Constraining Ultra Light Dark Matter with the Stellar Kinematics in the Galactic Centre

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I, Firat Toguz, confirm that the work presented in this thesis is my own. Where information has been derived from other sources, I confirm that this has been indicated in the work.

Abstract

We use the stellar kinematics in the Milky Way's Galactic centre to test the existence of a dark matter 'soliton core', as predicted in ultra-light dark matter (ULDM) models, which are currently an attractive model to resolve the small-scale problems of the cold dark matter model. We first use the Milky Way's nuclear star cluster (NSC) to test the existence of the ULDM by applying a spherical isotropic Jeans model to fit the NSC line-of-sight velocity dispersion data, assuming priors on the precisely measured Milky Way's supermassive black hole (SMBH) mass and the well-measured NSC density profile. We find that the current observational data reject the existence of a soliton core for a single ULDM particle with mass in the range $10^{20:40}$ eV . m_{DM} . $10^{18:50}$ eV, assuming that the soliton core structure is not affected by the Milky Way's SMBH. We then fix the NSC as an external component and used Action-based Galaxy Modelling Architecture (AGAMA) package to model the nuclear stellar disk (NSD) to test the existence of a soliton core at a broader range of m_{DM}. We assess the existence of a soliton core in the centre of the Milky Way by fitting the surface density, mean line-of-sight velocity, and vertical velocity dispersion of the NSD stars. We use the surface density and proper motion data from the updated version of the VISTA Variables in the Vía Láctea Infrared Astrometric Catalogue data, and the line-of-sight velocities provided by Apache Point Galactic Evolution Experiment. We find that these observational data reject the ULDM particle mass range between $10^{23:20}$ eV and $10^{20:0}$ eV. Hence, combining the constraints from the NSC, we conclude that the current observational data reject the ULDM particle mass range from $10^{23:20}$ eV and $10^{18:50}$ eV. Overall, this work clearly indicates that a model explaining the dark matter as a single

ULDM particle mass is not a viable solution to explain the nature of the dark matter in the Universe.

Impact Statement

The research presented in this thesis extends our knowledge in the nature of dark matter. Dark matter stands as the primary component responsible for the formation and existence of galaxies, including our own Milky Way. The profound impact of dark matter on cosmic structures, coupled with its potential to address significant problems in physics, such as the unification of gravity with other fundamental forces described by quantum mechanics. Consequently, this profound significance motivates and inspires a new generation of scientists and engineers in future space explorations, as well as providing an intriguing topic for public outreach. Hence, this research has a substantial positive impact on our culture. With regards to the academic impact, the work in this thesis suggests that a type of dark matter, called ultra-light dark matter (ULDM) is unlikely to be a viable candidate of dark matter particles. Hence, this research contributes to rejecting one of the current attractive dark matter scenarios, and encourages the exploration of other dark matter scenarios. Our work also highlights that the astronomy data are valuable to study the nature of dark matter. The statistical methods applied to use the large observational data, as well as carefully taking into account their measurement uncertainties, are also demonstrated to be important to assess the significance of the data to constrain the theoretical models. Therefore, this thesis also provides a valuable impact on highlighting the importance of the statistical data analysis technique, and its transferable techniques to the applications of data science for many other disciplines, including more societal topics, such as finance, economics, and education.

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Chapter 1

Introduction

1.1 Dark Matter

To this day, the source of the 85% of gravity in our universe is still a mystery. There are many different ideas that could possibly explain this mystery. One idea is that Einstein's theory of general relativity may be incomplete and that a new, upgraded theory of gravity is required. Majority of astrophysicists however, believe the existence of an invisible matter that does not emit, re ect or absorb electromagnetic radiation and therefore cannot be detected directly from observation. This matter is called dark matter and it is the main component that makes up all the structures we see in the universe today.

1.2 Evidence of Dark Matter

Although dark matter is unseen, the effects of dark matter can be seen indirectly through gravitational effects on luminous matter. In this section, some of the main evidences of dark matter will be brie y explained.

1.2.1 Coma Cluster

The Coma Cluster is a group of about thousand galaxies that is gravitationally bound together. The rst evidence of dark matter came in 1933 when the Swiss astrophysicist Fritz Zwicky measured the radial velocities of the galaxies in the Coma Cluster and used the virial theorem to compute the mass of the Coma Cluster (Zwicky, 1933, 2009). Zwicky realised that the velocity of the galaxies are very high such that the gravitational strength from the visible galaxies are not strong enough to hold the galaxies bound together. The mass of the cluster should be over about 400 times more in order for the gravitational strength to hold the clusters together. Zwicky concluded that there must be an extra component of mass that is non-luminous which provides the enough mass to hold the visible galaxies together, this non-luminous mass is called “dark matter”.

1.2.2 Rotation Curves

Most of the matter in galaxies, particularly spiral galaxies, is concentrated at the centre. Due to this, it was expected for the orbital velocity of stars to decrease as the distance from the centre increases. The rotation curve is given by the following formula

$$v(R) = \sqrt{\frac{GM(R)}{R}}, \tag{1.1}$$

where $v(R)$ is the velocity of stars, R is the radial distance from the centre and $M(R)$ is the mass contained in a sphere within R. The mass is expected to increase with R at the inner parts of the galaxy since the inner parts stars are clearly present and contribute to the dynamics significantly. However, $M(R)$ should start to increase slower than $\propto R$ in the outskirts of the galaxy where there are fewer observed stars. Therefore, we expect the relationship between the rotation velocity of the gas and distance from the centre of the galaxy to follow the red dashed line in Fig. 1.1. In 1970s, Rubin & Ford (1970) measured the rotation curves of spiral galaxies and discovered that the velocity of gas does not decrease as increase in distance from the centre of galaxy. Instead, the velocities of gas stays the same, giving a flat rotation curve as seen by the blue solid line in Fig. 1.1. The gas is clearly moving significantly faster than expected and should be able to escape the gravitational potential of the galaxy since the gravitational contribution from the luminous matter alone is inadequate to explain the high velocities of gas in the outer parts of the galaxy. To be able to explain the flatness of the curve and how the gas are held together, there needs to be a new mass component that is distributed as $M(R) \propto R$.

Figure 1.1: Red dashed line: Expected rotation curve from the mass distribution of visible matter. Blue solid line: Observed rotation curve.

1.3 Lambda-Cold Dark Matter

Lambda-Cold dark matter (ΛCDM) with Λ being the cosmological constant, and CDM is a type of dark matter that is composed of particles travelling slowly compared to speed of light - this is why this type of dark matter is called "cold". The primary candidate of ΛCDM is weakly interacting massive particle (WIMP) which is a stable, neutral particle with mass in the order of from GeV to more than TeV (see Fig. 1.2). In recent works, it has been shown that ΛCDM model successfully describes the cosmic microwave background (CMB) (e.g. Bennett et al., 2013; Planck Collaboration et al., 2020) and large scale structure (e.g. Percival et al., 2001; Tegmark et al., 2004; Weinberg et al., 2015). However, tensions between theory and observations persist at small scales (e.g. Bullock & Boylan-Kolchin, 2017, for a review).

Figure 1.2: Mass ranges of different dark matter models. ULDM is the ultra-light dark matter, WDM is the warm dark matter and CDM is cold dark matter. M_{pl} is the Planck scale.

1.4 Problems of CDM at Small Scales

1.4.1 Missing Satellite Problem

Mateo (1998) used the most recent distance and radial velocity data at the time to update the number of satellite dwarf galaxies in the Local Group, which was found close to 40 satellite galaxies are in the Local Group. A year later, Klypin et al. (1999) and Moore et al. (1999) used numerical simulations of hierarchical universe models with ΛCDM to study the substructure of galaxies. It was found that numerical simulations of a Milky Way-like galaxy in ΛCDM predict that a thousand dark matter subhalos large enough to host a visible galaxy ($M_{halo} > 10^7 M_\odot$) should be found orbiting within the Milky Way. However, this is disagreeing with observations and hence this problem is called the missing satellite problem - an image representation can be seen in Fig. 1.3. According to Tollerud et al. (2008) there could be hundreds of faint dwarf galaxies and that future surveys could prove this. More recently, Drlica-Wagner et al. (2020) used the Dark Energy Survey (DES) and Pan-STARRS1 (PS1) data which covers about 80% of the sky in which the number of satellite dwarf galaxies has now increased to about 70, still in serious tension with the theoretical predictions.

1.4.2 Cusp-Core Problem

A cuspy density profile means a density profile rising quickly towards the centre whereas core-like density profile is when the density is flattened at the centre of the galaxy (Fig. 1.4). The rotation curve of a galaxy with a cuspy profile rises rapidly, whereas in galaxies with core-like density profile, the rotation curve slowly rises, as shown in Fig. 1.5.

The pure dark matter N-body simulations of structure formation in ΛCDM

Figure 1.3: Left: A representation of spatial distribution of the satellites of the Milky Way according to simulations of a Universe with ΛCDM. Right: A representation of spatial distirbution of the satellites of the Milky Way according to observations.

were done in the early 1990s where the N-body simulations predict that bound dark matter halos have a centrally divergent `cuspy' density profile with inner distribution following a power law, $\rho \propto r^{\alpha}$, with $\alpha = -1$ (Dubinski & Carlberg, 1991; Navarro et al., 1997).

However, in the mid 1990s, observations showed that the inner distribution follows a core-like profile, $\alpha = 0$. This tension between the cuspy density profile of dark matter halo predicted by the numerical simulations and the cored density profile found by observation rose the so-called cusp-core problem. For example, Flores & Primack (1994) and Moore (1994) compared the rotation curve data of dwarf spirals with the theoretically predicted rotation curve and discovered that the theoretical rotation curve rises faster with the increasing radius in the inner region of the galaxy with respect to the observed rotation curve data. In more recent results, although some observations of the rotation curves of nearby low-surface brightness galaxies still favour a much lower density inner `core' (e.g de Blok et al., 2001; Kuzio de Naray et al., 2008), some studies also argued that the central density profiles in galaxies are cuspy-like (e.g. Richardson & Fairbairn, 2014). This dispute is due to the current data available not strong enough to make a crystal clear distinction whether the galaxies have a central cusp or core-like profile.

Figure 1.4: Density pro le that is cuspy (blue) and core-like (red).

Interestingly, future advanced astrometric telescopes can provide proper motion data strong enough to clearly distinguish between a central cuspy and core pro le of a galaxy. For example, de Martino et al. (2022) constructed mock data representing the expected proper motion data from the future astrometric Theia-like missions and found that proper motion of about 2000 stars in dwarf galaxies is enough to tell us in full accuracy whether dwarf galaxies have a cuspy or core-like pro le.

1.4.3 Too-Big-To-Fail

The mass of the Milky Way satellites from simulations should be about the same as the observed satellites. The Milky Way subhalo satellites found to be in similar mass as some of the satellite galaxies from simulations. However, simulations also predicts satellite halos with much larger mass than observed halos. This means that we should be able to also detect more of these larger mass subhalos, because these halos are too big to fail to form the stars. However, these halos are not observed in

Figure 1.5: Rotation curves for cuspy (blue) and core density pro le (red).

the Milky Way and it seems that these larger mass subhalos are not formed, although the smaller dark matter halos seem to form with a large number of stars, which are luminous enough to be detected.

The problem is the following: How can larger mass subhalos with deeper gravitational well fail to form galaxies where as smaller mass subhalos with less deeper gravitational well is able to form the galaxies? The larger mass subhalos should be able to form the galaxies as it is “too big to fail (TBTF)” (e.g. Boylan-Kolchin et al., 2011, 2012; Garrison-Kimmel et al., 2014). There is an interesting relationship between the TBTF problem and the cusp-core problem. Ogiya & Burkert (2015) found that the dark matter models with densities being core-like solves the TBTF problem. The idea is that the density of dark matter halo are initially cuspy and then transitions to core-like pro le through stellar feedback which led to reduction of galaxy formation (Kato et al., 2016). This suggests that nding a solution to the cusp-core problem could solve the TBTF problem. More discussion about the baryonic effects are provided in the following sections.

1.5 Possible Solutions to Small Scale Problems

1.5.1 Baryonic Effects

The above small scale issues of ΛCDM model may owe entirely to 'baryonic effects' (i.e. due to gas cooling, star formation and stellar feedback) not included in early structure formation models. Galaxy formation is expected to become increasingly inefficient at low mass due to a combination of stellar feedback and ionising radiation from the massive stars and quasars (e.g. Efstathiou, 1992; Benson et al., 2002; Sawala et al., 2016). Indeed, recent dynamical estimates of the masses of the Milky Way's dwarf companions suggests that there is no missing satellite problem at least down to a halo mass of $M_{200} \sim 10^9\,M_\odot$ (Read & Erkal, 2019), where M_{200} is the mass of the dark matter halo that is defined as the spherical region with the density being approximately about 200 times the critical density of the universe.

Furthermore, repeated gas inflow/outflow, driven by gas cooling and stellar feedback, can cause the central gravitational potential in dwarf galaxies to fluctuate with time. This pumps energy into the dark matter particle orbits which prevents the cuspy dark matter to form, but leading to cored dark matter density profile (Navarro et al., 1996; Read & Gilmore, 2005; Pontzen & Governato, 2012; Di Cintio et al., 2014). There is mounting observational evidence that this 'dark matter heating' effect has occurred in nearby dwarf galaxies; this may be sufficient to fully solve the cusp-core problem (e.g. Read et al., 2019).

1.5.2 Warm Dark Matter

ΛCDM's small scale issues have inspired a host of novel dark matter models designed to lower the inner density of dark matter halos and/or reduce the number of dark matter subhalos. These include warm dark matter (WDM e.g. Dodelson & Widrow, 1994; Bode et al., 2001) and ultra-light dark matter (ULDM e.g. Ferreira, 2020; Hui, 2021). In WDM, dark matter is assumed to be relativistic for a time in the early Universe, suppressing the small scale power spectrum and leading to fewer, lower-density, satellite galaxies as compared to CDM. This can naturally occur if, for example, dark matter is a light thermal relic particle.

For thermal relic masses of about 1 keV (Fig. 1.2), WDM has the potential to resolve the missing satellite problem (e.g. Knebe et al., 2002; Lovell et al., 2021, 2014), although this depends on the assumed total mass of the Milky Way (e.g. Kennedy et al., 2014). Indeed, the observed number of the Milky Way satellite galaxies puts a lower limit of the WDM mass (e.g. Polisensky & Ricotti, 2011). Newton et al. (2020) favoured a lower limit of 3.99 keV, marginalising the uncertainty in the Milky Way mass, and taking into account the expected inefficiency of dwarf galaxy formation (see also an even stronger constraint of > 6.5 keV in Nadler et al., 2021). A similar lower limit on the WDM mass was imposed by the other astronomical probes, such as Lyman-α forest data (Iršič et al., 2017), strong gravitational lensing (Gilman et al., 2020) and density fluctuations in Galactic stellar streams (Banik et al., 2019).

However, keV-scale WDM is not able to solve the cusp-core problem on its own (see e.g. Weinberg et al., 2015, for a review). Macciò et al. (2012) showed that a WDM mass of about 0.1 keV is required to generate kpc-sized cores in dwarf galaxies, but such a low mass WDM particle is incompatible with the above observational constraints.

1.5.3 Ultra-Light Dark Matter

ULDM has emerged as a novel dark matter model that can solve both the cusp-core and missing satellite problems on its own, without recourse to baryonic effects. ULDM is a type of dark matter that is made up of bosons with mass in the range of $10^{-22.0}$ eV $< m_{DM} <$ 1 keV (e.g. Ferreira, 2020; Hui, 2021; Lin, 2019; Battaglieri et al., 2017, for a review[1]). On large scales, ULDM behaves just like CDM, i.e. it successfully explains large scale structure and the CMB. However, in high density regions like the centres of dark matter halos, the de Broglie wavelength of the ULDM particles becomes larger than the mean inter-particle separation, and the ULDM undergoes Bose-Einstein condensation. Consequently, ULDM introduces a new scale length – the Jeans length, l_J, – set by the de Broglie wavelength and the

[1] ULDM is also often called Fuzzy dark matter. However, throughtout this thesis, we use ULDM only to avoid a confusion.

dark matter density (Hu et al., 2000a):

$$\lambda_J \simeq 55[m_{DM}/(10^{-22}\,\mathrm{eV})]^{-1/2}(\rho/\rho_b)^{-1/4}$$
$$(\Omega_{ULDM}h^2)^{-1/4}\,\mathrm{kpc}, \qquad (1.2)$$

where ρ is the matter density, Ω_{ULDM} is the mass fraction for the ULDM particle with respect to the critical density, and $\rho_b \simeq 2.8 \times 10^{11}(\Omega_{ULDM}h^2)\,M_\odot\,\mathrm{Mpc}^{-3}$ is the background density.

Perturbations larger than λ_J will collapse similarly to CDM, while perturbations smaller than λ_J are stabilized by quantum pressure due to the uncertainty principle (e.g. Hu et al., 2000b). At low dark matter density, close to the background density of the Universe, the Jeans mass can be computed from the Jeans length, as follows (e.g. Hui et al., 2017):

$$M_J = \frac{4\pi}{3}\rho\left(\frac{1}{2}\lambda_J\right)^3$$
$$\simeq 1.5 \times 10^7 M_\odot\,(1+z)^{3/4}\left(\frac{\Omega_{ULDM}}{0.27}\right)^{1/4}$$
$$\left(\frac{H_0}{70\,\mathrm{km\,s^{-1}\,Mpc^{-1}}}\right)^{1/2}\left(\frac{10^{-22}\mathrm{eV}}{m}\right)^{3/2}, \qquad (1.3)$$

where H_0 is the Hubble constant. This Jeans mass corresponds to the minimum halo mass which can collapse in the ULDM model; it leads to a smaller number of dwarf galaxies as compared to the CDM model. In this way, ULDM can resolve the missing satellite problem (e.g. Kulkarni & Ostriker, 2020). According to Nadler et al. (2021), the observed number of Milky Way satellites required a ULDM particle mass higher than $2.9 \times 10^{-21.0}$ eV.

Another consequence of ULDM is that, at the scale of the de Broglie wavelength within the collapsed halo, the Bose-Einstein condensation develops a 'soliton core' at the centres of galaxies (e.g. Hu et al., 2000b; Schive et al., 2014b). The soliton core has a half-mass radius of about 300 pc in a $M_{200} \sim 10^9 M_\odot$ dwarf galaxy halo for a ULDM model with $m_{DM} = 10^{-22.0}$ eV (see eq. 2.11 in Section 2.2.4). This soliton core can mitigate the cusp-core problem. Schive et al. (2014b) suggested that $m_{DM} = 8 \times 10^{-23.0}$ eV ULDM can explain the observed mass profile of the Fornax dwarf galaxy (e.g. Amorisco et al., 2013; Read et al., 2019). However, Safarzadeh & Spergel (2020) argued that no single ULDM particle mass can explain the current observations of the ultra-faint dwarfs and the Fornax and Sculptor dwarf spheroidal galaxies simultaneously (see also Hayashi et al., 2021), unless the baryonic physics changes the density profile of the dark matter halo (see above) or the observational constraints are relaxed.

Many different astronomical techniques has been used to constrain ULDM as summarised in Fig. 1.6. Kobayashi et al. (2017) used the Lyman-α forest to constrain the ULDM which they obtained a rejection limit of $m_{DM} > 10^{-21.0}$ eV. Black holes can constrain ULDM through the process of superradiance. Essentially, superradiance is when the angular momentum of the spinning black hole will be lost efficiently if a boson with a particular mass surrounds it. Stott & Marsh (2018) observed the spin of black holes to constrain the superradiance of black holes and found a rejection limit of $m_{DM} > 10^{-19.20}$ eV. Similarly, Davoudiasl & Denton (2019) used the Event Horizon Telescope observation of M87 that provided the first direct image of black hole. From this information, they obtained a rejection range of $10^{-21.07} < m_{DM} < 10^{-20.34}$ eV. Bar et al. (2019a) used the rotation curves of nearby galaxies and obtained a rejection limit of $m_{DM} < 10^{-21.0}$ eV. Through the use of satellite luminosity function inferred from the perturbed stellar streams (Banik et al., 2019) and lensed images (Gilman et al., 2020), Schutz (2020) found a range of $m_{DM} < 10^{-20.70}$ eV being rejected. González-Morales et al. (2017) used stellar kinematics of the Fornax and Sculptor dwarf spheroidal galaxies and rejected ULDM mass limit of $m_{DM} > 10^{-22.40}$ eV. Likewise, Hayashi et al. (2021) used Jeans analysis of the stellar kinematics of 18 ultra-faint dwarf (UFD) galaxies and concluded a range of $10^{-19.40} < m_{DM} < 10^{-18.0}$ eV to be consistent with the stellar kinematics. Zoutendijk et al. (2021) rejected $m_{DM} < 10^{-20.40}$ eV from the stellar kinematics of the ultra-faint dwarf galaxy, Eridanus.

Taking these results at a face value, no single particle ULDM model can satisfy all current observational constraints. Thus – at least as a full solution to ΛCDM's small scale puzzles – ULDM appears to be on the ropes. However, all of the current constraints on ULDM come with their own potential systematics. As such, independent observational constraints are invaluable in determining once and for all whether we can discard ULDM as a full solution to ΛCDM's small scale puzzles. In this thesis, we consider whether the Milky Way's nuclear star cluster (NSC) and nuclear stellar disk (NSD) can provide a new and competitive probe of ULDM models. Due to it being only 8 kpc away from us, the stellar kinematics of the central region of the Milky Way can be more precisely measured than for more distant dwarf galaxies (d 100 kpc). Hence, the inner gravitational potential of the Milky Way can be derived from precise measurements of the stellar kinematics and density distribution of tracer stars in the centre of the Galaxy.

1.6 Nuclear Star Cluster

The study of the NSC and the NSD (see Section 1.7) is also important because it can help constrain the evolution and formation of galaxies. The NSC was first discovered by Becklin & Neugebauer (1968) through detecting an infrared source that is centred around the supermassive black hole (SMBH) in our Milky Way. The Milky Way's NSC is a dense and massive star cluster (e.g. Bland-Hawthorn & Gerhard, 2016, for a review) that harbours the Milky Way's SMBH, called “Sagittarius A*” (e.g. Genzel et al., 1996; Ghez et al., 2008), whose mass of $M_{BH} = 4.261 \pm 0.012 \times 10^6\,M_\odot$, is now precisely measured by the GRAVITY collaboration (Gravity Collaboration et al., 2020), a cryogenic, interferometric beam combiner of all four UTs of the European Southern Observatory (ESO), Very Large Telescope (VLT) with adaptive optics. The coexistence of SMBH and NSC also occurs outside of our Milky Way within galaxies such as elliptical and spiral galaxies (Seth et al., 2008). However, it is more unlikely to find a NSC in early-type galaxies such as the elliptical galaxies. This could be due to the way elliptical galaxies are formed through mergers of relatively massive galaxies. Merging of galaxies can lead to the

Figure 1.6: Summary of rejected ULDM particle masses from various astronomical probes. The Lymanα forest observation rejects $m_{DM} < 10^{20.5}$ eV (Irsic et al., 2017; Kobayashi et al., 2017; Armengaud et al., 2017). The observed spin of black holes constrain the superradiance of black holes, and rejects $m_{DM} > 10^{19.20}$ eV (Stott & Marsh, 2018), including the Event Horizon Telescope observation of M87, which rejects $10^{21.07} < m_{DM} < 10^{20.34}$ eV (Davoudiasl & Denton, 2019). Rotation curves of nearby galaxies also reject $m_{DM} < 10^{21.0}$ eV (Bar et al., 2019a). Schutz (2020) suggests that $m_{DM} < 10^{20.70}$ eV is rejected by the satellite luminosity function inferred from the perturbed stellar streams (Banik et al., 2019) and lensed images (Gilman et al., 2020), similarly to constraints on the WDM mass (Section 1.5.2). González-Morales et al. (2017) reject $m_{DM} > 10^{22.40}$ eV from the stellar kinematics of the Fornax and Sculptor dwarf spheroidal galaxies. Hayashi et al. (2021) find that the stellar kinematics of Segue I is consistent with $10^{19.40} < m_{DM} < 10^{18.0}$ eV. We naively take this as the required ULDM mass range, and consider that the other mass ranges are rejected, if the Segue I stellar kinematics is purely due to the soliton core. Zoutendijk et al. (2021) reject $m_{DM} < 10^{20.40}$ eV from the stellar kinematics of the ultra-faint dwarf galaxy, Eridanus.

SMBHs from each galaxy to become a binary black holes. This process could send outwards energy to the surroundings causing the central stellar density to decrease and hence wiping out the NSC (Milosavljević & Merritt, 2001).

Axisymmetric Jeans models and two integral distribution function models were constructed based on stellar number counts, proper motions, and line-of-sight velocity data. Based on these models, a NSC mass of about 10^7 M$_\odot$ was derived (e.g. Fritz et al., 2016; Bland-Hawthorn & Gerhard, 2016; Chatzopoulos et al., 2015; Feldmeier-Krause et al., 2017b). The formation of the NSC is still unclear. There are two particular ideas. One proposed idea is mergers of star clusters. Star clusters formed outside of the centre of the galaxy fell into the centre of the galaxies and merged with each other, leading to the formation of the NSC (Tremaine et al., 1975). The other proposed idea is the NSC being formed in situ, from gas flows towards the centre of the galaxy (Milosavljević, 2004).

1.6.1 Structure and the Chemical Composition of the NSC

Fritz et al. (2016) constructed a stellar density map of the Galactic central $1000''$ by using the star counts from Visible and Infrared Survey Telescope for Astronomy (VISTA), Wide Field Camera 3 with Infrared (WFC3/IR), and Very Large Telescope (VLT) with NAOS-CONICA (NACO) data. The stellar density map showed that the structure of the NSC is not a perfect sphere, it is rather a flattened sphere with a minor to major axis ratio of $q = 0.80 \pm 0.04$. Not only the NSC in the Milky-Way Galaxy is flattened, many NSCs are found to be flattened in many edge-on spiral galaxies (Seth et al., 2006). The diameter of the NSC in the Milky Way is approximately about a few arcminutes, about 4.8 pc (e.g. Gallego-Cano et al., 2020), with an effective radius of about 3.5 pc (Böker et al., 2004). Through Mid-Infrared (MIR) wavelengths at 2.15 μm and 4.5 μm, the luminosity of the NSC was found to be about $4.1 \pm 0.4 \times 10^7$ L$_\odot$ (Schödel et al., 2014).

The NSC is surrounded by the NSD (see Fig. 1.8 and Section 1.7), with radius of about 230 pc (e.g. Launhardt et al., 2002; Bland-Hawthorn & Gerhard, 2016). A lot of theoretical and analytical studies have been made in how the stars in the NSC are distributed. For a dynamically relaxed cluster around a SMBH, the theoretical

Figure 1.7: The density profile for the Milky Way's NSC (blue dashed), and for the central dark matter density profile assuming ΛCDM (brown, Navarro et al., 1996) and dark matter with a ULDM particle mass of $10^{-23.0}$ eV (light blue), $10^{-21.0}$ eV (magenta), $10^{-20.0}$ eV (orange), $10^{-19.0}$ eV (green), $10^{-18.0}$ eV (red) and $10^{-16.0}$ eV (purple). Notice that over the fixed radial range probed by the NSC stellar kinematic data (vertical black lines of $r = 0.1$ pc and $r = 3$ pc), only ULDM models with mass in a specific range will affect the stellar kinematics. The red horizontal solid line indicates the size of the NSD.

work predicted the distribution of stars to follow a cuspy profile (e.g. Bahcall & Wolf, 1976; Murphy et al., 1991; Preto & Amaro-Seoane, 2010). However, observations showed that it is core-like (Fritz et al., 2016).

Most of the stars in the NSC are metal rich stars, with a metal rich to metal poor star ratio of about 2.6 (Schultheis et al., 2021). The majority (80%) of the stellar mass of the NSC formed more than 5 Gyrs ago (e.g. Gallego-Cano et al., 2018). Thus, we can expect that the NSC is dynamically relaxed and therefore, a good target for equilibrium mass modelling (e.g. Binney & Tremaine, 2008). Most of these stars are spectral type of K and M giant helium burning stars in the category of “red clump” with temperatures of about 3000 - 5000 K (e.g. Genzel et al., 2010; Feldmeier-Krause et al., 2017a). There are also young stars in the NSC that are concentrated more towards the centre (within the central 0.5 pc), where some of these young stars are observed as Wolf-Rayet and O and B-type whose ages around 3–8 Myr. The lower limit for the total mass of the young cluster in the NSC is about 12,000 M (e.g. Feldmeier-Krause et al., 2015; Lu et al., 2013; Paumard et al., 2006). Note that the Wolf-Rayet and OB stars are not all the stars in the NSC, but the total mass comes from many fainter stars which are not possible to be observed easily, but their contribution to the total mass of the NSC is significant.

1.6.2 NSC and ULDM

The number density of NSC stars dominate over other Milky Way stellar components up to about 3 pc, and within 0.1 pc it is difficult to resolve the density profile or stellar kinematics (e.g. Fritz et al., 2016; Gallego-Cano et al., 2020). As such, we can assume that almost all of the stars observed within 3 pc from the Milky Way's SMBH are NSC stars, and use these to trace the inner dynamical mass profile of the Galactic centre. In ULDM models, the dark matter mass profile on this small scale can be affected by the soliton core if the ULDM mass is less than about $10^{19.0}$ eV, as suggested by Fig. 15 of Bar et al. (2018). In addition to this, in Fig. 1.7, we show the NSC density profile obtained in Toguz et al. (2022) and the ULDM dark matter density profile with $m_{DM} = 10^{23.0}$ eV, $10^{21.0}$ eV, $10^{20.0}$ eV, $10^{19.0}$ eV, $10^{18.0}$ eV, $10^{16.0}$ eV and the so-called Navarro-Frenk-White (NFW) profile (Navarro et al., 1996) representing the CDM. It can be seen from Fig. 1.7 that an ULDM soliton core with a mass range of about $10^{20.0} < m_{DM} < 10^{19.0}$ eV can affect the stellar kinematics in the NSC in the radial range of $0.1 < r < 3$ pc. Hence, a dynamical model of the NSC promises a new and competitive probe of ULDM. It is intriguing to note that as depicted in Fig. 1.7, there is a potential for ULDM to manifest as a composite comprising mixtures of distinct soliton cores of the same type. Nevertheless, for the purpose of simplicity, in this thesis our focus is exclusively on a single ULDM particle mass.

1.7 Nuclear Stellar Disk

The NSD is a dense stellar structure that is flattened and dominates gravitationally in the Milky Way between the Galactocentric radius of about 30 pc $< R <$ 300 pc (e.g. Launhardt et al., 2002; Bland-Hawthorn & Gerhard, 2016). The NSD is also found in the extragalactic systems, particularly in barred galaxies (e.g. Erwin & Sparke, 2002; Gadotti et al., 2019), and considered to form when the bar formed (Baba & Kawata, 2020). The NSD has a total mass of about $M_{NSD} = 6.91 \pm 2 \times 10^8$ $M_\odot$ (Sormani et al., 2020b) and a radius of about 230 pc and a scale height of 45 pc (e.g. Launhardt et al., 2002). The SMBH, NSC and NSD are the main components of the centre of the Milky Way with SMBH and NSC being embedded in the NSD (see Fig. 1.8). A ring-like molecular gas, namely, central molecular zone (CMZ) with mass about 3×10^7 $M_\odot$ (Molinari et al., 2011), occupies the same space as the NSD with similar scale-height.

According to simulations of the Milky-Way like galaxies, the NSD is formed by the galactic bar (Carles et al., 2016; Seo et al., 2019). Essentially, the galactic bar causes gas and dust funneling to the centre of the galaxy with a rate of about 1 $M_\odot$ yr^{-1}. This gas and dust overtime gather and grows and as a consequence forms the CMZ (Kim et al., 2012; Shin et al., 2017; Sormani et al., 2020a). Continuous star formation takes place in the CMZ and eventually forming the NSD. There are strong evidence that there is a link between the CMZ and the NSD. For example, simulations of the CMZ has shown that the stars in NSD may come from the stars born in the dense gas of CMZ (e.g. Sormani et al., 2020a). Further to confirm this, Schönrich et al. (2015) found that the gas rotating in the CMZ is similar to what is expected for the NSD, suggesting a possibility of NSD being originated by the stars being born in the CMZ (Sormani et al., 2020b). Since the formation of the NSD is directly linked to the galactic bar, the age of the galactic bar can be derived through using the oldest population of stars in the NSD (Baba & Kawata, 2020).

1.7.1 Structure and the Chemical Composition of the NSD

Most of the stars in the NSD are metal rich stars with velocity dispersion of stars decreasing with increasing metallicity (Schultheis et al., 2021). The metal rich to

Figure 1.8: The Galactic centre components: SMBH (the size of the symbol does not indicate the size of SMBH), NSC and NSD.

metal poor star ratio of the NSD is about 1.6, much lower value than 2.6 for NSC. This indicates that the chemistry and formation of the NSC and NSD is different form each other (Schultheis et al., 2021).

Schönrich et al. (2015) used the line-of-sight kinematics of the NSD stars and discovered the NSD to be an axisymmetric, kinematically cool rotating component with rotation velocity of about 120 $\mathrm{km\,s^{-1}}$. Most of the stars (80%) in the NSD are old stars and formed about 8 Gyrs ago (Nogueras-Lara et al., 2020). However, there are important amount of stars with age about 1 Gyrs that takes in to account about 5% of the NSD mass. Furthermore, the NSD appears to be the region with most active star formation (Nogueras-Lara et al., 2020).

Since the majority of the stars are old stars, the NSD can also be assumed to be dynamically equilibrium (e.g. Sormani et al., 2020b, 2022), making it a great component to be used to test the existence of the ULDM just like the NSC. According to Fig. 15 of Bar et al. (2018) and Fig. 1.7, $10^{-21.0}$ eV soliton core becomes dominant around 200 pc which means the stellar kinematics in the galactic centre with the NSD will be able to constrain this type of soliton core since the NSD is dominant in this region.

1.8 The Work of this Thesis

In Chapter 2, we use the Milky Way's NSC to test the existence of a dark matter `soliton core', as predicted in ULDM models. We will show that the soliton core size is proportional to m_{DM}^{-1}, while the core density grows as m_{DM}^{2}, the NSC (dominant stellar component within 3 pc) is sensitive to a specific window in the dark matter particle mass, as discussed in subsection 1.6.2. We apply a spherical isotropic Jeans model to fit the NSC line-of-sight velocity dispersion data, assuming priors on the precisely measured Milky Way's SMBH mass and the well-measured NSC density profile. We find that the current observational data reject the existence of a soliton core for a ULDM particle with a single mass in the range $10^{-20.40}$ eV $\lesssim m_{DM} \lesssim 10^{-18.5}$ eV, assuming that the soliton core structure is not affected by the Milky Way's SMBH. We test our methodology on mock data, confirming that we are sensitive to the same range in ULDM mass as for the real data. Dynamical modelling of a larger region of the Galactic centre, including the NSD, promises tighter constraints over a broader range of m_{DM}, which we consider and explain in Chapter 3 and Chapter 4.

In Chapter 3, we discuss the observational data for the NSD and we describe our analysis of the surface density of stars and the distribution of velocity dispersion within 100 pc of the Galactic center region. This analysis utilizes the updated VISTA Variables in the Vı́a Láctea Infrared Astrometric Catalogue data (VIRAC2) and the line-of-sight velocity data from Apache Point Galactic Evolution Experiment Data Release 17 (APOGEE DR17).

In Chapter 4, we use the Milky Ways's NSD to test the existence of a dark matter `soliton core'. We fit the observational data with the theoretical model of the same distribution function model as what is used in Sormani et al. (2022), and examine if or not there is any sign of the existence of ULDM in the observational data of stellar density and kinematics in the Galactic centre. We nd that the current observational data of the NSD can reject a ULDM particle mass between $10^{23:20}$eV . m_{DM} . $10^{20:0}$ eV.

In Chapter 5, we discuss the conclusion of the studies in this thesis and we discuss the direction of the future work.

Chapter 2

Constraining Ultra Light Dark Matter with the Galactic Nuclear Star Cluster

This chapter is based on Toguz et al. (2022)

2.1 Introduction

In this chapter, we consider whether the Milky Way's nuclear star cluster (NSC) can provide a new and competitive probe of ultra-light dark matter (ULDM) models. Due to it being only about 8 kpc away from us, the stellar kinematics of the central region of the Milky Way can be more precisely measured than for more distant dwarf galaxies ($d \sim 100$ kpc). Hence, the inner gravitational potential of the Milky Way can be derived from precise measurements of the stellar kinematics and density distribution of tracer stars in the centre of the Galaxy. Taking advantage of the recent precise measurement of the Milky Way's supermassive black hole (SMBH) mass, and the density profile of the NSC, in this chapter we study if a ULDM soliton core can be detected or rejected by the existing kinematical data for NSC stars, as measured by Fritz et al. (2016). Bar et al. (2019b) rejected $1 \times 10^{-20.0} < m_{DM} < 8 \times 10^{-19.0}$ eV from the stellar dynamics around Sgr A* (~ 0.3 pc) of the Milky Way. As mentioned in Section 1.6.2, the NSC stars dominate over other Milky Way stellar components within about 3 pc (Gallego-Cano et al., 2020), and we can assume that the stars within 3 pc belong to the NSC. Therefore, our study is expected to provide a stronger constraint compared to Bar et al. (2019b) by using the NSC stellar kinematics within about 3 pc. Through the use of Bayesian Statistics and Markov Chain Monte Carlo (MCMC) algorithm, we compare and fit the model with the observed mean line-of-sight velocity dispersion at different radial bins to constrain the ULDM soliton core with a mass range of about $10^{20.0} < m_{DM} < 10^{19.0}$ eV. This chapter is structured as follows. In Section 2.2, we describe the observational data and our fitting methodology. In Section 2.3, we describe our results. In Section 2.4, we use mock data to test the validity of our results. Finally, in Section 2.5 we present our conclusions. Throughout this chapter, we consider that dark matter consists of a single mass ULDM particle.

2.2 Method

To derive the total mass distribution in the NSC, we use a spherically symmetric and isotropic dynamical model. Because the NSC is dominant only within about 3 pc (Gallego-Cano et al., 2018), we focus on the mass distribution within 3 pc in this chapter. As mentioned in Section 1.6, the structure of the NSC is not a perfect sphere. However, in this chapter we consider that the NSC is nearly spherical, and can be approximated, therefore, by a spherical model (e.g. Read & Steger, 2017). Fritz et al. (2016) used the projected radial and tangential velocity dispersion from the proper motions of the NSC stars to show that the NSC is close to isotropic. Hence, we also assume the NSC stellar kinematics is isotropic. Using the spherical isotropic Jeans equation, we can derive the total mass of the Galactic centre as a function of the 3D radius, r, from the surface density profile and projected velocity dispersion profile of the stars within the NSC. Although Fritz et al. (2016) also provide the proper motions of the NSC stars, we use only the line-of-sight velocity dispersion because we assume an isotropic spherical model and the uncertainties of the line-of-sight velocities are clearly defined, while the uncertainties of the tangential velocities from the proper motions are difficult to be properly assess due to their dependence on the unknown distances. The components of the Galactic centre that affect the stellar kinematics are the SMBH, NSC and any central dark matter, including a soliton core if the correct dark matter model is ULDM. The total mass, $M_{tot}(< r)$, in the Galactic centre is given by $M_{tot}(< r) = M_{BH} + M_{NSC}(< r) + M_{DM}(< r)$[1].

We adopt the recently precisely measured mass of the SMBH of $M_{BH} = 4.261 \pm 0.012 \times 10^6 M_\odot$ (Gravity Collaboration et al., 2020) as a strong prior (see Section 2.2.5). Gravity Collaboration et al. (2020) noted that the systematic uncertainty is larger than this statistical uncertainty. In Section 2.4.2, we demonstrate that the results of this chapter do not change if the black hole mass is varied over this larger systematic uncertainty of about $0.06 \times 10^6 M_\odot$. The stellar mass of the NSC within r, $M_{NSC}(< r)$, can be computed from the observed stellar number density profile, fitting a constant stellar mass and number density ratio. Although a CDM halo (Navarro et al., 1997) provides a negligible mass contribution within the NSC ($< 0.1\%$), if the dark matter is ULDM, with a particle mass of around $10^{-20.0}$ eV, there should be a significant contribution of the soliton core of ULDM within the NSC. In the following subsections, we describe Jeans equation (Section 2.2.1), the velocity dispersion data of the NSC (Section 2.2.2), the stellar density profile of the NSC (Section 2.2.3), our ULDM model (Section 2.2.4), and our fitting methodology (Section 2.2.5).

2.2.1 Jeans Equation

For a steady state spherical stellar system that is isotropic, the Jeans equation is given by (e.g. Binney & Tremaine, 2008)

$$\frac{1}{n(r)}\frac{\partial(n(r)\sigma(r)^2)}{\partial r} = -\frac{GM_{tot}(< r)}{r^2}, \tag{2.1}$$

where $\sigma(r)$ is the velocity dispersion of stars in NSC, $n(r)$ is the 3D number density profile of NSC stars, and $M_{tot}(< r)$ is the enclosed total mass of the system within

[1] There is a circumnuclear gas disk within 3 pc, whose mass could be as large as $10^6 M_\odot$ (e.g. Christopher et al., 2005). Since the estimate of the gas mass is uncertain, and this mass is about 10 % of our derived total mass within 3 pc, we do not include the contribution of the gas component to the total potential. This simplification makes more room for the ULDM soliton core to contribute the total mass, which leads to more conservative bounds on the ULDM particle mass.

Figure 2.1: The distribution of stars whose line-of-sight velocities are measured in Fritz et al. (2016). The data are decomposed into 32 bins, with approximately 79 stars per bin.

r.

Integrating both sides of equation (2.1) gives a velocity dispersion profile of

$$\sigma(r) = \sqrt{\frac{1}{n(r)}\int_r^\infty \frac{GM_{tot}(<r)n(r)}{r^2}dr}. \tag{2.2}$$

Through an Abel transformation of equation (2.2), the line-of-sight velocity dispersion is described as (Binney & Mamon, 1982)

$$\frac{1}{2}\sigma^2_{LOS}(R)\Sigma(R) = \int_R^\infty \frac{n(r)\sigma^2(r)r}{\sqrt{r^2-R^2}}dr. \tag{2.3}$$

Then, the line-of-sight velocity dispersion is derived as

$$\sigma_{LOS}(R) = \sqrt{\frac{2}{\Sigma(R)} \int_R^{\infty} \frac{n(r)\sigma^2(r)r}{\sqrt{r^2 - R^2}} dr}, \tag{2.4}$$

where R is the projected 2D radius, and $\Sigma(R)$ is the projected NSC surface number density profile, which is given by

$$\Sigma(R) = 2 \int_R^{\infty} \frac{rn(r)}{\sqrt{r^2 - R^2}} dr. \tag{2.5}$$

2.2.2 Velocity Dispersion Data

We use the line-of-sight velocity data measured by Fritz et al. (2016) with the integral field spectrometer VLT/SINFONI. Fritz et al. (2016) obtained the line-of-sight velocities for 2,513 late-type giant stars within $R < 95''$ from Sgr A*. Note that in this chapter, we use the notation r for the 3D spherical radius, and R for the projected 2D radius from Sgr A*. The distribution of stars whose line-of-sight velocities are provided by Fritz et al. (2016) is shown in Galactic coordinates in Fig. 2.1. The stellar distribution appears to be clumpy, because this shows only stars whose line-of-sight velocity is observed, which depends on the telescope's field of view and the observational strategy. When analysing the Fritz et al. (2016) data, we assume 1 degree corresponds to 144 pc at the distance to the Galactic centre and the radial velocity of the Sun is 11.1 km s^{-1} (Schönrich et al., 2010). We use a KD-Tree decomposition to bin the data (Fig. 2.1), so that there are 32 bins, and each bin has about 79 stars. We found that this is a good compromise to maximise the number of bins, but minimise the Poisson noise in each bin.

For the sample of stars in each bin, the line-of-sight velocity dispersion is normally computed using the following formula

$$\sigma = \sqrt{< v_{LOS}^2 > - < v_{LOS} >^2}, \tag{2.6}$$

where v_{LOS} is the line-of-sight velocity of the star. Following Fritz et al. (2016), to take into account the contribution of the rotation approximately, we instead use

$$\sigma_{LOS} = \sqrt{<v_{LOS}^2>}, \tag{2.7}$$

i.e. ignoring $<v_{LOS}>^2$ in equation (2.6). This is based on the approximation often used as effective velocity dispersion in the kinematical analysis of the external galaxies (e.g. Gültekin et al., 2009). The effective velocity dispersion of galaxies consists of $\sigma_c^2 + V_{rot}^2$, where σ_c is the velocity dispersion of the central bulge of galaxy and V_{rot} is the rotational component of the bulge. We assume V_{rot}^2 being $<v_{LOS}>^2$ since $<v_{LOS}>$ corresponds to the projected rotation curve and therefore from equation (2.6), $<v_{LOS}^2> = \sigma^2 + <v_{LOS}>^2 = \sigma^2 + V_{rot}^2$, considering the kinetic energy being proportional to $\sigma^2 + <v_{LOS}>^2$ (Binney & Tremaine, 2008).

We find that the mean uncertainty of the velocity dispersion measurements from the observational errors of line-of-sight velocities is about 1.7 km s^{-1}, which is smaller than the mean Poisson error of about 8 km s^{-1}. For this reason, we assume that the error on each bin owes solely to the Poisson error. Following Fritz et al. (2016), we measure the Poisson error of the velocity dispersion with $\sigma_{LOS,err,i} = \sigma_{LOS,fit}(R_i)/\sqrt{2N_i}$, where N_i is the number of stars in i-th bin and R_i is the mean projected radius of the stars in i-th bin. $\sigma_{LOS,fit}(R)$ is the fitted 3rd order polynomial velocity dispersion profile. Because $\sigma_{LOS,err}$ changes depending on $\sigma_{LOS,fit}(R)$, we iteratively derive $\sigma_{LOS,err,i}$.

We compute the line-of-sight velocity dispersion and uncertainties as described above, which are plotted against the mean radius of the stars within each bin in Fig. 2.2. We fit these observed velocity dispersion with the model described in Section 2.2.1.

2.2.3 NSC Density profiles

Following Gallego-Cano et al. (2018), we describe the 3D density profile, $\rho_{NSC}(r)$, of the NSC with a 3D Nuker law (Lauer et al., 1995)

$$\rho_{NSC}(r) = \rho_{b,NSC} 2^{(\beta-\gamma)/\alpha} \left(\frac{r}{r_b}\right)^{-\gamma} \left[1 + \left(\frac{r}{r_b}\right)^{\alpha}\right]^{(\gamma-\beta)/\alpha}, \tag{2.8}$$

where r_b is the break radius, $\rho_{b,NSC} = \rho_{NSC}(r_b)$ is the mass density of the NSC at the break radius, γ and β are the exponent of the inner and outer power-law slope, respectively, and α describes the sharpness of the transition between the inner and outer power-law profiles. Gallego-Cano et al. (2018) fitted the NSC stellar distribution from the high-resolution near-infrared photometric data with the 2D projected density profile of equation (2.8). We rely on the precise measurement of the NSC density profile from Gallego-Cano et al. (2018), and when we fit the velocity dispersion, we fix the density profile parameters with their best fit profile.

Gallego-Cano et al. (2018) demonstrated that the NSC number density profile depends on the selection of the observational data, which indicates the systematic uncertainties of the measurements of the density profile of the NSC. We take one of the best fitting models from Gallego-Cano et al. (2018): $\alpha = 10$, $\beta = 3.4$, $\gamma = 1.29$ and $r_b = 4.3$ pc (ID10 of Table 5 in Gallego-Cano et al., 2018). This is the case that excludes contamination from pre-main sequence stars. We consider this to be most appropriate for our kinematic sample, since the kinematic data of Fritz et al. (2016) are for late-type giants. This model also leads to the smallest γ value, allowing for the maximal amount of dark matter within the NSC and, thereby, ensuring maximally conservative constraints on the ULDM mass. However, we tested also a value of $\gamma = 1.43$, taken from a different best-fitting model from Gallego-Cano et al. (2018), and find that our results are not sensitive to these choices.

Although the stellar number density profile is well observed by Gallego-Cano et al. (2018), we need to convert it to the mass density profile to obtain the NSC mass contribution to the gravitational potential in the Jeans equation. Because the mass to light ratio of the observed stars are uncertain, we adopt $\rho_{b,NSC}$ as a parameter when fitting the velocity dispersion profile, and marginalise over the mass scaling of the density profile. To take into account the observational uncertainty of the number density profile, we also take γ, which controls the profile in the radial range of our interest, as a fitting parameter with the prior of $\gamma = 1.29 \pm 0.05$ (Gallego-Cano et al., 2018).

2.2.4 Dark Matter Density profiles

Dark matter halos in ULDM are well described by a Navarro–Frenk–White (NFW Navarro et al., 1997) density profile at large radii, ρ_{NFW}, and a `soliton core' density profile, $\rho_{DM,s}$, at small radii (Schive et al., 2014b). The NFW profile is given by

$$\rho_{NFW}(r) = \frac{\rho_0}{\frac{r}{r_s}\left(1+\frac{r}{r_s}\right)^2}, \tag{2.9}$$

where ρ_0 is the characteristic density and r_s is the scale radius. The cumulative mass of the NFW profile is given by

$$\begin{aligned} M_{NFW}(<r) &= \int_0^r 4\pi r'^2 \rho_{NFW}(r')dr' \\ &= 4\pi\rho_0 r_s^3 \left[\ln\left(\frac{r_s+r}{r_s}\right) + \frac{r_s}{r_s+r} - 1\right]. \end{aligned} \tag{2.10}$$

Schive et al. (2014a) suggested that the density profile of the soliton core obeys the following equation (e.g. Safarzadeh & Spergel, 2020)

$$\rho_{DM,s}(r) = \frac{1.9\{10[m_{DM}/(10^{-22}\,\mathrm{eV})]\}^{-2} r_c^{-4}}{[1+9.1\times10^{-2}(r/r_c)^2]^8} 10^9\,M_\odot\,\mathrm{kpc}^{-3}, \tag{2.11}$$

$$r_c \approx 1.6[m_{DM}/(10^{-22}\,\mathrm{eV})]^{-1}\left(\frac{M_h}{10^9 M_\odot}\right)^{-1/3}\mathrm{kpc}, \tag{2.12}$$

where M_h is the virial mass of the halo (Schive et al., 2014b). These relations lead to a soliton core mass of

$$M_c \simeq \frac{1}{4} M_h^{1/3}(4.4\times10^7[m_{DM}/(10^{-22}\,\mathrm{eV})]^{-3/2})^{2/3}, \tag{2.13}$$

where $M_c \equiv M(<r_c)$ gives the central core mass (see also Safarzadeh & Spergel, 2020).

The total cumulative dark matter mass is, therefore, given by

$$M_{DM}(<r) = M_{NFW}(<r) + \int_0^r 4\pi r'^2 \rho_{DM,s}(r')dr', \tag{2.14}$$

where $r_{DM;s}$ is the soliton core density pro le of equation (2.11). We adopt a total mass of the Milky Way of $M_h = 1{:}4 \quad 10^{12}$ M , with $r_0 = 0{:}00854$ M pc ^{3} and $r_s = 19{:}6$ kpc, obtained from McMillan (2017). Once these parameters are xed, the only free parameter is m_{DM} which controls the shape of the soliton core. As mentioned above, the NFW pro le provides a negligible contribution to the total mass within 3 pc, and therefore our analysis is insensitive to r_0 or r_s. However, M_h contributes to the soliton core radius and therefore density pro le, and it scales as $r_{DM;s}$ µ $M_h^{4=3}$ within the core radius. Hence, a larger Milky Way mass produces a denser soliton core, and a larger mass range of the ULDM can, therefore, contribute to the mass within the NSC region – i.e. a larger mass range of the ULDM can be constrained by the NSC data. In fact, the total mass of the Milky Way is still in debate (e.g. Erkal et al., 2020). Recently, Vasiliev et al. (2021) claims that the virial mass of the Milky Way is as small as 9 10^{11} M . In Section 2.4.3, we show the results with $M_h = 9 \quad 10^{11}$ M , and demonstrate that our results are not sensitive to M_h as long as it is within the current expected range of M_h.

2.2.5 Fitting Methodology

We t the measured line-of-sight velocity dispersion data in Fig. 2.2 with equation (2.4) with our tting parameters of m_{DM}, $r_{b;NSC}$, g and m_{BH}. We include the SMBH mass of m_{BH} as a tting parameter, because the SMBH mass is dominant at radii r 1 pc. We use Bayesian statistics to obtain the marginalised probability distribution function for these parameters, $q_m = (m_{DM}; r_{b;NSC}; g; m_{BH})$

$$P(q_m jD) = L \ (Djq_m)P(q_m)=P(D); \tag{2.15}$$

where D is the data, i.e. the line-of sight velocity dispersion in different radial bins (Fig. 2.2), $P(q_m jD)$ is the posterior probability of the parameters q_m given data D, $L \ (Djq_m)$ is the likelihood, $P(q_m)$ is prior and $P(D)$ is the model evidence. Since it does not depend on q_m, and it is considered to be constant under a single model hypothesis, we ignore the model evidence.

To obtain $P(\theta_m|D)$, we run a MCMC fit, with a likelihood function given by

$$\mathcal{L}(D|\theta_m) = \prod_{i}^{N_D} \frac{1}{\sqrt{2\pi\sigma_{err,i}^2}} \exp\left(-\frac{(\sigma_m(R_i;\theta_m) - \sigma_{obs,i})^2}{2\sigma_{err,i}^2}\right), \tag{2.16}$$

where $\sigma_{obs,i}$ is the observed line-of-sight velocity dispersion data at R_i, $\sigma_{err,i}$ is the measurement error on each bin, N_D is the number of the data points, and $\sigma_m(R_i;\theta_m)$ is the model line-of-sight velocity dispersion at R_i (with parameters θ_m).

We use $\log(\rho_{b;NSC})$ and $\log(m_{DM})$ as our fitting parameters with flat priors of $3 < \log[\rho_{b;NSC}(M_\odot\ \mathrm{pc}^{-3})] < 7$ and $-23 < \log[m_{DM}(\mathrm{eV})] < -16$, since we find that the likelihood changes more smoothly in $\log(\rho_{b;NSC})$ and $\log(m_{DM})$. The range of $\log[\rho_{b;NSC}(M_\odot\ \mathrm{pc}^{-3})]$ is chosen as above, because outside of this range is unrealistic from the NSC photometric observations (e.g. Schödel et al., 2014). Since γ and m_{BH} are well-constrained by the other observations, as described above, we adopt Gaussian priors for these two parameters. The Gaussian prior for γ has a mean and dispersion of 1.29 and 0.05, respectively. The mean and dispersion for the Gaussian prior on m_{BH} are set to be $4.26 \times 10^6\ M_\odot$ and $0.012 \times 10^6\ M_\odot$, respectively.

As explained in Section 1.6, Fig. 1.7, the NSC could be affected by a ULDM soliton core with a mass range of about $10^{-20.0} < m_{DM} < 10^{-19.0}$ eV within the radial range of $0.1 < r < 3$ pc. In other words, the NSC kinematics in this radial range has the potential to constrain the existence of $10^{-20.0} < m_{DM} < 10^{-19.0}$ eV ULDM, as discussed in Bar et al. (2018). Fig. 1.7 also shows that the soliton core with $m_{DM} < 10^{-23.0}$ eV or $m_{DM} > 10^{-16.0}$ eV has negligible density within $0.1 < r < 3$ pc as compared to the expected NSC density. Hence, we consider that our prior range on $\log(m_{DM})$ is large enough to capture the region we hope to constrain.

We use emcee (Foreman-Mackey et al., 2013) for our MCMC sampler, with 32 walkers and 4000 steps per walker. We discard the first 1000 steps as our 'burn-in'. We confirm that after 1000 steps the MCMC results are stable.

Figure 2.2: The observed line-of-sight velocity dispersion profile data (black dots with error bars). Overplotted is the velocity dispersion profile from 100 randomly selected model parameters sampled by the MCMC (red lines).

2.3 Results

Fig. 2.2 shows our modelled line-of-sight velocity dispersion profiles (equation 2.4) for 100 random parameter values sampled from our MCMC steps, as compared to the observed velocity dispersion data. Notice that there is a good agreement between the sampled line-of-sight velocity dispersion profiles and the observational data.

Fig. 2.3 shows the marginalised posterior probability distribution of our fitting parameters of $\log(\rho_{b;NSC})$, $\log(\rho_{DM})$, γ and m_{BH}. Notice that γ and m_{BH} are well constrained. We compute the mean and standard deviation of the posterior probability distributions of these parameters and obtain the best-fitting parameter values and 1σ uncertainties of $\gamma = 1.28 \pm 0.04$ and $m_{BH} = (4.26 \pm 0.01) \times 10^6 M_\odot$. Our results show that the best-fitting values of γ and m_{BH} are consistent with our priors, i.e. the observed inner slope of the NSC measured by Gallego-Cano et al. (2018)

Figure 2.3: Marginalised posterior probability distribution of the model parameters of $\log(r_{b,NSC})$, $\log(m_{DM})$, g and m_{BH} obtained by MCMC fitting to the observed velocity dispersion from the line-of-sight velocity data in Fritz et al. (2016).

and the black hole mass measured by Gravity Collaboration et al. (2020).

Fig. 2.4 shows a close-up view of the marginalised probability distribution of $\log(m_{DM})$ with a histogram with a smaller bin size, where we can see two interesting results. First is the gap of the posterior probability distribution of $\log(m_{DM})$ around the range of $20.40 \lesssim \log[m_{DM}(\mathrm{eV})] \lesssim 18.50$, which is highlighted by the black vertical lines of $\log[m_{DM}(\mathrm{eV})] = 20.40$ and 18.50 in Fig. 2.4. This result indicates that the observational data reject the ULDM particle mass between about $10^{20.40}$ eV and $10^{18.50}$ eV.

Note that the upper and lower limits of $\log(m_{DM})$ in Fig. 2.4 come from the upper and lower limit of the flat prior we imposed. The roughly flat probability

Figure 2.4: Marginalised posterior probability distribution of the model parameter $\log(m_{DM})$ from Fig. 2.3, but with finer bins. The solid black lines demark $\log[m_{DM}(eV)] = -20.40$ and -18.50.

distributions at higher than about -18.50 and lower than about -21.0 mean that the observational data cannot distinguish the difference in the ULDM particle mass in these ranges. Fig. 2.5 shows the cumulative mass profiles of the NSC, dark matter and the total mass as a function of the Galactocentric 3D radius. For the NSC profile, we take $\log[\rho_{b;NSC}(M_\odot\,pc^{-3})] = 4.21$, which is the mean $\log(\rho_{b;NSC})$ of our MCMC samples with $\log[m_{DM}(eV)] > -18.0$ or $\log[m_{DM}(eV)] < -21.0$. This leads to a NSC mass within $r = 3$ pc of about $5.03 \times 10^6\,M_\odot$, which is larger than the value of about $3.65 \times 10^6\,M_\odot$ measured by Fritz et al. (2016) within 75 arcsec ($r \approx 3$ pc). This is likely due to different density profiles we are using. For example, Fritz et al. (2016) used a lower γ value of $\gamma = 0.81$. We tested our results with a Gaussian prior for γ with the mean value of 0.81 and we confirmed that the NSC mass within 3 pc reduced to $3.91 \times 10^6\,M_\odot$, which is similar to the measured value by Fritz et al. (2016).

Figure 2.5: The cumulative mass profile, $M_{tot}(< r)$, for the total (black solid line), NSC (blue solid line) and dark matter with a ULDM particle mass of $10^{-18.50}$ eV and $10^{-21.0}$ eV (orange solid and magenta dot-dashed lines, respectively). The solid vertical black line shows $r = 3$ pc. The NSC mass profile is computed with $\log[\rho_{b,NSC}(M_\odot\ \mathrm{pc}^{-3})] = 4.21$. The total mass is computed for the case of the ULDM mass of $m_{DM} = 10^{-21.0}$ eV, including the SMBH.

Fig. 2.5 also shows the cumulative mass profile of the ULDM with $m_{DM} = 10^{-21.0}$ eV and $m_{DM} = 10^{-18.50}$ eV, where both cumulative masses reach about $4.4 \times 10^5\ M_\odot$ at 3 pc. These two ULDM soliton cores are much smaller than both the NSC mass within the same radius and the SMBH mass. Because the size of the soliton core increases with decreasing particle mass of the ULDM (equation 2.12), the soliton core mass within $r < 3$ pc decreases with the decreasing ULDM particle mass. Consequently, the ULDM particles mass with $m_{DM} < 10^{-21.0}$ eV does not affect the velocity dispersion of the NSC. This explains the equally accepted probability distribution of $m_{DM} < 10^{-21.0}$ eV in Fig. 2.4. On the other hand, the ULDM particles mass with $m_{DM} > 10^{-18.50}$ eV leads to too small of a soliton core to affect the stellar dynamics in the central region. This explains the equally accepted

Figure 2.6: The cumulative mass pro le, $M_{tot}(< r)$, for the total (black), NSC (blue) and dark matter mass with the particle mass of $10^{20:50}$ eV (orange). The solid vertical black line shows $r = 3$ pc. The total mass is computed for the case of the ULDM mass of $m_{DM} = 10^{20:50}$ eV, including the SMBH.

Table 2.1: Model parameters of the mock data.

Model name	$\log[m_{DM}(eV)]$	$\log[r_{b;NSC}(M\ pc^{3})]$	g	m_{BH}
A	$20:50$	$3:60$	1.29	4.26
B	$19:50$	$4:50$	1.29	4.26
C	$23:0$	$4:21$	1.29	4.26

probability distribution at $m_{DM} > 10^{18:50}$ eV. Hence, if the ULDM particle mass is larger than $m_{DM} = 10^{18:50}$ eV or smaller than $m_{DM} = 10^{21:0}$ eV, our current data of the NSC stellar dynamics cannot nd or reject their existence.

The second striking result of Fig. 2.4 is the peak around $\log[m_{DM}(eV)] = 20:50$. At rst sight, this appears to statistically favour a soliton core due to ULDM with a mass of $m_{DM} = 10^{20:50}$ eV. Fig. 2.6 shows the cumulative mass pro le for the total mass, dark matter halo mass including the soliton core with $m_{DM} = 10^{20:50}$ eV and the NSC mass with $\log[r_{b;NSC}(M\ pc^{3})] = 3:60$, which

Figure 2.7: Upper panel: Observed line-of-sight velocity dispersion as a function of the projected radius (black dots with error bars). Orange solid/red dot-dashed/yellow dotted/blue dashed line indicates the velocity dispersion profile expected from the combination of the soliton core with $m_{DM} = 10^{20.50}$ eV, NSC and SMBH/NSC and SMBH/SMBH only/NSC only. NSC contribution is computed with $\log[r_{b;NSC}(M\ \mathrm{pc}^{3})] = 3.60$. Lower panel: Same as the upper panel, but the soliton core with $m_{DM} = 10^{19.50}$ eV and $\log[r_{b;NSC}(M\ \mathrm{pc}^{3})] = 4.21$ are used for the soliton core and NSC contributions.

Figure 2.8: The mock line-of-sight velocity dispersion profile data of model A (black dots with error bars) overplotted with the velocity dispersion profile from 100 randomly selected model sampled by the MCMC (red lines) and the true velocity dispersion profile (blue line).

is the mean of $\log(r_{b;NSC})$ of the MCMC sample with $-21.0 < \log[m_{DM}(\mathrm{eV})] < -20.40$. Fig. 2.6 shows that the NSC mass is smaller than that in Fig. 2.5, and the $m_{DM} = 10^{-20.50}$ eV soliton core has a suitable size to compensate the deficit of the mass within $r < 3$ pc. The upper panel of Fig. 2.7 also shows that the additional mass from the $m_{DM} = 10^{-20.50}$ eV soliton core helps to increase the velocity dispersion at an outer radius ($r \gtrsim 0.5$ pc) to match with the observational data more than the expected velocity dispersion from the NSC and SMBH only.

Consequently, the NSC mass within 3 pc is about $2.51 \times 10^6\ M_\odot$, which is significantly smaller than the aforementioned NSC mass measured by Fritz et al. (2016). The cumulative mass of the NSC in Fig. 2.6 is also much smaller than the NSC mass of $(2.1 \pm 0.7) \times 10^7\ M_\odot$ within about 8.4 pc, as measured in Feldmeier-Krause et al. (2017b). Although these studies use dynamical models that assume that the NSC is the dominant source of the central gravitational potential, the pho-

Figure 2.9: Marginalised posterior probability distribution of the model parameters of $\log(r_{b;NSC})$, $\log(m_{DM})$, g and m_{BH} obtained by the MCMC fit to the velocity dispersion data of model A. The cyan line with cyan solid square shows the true values of the parameters.

tometric observations of Schödel et al. (2014) also suggested a total NSC mass of $(2.5 \pm 0.4) \times 10^7\,M_\odot$, assuming a constant mass to light ratio. Hence, it is unlikely that the NSC mass is as small as the case of Fig. 2.6. Thus, the peak of $m_{DM} = 10^{-20.50}$ eV is not likely to be a viable solution. Still, it is difficult to measure the mass to light ratio precisely, and there could be some systematic biases in these previous measurements. Also, it is unlikely that the peak at $m_{DM} = 10^{-20.50}$ eV is due to the artificially trapped MCMC walkers close to the gap of the posterior. This is because when we observe the m_{DM} values of the walkers as a function of steps, we see some of the walkers occasionally cross the gap and move to the other side of the gap. Also, if the posterior probability is flat at

$m_{DM} < 10^{20.40}$ eV, we should see the flat posterior probability as seen between the lowest limit of $m_{DM} = 10^{23.0}$ eV and around $m_{DM} = 10^{21.50}$ eV. Hence, higher probability at $m_{DM} = 10^{20.50}$ eV than any other values at $m_{DM} < 10^{20.40}$ eV should be real. Hence, we consider that we cannot (yet) reject the existence of the $m_{DM} = 10^{20.50}$ eV soliton core.

The constraining power of the observed velocity dispersion data to reject the ULDM mass between about $10^{20.40}$ eV and $10^{18.50}$ eV in Fig. 2.4 can be demonstrated in the lower panel of Fig. 2.7. The lower panel of Fig. 2.7 shows that the velocity dispersion profile expected from the $m_{DM} = 10^{19.50}$ eV soliton core and SMBH even without NSC (orange line) is systematically higher than the observational data within $r = 1$ pc. Hence, the data can reject the soliton core with the ULDM mass around $10^{19.50}$ eV. On the other hand, the velocity dispersion profile expected from the SMBH and NSC with $\log[_{b;SMC}(M \ \mathrm{pc}^{3})] = 4.21$, i.e. without any soliton core (red doted-dashed line), agrees well with the observational data. Hence, NSC and SMBH are enough to describe the observed stellar kinematics.

2.4 Discussion

2.4.1 Mock Data Validation

In Section 2.3, we found a gap in the probability distribution function of ULDM masses that rejects a ULDM particle in the mass range $20.40 \lesssim \log[m_{DM}(\mathrm{eV})] \lesssim 18.50$. We also found a peak in the probability distribution around $\log[m_{DM}(\mathrm{eV})] = 20.50$ that we argued owed to a degeneracy between $r_{b;NSC}$ and m_{DM}.

To test the validity of above results, we construct mock velocity dispersion data similar to the observational data, using the same model as in Section 2.2. We then fit the data as in Section 2.3. We adopt the same parameters for the SMBH, NSC and dark matter model as in Section 2.2.

We construct three different models with different values of $r_{b;NSC}$ and m_{DM}, as shown in Table 2.1. We then generate the mock velocity dispersion profile data for each model by solving equation (2.4) for 32 bins spaced out in exactly the same

Figure 2.10: Marginalised posterior probability distribution of the model parameter $\log(m_{DM})$ for model A from Fig. 2.9, but with finer bins. The solid black lines demark the range $\log[m_{DM}(\mathrm{eV})] = -20.40$ and -18.50.

way as for the observational data. We then add a random displacement to the velocity dispersion of each bin, within the measurement error of each bin, taken to be the same as for the observational data.

We use the same fitting methodology with the same priors, as described in Section 2.2.5, except that now the observational data are replaced by mock data for three models, labelled A, B and C (Table 2.1).

Model A employs $m_{DM} = 10^{-20.50}$ eV and $\log[\rho_{b;NSC}(M_\odot\ \mathrm{pc}^{-3})] = 3.60$, which is the mean value of our MCMC samples around $m_{DM} = 10^{-20.50}$ eV found in Section 2.3. This model is to test if the probability distribution of $\log[m_{DM}(\mathrm{eV})]$ would be similar to what is obtained in Fig. 2.4, when a soliton core of the $m_{DM} = 10^{-20.50}$ eV UDLM exists.

Fig. 2.8 overplots the model line-of-sight velocity dispersion profiles from the 100 random parameter values sampled from the results of MCMC with the mock velocity dispersion data for model A. Fig. 2.8 shows that there is a good agreement between the sampled line-of-sight velocity dispersion profiles and the mock data roughly within the uncertainties of the mock data. Fig. 2.9 shows the marginalised posterior probability distribution of our fitting parameters of $\log(\rho_{b;NSC})$, $\log(m_{DM})$, g and m_{BH} for model A with the cyan line with the cyan solid square representing the true values of the parameters.

The obtained best-fitting parameter values and 1σ uncertainties are $g = 1.29 \pm 0.05$ and $m_{BH} = (4.26 \pm 0.01) \times 10^6\ M_\odot$, which are consistent with the true values within our 1σ uncertainty regions. Just like the results in Section 2.3, there is a degeneracy between $\log(\rho_{b;NSC})$ and $\log(m_{DM})$. In the probability distribution between $\log(\rho_{b;NSC})$ and $\log(m_{DM})$, when $\log[m_{DM}(\mathrm{eV})]$ is around the true value of -20.50, $\log(\rho_{b;NSC})$ corresponds to $\log[\rho_{b;NSC}(M_\odot\ \mathrm{pc}^{-3})] = 3.74 \pm 0.37$, which is within one sigma of the true value of $\log[\rho_{b;NSC}(M_\odot\ \mathrm{pc}^{-3})] = 3.60$.

The close-up plot of the marginalised probability distribution of $\log(m_{DM})$ is shown in Fig. 2.10, and there is a similar peak around about $10^{-20.50}$ eV when compared to Fig. 2.4. Also, Fig. 2.10 shows the gap between $-20.40 \lesssim \log[m_{DM}\ (\mathrm{eV})] \lesssim -18.50$, and roughly flat probability distribution at $\log[m_{DM}\ (\mathrm{eV})] < -21.0$ and $\log[m_{DM}\ (\mathrm{eV})] > -18.50$, as seen in Fig. 2.10. This implies that the result in Section 2.3 is consistent with the expected result when there is a soliton core with ULDM particle mass around $10^{-20.50}$ eV.

Model B adopts $\log[\rho_{b;NSC}(M_\odot\ \mathrm{pc}^{-3})] = 4.50$ and $m_{DM} = 10^{-19.50}$ eV, to see if the data are capable of detecting a soliton core with $m_{DM} = 10^{-19.50}$ eV. If it is confirmed, we can be confident that the gap we obtained in Fig. 2.4 in Section 2.3 is not due to an artificial feature, but rather it is meaningful to reject the existence of a soliton core over this mass range. The choice of this higher $\log[\rho_{b;NSC}(M_\odot\ \mathrm{pc}^{-3})]$ compared to models A and C is to make the NSC more gravitationally dominant, i.e. to make it more challenging to recover the soliton core contribution.

Although not shown for brevity, we confirm that there is a good agreement between the sampled line-of-sight velocity dispersion profiles and the mock data of model B within the uncertainties of the mock data. Fig. 2.11 shows the

Figure 2.11: Marginalised posterior probability distribution of the model parameters of $\log(\rho_{b,NSC})$, $\log(m_{DM})$, γ and m_{BH} obtained by the MCMC fitting to the velocity dispersion data of model B. The cyan line with cyan solid square shows the true values of the parameters.

marginalised posterior probability distribution of our fitting parameters for model B with the cyan line with the cyan solid square representing the true values of the parameters. The best fitting values and the respective uncertainties of the parameters are $\log[\rho_{b,NSC}(M_\odot\ pc^{-3})] = 4.56 \pm 0.07$, $\log[m_{DM}(eV)] = -19.51 \pm 1.09$, $\gamma = 1.30 \pm 0.05$ and $m_{BH} = (4.26 \pm 0.01) \times 10^6\ M_\odot$, which are consistent with the true value within our 1σ uncertainty regions. This demonstrates that our MCMC fitting can recover the true parameter values well, especially the ULDM particle mass, which is the main focus of this chapter. This means that the current observational data are good enough to identify a soliton core of $m_{DM} = 10^{-19.50}$ eV, if it exists.

Figure 2.12: Marginalised posterior probability distribution of the model parameters of $\log(\rho_{b;NSC})$, $\log(m_{DM})$, g and m_{BH} obtained by the MCMC fit to the velocity dispersion data of model C. The cyan line with cyan solid square shows the true values of the parameters.

Model C employs $m_{DM} = 10^{-23.0}$ eV. As we discussed in Section 2.3, this particle mass of ULDM produces a negligible soliton core mass compared to the SMBH and NSC mass (see also Fig. 1.7), i.e. mimicking the case of no detectable soliton core. Hence, this model is designed to test what our MCMC fitting results will look like if there is no soliton core. Model C adopts $\log[\rho_{b;NSC}(M_\odot\ pc^{-3})] = 4.21$, which is found to be the best fitting parameter in Section 2.3, when the soliton core is negligible.

Although not shown for brevity, we confirm that there is a good agreement between the sampled line-of-sight velocity dispersion profiles and the mock obser-

Figure 2.13: Marginalised posterior probability distribution of the model parameter $\log(m_{DM})$ for model C from Fig. 2.12, but with finer bins. The solid black lines demark $\log[m_{DM}(\mathrm{eV})] = 20.40$ and 18.50.

vational data for model C. Fig. 2.12 shows the marginalised posterior probability distribution of our fitting parameters for model C with the cyan line with the cyan solid square representing the true values of the parameters. Except for $\log(m_{DM})$ (that is now expected to be challenging to detect), the true parameter values are well recovered.

Contrary to our MCMC results for the observational data (Fig. 2.3), the probability distribution of $\log(m_{DM})$ does not show a clear degeneracy with $\log({}_{b,NSC})$. The close-up view of the marginalised probability distribution of $\log(m_{DM})$ is shown in Fig. 2.13. Similar to model A, Fig. 2.13 shows a clear gap between about $\log[m_{DM}(\mathrm{eV})] = 20.40$ and 18.50, unlike model B that has a soliton core with $m_{DM} = 10^{19.50}$ eV. Hence, we can confidently conclude that the gap can be used to reject a soliton core with ULDM particle mass in the range between $m_{DM} = 10^{20.40}$ eV and $10^{18.50}$ eV. On the other hand, comparing with model A (Fig. 2.10), there is no clear peak of the probability distribution around $\log[m_{DM}(\mathrm{eV})] = -20.50$ in model C. This means that the $10^{-20.50}$ eV ULDM particle mass is equally possible to be $m_{DM} < 10^{-21.0}$ eV or $m_{DM} > 10^{-18.50}$ eV. In other words, the current quality of the data cannot identify or reject the ULDM particle mass outside of the gap, i.e. $m_{DM} < 10^{-20.40}$ eV or $m_{DM} > 10^{-18.50}$ eV, including $10^{-20.50}$ eV.

Interestingly, the fact that the result for the observational data (Fig. 2.4) has a clear peak around $\log[m_{DM}(\mathrm{eV})] = -20.50$ indicates two potential scenarios: there is a soliton core with $m_{DM} = 10^{-20.50}$ eV, or there is an extra mass contribution, compared to the pure NSC model of model C, to mimic the $m_{DM} = 10^{-20.50}$ eV soliton core. The extra mass contribution could be the nuclear stellar disk (NSD) because the mass of the NSD might become significant around 3 pc (Gallego-Cano et al., 2018). It is noteworthy that the additional mass needed are unlikely to be attributed to the black holes falling within 3 pc due to dynamical friction. The mass contributed by such clusters of black holes is insufficient to invalidate the necessity of the NSD.

2.4.2 Systematic uncertainty of the black hole mass

There is a strong correlation between the distance to the Galactic centre, R_0, and m_{BH} measurements by Gravity Collaboration et al. (2020), as shown in their Fig. E2. Gravity Collaboration et al. (2020) estimated that there is a systematic uncertainty of 45 pc for R_0, which propagates to a larger systematic uncertainty on the SMBH mass than the uncertainty considered in this chapter. We test the effect of this relatively large systematic uncertainty by considering two cases. The first case takes a distance to the Galactic centre of $R_0 = 8.20$ kpc, which is systematically shorter than our fiducial assumed distance. By fitting the correlation between R_0 and m_{BH} by eye from Fig. E2 of Gravity Collaboration et al. (2020), this corresponds to a SMBH mass of $m_{BH} = 4.20 \times 10^6 M_\odot$. The different R_0 also affects the conversion of arcsec to pc, and we adjust the project radial distance of the stars from Sgr A and the break radius of the NSC density profile. The second case applies a larger distance to the Galactic centre of $R_0 = 8.29$ kpc. This leads to $m_{BH} = 4.32 \times 10^6 M_\odot$.

Figure 2.14: Marginalised posterior probability distribution of the model parameter $\log(m_{DM})$ for lower black hole mass case. The marginalised posterior probability distribution is divided in to 250 bins. Solid black line indicates $\log[m_{DM}(\mathrm{eV})] = 20{:}40$ and $18{:}50$.

Figs. 2.14 and 2.15 show the marginalised probability distribution of $\log(m_{DM})$ for the former and latter cases, respectively, after fitting the data with the same method as in Section 2.2. These results show almost identical results to Fig. 2.4. This confirms that the systematic uncertainty on R_0 and m_{BH} in Gravity Collaboration et al. (2020) is still small enough that it does not affect our conclusions.

2.4.3 The lower Milky Way mass case

Vasiliev et al. (2021) recently suggested that the Milky Way's virial mass is as small as $9 \quad 10^{11}$ M . Fig. 2.16 shows the marginalised probability distribution of $\log(m_{DM})$ obtained by the MCMC fitting to the observed velocity dispersion with adapting $M_h = 9 \quad 10^{11}$ M . The result is similar to our fiducial result of Fig. 2.4 with $M_h = 1{:}4 \quad 10^{12}$ M , which is rather high side of the current estimates of the Milky Way mass. This demonstrates that our result is not sensitive to the assumed

Figure 2.15: Marginalised posterior probability distribution of the model parameter $\log(m_{DM})$ for higher black hole mass case. The marginalised posterior probability distribution is divided in to 250 bins. Solid black line indicates $\log[m_{DM}(eV)] = -20.40$ and -18.50.

M_h value within the current expected range of M_h of the Milky Way.

2.5 Conclusions

We test the existence of a soliton core due to ULDM in the centre of the Milky Way by fitting the line-of-sight velocity dispersion data of the NSC stars, taken from Fritz et al. (2016). We assume a spherical isotropic Jeans model, using strong priors on the accurately measured NSC stellar number density profile and the mass of the SMBH. We fit the NSC density, $\rho_{b,NSC}$, ULDM particle mass, m_{DM}, the inner slope of the NSC density profile, γ, and the SMBH mass, m_{BH}. The resultant marginalised probability distribution function of m_{DM} shows a peak around about $10^{-20.50}$ eV and a gap between about $10^{-20.40}$ eV and $10^{-18.50}$ eV, rejecting ULDM over this mass range. We show that this result is insensitive to our model assumptions and priors (see Sections 2.4.2 and 2.4.3). We also construct mock velocity dispersion data with

Figure 2.16: Marginalised posterior probability distribution of the model parameter $\log(m_{DM})$ for the MCMC fitting result with a lower Milky Way mass of $M_h = 9 \times 10^{11}\,M_\odot$, taken from Vasiliev et al. (2021). Solid black line indicates $\log[m_{DM}(\mathrm{eV})] = -20.40$ and -18.50.

the same radial bins and uncertainties as the observational data with different m_{DM}, further validating our observational constraints.

The $10^{-20.50}$ eV soliton core is unlikely to be a feasible solution because it results in an NSC mass significantly smaller than what has been observed in previous studies. It is also possible that the gravitational potential of $10^{-20.50}$ eV soliton core could be a representation of another component within the centre of our Galaxy, namely, the NSD. Because of this, in order to verify the existence of the $10^{-20.50}$ eV soliton core and to explore a broader range of ULDM masses, Chapter 3 and 4 detail our approach of utilizing the NSD to impose constraints on the ULDM particle mass.

Chapter 3

Observational Data for the Nuclear Stellar Disk

3.1 Introduction

As discussed at the end of Chapter 2, to constrain the ultra-light dark matter (ULDM) mass less than $10^{20:0}$ eV from the Milky Way data, we need to look at the structure larger than the nuclear star cluster (NSC). As summarised in Section 1.7, there is the nuclear stellar disk (NSD) in the central region of the Milky Way, and the NSD is the dominated stellar system in the radial range from 30 pc to 300 pc. Both photometric and spectroscopic observations indicate that the NSD is a kinematically relaxed (close to) axisymmetric stellar system (e.g. Launhardt et al., 2002; Nishiyama et al., 2013; Schönrich et al., 2015; Gallego-Cano et al., 2020). Hence, it is a good stellar system to study the gravitational potential within a few 100 pc of the Milky Way (e.g. Sormani et al., 2020a, 2022). This is similar size to the expected soliton core sizes if ULDM consists of the particle mass of less than $10^{20:0}$ eV. Therefore, in this and next chapters we study how the dynamics of the NSD constrain the existence of the ULDM in these mass range.

3.2 Observational Data

3.2.1 VIRAC2

The VISTA Variables in the Vía Láctea (VVV) Survey (Minniti et al., 2010) uses the Visible and Infrared Survey Telescope for Astronomy (VISTA) which is a reflecting telescope that is located at the European Southern Observatory's Paranal Observatory in Chile. VISTA has a primary mirror with a diameter of 4.1 meters. It covers a wavelength range of about $0.8\,\mu m$ to $2.3\,\mu m$ for infrared which includes five near-infrared bands Z, Y, J, H, and Ks. The VVV survey observations started in 2010 with the aims of studying the structure of the Milky Way Galaxy and to study the stellar populations of the Milky Way bulge. It has covered about over 500 deg^2 along the Galactic plane and observable from the Southern hemisphere.

The VVV Infrared Astrometric Catalogue (VIRAC1) is a proper motion catalogue of the VVV survery constructed by Smith et al. (2018). Smith et al. (2018) measured the proper motion of stars in the Milky Way by epoch astrometry for Ks band, where the positions of stars in the VVV survey measured over the span of 5 years from 2010 to 2015 were compared and proper motion of the stars were able to be measured. VIRAC2 is an updated version of VIRAC1. It is more precise and covers larger area, including the additional epoch of the VISTA Variables in the Vía Láctea Extended (VVVX) survey which are taken since 2016. The astrometric accuracy is improved by using the absolute reference frame provided by the *Gaia* mission (Sanders et al., 2019). VIRAC2 catalogue within $|l| < 1.5^\circ$ and $|b| < 1.5^\circ$ is provided by Jason L. Sanders and Leigh Smith.

We use the VIRAC2 proper motion data for 879,028 stars in the asymptotic giant branch bump (AGBb). AGB stars are with low to intermediate mass stars where these stars finished burning helium in the core and started to cool and expand as luminosity increases in which the path of these stars almost align with the red-giant branch in the Hertzsprung Russell diagram, hence why these stars are called asymptotic giant branch stars (Siess, 2006). AGBb stars are a particular feature in the colour magnitude diagram of stars in the AGB phase, and it is a state for some of AGB stars just before a sudden increase in luminosity due to rate of helium burning in the shell around the core increasing. Furthermore, the luminosity of the AGBb stars is almost constant with a weak relationship with its metallicity, suggesting that the AGBb stars can be treated as a standard candle (Pulone, 1992). It is also found more recently that the calibrated metallicity dependence makes it satisfactory to be a standard candle (Dau et al., 2022). AGBb is a good stellar population to select the stars at similar distance to study their stellar distribution and kinematics.

We select AGBb stars from VIRAC2 catalogue by selecting stars within $10.8 < K_s - 1.33(H - K_s) < 11.8$. Additionally, stars within $H - K < 0.3$ are ignored, to remove the foreground stars. Fig. 3.1 shows the distribution of the colour and magnitude of VIRAC1 stars within $|l| < 1.5$ and $|b| < 1.5$ from Smith et al. (2018), overplotting the VIRAC2 stars selected with the above color and magnitude criteria. VIRAC2 data are not publicly available yet, and we received the VIRAC2 data for the AGBb stars selected by the above criteria from J. Sanders. Hence, in Fig. 3.1, we use VIRAC1 data, which is publicly available, to show the colour magnitude distribution of the other populations of the VVV survey data. There is a clear sequence of higher number of stars from $K_s \approx 12$ mag and $H - K_s \approx 0.5$ mag to $K_s \approx 15$ mag and $H - K_s \approx 2.0$ mag as highlighted by the region enclosed by the red lines. This is the sequence of AGBb. These stars are likely to be located at the similar distance to the distance to the Galactic centre. However, their colours and apparent magnitudes are different because of the dust extinction, which makes the stars fainter and redder. The difference in extinction makes this sequence of the colour and magnitude of the AGBb. The high number density of the stars in this region expected by the AGBb stars at the distance to the Galactice centre means that the stars in this region is dominated by the AGBb stars in the Galactic centre. We assume that the majority of the selected stars in this region are at the distance of the NSD, and we will use these stars to analyse the stellar density distribution in Section 3.3 and the velocity dispersion distribution in Section 3.4.

Fig. 3.1 also shows the sequence of red clump stars, which are enclosed by green lines. Red clump stars are stars that have exhausted hydrogen in their cores and now burning helium. Red clump stars have a similar intrinsic luminosity irre-

Figure 3.1: Colour-magnitude diagram for stars from VIRAC1 and VIRAC2 (only in the region enclosed by red lines). Red open box highlights our selection of the NSD AGBb star candidates from VIRAC2. Green open area: highlights the sequence of red clump stars expected in the NSD. Red are the areas with the highest number density, blue and black are the lowest number density.

spective of their age or metallicity. We are not using the red clump stars as they are less luminous and therefore we may be missing the stars suffering from higher extinction, where the NSD is more dominant.

3.2.2 APOGEE DR17

The Sloan Digital Sky Survey (SDSS, York et al., 2000) is an astronomical survey that aims to create a detailed 3D map of the Universe by describing the distribution of luminous and non-luminous matter, for a better understanding of dark matter. SDSS started with a 2.5-meter telescope at Apache Point Observatory (APO) in New Mexico, USA. The operation of SDSS first started in 2000 and has evolved through 5 stages, with currently the SDSS-V being most recent one (Kollmeier et al., 2019). The SDSS-V aims to provide optical and infra-red spectra for over 6 million objects. It is currently an all-sky, multi-epoch spectroscopic survey. The survey started its observation from 2020 and expects to finish after five years.

Schönrich et al. (2015) used SDSS-III Apache Point Galactic Evolution Experiment (APOGEE) data which is a near-infrared spectroscopic survey that measures the line-of-sight velocity of stars. From this, Schönrich et al. (2015) computed the average line-of-sight velocity at both positive and negative longitude in $|l| < 1$ deg and latitude $|b| < 1$ deg. They found that the stars with higher K-band extinction, $A_K > 3$, show a clear rotation like features, i.e. the stars are moving away from us in the positive longitude, while the stars are moving toward us in the negative longitude after correcting the solar motion. This means that the stars with $A_K > 3$ are dominated by the stars in the NSD, and the NSD has a clear rotation velocity of about 120 km s^{-1} (Schönrich et al., 2015).

This rotation of the NSD is important information to construct the dynamical model of the NSD and infer the total enclosed mass as a function of radius from the Galactic centre. We use the SDSS-IV APOGEE Data Release 17 (APOGEE DR17) (Abdurro'uf et al., 2022; Majewski et al., 2017) data to analyse the line-of-sight velocity distribution of the NSD stars to derive the mean line-of-sight velocity as a function of the Galacto-centric radius, following Schönrich et al. (2015) and Sormani et al. (2020b). We select bright giant stars likely in the NSD with the criteria where the stars are within $H - K > -0.0233K + 1.63$ (Nogueras-Lara et al., 2020) and VSCATTER < 30 km s^{-1}, to eliminate binary stars or spurious data. This colour selection ensures that the selected stars are deeply enshrouded by the dust and has a redder colour expected for the stars in the NSD, like Schönrich et al. (2015) chose the stars with high K-band extinction. We consider that the selected stars are dominated by the NSD stars and analyse the line-of-sight velocity as a function the radius in Section 3.6.

3.2.3 Previous Dynamical Modelling of the NSD

Sormani et al. (2020b) modelled the NSD using Jeans dynamical models to fit the line-of-sight velocity data of APOGEE and SiO maser stars with several 3D stellar density distribution obtained in the previous studies which includes Launhardt et al. (2002); Chatzopoulos et al. (2015); Gallego-Cano et al. (2020). As a result, they measured the mass of the NSD to be $M(r < 100\,\mathrm{pc}) = (4 \pm 1) \times 10^8\,M_\odot$ which is consistent with Launhardt et al. (2002). They also found that the NSD seems to have a vertically biased velocity structure with $\sigma_z/\sigma_R > 1$, where σ_z and σ_R are vertical and radial velocity dispersions, respectively.

Sormani et al. (2022) used the distribution function modelling which overcomes the limitations of the Jeans equations, to model an axisymmetric self-consistent NSD. Sormani et al. (2022) also modelled the Galactic bar to take into account the contamination of the stars in the Galactic bar. By fitting the models to the line-of-sight velocities from the Very Large Telescope (VLT) K-band Multi Object Spectrograph (KMOS) survey of the NSD stars (Fritz et al., 2021) and VIRAC2 proper motions, they found that the NSD mass is about $M = 10.5 \times 10^8\,M_\odot$. Similarly to Sormani et al. (2020b), Sormani et al. (2022) also showed that the NSD velocity dispersion is vertically biased.

In the following chapters, we model the NSD by following the footsteps of Sormani et al. (2022). However, we introduce the ULDM soliton core to their distribution function model. Then we fit the surface stellar density distribution and the velocity dispersion distribution from the VIRAC2 data, and line-of-sight velocity data of APOGEE DR17 to constrain the ULDM particle mass. It is important to note that Sormani et al. (2022) did not fit the density distribution explicitly. However, we fit the stellar density and kinematic distribution simultaneously. In the rest of this chapter, we describe the parameters we assume (Section 3.3), how we measure the surface density distribution (Section 3.4), the velocity dispersion distribution (Section 3.5), and the mean rotation profile (Section 3.6) of the NSD to compare with our distribution function dynamical model in the next chapter.

3.3 Assumed Parameters

In this chapter, we assume the distance from us to the centre of our Galaxy to be $R_0 = 8.275\,\mathrm{kpc}$ (GRAVITY Collaboration et al., 2021). The Sgr A* is not at the centre of the Galactic coordinate $(l, b) = (0, 0)^\circ$. Rather it is at $(l_{\mathrm{SgrA*}}, b_{\mathrm{SgrA*}}) = (-0.05576432, -0.04616002)^\circ$. We consider that Sgr A* is located at the centre of the Galaxy. To redefine the new coordinate to set Sgr A* $(l^*, b^*) = (0, 0)°$, we subtract the position of the stars in the Galactic coordinate by the position of the Sgr A* as follows $(l^*, b^*) = (l - l_{\mathrm{SgrA}^*}, b - b_{\mathrm{SgrA}^*})$.

The motion of the stars obtained in the VIRAC2 proper motion data (Section 3.2.1) and the APOGEE DR17 line-of-sight velocity (Section 3.2.2) are the velocity of the stars with respect to the Sun's motion. To obtain their velocity in the Galactic rest-frame, which is required to compare with a dynamical model of the NSD, we need to subtract the Sun's motion. We first consider that the apparent motion of the Sgr A* is purely due to the Sun's motion with respect to the supermassive black hole (SMBH) fixed at the centre of the Galaxy. The SMBH has an apparent motion of about $(\mu_{l,\mathrm{SgrA}^*}, \mu_{b,\mathrm{SgrA}^*}) = (-6.411, -0.291)$ mas yr^{-1}, where μ_{l,SgrA^*} and μ_{b,SgrA^*} are the apparent motion in the longtitude and latitude direction for the Sgr A*, respectively (Reid & Brunthaler, 2020). We then subtract the proper motion of stars along the longitude, $\mu_{l,\mathrm{star}}$, and latitude, $\mu_{b,\mathrm{star}}$, by the apparent motion of the Sgr A* as follows

$$(\mu_{l^*}, \mu_{b^*}) = (\mu_{l,\mathrm{star}}, \mu_{b,\mathrm{star}}) - (\mu_{l,\mathrm{SgrA}^*}, \mu_{b,\mathrm{SgrA}^*}). \tag{3.1}$$

In terms of line-of-sight velocity, we need to take into account the Sun's motion toward the Galactic centre. We consider that the Sun is moving toward the Galactic centre by 11.1 km s^{-1} (Schönrich et al., 2010). To obtain the line-of-sight velocity with respect to the Galactic rest-frame, we take this into account by adding this line-of-sight velocity to the observed line-of-sight velocities of the stars.

In this thesis, for simplicity, we ignore the apparent angle difference, because we focus on the small region around the Galactic centre. We consider that the line-of-sight direction is always parallel to the Sun-Galactic centre line irrespective of their longitudinal or latitudinal position. When we compare with the model, we use a Galactocentric Cartesian X, Y and Z coordinate. We set X is the direction of the Galactic longitude l^*; Y is aligned with the Sun and the Galactic centre line, and toward the Galactic centre from the Sun is positive; Z is the direction of the Galactic latitude, and toward the Galactic North pole is positive. We also consider that all the

Figure 3.2: The number density distribution of the stars selected as the AGBb candidates in Fig. 3.1. The red open rectangle highlights our selection region of this study.

stars are at the distance of the Galactic centre, i.e. $R_0 = 8.275$ kpc, which assumes that all the stars are at $Y \approx 0$. We compute the velocity toward X and Z from the proper motion as follows

$$
\begin{aligned}
V_X\,(\mathrm{km\ s^{-1}}) &= 4.74047\,\mu_l\,(\mathrm{mas\ yr^{-1}})\,R_0(\mathrm{kpc}); && (3.2)\\
V_Z\,(\mathrm{km\ s^{-1}}) &= 4.74047\,\mu_b\,(\mathrm{mas\ yr^{-1}})\,R_0(\mathrm{kpc}). && (3.3)
\end{aligned}
$$

We also consider $V_Y = V_{LOS}$. We convert $(l, b)^\circ$ to (X, Z) kpc with the angular distance scale at $R_0 = 8.275$ kpc, i.e. $(X, Z)\ (\mathrm{kpc}) = 0.144\,(\mathrm{kpc\ deg^{-1}}) \times (l, b)$. The number density distribution of the selected NSD stars from the VIRAC2 data can be seen in Fig. 3.2. Also, the distribution of selected NSD stars from the APOGEE DR17 data colour-coded with the line-of-sight velocities of stars, V_{LOS}, can be seen in Fig. 3.3.

Figure 3.3: The distribution of the APOGEE DR17 selected in Section 3.2.2. The filled circles are colour coded by V_{LOS}.

3.4 Surface Density Distribution of the NSD

Fig. 3.2 shows that the NSD is clearly seen in the central region, and their stellar density is higher especially in the area enclosed by the red open rectangle within $|l| < 0.7°$ and $|b| < 0.35°$. Sormani et al. (2020b) also estimated that the NSD population is dominant (more than 75 % in their selected Galactic centre stars) in this region. Hence, in this thesis, to minimise the contamination from the stars in the other components, such as the Galactic bar, we focus on the NSD data within $|l| < 0.7°$ and $|b| < 0.35°$, which corresponds to about $|X| < 0.1011$ kpc and $|Z| < 0.0505$ kpc. Fig. 3.4 shows the surface number density of our selected VIRAC2 stars (Section 3.2.1) in a two dimensional grid from about -0.1011 kpc $< X < 0.1011$ kpc with 20 grids and about -0.0505 kpc $< Z < 0.0505$ kpc with 10 grids. We decided to use this large square grids of the data with each grid of the size of about 0.01 kpc $\times$ 0.01 kpc. Hereafter, for brevity, we will assume that we are focusing in a two dimensional grid of -0.10 kpc $\lesssim X \lesssim 0.10$ kpc and -0.05 kpc $\lesssim Z \lesssim 0.05$ kpc. The number of stars indicated in Fig. 3.4 is the number of stars within this size of grid, and therefore it shows the surface density of the stars within about 0.01×0.01 kpc. We use the same grids for the velocity dispersion data where this large grid ensures that there are enough number of stars in each grid to confidently obtain the velocity dispersion. Also, this large size grids ensures that both the density and velocity dispersion changes relatively smoothly both in the X and Z directions, after making the necessary correction as described below.

Fig. 3.2 and Fig. 3.4 clearly show that the data is affected by extinction as highlighted as "dark patches". To mitigate the lack of the stars in these dark patches, we use the assumption of the NSD being axisymmetry, so that the projected edge-on density distribution is symmetric about both X and Z-axes. We first split these grids into four quadrants of $X > 0$ and $Z > 0$; $X < 0$ and $Z > 0$; $X < 0$ and $Z < 0$; $X > 0$ and $Z < 0$. We then compare the number of stars among the grids at the symmetric position about X and Z axes, and take the maximum number of the among these grids. One example is that we compare the number of stars at the grids of $(X, Y) = (0.05, 0.15)$; $(-0.05, 0.15)$; $(-0.05, -0.15)$ and $(0.05, -0.15)$. Then, we take the maximum number of stars among these grids and reassign this number to these four grids. Assuming that the NSD is an axisymmetric structure, the number density of stars should be the same at the grids at the symmetric position. The difference in number is considered to be due to the difference in extinction, and we assume that the grid with the maximum number of the stars are the ones that is least suffering from extinction. After applying this process, the stellar density distribution has no dark patch, it changes more smoothly which can be seen in Fig. 3.5. The NSD is assumed to be axisymmetric in this thesis, because the stellar number density distribution in Fig. 3.4 and Fig. 1 of Nishiyama et al. (2013) show that the NSD is highly symmetric.

Since the data is now symmetric, we focus on one quarter of the distribution as seen in Fig. 3.6. The observed density distribution is number density, while the stellar density in dynamical models is described as mass density. To be able to compare the observed density distribution to the dynamical model, we introduce the stellar number density to mass ratio to convert the mass density of the model at position of the grid to the observed number of stars in these grid. Hence, we use the

Figure 3.4: The number density distribution of the stars selected in Section 3.2.1 in a two dimensional grid from 0:1 kpc. X . 0:1 kpc and 0:05 kpc. Z . 0:05 kpc with Sgr A being at the centre, at (0, 0) kpc. The number density distribution of the stars is not smooth, where green square open boxes are some examples showing dark patches caused by the extinction. The colour bar indicates the number of stars in each approximately sized 0:01 0:01 kpc grid.

Figure 3.5: The number density distribution of the stars selected in Section 3.2.1 in a two dimensional grid from 0:1 kpc. X . 0:1 kpc and 0:05 kpc. Z . 0:05 kpc that is symmetrical after the number of stars are re-asigned to take the highest number of stars among the symmetric grids (see the text of Section 3.4 for more details). The colour bar indicates the number of stars in each approximately sized 0:01 0:01 kpc grid.

Figure 3.6: Same as Fig. 3.5, but shown a quarter of the data. The X and Z direction has 10 and 5 grids respectively. The colour bar indicates the number of stars in each approximately sized 0.01×0.01 kpc grid.

number of stars in this fixed size of the grid as the surface number density, N. The uncertainty of the number of stars in each grid, is computed by $\sigma_N = \sqrt{N}$, where N is the number of stars in a grid. We can see that the uncertainty increases with the number of stars in each grid. However, the relative uncertainty, σ_N/N, decreases as the number of stars in each grid increases.

Fig. 3.7 shows the number density in each grid as a function of X at different position in Z. It can be seen that the density distribution changes smoothly in both X and Z directions. Also, the density is the highest at the centre, and the highest at the disk mid-plane ($Z = 0$) at a fixed X. The density decreases as increasing of both X and Z. However, it decreases more rapidly in the Z direction which is expected due to the disky shape of the NSD.

3.5 The Vertical Velocity Dispersion Distribution of the NSD

To examine the existence of the ULDM soliton core in the Galactic centre, in the next chapter we compare the observed velocity dispersion with an analytical dynamical model of the NSD with the soliton core of the ULDM with the different particle masses. In this section we present how we obtain the observed velocity distribution

Figure 3.7: Observed number density distributions as a function of X at different position in Z of $Z = 0.005$ kpc (black dots), $Z = 0.015$ kpc (red dots), $Z = 0.025$ kpc (yellow dots), $Z = 0.035$ kpc (green dots), $Z = 0.045$ kpc (blue dots). The uncertainty is too small and it is smaller than the symbols. The largest uncertainty of a density grid is the density grid at $Z = -0.005$ kpc and $X = -0.005$ kpc.

at different positions of the NSD from the VIRAC2 proper motion data for the stars selected in Section 3.2.1. As mentioned in Section 3.4, to analyse the 2D velocity structure, we use the same 20×10 grids in the region of $-0.1 \leq X \leq 0.1$ kpc and $-0.05 \leq Z \leq 0.05$ kpc as the ones used to analyse the density distribution. Section 3.2.1 shows that even in this large grid the VIRAC2 data are affected by extinction at different levels at the different positions of the sky. It is therefore important to assess how extinction has affected both the V_X and V_Z distributions of our selected VIRAC2 stars (Section 3.2.1). First we discuss that the distribution of V_X is too sensitive to the level of the extinction and it is difficult to obtain the velocity distribution reliably. In the left panel of Fig. 3.8, the distribution of V_X at $X = -0.086$ kpc, $Z = -0.005$ kpc grid is shown. The left panel of Fig. 3.8 shows two peak distribution for V_X. This is expected for a rotating disk. Fig. 3.9 schematically describes the velocity structure of the NSD and how the projected V_X distribution looks like depending on the location of the extinction becoming significant. Fig. 3.9 shows that due to the clock-wise rotation, when we look down at the NSD from the North Galactic pole, the stars closer to us than the distance to the Galactic centre (near-side of the NSD, $Y < 0$ in the coordinate shown in Fig. 3.9) are moving towards the positive X direction ($V_X > 0$). On the other hand, stars further away from us than the distance to the Galactic centre (far-side of the NSD, $Y > 0$) are moving towards the negative X direction ($V_X < 0$). Also, although the density is greater at the centre at $Y = 0$, the volume that enclose stars with $V_X = 0$ is smaller than the volume that enclose stars with non-zero V_X. Hence, as shown in the example of the vertical green arrow pointing toward the Sun in Fig. 3.9, if we could see both side of the NSD stars, we can obtain the two peaks of the V_X distribution of the stars. However, if we have dust blocking the lights from the stars in the far side of the NSD as shown in the example of the vertical purple arrow pointing toward the Sun in Fig. 3.9, then stars behind the dust cannot be seen and only the stars in front of the dust can be seen. As a result, this gives a one peak distribution of V_X. The right panel of Fig. 3.8 shows the distribution of V_X at $X = 0.086$ kpc, $Z = 0.005$ kpc grid which is the symmetric point to $X = 0.086$ kpc, $Z = 0.005$ kpc grid shown in the left panel. We can see that the expected two peak distribution of V_X now looks more like a one peak distribution and only the peak of the positive V_X is seen. This example demonstrates that the V_X distribution is sensitive to the significance of the extinction and at which distance the dust extinction becomes severe. As we can see in Fig. 3.2, the strength of extinction varies significantly at different positions even in this small region of the sky, which means that the observed V_X distributions at the different line-of-sight are affected by the different selection functions. To compare such observations with the model, we need to apply the correct selection function for each line-of-sight to the model. However, because the distance of the stars are unknown, it is difficult to construct such selection function as a function of the positions in the sky. Therefore, in this thesis we will not use the velocity distribution of V_X, but will focus on the V_Z distribution.

The top panels of Figs. 3.10, 3.11 and 3.12 show examples of the V_Z distri-

Figure 3.8: Left panel: An example of the V_X distribution at $X = 0.086$ kpc and $Z = 0.005$ kpc grid that appears to be less affected by extinction. Right panel: An example of the V_X distribution at $X = 0.086$ kpc and $Z = 0.005$ kpc grid that appears to miss the significant number of stars in the negative V_X, likely due to the extinction blocking the light from the stars in the far side of the NSD like the example of the line-of-sight shown in the purple vertical arrow in Fig. 3.9.

bution at different grids where some of these are grids close to the centre of the NSD, and some grids close to the outer regions of the NSD. We can see that the V_Z distribution displays a single peak around $V_Z = 0$ and the broad distribution to both positive and negative sides irrespective of the position of the grid. These different grids are at the different positions, and has the different extinction levels. The similar and also symmetric velocity distribution means that the V_Z distribution is less sensitive to the level of extinction. For example, Fig. 3.10 shows the V_Z distribution at the two different symmetric grid positions used in Fig. 3.8 where the V_X distribution shows significantly different shapes of the distribution. The top panels of Fig. 3.10 demonstrate that the V_Z distribution of these two grids is rather similar. This is because V_Z distribution is a vertical oscillation of stars where the vertical oscillations of stars in front side of the disk (at $Y < 0$) is the same for stars at the far side of the disk ($Y > 0$). Therefore, as long as we have a majority of the stars at $Y < 0$, the V_Z distribution should be the same as the case of all the stars in the line-of-sight are observed. Strictly speaking, this is incorrect, because the projected V_Z distribution at $X = X_{los}$ are contributed from the stars at the radius from $R = X_{los}$ to the edge of the NSD, where $R = \sqrt{X^2 + Y^2}$ is the two dimensional radius from

Figure 3.9: Schematic view of kinematics of the NSD stars when looked down from the Galactic North Pole. Red arrow shows the direction of rotation of stars. The examples of the two line-of-sights are shown by the green and purple arrows pointing toward the Sun's direction. The distribution of V_X along the green arrow is expected to be the two peak as shown in the bottom left, because the extinction is less severe and the stars in the near-side ($Y < 0$) and farther-side ($Y > 0$) can be observed. On the other hand, the distribution of V_X along the purple arrow is expected to be a single peak as shown in the bottom right, because the extinction is more severe as indicated with the grey clouds highlighted as "Dust and gas" and the stars in the far-side ($Y > 0$) are difficult to be observed. The grey circle indicates the circle with the radius of R_0, which is the smallest NSD radius for the stars with line-of-sight along the green arrow, where X_{los}

the Galactic centre (see Fig. 3.9), and the vertical velocity dispersion at different radii of the NSD should be different. The distance limit of the stars observed in a magnitude limit survey like VIRAC2 depend on their intrinsic luminosity and the 3D distribution and strength of the dust extinction, and the resultant V_Z distribution is a complex mixture of the stars at different radius of the NSD. However, in this thesis, we simply consider that V_Z distribution are less affected by the extinction and the correct V_Z probability distribution is obtained in all the grids we consider. Since the V_Z distribution is symmetric about $V_Z = 0$, we analyse only the velocity dispersions, σ_Z, to compare with the model.

To compute the velocity dispersion, $\sigma_{Z,i,j}$, of the V_Z distribution where i and j represents the i-th grid in the X direction and j-th grid in the Z direction, respectively, we fit the observed velocity distribution with a Gaussian profile with the mean of $\mu_{Z,i,j}$ and the standard deviation of $\sigma_{Z,i,j}$. To obtain the posterior probability distribution for the Gaussian parameters of $\theta_m = (\sigma_{Z,i,j}, \mu_{Z,i,j})$, Bayesian statistics is used which is given by

$$P(\theta_m|D) = \mathcal{L}(D|\theta_m)P(\theta_m)/P(D), \tag{3.4}$$

with D being the V_Z distribution within a grid, $P(\theta_m|D)$ is the posterior probability of the parameters θ_m given data D, $\mathcal{L}(D|\theta_m)$ is the likelihood, $P(\theta_m)$ is prior and $P(D)$ is the model evidence. Since it does not depend on θ_m, and it is considered to be constant under a single model hypothesis, we ignore the model evidence. For each grid of the V_Z distribution, we run a Markov Chain Monte Carlo (MCMC) sampling to obtain the posterior distribution of the parameter θ_m with the likelihood function, $\mathcal{L}_{ij}(D|\theta_m)$, which is given as follows

$$\mathcal{L}_{ij}(D|\theta_m) = \prod_k^{N_D} \frac{1}{\sqrt{2\pi(\sigma^2_{Zerr,i,j,k} + \sigma^2_{Z,i,j})}} \exp\left(-\frac{(\mu_{Z,i,j} - V_{Z,i,j,k})^2}{2(\sigma^2_{Zerr,i,j,k} + \sigma^2_{Z,i,j})}\right), \tag{3.5}$$

where $V_{Z,i,j,k}$ is the V_Z of the k-th stars in the i,j grid, N_D is the number of the stars in the i,j grid, and $\sigma_{Zerr,i,j,k}$ is the observational uncertainty of V_Z for the k-th star in each grid. We chose to use flat priors for our parameters that we are fitting as follows: $0 < \sigma_Z$ (km s^{-1}) < 500, $-500 < \mu_Z$ (km s^{-1}) < 500. The prior range is chosen to have a large range for the MCMC algorithm to search around and hence avoiding results biased by the prior assumption. We use emcee (Foreman-Mackey et al., 2013) for our MCMC sampler with 10 walkers with each walker having 1000 steps for the MCMC sampler. We chose to discard the first 500 steps as our 'burn-in'.

The top panels of Figs. 3.10, 3.11 and 3.12 also show a Gaussian profile with the best-fit mean and standard deviation within the grid computed by the MCMC for the V_Z data shown in the blue filled histogram. It can be seen that the fit is poor as the dispersion of the Gaussian is overestimated. This could be due to V_Z the distribution not well described with a Gaussian profile, because of the large tail of the high absolute value of V_Z. These outliers are unlikely a part of the NSD, but likely a contamination of stars in the Galactic bar component. Indeed, the effect of increase in velocity dispersion from the Galactic bar was confirmed by Sormani et al. (2022). Sormani et al. (2022) showed that the observed velocity distribution at the direction of the NSD cannot be described purely by the NSD kinematics. Then, Sormani et al. (2022) used a Galactic bar model from Portail et al. (2017) and showed that the increased observed velocity dispersion with radius can be explained with the increasing contribution of the Galactic bar contamination. In particular, the top left panel of Fig. 3.10 shows a very broad Gaussian profile as the best fit function. This is because the non-negligible number of stars with $|V_Z| > 100$ km s^{-1} exist in this grid.

To remove the outliers of the high $|V_Z|$ stars, we use sigma clipping method. Sigma clipping is an iterative process as follows: first the data are fitted with a Gaussian profile with the mean, μ, and standard deviation, σ. Then, the data that is outside of the mean $\pm n\sigma$ are removed, and the remaining data are fitted with a Gaussian again. This process is then repeated until the μ and σ values converges. We can control the size of the clipping by selecting the parameter value n. Note that we are not taking into account the uncertainty of the individual V_Z of each star for simplicity as uncertainties is more difficult to handle in this process. Figs. 3.10,

Figure 3.10: Examples of V_Z distribution (blue filled histogram) at different grids overplotting with a best fit Gaussian profile (orange line). N_s is the number of stars in a grid. Top panels: the observed V_Z distribution for all the stars in each grid. Second panels: the V_Z distribution of stars in each grid after applying 2σ sigma clipping. Third panels: the V_Z distribution of stars in each grid after applying 2.25σ sigma clipping. Bottom panels: the V_Z distribution of stars in each grid after applying 2.5σ sigma clipping.

3.11 and 3.12 show some examples of the V_Z distribution histogram where the V_Z data is clipped outside of 2σ, 2.25σ and 2.5σ, i.e. applying n=2, 2.25 and 2.5, from the mean at different grids. We have then fitted the V_Z distribution after sigma clipping is applied at each grid with a Gaussian profile with MCMC sampling as described above. The resultant best fit Gaussian profile is shown with the orange lines in Figs. 3.10, 3.11 and 3.12. We can see that for 2.5σ sigma clipping, the V_Z distribution histogram does not have a good fit with the Gaussian profile because there are excess number of stars at higher $|V_Z|$ compared to the Gaussian profile, and the contamination of the stars in the bar component with higher V_Z still remains. As a consequence, it leads to the velocity dispersion to be overestimated. We then applied the same procedure with more aggressive clipping with 2σ and 2.25σ sigma clipping which are shown respectively in the second and third panels of Figs. 3.10, 3.11 and 3.12. Both of these give a good fit. However, 2σ sigma clipping result shows an unnecessary over-cut of the data because the Gaussian peak is larger than the V_Z distribution histogram. This as a consequence leads to an underestimation of the velocity dispersion. Figs. 3.10, 3.11 and 3.12 shows that 2.25σ sigma clipping provides the most reasonable single Gaussian fit to the data irrespective of the positions of the grid. Fig. 3.13 shows an example marginalized posterior probability distribution of the parameters σ_Z and μ_Z at $X = 0.025$ kpc and $Z = 0.005$ kpc grid after 2.25σ sigma clipping which is shown in the third top left panel of 3.11. Clearly, Fig. 3.13 shows a reasonable single peak probability distribution. We then take the mean of the posterior of σ_Z parameter as the best fit velocity dispersion, and we take standard deviation of the posterior of σ_Z parameter for the uncertainty. The mean of the posterior of σ_Z parameter for this grid is $\sigma_Z = 57.2$ km s^{-1} with uncertainty of about 0.5 km s^{-1}. The mean of the posterior of μ_Z parameter for this grid is $\mu_Z = 1.9$ km s^{-1} with uncertainty of about 0.7 km s^{-1}.

As mentioned in Section 3.4, we focus on one quarter of the distribution at $X >$ 0 and $Z >$ 0 because we consider that the NSD is an axisymmetric disk, and their projected properties are symmetric about both X and Z axes. Taking an advantage of this assumption of symmetry, to maximise the signal of the information we sum

Figure 3.11: Examples of V_Z distribution (blue filled histogram) at different grids overplotting with a best fit Gaussian profile (orange line). N_s is the number of stars in a grid. Top panels: the observed V_Z distribution for all the stars in each grid. Second panels: the V_Z distribution of stars in each grid after applying 2σ sigma clipping. Third panels: the V_Z distribution of stars in each grid after applying 2.25σ sigma clipping. Bottom panels: the V_Z distribution of stars in each grid after applying 2.5σ sigma clipping.

Figure 3.12: Examples of V_Z distribution (blue filled histogram) at different grids overplotting with a best fit Gaussian profile (orange line). N_s is the number of stars in a grid. Top panels: the observed V_Z distribution for all the stars in each grid. Second panels: the V_Z distribution of stars in each grid after applying 2σ sigma clipping. Third panels: the V_Z distribution of stars in each grid after applying 2.25σ sigma clipping. Bottom panels: the V_Z distribution of stars in each grid after applying 2.5σ sigma clipping.

Figure 3.13: Marginalized posterior probability distribution of the parameters of the Gaussian distribution when fitted to the V_Z distribution data at $X = 0.025$ kpc and $Z = 0.005$ kpc grid after 2.25σ sigma clipping.

up the V_Z data at all 4 symmetric grid positions. For example, when we compute the velocity dispersion at $X = 0.025$, $Z = 0.005$, we combine all V_Z data at all 4 grids of $X = \;\; 0.025$, $Z = \;\; 0.005$. We then apply the process of 2.25σ sigma clipping and MCMC fit of the Gaussian profile to obtain the velocity dispersion and their uncertainties. Fig. 3.14 shows the two dimensional distribution of V_Z velocity dispersion at the different positions of the X Z grid. We can see that σ_Z is highest at the central region of the NSD and decreases smoothly as increase in the X. However, the σ_Z stays high as increase in Z, which is an unexpected feature for a stable disk. This feature can be seen more clearly in Fig. 3.15. Fig. 3.15 shows

σ_Z as a function of X at different Z. There is an increase from about 64 km s^{-1} to 66 km s^{-1} between about $X = 0.03$ kpc and $X = 0.07$ kpc for $Z = 0.045$ kpc and the V_Z velocity dispersion stays high at larger X, unlike the velocity dispersion profile at lower Z, which decreases with increasing X. The increase is about 3%, which could still be due to the contamination of Galactic bar stars and/or the NSD consists of multiple populations with different kinematics. Since the change of the value is relatively small, we use the obtained velocity dispersion as it is. However, we will still keep this problem in our mind and will discuss it more when we interpret the results of the comparison between the data and the model. It is important to note that the uncertainty for σ_Z is small. For example, the minimum values of σ_Z in a grid is about 43.4 km s^{-1}, whereas the highest uncertainty for σ_Z is about 1.0 km s^{-1}.

Note that the stars selected for the line-of-sight velocity analysis are of different populations. Their age distributions are different from AGBb stars used for the surface density distribution and the vertical velocity dispersion distribution. The NSD is likely to consist of the multiple populations of the stars with different age populations having different kinematics and/or spatial distribution (e.g. Nogueras-Lara et al., 2023). Hence, strictly speaking we need to take into account the effects of the multiple population. However, for simplicity we consider that the NSD is overall able to be described with a single component.

3.6 The Rotation Curve of the NSD

In addition to the use of the observed velocity dispersion from the VIRAC2 proper motion data, we will also compare the observed mean line-of-sight velocity with an analytical dynamical model of the NSD with ULDM of different particle masses to test the existence of the ULDM soliton core in the Galactic centre. As discussed in Section 3.2.2, Schönrich et al. (2015) found the rotation of the NSD from the line-of-sight velocity of the APOGEE data. Like the rotation curve of the external galaxies, the mean line-of-sight velocity profile provides the information of the rotation curve. The rotation is an important indicator for the total dynamical mass, and it is crucial to include the mean rotation curve in the observational constraints

Figure 3.14: σ_z in a two dimensional grid with number of grids in X and Z direction being 10 and 5, respectively.

to the dynamical model. In this section, we explain how we obtain the mean line-of-sight velocity at different positions of the NSD from the APOGEE DR17 data for the stars selected in Section 3.2.2. As mentioned in Section 3.4, we focus on one quarter of the distribution at $X > 0$ and $Z > 0$. To apply this concept to the selected stars of the APOGEE DR17 data whilst maximising the signal of the information, we map the stars at $X < 0$ or $Z < 0$ to its symmetric points at $X > 0$ and $Z > 0$, by taking $(X_{new}, Z_{new}) = (|X|, |Z|)$ and $V_{LOS} \rightarrow V_{LOSnew} = \mathrm{sign}(X) V_{LOS}$. Fig. 3.16 shows the results after this remapping of the V_{LOS} data. Since the selected stars from the APOGEE DR17 is small (see Section 3.2.2), we used a smaller grid size of 5 grid in the X-direction and no division in the Z-direction within our focused region of $0 < X \lesssim 0.1$, $0 < Z \lesssim 0.05$.

To assess if there are outliers such as contamination of Galactic bar stars within our selected stars of the APOGEE DR17 data, we apply the same assessment process as Section 3.5. The top panels of Fig. 3.17 show examples of the V_{LOSnew} distribution at different grids. We can see from the top panels of Fig. 3.17 that the V_{LOSnew} distribution displays a peak with mean about 24.3 km s^{-1} and 56.5 km s^{-1},

Figure 3.15: Vertical velocity dispersion as a function of X at different positions in Z of Z = 0.005 kpc (black dots with error bars), Z = 0.015 kpc (red dots with error bars), Z = 0.025 kpc (yellow dots with error bars), Z = 0.035 kpc (green dots with error bars), Z = 0.045 kpc (blue dots with error bars).

respectively. However, V_{LOSnew} does not have a tail distribution like a Gaussian distribution because the number of stars in each grid is small.

To compute the mean line-of-sight velocity, V_{LOS}, we use the same Bayesian statistics method as described in Section 3.5. The top panels of Fig. 3.17 also show a Gaussian profile with the best-fit mean and standard deviation within the grid computed by the MCMC for the V_{LOSnew} data shown in the blue filled histogram. The top panels of Fig. 3.17 show that the fit is poor as the peak Gaussian does not overlap with the peak of the blue filled histogram. This is due to the stars with line-of-sight velocity of $V_{LOSnew} < -50$ km s^{-1}, which is very far off from the mean. These stars are most likely not part of the NSD but rather a contamination of Galactic bar or halo stars.

To remove the outliers of the high V_{LOSnew} stars, we again use the sigma clipping method. Fig. 3.17 shows some examples of the V_{LOSnew} distribution histogram

Figure 3.16: The distribution of the APOGEE DR17 selected in Section 3.2.2 in two dimension. The filled circles are colour coded by V_{LOSnew}.

where the V_{LOSnew} data is clipped outside of 2.25σ, 2.5σ and 2.75σ from the mean at different grids. We have then used the MCMC sampling as described in Section 3.5 to fit the V_{LOSnew} distribution at each grid with a Gaussian profile. The resultant best fit Gaussian profile is shown with the orange lines in Fig. 3.17. We can see that for 2.75σ sigma clipping, the V_{LOSnew} distribution histogram has slightly poorer fit with the Gaussian profile at both $X = 0.05$ kpc and $X = 0.091$ kpc due to excess number of stars at negative V_{LOSnew}, and as a consequence it leads to the mean of the Gaussian profile not overlapping with the peak of the blue filled histogram. Furthermore, Fig. 3.18 shows V_{LOS} evaluated at all 5 grids after 2.75 sigma clipping, where it can be seen that the rate of change of the mean line-of-sight is not smooth as increase in distance because there is a sharp decrease of the mean line-of-sight velocity at about $X = 0.05$ kpc. The V_{LOSnew} distribution for 2.75σ sigma clipping at $X = 0.05$ kpc can be seen more closely in Fig. 3.17. The sharp decrease of the mean line-of-sight velocity at about $X = 0.05$ kpc is due to many stars with velocities of $V_{LOSnew} < -50$ km s^{-1}. Due to this, we reject to use 2.75σ sigma clipping. From Fig. 3.17, we can see that the mean of V_{LOSnew} for 2.25σ

Figure 3.17: Examples of V_{LOSnew} distribution (blue filled histogram) at different grids overplotting with a best fit Gaussian profile (orange line). N_s is the number of stars in a grid. Top panels: the observed V_{LOSnew} distribution for all the stars in each grid. Second panels: the V_{LOSnew} distribution of stars in each grid after applying 2.25σ sigma clipping. Third panels: the V_{LOSnew} distribution of stars in each grid after applying 2.5σ sigma clipping. Bottom panels: the V_{LOSnew} distribution of stars in each grid after applying 2.75σ sigma clipping.