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An improved model-free adaptive integral sliding mode control for a class of nonlinear discrete-time systems ⋆

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Abstract: For a class of uncertain single-input single-output (SISO) nonlinear discrete-time systems, an improved model-free adaptive integral sliding mode control (MFA-ISMC) algorithm is proposed based on a disturbance observer. The pseudo-partial derivative (PPD) estimation algorithm used to describe the system dynamics is modified and the stability of the closed-loop system is guaranteed. It is shown that the corresponding closed-loop system performance is improved and the effectiveness of the proposed method is demonstrated by a simulation example.

Keywords: Model-free adaptive control, Tracking performance, Disturbance observer, Robust PPD estimation algorithm.

1. INTRODUCTION

Sliding mode control (SMC) is known to exhibit robustness to matched uncertainty in the sliding phase and has been widely applied in mechatronic systems as in Utkin (1993), Bartolini et al. (1995), Yao et al. (2018). To enhance the performance in the reaching phase, Utkin and Shi (1996) proposed integral sliding mode control (ISMC) which provides robustness across the whole state space. The problem of unmatched disturbances has been studied by Castanos and Fridman (2006), Rubagotti et al. (2010).

Although such ISMC schemes have global robustness, and render the system robust to external disturbances from the initial time, the design of the ISMC depends on the availability of a known mathematical model. This may be difficult to obtain in practice. In addition, the associated discontinuous control can be perceived as problematic.

To remove the dependence on the existence of a system model, researchers have considered the development of a corresponding data-driven control as in Hjalmarsson et al. (1998), Chien (1998), Campi and Savaresi (2006), Hou and Xiong (2019). Among the available approaches, model free adaptive control (MFAC) is widely used for general nonlinear discrete-time systems due to its straightforward computational overhead as well as the attractive features of self-adaptation of parameters and structure as in Xu et al. (2014), Hou and Zhu (2013), Tutsoy et al. (2018). Corresponding results are now emerging in the area of model-free adaptive integral sliding mode control (MFA-ISMC) as in Xia and Zhao (2022). However, the control performance in these studies can exhibit problems due to the estimation of the pseudo-partial derivative (PPD) parameters in the MFAC algorithm, which can produce undesirable transients during plant operation.

In the sliding mode problem formulation, chattering problems may result from the use of a potentially high gain discontinuous control. An effective way to reduce this gain is to design a disturbance observer. In Su et al. (2021), an integral sliding mode disturbance observer is suggested to estimate compound disturbances. In Hwang and Kim (2020), an extended disturbance observer-based ISMC is formulated using a Takagi-Sugeno fuzzy model approach. Wang and Hou (2019) combined integral terminal sliding mode control with MFAC to improve the performance of discrete-time systems in the presence of disturbances. In Xia and Zhao (2022) an ISMC based on a sliding mode disturbance observer is designed to weaken the influence of chattering, and the stability of the closed-loop system is proved.

Motivated by the above literature, an MFA-ISMC algorithm is presented. By reducing potentially large variations during the estimation of the model parameters, the control gain is also consequently reduced, and the control performance of the system is improved. An ISM-C which integrates a disturbance observer improves the

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global robustness of the system and reduces chattering. The convergence of the disturbance observation error, the PPD estimation error, and the stability of the closed-loop system are proved mathematically. Simulation results show the effectiveness of the proposed algorithm and the improvement in the system performance.

The remainder of the paper is organized as follows. The system description and basic assumptions are formulated in Section 2. The proposed MFA-ISMC algorithm is presented in Section 3. In Section 4, the stability analysis of the system is validated. A simulation example is presented to demonstrate the proposed approach in Section 5. Concluding remarks appear in Section 6.

2. SYSTEM DESCRIPTION AND BASIC ASSUMPTIONS

Consider a class of uncertain single-input single-output (SISO) discrete-time systems given by:

$$y(k+1) = f[y(k),\cdots,y(k-k_y),u(k),\cdots,u(k-k_u), d(k),\cdots,d(k-k_d)] \tag{1}$$

where $f(\cdot)$ is an unknown nonlinear function and $u(k)$, $y(k)$ and $d(k)$ represent the input, output and uncertainty respectively. k_y, k_u and k_d are unknown.

It is assumed that the following assumptions hold.

Assumption 1. The system (1) satisfies the generalized Lipschitz condition so that $|\Delta y(k+1)| \le b|\Delta u(k)|$ for any fixed k, $|\Delta u(k)| \ne 0$, $b \in R^+$ (R^+ denotes the positive numbers), where $\Delta y(k+1) = y(k+1) - y(k)$, $\Delta u(k) = u(k) - u(k-1)$.

Assumption 2. The partial derivatives of $f(\cdot)$ with respect to $u(k)$ and $d(k)$ are continuous.

Assumption 3. The external disturbance $d(k)$ satisfies $|\Delta d(k)| \le \bar{d} \in R^+$, where $\Delta d(k) = d(k) - d(k-1)$.

Lemma 1. (from Xia and Zhao (2022)) Suppose that Assumptions 1, 2 and 3 hold. Then there exists a bounded parameter $\xi(k) \in R$ (R denotes the set of real numbers), when $|\Delta u(k)| \ne 0$, such that system (1) can be transformed into the following Compact Form Dynamic Linearization (CFDL) representation:

$$\Delta y(k+1) = \xi(k)\Delta u(k) + D(k), \tag{2}$$

where $\xi(k)$ is an unknown parameter denoted as the PPD, $D(k)$ is the external disturbances acting on the system.

Assumption 4. The PPD parameter satisfies the conditions that $|\xi(k)| \ne 0$, $|\xi(k) - \xi(k-1)| < \bar{\xi} \in R^+$, and considering the physical meaning of $\xi(k)$, the sign of $\xi(k)$ is invariant. In this work, it is considered that $\xi(k) > 0$. $D(k)$ satisfies the boundedness condition $|D(k)| \le \bar{D} \in R^+$. $\bar{\xi}$ and $\bar{D}$ are unknown parameters.

Remark 1. The constraints in Assumption 4 ensure that the slowly time-varying parameter $\xi(k)$ is greater than zero, which means that the system is controllable and singularity problems are avoided.

The expected output of the system is expressed by $y_d(k)$, and the output tracking error is denoted by $e(k) = y(k) - y_d(k)$. In this work, the control objective is to design an ISMC scheme using only the input and output information from the system (1), so that the tracking error converges to a bounded range.

3. ALGORITHM FRAMEWORK

3.1 Design of the Integral Sliding Surface

In this scheme, the controller is divided into two parts:

$$u(k) = u_{ISM}(k) + u_{MFA}(k), \tag{3}$$

where $u_{ISM}(k)$ is the discontinuous ISMC and $u_{MFA}(k)$ is the model-free adaptive controller which controls the nominal system.

The integral sliding surface for (2) is defined by:

$$\begin{cases} s(k) = e(k) - e(0) + h(k) \\ h(k) = h(k-1) - \hat{\xi}(k-1)\Delta u_{MFA}(k-1) + \Delta y_d(k) \\ h(0) = 0 \end{cases}, \tag{4}$$

where $s(k)$ is the sliding surface, $h(k)$ is the integral term, $\hat{\xi}(k-1)$ is the estimate of $\xi(k-1)$ and $\Delta u_{MFA}(k-1) = u_{MFA}(k) - u_{MFA}(k-1)$.

Remark 2. In this work, $u_{ISM}(k)$ is a non-smooth control to eliminate the effects of matched disturbances and ensure that the control drives the system trajectory to the sliding mode. The component $u_{ISM}(k)$ is determined from $\xi(k)\Delta u_{ISM}(k) + D(k) = 0$, where $\Delta u_{ISM}(k) = u_{ISM}(k) - u_{ISM}(k-1)$. $u_{MFA}(k)$ is a high-level control which defines the dynamic performance of the nominal system. The sliding mode dynamics are given by $\Delta y(k+1) = \xi(k)\Delta u_{MFA}(k)$.

3.2 Design of the Disturbance Observer based MFA-ISMC

Only relying on the robustness of the ISMC to overcome the disturbances may involve using a high gain switching control. For some systems this can be undesirable. In this work, a disturbance observer is designed to reduce the high gain of the discontinuous control element and hence reduce the chattering effects.

$$\begin{aligned} \hat{D}(k) &= D(k-1) \\ &= \Delta y(k) - \hat{\xi}(k-1)\Delta u(k-1). \end{aligned} \tag{5}$$

The initial observation is set to $\hat{D}(0) = 0$.

For $\hat{\xi}(k) \ne 0$, the ISMC is designed as:

$$\Delta u_{ISM}(k) = -\frac{[\hat{D}(k) + p_1 T s(k) + p_2 T sat(s(k))]}{\hat{\xi}(k)}, \tag{6}$$

where $p_1 > 0$, $p_2 > 0$, $sat(s(k)) = \begin{cases} 1 & s(k) > \bar{\Delta} \\ \frac{s(k)}{\bar{\Delta}} & |s(k)| \le \bar{\Delta} \\ -1 & s(k) < -\bar{\Delta} \end{cases}$ is the saturation function of $s(k)$, $\bar{\Delta}$ is the boundary layer.

The MFAC is obtained by minimizing the cost function:

$$\begin{aligned} J(u_{MFA}(k)) = &|y_d(k+1) - y(k+1)|^2 \\ &+ \lambda |u_{MFA}(k) - u_{MFA}(k-1)|^2, \end{aligned} \tag{7}$$

where $\lambda > 0$ is the regularization coefficient.

To design the controller for the nominal system, take the derivative of $u_{MFA}(k)$, and then set it to zero, to obtain:

$$\Delta u_{MFA}(k) = \frac{\kappa\hat{\xi}(k)}{\lambda + \hat{\xi}(k)^2}(y_d(k+1) - y(k)), \tag{8}$$

where $\kappa \in (0, 1]$ is the step factor.

From (3), (6) and (8) it can be obtained that:

$$\begin{aligned} u(k) =& u(k-1) + \frac{\kappa\hat{\xi}(k)}{\lambda + \hat{\xi}(k)^2}(y_d(k+1) - y(k)) \\ &- \frac{1}{\hat{\xi}(k)}(\hat{D}(k) + p_1 T s(k) + p_2 T sat(s(k))). \end{aligned} \tag{9}$$

3.3 Design of the Robust PPD Estimation Algorithm

As the system model is unknown, the time-varying PPD in (2) needs to be determined. This is achieved by minimizing the following cost function:

$$\begin{aligned} J(\xi(k)) =& |\Delta y(k) - D(k-1) - \xi(k)\Delta u(k-1)|^2 \\ &+ \beta\left|\xi(k) - \hat{\xi}(k-1)\right|^2, \end{aligned} \tag{10}$$

where $\beta > 0$ is a positive weighting factor.

By minimizing (10) with respect to $\xi(k)$ and considering Assumption 4, the estimate is derived as:

$$\begin{aligned} \hat{\xi}(k) =& \hat{\xi}(k-1) + \Omega_k \Delta u(k-1)[\Delta y(k) \\ &- \hat{\xi}(k-1)\Delta u(k-1) - \hat{D}(k-1)], \end{aligned} \tag{11}$$

with $\Omega_k = \frac{\rho_k}{\beta + \Delta u(k-1)^2}$ and

$$\rho_k = \alpha^i \rho_0, \tag{12}$$

where $\alpha \in (0, 1)$ and i is the minimal natural number to ensure the PPD estimate satisfies Assumption 4:

$$i = \min\left\{ i \in \mathrm{N} \,\middle|\, |\Delta\hat{\xi}(k)| < l\xi_0 \right\}, \tag{13}$$

where $\Delta\hat{\xi}(k) = \hat{\xi}(k) - \hat{\xi}(k-1)$. $l \in (0, 1]$ and ξ_0 are adjustable parameters.

If $\hat{\xi}(k) \le \varpi$ or $|\Delta u(k-1)| \le \varpi$ or $sign(\hat{\xi}(k)) \neq sign(\xi(k))$:

$$\hat{\xi}(k) = \hat{\xi}(k-1), \tag{14}$$

where ϖ is a small positive number.

The following two improvements are made to enhance the performance of the PPD estimator with respect to Hou and Zhu (2013), Xia and Zhao (2022). Improvement 1: if $\hat{\xi}(k) \le \varpi$ or $|\Delta u(k-1)| \le \varpi$ or $sign(\hat{\xi}(k)) \neq sign(\xi(k))$, then $\hat{\xi}(k) = \hat{\xi}(k-1)$ rather than $\hat{\xi}(k) = \hat{\xi}(1)$. Improvement 2: ρ_k is an online variable parameter determined by the size of $\Delta\hat{\xi}(k)$, while in Hou and Zhu (2013), Xia and Zhao (2022), $\rho_k = \rho_0$ is a constant. Based on this, the MFA-ISMC algorithm is shown in Algorithm 1.

Remark 3. In Improvement 1, when $\hat{\xi}(k) \le \varpi$, it follows $\hat{\xi}(k) = \hat{\xi}(k-1)$, then $\hat{\xi}(k) > \varpi$ is always guaranteed, which avoids the problem of singularity in the control.

Remark 4. In Improvement 2, an exponential adjustment method is presented. ρ_k is a soft coefficient to adjust the rate of change of $\hat{\xi}(k)$ when $\left|\Delta\hat{\xi}(k)\right| > \xi_0$, thus ensuring that $\Delta\hat{\xi}(k)$ satisfies Assumption 4. The proposed improved robust PPD estimation algorithm has better adjustment ability as well as high computational efficiency. The core idea of the robust PPD algorithm is to improve the resulting control effort by reducing the non-smoothness of the estimated parameters. The effect of the improvement will be verified by later numerical simulations.

Algorithm 1 MFA-ISMC based on disturbance observation

1: **Initialization:** Set the initial value of $u(0)$, $\Delta u(0)$, $y(0)$, $y(1)$, $\Delta y(0)$, $\Delta y(1)$, $e(0)$, $s(0)$, $\hat{\xi}(0)$, $\hat{D}(0)$; Set the control parameters: N, β, ρ_0, α, ϖ, $\bar{\Delta}$, p_1, p_2, κ, λ;
2: **While** $k \le N$**, do**
3: Calculate PPD estimation $\hat{\xi}(k)$ using (11)-(14);
4: Calculate sliding function $s(k)$ using (4);
5: Calculate disturbance observation $\hat{D}(k)$ using (5);
6: Calculate $u(k)$ using (9).
7: Apply the control $u(k)$ and obtain the output $y(k)$ and tracking error $e(k)$. Then, $k = k + 1$;
8: **End**

4. STABILITY ANALYSIS

In this section, the convergence of the disturbance observation error and the PPD observation error, the stability of the closed-loop system and the boundedness of the tracking error are discussed.

4.1 Convergence Analysis

Theorem 1. Consider the system (1) which is assumed to satisfy Assumptions 1 - 4. The disturbance observer (5) and the PPD estimation algorithm (11) - (14) ensure that both the disturbance observation error and the PPD estimation error converge to the following bounded domains:

$$\left|\tilde{D}(k)\right| \le 2\bar{D},\ \left|\tilde{\xi}(k)\right| \le \frac{\tau}{1 - r_1}, \tag{15}$$

where $\tau = \bar{\xi} + \frac{\bar{D}\rho_0}{\sqrt{\beta}}$, $r_1 = \left|1 - \frac{\rho_k \Delta u(k-1)^2}{\beta + \Delta u(k-1)^2}\right|$.

Proof. Define the disturbance observation error as $\tilde{D}(k) = D(k) - \hat{D}(k)$. From (5),

$$\begin{aligned} \left|\tilde{D}(k)\right| &= \left|D(k) - \hat{D}(k)\right| \\ &= |D(k) - D(k-1)| \le 2\bar{D}. \end{aligned} \tag{16}$$

From (16), the disturbance observation error is bounded.

Let the PPD estimation error be $\tilde{\xi}(k) = \xi(k) - \hat{\xi}(k)$. Then:

$$\begin{aligned} \tilde{\xi}(k) =& \xi(k) - \hat{\xi}(k-1) - \frac{\rho_k \Delta u(k-1)}{\beta + \Delta u(k-1)^2} \\ &\times \left[\Delta y(k) - \hat{D}(k-1) - \hat{\xi}(k-1)\Delta u(k-1)\right] \\ =& \left(1 - \frac{\rho_k \Delta u(k-1)^2}{\beta + \Delta u(k-1)^2}\right)\tilde{\xi}(k-1) + \xi(k) \\ &- \xi(k-1) - \frac{\rho_k \Delta u(k-1)}{\beta + \Delta u(k-1)^2}\tilde{D}(k-1). \end{aligned} \tag{17}$$

Let $\left|1 - \frac{\rho_k \Delta u(k-1)^2}{\beta + \Delta u(k-1)^2}\right| = r_1$, then $0 \le r_1 < 1$.

$$\begin{aligned}\left|\tilde{\xi}(k)\right| \leq & r_1\left|\tilde{\xi}(k-1)\right| + |\xi(k)-\xi(k-1)| \\ & + \left|\frac{\rho_k \Delta u(k-1)}{\beta+\Delta u(k-1)^2}\right|\left|\tilde{D}(k-1)\right| \\ \leq & r_1\left|\tilde{\xi}(k-1)\right| + \bar{\xi} + 2\bar{D}\frac{\rho_0}{2\sqrt{\beta}} \\ = & r_1\left|\tilde{\xi}(k-1)\right| + \tau,\end{aligned} \tag{18}$$

$$\begin{aligned}\lim_{k\to\infty}\left|\tilde{\xi}(k)\right| \leq & r_1 \lim_{k\to\infty}\left|\tilde{\xi}(k-1)\right| + \tau \\ \leq & r_1\left(r_1 \lim_{k\to\infty}\left|\tilde{\xi}(k-2)\right| + \tau\right) + \tau \\ \leq & \cdots \\ = & \frac{\tau}{1-r_1}.\end{aligned} \tag{19}$$

From (19), the PPD estimation error is bounded.

4.2 Stability Analysis

Theorem 2. Consider the system (1), which satisfies Assumptions 1 - 4, the disturbance observer (5) and the PPD estimation algorithm (11) - (14), then the control algorithm (9) can drive the system dynamics to maintain a sliding motion for all time and the tracking error converges to a bound dependent on $\tilde{D}(k) - \frac{\tilde{\xi}(k)}{\hat{\xi}(k)}\hat{D}(k)$.

Proof. From (4),

$$\begin{aligned}s(k+1)-s(k) = & e(k+1)-e(k)+h(k+1)-h(k) \\ = & \xi(k)\Delta u(k) + D(k) - \hat{\xi}(k)\Delta u_{MFA}(k) \\ = & \xi(k)\Delta u_{ISM}(k) + D(k) - \tilde{\xi}(k)\Delta u_{MFA}(k) \\ = & -\frac{\xi(k)}{\hat{\xi}(k)}[\hat{D}(k) + p_1 T s(k) + p_2 T sat(s(k))] \\ & + D(k) - \frac{\kappa\hat{\xi}(k)\tilde{\xi}(k)}{\lambda+\hat{\xi}(k)^2}[y_d(k+1)-y(k)],\end{aligned} \tag{20}$$

then:

$$\begin{aligned}s(k+1) = & \left(1 - \frac{\xi(k)}{\hat{\xi}(k)}\left(p_1 T + \frac{p_2 T sat(s(k))}{s(k)}\right)\right)s(k) \\ & + \tilde{\xi}(k)\frac{\kappa\hat{\xi}(k)}{\lambda+\hat{\xi}(k)^2}e(k) + \tilde{D}(k) - \frac{\tilde{\xi}(k)}{\hat{\xi}(k)}\hat{D}(k) \\ = & (1-r_2)s(k) + r_3 e(k) + r_4,\end{aligned} \tag{21}$$

where $r_2 = \frac{\xi(k)}{\hat{\xi}(k)}\left(p_1 T + \frac{p_2 T sat(s(k))}{s(k)}\right)$, $r_3 = \frac{\kappa\hat{\xi}(k)\tilde{\xi}(k)}{\lambda+\hat{\xi}(k)^2} \in (0,1)$, $r_4 = \tilde{D}(k) - \frac{\tilde{\xi}(k)}{\hat{\xi}(k)}\hat{D}(k)$. Considering Assumption 4, it can be seen that $r_2 > 0$, then the maximum value of r_2 is $\frac{\xi(k)}{\hat{\xi}(k)}\left(p_1 T + \frac{p_2 T}{\Delta}\right)$. Thus reasonable choice of p_1 and p_2 can ensure $r_2 \in (0,1)$ is always satisfied.

In this work, it is assumed that the reference trajectory $y_d(k+1) = const$, so that $\Delta y_d(k+1) = 0$. Then:

$$\begin{aligned}e(k+1) = & y(k+1) - y_d(k+1) \\ = & \Delta y(k+1) + y(k) - y_d(k+1) \\ = & \xi(k)\Delta u(k) + D(k) + y(k) - y_d(k+1) \\ = & \xi(k)[\frac{\kappa\hat{\xi}(k)}{\lambda+\hat{\xi}(k)^2}(yd(k+1)-y(k)) \\ & - \frac{1}{\hat{\xi}(k)}(\hat{D}(k) + p_1 T s(k) + p_2 T sat(s(k))] \\ & + D(k) + y(k) - y_d(k+1),\end{aligned} \tag{22}$$

From (22),

$$\begin{aligned}e(k+1) = & [1 - \frac{\kappa\hat{\xi}(k)\xi(k)}{\lambda+\hat{\xi}(k)^2}]e(k) - \frac{\xi(k)}{\hat{\xi}(k)}[\hat{D}(k) + p_1 T s(k) \\ & + p_2 T sat(s(k))] + D(k) \\ = & [1 - \frac{\kappa\hat{\xi}(k)\xi(k)}{\lambda+\hat{\xi}(k)^2}]e(k) \\ & - \frac{\xi(k)}{\hat{\xi}(k)}[p_1 T s(k) + p_2 T\frac{sat(s(k))}{s(k)}]s(k) \\ & + \tilde{D}(k) - \frac{\tilde{\xi}(k)}{\hat{\xi}(k)}\hat{D}(k) \\ = & [1 - \frac{\kappa\hat{\xi}(k)\xi(k)}{\lambda+\hat{\xi}(k)^2}]e(k) - r_2 s(k) + r_4 \\ = & (1-r_5)e(k) - r_2 s(k) + r_4,\end{aligned} \tag{23}$$

where $r_5 = \frac{\kappa\hat{\xi}(k)\xi(k)}{\lambda+\hat{\xi}(k)^2} \in (0,1)$.

From (21) and (23):

$$\begin{bmatrix} s(k+1) \\ e(k+1) \end{bmatrix} = \begin{bmatrix} 1-r_2 & r_3 \\ -r_2 & 1-r_5 \end{bmatrix}\begin{bmatrix} s(k) \\ e(k) \end{bmatrix} + \begin{bmatrix} r_4 \\ r_4 \end{bmatrix}. \tag{24}$$

Let $A = \begin{bmatrix} 1-r_2 & r_3 \\ -r_2 & 1-r_5 \end{bmatrix}$, then the eigenvalues of matrix A are:

$$\begin{aligned}\lambda_{A1} = & 1 + \frac{-(r_2+r_5) + \sqrt{(r_2+r_5)^2 - 4r_2(r_5+r_3)}}{2}, \\ \lambda_{A2} = & 1 + \frac{-(r_2+r_5) - \sqrt{(r_2+r_5)^2 - 4r_2(r_5+r_3)}}{2}.\end{aligned} \tag{25}$$

It can be clearly observed that $\lambda_{A1} \in (0,1)$. For $r_2, r_3, r_5 > 0$, it is clear that $(\lambda_{A2})_{\max} < 1$. While $(\lambda_{A2})_{\min} > 1-(r_2+r_5) \in (-1,1)$, so the values of λ_{A1} and λ_{A2} are in the unit circle. It follows that $\begin{bmatrix} s(k+1) \\ e(k+1) \end{bmatrix}$ converges to a bound which is dependent on r_4.

5. SIMULATION EXAMPLE

Consider the system described by:

$$\begin{aligned}y(k+1) = & \frac{2.5y(k)y(k-1)}{1+y(k)^2+y(k-1)^2} \\ & + 0.7\sin(0.5(y(k)+y(k-1))) \\ & \times \cos(0.5(y(k)+y(k-1))) \\ & + 1.2u(k) + 1.4u(k-1) + d(k) \\ & 1 \leq k \leq 11000,\end{aligned} \tag{26}$$

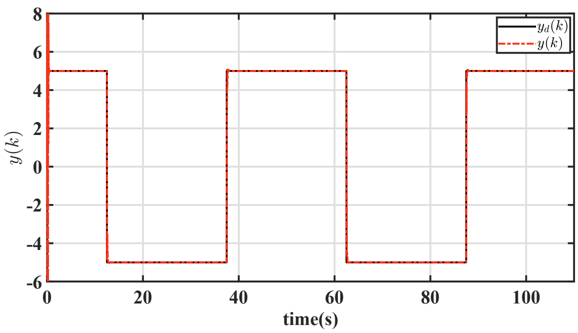

Fig. 1. Tracking performance of the system

where $d(k) = 30\sin(0.001k\pi)$. In this system, the sampling period is $T = 0.01s$. The expected output trajectory is $y_d(k+1) = 5 \times (-1)^{round(k/2500)}$.

The initial conditions of the system and the control parameters are defined in Table 1 and Table 2 respectively.

Table 1. Initial conditions

$u(0)$	y(0)	y(1)	$\Delta y(1)$	$e(0)$	$s(0)$	$\hat{\xi}(0)$	$\hat{D}(0)$
0	0	0	0	-5	0	0.5	0

Table 2. Control parameters

	$\hat{\xi}(k)$		$u_{ISM}(k)$		$u_{MFA}(k)$
β	2	Δ	0.02	κ	0.25
ρ_0	1.1	p_1	20	λ	0.01
α	0.57	p_2	0.6		
l	1				
ξ_0	0.008				
ϖ	1×10^{-5}				

The response of the system with the proposed control is shown in Figs. 1-6. From Fig. 1 and Fig. 2, the tracking error converges to a bounded range. The control signal is shown in Fig. 3. It can be seen from Fig. 4 that, when the expected output of the system changes, the system model makes some adjustments, and the PPD estimation parameters change significantly. The improved algorithm proposed in this paper avoids sudden changes when the system is adjusted, so the system behavior is smoother and the tracking accuracy is improved. It is clear that the sliding variable $s(k)$ in Fig. 5 and the disturbance observation error $\tilde{D}(k)$ in Fig. 6 converge to a bounded domain.

Fig. 7 and Fig. 8 compare the results of different PPD algorithms. The original PPD algorithm in Xia and Zhao (2022) is denoted by PPD_O. The robust PPD estimation proposed in this paper is denoted by PPD_N. It can be seen from Fig. 8 that the estimate obtained by the proposed algorithm fluctuates less. As a consequence, the tracking performance of the proposed PPD algorithm in Fig. 7 is better than that obtained using the original algorithm.

6. CONCLUSION

In this paper, a MFA-ISMC algorithm is proposed for a class of nonlinear discrete-time systems. The use of a

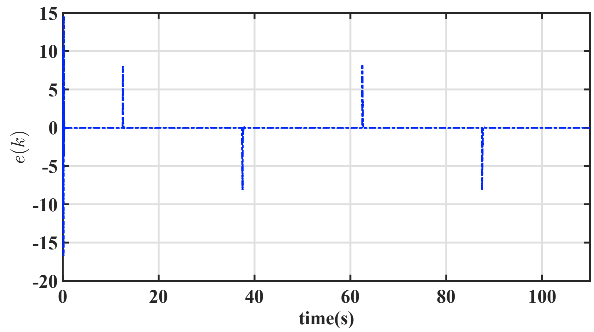

Fig. 2. Tracking error $e(k)$

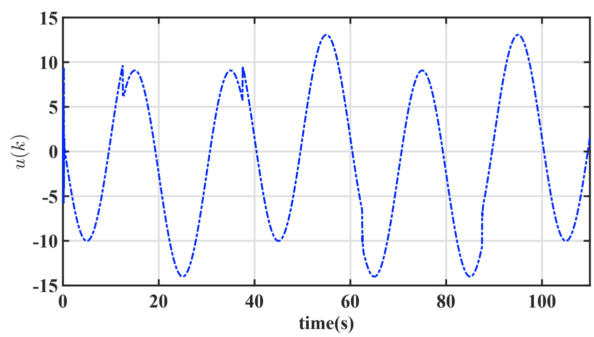

Fig. 3. Control signal $u(k)$

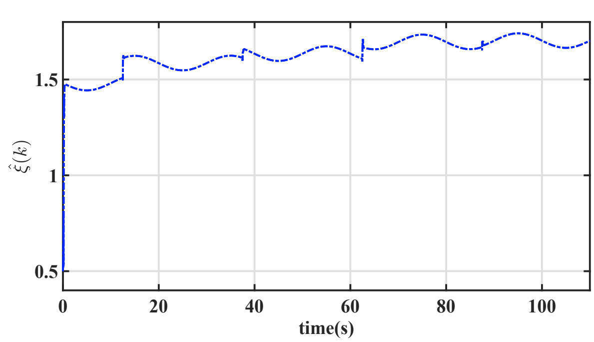

Fig. 4. PPD estimation $\hat{\xi}(k)$

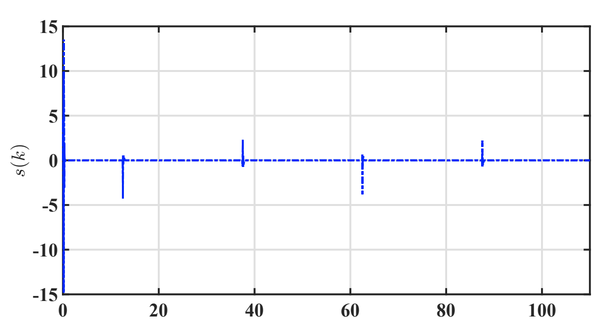

Fig. 5. Sliding variable $s(k)$

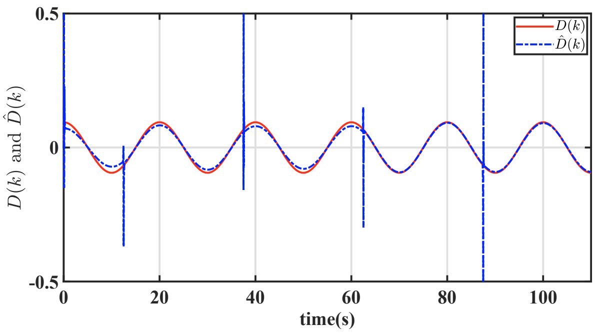

Fig. 6. Performance of the disturbance observer

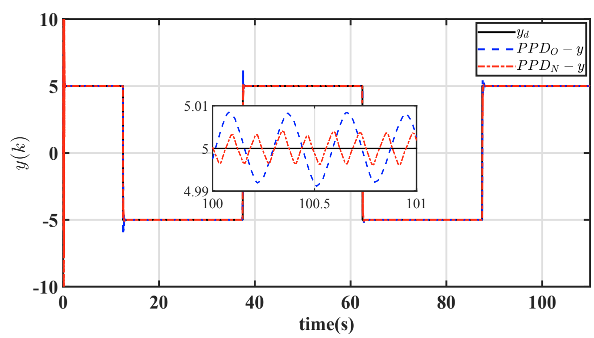

Fig. 7. System tracking performance of different PPD algorithms

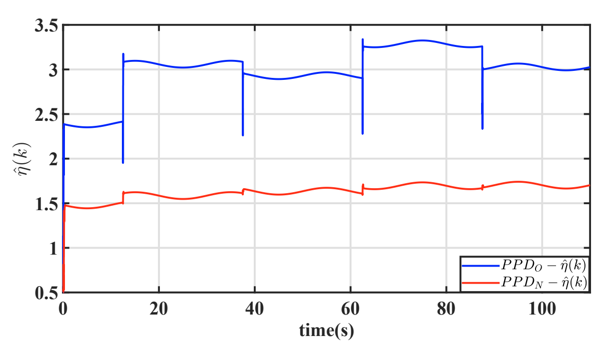

Fig. 8. Comparison of PPD estimates

disturbance observer reduces the gain of the discontinuous control, thereby reducing chattering. Two improvements have been made to the PPD algorithm to enhance the control performance of the system. The core idea is to prevent unnecessary fluctuations on the estimated parameters. The simulation results verify the effectiveness of the improved algorithm.

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