Essence and Necessity

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Abstract

What is the relation between metaphysical necessity and essence? This paper defends the view that the relation is one of identity: metaphysical necessity is a special case of essence. My argument consists in showing that the best joint theory of essence and metaphysical necessity is one in which metaphysical necessity is just a special case of essence. The argument is made against the backdrop of a novel, higher-order logic of essence (HLE), whose core features are introduced in the first part of the paper. The second part investigates the relation between metaphysical necessity and essence in the context of HLE. Reductive hypotheses are among the most natural hypotheses to be explored in the context of HLE. But they also have to be weighed against their non-reductive rivals. I investigate three different reductive hypotheses and argue that two of them fare better than their non-reductive rivals: they are simpler, more natural, and more systematic. Specifically, I argue that one candidate reduction, according to which metaphysical necessity is truth in virtue of the nature of all propositions, is superior to the others, including one proposed by Kit Fine, according to which metaphysical necessity is truth in virtue of the nature of all *objects*. The paper concludes by offering some reasons to think that the best joint theory of essence and metaphysical necessity is one in which the logic of metaphysical necessity includes S4, but not S5.

1 Introduction

What is the relation between metaphysical necessity and essence? In this paper, I argue that the relation is one of identity: metaphysical necessity is a special case of essence. The idea

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that necessity derives from, or is based on, essence can be traced back to Aristotle and has recently been revived by Kit Fine (1994).[1] The intuitive motivation for this view can be brought out by examples. For instance, it seems natural to suppose that the necessity of all triangles having three sides flows from the nature of triangularity and that the necessity of all red things being colored flows from the nature of redness.[2] But while examples like these may lend some plausibility to the view that all necessity derives from essence, they obviously do not suffice to justify the view in full generality. In order to justify the view in full generality, a more general and systematic argument is needed.

The aim of this paper is to provide such an argument. My argument consists in showing that the best joint theory of essence and metaphysical necessity is one in which metaphysical necessity is just a special case of essence. The argument is made against the background of a novel logic of essence (HLE). This logic differs in a number of important respects from Fine's logic of essence (LE) (Fine, 1995, 2000), the only fully developed logic of essence previously available in the literature. One central difference concerns their subject matter. The subject matter of LE is not the conceptually *basic* notion of essence characterized in Fine (1994), but what Fine calls the 'constrained consequential notion' of essence—an idealized notion of essence that, roughly speaking, results from the basic notion of essence by closure under (a restricted form of) logical consequence. In contrast, HLE is concerned with the conceptually basic notion of essence. Another central difference is that unlike LE, which is formulated in a first-order language, HLE is formulated in a higher-order language. The higher-order framework has the advantage of allowing us not only to regiment talk about the essences of *objects* in a straightforward way; it also provides a convenient and straightforward way of regimenting talk about the essences of properties, propositions and logical operations, such talk being central to our theorizing about essence.[3]

The framework of HLE suggests various different ways in which metaphysical necessity might be reducible to essence. As we will see, reductive hypotheses are among the most natural hypotheses to be explored in the context of HLE. But they also have to be weighed against their non-reductive rivals. I argue that some of the reductive hypotheses fare better than

[1] For Aristotle's view about the relation between essence and necessity, see e.g., Barnes (1994, p. 120), Malink (2013, p. 126) and Bronstein (2016, ch. 3). Some version of the view that necessity is based on essence was also held by Spinoza, Descartes and Leibniz; see Newlands (2013). For discussions of Fine's thesis of the reducibility of necessity to essence, see e.g., Hale (1996, 2002), Correia (2006, 2012), Cameron (2010), Vetter (2011, 2020), Wildman (2018), Teitel (2019) and Ditter (2020). A non-reductive essentialist account of modality is defended in Hale (2013).

[2] I will use 'essence' and 'nature' interchangeably throughout.

[3] The higher-order approach also allows us to sidestep the property-theoretic paradoxes that arise in the context of a first-order theory of properties. Any first-order theory of essence that is supplemented with a theory of properties would have to come up with a solution to these paradoxes. An analogous problem does not obviously arise in the higher-order framework. Even someone who ultimately rejects the higher-order approach, or thinks that the higher-order approach is less fundamental than the first-order approach (Fine, 1980), may want to take the theory presented here as a starting point and attempt to translate it into their preferred first-order framework.

their non-reductive rivals: they are simpler, more natural, and more systematic. Specifically, I argue that one candidate reduction, according to which metaphysical necessity is truth in virtue of the nature of all propositions, is superior to the others, including the one proposed by Fine (1994), according to which metaphysical necessity is truth in virtue of the nature of all *objects*.[4] My abductive argument for this reductive hypothesis is centrally based on the standard characterization of metaphysical necessity as the broadest objective necessity. However, even if metaphysical necessity is not the broadest objective necessity, I argue that truth in virtue of the nature of all propositions is the broadest objective necessity, at least up to necessary coextensiveness.

The theory proposed here also sheds new light on the question of the correct logic of metaphysical necessity. I contend that, regardless of whether we adopt any of the reductive accounts, there is good reason to believe that metaphysical necessity obeys all the principles of S4, but not those of S5. I have recently suggested the same conclusion in Ditter (2020), though my argument there rests on the acceptance of Fine's reduction thesis in the context of LE. The considerations put forward here show that the case for this revisionary conclusion does not essentially depend on the acceptance of Fine's reduction thesis or LE; in fact, it would seem to carry over to any joint theory of essence and metaphysical necessity based on HLE provided that metaphysical necessity is the broadest objective necessity.

The paper is in two parts. The first part (§2) outlines some of the core features of the theory HLE, which will serve as our background essentialist theory. In the second part (§3), I turn to the question of the relation between essence and metaphysical necessity. In Sections 3.1-3.3, I explore three different reductive hypotheses and assess their respective strenghts and weaknesses. Section 3.4 provides an argument for the reducibility of metaphysical necessity to essence based on the results of the foregoing sections and the premise that metaphysical necessity is the broadest objective necessity. In Section 3.5, I present some difficulties for maintaining S5 for metaphysical necessity in the context of a joint theory of essence and metaphysical necessity and argue that there is good reason to reject the standard view that the correct logic of metaphysical necessity is S5. Section 4 concludes. A technical appendix provides some further formal details and establishes some of the formal results appealed to in the main text.

[4]Fine has offered his most detailed defense of his reduction thesis in the context of LE. See Fine (1995), and see Ditter (2020) for discussion.

2 The Theory HLE

In this section, I will introduce some of the core features of the theory HLE that will serve as the base theory for our discussion.[5] Since the prospects for an essentialist reduction of metaphysical necessity crucially depend on what logical principles the notion of essence obeys, it is worth stating explicitly those principles that will be central to all of the reduction theses to be investigated later. The theory HLE contains some additional principles as well, but the reduction theses I will investigate here only depend on the principles explicitly stated in this section. The theory HLE as a whole should be judged by its overall systematicity, simplicity and explanatory power. The motivations and justifications I provide for the individual principles to be listed should therefore not be viewed as exhaustive; every principle of the system is at least partially justified, in addition, by its contribution to the systematicity and elegance of the overall theory.

2.1 Regimentation

The language I will be using to regiment essentialist statements is a generalization of Fine's regimentation in LE (Fine, 1995). In LE, essentialist statements are expressed by means of operators subscripted with a one-place predicate, written $\Box_F$, where F picks out the subjects of the essentialist attribution. For example, the intended meaning of a sentence of the form $\Box_F\phi$ is that it lies in the nature of the things which F that ϕ.[6] This form of expression allows us to state essentialist statements in which the subject of the essentialist attribution is a collection of objects without having to reify "collections" as objects in their own right. For example, we can formally render the sentence 'It lies in the nature of all objects (taken together) that everything is self-identical' by subscripting the essentialist operator with a predicate that applies to all objects.[7]

However, Fine's formalism only allows us to pick out *objects* as the subjects of essentialist attributions: LE is stated in a first-order language in which predicates only combine with terms in name position. In order to straightforwardly express essentialist statements in which properties, propositions, and logical operations can be the subjects of essentialist attributions, it will be convenient to use a higher-order language. In such a language, predicates can also combine with other predicates, sentences or operators to make a sentence. This allows us to pick out the entities expressed by terms of these grammatical categories as subjects of essentialist attributions in the same way in which Fine's approach allows us to pick out

[5] A more detailed development of HLE, including a model-theoretic semantics, is provided in Ditter (MS). All consistency claims made in the present paper can be proved by using the model-theory developed there.

[6] Equivalently: It is essential to the things which F that ϕ. Or: It is true in virtue of the nature of the things which F that ϕ.

[7] Note that I will be using all of the following expressions interchangeably: (i) 'the collective nature of ...'; (ii) 'the nature of the collection of ...'; (iii) 'the nature of ... taken together'.

objects for this purpose, though we may need to use *lists* of subscripts in order to talk about the collective nature of entities of different types. So, for example, if F is used to pick out Socrates and G is used to pick out negation (i.e. what is expressed by 'not'), we can use $\Box_{F,G}$ to express 'it is true in virtue of the collective nature of Socrates and negation that'.

Let me make this a bit more precise. The language we will be working with is a relationally typed language with lambda abstraction.[8] The set of *types* is generated by the following rules:

(i) e is a type;

(ii) if $\tau_1, ..., \tau_n$ are types, $n \geq 0$, then $\langle \tau_1, ..., \tau_n \rangle$ is a type;

(iii) if τ is a type, then $[\tau]$ is a type.

Informally speaking, e is the type of objects, $\langle \tau_1, ..., \tau_n \rangle$ is the type of ordinary relations between entities of type $\tau_1, ..., \tau_n$ with the special case $\langle\rangle$ being the type of propositions, and $[\tau]$ is the type of *rigid properties* of entities of type τ. Intuitively, a rigid property is a property of being one of $x_1, x_2, ...$, for certain specific entities $x_1, x_2, ...$. We can thus think of rigid properties as predicative analogs of pluralities. Rigid properties are, like pluralities and sets, "extensional" in that their identity is completely determined by the entities they apply to.[9]

Every syntactic item of our language is assigned a unique type from the above hierarchy of types representing its syntactic category. The syntactic items of our language are called *terms*. We call terms of type $\langle\rangle$ *formulas* (or *sentences* if they don't contain free variables), terms of type $\langle \tau_1, \ldots, \tau_n \rangle$ (ordinary) *predicates*, and terms of type $[\tau]$ *rigid predicates*.[10] We assume that our language contains infinitely many variables of every type as well as a fixed stock of typed constants. The set of *terms* of the language is recursively defined as follows:

(i) Every constant or variable of type τ is a term of type τ;

(ii) if $n \geq 1$, A is a term of type $\langle \tau_1, \ldots, \tau_n \rangle$ and $B_1, \ldots, B_n$ are terms of type $\tau_1, \ldots, \tau_n$, respectively, then $A(B_1, \ldots, B_n)$ is a term of type $\langle\rangle$;

(iii) if A is a term of type $[\tau]$ and B is a term of type τ, then $A(B)$ is a term of type $\langle\rangle$;

[8] The use of relationally typed languages goes back to Orey (1959). See Gallin (1975) for applications to formal semantics and Fine (1977), Dorr (2016) and Williamson (2013) for applications in metaphysics. The particular type system including rigid types (clause (iii) below) that is used here is closely related to the systems in Myhill (1958) and Fine (1977), which distinguish between extensional and intensional types. The rigid types employed here correspond to their extensional types, while the non-rigid types generated by clause (ii) below correspond to their intensional types. Fritz et al. (2021) use an intensional relational type theory with a single extensional type for pluralities of propositions.

[9] See the axioms for rigidity at the end of Section 2.2 and the appendix for further details.

[10] Our use of rigid predicates is a higher-order generalization of the category of rigid predicates in LE; see Fine (1995, 2000).

(iv) if $n \geq 1$, ϕ is a term of type $\langle\rangle$ and $v_1, \ldots, v_n$ are pairwise distinct variables of type $\tau_1, \ldots, \tau_n$, respectively, all of which are free in ϕ, then $(\lambda v_1 \ldots v_n.\phi)$ is a term of type $\langle \tau_1, \ldots, \tau_n \rangle$.[11]

We call types of the form $\langle\sigma\rangle$ or $[\sigma]$ *one-place predicate types* and terms of this type *one-place predicates*. Our language contains the logical constants $\neg$ (of type $\langle\langle\rangle\rangle$), $\wedge, \vee$, $\rightarrow$ and $\leftrightarrow$ (of type $\langle\langle\rangle, \langle\rangle\rangle$); for every type σ, $\forall_\sigma$ and $\exists_\sigma$ (of type $\langle\langle\sigma\rangle\rangle$, and identity predicates $\equiv_\sigma$ (of type $\langle\sigma, \sigma\rangle$); for any $n \geq 0$ and any one-place predicate types $\tau_1, \ldots, \tau_n$, $\Box_{\tau_1,\tau_2,\ldots,\tau_n}$ (of type $\langle\tau_1, ..., \tau_n, \langle\rangle\rangle$). Constants of this latter kind are *essentialist operators*. The case where $n = 0$ is written $\Box_\varnothing$.[12] The operation of lambda-abstraction (clause (iv)) allows us to form complex predicates from formulas with free variables. For example, we can form the predicate $\lambda x^e.x \equiv_e x$ ('is self-identical') from the formula $x \equiv_e x$ by applying clause (iv) above.

When $F_1, F_2, \ldots, F_n$ are one-place predicates and ϕ is a formula, we write $\Box_{F_1,F_2,\ldots F_n}\phi$ for $\Box_{\tau_1,\ldots,\tau_n}(F_1, ..., F_n, \phi)$. We also write $\forall x^\tau(\phi)$ for $\forall_\tau((\lambda x^\tau.\phi))$, and $\phi \wedge \psi$ for $\wedge(\phi, \psi)$, and similarly for other logical constants. Note that in this setting, quantifiers don't bind variables, and they combine with predicates—and not formulas—to make sentences. For instance, if F is a predicate of type $\langle\sigma\rangle$, then $\forall_\sigma(F)$ states that F applies to every entity of type σ (or, equivalently, that F is universal). Thus, to express that everything of type e is self-identical, we can write $\forall_e((\lambda x.x \equiv_e x))$, which can be abbreviated to the more usual $\forall x^e(x \equiv_e x)$ given our convention above. I will generally omit parentheses and type annotations whenever no confusion can arise and follow other standard conventions. For example, we may write $\forall x(x \equiv x)$ instead of $\forall x^e(x \equiv_e x)$ whenever it is clear from the context what the appropriate types are.

Here is how essentialist statements can be represented in this language. Let $H(s)$ stand for 'Socrates is human'. We can form the predicate $\lambda x.x \equiv s$, which invariably applies to Socrates, to form the sentence $\Box_{\lambda x.x\equiv s}H(s)$ expressing that it is essential to Socrates that Socrates is human. Analogously, we can talk about the nature of negation by using a predicate of type $\langle\langle\langle\rangle\rangle\rangle$. One such predicate is $\lambda O.\neg \equiv O$. We can then express that it lies in the nature of negation that Socrates is human by the sentence $\Box_{\lambda O.\neg\equiv O}H(s)$.[13] As mentioned above, in order to express essentialist statements whose subjects are collections of entities of different types we need to use lists of subscripts, since our predicates can only combine with terms of a fixed type. So, for example, we can formalize the claim that it lies in the nature of the collection of Socrates and negation that Socrates is human by subscripting the essentialist

[11] The last clause indicates that we are working with a so-called *λI-language* in which vacuous variable binding is not well-formed. This is mainly a matter of convenience. We could instead allow for vacuous variable binding and restrict the principle of β-conversion introduced below to its non-vacuous instances.

[12] We will later use the symbol '$\Box$' to express metaphysical necessity, hence the special notation for the essentialist operator where $n = 0$.

[13] Note that this claim is no more about the *expression* '$\neg$' than claims about Socrates' nature are about the *name* 'Socrates', and *mutatis mutandis* for other claims like this.

operator with the predicates $\lambda x.x \equiv s$ and $\lambda O.\neg \equiv O$ from above: $\Box_{\lambda x.x\equiv s,\lambda O.\neg\equiv O} H(s)$.

In order to succinctly express claims about the nature of particular entities, such as Socrates, negation, or redness, we use expressions of the form $[A]$ to abbreviate expressions of the form $\lambda x^{\sigma}.x \equiv A$, where A is a term distinct from x in which x does not occur free. Thus, truth in virtue of the nature of Socrates can be expressed by $\Box_{[s]}$, and truth in virtue of the nature of negation by $\Box_{[\neg]}$.[14] The claim that it lies in the collective nature of Socrates and negation that Socrates is human can then be simply written $\Box_{[s],[\neg]} H(s)$.

The formalism also allows us to express truth in virtue of the nature of all objects (entities of type e) and truth in virtue of the nature of all propositions (entities of type $\langle\rangle$) by subscripting the relevant essentialist operators with the predicates $\lambda x^{e}.x \equiv x$ and $\lambda p^{\langle\rangle}.p \equiv p$, which apply to all objects and all propositions, respectively. The essentialist operators $\Box_{\lambda x^{e}.x\equiv x}$ and $\Box_{\lambda p^{\langle\rangle}.p\equiv p}$, whose respective subjects are the collection of all objects and propositions, will be of special interest in our discussion of the reducibility of necessity to essence later.

Here are some more examples of formalized sentences:

1. (a) $\Box_{[s]} H(s)$

   (It is essential to Socrates to be human)

   (b) $\Box_{[E]} \forall x(E(x) \to C(x))$

   (It lies in the nature of *being an electron* that all electrons are negatively charged)

   (c) $\neg\Box_{N} H(s)$

   (It is not in the collective nature of the natural numbers that Socrates is human)

   (d) $\Box_{[\forall],[\neg],[\vee],[R]} \forall x(R(x) \vee \neg R(x))$

   (It lies in the collective nature of *being red*, universality, negation and disjunction that everything is either red or not red)

   (e) $\Box_{\lambda p^{\langle\rangle}.p\equiv p} \exists p\, p$

   (It is true in virtue of the collective nature of all propositions that there is a true proposition)

I will often pronounce sentences of our higher-order language by using terms such as 'entities', 'properties' and 'propositions'. Since it is often the case that sentences in a higher-order language cannot be synonymously translated into natural language, this talk of 'entities', 'properties' and 'propositions' should only be taken to be a rough ordinary language approximation of what is more perspicuously expressed in our higher-order language. However,

[14] Note that there are two distinct uses of the square brackets '[ ]': one for rigid types, and one for abbreviating predicates of the form $\lambda x^{\sigma}.x \equiv y$. The uses are systematically related in that the latter predicates 'rigidly' pick out the entities to which they apply; although they are not rigid in the syntactic sense of having rigid type, their logic is in many ways analogous to the logic of predicates of rigid type.

according to the view embraced here, this does not cast any doubt on whether the sentences are meaningful and capable of expressing truths.[15]

2.2 Some logical principles

I will now introduce the logical principles of HLE that will be presupposed in our discussion of the reduction of necessity to essence in Section 3. A full list of principles is given in Appendix A.1.

Our background logic will be classical higher-order logic with identity. The only principles explicitly about identity that will be assumed here are reflexivity and the substitutivity of identicals. We further assume that essence is factive and monotonic, in the following sense:

(T) $\Box_{F_1,\ldots,F_n}\phi \rightarrow \phi$

(Monotonicity) If X is a subcollection of Y, then if it lies in the nature of X that ϕ then it lies in the nature of Y that ϕ[16]

Monotonicity entails that the essence of a collection contains all the truths that are true in virtue of the nature of the members of the collection. For example, if it lies in the nature of Plato to be human, then it lies in the nature of all humans, and indeed all objects, that Plato is human, because Plato is one of the humans. We can think of the addition of new entities to the subject of an essentialist attribution as roughly analogous to the addition of new truths to some true theory, where the theorems of the old theory correspond to the propositions that are true in virtue of the collective nature of the entities in the original essentialist attribution.

Our use of lambda-abstraction is guided by the following standard principle:[17]

(β-conversion) $\phi \leftrightarrow \phi^*$, provided ϕ and ϕ^* are β-equivalent[18]

For example, β-conversion and the reflexivity of identity imply that the propositions expressed by the sentences $H(s)$ ('Socrates is human') and $(\lambda X.X(s))(H)$ ('Being human applies to Socrates') are identical.[19] The main role β-conversion plays in our theory is in enabling us to

[15]For a defense of this attitude toward higher-order languages, see Prior (1971) and Williamson (2013). See also Rayo and Yablo (2001), Dorr (2016) and Goodman (2017).

[16]In order to capture Monotonicity in our formal language we have to make sure that if the predicates $F_1, \ldots, F_n$ pick out some entities that form a (proper or improper) subcollection of the entities picked out by $G_1, \ldots, G_m$, then $\Box_{F_1,\ldots,F_n}\phi$ materially implies $\Box_{G_1,\ldots,G_m}\phi$. This is jointly captured by the principles MON1, MON2, Permutation, Idempotence, Separation, and Subtraction, listed in Appendix A.1. Whenever I appeal to Monotonicity in the text in a formal argument, the appeal should be understood as referring to one or more of these formal principles. Both (T) and Monotonicity are higher-order generalizations of the analogous first-order principles in Fine's LE. The first-order analog of Monotonicity can be expressed by a single axiom; see Fine (1995, p. 247).

[17]See Dorr (2016) for a detailed discussion of β-conversion.

[18]ϕ and ϕ^* are β-equivalent if ϕ^* is derived from ϕ by replacing some constituent of the form $(\lambda v_1 \ldots v_n.\psi)(t_1, ..., t_n)$ with $\psi[t_i/v_i]$, where $\psi[t_i/v_i]$ is the sentence that results from replacing each free occurrence of v_1 in ψ with t_1, and each free occurrence of v_2 with t_2, and so on, replacing bound variables in such a way that no free variables in any t_i become bound.

[19]Because $(H(s) \equiv H(s)) \leftrightarrow (H(s) \equiv (\lambda X.X(s))(H))$ is an instance of β-conversion.

introduce a natural and well-behaved notion of *essential involvement*. Intuitions about what is or isn't involved in the nature of some entity often guide verdicts about what is essential to what. For example, one reason for denying that it is essential to Socrates that he be a member of his singleton set is that, intuitively, his singleton set is not *involved* in his nature. Similarly, it is natural to suppose that it's not in the nature of being an electron that all red things are colored, on the grounds that redness or *being colored* are plausibly not involved in (or don't "figure in") the nature of being an electron. Unlike Fine's LE, which construes the relation of essential involvement (which he calls 'dependence') as one that can only obtain between *objects*, the framework of HLE allows us to construe the relation as one that can obtain between entities expressed by any grammatical category, as illustrated by the last example. As we will see below, the ability to capture this unrestricted notion of essential involvement turns out to be of central systematic importance.

Formally, we define, for any $n \geq 0$, type τ and one-place predicate types $\sigma_1, \ldots, \sigma_n$, a predicate $\succeq_{\sigma_1,\ldots,\sigma_n,\tau}$ of type $\langle \sigma_1, \ldots, \sigma_n, \tau \rangle$:

$$(F_1, \ldots, F_n) \succeq_{\sigma_1,\ldots,\sigma_n,\tau} x =_{df} \exists P^{\langle \tau \rangle} \Box_{F_1,\ldots,F_n} P(x)$$

In words: for the nature of the collection of entities picked out by $F_1, \ldots, F_n$ to involve x^τ is for there to be some property $P^{\langle \tau \rangle}$ such that it lies in the nature of the collection that x has P. In what follows, we will normally suppress the types of the predicates on the left-hand side and simply write $\succeq_\tau$ for $\succeq_{\tau_1,\ldots,\tau_n,\tau}$. We further define:

$$c_\tau(F_1, \ldots, F_n) =_{df} \lambda x^\tau.(F_1, \ldots, F_n) \succeq_\tau x.$$

The predicate $c_\tau(F_1, \ldots, F_n)$ is satisfied by all and only those entities of type τ that are involved in the nature of the collection of entities picked out by $F_1, \ldots, F_n$. For example, $c_e([s])$ is satisfied by all and only the *objects* involved in Socrates' nature and $c_{\langle\rangle}([\wedge])$ is satisfied by all and only the *propositions* involved in the nature of conjunction. It will be useful to introduce a special notation for the case in which an entity is involved in the nature of a *particular* entity. To this end, we define, for any types σ and τ, a predicate $\geq_{\sigma,\tau}$ of type $\langle \sigma, \tau \rangle$:

$$x \geq_{\sigma,\tau} y =_{df} \exists P^{\langle \tau \rangle} \Box_{[x]} P(y)$$

For example, for y^τ to be involved in Socrates' nature is for there to be some property $P^{\langle \tau \rangle}$ such that it lies in the nature of Socrates that y has P.[20] In what follows, I will often omit type subscripts and write $\geq$ instead of $\geq_{\sigma,\tau}$.

To illustrate, suppose that Socrates is essentially human ($\Box_{[s]} H(s)$). Then since $H(s)$ and $(\lambda X.X(s))(H)$ are β-equivalent, it follows that the nature of Socrates involves the

[20] Notice that $x \geq_{\sigma,\tau} y$ is the same as $([x]) \succeq_{\langle \sigma \rangle,\tau} y$.

property of being human (because by existential generalization, $\Box_{[s]}(\lambda X.X(s))(H)$ implies $\exists Y^{\langle\langle e\rangle\rangle}\Box_{[s]}Y(H)$, which is just $s \geq H$). Suppose now that the nature of Socrates does not involve sethood ($\neg(s \geq \text{set})$). It then follows that it is not in the nature of Socrates that he be a member of some set, since otherwise Socrates' nature would involve sethood.[21] We can also capture the perhaps less familiar idea that a *logical* operation may or may not be involved in the nature of a collection of entities. For example, if Socrates is essentially human and wise ($\Box_{[s]}(H(s) \wedge W(s))$), then his nature involves conjunction ($s \geq \wedge$).[22] As we will see in Section 2.3, the idea that a logical operation may or may not be involved in the nature of some entities plays an important part in explaining under what conditions the essence of a collection of entities manifests any type of logical closure.

Two formal features of essential involvement are worth mentioning before we move on. First, essential involvement is transitive:

($\geq$-Transitivity) $t \geq_{\tau,\sigma} s \wedge s \geq_{\sigma,\rho} r \rightarrow t \geq_{\tau,\rho} r$

If the nature of x involves y and the nature of y involves z, then the nature of x involves z as well. $\geq$-Transitivity follows from the principle of Inheritance introduced in the next section. Essential involvement is also reflexive:

($\geq$-Reflexivity) $s \geq_{\sigma,\sigma} s$

This should be relatively uncontroversial given our understanding of essential involvement. Among the propositions that are true in virtue of the nature of x there should be at least one that predicates something of x. In HLE, $\geq$-Reflexivity follows from the following two principles guiding the logic of rigid predicates:

(R-Comp) $\forall X^{\langle\sigma\rangle}\exists Y^{[\sigma]}\forall x^{\sigma}(X(x) \leftrightarrow Y(x))$

(Rigidity) $F^{[\sigma]}(x) \rightarrow \Box_F F(x)$

R-Comp says that for any property of any type there is a rigid property coextensive with it.[23] Rigidity says that if x is one of some entities, then it lies in the nature of those entities that x is one of them. Let us call the rigid property that applies to x and only x the *haecceity* of x.[24]

[21] *Proof*. $\Box_{[s]}\exists x(\text{set}(x) \wedge s \in x)$ implies $\Box_{[s]}(\lambda X.\exists x(X(x) \wedge s \in x))(\text{set})$ by β-conversion, whence $\exists Y \Box_{[s]}Y(\text{set})$ follows by existential generalization.

[22] *Proof*. Recall that $\phi \wedge \psi$ is shorthand for $\wedge(\phi, \psi)$. By β-conversion, we infer (b) $\Box_{[s]}(\lambda X.X(H(s), W(s)))(\wedge)$ from (a) $\Box_{[s]}\wedge(H(s), W(s))$. By existentially generalizing on $(\lambda X.X(H(s), W(s)))$ in (b) we obtain $\exists X \Box_{[s]}X(\wedge)$, which is just $s \geq \wedge$.

[23] R-Comp is analogous to a comprehension principle for pluralities, according to which for every property, there is a plurality coextensive with it. The relevant principle for pluralities is often restricted to properties with a non-empty extension, a restriction we don't impose for rigid properties. Compare Linnebo (2017) and Burgess (2004).

[24] The uniqueness presupposition is justified by the aforementioned extensionality principle for rigid properties. Although this principle is officially included in HLE, it will not be presupposed in our discussion of the reduction theses in the next section, and is therefore not listed here.

When F is the haecceity of x, we say that $F(x)$ is the *haecceity proposition* of x. Rigidity and the fact that every entity is in the extension of its haecceity entail that it is essential to x that its haecceity proposition be true. So for every x, the essentiality of its haecceity proposition witnesses the reflexivity of essential involvement.[25]

2.3 Logical closure

One of the key differences between metaphysical necessity and essence is that unlike metaphysical necessity, the notion of essence is not in general closed under logical consequence. For example, we may plausibly deny that the number two's being essentially even entails that it is essentially even or human, on the grounds that the nature of the number two presumably doesn't involve the property of being human. The Problem of Logical Closure is the problem of specifying under what conditions the nature of a collection manifests any type of logical closure. HLE provides a general and systematic solution to this problem. In HLE, any type of logical closure in the essence of a collection derives from the logical operations that are involved in the essence of the collection. Thus, for example, the essence of a collection will be closed under modus ponens or conjunction introduction if and only if the essence of the collection involves the conditional or conjunction, respectively.[26]

The way in which essences are closed under logical consequence is roughly analogous to the way in which a deductively closed theory in a given language is closed under logical consequence. Think of the entities that are involved in the nature of a given collection as a kind of metaphysical analog of the language of a deductively closed theory; these entities can be thought of as the "language" in which the essence is stated. A deductively closed theory is closed under precisely those logical consequences that are expressible in the language of the theory. If its language does not include the conjunction symbol, say, then no sentence in which that symbol occurs will be part of the theory; if it does contain the conjunction symbol and we assume that the symbol is guided by the usual classical laws for conjunction, then the theory will be closed under conjunction elimination and introduction, and similarly for other logical constants. Assuming that we have names for every entity involved in the nature of a given collection C, let $\mathcal{L}_C$, the *language of* C, be the collection of terms designating entities involved in the nature of C. The essence of C is then closed under precisely those logical consequences that can be expressed in $\mathcal{L}_C$. For example, if $\mathcal{L}_C$ contains $\forall_e$ and it is

[25] A formal proof $\geq$-Reflexivity, which also invokes R-Comp, can be found in Appendix A.2 under Proposition 2.

[26] The way HLE handles the Problem of Logical Closure marks a crucial difference to LE. Since LE deals with the idealized constrained consequential notion of essence, it licenses a very permissive type of logical closure. Roughly speaking, in LE the essence of every entity is closed under qualitative consequence, where a sentence ϕ is a *qualitative consequence* of a class of sentences Γ if ϕ is a consequence of Γ and ϕ expresses a qualitative proposition. Thus, e.g. the proposition that everything is either human or not human is true in virtue of the nature of any given entity according to LE. In LE it also follows from the number two's being essentially even that it is essentially even or human.

true in virtue of the nature of C that $\forall x F(x)$, then it is also true in virtue of the nature of C that $F(a)$, provided a is in $\mathcal{L}_C$, and similarly for other logical constants. Moreover, just as a deductively closed theory does not contain sentences involving vocabulary that is not part of the language of the theory, so too the essence of a collection does not contain "extraneous entities", i.e. entities that are not involved in the nature of the collection.

Formally, this account of the logical closure of essences is jointly captured by the following two principles ($\vdash$ denotes theoremhood in HLE):

(RC) If $\vdash \phi_1 \wedge \ldots \wedge \phi_n \to \psi$, then $\vdash \Box_{F_1,\ldots,F_k}\phi_1 \wedge \ldots \wedge \Box_{F_1,\ldots,F_k}\phi_n \to \Box_{F_1,\ldots,F_k,[A_1],\ldots,[A_m]}\psi$, where $A_1, \ldots, A_m$ are all the constants and free variables occurring in ψ but not any of $\phi_1, \ldots, \phi_n$

(CH) $\Box_{G_1,\ldots,G_n,c_\sigma(F_1,\ldots,F_k)}\phi \to \Box_{G_1,\ldots,G_n,F_1,\ldots,F_k}\phi$;

Recall that $c_\sigma(F_1, \ldots, F_k)$ picks out the collection of entities of type σ involved in the nature of the collection picked out by $F_1, \ldots, F_n$. CH says that anything that is true in virtue of the nature of the former collection is true in virtue of the nature of the latter.[27] In other words, whatever is true in virtue of the collective nature of the entities involved in the nature of a given collection is true in virtue of the collection. The following consequence of CH is of special importance:

(Inheritance) $(F_1, \ldots, F_n) \succeq_\sigma x \wedge \Box_{G_1,\ldots,G_k,[x]}\phi \to \Box_{G_1,\ldots,G_k,F_1,\ldots,F_n}\phi$.

Inheritance says that if the nature of some collection involves an entity x, then whatever is true in virtue of the nature of x is also true in virtue of the nature of the collection; the collection "inherits" the nature of the entities that are involved in its nature.

RC and Inheritance jointly entail that the nature of a collection is closed under all and only those logical consequences that are statable in terms of vocabulary expressing entities that are involved in the nature of the collection. For example, the nature of a collection is closed under conjunction elimination/introduction just in case the nature of the collection involves conjunction. Closure under conjunction elimination follows from RC alone. Suppose it lies in the nature of Socrates to be human and wise ($\Box_{[s]}H(s) \wedge W(s)$). It then follows immediately by RC that Socrates is both essentially human ($\Box_{[s]}H(s)$) and essentially wise ($\Box_{[s]}W(s)$). The rationale for this is that the fact that Socrates is essentially human and wise entails that his nature involves conjunction ($s \geq \wedge$). To show closure under conjunction introduction, we need to invoke Inheritance. For suppose (a) that Socrates is essentially human and (b) that he is essentially wise. RC then only allows us to infer (c) that it is essential to Socrates *together with conjunction* that Socrates is essentially human and wise ($\Box_{[s],[\wedge]}H(s) \wedge W(s)$).

[27] CH is a higher-order generalization of the "chaining axiom" from LE; see Fine (1995).

If we additionally assume that Socrates's nature involves conjunction, Inheritance permits us to infer from (c) that he is essentially human and wise.

In HLE, it is consistent to deny, for example, that the number two's being essentially even entails that the number two is essentially even or human ($\Box_{[2]}(E(2) \vee H(2))$. RC merely allows us to infer $\Box_{[2],[\vee],[H]}(E(2) \vee H(2))$ from $\Box_{[2]}E(2)$; so unless the nature of the number two involves both disjunction and the property of being human, the problematic entailment is blocked. However, even if its nature involves disjunction, which would entail $\Box_{[2],[H]}(E(2) \vee H(2))$, we can still consistently deny $\Box_{[2]}(E(2) \vee H(2))$, so long as the nature of the number two does not involve the property of being human, since no "extraneous entities" can be introduced via logical closure.

A special case of the Problem of Logical Closure concerns the logical truths. It would be natural to assume that those necessities that are expressed by logical truths, such as 'Everything is self-identical' or 'Everything is either red or not red', have their source at least in part in the essences of the relevant logical operations. The special case of RC where $n = 0$ predicts just that:

> (RC^0) If $\vdash \psi$, then $\vdash \Box_{[A_1],\ldots,[A_m]}\psi$, where $A_1, \ldots, A_m$ are all the constants and free variables occurring in ψ

It is a consequence of RC^0 that every logical truth whose only constants are logical constants is true in virtue of the nature of the logical operations expressed by these constants. So, for example, since $\forall x(x \equiv x)$ is a logical truth whose only constants are $\forall$ and $\equiv$, RC^0 entails $\Box_{[\forall],[\equiv]}\forall x(x \equiv x)$. RC^0 also entails $\Box_{[\forall],[\neg],[\vee],[R]}\forall x(R(x) \vee \neg R(x))$, since $\forall x(R(x) \vee \neg R(x))$ ('Everything is either red or not red') is a logical truth whose only constants are $\forall$, $\neg$, $\vee$, and R. However, RC^0 does *not* entail that this logical truth is true *solely* in virtue of the nature of universality, negation, and disjunction; it only entails that it is true in virtue of the nature of these logical operations *together with* redness. In general, RC^0 does not entail that every logical truth is true *solely* in virtue of some logical operations, since this would implausibly entail that the collective nature of the logical operations involves absolutely every entity of any type.

Our discussion of the Problem of Logical Closure has illustrated what might be the most salient difference between the logic of metaphysical necessity and the logic of essence: metaphysical necessity is closed under logical entailment, whereas essence is, in general, not. Indeed, the account of logical closure presented here allows for there to be essences that are "logically inert", in the sense of not being closed under logical consequence at all. But we have also seen that, although the essentialist operators fail to obey certain modal principles, such as the closure under conjunction introduction, they do obey qualified versions of these principles. For example, while $\Box_{[x]}\phi$ and $\Box_{[x]}\psi$ together do not generally imply $\Box_{[x]}(\phi \wedge \psi)$, they do imply $\Box_{[x],[\wedge]}(\phi \wedge \psi)$. In general, exactly which logical principles a given subscripted essentialist operator $\Box_{F_1,\dots,F_n}$ obeys crucially depends on which entities are picked out by $F_1, \dots, F_n$. And nothing I've said rules out that there *are* subscripted essentialist operators that are closed under logical consequence in the same way as metaphysical necessity is. In fact, our solution to the Problem of Logical Closure has provided a necessary and sufficient condition for this: the nature of a collection of entities is closed under unrestricted consequence just in case it involves absolutely every entity of any type.[28]

2.4 Iteration principles

Given that essentiality is expressed by means of operators in HLE, it is natural to ask whether these operators can be iterated. For example, do we have an essentialist analog of the 4-schema for metaphysical necessity, according to which every necessary truth is necessarily necessary? That is, does it lie in the nature of x that it lies in the nature of x that ϕ whenever it lies in the nature of x that ϕ? Formally: Do we have $\Box_F\phi \to \Box_F\Box_F\phi$, where F is the haecceity of x? One reason to doubt that this holds in general is that it would entail that essentiality (i.e. what is expressed by the essentialist operator $\Box_\sigma$) is involved in the nature of any x, because $\Box_F\Box_F\phi$ implies $x \geq \Box_\sigma$, provided F is the haecceity of x.[29]

But even if the exact analog of the 4-schema for metaphysical necessity doesn't hold for essence, it is worth asking whether a qualified version of it holds. One natural qualification to consider is to add essentiality to the subject of the essentialist attribution in the consequent: $\Box_F\phi \to \Box_{F,[\Box_\sigma]}\Box_F\phi$.[30] Thus qualified, and assuming that F is the haecceity of x, the principle states that whenever it lies in the nature of x that ϕ, then it lies in the nature of x *together with essentiality* that it lies in the nature of x that ϕ. This qualified version of the principle avoids the consequence that essentiality is involved in the nature of x.

A promising argument for the qualified principle stems from the fact that it provides a natural essentialist explanation of the plausible assumption that it is not possible for anything to lose any of its essence: if $\Box_F\phi$, then necessarily, $\Box_F\phi$. For $\Box_{F,[\Box_\sigma]}\Box_F\phi$ implies that it is necessary that $\Box_F\phi$ by the principle that essence implies necessity: whatever is true in virtue of the nature of some entities is necessarily true. It is natural to suppose that the impossibility of an entity to lose any of its essence flows in part from the nature of essentiality itself. We will therefore adopt the following generalization of our qualified 4-schema, formulated in our

[28] Sufficiency follows at once from RC, Inheritance and Monotonicity. For necessity, let x be some entity whose nature doesn't involve every entity of every type (the argument for non-single membered collections is exactly analogous). Let y be an entity such that $\neg(x \geq y)$. Then $\neg\Box_{[x]}y \equiv y$, since otherwise $\Box_{[x]}(\lambda z.z \equiv z)(y)$ by β-conversion, and thus $\exists P\Box_{[x]}P(y)$, which is just $x \geq y$, in contradiction to our hypothesis.

[29] By the β-equivalence of $\Box_F\phi$ and $(\lambda O.O_F\phi)(\Box_\sigma)$.

[30] One might, in addition, add the property of being F, so that the principle would read: $\Box_F\phi \to \Box_{F,[\Box_\sigma],[F]}\Box_F\phi$. However, in the presence of the principle R-Equiv (see Appendix A.1), this weakening implies the seemingly stronger principle in the text. R-Equiv says that whatever lies in the nature of a rigid property lies in the collective nature of the entities in its extension, and vice versa; the nature of a rigid property is thus exhausted by the collective nature of the entities that it applies to.

formal language:

(4) $\Box_{F_1,\ldots,F_n}\phi \rightarrow \Box_{F_1,\ldots,F_n,[\Box_{\sigma_1,\ldots,\sigma_n}]}\Box_{F_1,\ldots,F_n}\phi$, whenever $F_1^{\sigma_1}, ..., F_n^{\sigma_n}$ are rigid predicates[31]

We can similarly ask whether we have an essentialist analog of the 5-schema for metaphysical necessity, according to which what is not necessary is necessarily not necessary. As in the case of the 4-schema, an exact analog for essence is implausible. Such an exact analog would say that if it is not essential to x that ϕ, then it is essential to x that it is not essential to x that ϕ. But this would implausibly entail not only that the nature of any x involves essentiality, but also that it involves negation, and most problematically, any proposition ϕ not true in virtue of the nature of x. However, we can again consider a qualified version of this principle. The following principle, stated in our formal language, is a natural candidate:

(5) $\neg\Box_{F_1,\ldots,F_n}\phi \rightarrow \Box_{F_1,\ldots,F_n,[\neg],[\Box_{\sigma_1,\ldots,\sigma_n}],[\phi]}\neg\Box_{F_1,\ldots,F_n}\phi$, whenever $F_1^{\sigma_1}, ..., F_n^{\sigma_n}$ are rigid predicates[32]

In (5) we have added essentiality as well as two additional elements to the subject of the outer essentialist operator in the consequent of (5), namely negation and the proposition ϕ. The latter two entities have to be added in order to avoid the problem, mentioned above, that the nature of the entities picked out by $F_1, \ldots, F_n$ may not involve ϕ or negation.[33]

I take (5) to be less evident than (4). We can give an argument for (5) that is parallel to the argument for (4) above, although the first step of the argument is less straightforward in the case of (5) than it is in the case of (4). The reason is that it is less clear that if ϕ is *not* essential to the entities picked out by $F_1, \ldots, F_n$, then this is necessarily so. While it is very plausible to hold that it is not possible that entities *lose* any of their essence, it seems less obvious that their essence couldn't have been richer. This line of thought is closely related to denials of the B-schema for metaphysical necessity—the principle that whatever is the case is necessarily possibly the case—and corresponding denials of the necessity of distinctness. Suppose x and y are actually distinct but could have been identical. Then whatever is essential to x would be essential to y and vice versa in a circumstance in which

[31]The restriction to rigid predicates is required here, since the inner essentialist operator in the consequent should have the same subject as the operator in the antecedent. The schema does not in general hold for non-rigid predicates. For example, suppose that Mary is in fact the only person wearing a hat and that it is essential to the persons wearing a hat that Mary is human. Then it needn't be true that it is essential to the persons wearing a hat, taken together with essentiality, that it is essential to the persons wearing a hat that Mary is human, since it may not be the case that it is essential to Mary to be wearing a hat, and so it may not be essential to whoever wears a hat that Mary is human. Such a case is averted if the predicates in (4) rigidly pick out Mary.

[32]Principle (5), like (4), is subject to the restriction that the predicates be rigid. The motivation for this is analogous to the motivation in the case of (4).

[33]We didn't need to add ϕ in the case of (4) because, by Inheritance, it would be redundant given that the nature of the entities picked out by $F_1, \ldots, F_n$ already involves ϕ, in view of the antecedent of (4).

they are identical. In particular, it would be essential to x to be identical to y.[34] It is thus possibly essential to x to be identical to y, although x is, by hypothesis, not identical to y, and thus not essentially identical to y. This would be a counterexample to (5), since (5) implies that whenever ϕ is not true in virtue of the nature of x, then ϕ is necessarily not true in virtue of the nature of x, by the principle that essence implies necessity. The idea that it is possible for some entities to have more essential features than they actually have is intriguing, and it seems to me that it is not obviously false. Still, I will tentatively adopt (5) here, although it is worth emphasizing that it is not required for the reduction of necessity to essence I will defend in the next section. Whenever (5) is needed, I will explicitly say so.

This concludes the presentation of our theory. The next section examines the question of whether metaphysical necessity can be reduced to essence in the setting of HLE.

3 Reducing metaphysical necessity to essence

Now that we have our theory of essence in place, we can ask how to extend it to a joint theory of essence and metaphysical necessity. The theory HLE does not feature an operator for metaphysical necessity, but such an operator could of course be added, together with appropriate axioms and rules. Let $\Box$ be an operator expressing metaphysical necessity. At the very least, we should add the K- and T-schemas for $\Box$ and close the resulting theory under the rule of $\Box$-necessitation (RN$^\Box$):

(K$^\Box$) $\Box(\phi \to \psi) \to (\Box\phi \to \Box\psi)$

(T$^\Box$) $\Box\phi \to \phi$

(RN$^\Box$) If $\vdash \phi$, then $\vdash \Box\phi$

It is worth noting that the following controversial schemas become theorems upon adding the above principles:[35]

(CBF$^\Box$) $\Box\forall x^\sigma\phi \to \forall x^\sigma\Box\phi$

(NNE$^\Box$) $\Box\forall x^\sigma\Box\exists y(x \equiv y)$

CBF$^\Box$ is the converse Barcan formula (see Barcan (1946)). NNE$^\Box$ expresses (first- and higher-order) *necessitism*, the view that necessarily everything (of any type) is necessarily

[34] Or to x together with identity, if we want to avoid that x's nature involves the property of identity ($x \geq \equiv$). Note that $\Box_{[x],[\equiv]}x \equiv x$, but not $\Box_{[x]}x \equiv x$, is a theorem of HLE by RC^0.

[35] Note that the derivation of CBF$^\Box$ only requires K$^\Box$ and RN$^\Box$, and that of NNE$^\Box$ only RN$^\Box$, given a classical quantificational logic.

something.[36] Avoiding these consequences requires either adopting a free quantificational logic or giving up $\mathrm{RN}^{\Box}$.[37] I will pursue neither of those options here, though. As we will see below, $\mathrm{RN}^{\Box}$ enjoys independent support in the present context. And since HLE is based on a classical quantificational logic, adopting a free logic would amount to giving up HLE, and thus the background theory of essence for our investigation.[38]

Slightly more controversially, we should presumably also add the 4-schema for metaphysical necessity to obtain all the principles of S4 for $\Box$:[39]

($4^{\Box}$) $\Box\phi \to \Box\Box\phi$

If we take the logic of metaphysical necessity to further include all the principles of S5, we need to supplement the system with another principle. One standard principle that will do is the B-schema for metaphysical necessity:

($\mathrm{B}^{\Box}$) $\neg\phi \to \Box\neg\Box\phi$

In the presence of the $\mathrm{B}^{\Box}$-schema, the Barcan formula becomes derivable:

($\mathrm{BF}^{\Box}$) $\forall x^{\sigma}\Box\phi \to \Box\forall x^{\sigma}\phi$

$\mathrm{BF}^{\Box}$ entails that there could not have been more entities of any type. Note that $\mathrm{BF}^{\Box}$ is *not* a theorem of the above system without $\mathrm{B}^{\Box}$. Consequently, giving up $\mathrm{BF}^{\Box}$ does not require giving up any principles of HLE. The status of $\mathrm{BF}^{\Box}$ and $\mathrm{B}^{\Box}$ will come under scrutiny later.

In addition to these purely modal principles, it would be natural to consider principles of interaction between $\Box$ and the essentialist operators. One particularly plausible principle of this kind is the widely accepted principle that essence implies necessity, or, in other words, that every essential truth is necessary, which we can express by the following schema:

($\mathrm{EN}^{\Box}$) $\Box_{F_1,\ldots,F_n}\phi \to \Box\phi$

Given $\mathrm{EN}^{\Box}$, $\mathrm{RN}^{\Box}$ becomes redundant, since it immediately follows from RC^0 and $\mathrm{EN}^{\Box}$. A defense of $\mathrm{EN}^{\Box}$ will be provided in Section 3.4. For now, I will adopt the principle as a plausible working hypothesis.

Once we adopt $\mathrm{EN}^{\Box}$, it is very natural to wonder whether some kind of converse of it holds as well: Is it the case that whenever ϕ is metaphysically necessary, it is true in virtue of

[36] See Williamson (2013) for a book-length defense of first- and higher-order necessitism. See Kripke (1963) and Stalnaker (2003b) for first-order modal logics that invalidate first-order necessitism, and Stalnaker (2011) and Fine (1977) for defenses of higher-order *contingentism*, the negation of higher-order necessitism. See Fritz and Goodman (2016) for a detailed discussion of higher-order contingentism.

[37] See Williamson (2013, pp. 39 ff.).

[38] This is not to say that these options aren't worth exploring. Quite the contrary. I think the investigation of variants of HLE based on a free logic is of great interest. But such an investigation must be left for another occasion.

[39] See Salmon (1989) for a prominent critique of the 4-schema and Williamson (1990, ch. 8) for critical discussion.

the nature of some entities? Another question that arises in this connection is whether there is an operator expressible in the language of HLE without $\Box$ that plays the logical role of $\Box$. In the following sections, I will explore three theses that give an affirmative answer to both of these questions. All of these theses say that metaphysical necessity is just a special case of essence. The question of whether we should prefer one of these reductive hypotheses over the above-mentioned extensions of HLE with a primitive metaphysical necessity operator will be discussed in Section 3.4.

In what follows, it will be useful to introduce a shorthand for referring to certain characteristic modal principles where $\Box$ is uniformly replaced by another unary operator. When $\Phi^{\Box}$ is one of the schemas above and O is some unary operator, we use Φ^{O} to indicate the result of uniformly replacing $\Box$ by O in the schema. So, for example, $\mathrm{EN}^{\Box_F}$ denotes the schema $\Box_{F_1,\ldots,F_n}\phi \to \Box_F\phi$, and $\mathsf{4}^{\Box_F}$ the schema $\Box_F\phi \to \Box_F\Box_F\phi$.

3.1 Objectual Reduction

The first thesis I want to explore is due to Fine (1994, p. 9):

> Objectual Reduction
>
> For a proposition to be metaphysically necessary is for it to be true in virtue of the nature of all objects.

The Objectual Reduction offers a simple account of the connection between essence and metaphysical necessity: the connection is one of identity—metaphysical necessity just is truth in virtue of the nature of all objects, and thus a special case of essence. If the Objectual Reduction is true, we do not need to add a new operator to HLE, because one of the essentialist operators *is* metaphysical necessity. The essentialist operator that expresses truth in virtue of the nature of all objects (entities of type e) is $\Box_{\lambda x^e . x \equiv x}$, which I will abbreviate as $\Box_\Omega$. The Objectual Reduction can then be formally expressed as the claim that $\Box_\Omega$ expresses metaphysical necessity.

The tenability of the Objectual Reduction crucially depends on whether $\Box_\Omega$ obeys the same logical principles as metaphysical necessity as well as the principle that essence implies necessity. Given the Objectual Reduction, the latter principle says that every proposition that is true in virtue of the nature of any entities (of any type) is true in virtue of the nature of all *objects*. Unfortunately, HLE does not guarantee the truth of this principle. For example, it is consistent with HLE that it is not true in virtue of the nature of any *objects* that all red things are colored even if it lies in the nature of redness that all red things are colored. The reason for this is that nothing in HLE requires the nature of any object or collection of objects to involve such properties as being red or any of the logical operations. This entails further that it is consistent with HLE that necessities expressed by logical truths, such as ‘Everything is self-identical' or 'All red things are red', are not true in virtue of the nature of all objects, and similarly for many other logical truths that are uncontroversially necessary. Thus, $\Box_\Omega$ is not subject to necessitation, i.e. ϕ may be a theorem of HLE without $\Box_\Omega\phi$ being a theorem, even if we restricted necessitation to uncontroversially necessary logical truths like the above. Moreover, while the $\mathrm{T}^{\Box_\Omega}$- and $\mathrm{K}^{\Box_\Omega}$-schemas both hold in HLE,[40] the $\mathsf{4}^{\Box_\Omega}$- and $\mathsf{B}^{\Box_\Omega}$-schemas are not derivable. Thus, if we want to maintain the Objectual Reduction, we need to add further principles concerning the nature of objects to HLE.

Here is one principle that addresses the problem head-on:

(R) $\forall x^\tau \exists F^{\langle e \rangle} \Box_F x \equiv_\tau x$

The axiom says that for any entity x of any type, there are some *objects* such that it is true in virtue of the nature of these objects that x is self-identical. This entails that every entity is involved in the nature of some collection of objects. The problem that there may be no objects whose nature involves a given property or logical operation is thereby resolved.

However, the adoption of (R) may seem like an *ad hoc* fix. Is there any reason for assuming (R) that is independent of the Objectual Reduction? One natural way to motivate (R) would be to endorse some form of Platonism, positing for every entity of type $\neq e$ some objectual surrogate or *Form* of type e that stands in a specific relationship to the higher-order entity. For example, a Platonist might think that to be human is to exemplify the Form *humanity*. Where H stands for 'human', ε stands for 'exemplifies' and h stands for 'humanity', we can express this by the following identification: $H^{\langle e \rangle} \equiv_{\langle e \rangle} \lambda x^e.\varepsilon^{\langle e,e \rangle}(x, h)$. In addition, the Platonist could plausibly maintain that this identification is true in virtue of the nature of the Form *humanity*: $\Box_{[h]}(H \equiv \lambda x.\varepsilon(x, h))$. But by the substitutivity of identicals, this immediately implies $\Box_{[h]} H \equiv H$, witnessing an instance of (R).[41]

In order to justify (R), the Platonist would of course have to be able to give a more general argument. She would have to show that the argument above, or something analogous, generalizes to every entity of higher type. This could perhaps be done by endorsing some restricted version of the following comprehension schema:

(P-Comp) $\forall X^{\langle \sigma_1, \ldots, \sigma_n \rangle} \exists y^e (X \equiv \lambda x_1^{\sigma_1} \ldots x_n^{\sigma_n}.\varepsilon(x_1, \ldots, x_n, y))$, where $n \geq 0$

[40] The $\mathrm{K}^{\Box_\Omega}$-schema is an immediate consequence of RC, while the $\mathrm{T}^{\Box_\Omega}$-schema is just a special case of the T-schema of HLE.

[41] According to an alternative, ground-theoretic, version of Platonism, the Form *humanity* is such that one's exemplifying it is *grounded* in one's being human; see e.g., Dixon (2018). Formally: $\forall x(H(x) < \varepsilon(x, h))$, where $<$ stands for non-factive full ground. Such a Platonist might again plausibly maintain that this grounding claim is true in virtue of the nature of the Form *humanity*: $\Box_{[h]} \forall x(H(x) < \varepsilon(x, h))$, which implies $h \geq H$, witnessing an instance of a slight weakening of (R) to the effect that every entity is involved in the nature of some collection of objects. Such a weakening of (R) would also suffice for the purpose at hand, though of course this ground-theoretic version of Platonism—just like the version in the main text—would have to be generalized to every entity of higher type in order to justify (R) (or its weakening). The obvious generalization is, however, as paradox-prone as that of the Platonist theory in the main text. Thanks to an anonymous referee here.

Without any restriction, P-Comp leads to inconsistency, due to Russell's paradox.[42] But if the Platonist could come up with a plausible restriction, the argument above, together with the restricted version of P-Comp, might be used to derive (R). I will, however, not further pursue this question here because there is a weaker, consistent Platonist theory that yields (R) as well. I will only sketch the view here. The basic idea is that instead of positing the existence of objectual surrogates for entities of every type, we only posit objectual surrogates for propositions. Call this theory *Propositional Platonism*. Propositional Platonism can be characterized by introducing a primitive predicate, *true*, of type $\langle e \rangle$, governed by the following axiom:

(P-Truth) $\forall p^{\langle\rangle} \exists x^e \Box_{[x]}(true(x) \equiv_{\langle\rangle} p)$

P-Truth immediately implies $\forall p^{\langle\rangle} \exists X^{\langle e \rangle} \Box_X p \equiv_{\langle\rangle} p$, and thus all instances of (R) in which $\tau = \langle\rangle$. But it turns out that we can also infer all other instances of (R) from this alone.[43] In contrast to the stronger Platonist theory above, Propositional Platonism is provably consistent.[44]

There is thus at least one systematic and consistent way of motivating (R). But since (R) is clearly weaker than any of the Platonist theories above and there may also be other ways of motivating (R), I will remain neutral here on what justification a proponent of (R) should espouse and instead focus on the consequences of adopting (R).[45] Let me now turn to these consequences. Let HLE_R be the result of adding (R) to HLE. We first observe that $\mathrm{EN}^{\Box_\Omega}$ holds in HLE_R. Intuitively, the reason is that if p is true in virtue of the nature of some collection of entities $x_1, x_2, \ldots$, then these entities are involved in the nature of some objects, by (R). So these objects "inherit" the nature of $x_1, x_2, \ldots$, which entails that p is true in virtue of the nature of the objects. By Monotonicity, it follows that p is true in virtue of the nature of all objects.[46]

We can further show that $\Box_\Omega$ is subject to necessitation in HLE_R. Suppose ϕ is a theorem of HLE_R. Then by RC^0, $\Box_{[A_1],\ldots,[A_n]}\phi$ is a theorem, where $A_1, \ldots, A_n$ are all the constants and free variables occuring in ϕ. Hence, by $\mathrm{EN}^{\Box_\Omega}$ and modus ponens, $\Box_\Omega \phi$ is a theorem as well. As mentioned above, the $\mathrm{K}^{\Box_\Omega}$- and $\mathrm{T}^{\Box_\Omega}$-schemas, and hence also $\mathrm{CBF}^{\Box_\Omega}$ and $\mathrm{NNE}^{\Box_\Omega}$, already hold in HLE. The case of the $\mathsf{4}^{\Box_\Omega}$- and $\mathsf{B}^{\Box_\Omega}$-schemas is less straightforward, but the

[42] Instantiate the variable X with $\lambda x.\neg\varepsilon(x,x)$.

[43] *Proof*. Let x^τ be arbitrary. Then $x \equiv_\tau x$ is of type $\langle\rangle$. Hence, by P-Truth, $\exists y^e \Box_{[y]}(true(y) \equiv_{\langle\rangle} (x \equiv_\tau x))$, and thus $\exists y^e (y \geq x \wedge y \geq \equiv_\tau)$ by β-conversion. But $\Box_{[x],[\equiv_\tau]} x \equiv_\tau x$ is a theorem of HLE (by RC^0), and so $\exists y^e \Box_{[y]} x \equiv_\tau x$ follows by Inheritance and Monotonicity. So $\exists F^{\langle e \rangle} \Box_F x \equiv_\tau x$. Since x was arbitrary, we get $\forall x^\tau \exists F^{\langle e \rangle} \Box_F x \equiv_\tau x$.

[44] A semantic consistency proof is given in Ditter (MS).

[45] An alternative motivation for (R) might derive from certain plenitudinous views about material objects. Different versions of such views are described in, e.g. Bennett (2004), Fine (1999), Leslie (2011) and Yablo (1987), See Fairchild (2020) for an overview. As Fairchild (2019) shows, there are some delicate issues involved in the formulation of plenitudinous views about material objects.

[46] See Proposition 5 in Appendix A.2.

$\mathsf{4}^{\Box_\Omega}$-schema is in fact derivable in $\mathsf{HLE_R}$, though the $\mathsf{B}^{\Box_\Omega}$-schema is not. Thus, $\Box_\Omega$ obeys all the principles of $\mathsf{S4}$ in $\mathsf{HLE_R}$.[47] In order to obtain all the principles of $\mathsf{S5}$ we would need to add another axiom. The most promising candidate is Fine's 'domain axiom (ii)' from LE (Fine, 1995, p. 250):[48]

(DOM) $\forall x^e P^{[e]}(x) \to \Box_P \forall x P(x)$

DOM says that it is true in virtue of the nature of all objects that they are all the objects there are (note that the predicate P in DOM is rigid). In the presence of DOM, $\mathrm{B}^{\Box_\Omega}$, and therefore also $\mathrm{BF}^{\Box_\Omega}$, become derivable.[49] A detailed discussion of DOM is postponed until Section 3.5, where I will show that even without assuming the Objectual Reduction, the question of whether DOM is true is intimately connected with the Barcan formula and the B-schema for metaphysical necessity. Until then, I will remain neutral on whether a proponent of $\mathsf{HLE_R}$ should accept DOM.

The conjunction of $\mathsf{HLE_R}$ and the Objectual Reduction offers a simple and compelling joint theory of essence and metaphysical necessity. Yet, as it stands, it crucially relies on the controversial axiom (R). Although we have seen that there is a way of motivating (R) by adopting some form of Platonism, it is worth inquiring whether there are other ways of reducing necessity to essence in the setting of HLE that are not committed to (R). A natural suggestion would be to take metaphysical necessity to be based in the nature of *propositions* (entities of type $\langle\rangle$) instead of objects. There are different forms such a reduction could take. One proposal would be to take the metaphysical necessities to be the propositions that are true in virtue of the nature of *all* propositions, in parallel to the Objectual Reduction. Another natural proposal would be to identify the metaphysically necessary propositions with the propositions that are true in virtue of their own nature. The next two sections will discuss these two proposals in turn.

3.2 Universal propositional reduction

I begin with the propositional analog of the Objectual Reduction:

Universal Propositional Reduction

For a proposition to be metaphysically necessary is for it to be true in virtue of the nature of all propositions.

[47] For a proof of $\mathsf{4}^{\Box_\Omega}$, see Proposition 6 in Appendix A.2. We can in fact show by semantic methods that $\Box_\Omega$ obeys exactly the propositional modal logic $\mathsf{S4}$ in $\mathsf{HLE_R}$; see Ditter (MS).

[48] I have adopted the label 'DOM' from Ditter (2020), where the principle is discussed in the context of LE. One could of course also just add $\mathrm{B}^{\Box_\Omega}$ directly, but this would be rather *ad hoc*.

[49] See Proposition 10 in Appendix A.2. Note that the proof of $\mathrm{B}^{\Box_\Omega}$ requires the essentialist ($\mathsf{5}$)-schema from Section 2.4. It is again possible to show that $\Box_\Omega$ obeys exactly the propositional modal logic $\mathsf{S5}$ in $\mathsf{HLE_{R+DOM}}$.

The operator that expresses truth in virtue of the nature of all propositions is $\Box_{\lambda p^{\langle\rangle}.p\equiv p}$, which I will abbreviate as $\Box_\Pi$. Interestingly, $\Box_\Pi$ already obeys all the principles of the modal logic $\mathsf{S4}$ as well as $\mathrm{EN}^{\Box_\Pi}$ in HLE. In the case of the Objectual Reduction, we had to appeal to (R) in order to link the natures of entities of type $\neq e$ to the natures of objects. The reason why we needn't do the analogous thing in the case of the Universal Propositional Reduction is that the nature of every entity of any type is already reflected in the nature of some proposition by virtue of the principles of HLE alone. More specifically, let x be an entity of any type. Then there is a proposition—the *haecceity proposition of x* introduced in Section 2.2—whose nature involves x; hence, by Inheritance, everything that is true in virtue of the nature of x is true in virtue of the nature of its haecceity proposition.

To make this precise, we introduce the following abbreviation. Let $F_1, \ldots, F_n$ be one-place predicates. Then

$$\mathfrak{h}(F_1, \ldots, F_n) =_{df} \lambda p^{\langle\rangle}.(\exists x(F_1(x) \wedge \exists X^{[\sigma_1]}(\forall z(X(z) \leftrightarrow x \equiv_{\sigma_1} z) \wedge p \equiv X(x))) \vee \cdots \vee \exists x(F_n(x) \wedge \exists X^{[\sigma_n]}(\forall z(X(z) \leftrightarrow x \equiv_{\sigma_n} z) \wedge p \equiv X(x))))$$

The predicate $\mathfrak{h}(F_1, \ldots, F_n)$ applies to all and only the haecceity propositions of the entities picked out by $F_1, \ldots, F_n$.[50] In HLE, every essentialist claim is logically equivalent to one with a single subscript picking out the haecceity propositions of the entities that are the subject of the original essentialist claim.[51]

(Haec) $\Box_{F_1,\ldots,F_n}\phi \leftrightarrow \Box_{\mathfrak{h}(F_1,\ldots,F_n)}\phi$

$\mathrm{EN}^{\Box_\Pi}$ is an immediate consequence of Haec and Monotonicity. The fact that $\Box_\Pi$ is subject to necessitation is in turn an immediate consequence of RC^0 and $\mathrm{EN}^{\Box_\Pi}$. The $\mathrm{K}^{\Box_\Pi}$- and $\mathrm{T}^{\Box_\Pi}$-schemas, and thus also $\mathrm{CBF}^{\Box_\Pi}$ and $\mathrm{NNE}^{\Box_\Pi}$, are again immediate. Moreover, it turns out that the $\mathsf{4}^{\Box_\Pi}$-schema is derivable in HLE, but the $\mathrm{B}^{\Box_\Pi}$-schema is not. So $\Box_\Pi$ obeys all the principles of $\mathsf{S4}$, but not all the principles of $\mathsf{S5}$ in HLE. To additionally obtain all the principles of $\mathsf{S5}$ as well as $\mathrm{BF}^{\Box_\Pi}$, it is necessary to strengthen the system. In analogy to the case of $\Box_\Omega$, it is sufficient to add an axiom to the effect that it is true in virtue of the nature of all propositions that they are all the propositions:[52]

($\mathrm{DOM}^{\langle\rangle}$) $\forall x^{\langle\rangle} P^{[\langle\rangle]}(x) \rightarrow \Box_P \forall x P(x)$

[50] It is worth noting that it is not generally the case that the haecceity proposition of an entity x is identical to the haecceity proposition of an entity y (of the same type) only if x is identical to y. Otherwise there would be a one-one correspondence between the domain of properties of propositions and the domain of propositions, which is ruled out by the 'Russell-Myhill Antinomy'. The result, which can be proved by a Cantorian diagonalization argument, goes back to Russell (1903) (Appendix B) and was independently rediscovered by Myhill (1958). See Klement (2010a) for a succinct presentation of this result and its relation to Cantor's theorem.

[51] See Proposition 3 in Appendix A.2.

[52] Note that the proof of $\mathrm{B}^{\Box_\Pi}$ requires the essentialist (5)-principle from Section 2.4. One could again also just add $\mathrm{B}^{\Box_\Pi}$ directly, but this would again be rather *ad hoc*.

I will discuss the status of $\mathrm{DOM}^{\langle\rangle}$ and its bearing on the question of whether $\Box_\Pi$ obeys all the principles of $\mathsf{S5}$ in Section 3.5.

The Universal Propositional Reduction compares favorably with the Objectual Reduction in HLE. For one thing, it doesn't require any commitment to the controversial principle (R); and for another, it is more parsimonious in its posits, suggesting itself quite naturally from the principles of HLE alone, as we have just seen. The fact that $\Box_\Pi$, unlike $\Box_\Omega$, already obeys the characteristic logical principles of metaphysical necessity in HLE (or $\mathsf{HLE}_{\mathsf{DOM}^{\langle\rangle}}$ if we thought that metaphysical necessity obeys $\mathsf{S5}$), makes it a prima facie natural candidate for being metaphysical necessity. It is worth noting, however, that in the presence of (R), it is an immediate consequence of $\mathrm{EN}^{\Box_\Omega}$ and $\mathrm{EN}^{\Box_\Pi}$ that anything that is true in virtue of the nature of all objects is true in virtue of the nature of all propositions, and vice versa. Thus, given (R), the difference between the Objectual Reduction and the Universal Propositional Reduction becomes immaterial for most theoretical purposes, since the respective candidates for expressing metaphysical necessity are logically coextensive.

3.3 Intrinsic propositional reduction

Let me now turn to the other propositional reduction mentioned before.

> INTRINSIC PROPOSITIONAL REDUCTION
>
> For a proposition to be metaphysically necessary is for it to be true in virtue of its own nature.

Prima facie, the idea that the necessity of a proposition is determined solely by the nature of the proposition itself has a lot of intuitive appeal. To put it in epistemic terms, in order to know whether a proposition is necessary it suffices to have a fully transparent grasp of the nature of that proposition; we don't need to look beyond its nature to determine whether it is necessary. On this proposal, the necessity of a proposition is "intrinsic" to the proposition; every necessary proposition is its own source of necessity.[53]

This is in stark contrast to our previous two reduction theses for which it was not always possible to identify a *particular* object or proposition as the source of the necessity of a given proposition. For example, the Universal Propositional Reduction only entails that if p is necessary, then it is true in virtue of the nature of all propositions (equivalently, that it is true

[53] This is closely related to what Van Cleve (2018) calls 'intrinsically explained' or 'inherently intelligible' necessity—a kind of necessity that belongs 'only to those necessary truths that are true in virtue of the natures of their own constituents.' (p. 18). This idea is in turn related to the notion of *in se* necessity invoked by Leibniz, Wolff and Baumgarten; see Stang (2016). The appeal to "constituents" of propositions should be treated with caution, however, at least insofar as it presupposes a structured account of propositions, since such a theory is arguably inconsistent, due to the Russell-Myhill Antinomy (see note 50). For recent discussions of the implications of the Russell-Myhill Antinomy for theories of structured propositions, see Dorr (2016), Fritz et al. (2021), Goodman (2017), Klement (2010a,b), and Uzquiano (2015).

in virtue of the nature of some propositions), but not that it is true in virtue of the nature of some *specific* proposition. The Intrinsic Propositional Reduction, by contrast, always provides us with a particular such instance, namely the proposition itself whose necessity is in question.

The operator that expresses that a proposition is true in virtue of its own nature is $\lambda p.\Box_{[p]}p$, which I will abbreviate as $\Box_I$. Let us call a proposition that is true in virtue of its own nature *intrinsically necessary* for short. As in the case of the other reductions, we need to check whether $\Box_I$ obeys the logical principles of metaphysical necessity and the principle that essence implies necessity ($\mathrm{EN}^{\Box_I}$). It turns out that $\Box_I$ satisfies neither $\mathrm{EN}^{\Box_I}$ nor all the principles of $\mathsf{S4}$ in HLE. So, like the Objectual Reduction, the Intrinsic Propositional Reduction requires additional assumptions.

The most natural option would be to simply add $\mathrm{EN}^{\Box_I}$ to HLE. If, as suggested above, the idea that *all* necessities are intrinsically necessary enjoys intuitive support, then $\mathrm{EN}^{\Box_I}$ does so as well. Once we add $\mathrm{EN}^{\Box_I}$, we also obtain all the principles of $\mathsf{S4}$ for $\Box_I$, because by $\mathrm{EN}^{\Box_I}$, we have $\Box_\Pi\phi \to \Box_I\phi$, and by Monotonicity we have the converse as well. Hence, in $\mathsf{HLE}_{\mathsf{EN}^{\Box_I}}$, $\Box_I$ and $\Box_\Pi$ are logically coextensive, so the fact that $\Box_\Pi$ obeys all the principles of $\mathsf{S4}$ immediately implies that $\Box_I$ also obeys those principles.

Unfortunately, though, despite its initial appeal, $\mathrm{EN}^{\Box_I}$ seems to be subject to counterexamples. Consider the claim that it lies in the collective nature of two entities, say Socrates and his singleton set, that they are distinct. Given $\mathrm{EN}^{\Box_I}$, it follows that it lies in the nature of the proposition that there are at least two objects *that* there are at least two objects. Formally: Given $\mathrm{EN}^{\Box_I}$, $\Box_{[a],[b]}\neg(a \equiv b)$ implies $\Box_{[\exists x\exists y\neg(x\equiv y)]}\exists x\exists y\neg(x \equiv y)$.[54]

But this is prima facie implausible, at least if we take the usual principles of quantificational logic as our guide to the nature of a proposition that is expressed by a sentence whose only constants are logical. For the negation of the *sentence* 'There are at least two objects' is consistent in classical quantificational logic; the usual logical principles governing the existential quantifier, identity, and negation do not entail that there are at least two objects. So the fact that it is not a logical truth that there are at least two objects would seem to be good evidence for thinking that it is not in the collective nature of the existential quantifier, identity, and negation that there are at least two objects, and hence that it is not intrinsically necessary that there are at least two objects.[55] Many philosophers share the view that logic is not supposed to tell us how many objects there are,[56] and the claim that it does not lie in the nature of any logical operations alone that there are at least two objects is a natural

[54]*Proof*: Suppose $\Box_{[a],[b]}\neg(a \equiv b)$. By RC and existential generalization: $\Box_{[a],[b],[\exists],[\neg]}\exists x\exists y\neg(x \equiv y)$. By $\mathrm{EN}^{\Box_I}$, this implies $\Box_{[\exists x\exists y\neg(x\equiv y)]}\exists x\exists y\neg(x \equiv y)$.

[55]In HLE, $\Box_{[\exists x\exists y\neg(x\equiv y)]}\exists x\exists y\neg(x \equiv y)$ and $\Box_{[\exists],[\equiv],[\neg]}\exists x\exists y\neg(x \equiv y)$ are in fact logically equivalent by the axioms of Decomposition (see the appendix) and Inheritance, though I'm not relying on this logical equivalence here; even if they were not in general logically equivalent, it is plausible enough to suppose that the former is true only if the latter is.

[56]See Etchemendy (1990) and Hanson (1997) for discussion.

essentialist counterpart of this claim.[57] Even if it is *metaphysically* necessary that there are at least two objects, and indeed that there are infinitely many objects, one may want to deny that this is due solely to the nature of some logical operations (rather than, in addition, the nature of (say) all the natural numbers).

This judgment can be resisted, however, if one took the account of logical truth proposed by Tarski (1936) as a guide to the nature of the logical operations. One consequence of this conception of logical truth is that every sentence that contains only logical vocabulary is either logically true or logically false.[58] On such a view, a sentence like $\exists x \exists y \neg(x \equiv y)$ is logically true if true at all, and the claim that it is intrinsically necessary that there are at least two objects might be taken as a natural essentialist reflection of this view. The force of the above counterexample may therefore depend to some extent on what conception of logical truth one takes as a guide to the nature of the logical operations.

But there are other cases that, it seems, cannot be dealt with in this way. Consider the claim that it lies in the nature of Socrates to be human. Given $\text{EN}^{\Box_I}$, it follows from this that it lies in the nature of something being human that something is human; in other words, it is intrinsically necessary that there are humans.[59] Formally: Given $\text{EN}^{\Box_I}$, $\Box_{[s]}H(s)$ implies $\Box_{[\exists x H(x)]}\exists x H(x)$.[60] More generally, $\text{EN}^{\Box_I}$ entails that whenever some entity has a property P essentially, then it is intrinsically necessary that P is instantiated. Similarly, suppose (plausibly) that it lies in the nature of all propositions that something is not a natural number (because it lies in the nature of all propositions that I am not a natural number, for example). It then follows from $\text{EN}^{\Box_I}$ that it is intrinsically necessary that something is not a natural number.

These consequences seem quite implausible. Intuitively, it would seem to be perfectly compatible with the collective nature of *being human* and existence that there be no humans; the fact that there are humans can't plausibly be "read off" the collective nature of being human and existence.[61] Likewise, it is natural to think that it is perfectly compatible with the collective nature of *being a natural number* and all the logical operations that everything is a natural number; the nature of this collection doesn't demand there to be anything that's

[57] Fine (1995, p. 251) suggests a principle in the context of LE according to which, for any finite number of objects, it is 'logically possible' that there be exactly this number of objects. It is natural to interpret the appeal to 'logical possibility' there to be tantamount to something like 'compatibility with the nature of the logical operations'.

[58] See Williamson (2000) for a defense of this account of logical truth.

[59] Or perhaps more plausibly: suppose that Socrates is essentially human if concrete; it follows that it is intrinsically necessary that something is human if concrete.

[60] The proof is similar to the proof of the entailment in the previous example.

[61] The same considerations apply, *mutatis mutandis*, to the modified case suggested in a previous footnote where we suppose that Socrates is essentially human if concrete. Note that in HLE, $\Box_{[\exists x H(x)]}\exists x H(x)$ and $\Box_{[\exists],[H]}\exists x H(x)$ are in fact logically equivalent by the axioms of Decomposition (see the appendix) and Inheritance, though I'm not relying on their equivalence here, but merely on the plausible idea that if the latter is false, the former is false as well.

not a natural number. In explaining why it is metaphysically necessary that something is not a natural number, for instance, one would naturally point to some object whose nature precludes it from being a natural number, rather than to the natures of *being a natural number* and perhaps some logical operations.

Taken together, it seems to me that these consequences count considerably against $\mathrm{EN}^{\Box_I}$, and thus against the Intrinsic Propositional Reduction. I conclude that, in the absence of a compelling argument for accepting these consequences, the Intrinsic Propositional Reduction should be rejected. That being said, the intrinsically necessary truths still constitute a very interesting subclass of the metaphysically necessary truths that seems well worth investigating.

3.4 A general argument for an essentialist reduction of necessity

So far the case for the reduction theses above has consisted in showing that $\Box_\Pi$ and $\Box_\Omega$ are natural candidates for playing the role of metaphysical necessity on the grounds that they satisfy the same logical principles as metaphysical necessity. However, all by itself, this is consistent with $\Box_\Pi$ and $\Box_\Omega$ being not even coextensive with metaphysical necessity. This section provides an argument for their necessary coextensiveness, thereby providing further evidence for their identification. (For definiteness, I will focus on the Universal Propositional Reduction here, but the arguments below also apply to the Objectual Reduction.)

Metaphysical necessity is often characterized as the *broadest* necessity, or as necessity *in the highest degree*.[62] According to a standard way of spelling this out, the idea is that a proposition is metaphysically necessary if and only if it is necessary in every 'objective' (Williamson (2016)) or 'real' (Rosen (2006)) sense of necessity. Besides metaphysical necessity, paradigmatic examples of objective (or, equivalently, real) necessities include nomic necessity, biological necessity, and practical necessity. The objective modalities are taken to exclude the doxastic and epistemic modalities as well as the deontic and teleological modalities. According to Williamson, 'Objective modalities are non-epistemic, non-psychological, non-intentional. Thus they are not sensitive to the guises under which the objects, properties, relations and states of affairs at issue are presented' (ibid. p. 454).[63] Moreover, they are plausibly required to satisfy at least some minimal logical requirements, such as that (i) a conjunction is necessary just in case all of its conjuncts are necessary, and that (ii) every tautology is necessary.[64]

A key question in the characterization of objective necessity is whether the category of objective necessity respects essence, in the sense that, whenever something is an essential truth, it is necessary in every objective sense of necessity. If it does, then the role of the broadest

[62] See Kripke (1980) for a canonical reference. See also Fritz (2017), Hale (1996, 2013), Rosen (2006), Stalnaker (2003a), Williamson (2016) and Van Inwagen (1998). See Bacon (2018) and Clarke-Doane (2019a,b) for dissenting views.

[63] See also Rosen (2006, pp. 14 ff.), Nolan (2011, pp. 315 f.), and Rayo (2013, p. 49).

[64] See Williamson (2016, p. 456).

objective necessity is tightly constrained, since the assumption that objective necessity respects essence entails:[65]

(EON) $\Box_\Pi \phi \to L\phi$

for any objective necessity operator L. Say that an objective necessity operator L_1 is *at least as broad as* another objective necessity operator L_2 if and only if $L_3(L_1 p \to L_2 p)$, for every objective necessity operator L_3 and proposition p.[66] If true, (EON) is plausibly necessary in every objective sense of necessity. But $\Box_\Pi$ is clearly itself an objective necessity, so it is at least as broad as any other objective necessity.[67] Now if metaphysical necessity is the broadest objective necessity, then metaphysical necessity is at least necessarily coextensive with $\Box_\Pi$. So $\Box_\Pi$ is a very natural candidate for expressing metaphysical necessity. But even if we lift the assumption that metaphysical necessity is the broadest objective necessity, the argument would still establish that $\Box_\Pi$ is the broadest objective necessity up to necessary coextensiveness.

The principle (EON) strikes me as extremely compelling. Suppose that Socrates is essentially human. Could there be an objective sense of possibility according to which it is possible for Socrates not to be human? It would seem that any such possibility would not be a possibility *for* Socrates—it would not be an objective way for Socrates to be. One might be tempted to think that there is some kind of "logical possibility" according to which Socrates could fail to be human, a possibility that somehow corresponds to the sentential predicate of logical consistency. The paradigm cases of "logically necessary" truths would presumably be truths like $\forall p(p \vee \neg p)$ and $H(s) \to \neg\neg H(s)$ and a characteristic feature of this kind of possibility would be that contradictions are not possible. However, even if there is an operator that works like this and does not obey (EON)—which is not obvious—such a sense of possibility and necessity would seem to arbitrarily privilege the nature of the logical operations while ignoring the nature of other kinds of entities. This would prompt the question of whether there are other kinds of necessity that respect only Socrates' nature, for instance, or only the natures of concrete objects, and so on. As a limiting case, there would presumably also be a kind of necessity that respects absolutely *no* essences, which would lead to the conclusion that there is an objective sense of necessity according to which nothing whatever is necessary and everything is possible, contradicting the requirement that all tautologies be objectively necessary.

[65] A similar principle is endorsed by Rayo (2013, ch. 5), although Rayo presupposes a notion of essence that is not more fine-grained than metaphysical necessity.

[66] Bacon (2018) gives the same definition of *being at least as broad as* for necessity operators in general, i.e. without restricting it to objective necessity operators.

[67] Note that this is not to say that *every* essentialist operator is an objective necessity operator. For instance, if the nature of conjunction does not involve Socrates, then it is not true in virtue of the nature of conjunction that Socrates is human or not human.

One might object that it is not in fact arbitrary to privilege the (collective) nature of the logical operations in this way: respecting only the nature of the logical operations marks a natural, non-arbitrary stopping point for delineating the objective necessities, whereas respecting absolutely no essences does not. And it would seem as though there are many stricter necessities that respect the nature of the logical operations and some other entities. For example, the *mathematical necessities* might be taken to be the necessities that respect the (collective) nature of all mathematical entities. Indeed, Fine (1994) seems to suggest a view along these lines right after proposing his view that the metaphysically necessary truths are the propositions which are true in virtue of the nature of all objects:

> Other familiar concepts of necessity (though not all of them) can be understood in a similar manner. The conceptual necessities can be taken to be the propositions which are true in virtue of the nature of all concepts; the logical necessities can be taken to be the propositions which are true in virtue of the nature of all logical concepts; and, more generally, the necessities of a given discipline, such as mathematics or physics, can be taken to be those propositions which are true in virtue of the characteristic concepts and objects of the discipline. (Fine, 1994, pp. 9-10)

However, the concepts of 'conceptual'-, 'logical'-, and 'mathematical' necessity so defined are not concepts of necessity in the usual sense even according to Fine's own theory of essence. Take the propositions true in virtue of the logical operations (or concepts, if you will). If we assume that the nature of the logical operations does not involve Socrates, then it won't even be true in virtue of the nature of the logical operations that Socrates is identical to himself, or that he is either human or not human.[68] But this is clearly against the spirit of the intended notion of 'logical necessity', and it disqualifies the notion from being an objective necessity in the usual sense because it doesn't even apply to all tautologies. Exactly analogous considerations apply to 'conceptual'- and 'mathematical' necessity characterized in this way. Fine's remarks above are therefore more plausibly interpreted as characterizations of interesting *subclasses* of metaphysical necessities rather than different *notions* of *necessity* that are broader than metaphysical necessity.

That being said, my aim here is not to deny that there may be something that could be called a "modality" which is stricter than metaphysical necessity and which respects the essences of the logical operations while failing to respect all essences. The point is that, from an essentialist standpoint, there is good reason not to qualify any such "modality" as objective, i.e. as pertaining to how things could *genuinely* have been. Singling out the logical operations

[68] Note that this is not peculiar to HLE, but also predicted by Fine's LE. In fact, Fine (1995) suggests that the operator $\Box_{\lambda x.\neg(x=x)}$ in LE can be taken to express truth in virtue of the nature of the logical concepts, and it is a theorem of LE that $\neg\Box_{\lambda x.\neg(x=x)}(H(s) \vee \neg H(s))$.

as special is objectionably arbitrary because the essences of the logical operations do not have any special status compared to the essences of any other entities; the necessities that flow wholly or partly from their nature are not more "strictly" necessary than necessities flowing from the nature of non-logical entities such as redness. It is not more plausible to maintain, e.g., that it is objectively possible for there to be red things that are not colored than to think that it is objectively possible that not everything is self-identical. A view that privileges the essences of the logical operations in this way would seem rather unprincipled. It is far more natural and less arbitrary to require the category of objective necessity to respect all essences rather than just those of the logical operations.

The case for (EON) thus appears to be very strong. At the very least, it seems to me that the class of objective necessity operators that satisfy (EON) are of special interest, and the broadest such operator would appear to be an extremely natural candidate for deserving the name "metaphysical necessity", which is a term of art, after all. One could still question our argument for the necessary coextensiveness of $\Box_\Pi$ and metaphysical necessity by questioning whether metaphysical necessity is in fact the broadest objective necessity. If metaphysical necessity is not the broadest objective necessity, then the entailment from a proposition being metaphysically necessary to it being true in virtue of the nature of all propositions is blocked. But this option seems unpromising. If metaphysical necessity is not the broadest objective necessity, it is difficult to see what else it could be.[69] Metaphysical necessity would then seem to be akin to a kind of restricted quantification that is supposed to be of major importance for philosophical theorizing but for which the nature of the restriction remains obscure—unlike the qualification for other kinds of restricted necessity such as nomic necessity, the qualification "metaphysical" is not of much help here.[70]

The Universal Propositional Reduction offers a compelling explanation of the necessary coextensiveness between truth in virtue of the nature of all propositions and metaphysical necessity: they are necessarily coextensive because they are identical. Conversely, their necessary coextensiveness provides strong support for the reduction. For most theoretical purposes, the difference between the thesis that metaphysical necessity *just is* truth in virtue of the nature of all propositions and the thesis that they are merely necessarily coextensive doesn't matter. However, their identification *does* considerably simplify the joint theory of metaphysical necessity and essence. Instead of adopting one of the extensions of HLE containing the additional operator $\Box$ mentioned at the beginning of Section 3, we can simply

[69]Note that the truth of (EON) in every objective sense of necessity already rules out there being *no* broadest objective necessity (at least up to necessary coextensiveness). So it is no option to argue against identifying metaphysical necessity with the broadest objectve necessity by holding that there is no broadest such necessity.

[70]One might alternatively try to tie metaphysical necessity to some alleged paradigm cases proposed by Kripke (1980), such as the claim that Nixon is not an inanimate object, the necessity of origins, or certain supervenience theses. But this leaves the notion far too unconstrained and it furthermore problematically suggests that these alleged paradigm cases are in some sense uncontroversial. See Dorr (2016, p. 69) for relevant discussion.

work in HLE. Thus considerations of simplicity and elegance mark another point in favor of the Universal Propositional Reduction over the thesis of mere necessary coextensiveness.

3.5 Does metaphysical necessity obey S5?

What is the correct logic of metaphysical necessity, given either of our two remaining candidate reductions? We have seen that the propositional logic of both $\Box_\Omega$ and $\Box_\Pi$ contains at least all theorems of $\mathsf{S4}$, but I have left open in my discussion of the Objectual and Propositional reductions whether they also obey all of the principles of $\mathsf{S5}$. I now turn to this matter.

Recall that in order to obtain all the principles of $\mathsf{S5}$ for $\Box_\Omega$ or $\Box_\Pi$ we need to supplement the respective theories with the axioms DOM or DOM$^{\langle\rangle}$, respectively. Note that if we added B$^{\Box_\Omega}$ directly to $\mathsf{HLE_R}$, then DOM would become derivable, and likewise for B$^{\Box_\Pi}$ and DOM$^{\langle\rangle}$ in HLE. The logics $\mathsf{HLE_{R+DOM}}$ and $\mathsf{HLE_{R+B^{\Box_\Omega}}}$ thus have the same theorems, and likewise for $\mathsf{HLE_{DOM^{\langle\rangle}}}$ and $\mathsf{HLE_{B^{\Box_\Pi}}}$. Furthermore, in $\mathsf{HLE_{R+DOM}}$, DOM$^{\langle\rangle}$ becomes derivable, so the theory $\mathsf{HLE_{R+DOM}}$ properly includes the theory $\mathsf{HLE_{DOM^{\langle\rangle}}}$, but not vice versa. Given the necessary coextensiveness of metaphysical necessity and $\Box_\Pi$, rejecting DOM$^{\langle\rangle}$ is thus tantamount to rejecting the B-schema for metaphysical necessity in this setting.

In the framework of Fine's LE, the addition of DOM to the base theory $\mathsf{E5}$ is incompatible with certain true instances of the 'independence principle', which says that it is compatible with the nature of any object x that there be nothing except the entities involved in the nature of x (see Ditter (2020, p. 366 ff.)).[71] For example, let Simple be some object whose nature does not involve any other objects (Simple might be a mereological simple, for instance), and suppose that Simple is not the only object there is. In Fine's system $\mathsf{E5_{DOM}}$, it follows from this that it is essential to Simple that Simple is not the only object there is, and thus that there are objects that are not involved in Simple's nature, contradicting the independence principle. I have argued that the incompatibility of $\mathsf{E5_{DOM}}$ with even a single such instance of the independence principle constitutes a weighty reason for rejecting $\mathsf{E5_{DOM}}$. This raises the question of whether a similar problem arises for $\mathsf{HLE_{DOM^{\langle\rangle}}}$ or $\mathsf{HLE_{R+DOM}}$. Since the logic HLE differs importantly from Fine's LE—apart from the first-order vs. higher-order difference, they are concerned with different notions of essence—it is not immediately obvious whether $\mathsf{HLE_{DOM^{\langle\rangle}}}$ and $\mathsf{HLE_{R+DOM}}$ are subject to the same problem. I will now argue, however, that a very similar issue arises for them.

The reason why the above-mentioned instance of the independence principle is incompatible with $\mathsf{E5_{DOM}}$ is that it is a theorem of $\mathsf{E5_{DOM}}$ that whenever some objects are not all the objects there are, then it lies in their nature not to be all the objects there are. So, for example, it lies in the nature of Simple that Simple is not the only object there is. Although this same principle does not hold in either $\mathsf{HLE_{DOM^{\langle\rangle}}}$ or $\mathsf{HLE_{R+DOM}}$, a very close variant of

[71] In the language of HLE, we can express this as follows: $\forall x(\forall y(F^{[e]}(y) \leftrightarrow c_e([x])) \to \neg\Box_{[x]}\neg\forall y F(y))$.

it does hold in $\mathsf{HLE}_{\mathsf{R+DOM}}$. This variant says (roughly), that if some objects are not all the objects there are, then it lies in their nature *together with the nature of some logical operations* that they are not all the objects there are. Formally:[72]

$$\text{(N-DOM}^e\text{)}\ \neg\forall x R^{[e]}(x) \rightarrow \Box_{R,[\forall_e],[\neg],[\rightarrow],[\Box_{[e]}]}\neg\forall x R(x)$$

Given N-DOM^e, we can infer that it lies in the nature of Simple, together with some logical operations, that Simple is not the only object there is. This is prima facie implausible. Why should the collective nature of Simple and some logical operations demand that there be *objects* (entities of type e) that are distinct from Simple? The only plausible reason why this should be so seems to be that the natures of the relevant logical operations involve some objects distinct from Simple. However, despite the initial implausibility, the Platonist justification of (R) discussed in Section 3.1 should in fact lead us to expect just that. Intuitively, the reason is that the natures of negation, universality, etc. require there to be Forms corresponding to these higher-order entities, and these Forms are plausibly distinct from Simple. Hence, it lies in the nature of Simple together with these logical operations that Simple is not the only object there is. So at least on the Platonist underpinnings of $\mathsf{HLE}_{\mathsf{R+DOM}}$, the above consequence of N-DOM^e has a principled explanation that defuses its initial implausibility.

Nevertheless, both $\mathsf{HLE}_{\mathsf{R+DOM}}$ and $\mathsf{HLE}_{\mathsf{DOM}^{\langle\rangle}}$ entail the following *propositional* version of N-DOM^e, according to which, if some *propositions* are not all the propositions there are, then it lies in their nature together with the nature of some logical operations that they are not all the propositions there are:

$$\text{(N-DOM}^{\langle\rangle}\text{)}\ \neg\forall p R^{[\langle\rangle]}(p) \rightarrow \Box_{R,[\forall_{\langle\rangle}],[\neg],[\rightarrow],[\Box_{[\langle\rangle]}]}\neg\forall p R(p)$$

We can construct a counterexample to N-DOM$^{\langle\rangle}$ as follows. Let C be the collection of all propositions that are involved in the collective nature of the natural numbers and all the logical operations. C thus contains all the haecceity propositions of the natural numbers and the logical operations as well as all the propositions true in virtue of the nature of these entities; but it plausibly doesn't contain absolutely all propositions: for example, it arguably doesn't contain any propositions that concern particular humans. N-DOM$^{\langle\rangle}$ entails that it lies in the nature of the propositions in C that they are not all the propositions there are. But this prompts the question of why the nature of the propositions in C should demand that there be propositions not among them. Unlike in the case of Simple above, there is no obvious way of making this consequence more palatable; (R) is of no help here, and the propositions involved in the nature of the logical operations are, by hypothesis, already contained in C. So an explanation analogous to the Platonist explanation of the above consequence of N-DOM^e is not available here. Although this counterexample to N-DOM$^{\langle\rangle}$ is perhaps not altogether

[72]See Proposition 11 in Appendix A.2 for a proof. Note that N-DOM^e is not derivable in $\mathsf{HLE}_{\mathsf{DOM}^{\langle\rangle}}$.

decisive, it seems to me to carry enough weight to warrant considerable doubt about N-DOM$^{\langle\rangle}$, and consequently about $\mathsf{HLE}_{\mathsf{DOM}^{\langle\rangle}}$ (and $\mathsf{HLE}_{\mathsf{R+DOM}}$).

Does rejecting $\mathsf{HLE}_{\mathsf{DOM}^{\langle\rangle}}$ motivate the rejection of $\mathsf{S5}$ for metaphysical necessity? Not necessarily. The derivation of N-DOM$^{\langle\rangle}$ in $\mathsf{HLE}_{\mathsf{DOM}^{\langle\rangle}}$ crucially depends on an application of RC^0 to DOM$^{\langle\rangle}$;[73] N-DOM$^{\langle\rangle}$ is not derivable without applying RC^0 to DOM$^{\langle\rangle}$.[74] So instead of giving up DOM$^{\langle\rangle}$ and thus $\mathsf{S5}$, one might restrict the application of RC to the theorems of HLE, thereby barring its application to DOM$^{\langle\rangle}$ and, more generally, to those theorems of $\mathsf{HLE}_{\mathsf{DOM}^{\langle\rangle}}$ that are not already theorems of HLE.[75]

Let $\mathsf{HLE}_{\mathsf{DOM}^{\langle\rangle}-}$ be the logic that results from $\mathsf{HLE}_{\mathsf{DOM}^{\langle\rangle}}$ by replacing RC with the restricted version of RC. In $\mathsf{HLE}_{\mathsf{DOM}^{\langle\rangle}-}$, $\Box_\Pi$ still satisfies all the principles of $\mathsf{S5}$.[76] The main difference to the situation in $\mathsf{HLE}_{\mathsf{DOM}^{\langle\rangle}}$ is that the theorems of $\mathsf{S5}$ that are not also theorems of $\mathsf{S4}$ have a different logical status from the theorems of $\mathsf{S4}$. Consider, for example, a universal generalization of an instance of the $\mathsf{B}^{\Box_\Pi}$-schema:

$$(*)\ \forall p(\neg p \to \Box_\Pi \neg \Box_\Pi p)$$

The only constants that occur in (*) are logical constants (recall that Π is short for $\lambda p.p \equiv_{\langle\rangle} p$). Hence, by RC^0 we can infer in $\mathsf{HLE}_{\mathsf{DOM}^{\langle\rangle}}$:

$$(**)\ \Box_{[\forall],[\neg],[\Box_{\langle\langle\rangle\rangle}],[\equiv],[\to]}\forall p(\neg p \to \Box_\Pi \neg \Box_\Pi p)$$

In words, (**) says that (*) is true in virtue of the nature of the logical operations denoted by the logical constants occurring in (*). Recall that this is a status enjoyed by every theorem of HLE (and $\mathsf{HLE}_{\mathsf{DOM}^{\langle\rangle}}$) whose only constants are logical. In $\mathsf{HLE}_{\mathsf{DOM}^{\langle\rangle}-}$, by contrast, (**) is not a theorem; indeed, it is subject to counterexample if the counterexample to N-DOM$^{\langle\rangle}$ above is correct. In $\mathsf{HLE}_{\mathsf{DOM}^{\langle\rangle}-}$, we merely have the $\Box_\Pi$-necessitation of (*):

$$(***)\ \Box_\Pi \forall p(\neg p \to \Box_\Pi \neg \Box_\Pi p)$$

[73] The situation is thus analogous to the case of $\mathsf{E5{+}DOM}$ discussed in Ditter (2020, p. 369). There, too, the incompatibility with the independence principle crucially depends on applying a necessitation-like rule to DOM.

[74] Note that the derivation of N-DOM$^{\langle\rangle}$ also depends on the essentialist $\mathsf{5}$-schema from Section 2.4; see Proposition 12 in Appendix A.2. However, giving up the essentialist $\mathsf{5}$-schema in order to preserve $\mathsf{S5}$ for $\Box_\Omega$ and $\Box_\Pi$ is not an option, since the proofs of the $\mathsf{5}^{\Box_\Pi}$- and $\mathsf{5}^{\Box_\Omega}$-schemas also depend on the essentialist $\mathsf{5}$-schema.

[75] Formally, this would amount to replacing the rule RC with:

(RC$^-$) If $\vdash_{\mathsf{HLE}} \phi_1 \wedge ... \wedge \phi_n \to \psi$, then $\vdash \Box_{F_1,\dots,F_k}\phi_1 \wedge ... \wedge \Box_{F_1,\dots,F_k}\phi_n \to \Box_{F_1,\dots,F_k,[A_1],\dots,[A_m]}\psi$, where $A_1,\dots,A_m$ are all the constants and free variables occurring in ψ but not any of $\phi_1,\dots,\phi_n$.

In Ditter (2020), I have suggested (without endorsing) an analogous move to save $\mathsf{S5}$ for metaphysical necessity in the context of LE.

[76] One might think that one would also have to add the $\Box_\Pi$-necessitation of DOM$^{\langle\rangle}$ in order to derive all theorems of $\mathsf{S5}$ for $\Box_\Pi$. This is not necessary, however, because DOM$^{\langle\rangle}$ implies its own $\Box_\Pi$-necessitation already in HLE, from which the $\Box_\Pi$-necessitation of $\mathsf{B}^{\Box_\Pi}$ can be derived in $\mathsf{HLE}_{\mathsf{DOM}^{\langle\rangle}-}$.

This feature of $\mathsf{HLE}_{\mathsf{DOM}^{\langle\rangle-}}$ may not constitute a decisive reason for rejecting $\mathsf{HLE}_{\mathsf{DOM}^{\langle\rangle-}}$, but it clearly makes the theory less unified and elegant compared to HLE. Taken together with the failure of $\mathsf{HLE}_{\mathsf{DOM}^{\langle\rangle}}$, this underlines the costs of maintaining $\mathsf{S5}$ for metaphysical necessity in the setting of HLE. Our main motivation for introducing $\mathsf{DOM}^{\langle\rangle}$ in the first place was to explore the possibility of maintaining $\mathsf{S5}$ for metaphysical necessity. In view of the problems and complications involved in maintaining $\mathsf{S5}$, it is worth questioning the motivations for $\mathsf{S5}$.

Perhaps the strongest reason for thinking that $\mathsf{S5}$ is the correct logic of metaphysical necessity is the simplicity and elegance of $\mathsf{S5}$ in comparison with other candidate modal systems, such as $\mathsf{S4}$. However, these simplicity considerations usually stem from viewing the propositional logic of metaphysical necessity in isolation or in the context of a pure quantificational logic without any further logical operators that might affect the logic of necessity. But the question of whether $\mathsf{S5}$ is simpler and more elegant than some other logic of metaphysical necessity depends not only on considerations of simplicity and elegance in such relatively isolated contexts; we ultimately need to consider the overall simplicity and systematicity of theories that attempt to cover a wider class of phenomena—possibly using further logical and non-logical vocabulary. A joint theory of essence and metaphysical necessity is one such case. And if, as I've argued above, $\Box_\Pi$ expresses metaphysical necessity (or is at least necessarily coextensive with metaphysical necessity) then considerations of simplicity, systematicity and elegance favor a logic of metaphysical necessity that is weaker than $\mathsf{S5}$, but not weaker than $\mathsf{S4}$, because they favor the overall theory HLE over $\mathsf{HLE}+\mathsf{DOM}^{\langle\rangle-}$.[77]

Of course, considerations of simplicity and elegance are not the only aspects that we should take into account in our theory choice. If there were strong independent reasons for accepting DOM$^{\langle\rangle}$ for example, they might override such considerations. But it's not clear that there are such reasons. The argument for DOM in Fine (1995, p. 250) essentially takes for granted the Barcan formula for metaphysical necessity, an assumption that is itself highly contentious.[78] In fact, the most comprehensive defense of the Barcan formula in the literature in turn relies at least in part on considerations of the simplicity and elegance of (higher-order) quantified $\mathsf{S5}$, in which the Barcan formula is derivable (Williamson (2013)). But these considerations are based on the more isolated context of (higher-order) quantified modal logic and, as we have seen, they do not carry over to the present, broader context of a joint theory of essence and metaphysical necessity.

I conclude, therefore, that our examination of what a joint theory of essence and metaphysical necessity might look like reveals that there are good reasons to think that the propositional logic of metaphysical necessity is weaker than $\mathsf{S5}$, though not weaker than $\mathsf{S4}$.

[77]It is worth noting that due to the essentialist $\mathsf{5}$-schema, we still have the necessity of distinctness in HLE, since $\neg(x \equiv y) \rightarrow \Box_\Pi \neg(x \equiv y)$ is derivable in HLE from the essentialist $\mathsf{5}$-schema.

[78]See Ditter (2020, pp. 359 ff.) for a discussion of Fine's argument.

4 Conclusion

In this paper I have proposed a joint theory of essence and metaphysical necessity according to which metaphysical necessity is just a special case of essence: metaphysical necessity just is truth in virtue of the nature of all propositions. I have argued that this reduction thesis presents itself as the simplest and most natural hypothesis about the relation between metaphysical necessity and essence once we spell out a general and systematic joint theory of these notions. The argument I have presented for the reduction thesis rests on abductive considerations and on the characterization of metaphysical necessity as the broadest objective necessity. Taken together, these considerations have also been shown to motivate the view that the logic of metaphysical necessity is weaker than $\mathsf{S5}$.

This revisionary conclusion about the logic of metaphysical necessity serves to illustrate that the project of reducing metaphysical necessity to essence is by no means bound to preserve all of our prior views about metaphysical necessity—the account may well lead to revisionary consequences. Indeed, one of the most promising philosophical upshots of investigating reductive accounts of metaphysical necessity (and other notions) is their potential to lead us to rethink our prior views about the reduced notion. Of course, the essentialist reductions of metaphysical necessity considered in this paper leave open many questions about which propositions are metaphysically necessary given these reductions. While many paradigm cases of metaphysically necessary truths, such as uncontroversially necessary logical truths like 'Everything is self-identical' or truths like 'All red things are colored', plausibly come out as metaphysically necessary given the essentialist reduction, it is an open question whether other putatively metaphysically necessary truths like, e.g., the necessity of origins or various supervenience theses do so as well. But if truth in virtue of the nature of all propositions really is the broadest objective necessity, then the failure to provide a plausible essentialist source for some putative necessity need not count as evidence against the essentialist account; rather, it might well give us reason to doubt the metaphysical necessity of the proposition in question. In my view, the fact that such questions arise is a virtue of the account, since it invites us to investigate the status of such propositions through a novel conceptual lens. Equipped with a systematic theory of essence, such an investigation strikes me as an important research program that affords us a new perspective on a number of issues concerning metaphysical modality.

A Appendix

A.1 The theory HLE

The theory HLE is constituted by the following axioms and rules.

(i) *Background logic*:

(PC) ϕ, whenever ϕ is a tautology

(UI) $\forall_\tau(F) \to F(a)$

(EG) $F(a) \to \exists_\tau(F)$

(MP) If $\vdash \phi$ and $\vdash \phi \to \psi$, then $\vdash \psi$

(GEN) If $\vdash \phi \to \psi$ and v is free in ψ but not ϕ, then $\vdash \phi \to \forall_\tau v(\psi)$

(INST) If $\vdash \phi \to \psi$ and v is free in ϕ but not ψ, then $\vdash \exists_\tau v(\phi) \to \psi$

(Ref) $F \equiv_\sigma F$

(LL) $F \equiv_\sigma G \to (\phi[G/v] \to \phi[F/v])$

(β-conversion) $\phi \leftrightarrow \phi^*$, where ϕ^* is derived from ϕ by replacing some constituent of the form $(\lambda v_1 \dots v_n.\psi)(t_1, ..., t_n)$ with $\psi[t_i/v_i]$

(η-conversion) $\phi \leftrightarrow \phi^*$, where ϕ^* is derived from ϕ by replacing some constituent of the form $(\lambda v_1 \dots v_k.F(v_1, \dots, v_k))$, where none of $v_1, \dots, v_k$ is free in F, with F

(ii) *Background essentialist axioms*

(Permutation) $\Box_{F_1,...,F_n}\phi \to \Box_{F_{\pi(1)},...,F_{\pi(n)}}\phi$, where π is a permutation function on $\{1, ..., n\}$

(Idempotence) $\Box_{F_1,...,F_n}\phi \to \Box_{F_1,...,F_{n-1}}\phi$, if $F_n = F_i$, for some $i = 1, ..., n-1$

(Separation) $\Box_{F_1,...,F_n,\lambda x.(G(x)\vee H(x))}\phi \to \Box_{F_1,...,F_n,G,H}\phi$

(Subtraction) $\forall x^\sigma \neg F_{n+1}(x) \to (\Box_{F_1,...,F_{n+1}}\phi \leftrightarrow \Box_{F_1,...,F_n}\phi)$

(Decomposition) $\Box_{F_1,...,F_k,[B]}\phi \to \Box_{F_1,...,F_k,[A_1],...,[A_n]}\phi$, where $A_1, ..., A_n$ are all the constants and free variables of B

(MON1) $\forall x^\sigma(F_1(x) \to G(x)) \to (\Box_{F_1,...F_k}\phi \to \Box_{G,F_2,...,F_k}\phi)$, $1 \leq k$

(MON2) $\Box_{F_1,...F_k}\phi \to \Box_{F_1,...,F_n}\phi$, $k \leq n$

(iii) *Axioms for rigidity*

(R-Comp) $\forall X^{\langle\sigma\rangle}\exists Y^{[\sigma]}\forall x^\sigma(X(x) \leftrightarrow Y(x))$

(R-Ext) $\forall X^{[\sigma]}\forall Y^{[\sigma]}(\forall x^\sigma(X(x) \leftrightarrow Y(x)) \to X \equiv_{[\sigma]} Y)$

(Rigidity) $R^{[\sigma]}(x) \to \Box_R R(x)$

(R-Equiv) $\Box_{R,F_1,...,F_n}\phi \leftrightarrow \Box_{[R],F_1,...,F_n}\phi$, where R is a rigid predicate

(iv) *Core essentialist axioms and rules*

(CH) $\Box_{G_1,...,G_n,c_\sigma(F_1,...,F_k)}\phi \to \Box_{G_1,...,G_n,F_1,...,F_k}\phi$

(RC) If $\vdash \phi_1 \wedge \ldots \wedge \phi_n \rightarrow \psi$, then $\vdash \Box_{F_1,\ldots,F_k}\phi_1 \wedge \ldots \wedge \Box_{F_1,\ldots,F_k}\phi_n \rightarrow \Box_{F_1,\ldots,F_k,[A_1],\ldots,[A_m]}\psi$, where $A_1, \ldots, A_m$ are all the constants and free variables occurring in ψ but not any of $\phi_1, \ldots, \phi_n$

(T) $\Box_{F_1,\ldots,F_n}\phi \rightarrow \phi$

(4) $\Box_{F_1,\ldots,F_n}\phi \rightarrow \Box_{F_1,\ldots,F_n,[\Box_{\sigma_1,\ldots,\sigma_n}]}\Box_{F_1,\ldots,F_n}\phi$, whenever $F_1^{\sigma_1}, \ldots, F_n^{\sigma_n}$ are rigid predicates

(5) $\neg\Box_{F_1,\ldots,F_n}\phi \rightarrow \Box_{F_1,\ldots,F_n,[\neg],[\Box_{\sigma_1,\ldots,\sigma_n}],[\phi]}\neg\Box_{F_1,\ldots,F_n}\phi$, whenever $F_1^{\sigma_1}, \ldots, F_n^{\sigma_n}$ are rigid predicates

A.2 Some theorems

In this section, we prove some of the theorems appealed to in the main text.

Proposition 1 (Inheritance)**.** $\vdash (F_1, \ldots, F_n) \succeq_\sigma s \wedge \Box_{G_1,\ldots,G_k,[s]}\phi \rightarrow \Box_{G_1,\ldots,G_k,F_1,\ldots,F_n}\phi$

Proof. By β-conversion, we have $\vdash (F_1, \ldots, F_n) \succeq_\sigma s \rightarrow c_\sigma(F_1, \ldots, F_n)(s)$. By MON1, $\vdash \forall z^\sigma(\lambda x.(x \equiv s)(z) \rightarrow c_\sigma(F_1, \ldots, F_n)(z)) \rightarrow (\Box_{G_1,\ldots,G_k,[s]}\phi \rightarrow \Box_{G_1,\ldots,G_k,c_\sigma(F_1,\ldots,F_n)}\phi)$. By CH and classical logic we obtain $\vdash (F_1, \ldots, F_n) \succeq_\sigma s \wedge \Box_{G_1,\ldots,G_k,[s]}\phi \rightarrow \Box_{G_1,\ldots,G_k,F_1,\ldots,F_n}\phi$. □

Proposition 2. *(i)* $\vdash s \geq_{\sigma,\sigma} s$. *(ii)* $\vdash t \geq_{\tau,\sigma} s \wedge s \geq_{\sigma,\rho} r \rightarrow t \geq_{\tau,\rho} r$.

Proof. (i) By MON1, (1) $\vdash \forall y(X^{[\sigma]}(y) \leftrightarrow (\lambda z.z \equiv_\sigma s)(y)) \rightarrow ((\Box_X\phi \leftrightarrow \Box_{[s]}\phi)$. By classical reasoning, (2) $\vdash \forall y(X^{[\sigma]}(y) \leftrightarrow (\lambda z.z \equiv_\sigma s)(y)) \rightarrow X(s)$. From this we obtain (3) $\vdash \forall y(X^{[\sigma]}(y) \leftrightarrow (\lambda z.z \equiv_\sigma s)(y)) \rightarrow \Box_X X(s)$ by the instance $X^{[\sigma]}(s) \rightarrow \Box_X X(s)$ of Rigidity. Combining (1) and (3) we obtain (4) $\vdash \forall y(X^{[\sigma]}(y) \leftrightarrow (\lambda z.z \equiv_\sigma s)(y)) \rightarrow \exists X^{[\sigma]}\Box_{[s]}X(s)$. Hence, by classical reasoning, (5) $\vdash \exists X^{[\sigma]}\forall y(X^{[\sigma]}(y) \leftrightarrow (\lambda z.z \equiv_\sigma s)(y)) \rightarrow \exists X^{[\sigma]}\Box_{[s]}X(s)$. But the antecedent of (5) is an instance of R-Comp, so (6) $\vdash \exists X^{[\sigma]}\Box_{[s]}X(s)$. But (7) $\vdash \Box_{[s]}X(s) \leftrightarrow \Box_{[s]}(\lambda x.X(x))(s)$ by β-conversion. From this, $s \geq_{\sigma,\sigma} s$ follows by (6) and classical reasoning.

(ii) By Proposition 1 and classical reasoning, $\vdash \Box_{[s]}z(r) \wedge t \geq_{\tau,\sigma} s \rightarrow \exists z\Box_{[t]}z(r)$. From this, the claim follows by classical reasoning and the definition of $\geq$. □

Proposition 3 (Haec)**.** $\vdash \Box_{F_1,\ldots,F_n}\phi \leftrightarrow \Box_{\mathfrak{h}(F_1,\ldots,F_n)}\phi$.

Proof. Left-to-right direction: it follows at once from Proposition 2(i), the definition of $\mathfrak{h}(F_1, \ldots, F_n)$ and MON1 that for every F_i, $\vdash \forall x(F_i(x) \rightarrow (\mathfrak{h}(F_1, \ldots, F_n)) \succeq_{\sigma_i} x)$. Thus by MON1 and Permutation we obtain $\vdash \Box_{F_1,\ldots,F_n}\phi \rightarrow \Box_{c_{\sigma_i}(\mathfrak{h}(F_1,\ldots,F_n)),\ldots,c_{\sigma_n}(\mathfrak{h}(F_1,\ldots,F_n))}\phi$. From this, $\vdash \Box_{F_1,\ldots,F_n}\phi \rightarrow \Box_{\mathfrak{h}(F_1,\ldots,F_n)}\phi$ follows by CH and Idempotence. For the right-to-left direction, we have (1) $\vdash \forall y(X^{[\sigma]}(y) \leftrightarrow (\lambda z.z \equiv_\sigma x)(y)) \wedge (X(x) \equiv_{\langle\rangle} p) \rightarrow \Box_{[x]}p$ by Rigidity, MON1, and LL. From (1), we get (2) $\vdash \forall p(\mathfrak{h}(F_1, \ldots, F_n)(p) \rightarrow c_{\langle\rangle}(F_1, \ldots, F_n)(p))$. Hence, from (2), MON1 and CH we obtain $\vdash \Box_{\mathfrak{h}(F_1,\ldots,F_n)} \rightarrow \Box_{F_1,\ldots,F_n}\phi$. □

Proposition 4 (K)**.** $\vdash \Box_{F_1,\dots,F_n}(\phi \to \psi) \to (\Box_{F_1,\dots,F_n}\phi \to \Box_{F_1,\dots,F_n}\psi)$.

Proof. Immediate from RC. $\Box$

In what follows, we will use the following abbreviations: $F \approx G := \forall x(F(x) \leftrightarrow G(x))$; $F \subseteq G := \forall x(F(x) \to G(x))$; $\Omega := \lambda x^e.x \equiv x$; $\Pi := \lambda p^{\langle\rangle}.p \equiv p$. We now show that the logic of $\Box_\Omega$ is at least $\mathsf{S4}$ in $\mathsf{HLE_R}$.

Proposition 5 ($\mathrm{EN}^{\Box_\Omega}$)**.** $\vdash_{\mathsf{HLE_R}} \Box_{F_1,\dots,F_n}\phi \to \Box_\Omega\phi$.

Proof. By (R) and classical reasoning, we have $\vdash_{\mathsf{HLE_R}} \forall x^{\sigma_i}(F_i(x) \to \exists X^{\langle e\rangle}\Box_X x \equiv x)$. From this we get $\vdash_{\mathsf{HLE_R}} \forall x^{\sigma_i}(F_i(x) \to \Box_\Omega x \equiv x)$ by MON1 and classical reasoning. Hence, $\vdash_{\mathsf{HLE_R}}$ $\forall x^{\sigma_i}(F_i(x) \to c_{\sigma_i}(\Omega)(x))$. Therefore, $\vdash_{\mathsf{HLE_R}} \Box_{F_1,\dots,F_n}\phi \to \Box_{c_{\sigma_1}(\Omega),\dots,c_{\sigma_n}(\Omega)}\phi$ by Permutation and n applications of MON1, whence by CH and Idempotence, $\vdash_{\mathsf{HLE_R}} \Box_{F_1,\dots,F_n}\phi \to \Box_\Omega\phi$. $\Box$

Corollary 1 ($\mathrm{RN}^{\Box_\Omega}$)**.** *If* $\vdash_{\mathsf{HLE_R}} \phi$ *then* $\vdash_{\mathsf{HLE_R}} \Box_\Omega\phi$

Proposition 6 ($4^{\Box_\Omega}$)**.** $\vdash_{\mathsf{HLE_R}} \Box_\Omega\phi \to \Box_\Omega\Box_\Omega\phi$.

Proof. Let $X^{[e]}$ be a variable of rigid type that does not occur in ϕ. By MON1, the essentialist (4)-schema and classical reasoning, we have (1) $\vdash_{\mathsf{HLE_R}} X \approx \Omega \to (\Box_\Omega\phi \to \Box_{X,[\Box_{[e]}]}\Box_X\phi)$. From this we get (2) $\vdash_{\mathsf{HLE_R}} \Box_\Omega\phi \wedge X \approx \Omega \to \Box_{X,[\Box_{[e]},\langle\rangle]}\Box_X\phi$. By $\mathrm{EN}^{\Box_\Omega}$, (3) $\vdash_{\mathsf{HLE_R}} \Box_\Omega\phi \wedge X \approx \Omega \to \Box_\Omega\Box_X\phi$. By classical reasoning and $\mathrm{RN}^{\Box_\Omega}$ we have (4) $\vdash_{\mathsf{HLE_R}} \Box_\Omega X \subseteq \Omega$ and by MON1 and $\mathrm{RN}^{\Box_\Omega}$, (5) $\vdash_{\mathsf{HLE_R}} \Box_\Omega(X \subseteq \Omega \to (\Box_X\phi \to \Box_\Omega\phi))$. From (4), (5) and (K) we get (6) $\vdash_{\mathsf{HLE_R}} \Box_\Omega\Box_X\phi \to \Box_\Omega\Box_\Omega\phi$. So by (3), (6) and classical reasoning, (7) $\vdash_{\mathsf{HLE_R}} X \approx \Omega \to (\Box_\Omega\phi \to \Box_\Omega\Box_\Omega\phi)$. But X is not free in ϕ, so (8) $\vdash_{\mathsf{HLE_R}} \exists X(X \approx \Omega) \to (\Box_\Omega\phi \to \Box_\Omega\Box_\Omega\phi)$. But $\exists X(X \approx \Omega)$ is an instance of R-Comp, so (9) $\vdash_{\mathsf{HLE_R}} \Box_\Omega\phi \to \Box_\Omega\Box_\Omega\phi$. $\Box$

Corollary 2. *In* $\mathsf{HLE_R}$*, the logic of* $\Box_\Omega$ *is at least* $\mathsf{S4}$.

In $\mathsf{HLE_{R+DOM}}$, we can further prove:

Proposition 7 ($\mathrm{B}^{\Box_\Omega}$)**.** $\vdash_{\mathsf{HLE_{R+DOM}}} \neg\phi \to \Box_\Omega\neg\Box_\Omega\phi$.

Proof. Let $X^{[e]}$ be a variable of rigid type that does not occur in ϕ. By DOM we have (1) $\vdash_{\mathsf{HLE_{R+DOM}}} \forall x X(x) \to \Box_X\forall x X(x)$. By the essentialist (5)-schema, (2) $\vdash_{\mathsf{HLE_{R+DOM}}} \neg\Box_X\phi \to \Box_{X,[\neg],[\Box_{[e]}],[\phi]}\neg\Box_X\phi$. From (2), we obtain by MON1 and $\mathrm{EN}^{\Box_\Omega}$, (3) $\vdash_{\mathsf{HLE_{R+DOM}}} \neg\Box_\Omega\phi \wedge \forall x X(x) \to \Box_\Omega\neg\Box_X\phi$. By MON1 and $\mathrm{RN}^{\Box_\Omega}$, we obtain (4) $\vdash_{\mathsf{HLE_{R+DOM}}} \Box_\Omega(\forall x X(x) \to (\neg\Box_\Omega\phi \leftrightarrow \neg\Box_X\phi))$. From (4), (1) and (K), we can infer (5) $\vdash_{\mathsf{HLE_{R+DOM}}} \forall x X(x) \to (\Box_\Omega\neg\Box_\Omega\phi \leftrightarrow \Box_\Omega\neg\Box_X\phi)$. From (3) and (5), we obtain (6) $\vdash_{\mathsf{HLE_{R+DOM}}} \neg\Box_\Omega\phi \wedge \forall x X(x) \to \Box_\Omega\neg\Box_\Omega\phi$. So by classical reasoning, (7) $\vdash_{\mathsf{HLE_{R+DOM}}} \exists X\forall x X(x) \to (\neg\Box_\Omega\phi \to \Box_\Omega\neg\Box_\Omega\phi)$, because X is not free in ϕ. But the antecedent of (7) is equivalent to an instance of R-Comp, which gives us (8) $\vdash_{\mathsf{HLE_{R+DOM}}} \neg\Box_\Omega\phi \to \Box_\Omega\neg\Box_\Omega\phi$. By the contrapositive of $\mathrm{T}^{\Box_\Omega}$, (8) implies $\vdash_{\mathsf{HLE_{R+DOM}}} \neg\phi \to \Box_\Omega\neg\Box_\Omega\phi$. $\Box$

Corollary 3. *In* $\mathsf{HLE}_{\mathsf{R+DOM}}$*, the logic of* $\Box_\Omega$ *is at least* $\mathsf{S5}$*.*

By using semantic techniques, it is possible to show that the logic of $\Box_\Omega$ is *exactly* $\mathsf{S4}$ ($\mathsf{S5}$) in $\mathsf{HLE}_{\mathsf{R}}$ ($\mathsf{HLE}_{\mathsf{R+DOM}}$); see Ditter (MS). The analogs for $\Box_\Pi$ of the preceeding results are proved exactly like those for $\Box_\Omega$, except that we use $\mathrm{EN}^{\Box_\Pi}$ and $\mathrm{RN}^{\Box_\Pi}$ instead of $\mathrm{EN}^{\Box_\Omega}$ and $\mathrm{RN}^{\Box_\Omega}$ in the relevant places. By avoiding appeal to $\mathrm{EN}^{\Box_\Omega}$ and $\mathrm{RN}^{\Box_\Omega}$, both of which depend on (R), the proofs for $\Box_\Pi$ avoid making use of (R) and therefore go through in HLE.

Proposition 8 ($\mathrm{EN}^{\Box_\Pi}$)**.** $\vdash \Box_{F_1,\dots,F_n}\phi \to \Box_\Pi\phi$

Corollary 4 ($\mathrm{RN}^{\Box_\Pi}$)**.** *If* $\vdash \phi$ *then* $\vdash \Box_\Pi\phi$*.*

Proposition 9 ($\mathsf{4}^{\Box_\Pi}$)**.** $\vdash \Box_\Pi\phi \to \Box_\Pi\Box_\Pi\phi$*.*

Corollary 5. *In* HLE*, the logic of* $\Box_\Pi$ *is at least* $\mathsf{S4}$*.*

The $\mathrm{B}^{\Box_\Pi}$-principle becomes provable once we add $\mathrm{DOM}^{\langle\rangle}$ to HLE.

Proposition 10 ($\mathrm{B}^{\Box_\Pi}$)**.** $\vdash_{\mathsf{HLE}_{\mathsf{DOM}^{\langle\rangle}}} \neg\phi \to \Box_\Pi\neg\Box_\Pi\phi$*.*

Corollary 6. *In* $\mathsf{HLE}_{\mathsf{DOM}^{\langle\rangle}}$*, the logic of* $\Box_\Pi$ *is at least* $\mathsf{S5}$*.*

As in the case of $\Box_\Omega$, we can show by using semantic techniques that the logic of $\Box_\Omega$ is *exactly* $\mathsf{S4}$ ($\mathsf{S5}$) in HLE ($\mathsf{HLE}_{\mathsf{DOM}^{\langle\rangle}}$); see Ditter (MS).

Proposition 11 ($\mathrm{N\text{-}DOM}^e$)**.** $\vdash_{\mathsf{HLE}_{\mathsf{R+DOM}}} \neg\forall xR(x) \to \Box_{R,[\forall_e],[\neg],[\to],[\Box_{[e]}]}\neg\forall xR(x)$*, where* R *is of type* $[e]$

Proof. Let R be a rigid predicate of type $[e]$.

1. $\forall xR(x) \to \Box_R\forall xR(x)$	DOM
2. $\Box_{R,[\forall],[\to],[\Box_{[e]}]}(\forall xR(x) \to \Box_R\forall xR(x))$	1, RC^0, R-Equiv
3. $\Box_{R,[\forall],[\to],[\Box_{[e]}],[\neg]}(\neg\Box_R\forall xR(x) \to \neg\forall xR(x))$	2, RC
4. $\Box_{R,[\forall],[\to],[\Box_{[e]}],[\neg]}\neg\Box_R\forall xR(x) \to \Box_{R,[\forall],[\to],[\Box_{[e]}],[\neg]}\neg\forall xR(x)$	3, K
5. $\neg\forall xR(x) \to \neg\Box_R\forall xR(x)$	T
6. $\neg\Box_R\forall xR(x) \to \Box_{R,[\Box_{[e]}],[\neg],[\forall],[R]}\neg\Box_R\forall xR(x)$	(5), Decomposition
7. $\neg\Box_R\forall xR(x) \to \Box_{R,[\Box_{[e]}],[\neg],[\forall]}\neg\Box_R\forall xR(x)$	6, R-Equiv, Idempotence
8. $\neg\Box_R\forall xR(x) \to \Box_{R,[\forall],[\to],[\Box_{[e]}],[\neg]}\neg\Box_R\forall xR(x)$	7, MON2, Permutation
9. $\neg\forall xR(x) \to \Box_{R,[\forall],[\to],[\Box_{[e]}],[\neg]}\neg\forall xR(x)$	4, 5, 8

$\square$

Proposition 12 (N-DOM$^{\langle\rangle}$)**.** *$\vdash_{\mathsf{HLE}_{\mathsf{DOM}^{\langle\rangle}}} \neg\forall p R(p) \to \Box_{R,[\forall_{\langle\rangle}],[\neg],[\to],[\Box_{[\langle\rangle]}]}\neg\forall p R(p)$, where R is of type $[\langle\rangle]$*

Proof. Exactly analogous to the proof of Proposition 11. $\square$

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