives or exchanges can dominate in welfare terms, depending on market characteristics and derivative weights. Consider a derivative with arbitrary weights on L underlying assets. When asset covariances and trading needs are sufficiently symmetric, an exchange design tends to be more efficient due to the zero inference error among assets L (which does not hold with the derivative). When the market characteristics are heterogeneous, the derivative tends to be more efficient due to the symmetric per-unit price impact matrix. Furthermore, the welfare-maximizing derivative assigns a higher weight to the asset that is more strongly correlated with other assets or the asset with a larger trading need.

5 Discussion

Our results show that in imperfectly competitive markets, changes to market-clearing technology can make synthetic products that were previously redundant nonredundnat and affect efficient derivatives and vice versa. These results thus suggest when the growing complexity of the financial instruments may be justified on efficiency grounds. Joint design of synthetic products and market-clearing technology can further improve welfare. Our results imply that the effects of changes to the market structure implied by market-clearing technology innovation such as the introduction of a new exchange for the traded assets, a merger among exchanges, and a listing of an asset in exchanges where it was not listed before differ from those of, respectively, the introduction of a new synthetic product, listing a synthetic product while delisting the underlying assets, and including a new asset in a synthetic product. Analysis of the joint design of these instruments would be worthwhile.

In markets with bilateral transactions ($I = 2$) for multiple assets ($K > 1$), the nonneutrality of innovations will continue to apply, as will the welfare tradeoff due to the strategic interaction between the traders with two-sided private information. An exploration of how efficient innovation is shaped by the market structure is an exciting research direction.

In studies of decentralized trading, several authors have investigated the derivatives' effects on financial stability (Allen and Carletti (2006)) and hedging counterparty exposures in a financial network (Zawadowski (2013)). It would be worth exploring how the joint design of market-clearing technology and synthetic products can facilitate these objectives.

Budish, Cramton, Kyle, Lee, and Malec (2021) introduce a new design in which traders submit demands for trader-specific portfolios, which is defined by the weighted averages of assets instead of demands for individual assets. Understanding how such trader-specific portfolios alter the design trade-offs outlined in our study is an exciting research direction.

We conclude with a discussion of additional implications of our results.

5.1 Security Pricing

By Corollary 1, factor prices are generally not useful for either non-traded or traded derivatives in markets that clear securities independently. The implied representation of risk depends not only on $\mathbf{\Sigma}$, but also on the endogenous price impact.

5.2 Dynamic Trading

Demand conditioning on past prices of securities induces *asymmetric conditioning* across securities' demands.[33] Lyu, Rostek, and Yoon (2021a) examine efficient design with asymmetric demand conditioning, which corresponds to the problem of the design of market clearing with different disclosure rules that is outside of the current paper's framework. Lyu, Rostek, and Yoon (2021b) explore new financial products that are well-defined only in dynamic markets—their returns and prices are determined in different rounds (e.g., futures, repo contracts)—and demonstrate their nonredundancy with asymmetric demand conditioning in imperfectly competitive (but not competitive) markets. These papers show that the design of dynamic market clearing, transparency, and dynamic securities can improve efficiency relative to welfare achieved by innovations in static markets.

5.3 Markets Where Underlying Assets Cannot Be Traded

We have considered markets in which derivatives are traded along with underlying assets. Derivatives are often traded because the trade of the underlying assets is restricted. Suppose traders can trade synthetic products but not the underlying assets. In light of the results reported in this paper and in Rostek and Yoon (2021b), one can show that then, the bounds on welfare feasible with exchanges for the underlying assets are strictly tighter than those with derivatives, even in symmetric markets.

[33] For example, when asset 1 is traded in round 1 and asset 2 is traded in round 2, demand for asset 2 conditions on *past* price p_1; i.e., $q_2^i(p_1, p_2)$ but demand for asset 1 cannot condition on *future* price p_2, i.e., $q_1^i(p_1)$. Demand conditioning is asymmetric between assets 1 and 2 in the sense that one demand can be contingent on the other's asset price, but not vice versa.

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A Appendix

Equilibrium characterization (Theorem 1, Corollary 2, Eqs. (23)-(25)) follows Rostek and Yoon (2021a) by setting the initial asset holdings of L derivatives to zero and defining $\mathbf{W}$ as a weight matrix rather than an indicator matrix.[34] The appendices reference the relevant proofs from that paper and only present arguments that pertain to the current paper.

[34]With exchanges for multiple assets, but no derivatives, the weight matrix $\mathbf{W}$ becomes an indicator matrix $\mathbf{W} \in \{0,1\}^{K\times(\sum_n K(n))}$, given that the same asset traded in different exchanges can be treated as a replica of that asset, i.e., $w_{mk} = 1$ and $w_{m\ell} = 0$ for all $\ell \neq k$. The fact that indicator matrices are special cases of the weight matrix allows us to characterize the equilibrium in a unified framework (Theorem 1) and analyze the effects of innovations through the same sufficient statistics (Lemma 1).

A.1 Equilibrium Characterization

This section presents the characterization of the main equilibrium objects.

We allow heterogeneous risk preferences $\{\alpha^i\}_i$, an arbitrary set of $M \leq K + D$ securities, which may or may not include all K underlying assets, and an arbitrary market structure $N = \{K(n)\}_n$ (Definition 1, $K(n) \subseteq M$ for each n). By treating the same securities traded in multiple exchanges as different securities, the distribution of security returns is jointly Normal, $\mathcal{N}(\boldsymbol{\delta}^+, \boldsymbol{\Sigma}^+)$, where $\boldsymbol{\delta}^+ \in \mathbb{R}^{\sum_n K(n)}$ and $\boldsymbol{\Sigma}^+ \in \mathbb{R}^{(\sum_n K(n)) \times (\sum_n K(n))}$. The weight matrix $\mathbf{W} \equiv \left((w_{m,k,n})_{k,m \in K(n)}\right)_n \in \mathbb{R}^{K \times (\sum_n K(n))}$ represents the securities' weights and the market structure; $\boldsymbol{\delta}^+ = \mathbf{W}'\boldsymbol{\delta}$ and $\boldsymbol{\Sigma}^+ = \mathbf{W}'\boldsymbol{\Sigma}\mathbf{W}$ (Notation 1 in Section 4).

To ensure that the per capita aggregate asset holdings (equivalently, price) is random in the limit large market ($I \to \infty$), we allow for the common value component $\mathbf{q}_0^{cv} = (q_{0,k}^{cv})_k \in \mathbb{R}^K$ in traders' asset holdings. For each asset k, asset holdings $\{q_{0,k}^i\}_i$ are correlated among traders through $q_{0,k}^{cv} \sim \mathcal{N}(E[q_{0,k}^{cv}], \sigma_{cv}^2)$: for each i,

$$q_{0,k}^i = q_{0,k}^{cv} + q_{0,k}^{i,pv}, \qquad q_{0,k}^{i,pv} \sim \mathcal{N}(E[q_{0,k}^{i,pv}], \sigma_{pv}^2), \tag{13}$$

where $q_{0,k}^{i,pv}$ are independent across i and k.[35] Trader i knows his asset holdings $\mathbf{q}_0^i$, but not its components $\mathbf{q}_0^{cv}$ or $\mathbf{q}_0^{i,pv} = (q_{0,k}^{i,pv})_k \in \mathbb{R}^K$. The asset holdings $\{q_{0,k}^i\}_i$ and the common value $q_{0,k}^{cv}$ are independent across assets k.

Equilibrium: (Net) demands and price impacts. Theorem 1 characterizes equilibrium in demand schedules as a fixed point in price impacts $\{\boldsymbol{\Lambda}^i\}_i$. In particular, it endogenizes expected trades $\{E[q_\ell^i | \mathbf{p}_{K(n)}, \mathbf{q}_0^i]\}_{\ell \notin K(n), n}$ for all i (Eq. (6)) as functions of price impacts $\{\boldsymbol{\Lambda}^i\}_i$.

To write the fixed-point problem in matrix form, we parameterize the best response demand (6) of trader i for security m as

$$\mathbf{q}^i(\mathbf{p}) \equiv \boldsymbol{a}^i - \mathbf{B}^i\mathbf{q}_0^i - \mathbf{C}^i\mathbf{p} \qquad \forall \mathbf{p} \in \mathbb{R}^{\sum_n K(n)} \quad \forall \mathbf{q}_0^i \in \mathbb{R}^K, \tag{14}$$

with the demand intercept $\boldsymbol{a}^i \in \mathbb{R}^{\sum_n K(n)}$, the demand matrix coefficient $\mathbf{B}^i \in \mathbb{R}^{(\sum_n K(n)) \times K}$, and the demand slope $\mathbf{C}^i = diag(\mathbf{C}_{K(n)}^i)_n \in \mathbb{R}^{(\sum_n K(n)) \times (\sum_n K(n))}$, where $\mathbf{C}_{K(n)}^i \in \mathbb{R}^{K(n) \times K(n)}$ is the demand slope in each exchange n.

Notation 2. The operator $[\cdot]_N : \mathbb{R}^{(\sum_n K(n)) \times (\sum_n K(n))} \to \mathbb{R}^{(\sum_n K(n)) \times (\sum_n K(n))}$ maps a matrix $\mathbf{M}$ to a block-diagonal matrix $[\mathbf{M}]_N$ with $([\mathbf{M}]_N)_{K(n),K(n')} \equiv \mathbf{0}$ for $n \neq n'$ and $([\mathbf{M}]_N)_{K(n),K(n)} \equiv \mathbf{M}_{K(n),K(n)}$ for any n.

Theorem 1 (Equilibrium: Fixed Point in Demand Schedules) $I < \infty$ and consider a market structure $N = \{K(n)\}_n$ for M securities. A profile of (net) demand schedules, defined

[35] The common value component in $\{\mathbf{q}_0^i\}_i$ does not affect any results qualitatively and only impacts the magnitude of inference coefficients.

by matrix coefficients $\{\boldsymbol{a}^i\}, \boldsymbol{B}, \boldsymbol{C}$, and price impact $\boldsymbol{\Lambda}$ are an equilibrium if and only if the following conditions hold for each trader i:[36]

(i) (*Optimization, given price impact*) Given price impact matrix $\boldsymbol{\Lambda}$, the coefficients of (net) demands $\boldsymbol{a}^i, \mathbf{B}$, and $\mathbf{C}$ are characterized by

$$\boldsymbol{a}^i = \mathbf{C}\underbrace{\left(\boldsymbol{\delta}^+ - (\alpha\boldsymbol{\Sigma}^+ - \mathbf{C}^{-1}\mathbf{B})E[\overline{\mathbf{q}}_0]\right)}_{=\mathbf{p}-\mathbf{C}^{-1}\mathbf{B}\overline{\mathbf{q}}_0} + \underbrace{\left((\alpha\boldsymbol{\Sigma}^+ + \boldsymbol{\Lambda})^{-1}\mathbf{W}'\alpha\boldsymbol{\Sigma} - \mathbf{B}\right)(E[\overline{\mathbf{q}}_0] - E[\mathbf{q}_0^i])}_{\text{Adjustment due to cross-asset inference}}, \tag{15}$$

$$\mathbf{B} = \Big((1-\sigma_0)(\alpha\boldsymbol{\Sigma}^+ + \boldsymbol{\Lambda}) + \underbrace{\sigma_0\mathbf{C}^{-1}}_{\substack{\text{Adjustment due to}\\ \text{cross-asset inference}}}\Big)^{-1}\mathbf{W}'\alpha\boldsymbol{\Sigma}, \tag{16}$$

$$\mathbf{C} = \Big[(\alpha\boldsymbol{\Sigma}^+ + \boldsymbol{\Lambda})\underbrace{(\mathbf{B}Var[\overline{\mathbf{q}}_0]\mathbf{B}')[\mathbf{B}Var[\overline{\mathbf{q}}_0]\mathbf{B}']_N^{-1}}_{\substack{\text{Inference coefficient}\\ Var[\mathbf{s}^{-i}|\mathbf{q}_0^i][Var[\mathbf{s}^{-i}|\mathbf{q}_0^i]]_N^{-1}}}\Big]_N^{-1}, \tag{17}$$

where $\overline{\mathbf{q}}_0 \equiv \frac{1}{I}\sum_j \mathbf{q}_0^j \in \mathbb{R}^K$ is the aggregate asset holdings and $\sigma_0 \equiv \frac{\sigma_{cv}^2 + \frac{1}{I}\sigma_{pv}^2}{\sigma_{cv}^2 + \sigma_{pv}^2} \in \mathbb{R}_+$.

(ii) (*Correct price impact*) Price impact $\boldsymbol{\Lambda}$ equals the transpose of the Jacobian matrix of the trader's inverse residual supply function:

$$\boldsymbol{\Lambda} = \Big(\frac{\partial \mathbf{S}^{-i}(\cdot)}{\partial \mathbf{p}}\Big)' = \Big((\sum_{j\neq i}\mathbf{C}^j)^{-1}\Big)' = \frac{1}{I-1}(\mathbf{C}^{-1})'. \tag{18}$$

Remarks.

7. Pointwise optimization for each security $m \in K(n)$ in all exchanges n, with respect to $\mathbf{p}_{K(n)} \in \mathbb{R}^{K(n)}$ (i.e., optimization problem (5)), is necessary and sufficient for optimization in demand functions (i.e., optimization problem (4)) because for each security m, price realization $\mathbf{p}_{K(n)}$ maps one-to-one to the realization of residual supply intercept $\mathbf{s}_{K(n)}^{-i}$:

$$\mathbf{S}_{K(n)}^{-i}(\mathbf{p}_{K(n)}) \equiv \mathbf{s}_{K(n)}^{-i} + ((\boldsymbol{\Lambda}_{K(n)}^i)')^{-1}\mathbf{p}_{K(n)} \qquad \forall \mathbf{p}_{K(n)} \in \mathbb{R}^{K(n)}. \tag{19}$$

8. Security by security optimization is without loss of generality by the Fréchet differentiability of expected payoff (4) with respect to the profile of demands $\{q_{m,n}^i(\cdot)\}_{m,n}$. The second-order condition $(-\alpha\boldsymbol{\Sigma} - \boldsymbol{\Lambda}^i - (\boldsymbol{\Lambda}^i)' < 0)$ holds with downward-sloping demands, i.e., the Jacobian matrix $\frac{\partial \mathbf{q}^i(\cdot)}{\partial \mathbf{p}}$ of demands is negative definite, equivalently, $\boldsymbol{\Lambda}_{K(n)}^i > \mathbf{0}$ for all i and n.

[36]The numerical iteration that solves the equilibrium fixed-point equation in Theorem 1 converges to the same equilibrium in extensive simulations including random initial values, different forms of the fixed-point equation, fixed points with respect to $\boldsymbol{\Lambda}$ or $\mathbf{B}$.

Corollary 2 (Equilibrium Prices and Allocations) Given the equilibrium demand coefficients $\{\boldsymbol{a}^i\}_i, \mathbf{B}, \mathbf{C}$, and price impact $\mathbf{\Lambda}$ in Theorem 1, equilibrium prices and trades are

$$\mathbf{p} = \boldsymbol{\delta}^+ - (\mathbf{W}'\alpha\mathbf{\Sigma} - \mathbf{C}^{-1}\mathbf{B})E[\overline{\mathbf{q}}_0] - \mathbf{C}^{-1}\mathbf{B}\overline{\mathbf{q}}_0, \tag{20}$$

$$\mathbf{q}^i = \big((\alpha\mathbf{\Sigma}^+ + \mathbf{\Lambda})^{-1}\mathbf{W}'\alpha\mathbf{\Sigma} - \mathbf{B}\big)(E[\overline{\mathbf{q}}_0] - E[\mathbf{q}_0^i]) + \mathbf{B}(\overline{\mathbf{q}}_0 - \mathbf{q}_0^i). \tag{21}$$

Lemma 2 in Appendix A.1 shows that equilibrium price impact $\mathbf{\Lambda}^i$ converges to $\mathbf{0}$ for all i in the competitive market (i.e., $I \to \infty$). Its proof is analogous to the proof of Lemma 3 in Rostek and Yoon (2021a).

Lemma 2 (Price Impact in Competitive Markets) Consider market structure $N = \{K(n)\}_n$ for M securities and let $\{\alpha^i\}_i$ be the profile of traders' risk aversions. Suppose $\{\mathbf{\Lambda}^{i,I}\}_i$ is a profile of the equilibrium price impacts for all $I < \infty$ and in the limit market as $I \to \infty$. The equilibrium price impact becomes zero as $I \to \infty$ if $\alpha^{i,I} = \alpha^i\gamma^I \in \mathbb{R}_+$ increases slower than linearly by a common factor $\gamma^I \sim o(I^{1-\varepsilon})$ for some $\varepsilon > 0$: for each i, $\mathbf{\Lambda}^i = \lim_{I\to\infty} \mathbf{\Lambda}^{i,I} = \mathbf{0}$.

The proof of Lemma 1 shows that by considering a *single-exchange* counterfactual for the K underlying assets, one can characterize the equilibrium outcomes for arbitrary market structures and arbitrary securities in terms of the *per-unit price impact* $\widehat{\mathbf{\Lambda}} \in \mathbb{R}^{K\times K}$ (Definition 4) and *cross-asset inference* $\widehat{\mathbf{B}} \in \mathbb{R}^{K\times K}$ (Definition 6 below), which match the moments of each trader's total equilibrium trade $\widehat{\mathbf{q}}^i \equiv (\widehat{q}_k^i)_k = \mathbf{W}\mathbf{q}^i \in \mathbb{R}^K$ (Eq. (25)).

Definition 6 (Cross-Asset Inference) *Suppose $\widehat{\mathbf{q}}^i \in \mathbb{R}^K$ is trader i's total equilibrium trade in market $N = \{K(n)\}_n$ for securities M. The* cross-asset inference $\widehat{\mathbf{B}} \in \mathbb{R}^{K\times K}$ *is a positive semi-definite matrix, such that*

$$Var[\widehat{\mathbf{q}}^i] \equiv Var\big[\mathbf{W}\mathbf{q}^i\big] = \widehat{\mathbf{B}}Var[\overline{\mathbf{q}}_0 - \mathbf{q}_0^i]\widehat{\mathbf{B}}' = \frac{I-1}{I}\sigma_{pv}^2\widehat{\mathbf{B}}\widehat{\mathbf{B}}' \qquad \forall i. \tag{22}$$

The cross-asset inference $\widehat{\mathbf{B}}$ is the coefficient on the privately known asset holdings $\mathbf{q}_0^i$ in a trader's total demand that matches the variance of the equilibrium total trade (Eq. (22)). From Eq. (22), $(\widehat{\mathbf{B}}\widehat{\mathbf{B}}')_{k\ell}(\widehat{\mathbf{B}}\widehat{\mathbf{B}}')_{kk}^{-1}$ is the cross-asset inference coefficient in the expected total trade $E[\widehat{q}_\ell^i|\widehat{q}_k^i, \mathbf{q}_0^i]$ (see Eq. (21)).[37]

Although both per unit price impact $\widehat{\mathbf{\Lambda}}$ and cross-asset inference $\widehat{\mathbf{B}}$ are needed to compute a trader's equilibrium payoff, the per unit price impact $\widehat{\mathbf{\Lambda}}$ suffices to compare payoffs across market structures: the per unit price impact $\widehat{\mathbf{\Lambda}}$ changes if and only if cross-asset inference $\widehat{\mathbf{B}}$ does.[38] Step 2 in the proof of Lemma 1 prove this result.

[37] $\widehat{\mathbf{\Lambda}}$ and $\widehat{\mathbf{B}}$ are not defined as equilibrium variables in a single-exchange game.

[38] In the competitive market, $\widehat{\mathbf{C}}$ is sufficient for both payoff characterization and cross-market payoff comparison. This also implies that, unlike with large traders, the asymmetries in the asset covariance or the market

A.2 Proofs

Proof of Lemma 1 (Innovation and Sufficient Statistics).
(Step 1. $\widehat{\mathbf{\Lambda}}$ and $\widehat{\mathbf{B}}$ characterize the equilibrium payoffs) The per-unit price impact $\widehat{\mathbf{\Lambda}} \in \mathbb{R}^{K\times K}$, defined in Eq. (9), is characterized by

$$\widehat{\mathbf{\Lambda}} = \left(\mathbf{W}\mathbf{\Lambda}^{-1}\mathbf{W}'\right)^{-1}. \tag{23}$$

The cross-asset inference $\widehat{\mathbf{B}} \in \mathbb{R}^{K\times K}$, defined in Eq. (22), is characterized by

$$\widehat{\mathbf{B}} \equiv \mathbf{W}\left((1-\sigma_0)(\alpha\mathbf{\Sigma}^+ + \mathbf{\Lambda}) + \sigma_0(I-1)\mathbf{\Lambda}'\right)^{-1}\mathbf{W}'\alpha\mathbf{\Sigma}. \tag{24}$$

The expected equilibrium payoff of trader i is characterized as a function of $\widehat{\mathbf{\Lambda}}$ and $\widehat{\mathbf{B}}$:[39]

$$\begin{aligned} E[u^i(\mathbf{q}^i) - \mathbf{p}\cdot\mathbf{q}^i] \;=\; & \underbrace{E[\boldsymbol{\delta}\cdot\mathbf{q}_0^i - \frac{1}{2}\mathbf{q}_0^i\cdot\alpha\mathbf{\Sigma}\mathbf{q}_0^i]}_{\text{Payoff without trade}} + \underbrace{(E[\overline{\mathbf{q}}_0] - E[\mathbf{q}_0^i])\cdot\mathbf{\Upsilon}(\widehat{\mathbf{\Lambda}})(E[\overline{\mathbf{q}}_0] - E[\mathbf{q}_0^i])}_{\text{Equilibrium surplus from trade}} \\ & + \underbrace{\frac{1}{2}\frac{I-2}{I-1}\sigma_{pv}^2 tr\left(\alpha\mathbf{\Sigma}\right)}_{\text{Payoff term due to } Var[\overline{\mathbf{q}}_0|\mathbf{q}_0^i]>0} - \underbrace{\frac{I-1}{I}\sigma_{pv}^2 tr\Big((\mathbf{B}^c-\widehat{\mathbf{B}})'\alpha\mathbf{\Sigma}(\mathbf{B}^c-\widehat{\mathbf{B}}) + \frac{2}{I-1}\alpha\mathbf{\Sigma}(\mathbf{B}^c-\widehat{\mathbf{B}})\Big)}_{\text{Information loss}}, \end{aligned} \tag{25}$$

where $\mathbf{B}^c = \frac{I-2}{I-1}\mathbf{Id}$ and $\mathbf{\Upsilon}(\widehat{\mathbf{\Lambda}}) \equiv \frac{1}{2}\alpha\mathbf{\Sigma}(\alpha\mathbf{\Sigma}+\widehat{\mathbf{\Lambda}}')^{-1}(\alpha\mathbf{\Sigma}+\widehat{\mathbf{\Lambda}}+\widehat{\mathbf{\Lambda}}')(\alpha\mathbf{\Sigma}+\widehat{\mathbf{\Lambda}})^{-1}\alpha\mathbf{\Sigma} \in \mathbb{R}^{K\times K}$. The derivation of Eqs. (23)-(25) is analogous to that in Rostek and Yoon (2021a).

We next rewrite the equilibrium fixed point (17)-(18) to identify a term that is a function of $\widehat{\mathbf{\Lambda}}$ and $\widehat{\mathbf{B}}$ and a term that is a function of $\mathbf{\Lambda}$ but not $\widehat{\mathbf{\Lambda}}$ or $\widehat{\mathbf{B}}$. Applying the Woodbury Matrix Identity to $\mathbf{B}$ in Eq. (16) gives

$$\mathbf{B} \;=\; \left((1-\sigma_0)\mathbf{\Lambda} + \sigma_0\mathbf{\Lambda}'\right)^{-1}\mathbf{W}'\alpha\mathbf{\Sigma}(\mathbf{Id}-\widehat{\mathbf{B}}). \tag{26}$$

Substituting Eq. (26) into Eq. (18) simplifies the equilibrium fixed-point equation to

$$\left[\mathbf{W}'\mathbf{\Psi}_1(\widehat{\mathbf{B}})\mathbf{W}\right]_N + \mathbf{\Lambda}\left((1-\sigma_0)\mathbf{\Lambda}+\sigma_0\mathbf{\Lambda}'\right)^{-1}\left[\mathbf{W}'\mathbf{\Psi}_2(\widehat{\mathbf{B}})\mathbf{W}\right]_N = \mathbf{0}, \tag{27}$$

structure do not lead to innovation nonredundancy. Mathematically, this is because with zero (hence, symmetric) price impact matrix, the per-unit cross-asset inference $\widehat{\mathbf{B}}$ and demand Jacobian $\widehat{\mathbf{C}}$ are one-to-one *irrespective of* whether $\mathbf{C}$ is a symmetric matrix: As $I\to\infty$, the counterpart of Eq. (24) becomes

$$\widehat{\mathbf{B}} = \mathbf{W}((1-\sigma_0)(\alpha\mathbf{\Sigma}^+ + \mathbf{\Lambda}) + \sigma_0\mathbf{C}^{-1})^{-1}\mathbf{W}'\alpha\mathbf{\Sigma} \to ((1-\sigma_0)\alpha\mathbf{\Sigma} + \sigma_0\widehat{\mathbf{C}}^{-1})^{-1}\alpha\mathbf{\Sigma}.$$

Intuitively, the liquidity risk due to imperfect competition ($\mathbf{\Lambda}$) distorts how equilibrium prices incorporate the fundamental asset risks, which are represented by $\mathbf{\Sigma}$. $\mathbf{B}$ determines how prices aggregate traders' initial holdings (price distribution) and depends on traders' total demand adjustment ($\widehat{\mathbf{C}}$) and the resulting change in the per-unit price impact ($\widehat{\mathbf{\Lambda}}$) differently, as seen in Eq. (24). In the competitive market, the additional effect due to the change in price impact disappears.

[39] In the contingent market, the information loss in Eq. (25) is zero (i.e., equilibrium is *ex post*).

where $\mathbf{\Psi}_1(\cdot):\mathbb{R}^{K\times K}\to\mathbb{R}^{K\times K}$ and $\mathbf{\Psi}_2(\cdot):\mathbb{R}^{K\times K}\to\mathbb{R}^{K\times K}$ are matrix functions of $\widehat{\mathbf{B}}$:

$$\begin{aligned}\mathbf{\Psi}_1(\widehat{\mathbf{B}}) &\equiv (1-(I-2)\sigma_0)^{-1}\alpha\mathbf{\Sigma}\big(\sigma_0(\mathbf{Id}-\widehat{\mathbf{B}})^{-1}\widehat{\mathbf{B}}-\mathbf{Id})(\mathbf{Id}-\widehat{\mathbf{B}})(\mathbf{Id}-\widehat{\mathbf{B}})'\alpha\mathbf{\Sigma},\\ \mathbf{\Psi}_2(\widehat{\mathbf{B}}) &\equiv \alpha\mathbf{\Sigma}(\mathbf{Id}-\widehat{\mathbf{B}})(\mathbf{Id}-\widehat{\mathbf{B}})'\alpha\mathbf{\Sigma}.\end{aligned}$$

The terms $\big[\mathbf{W}'\mathbf{\Psi}_1(\widehat{\mathbf{B}})\mathbf{W}\big]_N$ and $\big[\mathbf{W}'\mathbf{\Psi}_2(\widehat{\mathbf{B}})\mathbf{W}\big]_N$ in Eq. (27) are functions of $\widehat{\mathbf{B}}$ alone, while the term $\mathbf{\Lambda}\big((1-\sigma_0)\mathbf{\Lambda}+\sigma_0\mathbf{\Lambda}'\big)^{-1}$ cannot be written as a function of $\widehat{\mathbf{\Lambda}}$ or $\widehat{\mathbf{B}}$ unless $\mathbf{\Lambda}$ is symmetric.

For each market N, we define a matrix operator $[\cdot]_{\widehat{N}}:\mathbb{R}^{K\times K}\to\mathbb{R}^{K\times K}$ as follows: $([\mathbf{X}]_{\widehat{N}})_{k,\ell}\equiv \mathbf{X}_{k,\ell}$ if there exists an exchange $n\in N$ such that $\{k,\ell\}\subseteq N$ or a derivative d such that $w_{dk}\neq 0$ and $w_{d\ell}\neq 0$; $([\mathbf{X}]_{\widehat{N}})_{k,\ell}\equiv 0$, otherwise. We remark that the matrix operator $[\cdot]_N$ is defined in the space of the $(\sum_n K(n))$ traded securities, while $[\cdot]_{\widehat{N}}$ is defined in the space of the K underlying assets.

(Step 2. An innovation changes $\widehat{\mathbf{\Lambda}}$ if and only if it changes $\widehat{\mathbf{B}}$) Consider a market structure $N=\{K(n)\}_n$ for M securities. Suppose an innovation in technology or derivatives is introduced, and the new market structure is N'. The innovation changes the per-unit price impact ($\widehat{\mathbf{\Lambda}}^N\neq\widehat{\mathbf{\Lambda}}^{N'}$) if and only if it changes the per-unit cross-asset inference ($\widehat{\mathbf{B}}^N\neq\widehat{\mathbf{B}}^{N'}$).

To prove this result, it suffices to consider innovations in market-clearing technology or securities that do not create additional cross-asset demand conditioning, i.e., an exchange $K(n')\in K$ or a security $K(n')=\{d\}$ such that all asset pairs k and $\ell\neq k$ in $K(n')$ are traded together in some existing exchanges $n''\in N$. Otherwise, the innovation changes both $\widehat{\mathbf{\Lambda}}$ and $\widehat{\mathbf{B}}$, because the $(k,\ell)^{\text{th}}$ elements of $\widehat{\mathbf{\Lambda}}$ and $\mathbf{W}\big((1-\sigma_0)\mathbf{\Lambda}+\sigma_0\mathbf{\Lambda}'\big)^{-1}\mathbf{W}'$ (Eqs. (23) and (26))—that were zeros in N—are nonzero with the innovation.

Consider an innovation that does not create new cross-asset demand conditioning that defines a new market structure $N'=N\cup\{n'\}$. The innovation applies to the assets in an existing exchange $n''\in N$. The equilibrium price impacts in markets N and N' are $\mathbf{\Lambda}^N$ and $\mathbf{\Lambda}^{N'}$, respectively.

(Only if) We prove the contrapositive: Suppose $\widehat{\mathbf{B}}^N=\widehat{\mathbf{B}}^{N'}$. Substituting Eq. (26) to $\widehat{\mathbf{B}}=\mathbf{WB}$, we have that $\widehat{\mathbf{B}}^N=\widehat{\mathbf{B}}^{N'}$ if and only if

$$\mathbf{W}^N\big((1-\sigma_0)\mathbf{\Lambda}^N+\sigma_0(\mathbf{\Lambda}^N)'\big)^{-1}(\mathbf{W}^N)'=\mathbf{W}^{N'}\big((1-\sigma_0)\mathbf{\Lambda}^{N'}+\sigma_0(\mathbf{\Lambda}^{N'})'\big)^{-1}(\mathbf{W}^{N'})'. \tag{28}$$

Combined with Eq. (27), Eq. (28) shows that the per-unit price impacts coincide in markets N and N':

$$\begin{aligned}&\mathbf{W}^N(\mathbf{\Lambda}^N)^{-1}(\mathbf{W}^N)'=-\mathbf{W}^N\big((1-\sigma_0)\mathbf{\Lambda}^N+\sigma_0(\mathbf{\Lambda}^N)'\big)^{-1}(\mathbf{W}^N)'\big[\mathbf{\Psi}_2(\widehat{\mathbf{B}})\big]_{\widehat{N}}\big[\mathbf{\Psi}_1(\widehat{\mathbf{B}})\big]_{\widehat{N}}^{-1} \\ &=-\mathbf{W}^{N'}\big((1-\sigma_0)\mathbf{\Lambda}^{N'}+\sigma_0(\mathbf{\Lambda}^{N'})'\big)^{-1}(\mathbf{W}^{N'})'\big[\mathbf{\Psi}_2(\widehat{\mathbf{B}})\big]_{\widehat{N}'}\big[\mathbf{\Psi}_1(\widehat{\mathbf{B}})\big]_{\widehat{N}'}^{-1}=\mathbf{W}^{N'}(\mathbf{\Lambda}^{N'})^{-1}(\mathbf{W}^{N'})'.\end{aligned} \tag{29}$$

The first and third equalities hold by the equilibrium fixed point (28) and the property of matrix operators that $[\mathbf{W}'\mathbf{\Psi}\mathbf{W}]_N = \mathbf{W}'[\mathbf{\Psi}]_{\widehat{N}}\mathbf{W}$ for any matrix $\mathbf{\Psi} \in \mathbb{R}^{K\times K}$. The second equality holds by Eq. (28) and the assumption that the new exchange n' does not create additional cross-asset demand conditioning (i.e., $[\cdot]_{\widehat{N}} = [\cdot]_{\widehat{N}'}$). From the per-unit price impact equation (23), we conclude that $\widehat{\mathbf{\Lambda}}^N = \widehat{\mathbf{\Lambda}}^{N'}$ when $\widehat{\mathbf{B}}^N = \widehat{\mathbf{B}}^{N'}$.

(If) Suppose that $\widehat{\mathbf{\Lambda}}^N = \widehat{\mathbf{\Lambda}}^{N'}$. We show that $\widehat{\mathbf{B}}^N = \widehat{\mathbf{B}}^{N'}$ holds (part (a) below) or $\widehat{\mathbf{\Lambda}}^{N'}$ cannot be an equilibrium per-unit price impact in N' (part (b) below). Namely, we first construct a block-diagonal matrix $\mathbf{\Lambda}^{N'} = diag(\mathbf{\Lambda}_n^{N'})_{n\in N'} \in \mathbb{R}^{(\sum_{n\in N'} K(n))\times(\sum_{n\in N'} K(n))}$ such that $\widehat{\mathbf{\Lambda}}^{N'} = \widehat{\mathbf{\Lambda}}^N$. By Eq. (18), this construction of $\mathbf{\Lambda}^{N'}$ is equivalent to constructing a block-diagonal matrix $\mathbf{C}^{N'} = diag(\mathbf{C}_n^{N'})_{n\in N'} \in \mathbb{R}^{(\sum_{n\in N'} K(n))\times(\sum_{n\in N'} K(n))}$ such that $\mathbf{W}^{N'}\mathbf{C}^{N'}(\mathbf{W}^{N'})' = \mathbf{W}^N\mathbf{C}^N(\mathbf{W}^N)'$. Second, we show that such matrix $\mathbf{C}^{N'}$ cannot be an equilibrium demand Jacobian matrix in market N' if $\widehat{\mathbf{B}}^{N'} \neq \widehat{\mathbf{B}}^N$.

Construction of demand Jacobian $\mathbf{C}^{N'}$ in N'. To construct $\mathbf{C}^{N'} = diag(\mathbf{C}_{K(n)}^{N'})_{n\in N'}$ such that $\widehat{\mathbf{C}}^{N'} = \widehat{\mathbf{C}}^N$, we define $\mathbf{C}_{K(n)}^{N'} = \mathbf{C}_{K(n)}^N$ for exchanges $n \neq n''$, $n \in N$. We pick an arbitrary positive semi-definite matrix $\mathbf{C}_{K(n')}^{N'} \in \mathbb{R}^{K(n')\times K(n')}$ in the new exchange n'. Then, because the new exchange n' does not create a new cross-asset demand conditioning relative to exchange n'', it suffices to find a matrix $\mathbf{C}_{K(n'')}^{N'} \in \mathbb{R}^{K(n'')\times K(n'')}$ that satisfies

$$\mathbf{W}_{K(n')}\mathbf{C}_{K(n')}^{N'}\mathbf{W}_{K(n')}' + \mathbf{W}^N\mathbf{C}_{K(N)}^{N'}(\mathbf{W}^N)' = \mathbf{W}^N\mathbf{C}_{K(N)}^N(\mathbf{W}^N)'. \tag{30}$$

Here, $\mathbf{C}_{K(N)}^{N'} = diag(\mathbf{C}_{K(n)}^{N'})_{n\in N}$ is a submatrix of $\mathbf{C}^{N'}$ that corresponds to the existing exchanges in N including n''. Such matrix $\mathbf{C}_{K(n'')}^{N'}$ exists because matrix equation (30) is equivalent to $\frac{|K(n'')|(|K(n'')|+1)}{2}$ linear equations for non-zero elements of $\mathbf{C}_{K(n'')}^{N'}$, whose number is equal to $\frac{|K(n'')|(|K(n'')|+1)}{2}$. The matrix $\mathbf{C}^{N'}$ constructed in Eq. (30) satisfies $\widehat{\mathbf{C}}^{N'} = \widehat{\mathbf{C}}^N$ or, equivalently, $\widehat{\mathbf{\Lambda}}^{N'} = \widehat{\mathbf{\Lambda}}^N$ by Eq. (23).

Part (a): When $\mathbf{C}_{K(n'')}^N$ is symmetric, there exists $\mathbf{C}^{N''}$ such that $\widehat{\mathbf{B}}^N = \widehat{\mathbf{B}}^{N'}$ holds. We now show that the constructed $\mathbf{C}^{N'}$ satisfies $\widehat{\mathbf{B}}^{N'} = \widehat{\mathbf{B}}^N$ when $\mathbf{C}_{K(n'')}^N$ is symmetric and we pick a symmetric matrix $\mathbf{C}_{K(n'')}^{N'}$. The characterization of the cross-asset inference (Eq. (24)) shows that $\widehat{\mathbf{B}}^{N'} = \widehat{\mathbf{B}}^N$ if and only if Eq. (28) holds, so we prove the equivalent equation (28). Let $\xi \equiv \frac{\sigma_0(I-1)}{(1-\sigma_0)}$. Given the symmetry of $\mathbf{C}_{K(n',n'')}^{N'} = diag(\mathbf{C}_{K(n')}^{N'}, \mathbf{C}_{K(n'')}^{N'})$, we can write Eq. (30) as

$$\mathbf{W}_{\{n',n''\}}^{N'}\big(\mathbf{C}_{K(n',n'')}^{N'} + \xi(\mathbf{C}_{K(n',n'')}^{N'})'\big)^{-1}(\mathbf{W}_{\{n',n''\}}^{N'})' = \mathbf{W}_{n''}^N\big(\mathbf{C}_{K(n'')}^N + \xi(\mathbf{C}_{K(n'')}^N)'\big)^{-1}(\mathbf{W}_{n''}^N)'. \tag{31}$$

Furthermore, since $\mathbf{C}_{K(n)}^{N'} = \mathbf{C}_{K(n)}^N$ and $\mathbf{W}_n^{N'} = \mathbf{W}_n^N$ for all exchanges $n \neq n', n \in N$, we have

$$\mathbf{W}_n^{N'}\big(\mathbf{C}_{K(n)}^{N'} + \xi(\mathbf{C}_{K(n)}^{N'})'\big)^{-1}(\mathbf{W}_n^{N'})' = \mathbf{W}_n^N\big(\mathbf{C}_{K(n)}^N + \xi(\mathbf{C}_{K(n)}^N)'\big)^{-1}(\mathbf{W}_n^N)' \quad \forall n \notin N'. \tag{32}$$

Eqs. (31)-(32) imply that Eq. (28) holds; equivalently $\widehat{\mathbf{B}}^{N'} = \widehat{\mathbf{B}}^N$ holds.

Part (b): When $\mathbf{C}_{K(n'')}^N$ is asymmetric, the constructed $\mathbf{C}^{N'}$ cannot be an equilibrium demand *coefficient.* The proof is by contradiction: Given the equilibrium fixed point (18) and (27), the system of matrix equations (27) and (30) gives

$$\mathbf{W}'_{n''}\big((\mathbf{C}^{N}_{K(n'')})'-\mathbf{C}^{N}_{K(n'')}\big)\mathbf{W}_{n''}=\mathbf{W}'_{n''}\big((\mathbf{C}^{N'}_{K(n'')})'-\mathbf{C}^{N'}_{K(n'')}\big)\mathbf{W}_{n''}+\mathbf{W}'_{n'}\big((\mathbf{C}^{N'}_{K(n')})'-\mathbf{C}^{N'}_{K(n')}\big)\mathbf{W}_{n'}. \tag{33}$$

However, there exists a block-diagonal matrix $diag(\mathbf{C}^{N''}_{K(n)})_n$ that satisfies Eq. (33) only if either $\mathbf{C}^{N}_{K(n'')}$ is symmetric or the $(k,\ell)^{\text{th}}$ element of $\mathbf{C}^{N'}_{K(n')}$ is zero for any (k,ℓ) such that the $(k,\ell)^{\text{th}}$ element of $\mathbf{C}^{N'}_{K(n'')}$ is zero. In the latter case, the new exchange n' is redundant, which implies that $\mathbf{C}^{N}_{K(n'')}$ is symmetric (Theorem 4 in Rostek and Yoon (2021a)). It follows that a matrix $\mathbf{C}^{N''}$ such that $\widehat{\mathbf{\Lambda}}^{N''}=\widehat{\mathbf{\Lambda}}^{N'}$ is an equilibrium only if $\mathbf{C}^{N}_{K(n'')}$ is symmetric. ■

Proof of Proposition 1 (Derivatives and Technology Innovations Are Generally Not Equivalent).

(Part (1): Necessary and Sufficient Conditions for Derivatives to Mimic Exchange Innovations) Suppose that new exchanges $\{K(n')\}_{n'}$ are introduced and $\mathbf{\Lambda}^{N'}$ satisfies Eq. (27) in $N'=N\cup\{K(n')\}_{n'}$, i.e., it is an equilibrium price impact in N'. If the new exchanges are redundant (condition (ii)), the result is trivial by setting $D=0$. In what follows, we assume that new exchanges $\{K(n')\}_{n'}$ are not redundant (condition (ii) does not hold), and show that there exist D derivatives such that equilibrium payoffs (25) are the same in markets N' and $N''=N\cup\{\{d\}\}_{d\in D}$ if and only if the equilibrium price impact in the new exchanges in market N' is symmetric, i.e., $\mathbf{\Lambda}^{N'}_{K(n')}=(\mathbf{\Lambda}^{N'}_{K(n')})'$ for all n' (condition (i)).

(If) Suppose that price impact in new exchanges $\mathbf{\Lambda}^{N'}_{K(n')}$ is symmetric. The proof is constructive: We construct D derivatives with which the per-unit price impact $\widehat{\mathbf{\Lambda}}^{N''}$ and the cross-asset inference $\widehat{\mathbf{B}}^{N''}$ in N'' coincide with $\widehat{\mathbf{\Lambda}}^{N'}$ and $\widehat{\mathbf{B}}^{N'}$ in N'. Then, we show that a positive definite block-diagonal matrix $\mathbf{\Lambda}^{N''}\in\mathbb{R}^{(\sum_n K(n)+D)\times(\sum_n K(n)+D)}$ such that $\widehat{\mathbf{\Lambda}}^{N''}=\widehat{\mathbf{\Lambda}}^{N'}$ and $\widehat{\mathbf{B}}^{N''}=\widehat{\mathbf{B}}^{N'}$ is an equilibrium price impact in $N''\equiv N\cup\{\{d\}\}_{d\in D}$. Let $\mathbf{W}^{N'}\in\mathbb{R}^{K\times(\sum_n K(n)+\sum_{n'}K(n'))}$ and $\mathbf{W}^{N''}\in\mathbb{R}^{K\times(\sum_n K(n)+D)}$ be weight matrices in N' and N'', respectively.

Construction of D derivatives. Consider D derivatives, each of which is a linear combination of two assets that are traded in a new exchange n':

$$D\equiv\{d|d=k\ \text{ or }\ r_d=w_{dk}r_k+w_{dm}r_m \text{ if } \{k,m\}\in K(n') \text{ for a new exchange } n'\}. \tag{34}$$

For each $d\in D$, $(w_{dk},w_{dm})\in\mathbb{R}^2$ are non-zero weights, $w_{dk}\neq 0$ and $w_{dm}\neq 0$. Derivative $d\in D$ maps to each pair of assets k and m that are cleared jointly in some new exchanges $\{K(n')\}_{n'}$.

Given the D derivatives in Eq. (34), we show that there exists a block-diagonal matrix $\mathbf{\Lambda}^{N''}=diag((\mathbf{\Lambda}^{N''}_{K(n)})_{n\in N},(\lambda^{N''}_d)_{d\in D})\in\mathbb{R}^{(\sum_n K(n)+D)\times(\sum_n K(n)+D)}$ such that $\widehat{\mathbf{\Lambda}}^{N''}=\widehat{\mathbf{\Lambda}}^{N'}$ and $\widehat{\mathbf{B}}^{N''}=\widehat{\mathbf{B}}^{N'}$: We define $\mathbf{\Lambda}^{N''}_{K(n)}=\mathbf{\Lambda}^{N'}_{K(n)}$ for $n\in N$ and $\{(\lambda^{N''}_d)^{-1}\}_{d\in D}$ as the solution to linear equations

$$\sum_{d\in D}(\lambda_d^{N''})^{-1}\mathbf{w}_d\mathbf{w}_d' = \sum_{n'}\mathbf{W}_{n'}(\mathbf{\Lambda}_{K(n')}^{N'})^{-1}\mathbf{W}_{n'}'. \tag{35}$$

Matrix equation (35) is a system of linear equations for D variables $\{(\lambda_d^{N''})^{-1}\}_{d\in D}$, one equation for each non-zero element of matrix $\sum_{n'}\mathbf{W}_{n'}(\mathbf{\Lambda}_{K(n')}^{N'})^{-1}\mathbf{W}_{n'}'$. Because $(\mathbf{\Lambda}_{K(n')}^{N'})^{-1}$ is symmetric for all new exchanges n', Eq. (35) gives D equations that are linearly independent when $\mathbf{\Sigma}$ is not singular. Hence, a solution $\{(\lambda_d^{N''})^{-1}\}_{d\in D}$ to Eq. (35) exists. The matrix $\mathbf{\Lambda}^{N''}$ constructed by Eq. (35) satisfies $\widehat{\mathbf{\Lambda}}^{N''} = \widehat{\mathbf{\Lambda}}^{N'}$ and $\widehat{\mathbf{B}}^{N''} = \widehat{\mathbf{B}}^{N'}$ by Eqs. (23)-(24).

The constructed $\mathbf{\Lambda}^{N''}$ is an equilibrium price impact in N''. We show that $\mathbf{\Lambda}^{N''}$ such that $\widehat{\mathbf{\Lambda}}^{N''} = \widehat{\mathbf{\Lambda}}^{N'}$ and $\widehat{\mathbf{B}}^{N''} = \widehat{\mathbf{B}}^{N'}$ is an equilibrium price impact in market N'' with derivatives (34).

We first show that, for any matrix $\mathbf{\Psi} \in \mathbb{R}^{K\times K}$, if $\left([(\mathbf{W}^{N'})'\mathbf{\Psi}\mathbf{W}^{N'}]_{N'}\right)_{\{k,m\}} = \mathbf{0}$, then $\left([(\mathbf{W}^{N''})'\mathbf{\Psi}\mathbf{W}^{N''}]_{N''}\right)_{dd} = 0$, where $r_d = w_{dk}r_k + w_{dm}r_m$. By the definition of derivatives (34), there exists a new exchange n' in market N' such that $k, m \in K(n')$ and in that exchange,

$$\begin{aligned}
\left([(\mathbf{W}^{N'})'\mathbf{\Psi}\mathbf{W}^{N'}]_{N'}\right)_{kk} &= ((\mathbf{W}^{N'})'\mathbf{\Psi}\mathbf{W}^{N'})_{kk} = \psi_{kk} = 0 \quad \forall k \in K(n'), &(36)\\
\left([(\mathbf{W}^{N'})'\mathbf{\Psi}\mathbf{W}^{N'}]_{N'}\right)_{km} &= ((\mathbf{W}^{N'})'\mathbf{\Psi}\mathbf{W}^{N'})_{km} = \psi_{km} = 0 \quad \forall k, m \in K(n') \quad m \neq k. &(37)
\end{aligned}$$

Eqs. (36)-(37) show that $\left([(\mathbf{W}^{N''})'\mathbf{\Psi}\mathbf{W}^{N''}]\right)_{dd} = 0$ holds for all $d \in D$:

$$\left([(\mathbf{W}^{N''})'\mathbf{\Psi}\mathbf{W}^{N''}]_{N''}\right)_{dd} = (w_{dk}, w_{dm})\begin{bmatrix}\psi_{kk} & \psi_{km}\\ \psi_{mk} & \psi_{mm}\end{bmatrix}(w_{dk}, w_{dm})' = 0. \tag{38}$$

This result (Eq. (38)) implies that $\mathbf{\Lambda}^{N''}$ defined in Eq. (35) is an equilibrium price impact in N'', given the equilibrium price impact $\mathbf{\Lambda}^{N'}$ in N'. This can be seen from the equilibrium fixed point (27) in N'': For each existing exchange $n \in N$,

$$\begin{aligned}
&\left(\left[(\mathbf{W}^{N''})'\mathbf{\Psi}_1(\widehat{\mathbf{B}}^{N''})\mathbf{W}^{N''}\right]_{N''} + \mathbf{\Lambda}^{N''}\left((1-\sigma_0)\mathbf{\Lambda}^{N''} + \sigma_0(\mathbf{\Lambda}^{N''})'\right)^{-1}\left[(\mathbf{W}^{N''})'\mathbf{\Psi}_2(\widehat{\mathbf{B}}^{N''})\mathbf{W}^{N''}\right]_{N''}\right)_{K(n)} &(39)\\
&= \mathbf{W}_n'\mathbf{\Psi}_1(\widehat{\mathbf{B}}^{N''})\mathbf{W}_n + \mathbf{\Lambda}_{K(n)}^{N''}\left((1-\sigma_0)\mathbf{\Lambda}_{K(n)}^{N''} + \sigma_0(\mathbf{\Lambda}_{K(n)}^{N''})'\right)^{-1}\mathbf{W}_n'\mathbf{\Psi}_2(\widehat{\mathbf{B}}^{N''})\mathbf{W}_n\\
&= \left(\left[(\mathbf{W}^{N'})'\mathbf{\Psi}_1(\widehat{\mathbf{B}}^{N'})\mathbf{W}^{N'}\right]_{N'} + \mathbf{\Lambda}^{N'}\left((1-\sigma_0)\mathbf{\Lambda}^{N'} + \sigma_0(\mathbf{\Lambda}^{N'})'\right)^{-1}\left[(\mathbf{W}^{N'})'\mathbf{\Psi}_2(\widehat{\mathbf{B}}^{N'})\mathbf{W}^{N'}\right]_{N'}\right)_{K(n)} = \mathbf{0}.
\end{aligned}$$

The first equality follows from $N \subset N'$ and $N \subset N''$; the second equality holds because $\mathbf{\Lambda}_{K(n)}^{N''} = \mathbf{\Lambda}_{K(n)}^{N'}$ for all $n \in N$ and $\widehat{\mathbf{B}}^{N''} = \widehat{\mathbf{B}}^{N'}$ by Eqs. (24) and (35); and the third equality holds because $\mathbf{\Lambda}^{N'}$ is an equilibrium price impact in N'.

On the other hand, for each derivative $d \in D$ defined in Eq. (34), we have

$$\Big([(\mathbf{W}^{N''})'\mathbf{\Psi}_1(\widehat{\mathbf{B}}^{N''})\mathbf{W}^{N''}]_{N''}+\mathbf{\Lambda}^{N''}\big((1-\sigma_0)\mathbf{\Lambda}^{N''}+\sigma_0(\mathbf{\Lambda}^{N''})'\big)^{-1}[(\mathbf{W}^{N''})'\mathbf{\Psi}_2(\widehat{\mathbf{B}}^{N''})\mathbf{W}^{N''}]_{N''}\Big)_{dd} \tag{40}$$
$$=\mathbf{w}_d'\mathbf{\Psi}_1(\widehat{\mathbf{B}}^{N''})\mathbf{w}_d+\mathbf{w}_d'\mathbf{\Psi}_2(\widehat{\mathbf{B}}^{N''})\mathbf{w}_d$$
$$=\Big((\mathbf{W}^{N'}_{K(n')})'\mathbf{\Psi}_1(\widehat{\mathbf{B}}^{N'})\mathbf{W}^{N'}_{K(n')}+(\mathbf{W}^{N'}_{K(n')})'\mathbf{\Psi}_2(\widehat{\mathbf{B}}^{N'})\mathbf{W}^{N'}_{K(n')}\Big)_{\{k,m\}}$$
$$=\Big((\mathbf{W}^{N'}_{K(n')})'\mathbf{\Psi}_1(\widehat{\mathbf{B}}^{N'})\mathbf{W}^{N'}_{K(n')}+\mathbf{\Lambda}^{N'}_{K(n')}\big((1-\sigma_0)\mathbf{\Lambda}^{N'}_{K(n')}+\sigma_0(\mathbf{\Lambda}^{N'}_{K(n')})'\big)^{-1}(\mathbf{W}^{N'}_{K(n')})'\mathbf{\Psi}_2(\widehat{\mathbf{B}}^{N'})\mathbf{W}^{N'}_{K(n')}\Big)_{\{k,m\}}=\mathbf{0}.$$

The first equality follows from $\lambda_d^{N''}((1-\sigma_0)\lambda_d^{N''}+\sigma_0\lambda_d^{N''})^{-1}=1$; the second equality holds by the argument around Eq. (38); the third equality holds by $\mathbf{\Lambda}^{N'}_{K(n')}\big((1-\sigma_0)\mathbf{\Lambda}^{N'}_{K(n')}+\sigma_0(\mathbf{\Lambda}^{N'}_{K(n')})'\big)^{-1}=\mathbf{Id}$ (condition (i)); and the fourth equality is satisfied because $\mathbf{\Lambda}^{N'}$ is an equilibrium price impact in N'.

Eqs. (39)-(40) imply that $\mathbf{\Lambda}^{N''}$ such that $\widehat{\mathbf{\Lambda}}^{N''}=\widehat{\mathbf{\Lambda}}^{N'}$ and $\widehat{\mathbf{B}}^{N''}=\widehat{\mathbf{B}}^{N'}$ (Eq. (35)) is an equilibrium price impact in N''. Lemma 1 completes the proof.

(Only if) Suppose there exist derivatives with which equilibrium payoffs in $N''=N\cup\{\{d\}\}_{d\in D}$ coincide with those in N', or equivalently, $\widehat{\mathbf{\Lambda}}^{N''}=\widehat{\mathbf{\Lambda}}^{N'}$ and $\widehat{\mathbf{B}}^{N''}=\widehat{\mathbf{B}}^{N'}$ (Lemma 1). The arguments in parts (a) and (b) of the proof of Lemma 1 apply to the comparison between two innovations (markets N' and N'') and show that one of the conditions (i) and (ii) is necessary for $\widehat{\mathbf{\Lambda}}^{N''}=\widehat{\mathbf{\Lambda}}^{N'}$ and $\widehat{\mathbf{B}}^{N''}=\widehat{\mathbf{B}}^{N'}$ to hold.

(Part (2): Necessary and Sufficient Conditions for Exchange Innovations to Mimic Derivatives) Suppose that $I<\infty, K>1$, and $N=\{K(n)\}_n$. $D\geq 1$ derivatives are introduced, i.e., $N''=N\cup\{\{d\}\}_{d\in D}$. We show that there exists a market structure $N'=N\cup\{K(n')\}_{n'}$ such that equilibrium payoffs in N' are the same as those in N'' if and only if the following conditions hold:

(i) There exists $D'=|K(n')|(|K(n')|+1)/2$ derivatives, each with at most two underlying assets (i.e., $r_{d'}=w_{d'k}r_k+w_{d'm}r_m$ for all d', for all pairs $k,m\in K(n')$), such that equilibrium payoffs in $N'''=N\cup\{\{d'\}\}_{d'\in D'}$ coincide with those in N''; and

(ii) Price impact $\mathbf{\Lambda}^{N'}_{K(n')}$ is symmetric for all n'.

(If) The proof is constructive. We introduce new exchanges $\{K(n')\}_{n'}$ such that each n' corresponds to one derivative $d'\in D'$ in condition (i):

$$K(n')\equiv\{k,m\},\quad \text{where } r_{d'}=w_{d'k}r_k+w_{d'm}r_m \text{ for some } d'\in D'.$$

If $\mathbf{\Lambda}^{N'}_{K(n')}$ for all n' (condition (ii) holds), the same argument as in Part (1) applies. Part "If" shows that equilibrium payoffs in $N'=N\cup\{K(n')\}_{n'}$ and $N'''=N\cup\{\{d'\}\}_{d'\in D'}$ are the same, and, by condition (i), also coincide with those in N''.

(Only if) We prove the necessity of conditions (i)-(ii) by contradiction: if either condition (i) or (ii) fails, then there is no market structure N' that gives the same equilibrium payoffs as N''; i.e., there is no market structure N' such that $\widehat{\mathbf{\Lambda}}^{N'} = \widehat{\mathbf{\Lambda}}^{N''}$ and $\widehat{\mathbf{B}}^{N'} = \widehat{\mathbf{B}}^{N''}$.

Suppose that condition (ii) does not hold: $\mathbf{\Lambda}^{N'}_{K(n')}$ is asymmetric for all n'. It is immediate that $\widehat{\mathbf{\Lambda}}^{N'} \neq \widehat{\mathbf{\Lambda}}^{N''}$, because $\widehat{\mathbf{\Lambda}}^{N''}$ is symmetric by Eq. (23) but $\widehat{\mathbf{\Lambda}}^{N'}$ is asymmetric.

Suppose that condition (i) does not hold, while condition (ii) does: A nonredundant derivative $d \in D$ with more than two underlying assets (i.e., $K(d) \equiv \{k \in K | w_{dk} \neq 0\}$ satisfies $|K(d)| > 2$) is traded. Then, we can pick three imperfectly correlated underlying assets $\{k_1, k_2, k_3\} \subseteq K(d)$ such that for at least one pair $\{k_1, k_2\}$, $\{k_2, k_3\}$, and $\{k_1, k_3\}$, d is the only derivative whose return is a linear combination of that pair's assets, e.g., $\{k_1, k_2\} \notin K(d')$ for all $d' \neq d, d' \in D$. This is because, if such a triple $\{k_1, k_2, k_3\}$ does not exist, then derivative d is redundant (by Proposition 2).

From Eq. (23), the inverse of the per-unit price impact $(\widehat{\mathbf{\Lambda}}^{N''})^{-1} = (I-1)\mathbf{W}^{N''}(\mathbf{C}^{N''})'(\mathbf{W}^{N''})'$ in market N'' has a non-zero off diagonal element $(\widehat{\mathbf{\Lambda}}^{N''})^{-1}_{k_1 k_2} \neq 0$, $(\widehat{\mathbf{\Lambda}}^{N''})^{-1}_{k_1 k_3} \neq 0$, and $(\widehat{\mathbf{\Lambda}}^{N''})^{-1}_{k_2 k_3} \neq 0$. Because $(\widehat{\mathbf{\Lambda}}^{N'})^{-1} = (\widehat{\mathbf{\Lambda}}^{N''})^{-1}$, the corresponding off diagonal elements of $(\widehat{\mathbf{\Lambda}}^{N'})^{-1} = (I-1)\mathbf{W}^{N'}(\mathbf{C}^{N'})'(\mathbf{W}^{N'})'$ are also non-zero. Hence, by the definition of demand coefficient $\mathbf{C}^{N'}$ (Eq. (14)), there exists an exchange in market N in which each pair of assets $\{k_1, k_2\}$, $\{k_1, k_3\}$, and $\{k_2, k_3\}$ is cleared jointly.

However, equilibrium price impact of market N' in which each pair $\{k_1, k_2\}$, $\{k_1, k_3\}$, and $\{k_2, k_3\}$ is cleared jointly in an exchange cannot satisfy $\widehat{\mathbf{\Lambda}}^{N'} = \widehat{\mathbf{\Lambda}}^{N''}$ and $\widehat{\mathbf{B}}^{N'} = \widehat{\mathbf{B}}^{N''}$. This can be seen in equilibrium fixed point (27) in market N':

$$\left[(\mathbf{W}^{N'})'(\mathbf{\Psi}_1(\widehat{\mathbf{B}}^{N'}) + \mathbf{\Psi}_1(\widehat{\mathbf{B}}^{N'}))\mathbf{W}^{N'}\right]_{N'} = \mathbf{0}. \tag{41}$$

Given $\widehat{\mathbf{B}}^{N'} = \widehat{\mathbf{B}}^{N''}$, equivalently $\widehat{\mathbf{\Lambda}}^{N'} = \widehat{\mathbf{\Lambda}}^{N''}$, price impacts $\lambda^{N''}_{k_1}, \lambda^{N''}_{k_2}, \lambda^{N''}_{k_3}$, and $\lambda^{N''}_d$ must be a solution to the matrix equation $\left(\mathbf{\Psi}_1(\widehat{\mathbf{B}}^{N''}) + \mathbf{\Psi}_1(\widehat{\mathbf{B}}^{N''})\right)_{\{k_1,k_2,k_3\}} = \mathbf{0}$, i.e., all elements of the LHS in Eq. (41) that correspond to $\{k_1, k_2, k_3\}$ are zero. The matrix equation $\left(\mathbf{\Psi}_1(\widehat{\mathbf{B}}^{N''}) + \mathbf{\Psi}_1(\widehat{\mathbf{B}}^{N''})\right)_{\{k_1,k_2,k_3\}} = \mathbf{0}$ is the system of six linearly independent equations for four variables $\lambda^{N''}_{k_1}, \lambda^{N''}_{k_2}, \lambda^{N''}_{k_3}$, and $\lambda^{N''}_d$. Because the number of equations exceeds the number of variables, (generically) there exists no solution $\lambda^{N''}_{k_1}, \lambda^{N''}_{k_2}, \lambda^{N''}_{k_3}$, and $\lambda^{N''}_d$. It follows that $\widehat{\mathbf{\Lambda}}^{N'} = \widehat{\mathbf{\Lambda}}^{N''}$ and $\widehat{\mathbf{B}}^{N'} = \widehat{\mathbf{B}}^{N''}$ cannot hold in market N' when condition (i) does not hold. ■

Corollary 4 provides a sufficient condition on the primitives for derivatives to substitute for innovation in market-clearing technology. Corollary 3 shows that a unique equilibrium exists and induces a symmetric price impact in a subclass of symmetric markets (Definition 5). The proofs of Corollaries 3 and 4 are immediate from Rostek and Yoon (2021b, Proposition 7) and Proposition 1.

Corollary 3 (Equilibrium Existence and Uniqueness: Sufficient Conditions) Let $K =$

$DL > 1, L \geq 1, D \geq 1$, and suppose that asset covariances are symmetric, i.e., $\sigma_{kk} = \sigma^2$ for all k and $\sigma_{k\ell} = \sigma^2\rho$ for all k and $\ell \neq k$. Apart from the K assets, for each disjoint set of L underlying assets, either a derivative whose return is defined by the unweighted average of L assets, i.e., $r_d = \frac{1}{L}\sum_{m=1}^{L} r_{L(d-1)+m}$, or an exchange $\{L(d-1)+m : m = 1, \cdots, L\}$ is introduced. Equilibrium exists and is unique.

Corollary 4 (Derivatives and Innovation in Market-Clearing Technology Are Generally Not Equivalent) Let $I < \infty$ and $N = \{K(n)\}_n$. Suppose that a new exchange n' is introduced in market N, i.e., $N' = N \cup \{n'\}$. There exists a set of D derivatives such that traders' *ex ante* equilibrium payoffs in market structure $N'' = N \cup \{\{d\} : d \in D\}$ are the same as those in market structure N for all distributions of asset holdings $F(\boldsymbol{q}_0^i)_i$ if one of the following conditions holds:

(i) The market structure N' is symmetric for any asset pair k and $\ell \neq k$ in the new exchange n' and for any asset pair k' and $\ell' \neq k'$ not in n'. Furthermore, the asset covariances are symmetric for any asset pair k and $\ell \neq k$ in the new exchange n' (i.e., $\sigma_{kk''} = \sigma_{\ell k''}$ for all k'') and for any asset pair k' and $\ell' \neq k'$ not in n' (i.e., $\sigma_{k'k''} = \sigma_{\ell' k''}$ for all k'').

(ii) An existing exchange $n'' \in N$ lists the same assets as the new exchange n', i.e., there exists $n'' \in N$ such that $K(n'') = K(n')$.

Condition (i) in Corollary 4 induces a symmetric price impact $\boldsymbol{\Lambda}^{N'}_{K(n')}$ in the new exchange n' (i.e., condition (i) in Proposition 1). Condition (ii) in Corollary 4 makes the new exchange n' redundant (i.e., condition (ii) in Proposition 1).

Proof of Proposition 2 (Nonredundancy of Innovations). Suppose that we introduce an exchange n'' with L securities and let $N'' = N \cup \{K(n'')\}$. We show that the introduction of exchange n'' is redundant if and only if conditions (i)-(ii) hold.

(If) Suppose conditions (i)-(ii) hold. The proof of Lemma 1 constructs a block-diagonal matrix $\boldsymbol{\Lambda}^{N''} = diag(\boldsymbol{\Lambda}_n^{N''})_{n \in N''} \in \mathbb{R}^{(\sum_{n \in N''} K(n)) \times (\sum_{n \in N''} K(n))}$ such that $\widehat{\boldsymbol{\Lambda}}^{N''} = \widehat{\boldsymbol{\Lambda}}^{N}$, and shows that such $\boldsymbol{\Lambda}^{N''}$ satisfies $\widehat{\mathbf{B}}^{N''} = \widehat{\mathbf{B}}^{N}$ if conditions (i)-(ii) hold. Lastly, we now show that $\boldsymbol{\Lambda}^{N''}$ is an equilibrium price impact in market N''.

The constructed $\boldsymbol{\Lambda}^{N''}$ is an equilibrium price impact. We show that equivalently, $\mathbf{C}^{N''} = \frac{1}{I-1}\big((\boldsymbol{\Lambda}^{N''})^{-1}\big)'$ constructed as Eq. (30) is the equilibrium demand coefficient. By applying the Woodbury Matrix Identity to $\mathbf{B}$ (Eq. (16)), equilibrium fixed-point equation (18) can be rewritten in terms of the per-unit price impact $\widehat{\boldsymbol{\Lambda}}^{N''}$ and the cross-asset inference $\widehat{\mathbf{B}}^{N''}$:

$$\Big[(\mathbf{W}^{N''})'(\alpha\boldsymbol{\Sigma} + \widehat{\boldsymbol{\Lambda}}^{N''} - (\widehat{\mathbf{C}}^{N''})^{-1})\widehat{\mathbf{B}}^{N''}(\widehat{\mathbf{B}}^{N''})'\mathbf{W}^{N''}\Big]_{N''} = \mathbf{0}. \tag{42}$$

Given that $\widehat{\mathbf{\Lambda}}^N = \widehat{\mathbf{\Lambda}}^{N''}$, $\widehat{\mathbf{B}}^N = \widehat{\mathbf{B}}^{N''}$, and $\mathbf{W}^{N''} = \begin{bmatrix} \mathbf{W}^N & \mathbf{W}_{n''} \end{bmatrix}$, the submatrix of the LHS of Eq. (42) that corresponds to existing exchanges N equals $\mathbf{0}$:

$$\begin{aligned}&\left(\left[(\mathbf{W}^{N''})'(\alpha\mathbf{\Sigma} + \widehat{\mathbf{\Lambda}}^{N''} - (\widehat{\mathbf{C}}^{N''})^{-1})\widehat{\mathbf{B}}^{N''}(\widehat{\mathbf{B}}^{N''})'\mathbf{W}^{N''}\right]_{N''}\right)_N \\ &= \left[(\mathbf{W}^{N})'(\alpha\mathbf{\Sigma} + \widehat{\mathbf{\Lambda}}^{N} - (\widehat{\mathbf{C}}^{N})^{-1})\widehat{\mathbf{B}}^{N}(\widehat{\mathbf{B}}^{N})'\mathbf{W}^{N}\right]_{N} = \mathbf{0}.\end{aligned}$$

The submatrix of the LHS of Eq. (42) that corresponds to the new exchange n'' equals zero by conditions (i) and (ii): For each security $\ell \in L$,

$$\mathbf{w}_\ell\big((\alpha\mathbf{\Sigma}+\widehat{\mathbf{\Lambda}}^{N''}-(\widehat{\mathbf{C}}^{N''})^{-1})\widehat{\mathbf{B}}^{N''}(\widehat{\mathbf{B}}^{N''})'\big)\mathbf{w}_\ell' = \sum_m\sum_{m'} w_{\ell m}w_{\ell m'}\big((\alpha\mathbf{\Sigma}+\widehat{\mathbf{\Lambda}}^{N''}-(\widehat{\mathbf{C}}^{N''})^{-1})\widehat{\mathbf{B}}^{N''}(\widehat{\mathbf{B}}^{N''})'\big)_{\ell,\ell'} = 0,$$

because $\big((\alpha\mathbf{\Sigma}+\widehat{\mathbf{\Lambda}}^{N''}-(\widehat{\mathbf{C}}^{N''})^{-1})\widehat{\mathbf{B}}^{N''}(\widehat{\mathbf{B}}^{N''})'\big)_{m,m'} + \big((\alpha\mathbf{\Sigma}+\widehat{\mathbf{\Lambda}}^{N''}-(\widehat{\mathbf{C}}^{N''})^{-1})\widehat{\mathbf{B}}^{N''}(\widehat{\mathbf{B}}^{N''})'\big)_{m',m} = 0$ for all m, m' by condition (i). Furthermore, for any pair of securities $\ell, \ell' \in L$, $\ell' \neq \ell$,

$$\mathbf{w}_\ell\big((\alpha\mathbf{\Sigma}+\widehat{\mathbf{\Lambda}}^{N''}-(\widehat{\mathbf{C}}^{N''})^{-1})\widehat{\mathbf{B}}^{N''}(\widehat{\mathbf{B}}^{N''})'\big)\mathbf{w}_{\ell'}' = \sum_m\sum_{m'} w_{\ell m}w_{\ell' m'}\big((\alpha\mathbf{\Sigma}+\widehat{\mathbf{\Lambda}}^{N''}-(\widehat{\mathbf{C}}^{N''})^{-1})\widehat{\mathbf{B}}^{N''}(\widehat{\mathbf{B}}^{N''})'\big)_{m,m'} = 0,$$

because $\big((\alpha\mathbf{\Sigma}+\widehat{\mathbf{\Lambda}}^{N''}-(\widehat{\mathbf{C}}^{N''})^{-1})\widehat{\mathbf{B}}^{N''}(\widehat{\mathbf{B}}^{N''})'\big)_{m,m'} = 0$ for all m, m' such that $w_{\ell m} \neq 0$ and $w_{\ell' m'} \neq 0$ by condition (ii). It follows that the constructed $\mathbf{C}^{N''}$ in Eq. (30) satisfies the equilibrium fixed point (42), and hence is the equilibrium demand coefficient. Therefore, $\mathbf{\Lambda}^{N''} = \frac{1}{I-1}((\mathbf{C}^{N''})^{-1})'$ is the equilibrium price impact. Because $\widehat{\mathbf{\Lambda}}^N = \widehat{\mathbf{\Lambda}}^{N''}$ and $\widehat{\mathbf{B}}^N = \widehat{\mathbf{B}}^{N''}$, Lemma 1 implies that equilibrium payoffs are the same in markets N and N''.

(Only if) We prove a contrapositive: if one of conditions (i)-(ii) does not hold, then the introduction of a new exchange n'' is nonredundant.

Suppose that there exists no collection of exchanges N' that provides locally contingent demands for assets L (condition (i) does not hold). Without loss of generality, we suppose that distinct assets $k, \ell \in L$, $k \neq \ell$ are not traded in any existing exchange in N (i.e., $\{k, \ell\} \not\subset K(n)$ for all $n \in N$) and assets k and ℓ do not both underlie a security in N (i.e., either $w_{mk} = 0$ or $w_{m\ell} = 0$ for any security m traded in N). By the definition of the weight matrix, the $(k, \ell)^{\text{th}}$ element of $\mathbf{W}^N\mathbf{C}^N(\mathbf{W}^N)'$ is zero. The introduction of exchange $\{k, \ell\}$ is not redundant, because the $(k, \ell)^{\text{th}}$ element of the equilibrium demand coefficient $\mathbf{C}^{N''}_{\{k,\ell\}}$ in exchange $\{k, \ell\}$ is non-zero, except when assets k and ℓ are independent mutually and with all other assets. Therefore, in market $N'' = N \cup \{\{k, \ell\}\}$, the $(k, \ell)^{\text{th}}$ element of $\mathbf{W}^{N''}\mathbf{C}^{N''}(\mathbf{W}^{N''})'$ is non-zero in equilibrium, so we get $\mathbf{W}^{N''}\mathbf{C}^{N''}(\mathbf{W}^{N''})' \neq \mathbf{W}^N\mathbf{C}^N(\mathbf{W}^N)'$, and equivalently, $\widehat{\mathbf{\Lambda}}^{N''} \neq \widehat{\mathbf{\Lambda}}^N$.

Suppose that there exists a collection of exchanges N' with locally contingent demands for assets L (condition (i) holds), but $\mathbf{\Lambda}^N_{K(N')}$ is not symmetric (condition (ii) does not hold). Without loss of generality, suppose that $K(n') \cup L \neq \emptyset$ for all $n' \in N'$. The asymmetry of matrix $\mathbf{\Lambda}^N_{K(N')}$ implies that there exists an exchange $n' \in N'$ with more than one security (i.e., $|K(n')| > 1$), and that the price impact in exchange n' is not symmetric (i.e., $\mathbf{\Lambda}^N_{K(n')} \neq (\mathbf{\Lambda}^N_{K(n')})'$). By Theorem 4 in Rostek and Yoon (2021a), introducing an exchange n'' such that

$K(n'') \subset K(n')$ is not redundant and $\mathbf{\Lambda}^{N''}_{K(n'')}$ is not symmetric for the new exchange n''. ■

Corollary 5 (Nonredundancy of Innovations: Competitive Market) Let $I \to \infty$, $M > 1$, and $N = \{K(n)\}_n$. Suppose that the equilibrium in the market structure N is not *ex post*. Introducing an exchange n' with $L \subsetneq K$ assets that does not replicate an existing exchange is redundant if and only if the following conditions hold:

(i) (*Locally contingent market*) There is a set of exchanges $N'' \subset N$ such that, for each pair of assets, there is an exchange in which they are traded or a derivative which bundles their returns; there must be a separate such derivative—but not an exchange—for each pair. In addition to (i), one of the following conditions holds:

(ii) (*Symmetric demand coefficient*) The submatrix of the per-unit demand Jacobian corresponding to exchanges N'', $\widehat{\mathbf{C}}_{K(N'')} \equiv \mathbf{W}_{K(N'')} diag(\mathbf{C}_{n''})_{n'' \in N''} \mathbf{W}'_{K(N'')}$, and the demand coefficient of the new exchange $\mathbf{C}_{K(n')}$ are symmetric; or

(iii) (*No increase in demand conditioning*) For each asset or derivative $\ell \in K(n')$ in the new exchange n', the number of the contingent variables of demand $q^i_{\ell,n'}(\cdot)$ in the new exchange n' is smaller than or equal to the number of the contingent variables of demand $q^i_{k,n}(\cdot)$ for all asset k such that $w_{\ell k} \neq 0$ in all existing exchanges n.

Proof of Corollary 5 (Nonredundancy of Innovations: Competitive Market). The proof is analogous to the proof of Proposition 2. A block-diagonal matrix $\mathbf{C}^{N''}$ such that $\widehat{\mathbf{C}}^{N''} = \widehat{\mathbf{C}}^{N}$ is constructed using Eq. (30) in the proof of Lemma 1. As seen in ft. 38, $\widehat{\mathbf{B}}^{N''} = \widehat{\mathbf{B}}^{N}$ also holds given the constructed $\mathbf{C}^{N''}$, regardless of the symmetry of $\mathbf{C}^{N''}$. We show that the constructed matrix $\mathbf{C}^{N''}$ satisfies the equilibrium fixed point,

$$\left[(\mathbf{W}^{N''})'(\alpha\mathbf{\Sigma} - (\widehat{\mathbf{C}}^{N''})^{-1})\widehat{\mathbf{B}}^{N''}(\widehat{\mathbf{B}}^{N''})'\mathbf{W}^{N''}\right]_{N''} = \mathbf{0}, \tag{43}$$

if and only if either conditions (i) and (ii) or conditions (i) and (iii) hold.

For simplicity, we define $\mathbf{\Psi}^{N''} \equiv (\alpha\mathbf{\Sigma} - (\widehat{\mathbf{C}}^{N''})^{-1})\widehat{\mathbf{B}}^{N''}(\widehat{\mathbf{B}}^{N''})'$ in Eq. (43), and similarly $\mathbf{\Psi}^{N} \equiv (\alpha\mathbf{\Sigma} - (\widehat{\mathbf{C}}^{N})^{-1})\widehat{\mathbf{B}}^{N}(\widehat{\mathbf{B}}^{N})'$. Given that $\widehat{\mathbf{C}}^{N''} = \widehat{\mathbf{C}}^{N}$ and $\widehat{\mathbf{B}}^{N''} = \widehat{\mathbf{B}}^{N}$ holds, we get $\mathbf{\Psi}^{N''} = \mathbf{\Psi}^{N}$. Hence, the proof can be completed by showing that

$$\left[(\mathbf{W}^{N''})'\mathbf{\Psi}\mathbf{W}^{N''}\right]_{N''} = \mathbf{0} \;\text{ holds for } \mathbf{\Psi} \in \mathbb{R}^{K\times K} \text{ s.t. } \left[(\mathbf{W}^{N})'\mathbf{\Psi}\mathbf{W}^{N}\right]_{N} = \mathbf{0} \tag{44}$$

is satisfied if and only if either conditions (i) and (ii) or conditions (i) and (iii) hold.

(If) Suppose conditions (i)-(ii) hold. By the same argument as in the proof of Proposition 2, $\left[(\mathbf{W}^{N''})'\mathbf{\Psi}\mathbf{W}^{N''}\right]_{N''} = \mathbf{0}$ holds whenever $\left[(\mathbf{W}^{N})'\mathbf{\Psi}\mathbf{W}^{N}\right]_{N} = \mathbf{0}$ does. Hence, equilibrium payoffs are the same in N'' and N.

Suppose conditions (i) and (iii) hold. Then, there exists an exchange $n \in N$ such that all securities in the new exchange n' are in the span of securities $K(n)$ in exchange n. In fact, there exists a weight matrix $\widetilde{\mathbf{W}} = (\widetilde{w}_{k\ell})_{\ell,k} \in \mathbb{R}^{K(n)\times K(n')}$ such that each security k's return in the new exchange n' can be written as a linear combination of the securities in exchange n: for each $k \in K(n')$, $r_k = \sum_{\ell\in K(n)} \widetilde{w}_{k\ell} r_\ell = \sum_{\ell \in K(n)} \widetilde{w}_{k\ell}(\mathbf{w}'_\ell \mathbf{r})$. Given $\widetilde{\mathbf{W}}$, in Eq. (44), the block-diagonal matrix $(\mathbf{W}_n^N)'\mathbf{\Psi}\mathbf{W}_n^N = (\mathbf{W}_n^{N''})'\mathbf{\Psi}\mathbf{W}_n^{N''} = \mathbf{0}$ implies that

$$(\mathbf{W}_{n'}^{N''})'\mathbf{\Psi}\mathbf{W}_{n'}^{N''} = \widetilde{\mathbf{W}}'(\mathbf{W}_n^N)'\mathbf{\Psi}\mathbf{W}_n^N\widetilde{\mathbf{W}} = \mathbf{0},$$

which proves that Eq. (44) holds. Hence, equilibrium payoffs in N'' are the same as in N.

(Only if) The necessity of condition (i) is proved in Proposition 2. For the necessity of condition (ii) or (iii), we prove the contrapositive. Suppose that conditions (ii) and (iii) are both violated: securities $x, y \in K(n')$ such that $x \neq y$ contain underlying assets k and ℓ that are not traded in the same exchange together, i.e., $w_{xk} \neq 0$, $w_{y\ell} \neq 0$, and $\{k, \ell\} \notin K(n')$ for all $n' \in N'$. However, condition (i) implies that there exists a security m in market N such that $w_{mk} \neq 0$ and $w_{m\ell} \neq 0$. The equilibrium demand coefficient $\mathbf{C}^{N''}$ in market $N'' = N \cup \{n''\}$ must satisfy the equilibrium fixed-point equation (Eq. (43)). Without loss of generality, we assume that new securities x and y have return $r_x = r_k$ and $r_y = r_\ell$, and the security m is such that $r_m = w_{mk}r_k + w_{m\ell}r_\ell$ for some $w_{mk} \neq 0$ and $w_{m\ell} \neq 0$. The argument below extends to securities x, y, and m, that contain other underlying assets.

Equilibrium fixed-point equation (43) in market N'' implies that both the $(k,\ell)^{\text{th}}$ element and the $(\ell,k)^{\text{th}}$ element of the LHS of Eq. (43) must be zero:

$$\big((\alpha\mathbf{\Sigma} - (\widehat{\mathbf{C}}^{N''})^{-1})\widehat{\mathbf{B}}^{N''}(\widehat{\mathbf{B}}^{N''})'\big)_{k,\ell} = \big((\alpha\mathbf{\Sigma} - (\widehat{\mathbf{C}}^{N''})^{-1})\widehat{\mathbf{B}}^{N''}(\widehat{\mathbf{B}}^{N''})'\big)_{\ell,k} = 0. \tag{45}$$

The corresponding equilibrium fixed-point equation in market N implies that $\mathbf{C}^N$ satisfies

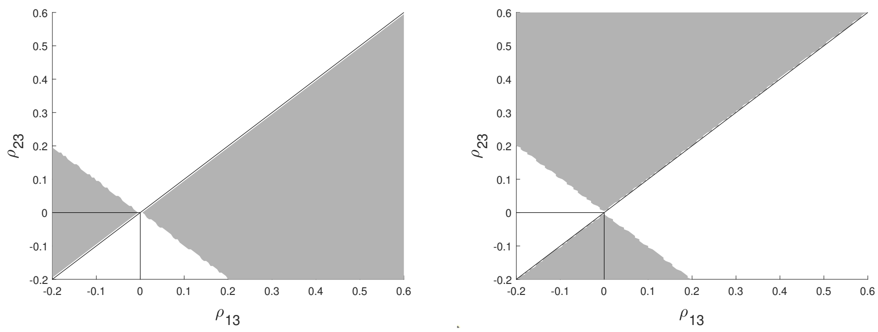
$$w_{mk}w_{m\ell}\Big(\big((\alpha\mathbf{\Sigma} - (\widehat{\mathbf{C}}^{N})^{-1})\widehat{\mathbf{B}}^{N}(\widehat{\mathbf{B}}^{N})'\big)_{k,\ell} + \big((\alpha\mathbf{\Sigma} - (\widehat{\mathbf{C}}^{N})^{-1})\widehat{\mathbf{B}}^{N}(\widehat{\mathbf{B}}^{N})'\big)_{\ell,k}\Big) = 0, \tag{46}$$

i.e., the sum of the $(k,\ell)^{\text{th}}$ and the $(\ell,k)^{\text{th}}$ elements must be zero. When the demand coefficient $\widehat{\mathbf{C}}^N$ satisfies Eq. (46), $\widehat{\mathbf{C}}^{N''} = \widehat{\mathbf{C}}^N$ does not satisfy Eq. (45), unless the matrix $(\alpha\mathbf{\Sigma} - (\widehat{\mathbf{C}}^N)^{-1})\widehat{\mathbf{B}}^N(\widehat{\mathbf{B}}^N)'$ is a symmetric matrix, i.e., unless equilibrium in market N is *ex post*. Hence, given that equilibrium in N is not *ex post*, the new exchange n' is not redundant. ■

A.3 Example

Example 8 (Nonlinear Pricing of Derivatives) Consider market structure $N = \{\{1,2\},\{3\},\{d\}\}$ and a derivative d that assigns the same weights to assets 1 and 2 (i.e., $r_d = \frac{1}{2}r_1 + \frac{1}{2}r_2$). Assets 1 and 2 are symmetric with respect to their return variances ($\sigma_{11} = \sigma_{22}$) and the realizations of traders' aggregate initial holdings ($|\overline{q}_{0,1}| = |\overline{q}_{0,2}|$), but they can be heterogeneously correlated with asset 3 (i.e., σ_{13} and σ_{23} can differ).

Figure 2: UNDERPRICED AND OVERPRICED DERIVATIVES



(A) (B)

Notes. With innovation in market-clearing technology, derivatives are not linearly priced, given the underlying asset prices. A derivative is generally either underpriced, i.e., $p_d < \sum_k w_{dk} p_k$ (grey areas), or overpriced, i.e., $p_d > \sum_k w_{dk} p_k$ (white areas), if the underlying assets' return distributions are asymmetric.
In both panels, the market structure is $N = \{\{1, 2\}, \{3\}, \{d\}\}$. Derivative d weights assets 1 and 2 equally (i.e., $r_d = (r_1 + r_2)/2$). In addition, $\sigma_{12} = -0.1$, $\sigma_{kk} = 1$ for all k, and $\overline{q}_{0,1} = \overline{q}_{0,3} = 10$. In Panel (A), the aggregate initial holdings satisfy $\overline{q}_{0,1} < \overline{q}_{0,2}$. In Panel (B), the aggregate initial holdings satisfy $\overline{q}_{0,1} > \overline{q}_{0,2}$.

(i) *(Derivatives are linearly priced when the market is either symmetric or uncontingent)* When $\sigma_{13} = \sigma_{23}$ (45-degree lines in Fig. 2), the market structure is symmetric for assets 1 and 2, and thus, price impact matrix in market N' is symmetric, i.e., $\lambda_{12} = \lambda_{21}$. Then, by Corollary 1, derivative d is linearly priced by the underlying asset prices, i.e., $p_d = \frac{1}{2}p_1 + \frac{1}{2}p_2$ (i.e., the 45-degree lines in the panels of Fig. 2). This result holds for an arbitrary derivative weight and for any realizations of traders' aggregate initial holdings.

(ii) *(Derivatives can be underpriced or overpriced, depending on heterogeneity in asset covariances and heterogeneity in aggregate initial holdings across assets)* When $\rho_{13} \neq \rho_{23}$, the price impact $\mathbf{\Lambda}$ is asymmetric, i.e., $\mathbf{\Lambda}_a \boldsymbol{C}_a \neq (I-1)\mathbf{Id}$. With an asymmetric price impact, Corollary 1 provides a necessary and sufficient condition for the derivative to be overpriced relative to the linear price based on the equilibrium prices of the underlying assets $\widehat{p}_d = w_{d1}p_1 + w_{d2}p_2$: with $\boldsymbol{M} \equiv \left((\alpha\mathbf{\Sigma})^{-1} + \boldsymbol{W}(\mathbf{\Lambda} + \frac{\sigma_0}{1-\sigma_0}\boldsymbol{C}^{-1})^{-1}\boldsymbol{W}'\right)^{-1} \in \mathbb{R}^{K \times K}$,

$$p_d - \widehat{p}_d = \boldsymbol{W}_d'\big(((1-\sigma_0)(I-1)\mathbf{Id} + \sigma_0\mathbf{Id})^{-1} - ((1-\sigma_0)\mathbf{\Lambda}_a\boldsymbol{C}_a + \sigma_0\mathbf{Id})^{-1}\big)\boldsymbol{M}\overline{\boldsymbol{q}}_0 > 0. \quad (47)$$

Due to the lack of closed form solution for $\mathbf{\Lambda}$, we cannot rewrite the condition (47) on price impact $\mathbf{\Lambda}$ as a condition on primitives $\mathbf{\Sigma}$ and $\overline{\boldsymbol{q}}_0$. Instead, Fig. 2 shows that the derivative can be underpriced (grey areas) or overpriced (white areas), depending on the heterogeneity in asset covariances (ρ_{13}, ρ_{23}) and the realization of aggregate initial holdings ($\overline{q}_{0,2} = 10$ in Fig. 2A; $\overline{q}_{0,2} = -10$ in Fig. 2B).