UAV-Assisted Edge computing with 3D Trajectory Design and Resource Allocation

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***Abstract*—With the explosive increase in computing demands and the rise of portable wearable devices, the concept of mobile-edge computing (MEC) has emerged and attracted a lot of attention from both academia and industry. Unmanned Aerial Vehicle (UAV) as flexible moving platform has been wide adopted as a edge computing server to help ground users compute their intensive tasks. Although UAV-assisted edge computing is capable to enhance the computing performance, there are still many challenges in this system, including UAV 3D trajectory design, the allocation of UAV computational resources and the communication time allocation between users and UAV. In this article, we try to solve these challenges in a UAV-assisted edge computing system, aiming at minimizing the completion time of computing users' tasks. Specially, we propose a combination algorithm of the alternating optimization method and the bisection search method to minimize the delay of the whole system. The whole algorithm can be described in two iterative steps. In the first step, with given total number of time slot N assuming each slot with fixed length, we check whether the current N can satisfy the computational demands of the whole system through the alternating optimization algorithm to obtain the computational and time allocation. In the second step, we use the resource allocation results obtained in the first step to choose whether to increase or decrease N via the bisection search method. Then we repeat the first and second steps until we find the the smallest N that best fits the current computational demand. Extensive experimental results demonstrate that our proposed algorithm greatly reduces the users' task completion time in comparison with traditional benchmarks. In addition, the convergence of the proposed algorithm can be guaranteed.**

I. Introduction

The emerging technology of edge computing caters to the needs of this era of rapid growth in computation task size and strict requirements for latency, thus attracting wide attention from researchers. The application scenarios of edge computing have been explored in [1] and [2], which include virtual reality, massive machine-like communication, intelligent unmanned vehicle, etc. Meanwhile, edge computing servers deployed on the user side can provide massive computing resources to support the vision of intelligent IoT. UAV is now seen as a cost-effective and reliable platforms for communications and computing due to its characteristics of flexible deployment and low-cost. The prospects and challenges of UAV in wireless communication systems have been extensively studied in [3]–[5].

In [6], UAV with edge computing and storage server is used to reduce the latency of online VR users, and the authors effectively reduce the latency of the whole system by optimizing the resource allocation and UAV deployment location. In [7], The authors use Lagrange multiplier and sub-gradient descent methods to optimize spectrum resource allocation, offloading task size, and computational resource allocation, and use the SCA method to optimize the 2D trajectory of the UAV to minimize the energy consumption of both the UAV and the users. In [8], The authors envision an extreme case of combining wireless charging and edge computing. The base station first charges the energy-poor user via wireless charging and then helps users to complete the computation task after the charging. The energy consumption of the base station is minimized by optimizing the user's offloading decisions, time allocation, and base station transmit power. In [9], the weighted energy minimization problem of a three-level edge computing architecture consisting of satellites, spacecrafts and users is investigated. Moreover, the authors used multiple antenna technique and classical WMMSE optimization method to overcome the path loss due to the long distance transmission caused by satellite communication. The problem of minimizing the mission completion time of an edge computing system composed of satellites, base stations and users is also introduced in [10], and the closed-form solution of the scaling factors for each level are derived and then solve the problem jointly with an intelligent algorithm. Besides, [11]–[13] investigated the problem of minimizing the completion time of UAV-enabled information collection and UAV-enabled broadcasting via trajectory design, respectively, [14] solved the problem of minimizing the completion time of an edge computing system with multi-UAV collaboration, but it only considering 2D UAV trajectory optimization. In addition, the time minimization problem was studied in different ways in [15]–[17], but these articles only considered scenarios where UAVs are used for communication and did not consider the scenario where UAV with edge computing capability.

Inspired by the above mentioned paper, an edge computing system consisting of a UAV and users is considered in this paper. In this scenario, UAV with its prominent mobility can move around the users to provide emergency computing support to the users to ensure that users can achieve low computing latency. The main contributions of this paper are summarized as follows. We incorporate the UAV into the edge computing system as a complement to the computational resources. We propose an alternating optimization algorithm that sequentially optimizes UAV 3D trajectory, time and computational resource allocation. In this algorithm, we use the quadratic transformation in the trajectory optimization, so that the horizontal and vertical trajectories of the UAV can be optimized together. And we find the optimal solution of this minimization time problem by combining the method of the bisection search method and the alternating algorithm. Simulation results prove that our proposed algorithm and scheme can greatly reduce user latency.

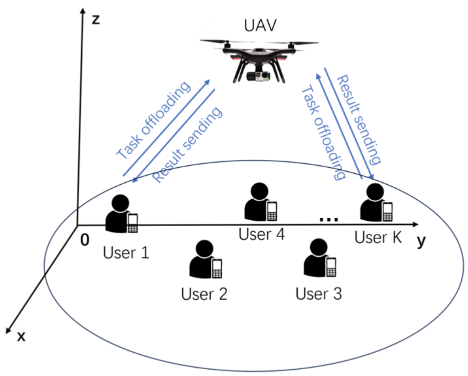


Fig. 1. UAV-enabled MEC system.

II. System Model and Problem Formulation

As shown in Fig. 1, a three-dimensional (3D) Euclidean coordinate system is adopted, whose coordinates are measured in meters. Consider a system consisting of a UAV and K users denoted by the set $\mathcal{K} = \{1, ..., K\}$, and the location of user k denoted as $\mathbf{w_k} \in \mathbb{R}^{1\times 2}$. For ease of discussion, the completion time T is divided into N small equal-size time slots with each size of δ, which are denoted by the set $\mathcal{N} = \{1, ..., N\}$. Therefore the trajectory of the UAV in time slot $n \in \mathcal{N}$ can be denoted by $(\mathbf{q}[n], z[n])$, in which $\mathbf{q}[n]$ is horizontal coordinates that can be denoted as $\mathbf{q}[n] \in \mathbb{R}^{1\times 2}$ and $z[n]$ is vertical coordinates. It is also assumed that each user has a bit-wise-independent and computation-intensive task, furthermore, UAV carries an edge computing servers, which can help users handle their required computing tasks. We minimize the completion time of the system by optimizing the 3D trajectory, time and computational resource allocation.

A. UAV Trajectory Model

Each time slot is $\delta = \frac{T}{N}$, which is so small that we assume that the position and channel condition of UAV are constant during each time slot. And the maximum horizontal and vertical speed denoted by $\mathrm{V}_{\max}^{\mathrm{hor}}$ and $\mathrm{V}_{\max}^{\mathrm{ver}}$ in meter/second (m/s), respectively. In order to be able to design an UAV trajectory that are feasible given the motion characteristics, the following constraints have to be ensured

$$\|\mathbf{q}[n+1] - \mathbf{q}[n]\|_2 \le \mathrm{V}_{\max}^{\mathrm{hor}} \cdot \delta, \quad (1)$$
$$|z[n+1] - z[n]| \le \mathrm{V}_{\max}^{\mathrm{ver}} \cdot \delta, \quad (2)$$
$$\mathbf{q}[1] = q_0, z[1] = z_0, \quad (3)$$
$$H_{\min} \le z[n] \le H_{\max}, \quad (4)$$

where constraints (1), (2), and (4) denote the speed and maneuverability constraint of the UAV, and constraint (3) denotes that the UAV starts from a fixed starting point in this paper.

B. UAV-Users Channel Model

Due to the high mobility of the UAV, the channel states between UAV and users are coherently changing, especially when the altitude of the UAV changes during its movement, the channel states is always switching back and forth between LoS and NLoS states. It should be noted that the LoS probability between UAV and users in the general scenario is given by

$$P_k[n]^{\mathrm{LoS}} = \frac{1}{1 + ae^{-(b(\theta_k[n]-a))}}, P_k^{\mathrm{NLoS}}[n] = 1 - P_k^{\mathrm{LoS}}[n], \quad (5)$$
$$n \in \mathcal{N}, k \in \mathcal{K},$$
$$\theta_k[n] = \frac{180}{\pi} \arcsin\left(\frac{z[n]}{D_{ku}^k[n]}\right), n \in \mathcal{N}, k \in \mathcal{K}, \quad (6)$$

where a and b are constant term depend on the environment.

The average channel power gain from UAV to user k in time slot n can be modeled as

$$h_k[n] = \left((1-\varrho) P_k^{\mathrm{LoS}}[n] + \varrho\right) \beta_0 D_{gu}^k [n]^{-\frac{\alpha}{2}}, n \in \mathcal{N}, k \in \mathcal{K}, \quad (7)$$
$$D_{ku}^k[n] = \sqrt{\|\mathbf{q}[n] - \mathbf{w_k}\|^2 - z^2[n]}, n \in \mathcal{N}, k \in \mathcal{K}, \quad (8)$$

where $D_{ku}^k[n]$ is the distance between UAV and user k, β_0 denotes the channel power gain at the reference distance of 1 meter and $\varrho < 1$ is the additional attenuation factor due to the NLoS condition. However $h_k[n]$ is quite complicated and difficult to deal with, fortunately, an approximate expression for the transmission rate in this case is given in [11], which is used throughout our paper to express the transmission rate between users and UAV, which is given by

$$R_k[n] = B\log_2\left(1 + \frac{r_k(C_1 + \frac{C_2}{1+e^{-(B_1+B_2V_k[n])}})}{(\|\mathbf{q}[n] - \mathbf{w_k}\|_2^2 + z[n]^2)^{\frac{\alpha}{2}}}\right) \quad (9)$$
$$n \in \mathcal{N}, k \in \mathcal{K},$$
$$V_k[n] = \frac{z[n]}{(\|\mathbf{q}[n] - \mathbf{w_k}\|_2^2 + z[n]^2)^{\frac{1}{2}}}, n \in \mathcal{N}, k \in \mathcal{K}, \quad (10)$$

where $r_k = \frac{P_k \beta_0}{\sigma^2 \Gamma}$ is the SNR of signal from user k to UAV, σ^2 is the receiver noise power and $\Gamma \ge 1$ denotes the signal-to-noise ratio (SNR) gap between the practical modulation-and-coding scheme and the theoretical Gaussian signaling. And C_1, C_2, B_1, and B_2 is constant term depend on environment, respectively.

C. Offloading Model

In this paper, we assume that all computing tasks can be arbitrarily divided to be transmitted to the UAV for computing and ignore the time delay for returning the computing result from UAV to users. Users can offload the computing tasks to the UAV, and UAV utilizes the edge computing servers it carries to help users to complete required task, therefore, leading to the following constraints:

$$\sum_{k=1}^{K} t_{k,u}[n] \le \delta, \qquad n = 1, 2, ..., N, \tag{11}$$

$$\frac{t_{k,u}[n] R_k[n] C_k}{f_k^{\mathrm{UAV}}[n+1]} \le \delta, \quad \forall k, \forall n, \tag{12}$$

$$\sum_{n=1}^{N-1} t_{k,u}[n] R_k[n] + \frac{N\delta f_k}{C_k} \ge I_k, \forall k, \tag{13}$$

$$0 \le t_{k,u}[n] \le \delta, 0 \le t_{u,b}[n] \le \delta, \forall k, \forall n, \tag{14}$$

$$t_{k,u}[N] = t_{u,b}[1] = 0, \forall k, \tag{15}$$

$$f_k^{\mathrm{UAV}}[n] \ge 0, \qquad \forall k, \forall n, \tag{16}$$

$$\sum_{k=1}^{K} f_k^{\mathrm{UAV}}[n] \le F_{\max}^{\mathrm{UAV}}, \qquad n = 2, 3, \ldots, N, \tag{17}$$

where $t_{k,u}[n]$ denotes the amount of allocated transmission time between user k and UAV in n-th time slot. $f_k^{UAV}[n]$ is the computing resources allocated by the UAV to user k in n-th time slot, C_k is the required CPU cycles for computing 1 bit of the required computing tasks of user k, f_k and $F_{\max}^{\mathrm{UAV}}$ denote the maximum CPU frequency of user k and UAV, respectively. Constraint (12) states that the computational task transmitted from user k to the UAV in the n-th time slot must be processed and return the result to user k in the $n+1$-th time slot. I_k denotes the size of the required computing tasks of each user k. Constraint (13) guarantees that each user's computational task must be completed in time T. Constraint (17) states that the computational resources allocated by the UAV to each user in the n-th time slot cannot exceed its maximum computational resources.

D. Problem Formulation

Our objective is to minimize the completion time by optimizing 3D UAV trajectory, computing resources and time allocation in each time slot, which is expressed as:

$$(\textbf{P1}) \quad \min_{\mathbf{q}[n], z[n], f_k^{\mathrm{UAV}}[n], t_{k,u}[n], N} \sum_{n=1}^{N} \delta[n] \tag{18a}$$

$$\text{s.t.} (1) - (4), (11) - (17). \tag{18b}$$

III. OPTIMIZATION ALGORITHM DESIGN

A. Reformulation of the problem P1

The formulated problem P1 is difficult to be sufficiently solved due to the non-convex constraints (12) and (13), and N is a constant term in the problem P1, which is contradictory to our purpose in this paper. Therefore, how to get a suitable N is a tricky problem. In order to deal with this problem, we introduce a auxiliary variable η , which denotes the relationship between task requirements and optimization values. If $\eta \ge 1$, with given N, denotes tasks requirements of (P1) can be completed by solving (P2) and $\eta < 1$ otherwise.

$$(\textbf{P2}) \quad \max_{\mathbf{q}[n], z[n], f_k^{\mathrm{UAV}}[n], t_{k,u}[n], N} \quad \eta \tag{19a}$$

$$\text{s.t.} (1) - (4), (11) - (12), (14) - (17), \tag{19b}$$

$$\frac{\sum_{n=1}^{N-1} t_{k,u}[n] R_k[n] + \frac{N\delta f_k}{C_k}}{I_k} \ge \eta, \quad \forall k. \tag{19c}$$

Theorem 1. *The optimal value $\eta(N)$ of problem (P2) is a increasing function of N.*

Proof. we assume that $N_1 > N_2$, the optimal solution of (P2), with given N_2, is denoted as $\eta^*(N_2)$. Obviously, the feasible solution of trajectory of probblem (P2), with given N_1, has a special solution, in which we firstly find N_2 time slots from N_1 time slots and let them satisfy the optimal solution of $\eta^*(N_2)$, the set of these N_2 points is denoted as N', and the values of remaining variables are arbitrarily given under the constraints. Therefore, a feasible solution η of problem (P2) is $\eta(N_1)$, which is determined by the value of $\min\limits_{k \in \mathcal{K}} \frac{\sum_{n \in N'} t_{k,u}[n] R_k[n] + \sum_{n \in N_1/N'} t_{k,u}[n] R_k[n] + \frac{N\delta f_k}{C_k}}{I_k}$. Because $t_{k,u}[n] \ge 0$ when $n \in N_1/N'$ and $\frac{N\delta f_k}{C_k}$ increases with the increase of N, so we can conclude that $\eta^*(N_1) \ge \eta(N_1) > \eta^*(N_2)$, where $\eta^*(N_1)$ is the optimal solution of problem (P2) with given N_1. Therefore the optimal solution of η of problem (P2) increases with the increase of N. □

Unlike P1 that is highly dependent on the initial trajectory due to constraint (13), which leads to the feasibility of problem 1 if N is small, P2 doesn't require a strict initial trajectory. We can know whether the current constant term N can meet the requirements of the whole system by solving P2 according to algorithm that we propose in the later part of this paper. Since the optimal solution of P2 increases as N increases, we can always find a suitable N using a combination of the method of bisection and the alternating optimization algorithm that we proposed below so that the optimal solution of P2, $\eta^*(N)$, is just around one, which means we find the minimum time to complete the whole task. The whole process will be described in detail in the rest parts of this paper.

B. UAV Trajectory Subproblem

For any given $t[n]$, $f_k^{\mathrm{UAV}}[n]$ and N, the UAV trajectory subproblem can be optimized by solving the following problem (P3):

$$(\textbf{P3}) \quad \max_{\mathbf{q}[n], z[n]} \quad \eta \tag{20a}$$

$$\text{s.t.} (1) - (4), (12), (19c). \tag{20b}$$

Problem (P3) is difficult to be handled due to constraints (12) and (13) are still non-convex w.r.t $\mathbf{q}$[n] and z[n]. Therefore, we introduce four slack variables β_k[n] ξ_k[n] ψ_k[n] and ζ_k[n], and thus, constraints (12) and (19c) can be converted as the following form:

$$\frac{t_{k,u}[n]\beta_k[n]C_k}{f_k^{\mathrm{UAV}}[n+1]} \leq \delta, \tag{21a}$$

$$\frac{\sum_{n=1}^{N-1} t_{k,u}[n]\beta_k[n] + \frac{N\delta f_k}{C_k}}{I_k} \geq \eta, \tag{21b}$$

$$\beta_k[n] \leq \mathrm{B}\log_2(1 + \frac{\mathrm{r_k}(\mathrm{C_1} + \frac{\mathrm{C_2}}{\xi_\mathrm{k}[\mathrm{n}]})}{\psi_\mathrm{k}[\mathrm{n}]^{\frac{a}{2}}}), \tag{21c}$$

$$\xi_k[n] \geq 1 + e^{-\zeta_k[n]}, \tag{21d}$$

$$\zeta_k[n] \leq B_1 + B_2 V_k[n], \tag{21e}$$

$$\psi_k[n] \geq \|\mathbf{q}[n] - \mathbf{w_k}\|_2^2 + z[n]^2, \tag{21f}$$

where the right side of constraint (21c) is a joint convex function w.r.t $\psi_k[n]^{\frac{a}{2}}$ and $\xi_k[n]$. Therefore, for given feasible point $\psi_k^m[n]^{\frac{a}{2}}$ and ${\xi_k}^m[n]$, we get the lower-bound of the right side of constraint (21c) as follow.

$$\beta_k[n] \leq \mathrm{B}\log_2(1 + \frac{r_k(C_1 + \frac{C_2}{\xi_k^m[n]})}{\psi_k^m[n]^{\frac{a}{2}}}) + \frac{B}{\ln 2}(\xi_k[n] - \xi_k^m[n]) \times$$
$$C_k^m[n] + \frac{B}{\ln 2}(\psi_k[n]^{\frac{a}{2}} - \psi_k^m[n]^{\frac{a}{2}})D_k^m[n], \tag{22}$$

where $C_k^m[n] = \frac{\frac{-\mathrm{C_2 r_k}}{\xi_k^m[n]^2}}{1+\frac{r_k(C_1+\frac{C_2}{\xi_k^m[n]})}{\psi_k^m[n]^{\frac{a}{2}}}}$ and $D_k^m[n] = \frac{\frac{-r_k(C_1+\frac{C_2}{\xi_k^m[n]})}{\psi_k^m[n]^a}}{1+\frac{r_k(C_1+\frac{C_2}{\xi_k^m[n]})}{\psi_k^m[n]}^{\frac{a}{2}}}$.

After the above analysis, we can see that constraints (21a) - (21c), (21d), and (21f) are convex constraints w.r.t $\mathbf{q}[n]$ and $z[n]$. However, constraint (21e) is still a non-convex constraint due to $V_k[n]$ is a fractional form and not a convex function for $z[n]$ and $\mathbf{q}[n]$, fortunately, according to [9] and [18], the method of quadratic transform can be used to handle $V_k[n]$. According to [18], we have the following theorem.

Theorem 2. *By applying the quadratic transformation, $V_k[n]$ can be equivalently written in the following form*

$$V_k[n] = 2e_k[n]\sqrt{z[n]} - e_k^2[n](\|\mathbf{q}[n] - \mathbf{w_k}\|_2^2 + z[n]^2)^{\frac{1}{2}}, \tag{23}$$

where $e_k[n]$ is the auxiliary variable we introduced. Given the $z[n]$ and $\mathbf{q}[n]$, at the m-th iteration, the optimal $e_k[n]$ at the m-th iteration can be updated by

$$e_k^m[n] = \frac{\sqrt{z^m[n]}}{(\|\mathbf{q}^m[n] - \mathbf{w_k}\|_2^2 + z^m[n]^2)^{\frac{1}{2}}}, \tag{24}$$

Proof. The detailed proof can be found in the [18] and will not be described in detail here. □

However, $V_k[n]$ is still non-convex, due to $(\|\mathbf{q}[n] - \mathbf{w_k}\|_2^2 + z[n]^2)^{\frac{1}{2}}$, next, we can use SCA method to solve it efficiently. The first order Taylor expression of $(\|\mathbf{q}[n] - \mathbf{w_k}\|_2^2 + z[n]^2)^{\frac{1}{2}}$ w.r.t $\|\mathbf{q}[n] - \mathbf{w_k}\|_2^2 + z[n]^2$ around feasible point $\mathbf{q}^m[n]$ and $z^m[n]$ at the m-th iteration is give by

$$\frac{\sqrt{(\|\mathbf{q}^m[n] - \mathbf{w_k}\|_2^2 + z^m[n]^2)}}{2} + \frac{\|\mathbf{q}[n] - \mathbf{w_k}\|_2^2 + z[n]^2}{2\sqrt{\|\mathbf{q}^m[n] - \mathbf{w_k}\|_2^2 + z^m[n]^2}}. \tag{25}$$

Now, constraint (21e) is converted to a convex constraint. And by substituting constraints (12) - (13) with constraints (21a) - (21b), (21d) - (21f). It can be noted that problem (P3) is convex with respect to $\mathbf{q}$[n] z[n] ξ_k[n] ψ_k[n] ζ_k[n] and ω_k[n], it can be sufficiently solved by the convex optimization tools such as CVX in matlab.

C. Time Allocation of Users to UAV Sub-problem

For any given $\mathbf{q}[n]$ $z[n]$ $f_k^{\mathrm{UAV}}[n]$ and N, the time allocation between users and UAV can be optimized by solving the following problem (P4):

$$(\mathbf{P4}) \qquad \max_{t_{k,u}[n]} \quad \eta \tag{26a}$$

$$\mathrm{s.t.}(11) - (15). \tag{26b}$$

When the other variables fixed, the constraints (11) - (15) are linear with respect to $t_{k,u}[n]$, thus P4 is a standard convex problem. Therefore, which can be sufficiently solved by the convex optimization tools such as CVX in matlab.

Algorithm 1: Proposed Algorithm for Problem (P2)

1: Initialize N_{up} as a suitable number, $N_{\mathrm{down}} = 0$, $N_{\mathrm{tore}} = 10$ and define the tolerance ε.
2: **repeat**
3: Update N = $\frac{N_{\mathrm{up}}+N_{\mathrm{down}}}{2}$, m = 0.
4: The initial t^{m} f^{m} $\mathbf{q}^{\mathrm{m}}$ and z^{m} are chosen according to the size of N.
5: **while** m $\leq N_{tore}$ or $\eta^{\mathrm{m}} < 1$ **do**
6: update m = m + 1
7: Given t^{m} f^{m} , update $\mathbf{q}^{\mathrm{m}}$ z^{m} by solving problem (P3).
8: Given $\mathbf{q}^{\mathrm{m}}$ z^{m} f^{m}, update t^{m} and η^{m} by solving problem (P4).
9: Given $\mathbf{q}^{\mathrm{m}}$ z^{m} t^{m}, update f^{m} by solving problem (P6).
10: **end while**
11: **if** $\eta^{\mathrm{m}} \geq 1$ **then**
12: Let N_{up} = N, N^{opt} = N.
13: **else**
14: Let N_{down} = N.
15: **end if**
16: **until** (N_{up} - N_{down}) $\leq \varepsilon$.
17: **Output**: N^{opt}

D. UAV computing resource allocation Subproblem

For any given $\mathbf{q}[n]$ $z[n]$ $t_{k,u}[n]$ and N, the UAV computing resources allocation subproblem can be optimized by solving the following problem (P5):

$$(\mathbf{P5}) \qquad \max_{f_k^{\mathrm{UAV}}[n]} \quad \eta \tag{27a}$$

$$\mathrm{s.t.}(12), (17). \tag{27b}$$

Problem (P5) is a typical resource allocation problem, We will use the following criteria for the allocation of computing resources of UAV

$$(\mathbf{P6}) \qquad \min_{f_k^{\mathrm{UAV}}[n]} \sum_{k=1}^{K} \frac{(t_{k,u}[n]R_k[n])C_k}{f_k^{UAV}[n+1]} \tag{28a}$$

$$\text{s.t.}(12),(17). \tag{28b}$$

It is worth noting that problem (P5) is a convex problem w.r.t $f_k^{UAV}[n]$, we can obtain the closed-form solution of $f_k^{UAV}[n]$ via KKT conditions [19].

$$f_k^{\mathrm{UAV}}[n+1] = \begin{cases} 0, & t_{k,u}[n]=0 \\ \frac{\sqrt{A_k[n]}}{\sum_{i=1}^{K}\sqrt{A_i[n]}} F_{\max}^{\mathrm{UAV}}, & \text{else}, \end{cases} \tag{28c}$$

where $A_k[n] = t_{k,u}[n]R_k[n]C_k$.

Theorem 3. *Following the criteria that we proposed above, the computational resource allocation in the m-th interation ensures that the optimal solution η^* with given N in the $m+1$-th iteration is greater than or equal to the optimal solution η^* with given N in the m-th interation.*

Proof. By solving P6, we can always know that for $\min_{k\in\mathcal{K}} \frac{\sum_{n=1}^{N-1} t_{k,u}[n]R_k[n]+\frac{N\delta f_k}{C_k}}{I_k}$, the completion time of the offload task transmitted to the UAV in each time slot of users in the next time slot less than or equal to delta, and in the next optimization, we can always find a suitable set of $t_{k,u}[n]$ $\mathbf{q}[n]$ and $z[n]$ such that $\min_{k\in\mathcal{K}} \frac{\sum_{n=1}^{N-1} t_{k,u}[n]R_k[n]+\frac{N\delta f_k}{C_k}}{I_k}$ increases. Otherwise, the optimal solution of Problem 2 has been reached. □

IV. Simulation Results

In this section, numerical results are presented to evaluate the performance of our proposed algorithm. We consider K = 5 users whose coordinates are (-30 -32), (-20 -14), (0 20), (24 -20) and (32 38), respectively. The minimum flight altitude for UAV set at ten meters. The transmit power of all users is assumed to be 0.1 W and their maximum CPU frequency is 2 GHz. The UAV carries an edge computing server, which has a maximum computing frequency of 10 GHz. The maximum horizontal speed and maximum vertical speed of the UAV are 20 m/s and 10 m/s, respectively. We assume that the initial path of the UAV is a straight line path from user 1 to user 5 and the flight altitude of the initial trajectory is 15 m. Set the value of C_k from user 1 to user 5 as [600 650 700 650 800] cycles/bit respectively. According to [11], let B_1 = - 4.3221, B_2 = 6.0750, C_1 = 0, C_2 = 1 and the path loss exponent α = 2. The referenced channel gain and noise power are β_0 = - 60 dB and σ^2 = - 90 dBm. The total bandwidth in this paper is 5 MHz.

In Fig. 2, we compare our algorithm with four benchmark for different task sizes and plot the completion time as a function of task size I_k in different scenarios : 1) No offloading: Users handle their own computational tasks without the help of the UAV; 2) Equal time: We assume that the time for the user to transmit the offloading task to UAV is divided equally in each time slot, i.e. $t_{k,u}[n] = \delta/k$; 3) Equal frequency: Assuming that the UAV allocates its computational resources equally to each user in each time slot 4) Initial trajectory: We fix the trajectory as the initial trajectory. As can be seen from the Fig. 1, we achieved extremely marvellous performance in reducing the overall system latency performance by jointly optimizing the 3D trajectory of the UAV, the user's time allocation and the UAV's computational resource allocation. With the task size of 140Mb, our proposed algorithm reduces the overall system latency by 53.5%.

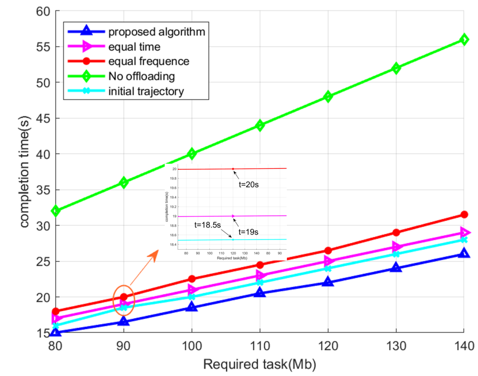


Fig. 2. Completion time versus the required task size.

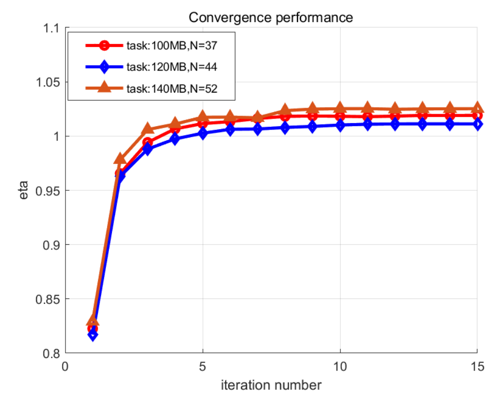
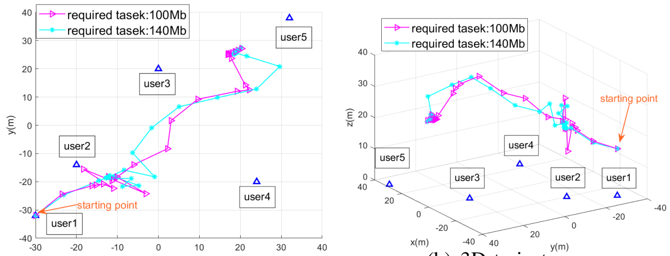


Fig. 3. Convergence performance.

In the Fig. 3 we plot the trend in the size of eta with the number of iterations for a given N used to demonstrate the convergence of our proposed algorithm. From Fig. 2 and Fig. 3 , we can conclude that the required N, can be found accurately by our proposed algorithm to fit the required task size. And as can be seen in Fig. 3, given a suitable N, eta monotonically increases to near 1 and finally stabilizes near and greater than 1 by our proposed alternating optimization algorithm. The efficiency and stability of the algorithm proposed in this paper can be proved by the above two points.

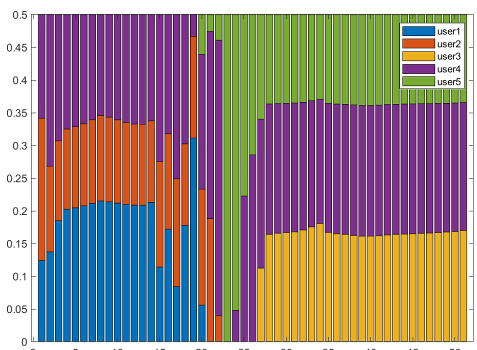
The 2D and 3D flight trajectories of the UAV are presented in Fig. 4.(a) and Fig. 4.(b), respectively. We sample the users as ▲. From Fig. 4, it can be seen that in order to reasonably allocate the computational resources of the UAV, which tends to choose a relatively centered route to fly regardless of whether the task size is 100MB or the task size is 140MB, and due to the larger task size of user 3 user 4 and user 5, the



(a) 2D trajectory (b) 3D trajectory

Fig. 4. 2D vs 3D UAV trajectory with different required task size.

UAV hovers around the second half of the flight at the point that simultaneously enables these three users to get a better communication rate, minimizing the overall completion time. In terms of changes in flight altitude, if the average elevation angle of the UAV to the simultaneously served users is not favorable for receiving the offloading task, the UAV will increase or decrease its flight altitude to achieve better communication quality. Overall, when we design the trajectory of the UAV, there is a trade-off between distance and elevation angle to simultaneously served users to achieve optimal communication quality, making the overall task completion time minimal.



(a) time allocation

Fig. 5. Time allocation chart for a task size of 140MB.

As shown in Fig. 5, since the UAV chooses a trajectory far away from user 4, the UAV maintains communication with user 4 throughout most of the whole time to share its computational tasks as compensation, and for the other users, the UAV chooses to connect with users relatively close to it to maximize the communication rate, thus reducing the delay of the whole system.

V. Conclusions

In this paper, we consider the resource allocation and 3D trajectory design for a UAV-assisted MEC system. we mainly use quadratic transformation and SCA method to solve the 3D trajectory design, time and computing resource allocation problem of the UAV-enabled MEC system. We convert the intractable Problem 1 into the form of Problem 2, which is still nonconvex. Furthermore, a combination of bisection search method and alternating optimization algorithms is leveraged to make Problem 2 solvable, which alternately optimizes the 3D trajectory, time and computational resource allocation. The simulation results show that our proposed algorithm has a significant performance improvement over the four benchmark schemes. In addition, the efficiency and stability of our proposed algorithm are also clearly verified.

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