Kullback-Leibler Divergence Analysis for Integrated Radar and Communications (RadCom)

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Abstract—In this paper, we provide performance analysis for an integrated radar-communication (RadCom) system based on the relative information (RE), also called the Kullback-Leibler divergence (KLD) theorem. The considered system model consists of a multiple-input-multiple-output (MIMO) base-station (BS) which aims at providing RadCom services to multiple communication user equipments (UEs) and detecting a target. The separated deployment, in which the base-station antennas are distributed among radar and communication subsystems, is considered with Zero forcing (ZF) and maximum ratio transmission (MRT) precoders are applied to precede the communication signal. Results show that the derived formulas in this paper are accurate and imply that MRT suffers from bad performance compared to ZF.

Index Terms—Integrated sensing and communication (ISAC), integrated RadCom system, relative information, Kullback-Leibler distance, zero forcing (ZF), maximum ratio transmission (MRT).

I. INTRODUCTION

The collection of measurements from the environment about a certain phenomenon can be seen in a massive number of every day applications, such as radar, LiDAR, and IoT applications [1], [2]. In recent types of radars, multiple antennas are employed to transmit/receive the electromagnetic waves. In such types of multiple-input-multiple-output (MIMO) radars, the separation between adjacent antennas is $d \geq \lambda/2$, where $d$ is the antenna separation and $\lambda$ is the wavelength. With this setup, spatial diversity can be obtained which leads to high detection capability of MIMO radar and phased array radar, which typically has antenna separation of $d < \lambda/2$, have been compared in [3]. In addition, non-orthogonal signalling for MIMO radars is proposed in [4]. The effect of system impairments such as interference, fluctuating targets and synchronization errors are investigated in [9]–[11]. The relative information, or Kullback-Leibler divergence theorem has been adopted in [12]–[14] to analyze and design MIMO radars.

The integration between radar and communication (RadCom) systems has been recently proposed in the literature, and has attracted the research community. In RadCom systems, the resources of the base-station (BS) are utilized for both sensing and communications [15], [16]. In addition, RadCom systems can be very beneficial for sensing assisted communications such as channel estimation and spectrum sensing for cognitive radio networks [21], [22]. The performance of integrated sensing and communication (ISAC) system is analyzed in [17] using the communication user’s rate and radar detection probability. In [18] and [19], ISAC systems are considered with uplink and downlink data transmissions, and the analysis are provided using the outage probability, ergodic communication rate and diversity order for the communication users, whereas the sensing rate is evaluated for the radar system. Besides, the probability of detection for the radar sensing part and the spectral efficiency for communication system are analyzed for a full-duplex ISAC system in [20], [23].

Obviously, RadCom systems are expected to play a pivotal role in future wireless networks. As can be depicted from the literature, the performance of RadCom systems is typically evaluated using different metrics, for example, the bit error rate for the communications and the detection probability for the sensing system. However, introducing a unified performance measure for both systems should be very useful especially when optimizing the available network resources. Consequently, the main objective of this work is to provide a unified performance measure applicable for measuring the performance of communication and radar subsystems at the same time using the Kullback-Leibler divergence theorem, also known as the relative information (RE). It is worth mentioning that although the RE theorem has been widely used to analyze and design radar systems, to the best of authors knowledge, it has not been considered to evaluate the MIMO communication systems. Moreover, we will show that RE can capture the detection performance for both systems by relating it to the symbol error rate (SER) of the communication system and the detection probability of the radar system [24], [25]. The obtained results show that derivaties in this paper are very accurate and the proposed RE is informative about the RadCom system holistically.

Sec. II presents the system model. In Secs. III and IV, the analysis based Kullback-Leibler divergence (KLD) is provided for the communication and radar subsystems, respectively. Sec. V shows numerical results for the considered model, and finally a conclusion is provided in Sec. VI.

II. SYSTEM MODEL

The system model in this paper includes $N$ antenna MIMO-BS serving a number of $K$ communication user equipments (UEs) using $N_C < N$ antennas in downlink and aims at detecting an unmanned aerial vehicles (UAV) target using the remaining $N_R = N - N_C$ antennas. As shown in 1, the
separated deployment, in which the BS antennas are divided between the communication and radar systems, is assumed. BS employs zero forcing (ZF) or maximum ratio transmission (MRT) to precode the UEs data information [26, 27]. The radar waveform matrix $S = [s_1, s_2, \ldots, s_L]$, where $L$ is the number of snapshots and $s_1 \in \mathbb{C}^{N_R \times 1}$ is the transmitted signals vector in the $l$th snapshot, is designed such that the resulting covariance matrix $R_s = \frac{1}{L} \sum_{l=1}^{L} s_l s_l^H$ satisfies a desired form. The total amount of power available at BS, $P_T$, is also distributed among the communication and radar subsystems with $P_C$ and $P_{rad} = P_T - P_C$, respectively.

For transmission interval $l$, a data symbol $d_k [l]$ intended for the $k$th UE is picked from a normalized $M$-ary phase shift keying (MPSK), i.e., $|d_k [l]|^2 = 1$, and precoded using a linear precoder with a precoding vector $w_k \in \mathbb{C}^{N_C \times 1}$, and thus the precoded information symbols for all users can be written as

$$d_k [l] = \sum_{i=1}^{K} \sqrt{p_k} w_k d_k [l]$$

where $p_k \forall k$ is a power control factor. Consequently, the received signal at the $k$th UE with a radar interference

$$y_k [l] = g_k^T d_k [l] + n_k$$

where $P_{rad}$ is the power allocated to the radar subsystem, $g_k \in \mathbb{C}^{N_C \times 1} \sim \mathcal{CN} (0, 2\sigma^2_n)$ is a flat Rayleigh channel gain vector from the communication antennas to UE $k$, $n_k \overset{iid}{\sim} \mathcal{CN} (0, \sigma^2_n)$ is the radar interference plus noise, $f_k^T \in \mathbb{C}^{N_R \times 1} \sim \mathcal{CN} (0, 2\sigma^2_f)$ is a flat Rayleigh channel gain vector which represents the channel from the radar antennas to UEs, and $n_k \sim \mathcal{CN} (0, \sigma^2_n)$ is the additive white Gaussian noise (AWGN). It can be shown that $n_k \sim \mathcal{CN} (0, 2\sigma^2_f)$ where $\sigma^2_n = P_{rad} / N_R \sigma^2_f + \sigma^2_n$. It worth noting that in case of massive MIMO-BS, the radar signal can be designed such that the radar signal $s_l$ falls into the null space of $f_k^T$, i.e., $f_k^T \mathbb{E} [s_l s_l^H] f_k^* = 0$, which cancels the radar interference at UEs [15].

On the other hand, the desired radar waveform $s_l \in \mathbb{C}^{N_R \times 1}$, $l \leq L$, where $L$ is the number of snapshots, and $a_T (\theta)$ and $a_R (\theta)$ are respectively the transmit and receive array manifold of a uniform linear array (ULA). Therefore, the received signals vector at BS can be expressed as a binary hypothesis testing problem

$$\hat{y}_{rad} [l] = \begin{cases} H_1 : \alpha \sqrt{\frac{P_{rad}}{N_R}} a_R (\theta) a_T (\theta)^T s_l + G_{rad} d_w [l] + n_{rad} [l] \\ H_0 : G_{rad} d_w [l] + n_{rad} [l] \end{cases}$$

where $H_0$ and $H_1$ denote the absence and existence of a target, respectively, $\alpha$ is the channel gain for BS-Target-BS path, the term $G_{rad} d_w [l]$ represents the interference caused by the communication signal, $G_{rad} \in \mathbb{C}^{N_R \times N_C}$ is the channel matrix from the communication antennas to the radar antennas, and $n_{rad} \in \mathbb{C}^{N_R \times 1} \sim \mathcal{CN} (0, 2\sigma^2_n I_{N_R})$ is the AWGN with $I_{N_R}$ the identity matrix. We assume that the channel side information (CSI) of $G_{rad}$ is available at BS, and thus $G_{rad} d_w [l]$ can be subtracted from $\hat{y}_{rad} [l]$. It is worth noting that the estimation of $G_{rad}$ can be performed at the BS in a previous phase. Therefore, given the estimated channel matrix $\hat{G}_{rad}$, the received signals in (3) after subtracting $G_{rad} d_w [l]$ can be rewritten as

$$y_{rad} [l] = \begin{cases} H_1 : \alpha \sqrt{\frac{P_{rad}}{N_R}} A (\theta) s_l + \omega_{rad} + n_{rad} [l] \\ H_0 : \omega_{rad} + n_{rad} [l] \end{cases}$$

where a monostatic radar is considered with $A (\theta) = a_T (\theta) \triangleq 1, e^{j 2\pi f_0 / \rho_0} \sin (\theta), \ldots, e^{j 2\pi (N_R - 1) f_0 / \rho_0} \sin (\theta) \}^T$, $A (\theta) \in \mathbb{C}^{N_R \times N_R} = A (\theta) A (\theta)^T$ is the equivalent array manifold, and $\omega_{rad} \in \mathbb{C}^{N_R \times 1} = \mathcal{G}_{err} d_w [l] = G_{err} \sum_{i=1}^{K} \sqrt{p_i} w_i d_i [l]$ is the remaining communication signal interference after the subtraction process mentioned above, and $\mathcal{G}_{err} \triangleq G_{rad} - \hat{G}_{rad}$ representing channel estimation errors. In this paper, we consider imperfect channel estimation, and the estimation error is modeled as a complex Gaussian random variable with a mean of 0 and a variance of $\sigma^2_{err}$.

### III. THE RELATIVE ENTROPY OF COMMUNICATION SYSTEM

In this section, KLD will be evaluated for the communication subsystem where ZF and MRT precoders are considered with long-term matrix power normalization.

For a pair continuous probability density functions, $f_m (x)$ and $f_n (x)$, KLD$_{n \rightarrow m}$ is defined as the relative entropy from $f_n (x)$ to $f_m (x)$ or a measure of how different a PDF $f_n (x)$ is from another PDF $f_m (x)$. In general, KLD is asymmetric metric, and mathematically KLD$_{n \rightarrow m}$ for continuous random variables can be represented as

$$\text{KLD} (f_m \parallel f_n) = \int_{-\infty}^{\infty} f_n (x) \log_2 \left( \frac{f_m (x)}{f_n (x)} \right) dx,$$

and for multivariate Gaussian distributed random variables having mean vectors of $\mu_m$ and $\mu_n$ and covariance matrices of $\Sigma_m$ and $\Sigma_n$, it can be derived as

$$\begin{align*}
\text{KLD}_{n \rightarrow m} &= \frac{1}{2 \ln 2} \left( \text{tr} (\Sigma_m^{-1} \Sigma_n) - 2 + \ln \frac{\Sigma_n}{\Sigma_m} \\
&\quad + (\mu_{k,n} - \mu_{k,m})^T \Sigma_m^{-1} (\mu_{k,n} - \mu_{k,m}) \right)
\end{align*}$$

Fig. 1. An ISAC system with 2 UEs and 2 targets.
A. ZF based Data Precoding

Here, we assume ZF is employed at BS, which is able to cancel out the interference between users. The precoding matrix \(W\) for ZF case is generally given by \(W = PG^H \left( GG^H \right)^{-1}\), where \(P\) is a diagonal matrix used to control the transmit power of each UE. Consequently, the received signals vector at UEs can be written as

\[
y[l] = Pd + \eta
\]

where \(P\) is

\[
P \triangleq \alpha_{ZF} P_{\text{com}} = \sqrt{N_C - K + 1} P_{\text{com}}
\]

\((8)\)

where \(\alpha_{ZF} = \sqrt{N_C - K + 1}\) is a normalization factor to ensure that the average transmit power is fixed and \(P_{\text{com}} \triangleq \text{diag}(\sqrt{P_{1,\text{com}}}, \sqrt{P_{2,\text{com}}}, \ldots, \sqrt{P_{K,\text{com}}})\) is used to control the amount of power for each UE. The total amount of power is limited to \(P_c\), \(\sum P_{k,\text{com}} = P_c\) \((27)\), and it can be selected such that \(P_{k,\text{com}} = \frac{P_c}{K}\) for uniform power allocation among UEs. The received signal at the \(k\)th user can be rewritten as

\[
y_k[l] = \sqrt{P_{k,\text{com}} \alpha_{ZF}} d_k[l] + \eta_k
\]

\((9)\)

Based on the received signal \(y_k[l]\), the conditional probability density function \(f(y_k[d_k[l]])\) is complex Gaussian and can be written as

\[
f(y_k[d_k[l]]) = \frac{1}{(2\pi)^{\frac{K}{2}} |\Sigma|} e^{-\frac{1}{2}(y_k - \mu_k)^T \Sigma^{-1} (y_k - \mu_k)}
\]

\((10)\)

where \(y_k \triangleq [y_{k,1}, y_{k,2}, \ldots, y_{k,M}]^T\) with \(y_{k,i} \triangleq \text{Re}(y_k)\) and \(y_{k,i} \triangleq \text{Im}(y_k)\) denote the real and imaginary components of \(y_k\), respectively. \(\mu_k \triangleq [\mu_{k,1}, \mu_{k,2}, \ldots, \mu_{k,M}]^T\) with \(\mu_{k,i} = \sqrt{P_{k,\text{com}} \alpha_{ZF}} \text{Re}(d_k[l])\) and \(\mu_{k,i} \triangleq \sqrt{P_{k,\text{com}} \alpha_{ZF}} \text{Im}(d_k[l])\), \(\Sigma = \sigma_k^2 I_2\), and \(|\Sigma| = \sigma_k^4\) and \(\Sigma^{-1} = \frac{1}{\sigma_k^2} I_2\).

For MPSK, KLD should be evaluated for each possible pair of unequal data symbols \(\{d_{k,1}[l], d_{k,2}[l], \ldots, d_{k,M}[l]\}\). Let us consider a pair of MPSK symbols, \(\{d_{k,n,1}[l], d_{k,n,2}[l]\}\) \(\forall n \neq m\), with density functions given by \(f_n \sim \mathcal{CN}(\mu_{k,n}, \Sigma_n)\) and \(f_m \sim \mathcal{CN}(\mu_{k,m}, \Sigma_m)\), thus KLD\(_{k,n-m}\) can be derived as

\[
\text{KLD}_{k,n-m} = \frac{1}{2} \ln \left( 1 + \frac{\Sigma_m^{-1} \Sigma_n^{-1}}{2} \right) + \ln \left( \frac{\Sigma_m}{\Sigma_n} \right)
\]

\((11)\)

Moreover, by noting that \(\Sigma_m = \Sigma_n = \sigma_k^2 I_2\), and given that \(\mu_{k,m} = \frac{\sqrt{P_{k,\text{com}} \alpha_{ZF}} \cos \phi_k}{\sqrt{P_{k,\text{com}} \alpha_{ZF}} \sin \phi_k}\) \(\forall k\), KLD\(_{k,n-m}\) for long-term normalization based ZF can be simplified to

\[
\text{KLD}_{k,n-m}^{ZF} = \frac{1}{2 \sigma_k^2 \ln 2} \left( \mu_{k,m} - \mu_{k,n} \right)^T \left( \mu_{k,m} - \mu_{k,n} \right)
\]

\((12)\)

where \(\gamma_{k,ZF} = \frac{\alpha_{ZF} P_{k,\text{com}}}{\sigma_k^2} \rho_k\).

As stated earlier, since KLD is measured for a pair of PDFs, the average KLD, KLD\(_{k,\text{avg}}\), should be evaluated by considering all possible pairs of dissimilar symbols, which can be represented as

\[
\text{KLD}_{k,\text{avg}}^{ZF} = \frac{\gamma_{ZF}}{\ln 2} \sum_{k=1}^{K} \sum_{m=1}^{M} \text{Pr}(\phi_{k,m} \neq \phi_k) |1 - \cos(\phi_{k,m} - \phi_k)|
\]

\((13)\)

where \(\gamma_{ZF} = \frac{\alpha_{ZF} P_{k,\text{com}}}{\sigma_k^2} \rho_k\).

B. MRT based Data Precoding

The MRT precoding vector for the \(k\)th user data, \(w_k\), is evaluated based on the channel vector \(h_k\) as \(w_k = \frac{\sqrt{P_{k,\text{com}}}}{2 \sigma_g^2} g_k^H\), where \(E[g_k^H g_k^H] = 2 N_C \sigma_g^2\) and the received signal at the \(k\)th UE is

\[
y_k[l] = g_k^T \sum_{k=1}^{K} w_k d_k[l] + \eta_k
\]

\(= \sqrt{\frac{P_{k,\text{com}}}{2 N_C \sigma_g^2}} \|g_k^H d_k[l]\| + \omega_{\text{MRT}}
\]

\((15)\)

where \(\omega_{\text{MRT}} = g_k^T \sum_{i=1, i \neq k}^{K} \sqrt{\frac{P_{i,\text{com}}}{2 N_C \sigma_g^2}} g_i^H d_i[l] + \eta_k\) represents the radar and communication interference plus noise. Given that \(g_k\) and \(g_i\) \(\forall i \neq k\) are independent identically distributed (i.i.d) with 0 mean and \(\eta_k \sim \mathcal{CN}(0, 2 \sigma^2)\), the central limit theorem (CLT) can be employed to asymptotically find the density of \(\omega_{\text{MRT}}\). Accordingly, \(\omega_{\text{MRT}}\) can be seen as a complex Gaussian random variable with 0 mean and a variance of

\[
\sigma^2 \omega_k = \text{var} \left[ g_k^T (n_c) g_k^H (n_c) d_i[l] + \eta_k \right]
\]

\((16)\)

where \(g_k^T (n_c)\) is the element number \(n_c\) of the channel vector \(g_k^H\). Given that \(|d_i[l]|^2 = 1\), as well as \(g_k^T\) and \(g_k^H\) are independent, \(\sigma^2 \omega_k\) can be then found as

\[
\sigma^2 \omega_k = \sum_{n_c=1}^{N_C} \text{var} \left[ g_k^T (n_c) g_k^H (n_c) d_i[l] + \eta_k \right]
\]

\((17)\)

Consequently, following the definition of KLD in (6), the relative entropy for MRT scenario can be found as

\[
\text{KLD}_{k,\text{MRT,avg}} = \frac{\lambda}{\ln 2} \sqrt{\frac{P_{k,\text{com}}}{M (M-1) 2 N_C \sigma_g^2} \sigma^2 \omega_k}
\]

\((18)\)
Given that \( |g_k(n_c)| \) is Rayleigh distributed with a scale parameter \( \sigma_g \), i.e., \( |g_k(n_c)| \sim \text{Rayleigh}(\sigma_g) \), therefore, \( \|g_k(n_c)\|^2 = \sum_{n_c=1}^{N_C} |g_k(n_c)|^2 \) has Gamma distribution with shape and scale parameters of \( N_C \) and \( 2\sigma_g^2 \), respectively, i.e., \( \|g_k\|^2 \sim \text{Gamma}(N_C, 2\sigma_g^2) \). Consequently, the average relative entropy for the \( k \)th UE is

\[
\text{KLD}_{k,\text{MRT,avg}} = \frac{2\sigma_g^2\lambda}{M(M-1)\sigma_r^2}[1 + N_C] P_{k,\text{com}}
\]

Finally, the average KLD for all communication UEs can be then written as

\[
\text{KLD}_{\text{MRT}} = \frac{1}{K} \sum_{k=1}^{K} \text{KLD}_{k,\text{MRT,avg}}
\]

IV. THE RELATIVE ENTROPY OF RADAR SYSTEM

Based on (4) and assuming that the error caused by imperfect IC scheme is complex Gaussian \( \omega_{\text{rad}} \sim \mathcal{C}\mathcal{N}(0, 2\sigma_r^2\mathbf{I}_{N_R}) \), the received radar signals can be expressed as

\[
y_{\text{rad}}[l] = \begin{cases} 
H_1 : & \alpha \sqrt{\frac{P_{\text{rad}}}{N_0}} \mathbf{A}(\theta) s_l + \omega_{\text{rad}} \\
H_0 : & \omega_{\text{rad}}
\end{cases}
\]

where \( \omega_{\text{rad}} \sim \mathcal{C}\mathcal{N}(0, 2\sigma_r^2\mathbf{I}_{N_R}) \) with \( \sigma_r^2 = \sigma_s^2 + \sigma_n^2 \) and \( \sigma_s^2 = \sigma_{\text{err}}^2 \sigma_{\text{rad}}^2 N_C \sum_{k=1}^{K} P_{k,\text{com}} \). The sufficient statistics of the generalized likelihood ratio test, denoted as \( \xi(\theta_k) \), is asymptotically Chi-squared distributed with the following statistics [3], [4], [8], [9], [28].

\[
\xi(\theta_k) \sim \begin{cases} 
H_1 : & \Lambda_2^2(\lambda) \\
H_0 : & \Lambda_2^2(0)
\end{cases}
\]

where \( \Lambda_2^2 \) denotes noncentral Chi-squared random variable with 2 degrees of freedom and \( \lambda_{\text{rad}} = \frac{\sigma_r^2}{\sigma_s^2} \mathbf{R}_s(a(\theta))^2 = \sigma_{\text{err}}^2 \sigma_{\text{rad}}^2 N_C \sum_{k=1}^{K} P_{k,\text{com}} \) is the noncentrality parameter of \( \xi(\theta_k) \) under hypothesis \( H_1 \). By using the definition of KLD, the KLD from \( \xi_{H_1} \) to \( \xi_{H_0} \) in this case can be derived as

\[
\text{KLD}_{H_1 \rightarrow 0} = \int_{-\infty}^{\infty} f_{\xi}(\xi|H_0) \log_2 \left( \frac{f_{\xi}(\xi|H_0)}{f_{\xi}(\xi|H_1)} \right) d\xi
\]

After substituting \( f_{\xi}(\xi|H_0) \) and \( f_{\xi}(\xi|H_1) \),

\[
\text{KLD}_{H_1 \rightarrow 0} = \lambda \log_2 e \int_{0}^{\infty} e^{-0.5\lambda \xi} d\xi - \frac{1}{2} \int_{0}^{\infty} e^{-0.5\lambda \xi} \ln \left( I_0 \left( \sqrt{\lambda \xi} \right) \right) d\xi,
\]

which can be simplified to

\[
\text{KLD}_{H_1 \rightarrow 0} = \frac{1}{2} \left( 14.427 \lambda - \frac{1}{2} \int_{0}^{\infty} e^{-0.5\lambda \xi} \ln \left( I_0 \left( \sqrt{\lambda \xi} \right) \right) d\xi \right)
\]

By using Maple, it can be observed that the limit of the integrand is 0 as \( \xi \to \infty \). Consequently, we solve this integral numerically by using trapezoidal method due to the space limit. Similarly, \( \text{KLD}_{H_0 \rightarrow 1} \) can be found as

\[
\text{KLD}_{H_1 \rightarrow 0} = \int_{-\infty}^{\infty} f_{\xi}(\xi|H_1) \log_2 \left( \frac{f_{\xi}(\xi|H_1)}{f_{\xi}(\xi|H_0)} \right) d\xi
\]

V. NUMERICAL RESULTS

The section presents the measured performance of the ISAC system introduced in this paper. Monte Carlo simulation with 10^6 realizations for each run is used to generate the simulation (Sim.) results and the derived formulas in this paper are used to generate the theoretical performance. Two UEs and a single target scenario have been considered with \( L = 100 \) snapshots and antenna separation of half the wavelength, \( \Delta = 0.5\lambda \). Moreover, the total transmit power is normalized to \( P_T = 1 \), the target is located at \( \theta = 35^\circ \), the radar covariance matrix is
designed such that \( \mathbf{R}_c = \mathbf{I}_{N_{P_c}} \), and the radar channel pathloss is normalized, \( \alpha = 1 \).

Figs. 2 and 3 present the measured theoretical and simulated performance of an ISAC system which consists of a 20 antenna BS serving 2 UEs through ZF (Fig. 2a) and MRT (Fig. 2b) precoding with BPSK signalling, and aiming at detecting the target. For these figures, each subsystem is allocated 10 antennas, the radar signal is well designed such that it does not cause interference with the communication signal received by UEs, i.e., \( \mathbf{f}_k^t \mathbf{E} \left[ s(t) s^H(t) \right] \mathbf{f}_k^t = 0 \), as well as, \( \mathbf{G}_{\text{rad}} \) is perfectly estimated at BS, i.e., \( \omega_{\text{rad}} \to 0 \). As can be observed from these figures, the derived equations for the radar and communication KLDs match the simulation results. In addition, Figs. 2a and 2b show that the KLD of the communication system is inversely proportional to \( P_c \), whereas it is directly proportional to the detection probability of the radar subsystem, \( P_{\text{D}} \), as can be realized by comparing Fig. 3a and Fig. 3b. Moreover, increasing \( P_{\text{rad}} \) would enhance the detection capability of the radar system through increasing KLD and \( P_{\text{D}} \); however, as the total transmission power is fixed, the portion allocated to the communication system decreases resulting in a higher error rate.

Fig. 4 presents the impact of interference caused from radar subsystem to UEs and the effect of estimation errors in \( \mathbf{G}_{\text{rad}} \). The total number of BS antennas is 20 with 10 antennas are assigned for each subsystem and QPSK signalling is employed for the communication subsystem. A number of 2 UEs and 100 snapshots are assumed and the power allocated for radar and communication are respectively \( P_{\text{Rt}} = 0.1 \) and \( P_{\text{C}} = 0.9 \). As can be observed from the figure, the simulation results confirm the accuracy of the derived equations in this paper for KLD_{\text{rad}}, KLD_{\text{ZF}} and KLD_{\text{MRT}}. It can be also seen that the interference caused by one subsystem to the other has sever impact and can limit the performance of the whole system. For example, for the case of ZF precoding, Fig. 4a shows that the probability of error for UEs suffers from an error floor at about \( 5 \times 10^{-6} \), and KLD_{\text{ZF}} also reaches an upper bound of about 54 bits for \( \frac{\sigma_{\text{err}}^2}{\sigma^2} \geq 30 \text{ dB} \). Moreover, MRT precoding suffers from relatively very bad performance which can be attributed to the fact that a certain UE suffers from inter-user interference in addition to radar signal interference. It can be also seen from 4b and 4c that the impact of channel estimation errors in \( \mathbf{G}_{\text{rad}} \) is very severe at considerable values of \( \sigma_{\text{err}}^2 \). For instance, a very low detection probability of about 0.2 is obtained when \( \sigma_{\text{err}}^2 = 0.1 \) even at very high SNRs, and a KLD of about 3.45 bits is achieved for the same values of \( \sigma_{\text{err}}^2 \).

VI. CONCLUSION

A RadCom system which consists of a single BS serving a number of UEs sensing a target was introduced in this paper, where the separated deployment was considered. ZF and MRT precoders were employed to multiplex multiple data symbols intended for UEs. The relative entropy or KLD was derived for both radar and communication subsystems. Moreover, the effect of interference caused by the radar subsystem on UEs and the effect of imperfect interference cancellation on the radar subsystem were studied. The derived KLD was emphasized by Monti Carlo simulations. The results showed that there is a trade-off between the radar and communication subsystems where enhancing the detection capability of one negatively affects the other as the amount of interference increases accordingly.

ACKNOWLEDGMENT

This project has received funding from the European Union’s Horizon 2020 research and innovation program under grant agreement No 812991.

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Fig. 3. The performance of radar subsystem versus the transmit SNR $P_T/\sigma_n^2$ for several values of $P_{\text{rad}}$: a) KLD$_{\text{rad}}$ vs. $P_T/\sigma_n^2$, b) The detection probability $P_D$ vs. $P_T/\sigma_n^2$, and c) The detection probability $P_D$ vs KLD$_{\text{rad}}$.

Fig. 4. The impact of radar-to-CUs interference and the estimation errors of $G_{\text{rad}}$ on the performance of CUs and radar subsystem, respectively, versus the transmit SNR $P_T/\sigma_n^2$: a) The error rate and KLD of CUs, b) The detection probability $P_D$ for radar subsystem, and c) The KLD for the radar subsystem.


