Decentralized robust adaptive backstepping control for a class of non-minimum phase nonlinear interconnected systems

Jiehua Feng[1], Dongya Zhao[1*], Xing-Gang Yan[2], Sarah K. Spurgeon[3]
1. College of New Energy, China University of Petroleum (East China), Qingdao, China, 266580;
2. School of Engineering & Digital Arts, University of Kent, United Kingdom, Canterbury CT2 7NT, UK;
3. Department of Electronic & Electrical Engineering, University College London, Torrington Place, London WC1E 7JE, UK.
∗ Corresponding author's Email: dyzhao@upc.edu.cn

Abstract

In this paper, a class of interconnected systems is considered, where the nominal isolated subsystems are fully nonlinear and non-minimum phase. A decentralized Extended Kalman Filter-Extended High Gain Observer (EKF-EHGO) is designed to observe the system states. Then, a systematic backstepping design procedure is employed to develop a novel decentralized robust adaptive output feedback control, in which the adaptive law is designed to counter the effects of the interconnections and uncertainties. The proposed decentralized dynamic output feedback control scheme can guarantee that all the signals in the closed-loop system are uniformly ultimately bounded (UUB). Both interconnections and uncertainties are allowed to be unmatched and bounded by an unknown high-order polynomial, which is a more general form when compared with existing work. Two MATLAB simulation examples are used to demonstrate the effectiveness of the proposed method including a system comprising translational oscillator with rotating actuator (TORA) sub-systems.

Key Words: Decentralized robust adaptive control; Non-minimum phase; Nonlinear systems; Dynamic output feedback; Backstepping control.

I. Introduction

Increasingly complex requirements are producing engineering systems which are formed from multiple sub-systems coupled into nonlinear large-scale interconnected systems [1], [2]. Such systems include networked microgrids [3], power systems [4] and mechanical systems [5]. In addition, non-minimum phase characteristics may appear in nonlinear interconnected systems, such as chemical networks composed of multiple non-minimum phase continuous stirred tank reactors [6] and mechanical systems formed by multiple one-link flexible manipulators [7]. The control of non-minimum phase interconnected systems is thus strongly motivated by the needs of practical applications.

In addition subsystem uncertainties will not only affect their own performance, but also affect the performance of the other subsystems through interaction. Managing the interconnections and uncertainties is an important issue in control of interconnected systems [8]. For non-minimum phase nonlinear interconnected systems, the problem becomes more complicated since the control not only needs to guarantee the stability of the external dynamics, but also realize the convergence of the unstable internal dynamics. It should be noted that the instability of the zero dynamics cannot be changed by the addition of feedback, so the control of non-minimum phase nonlinear interconnected systems is much more difficult than that of their minimum phase counterparts. To the best of the authors' knowledge, there is currently no solution to the stabilization problem for this class of interconnected systems, where the nominal isolated subsystems are fully nonlinear and non-minimum phase.

Interconnected systems may require high online computing power to implement controllers. This may be undesirable and has resulted in the development of decentralized control strategies [9] in which local control design only needs subsystem information. A decentralized control approach may improve both the computational efficiency and the overall security of the system; control computations are locally performed and the requirements for data exchange are reduced. These advantages have made decentralized control a popular choice for interconnected systems. Adaptive control is generally considered to be an effective method to deal with uncertainties due to its excellent performance characteristics and relatively simple design process [10], [11]. This has motivated study to apply decentralized adaptive techniques to nonlinear interconnected systems, and many interesting results have been obtained [12], [13], [14]. It should be noted that these contributions all require that the nominal isolated subsystems are minimum phase.

Research on non-minimum phase nonlinear systems is a challenging problem from the perspective of both control theory and engineering application. By assuming the existence of a dynamic stabilizing controller for an auxiliary system, a simple and very useful design tool has been proposed for non-minimum phase nonlinear systems in [15]. The method proposed in [15] is pioneering for the stabilization problem of non-minimum phase nonlinear systems and many related dynamic compensator-based output feedback control methods have been proposed [16], [17], [18]. However the assumption that the auxiliary system can be globally asymptotically stabilized by a known dynamic compensator may be difficult to satisfy for interconnected systems. The backstepping technique is also a good candidate to deal with the non-minimum phase nonlinear systems in normal form or strict-feedback form [19]. Based on the reduced-order observer proposed in [20] and the small-gain technique, an output feedback backstepping control has been designed for a class of nonlinear systems. However the considered system must be affine in the internal state [21]. Note that these methods all use a centralized control approach. Although such centralized control methods can be used to control non-minimum phase nonlinear systems, the interconnection terms are not considered and this limits their applicability to non-minimum phase nonlinear interconnected systems [22].

For a class of interconnected systems with non-minimum phase isolated nominal subsystems, a robust decentralised output feedback sliding mode controller has been designed to drive the system to a composite sliding surface and maintain a sliding motion on it thereafter [23]. Although the interconnected systems considered in [23] were allowed to be non-minimum phase and had unmatched interconnections and uncertainties, the nominal isolated subsystems were linear. Note that when nominal isolated subsystems are fully nonlinear, neither the observer nor the control method proposed in [23] are applicable due to the existence of unstable nonlinear internal dynamics.

In summary, when the nominal isolated systems of the considered interconnected system are nonlinear and non-minimum phase, problems arise from the nonlinear non-minimum phase characteristics, the interconnections and uncertainties. A decentralized dynamic output feedback robust adaptive backstepping control is proposed in this paper to tackle these problems. A decentralized state observer is first designed as a prerequisite for dealing with the nonlinear non-minimum phase characteristics. When the interconnections and uncertainties with unknown higher-order nonlinear bounds are considered, this increases the difficulty with observer design. Adaptive terms are designed to offset not only the interconnections and uncertainties in the considered system itself, but also deal with the destabilizing terms coming from the state observation error dynamics. Finally stability of the system is addressed. The convergence of the state observation error, system states and adaptive terms is guaranteed. In light of [24], [25], a decentralized Extended Kalman Filter-Extended High Gain Observer (EKF-EHGO) is first designed to observe the states of a class nonlinear non-minimum phase interconnected systems. By transforming the original stabilization problem into a time-varying tracking problem, the backstepping design procedure is employed to develop a novel decentralized robust adaptive control to deal with the nonlinear non-minimum phase characteristics, in which the adaptive law is used to counteract the effects of the interconnections and uncertainties, thereby reducing the conservatism and enhancing the robustness. A Lyapunov approach is used to address stability.

In the control of nonlinear interconnected systems, asymptotic stability or even exponential stability are often expected results. To this end, various constraints are imposed on the considered systems, such as that the bounds on the uncertainties and interconnections are all known functions as in [26], [27], [28]. In the case of output feedback control, the restrictions imposed on the systems will be more severe, such as that part or all of the interconnections are required to be known as in [8], [23] or even that the uncertainties are also known in [29]. In addition, the limitation of the constrained Lyapunov problem (CLP) is often required in static output feedback control [30], [31] and dynamic output feedback control [23]. In order to relax such restrictions, the results obtained in this paper deliver uniformly ultimately bounded (UUB) stability rather than asymptotic stability. Effectively a trade-off is made between achieving system stability and the generality of the system to which the results may be applied. Note that UUB can already meet production needs in practical industrial applications. There have also been many UUB results developed for nonlinear interconnected systems, such as [12], [13], [14], [32], although the minimum phase assumption is required.

In comparison with the existing decentralised adaptive control methods [12], [13], [14], the interconnected systems considered in this paper are allowed to be non-minimum phase, which extends both the potential practical application and theoretical development. In comparison with decentralised sliding mode control [23] for non-minimum phase linear interconnected system, the nominal isolated subsystems considered in this paper are fully nonlinear. The interconnections and uncertainties considered are allowed to be unmatched and bounded by an unknown high-order polynomial, which has a more general form when compared with most of the existing other control methods [1], [21]. The main theoretical contributions of this paper include: (i) a decentralized full order observer is designed for a class of non-minimum phase nonlinear interconnected large-scale systems; (ii) an adaptive backstepping method is proposed to deal with the non-minimum phase characteristics, high-order interconnections and uncertainties; (iii) sufficient conditions are given to guarantee the considered system is UUB.

The remainder of the paper is organized as follows. Section II formulates the problem and gives some assumptions that will be used in the following sections. In Section III, a decentralized EKF-EHGO based robust adaptive backstepping control is designed and the stability proof of the closed-loop system is given. Two simulation examples are presented to validate the proposed approach in Section IV while the conclusions are given in Section V.

Notation: For a square matrix A, $\lambda_{\min}(A)$ and $\lambda_{\max}(A)$ denote the minimum and maximum eigenvalue respectively.

II. System description and problem formulation

Consider a class of uncertain nonlinear interconnected systems

$$\begin{aligned}
&\dot{z}_i^b = \omega_i\left(z_i^b, z_{i1}^a\right) + \Delta\delta_i\left(t, z_i\right) + \sum_{\substack{j=1\\ j\neq i}}^N H_{ij}\left(z_j^b, z_{j1}^a\right)\\
&\dot{z}_{i1}^a = z_{i2}^a + \Delta f_{i1}\left(t, z_i\right) + \sum_{\substack{j=1\\ j\neq i}}^N \Gamma_{ij1}\left(z_j^b, z_{j1}^a\right)\\
&\dot{z}_{i2}^a = z_{i3}^a + \Delta f_{i2}\left(t, z_i\right) + \sum_{\substack{j=1\\ j\neq i}}^N \Gamma_{ij2}\left(z_j^b, z_{j1}^a\right)\\
&\vdots\\
&\dot{z}_{ir_i}^a = \xi_i\left(z_i^b, z_{i1}^a\right) + \mu_i\left(z_{i1}^a\right)u_i + \Delta f_{ir_i}\left(t, z_i\right) + \sum_{\substack{j=1\\ j\neq i}}^N \Gamma_{ijr_i}\left(z_j^b, z_{j1}^a\right)\\
&y_i = z_{i1}^a
\end{aligned} \tag{1}$$

where $z_i := col\left(z_i^b, z_{i1}^a, \cdots, z_{ir_i}^a\right) \in \mathrm{Z}_i \in R^{n_i}, u_i \in R, y_i \in R$ are the state, input and output of the i^{th} subsystem respectively with $i = 1, 2, \cdots, N$, $y := col\left(y_1, \cdots, y_N\right)$, $z^b := col\left(z_1^b, \cdots, z_N^b\right)$ with $z_i^b \in R^{n_i - r_i} \in \mathrm{X}_i$, Z_i and X_i are neighborhoods of the origin, $z := col\left(z_1, \cdots, z_N\right) \in \mathrm{Z} := \mathrm{Z}_1 \times \cdots \times \mathrm{Z}_N$ and $\sum_{\substack{j=1\\ j\neq i}}^N H_{ij}(\cdot), \sum_{\substack{j=1\\ j\neq i}}^N \Gamma_{ij1}(\cdot), \cdots, \sum_{\substack{j=1\\ j\neq i}}^N \Gamma_{ij,r_i-1}(\cdot)$ denote the unmatched interconnections, $\sum_{\substack{j=1\\ j\neq i}}^N \Gamma_{ijr_i}(\cdot)$ denote the matched interconnections, $\Delta\delta_i(\cdot), \Delta f_{i1}(\cdot), \cdots, \Delta f_{i,r_i-1}(\cdot)$ denote the unmatched system uncertainties, $\Delta f_{ir_i}(\cdot)$ denote the matched system uncertainties. The nonlinear functions $\omega_i\left(z_i^b, z_{i1}^a\right)$, $\xi_i\left(z_i^b, z_{i1}^a\right)$ and $\mu_i\left(z_{i1}^a\right)$ are all smooth enough and known, where $\mu_i\left(z_{i1}^a\right)$ represents the nonlinear gain function of the control input.

For simplicity, denote $\delta_i(\cdot) = \sum_{\substack{j=1\\ j\neq i}}^N H_{ij}\left(z_j^b, z_{j1}^a\right)$ and $\Delta_{il}(\cdot) = \sum_{\substack{j=1\\ j\neq i}}^N \Gamma_{ijl}\left(z_j^b, z_{j1}^a\right)$. Without loss of generality, all subsystems are assumed to have the same uniform relative degree, i,e., $r_i = r, 1 \leqslant i \leqslant N$. Then system (1) can be written in the following form

$$\begin{aligned}
&\dot{z}_i^b = \omega_i\left(z_i^b, z_{i1}^a\right) + \Delta\delta_i\left(t, z_i\right) + \delta_i\\
&\dot{z}_i^a = Az_i^a + B\left\{\xi_i\left(z_i^b, z_{i1}^a\right) + \mu_i\left(z_{i1}^a\right)u_i\right\} + \Delta f_i\left(t, z_i\right) + \Delta_i\\
&y_i = Cz_i^a
\end{aligned} \tag{2}$$

where $z_i^a := col\left(z_{i1}^a, \cdots, z_{ir}^a\right) \in R^r$, $\Delta f_i\left(t, z_i\right) := col\left(\Delta f_{i1}, \cdots, \Delta f_{ir}\right)$, $\Delta_i := col\left(\Delta_{i1}, \cdots, \Delta_{ir}\right)$ and

$$A = \begin{bmatrix} 0 & 1 & \cdots & 0\\ \vdots & \ddots & \ddots & \vdots\\ 0 & \cdots & 0 & 1\\ 0 & \cdots & \cdots & 0 \end{bmatrix}_{r\times r}, B = \begin{bmatrix} 0\\ \vdots\\ 0\\ 1 \end{bmatrix}_{r\times 1} \tag{3}$$

with $C = \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix}_{1\times r}$.

Remark 1. It should be noted that the nonlinear interconnected system (1) can be obtained from a general affine nonlinear interconnected system by local coordinate transformation and feedback linearization [27], [33], [34]. In addition, z_i^b in (1) can be viewed as unmodeled dynamics or dynamic uncertainty [35]. Hence system (1) is commonly seen in the literature across both centralized and decentralized control and many practical systems can be modeled as (1), such as the Translational Oscillator with Rotating Actuator (TORA) system [24].

Remark 2. In existing work on nonlinear interconnected systems, minimum phase requirements are necessary, i.e. the zero dynamics $\dot{z}_i^b = \omega_i\left(z_i^b, 0\right)$ is asymptotically stable [36] or exponentially stable [26] or the corresponding internal dynamics is input-to-state practically stable [12], [14], [37]. These assumptions are not needed in this paper.

The following Assumptions are imposed on system (1).

Assumption 1. $\mu_i\left(z_{i1}^a\right) \neq 0$.

Remark 3. Assumption 1 is a common assumption in the study of nonlinear systems, as seen in [12], [38], [39], [40]. In addition, Assumption 1 is always satisfied for many practical systems, such as the triple inverted pendulum system [40] and TORA systems [41].

Assumption 2. The uncertainties satisfy

$$\left\|\Delta\delta_i\left(t, z_i\right)\right\| \leq \varsigma_{ii}\left\|z_i^b\right\| + \sum\nolimits_{k=1}^{p_i} \tau_{iik}\left\|z_{i1}^a\right\|^k \tag{4}$$

$$\left\|\Delta f_{il}\left(t, z_i\right)\right\| \leq \omega_{iil}\left\|z_i^b\right\| + \sum\nolimits_{k=1}^{q_{il}} \upsilon_{iilk}\left\|z_{i1}^a\right\|^k \tag{5}$$

where $l = 1, \cdots, r$, the parameters ς_{ii}, ω_{iil}, τ_{iik} and υ_{iilk} are all unknown.

Assumption 3. The interconnections satisfy

$$\left\|H_{ij}\left(z_j^b, z_{j1}^a\right)\right\| \leq \varsigma_{ij}\left\|z_j^b\right\| + \sum\nolimits_{k=1}^{p_{ij}} \tau_{ijk}\left\|z_{j1}^a\right\|^k \tag{6}$$

$$\left\|\Gamma_{ijl}\left(z_j^b, z_{j1}^a\right)\right\| \leq \omega_{ijl}\left\|z_j^b\right\| + \sum\nolimits_{k=1}^{p_{ijl}} \upsilon_{ijlk}\left\|z_{j1}^a\right\|^k \tag{7}$$

where the parameters ς_{ij}, ω_{ijl}, τ_{ijk} and υ_{ijlk} are all unknown.

Note that $p_{ij}, p_{ijl}, p_i, q_{il}$ in Assumptions 2-3 are allowed to be unknown and only the quantity $pq = \max\left\{p_{ij}, p_{ijl}, p_i, q_{il}\right\}$ with $1 \leq l \leq r$ and $1 \leq i, j \leq N$ needs to be known.

Assumption 4. The functions $\omega_i\left(z_i^b, z_{i1}^a\right)$ satisfy the Lipschitz condition with respect to (w.r.t.) z_{i1}^a uniformly for z_i^b in the considered domain, that is, for any $col\left(z_i^b, z_{i1}^a\right) \in \mathrm{X}_i \times R$ and $col\left(z_i^b, \widehat{z}_{i1}^{a}\right) \in \mathrm{X}_i \times R$, there exists a nonnegative continuous function $\mathcal{L}_{\omega_i}$ such that

$$\left\|\omega_i\left(z_i^b, z_{i1}^a\right) - \omega_i\left(z_i^b, \widehat{z}_{i1}^{a}\right)\right\| \leqslant \mathcal{L}_{\omega_i}\left(z_i^b\right)\left\|z_{i1}^a - \widehat{z}_{i1}^{a}\right\| \tag{8}$$

Assumption 5. The functions $\xi_i\left(z_i^b, z_{i1}^a\right)$ are continuously differentiable with local Lipschitz derivatives.

Assumption 6. There exist a set of smooth functions $F_i\left(t, z_i^b\right) \in R$ and $V_{i0}\left(t, z_i^b\right) : R \times R^{n_i - r} \mapsto R$ and positive constants $c_{i1}, \cdots, c_{i4}$ such that

$$\begin{gathered} c_{i1}\left\|z_i^b\right\|^2 \leqslant V_{i0}\left(t, z_i^b\right) \leqslant c_{i2}\left\|z_i^b\right\|^2 \\ \frac{\partial V_{i0}}{\partial t} + \frac{\partial V_{i0}}{\partial z_i^b}\omega_i\left(z_i^b, F_i\left(t, z_i^b\right)\right) \leqslant -c_{i3}\left\|z_i^b\right\|^2 \\ \left\|\frac{\partial V_{i0}}{\partial z_i^b}\right\| \leqslant c_{i4}\left\|z_i^b\right\| \end{gathered} \tag{9}$$

and the functions $F_i\left(t, z_i^b\right)$ satisfy the Lipschitz condition w.r.t. z_i^b and uniformly for $t \in R^+$ in the considered domain. That is, for any $col\left(t, z_i^b\right) \in R^+ \times \mathrm{X}_i$ and $col\left(t, \widehat{z}_i^{b}\right) \in R^+ \times \mathrm{X}_i$, there exists a nonnegative continuous function $\mathcal{L}_{F_i}\left(t\right)$ such that

$$\left\|F_i\left(t, z_i^b\right) - F_i\left(t, \widehat{z}_i^{b}\right)\right\| \leqslant \mathcal{L}_{F_i}\left(t\right)\left\|z_i^b - \widehat{z}_i^{b}\right\| \tag{10}$$

Remark 4. In most existing decentralized control studies for nonlinear interconnected systems, only weak coupling is considered, whereby the nonlinear interconnections only contain the outputs of other subsystems, as seen for example in [42],

[43], [44]. Although there has been some research on the case of strong interconnections, there are many limitations. For example, in the system considered in [45], the nominal isolated subsystems are fully linear and in strict feedback form while a minimum phase requirement is typically necessary in all methods on strong coupling, as seen in [45], [46], [47]. It should be noted that the coupling considered in this paper is stronger than the general weak coupling, because the effect of the unmodeled dynamics z_j^b from the other subsystems on the interconnection has also been considered. The interconnections and uncertainties considered in this paper as shown in Assumptions 2-3 have more general forms than in other work; the terms $\Delta\delta_i$ and δ_i are not considered in [12], [13] and all the parameters of the bounded functions must be known in [21], [23], [27], [31]. These constraints are not required in this study.

Remark 5. Assumptions 4-5 require that the nonlinear terms $\omega_i\left(z_i^b, z_{i1}^a\right)$ and $\xi_i\left(z_i^b, z_{i1}^a\right)$ satisfy Lipschitz conditions as often appear in the relevant results concerning observer and controller design for nonlinear systems, such as [24], [25].

Remark 6. Assumption 6 shows there exists a known smooth function that can exponentially stabilize the dynamic equation $\dot{z}_i^b = \omega_i\left(z_i^b, z_{i1}^a\right)$, which is relatively straightforward to achieve, and similar assumptions can be seen in [21], [48], [49].

The difficulty from the control perspective with system (1) results from the nonlinear non-minimum phase characteristics, i.e. $\dot{z}_i^b = \omega_i\left(z_i^b, 0\right)$ is unstable, as well as the interconnections and uncertainties with unknown higher-order nonlinear bounds, as shown in Assumptions 2-3. In view of these challenges, the objective of this paper is to design a decentralised full order observer and a robust adaptive backstepping control to stabilize the origin of the system (1) using the measured output and observed states.

The following key lemmas are listed below.

Lemma 1: [27]

$$\sum\nolimits_{i=1}^{N}\sum\nolimits_{j=1}^{N}\Theta_{ij} = \sum\nolimits_{i=1}^{N}\sum\nolimits_{j=1}^{N}\Theta_{ji} \tag{11}$$

where Θ_{ij} denotes any function.

Lemma 2: [50]

$$\left|a+b\right|^k \le 2^{k-1}\left|a^k + b^k\right| \tag{12}$$

where $a \in R$, $b \in R$ and $k \ge 1$.

Lemma 3: [38]

$$\left(\sum\nolimits_{k=1}^{p} a_k b_k\right)^2 \le \left(\sum\nolimits_{k=1}^{p} a_k^2\right)\left(\sum\nolimits_{k=1}^{p} b_k^2\right) \tag{13}$$

where $p \ge 1$, $a_k \in R$ and $b_k \in R$ with $1 \le k \le p$.

III. A DECENTRALIZED EKF-EHGO BASED ROBUST ADAPTIVE BACKSTEPPING CONTROL

A. The decentralized full order observer design

This section aims to design a decentralized full order observer for system (2). Inspired by [24], [25], the following decentralized EKF-EHGO is proposed as

$$\begin{aligned} \dot{\hat{z}}_i^a &= A\hat{z}_i^a + B\left(\hat{\sigma}_i + \mu_i\left(z_{i1}^a\right)u_i\right) + H_i\left(\varepsilon_i\right)\left(y_i - C\hat{z}_i^a\right) \\ \dot{\hat{\sigma}}_i &= \bar{\xi}_i\left(\hat{z}_i^b, \hat{z}_i^a\right) + \frac{\eta_{i,r+1}}{{\varepsilon_i}^{r+1}}\left(y_i - C\hat{z}_i^a\right) \\ \dot{\hat{z}}_i^b &= \omega_i\left(\hat{z}_i^b, \hat{z}_{i1}^a\right) + L_i\left(t\right)\left(\hat{\sigma}_i - \xi_i\left(\hat{z}_i^b, \hat{z}_{i1}^a\right)\right) \end{aligned} \tag{14}$$

where $\bar{\xi}_i\left(\hat{z}_i^b, \hat{z}_i^a\right) = \left.\frac{\partial \xi_i}{\partial z_i^b}\right|_{\left(\hat{z}_i^b, \hat{z}_{i1}^a\right)}\omega_i\left(\hat{z}_i^b, \hat{z}_{i1}^a\right) + \left.\frac{\partial \xi_i}{\partial z_{i1}^a}\right|_{\left(\hat{z}_i^b, \hat{z}_{i1}^a\right)}\hat{z}_{i2}^a$, $\varepsilon_i > 0$ are small parameters, the EHGO gain $H_i\left(\varepsilon_i\right) = \begin{bmatrix} \frac{\eta_{i1}}{\varepsilon_i} & \frac{\eta_{i2}}{\varepsilon_i^2} & \cdots & \frac{\eta_{ir}}{\varepsilon_i^r} \end{bmatrix}^T$ and $\eta_{i1}, \cdots, \eta_{ir}, \eta_{i,r+1}$ are designed such that the polynomials $s^{i,r+1} + \eta_{i1}s^{i,r} + \cdots + \eta_{i,r+1}$ are all Hurwitz, the EKF gain $L_i\left(t\right) = P_i\left(t\right)\bar{C}_i(t)^T R_i^{-1}$ and $P_i\left(t\right)$ is generated by the following Riccati equation

$$\dot{P}_i = \bar{A}_i P_i + P_i\bar{A}_i^T + Q_i - 2P_i\bar{C}_i^T R_i^{-1}\bar{C}_i P_i \tag{15}$$

where the initial value $P_i(t_0) > 0$, $\bar{A}_i(t) = \frac{\partial \omega_i}{\partial z_i^b}\left(\hat{z}_i^b, \hat{z}_{i1}^a\right)$ and $\bar{C}_i(t) = \frac{\partial \xi_i}{\partial z_i^b}\left(\hat{z}_i^b, \hat{z}_{i1}^a\right)$ are time varying matrices, R_i and Q_i are symmetric positive definite matrices.

Define $\tilde{z}_i^b = z_i^b - \hat{z}_i^b$, $\tilde{z}_i^a = z_i^a - \hat{z}_i^a$, $\chi_{il} = \frac{z_{il}^a - \hat{z}_{il}^a}{\varepsilon_i^{r+1-l}}$ with $l = 1, 2, \cdots, r$ and $\chi_{i,r+1} = \xi_i\left(z_i^b, z_{i1}^a\right) - \hat{\sigma}_i$, $\varphi_i := col\left(\chi_{i1}, \cdots, \chi_{ir}\right)$, $\chi_i := \left[\begin{array}{cc} \varphi_i^T & \chi_{r+1} \end{array}\right]^T$, then

$$D_i(\varepsilon_i)\,\varphi_i = \bar{D}_i(\varepsilon_i)\,\chi_i = \tilde{z}_i^a \tag{16}$$

where $D_i(\varepsilon_i) = diag\left[\varepsilon_i^r, \cdots, \varepsilon_i\right]$ and $\bar{D}_i(\varepsilon_i) = \left[\begin{array}{cc} D_i(\varepsilon_i) & 0_{r\times 1} \end{array}\right]$.

It follows from (2) and (14) that

$$\varepsilon_i \dot{\chi}_i = \Lambda_i \chi_i + \varepsilon_i\left[\bar{B}_1 \Delta\xi_i + \bar{B}_2 D_i^{-1}\left(\Delta f_i + \Delta_i\right)\right] \tag{17}$$

$$\dot{\tilde{z}}_i^b = \omega_i\left(z_i^b, z_{i1}^a\right) - \omega_i\left(\hat{z}_i^b, \hat{z}_{i1}^a\right) - L_i(t)\left(\hat{\sigma}_i - \xi_i\left(\hat{z}_i^b, \hat{z}_{i1}^a\right)\right) + \Delta\delta_i + \delta_i \tag{18}$$

where $\Delta\xi_i = \dot{\xi}_i\left(z^b, y\right) - \bar{\xi}_i\left(\hat{z}_i^b, \hat{z}_i^a\right)$ with $\dot{\xi}_i\left(z^b, y\right) = \left.\frac{\partial \xi_i}{\partial z_i^b}\right|_{\left(z_i^b, z_{i1}^a\right)}\left\{\omega_i\left(z_i^b, z_{i1}^a\right) + \Delta\delta_i + \delta_i\right\} + \left.\frac{\partial \xi_i}{\partial z_{i1}^a}\right|_{\left(z_i^b, z_{i1}^a\right)}\left(z_{i2}^a + \Delta f_{i1} + \Delta_{i1}\right)$, $\bar{B}_1 = \left[\begin{array}{cc} 0_{1\times r} & 1 \end{array}\right]^T$, $\bar{B}_2 = \left[\begin{array}{cc} I_{r\times r} & 0_{r\times 1} \end{array}\right]^T$, and

$$\Lambda_i = \left[\begin{array}{ccccc} -\eta_{i1} & 1 & 0 & \cdots & 0 \\ -\eta_{i2} & 0 & 1 & \ddots & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots \\ -\eta_{ir} & 0 & 0 & \ddots & 1 \\ -\eta_{i,r+1} & 0 & 0 & \cdots & 0 \end{array}\right] \tag{19}$$

Since Λ_i is stable, for any symmetric matrix $\Upsilon_i > 0$, the Lyapunov equation

$$\Omega_i^T \Lambda_i + \Lambda_i^T \Omega_i = -\Upsilon_i \tag{20}$$

has unique symmetric solutions $\Omega_i > 0$.

For the state observation error dynamics (17), consider the Lyapunov function candidate

$$V_{i1}(\chi_i) = \chi_i^T \Omega_i \chi_i \tag{21}$$

where Ω_i is defined in (20).

For the state observation error dynamics (18), consider the Lyapunov function candidate

$$V_{i2}\left(t, \tilde{z}_i^b\right) = \left(\tilde{z}_i^b\right)^T P_i^{-1} \tilde{z}_i^b \tag{22}$$

where P_i is defined in (15).

Proposition 1: Suppose Assumptions 1-6 are satisfied and there exist compact sets $\aleph_i \in R^{n_i - r}$, containing the origin, which are the attraction region of the error dynamics (18). Then for any $\tilde{z}_i^b \in \aleph_i$, $\chi_i \in R^{r+1}$ and $z_i \in \mathrm{Z}_i$, when $\left\|\bar{C}_i(t)\right\|$ is bounded and there exist positive constants $\underline{p}_i$ and $\bar{p}_i$ such that the solutions $P_i(t)$ of (15) satisfy $\underline{p}_i I_{n_i - r} \leqslant P_i^{-1}(t) \leqslant \bar{p}_i I_{n_i - r}$, there exist constants α_{i5}, α_{i6} such that

$$\dot{V}_{i1}(\chi_i) + \dot{V}_{i2}(t, \tilde{\eta}) \leq \alpha_{i5}\left\|\tilde{z}_i^b\right\|^2 + \alpha_{i6}\left\|\chi_i\right\|^2 + \Xi_i\left(z^b, y\right) \tag{23}$$

where $\Xi_i = 4N\sum_{j=1}^N\left(\varsigma_{ij}^2\left\|z_j^b\right\|^2 + \sum_{k=1}^{pq}\tau_{ijk}^2\sum_{k=1}^{pq}\left\|z_{j1}^a\right\|^{2k}\right) + 2N\sum_{j=1}^N\left(\omega_{ij1}^2\left\|z_j^b\right\|^2 + \sum_{k=1}^{pq}\upsilon_{ij1k}^2\sum_{k=1}^{pq}\left\|z_{j1}^a\right\|^{2k}\right) + 2Nr\sum_{j=1}^N\sum_{l=1}^r\left(\omega_{ijl}^2\left\|z_j^b\right\|^2 + \sum_{k=1}^{pq}\upsilon_{ijlk}^2\sum_{k=1}^{pq}\left\|z_{j1}^a\right\|^{2k}\right)$, which represents the possibly destabilizing terms caused by the inter-connections and uncertainties.

***Proof*:**

The time derivative of V_{i1} along the trajectories of (17) is given as

$$\dot{V}_{i1}(\chi_i) = -\frac{1}{\varepsilon_i}\chi_i^T\Upsilon_i\chi_i + 2\chi_i^T\Omega_i\left[\bar{B}_1\Delta\xi_i + \bar{B}_2D_i^{-1}(\Delta f_i + \Delta_i)\right] \tag{24}$$

It follows from Assumptions 4-5, there exist positive constants α_{i1}, α_{i2} such that

$$\left|\left.\frac{\partial\xi_i}{\partial z_i^b}\right|_{(z_i^b,z_{i1}^a)}\omega_i\left(z_i^b, z_{i1}^a\right) + \left.\frac{\partial\xi_i}{\partial z_{i1}^a}\right|_{(z_i^b,z_{i1}^a)}z_{i2}^a - \left.\frac{\partial\xi_i}{\partial z_i^b}\right|_{(\hat{z}_i^b,\hat{z}_{i1}^a)}\omega_i\left(\hat{z}_i^b, \hat{z}_{i1}^a\right) - \left.\frac{\partial\xi_i}{\partial z_{i1}^a}\right|_{(\hat{z}_i^b,\hat{z}_{i1}^a)}\hat{z}_{i2}^a\right| \le \alpha_{i1}\left\|\tilde{z}_i^b\right\| + \alpha_{i2}\left\|\chi_i\right\| \tag{25}$$

It follows from the definition of $\Delta\xi_i$ and (25),

$$|\Delta\xi_i| \le \alpha_{i1}\left\|\tilde{z}_i^b\right\| + \alpha_{i2}\left\|\chi_i\right\| + \rho_{i1}\sum_{j=1}^{N}\left(\varsigma_{ij}\left\|z_j^b\right\| + \sum_{k=1}^{pq}\tau_{ijk}\left\|z_{j1}^a\right\|^k\right) + \rho_{i2}\sum_{j=1}^{N}\left(\omega_{ij1}\left\|z_j^b\right\| + \sum_{k=1}^{pq}\upsilon_{ij1k}\left\|z_{j1}^a\right\|^k\right) \tag{26}$$

where $\rho_{i1}\left(z_i^b, z_{i1}^a\right) = \left\|\left.\frac{\partial\xi_i}{\partial z_i^b}\right|_{(z_i^b,z_{i1}^a)}\right\|$ and $\rho_{i2}\left(z_i^b, z_{i1}^a\right) = \left\|\left.\frac{\partial\xi_i}{\partial z_{i1}^a}\right|_{(z_i^b,z_{i1}^a)}\right\|$.

It follows from Assumptions 2-3 and (24)-(26),

$$\begin{aligned}\dot{V}_{i1}(\chi_i) \le& \left\{-\frac{1}{\varepsilon_i}\lambda_{\min}(\Upsilon_i) + \left(1+\rho_{i1}^2+\rho_{i2}^2\right)\|\Omega_i\|^2 + 2\alpha_{i2}\|\Omega_i\| + \left\|\Omega_i\bar{B}_2D_i^{-1}\right\|^2\right\}\|\chi_i\|^2 + \alpha_{i1}^2\left\|\tilde{z}_i^b\right\|^2 \\ &+ 2N\sum_{j=1}^{N}\left(\varsigma_{ij}^2\left\|z_j^b\right\|^2 + \sum_{k=1}^{pq}\tau_{ijk}^2\sum_{k=1}^{pq}\left\|z_{j1}^a\right\|^{2k}\right) + 2N\sum_{j=1}^{N}\left(\omega_{ij1}^2\left\|z_j^b\right\|^2 + \sum_{k=1}^{pq}\upsilon_{ij1k}^2\sum_{k=1}^{pq}\left\|z_{j1}^a\right\|^{2k}\right) \\ &+ 2Nr\sum_{j=1}^{N}\sum_{l=1}^{r}\left(\omega_{ijl}^2\left\|z_j^b\right\|^2 + \sum_{k=1}^{pq}\upsilon_{ijlk}^2\sum_{k=1}^{pq}\left\|z_{j1}^a\right\|^{2k}\right)\end{aligned} \tag{27}$$

The time derivative of V_{i2} along the trajectories of (18) is given as

$$\dot{V}_{i2}(t,\tilde{\eta}) = 2\left(\tilde{z}_i^b\right)^TP_i^{-1}\left\{\Theta_i\left(\hat{z}_i^b, \hat{z}_{i1}^a, \tilde{z}_i^b, \chi_i, t\right) + \Delta\delta_i + \delta_i\right\} + \left(\tilde{z}_i^b\right)^T\dot{P}_i^{-1}\tilde{z}_i^b \tag{28}$$

where $\Theta_i\left(\hat{z}_i^b, \hat{z}_{i1}^a, \tilde{z}_i^b, \chi_i, t\right) \triangleq \omega_i\left(z_i^b, z_{i1}^a\right) - \omega_i\left(\hat{z}_i^b, \hat{z}_{i1}^a\right) - L_i(t)\left(\hat{\sigma}_i - \xi_i\left(\hat{z}_i^b, \hat{z}_{i1}^a\right)\right)$.

Considering Assumptions 4-5, the Riccati equation (15) and the boundedness of $\left\|\bar{C}_i(t)\right\|$, the following inequalities can be easily derived from [24], [25]:

$$2\left(\tilde{z}_i^b\right)^TP_i^{-1}\Theta_i\left(\hat{z}_i^b, \hat{z}_{i1}^a, \tilde{z}_i^b, \chi_i, t\right) + \left(\tilde{z}_i^b\right)^T\dot{P}_i^{-1}\tilde{z}_i^b \leqslant -\alpha_{i3}\left\|\tilde{z}_i^b\right\|^2 + \alpha_{i4}\|\chi_i\|^2 \tag{29}$$

where α_{i3}, α_{i4} are positive constants. Detailed proof of (29) is given in the Appendix.

It follows from Assumptions 2-3 and (28)-(29),

$$\begin{aligned}\dot{V}_{i2}(t,\tilde{\eta}) &\le -\alpha_{i3}\left\|\tilde{z}_i^b\right\|^2 + \alpha_{i4}\|\chi_i\|^2 + 2\left\|\tilde{z}_i^b\right\|\left\|P_i^{-1}\right\|\sum_{j=1}^{N}\left(\varsigma_{ij}\left\|z_j^b\right\| + \sum_{k=1}^{pq}\tau_{ijk}\left\|z_{j1}^a\right\|^k\right) \\ &\le \left(-\alpha_{i3} + \bar{p}_i^2\right)\left\|\tilde{z}_i^b\right\|^2 + \alpha_{i4}\|\chi_i\|^2 + 2N\left\{\sum_{j=1}^{N}\varsigma_{ij}^2\left\|z_j^b\right\|^2 + \sum_{j=1}^{N}\left(\sum_{k=1}^{pq}\tau_{ijk}^2\right)\left(\sum_{k=1}^{pq}\left\|z_{j1}^a\right\|^{2k}\right)\right\}\end{aligned} \tag{30}$$

Hence, Proposition 1 follows from the inequalities (27) and (30) with

$$\begin{aligned}\alpha_{i5} &= -\alpha_{i3} + \bar{p}_i^2 + \alpha_{i1}^2 \\ \alpha_{i6} &= -\frac{1}{\varepsilon_i}\lambda_{\min}(\Upsilon_i) + \left(1+\rho_{i1}^2+\rho_{i2}^2\right)\|\Omega_i\|^2 + 2\alpha_{i2}\|\Omega_i\| + \left\|\Omega_i\bar{B}_2D_i^{-1}\right\|^2 + \alpha_{i4}\end{aligned} \tag{31}$$

Based on Proposition 1, the stability proof of the proposed observer (14) will be given in the subsequent proof which considers the behaviour of the closed-loop system.

Remark 7. The observers developed in [24], [25] do not consider the interconnections and uncertainties, hence exponential convergence of the observer can be obtained. However, when the unmatched interconnections and uncertainties are considered as in this paper, as shown in (23), possibly destabilizing terms have arisen in the observer error dynamics (17) and (18). An adaptive law will be designed to deal with these terms in the following subsection to ensure that the observer error is bounded.

Remark 8. It should be pointed out that the observer (14) does not use information from other subsystems and it is completely decentralized. This means all the interconnections remain in the observation error dynamics, bringing challenges to the controller design, especially the stability proof. In some decentralized dynamic output feedback control schemes, in order to decouple the observation error dynamics and the interconnections, the observer design has used output information [23] or reference signals [29] from other subsystems so that partially decentralized filters (observers) have been constructed.

Remark 9. Note that the EKF is usually used in the study of nonlinear stochastic systems (see, [51]). However, it should be noted that there are also many researchers who directly use the EKF as an observer for nonlinear deterministic systems (see, e.g., [24], [25], [52]). In this paper, inspired by [24], [25], the EKF has been used to deal with the system internal dynamics relating to the partial system states z_i^b due to its simplicity and applicability to a wide range of nonlinear systems.

B. The decentralized robust adaptive backstepping control design

This section aims to design an output feedback control based on the step-by-step recursive backstepping algorithm to guarantee the closed-loop system is UUB.

Step 1

Define $\zeta_{i1} = z_{i1}^a - \psi_{i1}$ with $\psi_{i1} = F_i\left(t, \hat{z}_i^b\right)$. Consider the ζ_{i1} dynamics:

$$\begin{aligned} \dot{\zeta}_{i1} &= z_{i2}^a + \Delta f_{i1} + \Delta_{i1} - \dot{\psi}_{i1} \\ &= \hat{z}_{i2}^a + \tilde{z}_{i2}^a + \Delta f_{i1} + \Delta_{i1} - \dot{\psi}_{i1} \end{aligned} \tag{32}$$

where $\dot{\psi}_{i1} = \frac{\partial F_i\left(t, \hat{z}_i^b\right)}{\partial t} + \frac{\partial F_i\left(t, \hat{z}_i^b\right)}{\partial \hat{z}_i^b}\dot{\hat{z}}_i^b$.

Then define $\zeta_{i2} = \hat{z}_{i2}^a - \psi_{i2}$. For system (2), (17), (18) and (32), a Lyapunov function is chosen as:

$$W_1 = \sum\nolimits_{i=1}^{N}\left\{V_{i0}\left(t, z_i^b\right) + V_{i1}\left(\chi_i\right) + V_{i2}\left(t, \tilde{z}_i^b\right) + \zeta_{i1}^2 + \tau_i^{-1}\left(\hat{\beta}_i - \beta_i^*\right)^2\right\} \tag{33}$$

where V_{i0} is defined in Assumption 6, V_{i1} is defined in (21), V_{i2} is defined in (22), τ_i is a positive constant and $\hat{\beta}_i$ is a time-varying adaptation gain that will be designed later to counter the effects of the possibly destabilizing terms caused by the interconnections and uncertainties. Note that β_i^* is its desired value.

The time derivative of W_1 along the trajectories of (32) is given as:

$$\dot{W}_1 = \sum\nolimits_{i=1}^{N}\left\{\begin{array}{l} \dot{V}_{i0}\left(t, z_i^b\right) + \dot{V}_{i1}\left(\chi_i\right) + \dot{V}_{i2}\left(t, \tilde{z}_i^b\right) + 2\zeta_{i1}\left(\psi_{i2} + \zeta_{i2} + \tilde{z}_{i2}^a + \Delta f_{i1} + \Delta_{i1} - \dot{\psi}_{i1}\right) \\ +2\tau_i^{-1}\left(\hat{\beta}_i - \beta_i^*\right)\dot{\hat{\beta}}_i \end{array}\right\} \tag{34}$$

Then design $\psi_{i2} = -\lambda_{i1}\zeta_{i1} - \hat{\beta}_i\sum_{k=1}^{pq} 2^{2k-2}\zeta_{i1}^{2k-1} + \dot{\psi}_{i1}$ where $\lambda_{i1} > 0$ is a design parameter and the adaptive law is designed as:

$$\dot{\hat{\beta}}_i = \tau_{\mathrm{i}}\sum\nolimits_{k=1}^{pq} 2^{2k-2}\zeta_{i1}^{2k} - \gamma_i\tau_{\mathrm{i}}\hat{\beta}_i \tag{35}$$

where γ_i is a positive constant and the initial value $\hat{\beta}_i\left(t_0\right) > 0$.

For the term $\dot{V}_{i0}\left(t, z_i^b\right)$ in (34), it follows from Assumptions 2-4 and Assumption 6,

$$\begin{aligned} \dot{V}_{i0}\left(t, z_i^b\right) &= \frac{\partial V_{i0}}{\partial t} + \frac{\partial V_{i0}}{\partial z_i^b}\omega_i\left(z_i^b, z_{i1}^a\right) + \frac{\partial V_{i0}}{\partial z_i^b}\left(\Delta\delta_i + \delta_i\right) \\ &= \frac{\partial V_{i0}}{\partial t} + \frac{\partial V_{i0}}{\partial z_i^b}\left(\Delta\delta_i + \delta_i\right) \\ &+ \frac{\partial V_{i0}}{\partial z_i^b}\left\{\omega_i\left(z_i^b, z_{i1}^a\right) + \omega_i\left(z_i^b, F_i\left(t, z_i^b\right)\right) - \omega_i\left(z_i^b, F_i\left(t, z_i^b\right)\right) + \omega_i\left(z_i^b, F_i\left(t, \hat{z}_i^b\right)\right) - \omega_i\left(z_i^b, F_i\left(t, \hat{z}_i^b\right)\right)\right\} \\ &\le -c_{i3}\left\|z_i^b\right\|^2 + c_{i4}\left\|z_i^b\right\|\left\{\mathcal{L}_{\omega_i}\left\|z_{i1}^a - F_i\left(t, \hat{z}_i^b\right)\right\| + \mathcal{L}_{\omega_i}\mathcal{L}_{F_i}\left\|\tilde{z}_i^b\right\|\right\} + c_{i4}\left\|z_i^b\right\|\sum\nolimits_{j=1}^{N}\left(\varsigma_{ij}\left\|z_j^b\right\| + \sum\nolimits_{k=1}^{pq}\tau_{ijk}\left\|z_{j1}^a\right\|^k\right) \\ &\le \left(-c_{i3} + 2 + \frac{c_{i4}^2}{4}\right)\left\|z_i^b\right\|^2 + \frac{c_{i4}^2\mathcal{L}_{\omega_i}^2}{4}\left\|\zeta_{i1}\right\|^2 + \frac{c_{i4}^2\mathcal{L}_{\omega_i}^2\mathcal{L}_{F_i}^2}{4}\left\|\tilde{z}_i^b\right\|^2 \\ &+ 2N\left\{\sum\nolimits_{j=1}^{N}\varsigma_{ij}^2\left\|z_j^b\right\|^2 + \sum\nolimits_{j=1}^{N}\left(\sum\nolimits_{k=1}^{pq}\tau_{ijk}^2\right)\left(\sum\nolimits_{k=1}^{pq}\left\|z_{j1}^a\right\|^{2k}\right)\right\} \end{aligned} \tag{36}$$

It follows from Lemma 3, (27), (30) and (34)-(36) that

$$\begin{aligned}
\dot{W}_1 \leq & \sum\nolimits_{i=1}^{N}\left(-c_{i3}+2+\frac{c_{i4}^2}{4}\right)\left\|z_i^b\right\|^2+\sum\nolimits_{i=1}^{N}\left(-\alpha_{i3}+\bar{p}_i^2+\alpha_{i1}^2+\frac{c_{i4}^2\mathcal{L}_{\omega_i}^2\mathcal{L}_{F_i}^2}{4}\right)\left\|\tilde{z}_i^b\right\|^2 \\
& +\sum\nolimits_{i=1}^{N}\left\{-\frac{1}{\varepsilon_i}\lambda_{\min}\left(\Upsilon_i\right)+\left(1+\rho_{i1}^2+\rho_{i2}^2\right)\left\|\Omega_i\right\|^2+2\alpha_{i2}\left\|\Omega_i\right\|+\left\|\Omega_i\bar{B}_2D_i^{-1}\right\|^2+\alpha_{i4}+\left\|\bar{D}_i\right\|^2\right\}\left\|\chi_i\right\|^2 \\
& +\sum\nolimits_{i=1}^{N}\left\{6N\sum\nolimits_{j=1}^{N}\varsigma_{ij}^2\left\|z_j^b\right\|^2+4N\sum\nolimits_{j=1}^{N}\omega_{ij1}^2\left\|z_j^b\right\|^2+2Nr\sum\nolimits_{j=1}^{N}\sum\nolimits_{l=1}^{r}\omega_{ijl}^2\left\|z_j^b\right\|^2\right\} \\
& +\sum\nolimits_{i=1}^{N}\left\{\begin{array}{c} 6N\sum_{j=1}^{N}\left(\sum_{k=1}^{pq}\tau_{ijk}^2\right)\left(\sum_{k=1}^{pq}\left\|z_{j1}^a\right\|^{2k}\right)+4N\sum_{j=1}^{N}\left(\sum_{k=1}^{pq}\upsilon_{ij1k}^2\right)\left(\sum_{k=1}^{pq}\left\|z_{j1}^a\right\|^{2k}\right) \\ +2Nr\sum_{j=1}^{N}\sum_{l=1}^{r}\left(\sum_{k=1}^{pq}\upsilon_{ijlk}^2\right)\left(\sum_{k=1}^{pq}\left\|z_{j1}^a\right\|^{2k}\right)\end{array}\right\} \\
& +\sum\nolimits_{i=1}^{N}\left\{\left(-2\lambda_{i1}+2+\frac{c_{i4}^2\mathcal{L}_{\omega_i}^2}{4}\right)\zeta_{i1}^2+2\zeta_{i1}\zeta_{i2}\right\} \\
& +\sum\nolimits_{i=1}^{N}\left\{2\left(\hat{\beta}_i-\beta_i^*\right)\left(\sum\nolimits_{k=1}^{pq}2^{2k-2}\zeta_{i1}^{2k}-\gamma_i\hat{\beta}_i\right)-2\hat{\beta}_i\sum\nolimits_{k=1}^{pq}2^{2k-2}\zeta_{i1}^{2k}\right\}
\end{aligned} \tag{37}$$

For the final term, it follows from the inequality $-2\gamma_i\hat{\beta}_i\left(\hat{\beta}_i-\beta_i^*\right)\leq-\gamma_i\left(\hat{\beta}_i-\beta_i^*\right)^2+\gamma_i\beta_i^{*2}$,

$$\begin{aligned}
2\left(\hat{\beta}_i-\beta_i^*\right)\left(\sum\nolimits_{k=1}^{pq}2^{2k-2}\zeta_{i1}^{2k}-\gamma_i\hat{\beta}_i\right)-2\hat{\beta}_i\sum\nolimits_{k=1}^{pq}2^{2k-2}\zeta_{i1}^{2k} &= -2\beta_i^*\sum\nolimits_{k=1}^{pq}2^{2k-2}\zeta_{i1}^{2k}-2\gamma_i\hat{\beta}_i\left(\hat{\beta}_i-\beta_i^*\right) \\
&\leq -\beta_i^*\sum\nolimits_{k=1}^{pq}2^{2k-1}\zeta_{i1}^{2k}-\gamma_i\left(\hat{\beta}_i-\beta_i^*\right)^2+\gamma_i\beta_i^{*2}
\end{aligned} \tag{38}$$

Define $d_i=\sum_{j=1}^{N}\sum_{k=1}^{pq}\tau_{jik}^2$, $h_i=\sum_{j=1}^{N}\sum_{k=1}^{pq}\upsilon_{ji1k}^2$ and $m_i=\sum_{j=1}^{N}\sum_{l=1}^{r}\sum_{k=1}^{pq}\upsilon_{jilk}^2$. It follows from Lemmas 1-2, (37)-(38) that

$$\begin{aligned}
\dot{W}_1 \leq & \sum\nolimits_{i=1}^{N}\left(-c_{i3}+2+\frac{c_{i4}^2}{4}\right)\left\|z_i^b\right\|^2+\sum\nolimits_{i=1}^{N}\left(-\alpha_{i3}+\bar{p}_i^2+\alpha_{i1}^2+\frac{c_{i4}^2\mathcal{L}_{\omega_i}^2\mathcal{L}_{F_i}^2}{4}\right)\left\|\tilde{z}_i^b\right\|^2 \\
& +\sum\nolimits_{i=1}^{N}\left\{-\frac{1}{\varepsilon_i}\lambda_{\min}\left(\Upsilon_i\right)+\left(1+\rho_{i1}^2+\rho_{i2}^2\right)\left\|\Omega_i\right\|^2+2\alpha_{i2}\left\|\Omega_i\right\|+\left\|\Omega_i\bar{B}_2D_i^{-1}\right\|^2+\alpha_{i4}+\left\|\bar{D}_i\right\|^2\right\}\left\|\chi_i\right\|^2 \\
& +\sum\nolimits_{i=1}^{N}\left\{6N\sum\nolimits_{j=1}^{N}\varsigma_{ji}^2\left\|z_i^b\right\|^2+4N\sum\nolimits_{j=1}^{N}\omega_{ji1}^2\left\|z_i^b\right\|^2+2Nr\sum\nolimits_{j=1}^{N}\sum\nolimits_{l=1}^{r}\omega_{jil}^2\left\|z_i^b\right\|^2\right\} \\
& +\sum\nolimits_{i=1}^{N}\sum\nolimits_{k=1}^{pq}\left\{2^{2k-1}\left(6Nd_i+4Nh_i+2Nrm_i\right)\left(\left\|\zeta_{i1}\right\|^{2k}+\left\|\psi_{i1}\right\|^{2k}\right)\right\} \\
& +\sum\nolimits_{i=1}^{N}\left\{\left(-2\lambda_{i1}+2+\frac{c_{i4}^2\mathcal{L}_{\omega_i}^2}{4}\right)\zeta_{i1}^2+2\zeta_{i1}\zeta_{i2}\right\} \\
& +\sum\nolimits_{i=1}^{N}\left\{-\beta_i^*\sum\nolimits_{k=1}^{pq}2^{2k-1}\zeta_{i1}^{2k}-\gamma_i\left(\hat{\beta}_i-\beta_i^*\right)^2+\gamma_i\beta_i^{*2}\right\}
\end{aligned} \tag{39}$$

It can be seen from (39) that the interconnections and uncertainties in the considered system itself and the destabilizing terms arising from the state observation error dynamics are all represented in the Lyapunov function. This motivates the design of an adaptive law to offset these effects.

Remark 10. It should be noted that the adaptive law (35) proposed in this paper may not produce the true parameter values. In fact, in adaptive control, it is usually unnecessary to design adaptive laws to obtain/estimate the true parameter values, see for example, [12], [38], specifically when UUB is considered. Although there are many parameter estimation methods that can be used to obtain the true value of corresponding parameters, strong restrictions are needed on the considered system (see, e.g., [53], [54], [55]).

Step 2

Consider the ζ_{i2} dynamics:

$$\dot{\zeta}_{i2}=\hat{z}_{i3}^a+\frac{\eta_{i2}}{\varepsilon_i^2}\tilde{z}_{i1}^a-\frac{\partial\psi_{i2}}{\partial\zeta_{i1}}\left(\psi_{i2}+\zeta_{i2}-\dot{\psi}_{i1}\right)-\frac{\partial\psi_{i2}}{\partial\zeta_{i1}}\left(\tilde{z}_{i2}^a+\Delta f_{i1}+\Delta_{i1}\right)-\frac{\partial\psi_{i2}}{\partial\hat{\beta}_i}\dot{\hat{\beta}}_i-\ddot{\psi}_{i1} \tag{40}$$

Define $\omega_{i2}\left(\zeta_{i1},\hat{z}_{i2}^a,\hat{\beta}_i,\psi_{i1},\dot{\psi}_{i1},\ddot{\psi}_{i1}\right)=-\frac{\partial\psi_{i2}}{\partial\zeta_{i1}}$ and

$$\varpi_{i2}\left(\zeta_{i1},\hat{z}_{i2}^a,\hat{\beta}_i,\psi_{i1},\dot{\psi}_{i1},\ddot{\psi}_{i1}\right)=\frac{\eta_{i2}}{\varepsilon_i^2}\tilde{z}_{i1}^a-\frac{\partial\psi_{i2}}{\partial\zeta_{i1}}\left(\psi_{i2}+\zeta_{i2}-\dot{\psi}_{i1}\right)-\frac{\partial\psi_{i2}}{\partial\hat{\beta}_i}\dot{\hat{\beta}}_i-\ddot{\psi}_{i1} \tag{41}$$

A Lyapunov function is chosen as:

$$W_2=W_1+\sum\nolimits_{i=1}^N\zeta_{i2}^2 \tag{42}$$

The time derivative of W_2 along the trajectories of (40) is given as:

$$\dot{W}_2=\dot{W}_1+2\sum\nolimits_{i=1}^N\zeta_{i2}\left\{\hat{z}_{i3}^a+\varpi_{i2}+\omega_{i2}\left(\tilde{z}_{i2}^a+\Delta f_{i1}+\Delta_{i1}\right)\right\} \tag{43}$$

For the term $2\sum_{i=1}^N\zeta_{i2}\omega_{i2}\tilde{z}_{i2}^a$, it follows that

$$2\sum\nolimits_{i=1}^N\zeta_{i2}\omega_{i2}\tilde{z}_{i2}^a\leq\sum\nolimits_{i=1}^N\omega_{i2}^2\zeta_{i2}^2+\sum\nolimits_{i=1}^N\left\|\bar{D}_i\right\|^2\chi_i^2 \tag{44}$$

For the term $2\sum_{i=1}^N\zeta_{i2}\omega_{i2}\left(\Delta f_{i1}+\Delta_{i1}\right)$, it follows that

$$\begin{aligned}2\sum\nolimits_{i=1}^N\zeta_{i2}\omega_{i2}\left(\Delta f_{i1}+\Delta_{i1}\right)&\leq\sum\nolimits_{i=1}^N\omega_{i2}^2\zeta_{i2}^2+2N\sum\nolimits_{i=1}^N\sum\nolimits_{j=1}^N\left\{\omega_{ij1}^2\left\|z_j^b\right\|^2+\sum\nolimits_{k=1}^{pq}\upsilon_{ij1k}^2\sum\nolimits_{k=1}^{pq}\left\|z_{j1}^a\right\|^{2k}\right\}\\&=\sum\nolimits_{i=1}^N\omega_{i2}^2\zeta_{i2}^2+2N\sum\nolimits_{i=1}^N\sum\nolimits_{j=1}^N\omega_{ji1}^2\left\|z_i^b\right\|^2+2N\sum\nolimits_{i=1}^N h_i\sum\nolimits_{k=1}^{pq}\left\|z_{i1}^a\right\|^{2k}\end{aligned} \tag{45}$$

Then define $\zeta_{i3}=\hat{z}_{i3}^a-\psi_{i3}$ and design $\psi_{i3}=-\lambda_{i2}\zeta_{i2}-\zeta_{i1}-\varpi_{i2}-\omega_{i2}^2\zeta_{i2}$ where $\lambda_{i2}>0$ is design parameter. It follows that

$$\begin{aligned}\dot{W}_2\leq\dot{W}_1&+2\sum\nolimits_{i=1}^N\zeta_{i2}\left\{\zeta_{i3}-\lambda_{i2}\zeta_{i2}-\zeta_{i1}\right\}+\sum\nolimits_{i=1}^N\left\|\bar{D}_i\right\|^2\chi_i^2\\&+2N\sum\nolimits_{i=1}^N\sum\nolimits_{j=1}^N\omega_{ji1}^2\left\|z_i^b\right\|^2+2N\sum\nolimits_{i=1}^N h_i\sum\nolimits_{k=1}^{pq}\left\|z_{i1}^a\right\|^{2k}\end{aligned} \tag{46}$$

It follows from (39) and (43)-(46) that

$$\begin{aligned}\dot{W}_2\leq&\sum\nolimits_{i=1}^N\left(-c_{i3}+2+\frac{c_{i4}^2}{4}\right)\left\|z_i^b\right\|^2+\sum\nolimits_{i=1}^N\left(-\alpha_{i3}+\bar{p}_i^2+\alpha_{i1}^2+\frac{c_{i4}^2\mathcal{L}_{\omega_i}^2\mathcal{L}_{F_i}^2}{4}\right)\left\|\tilde{z}_i^b\right\|^2\\&+\sum\nolimits_{i=1}^N\left\{-\frac{1}{\varepsilon_i}\lambda_{\min}\left(\Upsilon_i\right)+\left(1+\rho_{i1}^2+\rho_{i2}^2\right)\left\|\Omega_i\right\|^2+2\alpha_{i2}\left\|\Omega_i\right\|+\left\|\Omega_i\bar{B}_2D_i^{-1}\right\|^2+\alpha_{i4}+2\left\|\bar{D}_i\right\|^2\right\}\left\|\chi_i\right\|^2\\&+\sum\nolimits_{i=1}^N\left\{6N\sum\nolimits_{j=1}^N\varsigma_{ji}^2\left\|z_i^b\right\|^2+6N\sum\nolimits_{j=1}^N\omega_{ji1}^2\left\|z_i^b\right\|^2+2Nr\sum\nolimits_{j=1}^N\sum\nolimits_{l=1}^r\omega_{jil}^2\left\|z_i^b\right\|^2\right\}\\&+\sum\nolimits_{i=1}^N\sum\nolimits_{k=1}^{pq}\left\{2^{2k-1}\left(6Nd_i+6Nh_i+2Nrm_i\right)\left(\left\|\zeta_{i1}\right\|^{2k}+\left\|\psi_{i1}\right\|^{2k}\right)\right\}\\&+\sum\nolimits_{i=1}^N\left\{\left(-2\lambda_{i1}+2+\frac{c_{i4}^2\mathcal{L}_{\omega_i}^2}{4}\right)\zeta_{i1}^2-2\lambda_{i2}\zeta_{i2}^2+2\zeta_{i2}\zeta_{i3}\right\}\\&+\sum\nolimits_{i=1}^N\left\{-\beta_i^*\sum\nolimits_{k=1}^{pq}2^{2k-1}\zeta_{i1}^{2k}-\gamma_i\left(\hat{\beta}_i-\beta_i^*\right)^2+\gamma_i\beta_i^{*2}\right\}\end{aligned} \tag{47}$$

Step $m\left(3\leqslant m\leqslant r-1\right)$

Consider the ζ_{im} dynamics:

$$\dot{\zeta}_{im}=\hat{z}_{i,m+1}^a+\varpi_{im}\left(\zeta_{i1},\hat{z}_{i2}^a,\cdots,\hat{z}_{i,m}^a,\hat{\beta}_i,\psi_{i1},\cdots,\psi_{i1}^{(m)}\right)+\omega_{im}\left(\zeta_{i1},\hat{z}_{i2}^a,\cdots,\hat{z}_{i,m}^a,\hat{\beta}_i,\psi_{i1},\cdots,\psi_{i1}^{(m)}\right)\left(\tilde{z}_{i2}^a+\Delta f_{i1}+\Delta_{i1}\right) \tag{48}$$

where $\zeta_{im}=\hat{z}_{im}^a-\psi_{im}\left(\zeta_{i1},\hat{z}_{i2}^a,\cdots,\hat{z}_{i,m-1}^a,\hat{\beta}_i,\psi_{i1},\cdots,\psi_{i1}^{(m-1)}\right)$ with $\psi_{i,m}=-\lambda_{i,m-1}\zeta_{i,m-1}-\zeta_{i,m-2}-\varpi_{i,m-1}-\zeta_{i,m-1}\omega_{i,m-1}^2$, ϖ_{im} and ω_{im} can be obtained by iteration where the specific expressions are given by

$$\begin{aligned}\omega_{im}&=-\tfrac{\partial\psi_{im}}{\partial\zeta_{i1}}\\\varpi_{im}&=\tfrac{\eta_{im}}{\varepsilon_i^m}\tilde{z}_{i1}^a-\tfrac{\partial\psi_{im}}{\partial\zeta_{i1}}\left(\psi_{i2}+\zeta_{i2}-\dot{\psi}_{i1}\right)-\sum\nolimits_{j=2}^{m-1}\left(\tfrac{\partial\psi_{im}}{\partial\hat{z}_{ij}^a}\dot{\hat{z}}_{ij}^a\right)-\tfrac{\partial\psi_{im}}{\partial\hat{\beta}_i}\dot{\hat{\beta}}_i-\sum\nolimits_{j=1}^m\left(\tfrac{\partial\psi_{i3}}{\partial\psi_{i1}^{(j-1)}}\psi_{i1}^{(j)}\right)\end{aligned} \tag{49}$$

A Lyapunov function is chosen as:

$$W_m=W_{m-1}+\sum\nolimits_{i=1}^N\zeta_{im}^2 \tag{50}$$

The time derivative of W_m along the trajectories of (48) is given as:

$$\dot{W}_m = \dot{W}_{m-1} + 2\sum_{i=1}^{N} \zeta_{im} \left\{ \hat{z}_{i,m+1}^a + \varpi_{im} + \omega_{im} \left(\tilde{z}_{i2}^a + \Delta f_{i1} + \Delta_{i1} \right) \right\} \tag{51}$$

Define $\zeta_{i,m+1} = \hat{z}_{i,m+1}^a - \psi_{i,m+1}$ and design $\psi_{i,m+1} = -\lambda_{im}\zeta_{im} - \zeta_{i,m-1} - \varpi_{im} - \zeta_{im}\omega_{im}^2$ where λ_{im} is design parameter. It follows that

$$\begin{aligned}
\dot{W}_m \leq & \sum_{i=1}^{N} \left(-c_{i3} + 2 + \frac{c_{i4}^2}{4} \right) \left\| z_i^b \right\|^2 + \sum_{i=1}^{N} \left(-\alpha_{i3} + \bar{p}_i^2 + \alpha_{i1}^2 + \frac{c_{i4}^2 \mathcal{L}_{\omega_i}^2 \mathcal{L}_{F_i}^2}{4} \right) \left\| \tilde{z}_i^b \right\|^2 \\
& + \sum_{i=1}^{N} \left\{ -\frac{1}{\varepsilon_i} \lambda_{\min} \left(\Upsilon_i \right) + \left(1 + \rho_{i1}^2 + \rho_{i2}^2 \right) \left\| \Omega_i \right\|^2 + 2\alpha_{i2} \left\| \Omega_i \right\| + \left\| \Omega_i \bar{B}_2 D_i^{-1} \right\|^2 + \alpha_{i4} + m \left\| \bar{D}_i \right\|^2 \right\} \left\| \chi_i \right\|^2 \\
& + \sum_{i=1}^{N} \left\{ 6N \sum_{j=1}^{N} \varsigma_{ji}^2 \left\| z_i^b \right\|^2 + 2\left(m+1 \right) N \sum_{j=1}^{N} \omega_{ji1}^2 \left\| z_i^b \right\|^2 + 2Nr \sum_{j=1}^{N} \sum_{l=1}^{r} \omega_{jil}^2 \left\| z_i^b \right\|^2 \right\} \\
& + \sum_{i=1}^{N} \sum_{k=1}^{pq} \left\{ 2^{2k-1} \left(6Nd_i + 2\left(m+1 \right) Nh_i + 2Nrm_i \right) \left(\left\| \zeta_{i1} \right\|^{2k} + \left\| \psi_{i1} \right\|^{2k} \right) \right\} \\
& + \sum_{i=1}^{N} \left\{ \left(-2\lambda_{i1} + 2 + \frac{c_{i4}^2 \mathcal{L}_{\omega_i}^2}{4} \right) \zeta_{i1}^2 - 2\sum_{j=2}^{m} \lambda_{ij} \zeta_{ij}^2 + 2\zeta_{im}\zeta_{i,m+1} \right\} \\
& + \sum_{i=1}^{N} \left\{ -\beta_i^* \sum_{k=1}^{pq} 2^{2k-1} \zeta_{i1}^{2k} - \gamma_i \left(\hat{\beta}_i - \beta_i^* \right)^2 + \gamma_i \beta_i^{*2} \right\}
\end{aligned} \tag{52}$$

Step r

Based on the above analysis,

$$\psi_{i,r} = -\lambda_{i,r-1}\zeta_{i,r-1} - \zeta_{i,r-2} - \varpi_{i,r-1} - \zeta_{i,r-1}\omega_{i,r-1}^2 \tag{53}$$

$$\begin{aligned}
\dot{W}_{r-1} \leq & \sum_{i=1}^{N} \left(-c_{i3} + 2 + \frac{c_{i4}^2}{4} \right) \left\| z_i^b \right\|^2 + \sum_{i=1}^{N} \left(-\alpha_{i3} + \bar{p}_i^2 + \alpha_{i1}^2 + \frac{c_{i4}^2 \mathcal{L}_{\omega_i}^2 \mathcal{L}_{F_i}^2}{4} \right) \left\| \tilde{z}_i^b \right\|^2 \\
& + \sum_{i=1}^{N} \left\{ -\frac{1}{\varepsilon_i} \lambda_{\min} \left(\Upsilon_i \right) + \left(1 + \rho_{i1}^2 + \rho_{i2}^2 \right) \left\| \Omega_i \right\|^2 + 2\alpha_{i2} \left\| \Omega_i \right\| + \left\| \Omega_i \bar{B}_2 D_i^{-1} \right\|^2 + \alpha_{i4} + \left(r-1 \right) \left\| \bar{D}_i \right\|^2 \right\} \left\| \chi_i \right\|^2 \\
& + \sum_{i=1}^{N} \left\{ 6N \sum_{j=1}^{N} \varsigma_{ji}^2 \left\| z_i^b \right\|^2 + 2rN \sum_{j=1}^{N} \omega_{ji1}^2 \left\| z_i^b \right\|^2 + 2Nr \sum_{j=1}^{N} \sum_{l=1}^{r} \omega_{jil}^2 \left\| z_i^b \right\|^2 \right\} \\
& + \sum_{i=1}^{N} \sum_{k=1}^{pq} \left\{ 2^{2k-1} \left(6Nd_i + 2rNh_i + 2Nrm_i \right) \left(\left\| \zeta_{i1} \right\|^{2k} + \left\| \psi_{i1} \right\|^{2k} \right) \right\} \\
& + \sum_{i=1}^{N} \left\{ \left(-2\lambda_{i1} + 2 + \frac{c_{i4}^2 \mathcal{L}_{\omega_i}^2}{4} \right) \zeta_{i1}^2 - 2\sum_{j=2}^{r-1} \lambda_{ij} \zeta_{ij}^2 + 2\zeta_{im}\zeta_{i,m+1} \right\} \\
& + \sum_{i=1}^{N} \left\{ -\beta_i^* \sum_{k=1}^{pq} 2^{2k-1} \zeta_{i1}^{2k} - \gamma_i \left(\hat{\beta}_i - \beta_i^* \right)^2 + \gamma_i \beta_i^{*2} \right\}
\end{aligned} \tag{54}$$

Then consider the ζ_{ir} dynamics:

$$\dot{\zeta}_{ir} = \hat{\sigma}_i + \mu_i \left(z_{i1}^a \right) u_i + \varpi_{ir} \left(\zeta_{i1}, \hat{z}_{i2}^a, \cdots, \hat{z}_{i,r}^a, \hat{\beta}_i, \psi_{i1}, \cdots, \psi_{i1}^{(r)} \right) + \omega_{ir} \left(\zeta_{i1}, \hat{z}_{i2}^a, \cdots, \hat{z}_{i,r}^a, \hat{\beta}_i, \psi_{i1}, \cdots, \psi_{i1}^{(r)} \right) \left(\tilde{z}_{i2}^a + \Delta f_{i1} + \Delta_{i1} \right) \tag{55}$$

with $\zeta_{ir} = \hat{z}_{ir}^a - \psi_{ir} \left(\zeta_{i1}, \hat{z}_{i2}^a, \cdots, \hat{z}_{i,r-1}^a, \hat{\beta}_i, \psi_{i1}, \cdots, \psi_{i1}^{(r-1)} \right)$.

A Lyapunov function is chosen as:

$$W = W_{r-1} + \sum_{i=1}^{N} \zeta_{ir}^2 \tag{56}$$

The time derivative of W along the trajectories of (55) is given as:

$$\dot{W} = \dot{W}_{r-1} + 2\sum_{i=1}^{N} \zeta_{ir} \left\{ \hat{\sigma}_i + \mu_i \left(z_{i1}^a \right) u_i + \varpi_{ir} + \omega_{ir} \left(\tilde{z}_{i2}^a + \Delta f_{i1} + \Delta_{i1} \right) \right\} \tag{57}$$

Then design the output feedback control:

$$u_i = -\frac{1}{\mu_i \left(z_{i1}^a \right)} \left(\hat{\sigma}_i + \varpi_{ir} + \lambda_{ir}\zeta_{i,r} + \zeta_{i,r-1} + \omega_{i,r}^2 \zeta_{ir} \right) \tag{58}$$

where $\lambda_{ir} > 0$ is a design parameter.

It follows from (54)-(58) that

$$\begin{aligned}\dot{W} \leq & \sum_{i=1}^{N}\left(-c_{i3}+2+\frac{c_{i4}^{2}}{4}+6N\sum_{j=1}^{N}\varsigma_{ji}^{2}+2\left(r+1\right)N\sum_{j=1}^{N}\omega_{ji1}^{2}+2Nr\sum_{j=1}^{N}\sum_{l=1}^{r}\omega_{jil}^{2}\right)\left\|z_{i}^{b}\right\|^{2} \\ & +\sum_{i=1}^{N}\left(-\alpha_{i3}+\bar{p}_{i}^{2}+\alpha_{i1}^{2}+\frac{c_{i4}^{2}\mathcal{L}_{\omega_i}^{2}\mathcal{L}_{F_i}^{2}}{4}\right)\left\|\tilde{z}_{i}^{b}\right\|^{2}+\sum_{i=1}^{N}\left\{\left(-2\lambda_{i1}+2+\frac{c_{i4}^{2}\mathcal{L}_{\omega_i}^{2}}{4}\right)\zeta_{i1}^{2}-2\sum_{j=2}^{r}\lambda_{ij}\zeta_{ij}^{2}\right\} \\ & +\sum_{i=1}^{N}\left\{-\frac{1}{\varepsilon_i}\lambda_{\min}\left(\Upsilon_i\right)+\left(1+\rho_{i1}^{2}+\rho_{i2}^{2}\right)\left\|\Omega_i\right\|^{2}+2\alpha_{i2}\left\|\Omega_i\right\|+\left\|\Omega_i\bar{B}_2D_i^{-1}\right\|^{2}+\alpha_{i4}+r\left\|\bar{D}_i\right\|^{2}\right\}\left\|\chi_i\right\|^{2} \\ & +\sum_{i=1}^{N}\sum_{k=1}^{pq}\left\{2^{2k-1}\left(6Nd_i+2\left(r+1\right)Nh_i+2Nrm_i\right)\left(\left\|\zeta_{i1}\right\|^{2k}+\left\|\psi_{i1}\right\|^{2k}\right)\right\} \\ & +\sum_{i=1}^{N}\left\{-\beta_i^{*}\sum_{k=1}^{pq}2^{2k-1}\zeta_{i1}^{2k}-\gamma_i\left(\hat{\beta}_i-\beta_i^{*}\right)^{2}+\gamma_i\beta_i^{*2}\right\}\end{aligned} \tag{59}$$

Theorem 1: Suppose Assumptions 1-6 are satisfied. In the considered domain $\tilde{z}_i^b \in \aleph_i$ and $z_i \in \mathrm{Z}_i$, all signals in the closed-loop system formed by system (1) and the state observers (14) associated with the control (58) are UUB if $\kappa > 0$ with κ defined by

$$\kappa = \min_{1\leq i\leq N}\left(\min\left\{-\kappa_{i1}\lambda_{\max}(\Omega_i)^{-1}, -\kappa_{i2}\bar{p}^{-1}, -\kappa_{i3}c_{i2}^{-1}, -\kappa_{i4}, 2\lambda_{i2}, \cdots, 2\lambda_{ir}, \gamma_i\tau_i\right\}\right) \tag{60}$$

where $\kappa_{i1} = -\frac{1}{\varepsilon_i}\lambda_{\min}\left(\Upsilon_i\right)+\left(1+\rho_{i1}^{2}+\rho_{i2}^{2}\right)\left\|\Omega_i\right\|^{2}+2\alpha_{i2}\left\|\Omega_i\right\|+\left\|\Omega_i\bar{B}_2D_i^{-1}\right\|^{2}+\alpha_{i4}+r\left\|\bar{D}_i\right\|^{2}$, $\kappa_{i2} = -\alpha_{i3}+\bar{p}_i^2+\alpha_{i1}^2+\frac{c_{i4}^2\mathcal{L}_{\omega_i}^2\mathcal{L}_{F_i}^2}{4}$, $\kappa_{i3} = -c_{i3}+2+\frac{c_{i4}^2}{4}+6N\sum_{j=1}^{N}\varsigma_{ji}^2+2\left(r+1\right)N\sum_{j=1}^{N}\omega_{ji1}^2+2Nr\sum_{j=1}^{N}\sum_{l=1}^{r}\omega_{jil}^2$ and $\kappa_{i4} = -2\lambda_{i1}+2+\frac{c_{i4}^2\mathcal{L}_{\omega_i}^2}{4}$.

***Proof*:**

Choose the desired value β_i^* satisfying

$$\beta_i^* \geq 6Nd_i+2\left(r+1\right)Nh_i+2Nrm_i \tag{61}$$

Then the possibly destabilizing term $\sum_{i=1}^{N}\sum_{k=1}^{pq}\left(\cdot\right)\left\|\zeta_{i1}\right\|^{2k}$ can be fully compensated by the adaptive law. It follows from (59) and (61) that

$$\begin{aligned}\dot{W} \leq & \sum_{i=1}^{N}\left(\kappa_{i1}\left\|\chi_i\right\|^{2}+\kappa_{i2}\left\|\tilde{z}_i^b\right\|^{2}+\kappa_{i3}\left\|z_i^b\right\|^{2}\right)+\sum_{i=1}^{N}\left(\kappa_{i4}\zeta_{i1}^{2}-2\sum_{j=2}^{r}\lambda_{im}\zeta_{im}^{2}\right)-\sum_{i=1}^{N}\gamma_i\left(\hat{\beta}_i-\beta_i^{*}\right)^{2} \\ & +\sum_{i=1}^{N}\sum_{k=1}^{pq}\left\{2^{2k-1}\left(6Nd_i+2\left(r+1\right)Nh_i+2Nrm_i\right)\left\|\psi_{i1}\right\|^{2k}\right\}+\sum_{i=1}^{N}\gamma_i\beta_i^{*2}\end{aligned} \tag{62}$$

It follows from Assumption 6 that the functions $F_i\left(t, z_i^b\right)$ satisfy the Lipschitz condition. Hence in the considered domain $\tilde{z}_i^b \in \aleph_i$ and $z_i \in \mathrm{Z}_i$, there exist constants $\bar{\psi}_{i1}$ such that $\left\|\psi_{i1}\right\| \leq \bar{\psi}_{i1}$.

Then define

$$\prod = \sum_{i=1}^{N}\left\{\sum_{k=1}^{pq}\left\{2^{2k-1}\left(6Nd_i+2\left(r+1\right)Nh_i+2Nrm_i\right)\bar{\psi}_{i1}^{2k}\right\}+\gamma_i\beta_i^{*2}\right\} \tag{63}$$

It follows from (62) and (63) that

$$\dot{W} \leq -\kappa W+\prod \tag{64}$$

Hence, $W\left(t, z^b, \tilde{z}^b, \chi, \zeta, \hat{\beta}\right)$ decreases monotonically until $\left(t, z^b, \tilde{z}^b, \chi, \zeta, \hat{\beta}\right)$ reaches the compact set

$$R_s = \left\{\left(t, z^b, \tilde{z}^b, \chi, \zeta, \hat{\beta}\right) \in R^{+}\times R^{n-Nr}\times R^{n-Nr}\times R^{N(r+1)}\times R^{Nr}\times R^{N} : W\left(t, z^b, \tilde{z}^b, \chi, \zeta, \hat{\beta}\right) \leq \kappa^{-1}\prod\right\} \tag{65}$$

where $n = \sum_{i=1}^{N} n_i$, $\tilde{z}^b := col\left(\tilde{z}_1^b, \cdots, \tilde{z}_N^b\right)$, $\chi := col\left(\chi_{i1}, \cdots, \chi_{iN}\right)$, $\zeta := col\left(\zeta_1, \cdots, \zeta_N\right)$ with $\zeta_i := col\left(\zeta_{i1}, \cdots, \zeta_{ir}\right)$ and $\hat{\beta} := col\left(\hat{\beta}_{i1}, \cdots, \hat{\beta}_{iN}\right)$.

Hence, Theorem 1 follows form the condition $\kappa > 0$.

Remark 11. It can be seen from (64) that the closed-loop system is UUB, rather than exponentially stable, due to the presence of the term $\prod$. $\prod$ is mainly determined by $\bar{\psi}_{i1}$ and β_i^*, which are generated when dealing with the nonlinear non-minimum phase characteristics, the unmatched uncertainties and interconnections bounded by an unknown high-order polynomial, respectively.

Remark 12. Note that if system (1) is minimum phase, the proposed method is still feasible. In that case, Assumption 6 will be satisfied with $F_i = 0$, hence the observer (14) and control (58) do not need to be modified, and the closed-loop system is also UUB. In this case (63) will become $\prod = \sum_{i=1}^{N} \gamma_i {\beta_i^*}^2$.

Remark 13. It should be noted that although the zero dynamics $\dot{z}_i^b = \omega_i\left(z_i^b, 0\right)$ is allowed to be unstable in this paper, when the dynamic equation $\dot{z}_i^b = \omega_i\left(z_i^b, z_{i1}^a\right)$ is controllable with respect to the virtual control input z_{i1}^a, a smooth function $z_{i1}^a = F_i\left(t, z_i^b\right)$ can always be found that makes the term $\omega_i\left(z_i^b, z_{i1}^a\right)$ converge exponentially. This provides a solution whereby, if the designed control can ensure that z_{i1}^a converges to $F_i\left(t, z_i^b\right)$, then the non-minimum phase problem may be solved. However, z_i^b is unmeasurable, so the observers (14) are first designed to obtain its estimated value. Then by defining $\zeta_{i1} = z_{i1}^a - F_i\left(t, \hat{z}_i^b\right)$, a decentralized robust adaptive backstepping control is proposed to guarantee ζ_{i1} is convergent. Finally, a Lyapunov approach is used to ensure that all signals in the closed-loop system formed by system (1) and the state observers (14) associated with the control (58) are UUB.

Remark 14. An "explosion of terms" problem is induced by calculating the high-order time derivatives of virtual inputs, which is a limitation of the standard backstepping control, as shown in (49). It should be pointed out that the Dynamic Surface Control (DSC) proposed in [56], [57] can be used to avoid this problem. Multiple derivative calculations of virtual inputs can be avoided by adding some first-order, low-pass filters as in [56], [57], which does not affect the closed-loop system stability in the sense of UUB. To improve control quality, on the one hand, the accuracy of the filter can be improved to meet the design requirements, which is relatively straightforward to implement, and on the other hand, other types of filters can be used if necessary.

Remark 15. It should be pointed out that the condition developed in Theorem 1 is sufficient. In the process of controller design and in the stability proof, the inequalities relating to Lyapunov functions, such as (36) – (39), will result in the final conclusion being conservative. Theorem 1 above indicates that the sufficient condition for the closed-loop system to be UUB is $\kappa > 0$, which implies that $\kappa_{i1} < 0$, $\kappa_{i2} < 0$, $\kappa_{i3} < 0$ and $\kappa_{i4} < 0$. However, due to the conservativeness in the method of proof, the system may still be UUB when these conditions do not hold. Although there is conservativeness in the analysis, the method can be applied to a large class of interconnected systems, especially those with unstable zero dynamics.

IV. Simulation Examples

This section will test the effectiveness of the designed method by two simulation examples.

Case 1:

Consider the multiple TORA system studied in [41], as shown in Fig.1. The dynamical model can be described by:

$$\begin{array}{l} \left(M_i + m_i\right)\ddot{x}_i - m_i L_i \dot{\theta}_i^2 \sin\theta_i + m_i L_i \ddot{\theta}_i \cos\theta_i + k_i\left(x_i - x_{i-1}\right) + k_{i+1}\left(x_i - x_{i+1}\right) = 0 \\ \left(J_i + m_i L_i^2\right)\ddot{\theta}_i + m_i L_i \ddot{x}_i \cos\theta_i = \tau_i \end{array} \tag{66}$$

where m_i is the rotor mass, J_i is the inertia of the rotational centre, L_i is the rotor rotational radius, M_i is the mass of the cart, k_i is the spring constant, θ_i is the rotor angular position and x_i is the cart translational position, τ_i is the input torque of the i^{th} subsystem respectively with $i = 1, 2, \cdots, N$.

Note that the multiple TORA system (66) is non-minimum phase.

Inspired by [24], introduce the following change of variables:

$$\begin{array}{l} z_{i1}^b = x_i + \frac{m_i L_i \sin\theta_i}{M_i + m_i} \\ z_{i2}^b = \dot{x}_i + \frac{m_i L_i \dot{\theta}_i \cos\theta_i}{M_i + m_i} \\ z_{i1}^a = \theta_i \\ z_{i2}^a = \dot{\theta}_i \end{array} \tag{67}$$

In practice, often only the rotor angular position can be measured, that is, $y_i = z_{i1}^a$. Then define $z_i^a := col\left(z_{i1}^a, z_{i2}^a\right)$ and

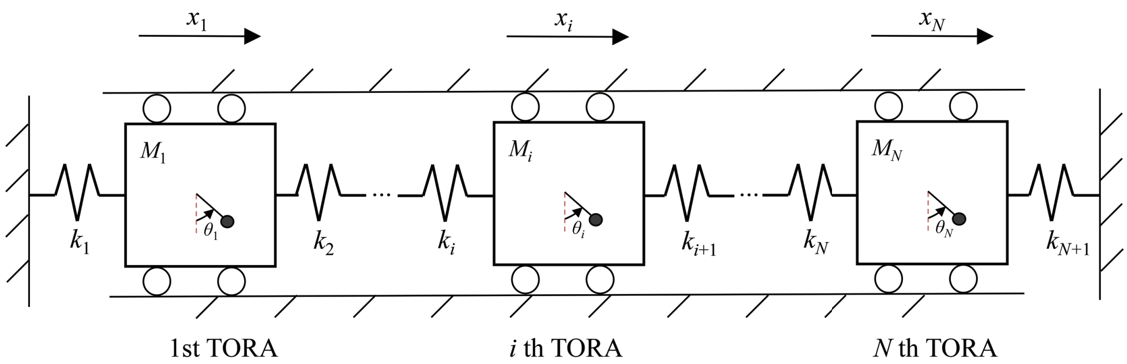


Fig. 1: Structure of multiple TORA system

$z_i^b := col\left(z_{i1}^b, z_{i2}^b\right)$. In the new coordinate system $\left(z_i^a, z_i^b\right)$, the system (66) can be described as:

$$\begin{aligned} \dot{z}_i^b &= \begin{bmatrix} z_{i2}^b \\ -\frac{k_i+k_{i+1}}{M_i+m_i} z_{i1}^b + \frac{(k_i+k_{i+1}) m_i L_i \sin z_{i1}^a}{(M_i+m_i)^2} \end{bmatrix} + \delta_i \\ \dot{z}_i^a &= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} z_i^a + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \left\{\xi_i + \mu_i\left(z_{i1}^a\right)\tau_i\right\} + \begin{bmatrix} 0 \\ \Delta f_{i2} \end{bmatrix} + \begin{bmatrix} 0 \\ \Delta_{i2} \end{bmatrix} \end{aligned} \tag{68}$$

where $\mu_i\left(z_{i1}^a\right) = \frac{(M_i+m_i)}{\left(J_i+m_i L_i^2\right)(M_i+m_i)-m_i^2 L_i^2 \cos^2 z_{i1}^a}$, $\xi_i = \frac{m_i L_i \cos z_{i1}^a (k_i+k_{i+1}) z_{i1}^b - m_i^2 L_i^2 (z_{i2}^a)^2 \sin z_{i1}^a \cos z_{i1}^a}{\left(J_i+m_i L_i^2\right)(M_i+m_i)-m_i^2 L_i^2 \cos^2 z_{i1}^a}$, the uncertainty Δf_{i2} and interconnections δ_i, Δ_{i2} are described by

$$\begin{aligned} \Delta f_{i2} &= -\frac{m_i^2 L_i^2 \sin z_{i1}^a \cos z_{i1}^a}{\left(J_i+m_i L_i^2\right)(M_i+m_i)-m_i^2 L_i^2 \cos^2 z_{i1}^a} \cdot \frac{k_i+k_{i+1}}{M_i+m_i} \\ \delta_i &= \begin{bmatrix} 0 \\ \frac{k_i}{M_i+m_i}\left(z_{i-1,1}^b - \frac{m_{i-1} L_{i-1} \sin z_{i-1,1}^a}{M_{i-1}+m_{i-1}}\right) + \frac{k_{i+1}}{M_i+m_i}\left(z_{i+1,1}^b - \frac{m_{i+1} L_{i+1} \sin z_{i+1,1}^a}{M_{i+1}+m_{i+1}}\right) \end{bmatrix} \\ \Delta_{i2} &= -\frac{m_i L_i \cos z_{i1}^a k_i \left(z_{i-1,1}^b - \frac{m_{i-1} L_{i-1} \sin z_{i-1,1}^a}{M_{i-1}+m_{i-1}}\right)}{\left(J_i+m_i L_i^2\right)(M_i+m_i)-m_i^2 L_i^2 \cos^2 z_{i1}^a} - \frac{m_i L_i \cos z_{i1}^a k_{i+1} \left(z_{i+1,1}^b - \frac{m_{i+1} L_{i+1} \sin z_{i+1,1}^a}{M_{i+1}+m_{i+1}}\right)}{\left(J_i+m_i L_i^2\right)(M_i+m_i)-m_i^2 L_i^2 \cos^2 z_{i1}^a} \end{aligned} \tag{69}$$

Two TORA sub-systems are used as the simulation test. The system parameters are given by:

$$\begin{aligned} M_1 = 1.3608\,kg, m_1 = 0.096\,kg, L_1 = 0.0592\,m, J_1 = 0.0002175\,kg \cdot m^2, k_1 = 186.3\,N \cdot m \\ M_2 = 1.2985\,kg, m_2 = 0.108\,kg, L_2 = 0.0604\,m, J_2 = 0.0001298\,kg \cdot m^2, k_2 = 186.3\,N \cdot m \end{aligned} \tag{70}$$

Choose $V_{i0} = \left(z_i^b\right)^T \left(z_i^b\right)$ with $F_1 = \arcsin\left(-0.52 z_{11}^b - 0.51 z_{12}^b\right)$ and $F_2 = \arcsin\left(-0.45 z_{21}^b - 0.40 z_{22}^b\right)$ such that Assumptions 1-6 are all satisfied.

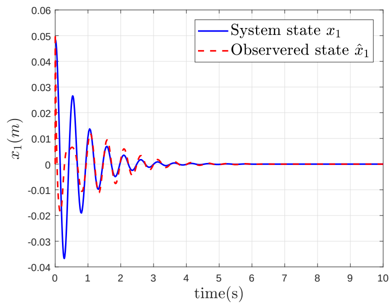
Denote the virtual system state $\sigma_i = \frac{(k_i+k_{i+1}) m_i L_i z_{i1}^b \cos z_{i1}^a}{\left(J_i+m_i L_i^2\right)(M_i+m_i)-m_i^2 L_i^2 \cos^2 z_{i1}^a}$, then the main parameters of the decentralized EKF-EHGO (14) are given by:

$$\begin{aligned} \varepsilon_1 = 0.003, \eta_{11} = 25, \eta_{12} = 2, \eta_{13} = 0.03, R_1 = 1, Q_1 = \begin{bmatrix} 0.8 & 0 \\ 0 & 0.8 \end{bmatrix} \\ \varepsilon_2 = 0.003, \eta_{21} = 3, \eta_{22} = 3, \eta_{23} = 1, R_2 = 1, Q_2 = \begin{bmatrix} 0.8 & 0 \\ 0 & 0.8 \end{bmatrix} \end{aligned} \tag{71}$$

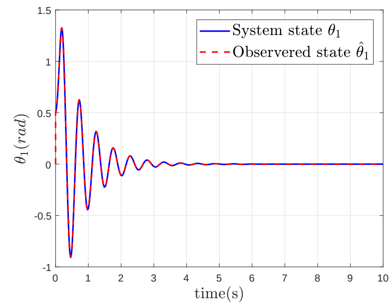
The main parameters of the control (58) are given by:

$$\begin{aligned} \gamma_1 = 0.01, \tau_1 = 100, \lambda_{11} = 1, \lambda_{12} = 2 \\ \gamma_2 = 0.02, \tau_2 = 100, \lambda_{21} = 2, \lambda_{22} = 3 \end{aligned} \tag{72}$$

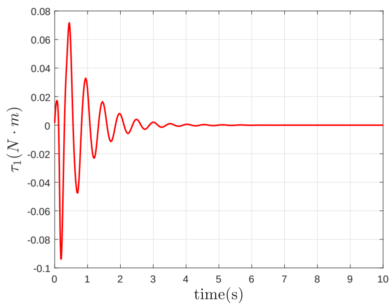
The time response of the original system states (x_i, θ_i) with their estimates, the system input torques and adaptation gains are shown in Fig.2 and Fig.4. The system states $\left(z_{i1}^b, z_{i2}^b, z_{i2}^a, \sigma_i\right)$ and their estimates are shown in Fig.3 and Fig.5. As shown in Figs.2-5, all signals in the nonlinear multiple TORA system (66) and the adaptation gains are all UUB despite the presence of unmatched interconnections and uncertainties while the designed EKF-EHGO can quickly observe the system states. The simulation results demonstrate that the designed method can effectively stabilize the multiple TORA system. Note that in this specific example, by direct calculation, the minimum values of the parameters β_i^* defined in (61) are $\beta_1^* = 1286.45$ and $\beta_2^* = 2435.26$, respectively. It is clear to see from the simulation that the adaptation gains β_1 and β_2 cannot estimate their corresponding desired values, but the proposed adaptive control can still guarantee the UUB stability of the controlled system.



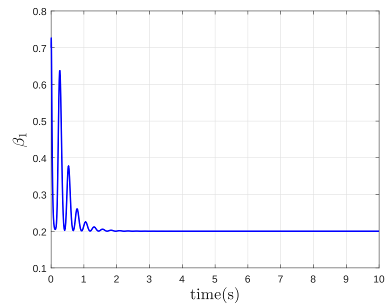
(a) The system state x_1 and its estimate $\hat{x}_1$



(b) The system state θ_1 and its estimate $\hat{\theta}_1$

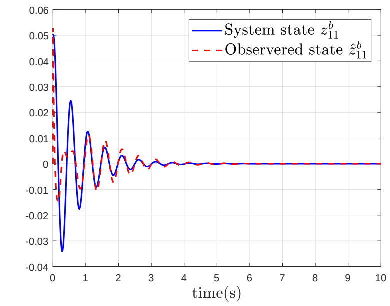


(c) The time response of the system input torque τ_1

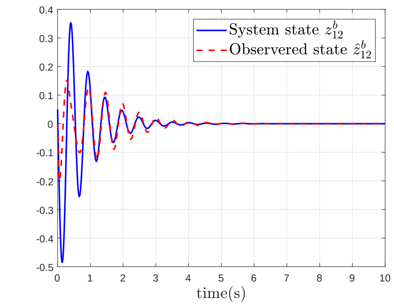


(d) The time response of the adaptation gain β_1

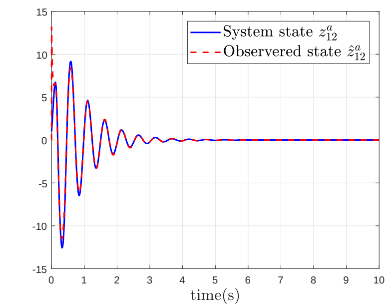
Fig. 2: The time response of the original system states (x_1, θ_1) with their estimates, the system input torque and adaptation gain



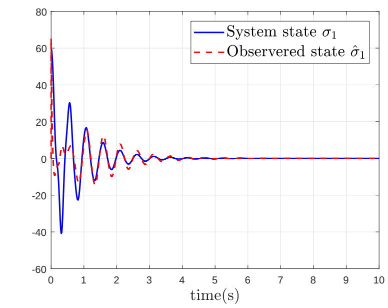
(a) The system state z_{11}^b and its estimate $\hat{z}_{11}^b$



(b) The system state z_{12}^b and its estimate $\hat{z}_{12}^b$

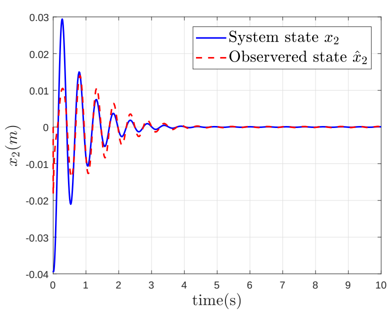


(c) The system state z_{12}^a and its estimate $\hat{z}_{12}^a$

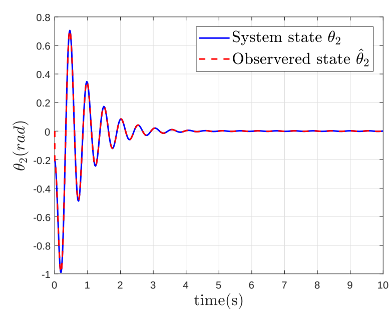


(d) The time response of σ_1 and its estimate $\hat{\sigma}_1$

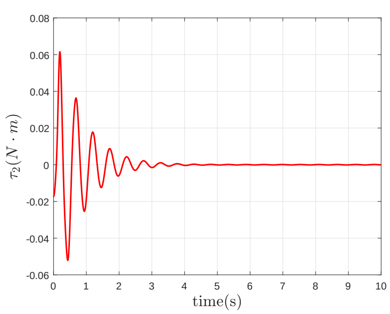
Fig. 3: The time response of the system states $\left(z_{11}^b, z_{12}^b, z_{12}^a, \sigma_1\right)$ and their estimates



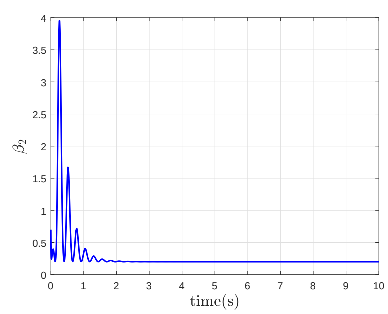
(a) The system state x_2 and its estimate $\hat{x}_2$



(b) The system state θ_2 and its estimate $\hat{\theta}_2$

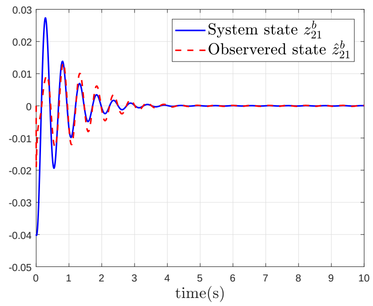


(c) The time response of the system input torque τ_2

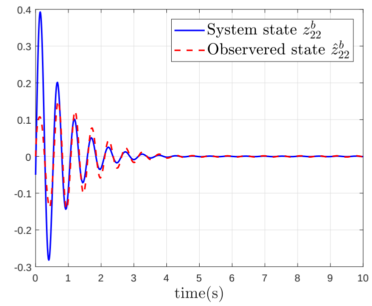


(d) The time response of the adaptation gain β_2

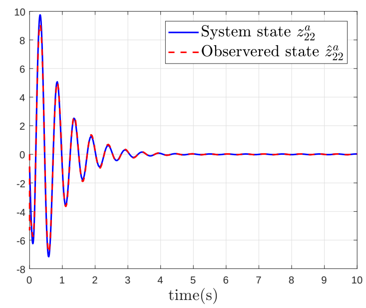
Fig. 4: The time response of the original system states (x_2, θ_2) with their estimates, the system input torque and adaptation gain



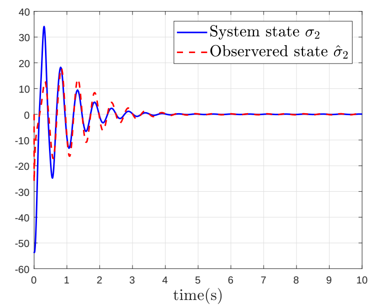
(a) The system state z_{21}^b and its estimate $\hat{z}_{21}^b$



(b) The system state z_{22}^b and its estimate $\hat{z}_{22}^b$



(c) The system state z_{22}^a and its estimate $\hat{z}_{22}^a$



(d) The time response of σ_2 and its estimate $\hat{\sigma}_2$

Fig. 5: The time response of the system states $\left(z_{21}^b, z_{22}^b, z_{22}^a, \sigma_2\right)$ and their estimates

To further test the proposed decentralized robust adaptive control, the result in [23] will be compared with the method proposed in this paper. A robust decentralised output feedback sliding mode controller has been designed in [23], although the considered interconnected systems in [23] were non-minimum phase, the nominal isolated subsystems were required to be linear. In order to use the method proposed in [23] on nonlinear interconnected systems (68), it is necessary to first linearize the considered system at the origin. By using a Taylor expansion around the origin and neglecting the higher order terms, the following linearized model of the nominal isolated systems of (68) can be obtained:

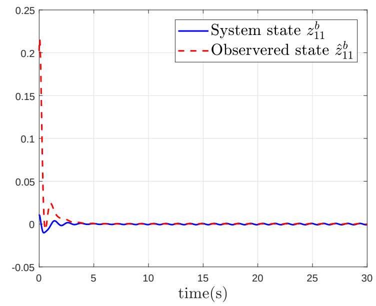
$$\begin{aligned}
&\begin{bmatrix} \dot{z}_{11}^a \\ \dot{z}_{12}^a \\ \dot{z}_{11}^b \\ \dot{z}_{12}^b \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 2733.433 & 0 \\ 0 & 0 & 0 & 1 \\ 0.998 & 0 & -255.766 & 0 \end{bmatrix} \begin{bmatrix} z_{11}^a \\ z_{12}^a \\ z_{11}^b \\ z_{12}^b \end{bmatrix} + \begin{bmatrix} 0 \\ 1880.497 \\ 0 \\ 0 \end{bmatrix} \tau_1 \\
&y_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} z_{11}^a & z_{12}^a & z_{11}^b & z_{12}^b \end{bmatrix}^T
\end{aligned} \tag{73}$$

$$\begin{aligned}
&\begin{bmatrix} \dot{z}_{21}^a \\ \dot{z}_{22}^a \\ \dot{z}_{21}^b \\ \dot{z}_{22}^b \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 3501.346 & 0 \\ 0 & 0 & 0 & 1 \\ 1.229 & 0 & -2.64.913 & 0 \end{bmatrix} \begin{bmatrix} z_{21}^a \\ z_{22}^a \\ z_{21}^b \\ z_{22}^b \end{bmatrix} + \begin{bmatrix} 0 \\ 2026.148 \\ 0 \\ 0 \end{bmatrix} \tau_2 \\
&y_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} z_{21}^a & z_{22}^a & z_{21}^b & z_{22}^b \end{bmatrix}^T
\end{aligned} \tag{74}$$

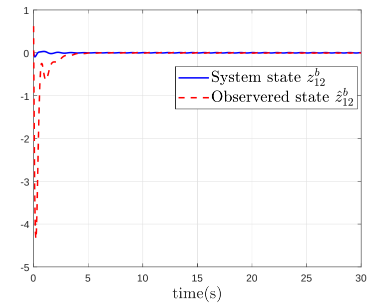
The main parameters of the reduced-order observer and sliding mode control in [23] are given by:

$$\begin{aligned}
L_1 &= \begin{bmatrix} 37 & 166.234 & -2.818 & -13.679 \end{bmatrix}^T, S_1 = \begin{bmatrix} 60 & 1 & -9.366.734 & 846.110 \end{bmatrix} \\
L_2 &= \begin{bmatrix} 37 & 157.087 & -2.297 & -9.971 \end{bmatrix}^T, S_2 = \begin{bmatrix} 37 & 1 & -6850.727 & 119.716 \end{bmatrix}
\end{aligned} \tag{75}$$

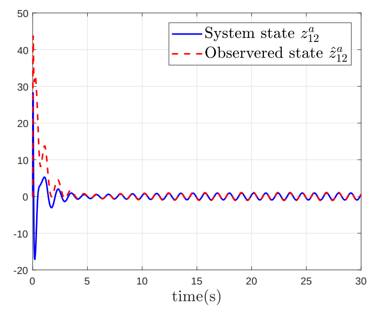
The time response of the system states $\left(z_{i1}^b, z_{i2}^b, z_{i2}^a\right)$ with their estimates and the system input torques using the method proposed in [23] are shown in Fig.6 and Fig.8. The corresponding time response of the original system states x_i with their estimates and θ_i using the method proposed in [23] are shown in Fig.7 and Fig.9. The time response of the sliding functions using the method proposed in [23] are shown in Fig.10. Comparing Figs.2-10, it can be seen that the approach proposed in this paper exhibits better control performance due to its direct consideration of the nonlinear characteristics while the method proposed in [23] shows poor system response. In fact, the method proposed in [23] delivers a controller that is only valid in a small region near the origin.



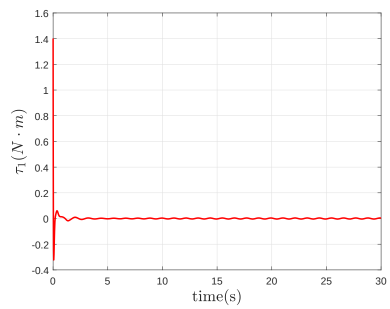
(a) The system state z_{11}^b and its estimate $\hat{z}_{11}^b$



(b) The system state z_{12}^b and its estimate $\hat{z}_{12}^b$

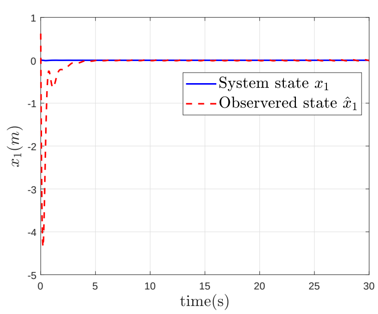


(c) The system state z_{12}^a and its estimate $\hat{z}_{12}^a$

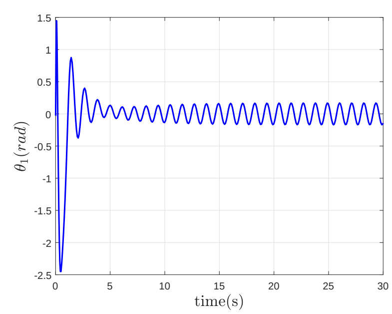


(d) The time response of the system input torque τ_1

Fig. 6: The time response of the system states $\left(z_{11}^b, z_{12}^b, z_{12}^a\right)$ with their estimates and the system input torque using the method proposed in [23]

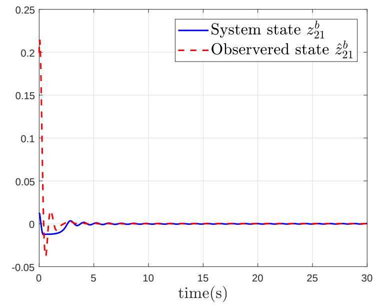


(a) The system state x_1 and its estimate $\hat{x}_1$

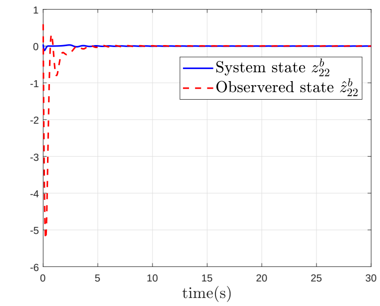


(b) The system state θ_1

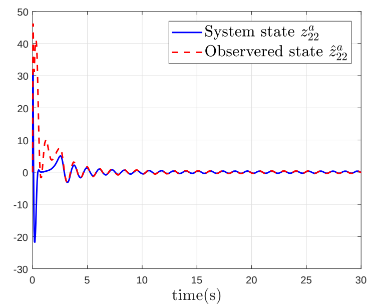
Fig. 7: The time response of x_1 with its estimate and θ_1 using the method proposed in [23]



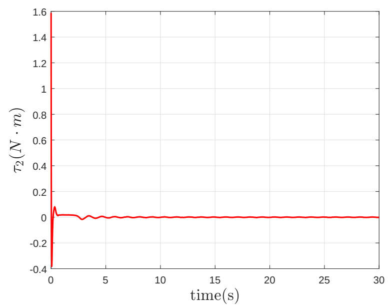
(a) The system state z_{21}^b and its estimate $\hat{z}_{21}^b$



(b) The system state z_{22}^b and its estimate $\hat{z}_{22}^b$

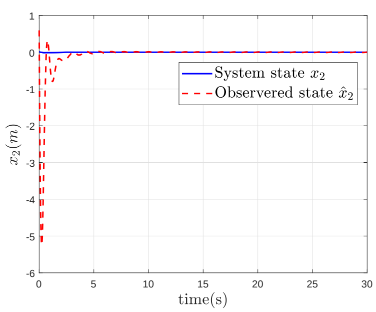


(c) The system state z_{22}^a and its estimate $\hat{z}_{22}^a$

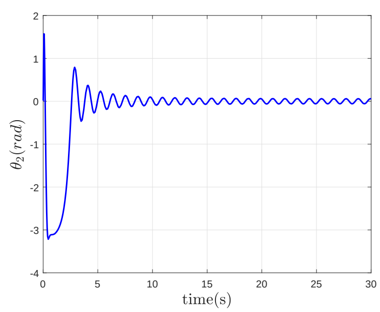


(d) The time response of the system input torque τ_2

Fig. 8: The time response of the system states $\left(z_{21}^b, z_{22}^b, z_{22}^a\right)$ with their estimates and the system input torque using the method proposed in [23]

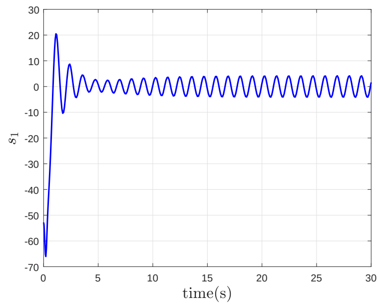


(a) The system state x_2 and its estimate $\hat{x}_2$

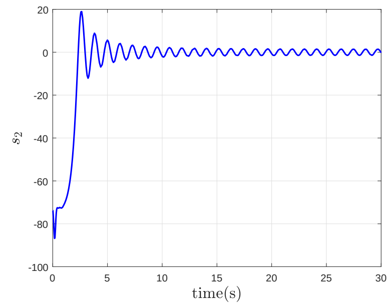


(b) The system state θ_2

Fig. 9: The time response of x_2 with its estimate and θ_2 using the method proposed in [23]



(a) The sliding functions s_1



(b) The sliding functions s_2

Fig. 10: The time response of the sliding functions using the method proposed in [23]

Case 2:

Consider the nonlinear interconnected system formed of two subsystems described by:

$$\begin{aligned} \dot{z}_1^b &= 3\left(z_1^b\right)^2 + z_1^b z_{11}^a + z_{11}^a + \Delta\delta_1 + \delta_1 + \Delta d_1 \\ \dot{z}_{11}^a &= z_{12}^a + \Delta f_{11} + \Delta_{11} + \Delta d_2 \\ \dot{z}_{12}^a &= z_1^b + z_{11}^a + u_1 + \Delta f_{12} + \Delta_{12} \\ y_1 &= z_{11}^a \end{aligned} \tag{76}$$

$$\begin{aligned} \dot{z}_2^b &= 2\left(z_2^b\right)^2 + 2z_2^b z_{21}^a + 2z_{21}^a + \Delta\delta_2 + \delta_2 + \Delta d_3 \\ \dot{z}_{21}^a &= z_{22}^a + \Delta f_{21} + \Delta_{21} \\ \dot{z}_{22}^a &= z_2^b + z_{21}^a + u_2 + \Delta f_{22} + \Delta_{22} + \Delta d_4 \\ y_2 &= z_{21}^a \end{aligned} \tag{77}$$

where $z := col\left(z_1, z_2\right) \in \mathrm{Z} = \left\{ \left(z_1, z_2\right) \middle| \left|z_1^b\right| \leq 0.1, \left|z_2^b\right| \leq 0.1 \right\}$, the interconnections and uncertainties satisfy

$$\begin{aligned} \Delta\delta_1 + \delta_1 &\leq 0.2\left|z_{11}^a\right| + 0.2\left|z_{11}^a\right|^2 + 0.1\left|z_2^b\right| + 0.2\left|z_{21}^a\right| + 0.2\left|z_{21}^a\right|^2 \\ \Delta f_{11} + \Delta_{11} &\leq 0.1\left|z_1^b\right| + 0.2\left|z_{11}^a\right| + 0.1\left|z_2^b\right| + 0.2\left|z_{21}^a\right| + 0.2\left|z_{21}^a\right|^2 \\ \Delta f_{12} + \Delta_{12} &\leq 0.1\left|z_1^b\right| + 0.2\left|z_{11}^a\right|^2 + 0.1\left|z_2^b\right| + 0.2\left|z_{21}^a\right| + 0.2\left|z_{21}^a\right|^2 \\ \Delta\delta_2 + \delta_2 &\leq 0.1\left|z_2^b\right| + 0.2\left|z_{21}^a\right| + 0.2\left|z_{21}^a\right|^2 + 0.1\left|z_1^b\right|^2 + 0.2\left|z_{11}^a\right| \\ \Delta f_{21} + \Delta_{21} &\leq 0.1\left|z_2^b\right| + 0.2\left|z_{21}^a\right| + 0.2\left|z_{21}^a\right|^2 + 0.1\left|z_1^b\right|^2 + 0.2\left|z_{11}^a\right|^2 \\ \Delta f_{22} + \Delta_{22} &\leq 0.1\left|z_2^b\right| + 0.2\left|z_{21}^a\right| + 0.1\left|z_1^b\right| + 0.2\left|z_{11}^a\right| + 0.2\left|z_{11}^a\right|^2 \end{aligned} \tag{78}$$

and $\Delta d_1, \Delta d_2, \Delta d_3, \Delta d_4$ represent the interconnections and uncertainties with nonzero steady-state values, which are defined by:

$$\begin{aligned} \Delta d_1 &= 0.002\cos\left(t\right), \Delta d_2 = 0.006\cos\left(z_1^b\right) \\ \Delta d_3 &= 0.005\sin\left(t\right), \Delta d_4 = 0.001\cos\left(z_{21}^a\right) \end{aligned} \tag{79}$$

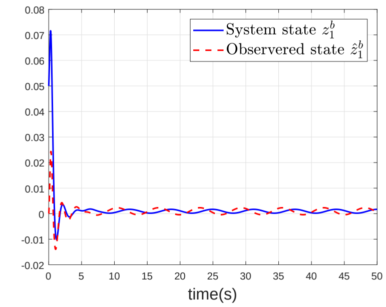
Note that the nonlinear interconnected system (76) – (77) is non-minimum phase and exhibits strong nonlinear coupling.

Choose $V_{i0} = \left(z_i^b\right)^2$ with $F_1 = -3z_1^b$ and $F_2 = -2z_2^b$. Denote the virtual system state $\sigma_1 = z_1^b + z_{11}^a$ and $\sigma_2 = z_2^b + z_{21}^a$, then the main parameters of the decentralized EKF-EHGO (14) are given by:

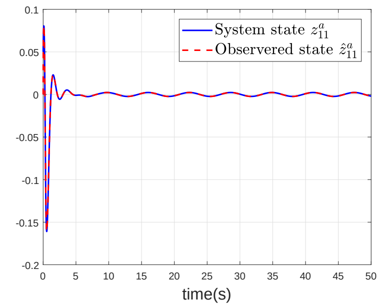
$$\begin{aligned} \varepsilon_1 &= 0.001, \eta_{11} = 3, \eta_{12} = 3, \eta_{13} = 1, R_1 = 1, Q_1 = 0.1 \\ \varepsilon_2 &= 0.001, \eta_{21} = 3, \eta_{22} = 5, \eta_{23} = 3, R_2 = 2, Q_2 = 0.2 \end{aligned} \tag{80}$$

The main parameters of the control (58) are given by:

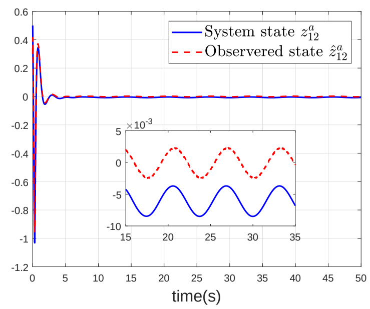
$$\begin{aligned} \gamma_1 &= 0.01, \tau_1 = 100, \lambda_{11} = 2, \lambda_{12} = 2, \lambda_{13} = 3 \\ \gamma_2 &= 0.01, \tau_2 = 100, \lambda_{21} = 2, \lambda_{22} = 3, \lambda_{23} = 3 \end{aligned} \tag{81}$$



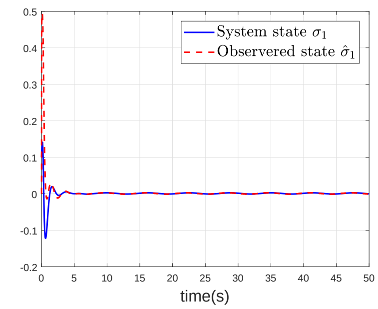
(a) The system state z_1^b and its estimate $\hat{z}_1^b$



(b) The system state z_{11}^a and its estimate $\hat{z}_{11}^a$

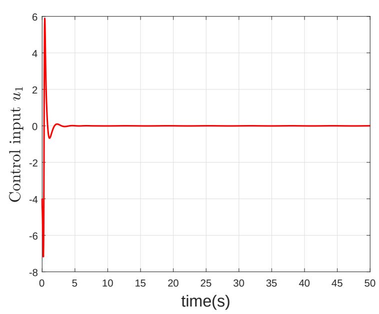


(c) The system state z_{12}^a and its estimate $\hat{z}_{12}^a$

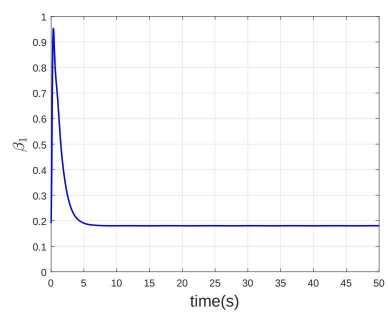


(d) The time response of σ_1 and its estimate $\hat{\sigma}_1$

Fig. 11: The system states z_1 with σ_1 and their estimates

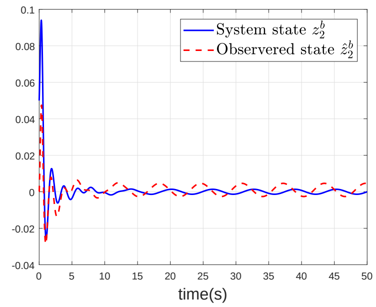


(a) The time response of the system control signal u_1

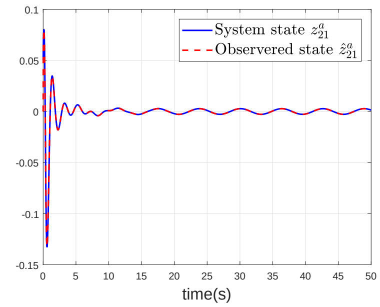


(b) The time response of the adaptation gain β_1

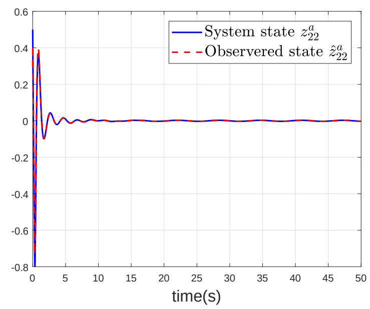
Fig. 12: The time response of the system control signal u_1 and the adaptation gain β_1



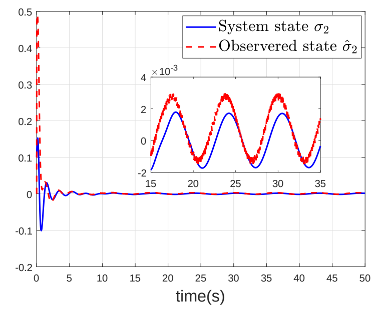
(a) The system state z_2^b and its estimate $\hat{z}_2^b$



(b) The system state z_{21}^a and its estimate $\hat{z}_{21}^a$

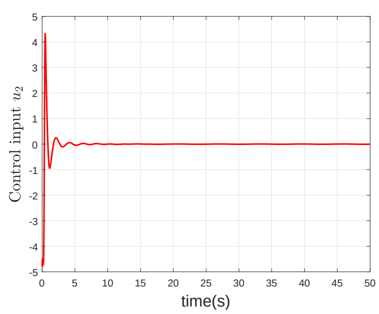


(c) The system state z_{22}^a and its estimate $\hat{z}_{22}^a$

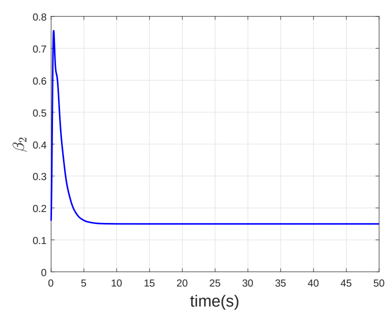


(d) The time response of σ_2 and its estimate $\hat{\sigma}_2$

Fig. 13: The system states z_2 with σ_2 and their estimates



(a) The time response of the system control signal u_2



(b) The time response of the adaptation gain β_2

Fig. 14: The time response of the system control signal u_2 and the adaptation gain β_2

The system states and their estimates are shown in Fig.11 and Fig.13. The time response of the system control signals and the adaptation gains are shown in Fig.12 and Fig.14. As shown in Figs.11-14, although the interconnections and uncertainties have nonzero steady-state values, all signals in the nonlinear interconnected system (76) – (77) and the adaptation gains are all UUB while the designed EKF-EHGO can quickly observe the system states within a small error bound. The simulation results further verify the effectiveness of the proposed method. In this case, by direct calculation, the minimum values of β_i^* defined in (61) are $\beta_1^* = 4$ and $\beta_2^* = 5.28$, respectively. Similarly, the adaptation gains β_1 and β_2 cannot track their corresponding desired values. In fact, if the true values of the parameters are to be obtained by proper adaptive laws, additional requirements on the considered system may be needed.

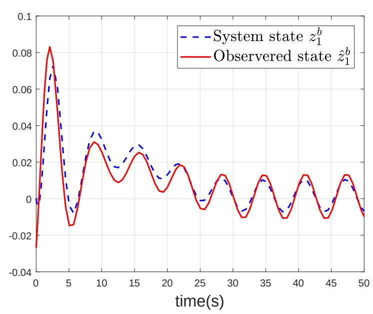
The result in [23] will be compared with the method proposed in this paper. In order to apply the method proposed in [23] to the nonlinear interconnected system (76) – (77), it is necessary to first linearize the considered system at the origin. By using the Taylor expansion around the origin and neglecting higher order terms, the following linearized model of the nominal isolated systems of (76) – (77) can be obtained:

$$\begin{bmatrix} \dot{z}_{11}^a \\ \dot{z}_{12}^a \\ \dot{z}_1^b \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} z_{11}^a \\ z_{12}^a \\ z_1^b \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} u_1$$
$$y_1 = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} z_{11}^a & z_{12}^a & z_1^b \end{bmatrix}^T \tag{82}$$

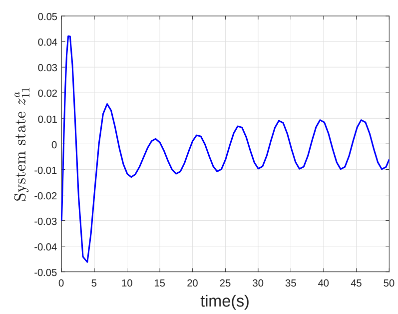
$$\begin{bmatrix} \dot{z}_{21}^a \\ \dot{z}_{22}^a \\ \dot{z}_2^b \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 2 & 0 & 0 \end{bmatrix} \begin{bmatrix} z_{21}^a \\ z_{22}^a \\ z_2^b \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} u_2$$
$$y_2 = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} z_{21}^a & z_{22}^a & z_2^b \end{bmatrix}^T \tag{83}$$

The main parameters of the reduced-order observer and sliding mode control in [23] are given by:

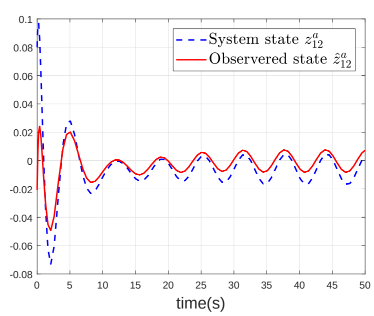
$$L_1 = \begin{bmatrix} 6 & 12 & 7 \end{bmatrix}^T, S_1 = \begin{bmatrix} -3 & -1 & -2 \end{bmatrix}$$
$$L_2 = \begin{bmatrix} 6 & 12 & 8 \end{bmatrix}^T, S_2 = \begin{bmatrix} -3 & -1 & -1 \end{bmatrix} \tag{84}$$



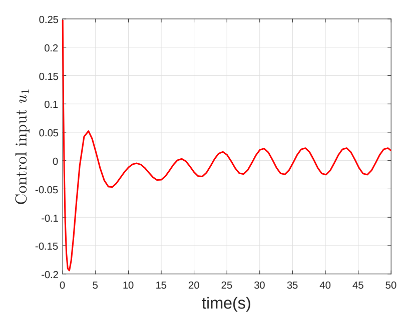
(a) The system state z_1^b and its estimate $\hat{z}_1^b$



(b) The system state z_{11}^a

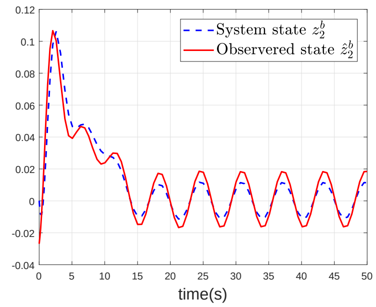


(c) The system state z_{12}^a and its estimate $\hat{z}_{12}^a$

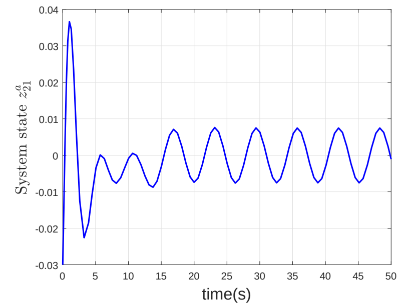


(d) The time response of the system control signal u_1

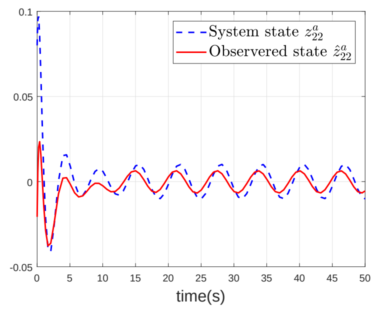
Fig. 15: The time response of the system states z_1 and system control signal u_1 using the method proposed in [23]



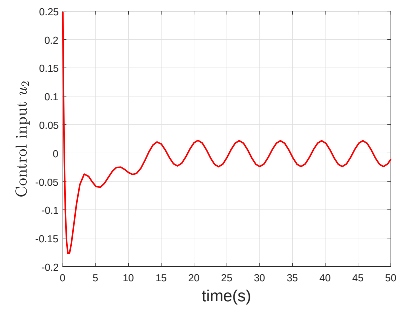
(a) The system state z_2^b and its estimate $\hat{z}_2^b$



(b) The system state z_{21}^a

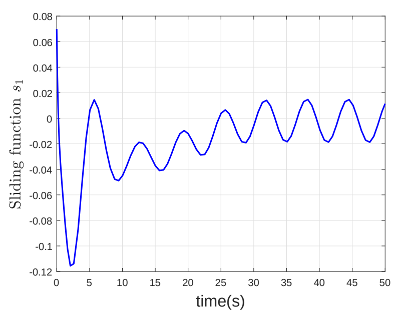


(c) The system state z_{22}^a and its estimate $\hat{z}_{22}^a$

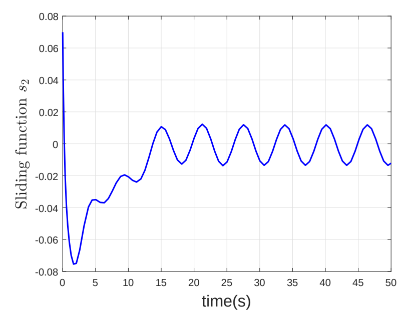


(d) The time response of the system control signal u_2

Fig. 16: The time response of the system states z_2 and system control signal u_2 using the method proposed in [23]



(a) The sliding functions s_1



(b) The sliding functions s_2

Fig. 17: The time response of the sliding functions using the method proposed in [23]

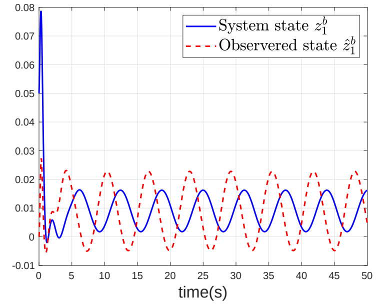
The corresponding time response of the system states with their estimates and the system control signals using the method proposed in [23] are shown in Figs.15-16. The time response of the sliding functions using the method proposed in [23] are shown in Fig.17. Comparing Figs.11-14 and Figs.15-17, it can be seen that the final convergence bound of the system states and observer errors are all bigger than those using the method proposed in this paper while the method proposed in [23] shows poorer system response. This is because the method proposed in [23] is developed based on a linearized model of (76), which has lost the nonlinear characteristics of the original system while the method proposed in [23] can not readily handle the uncertainties and interconnections bounded by an unknown high-order polynomial, which further shows the superiority of the proposed method.

To demonstrate the conservativeness discussed in Remark 15, the values of the parameters λ_{i1} are now chosen as:

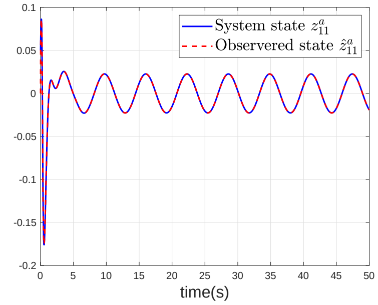
$$\lambda_{11} = 0.5, \lambda_{21} = 0.8 \tag{85}$$

By direct calculation, the sufficient condition $\kappa_{i4} < 0$ proposed in Theorem 1 does not hold while the parameters given in

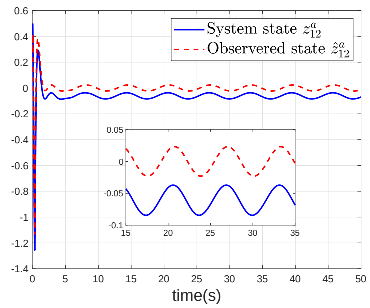
(81) can guarantee the corresponding conditions proposed in Theorem 1 hold. When using the values given in (85) and keeping all other relevant parameters the same as (80) – (81), the system states and their estimates are shown in Fig.18 and Fig.20 while the time response of the system control signals and the adaptation gains are shown in Fig.19 and Fig.21. As shown in Figs.18-21, although the sufficient condition proposed in Theorem 1 do not hold in this case, all signals in the nonlinear interconnected system (76) – (77) and the adaptation gains are still UUB while the final convergence bound of the system will increase. The simulation results further verify the conservativeness of analytical methods underpinning the theoretical analysis of the proposed method.



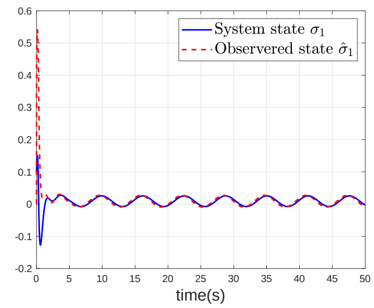
(a) The system state z_1^b and its estimate $\hat{z}_1^b$



(b) The system state z_{11}^a and its estimate $\hat{z}_{11}^a$

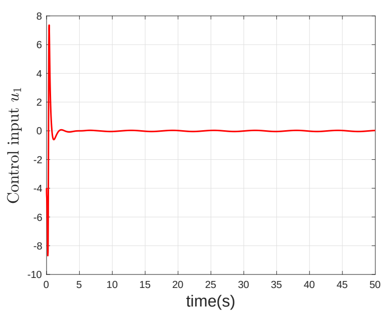


(c) The system state z_{12}^a and its estimate $\hat{z}_{12}^a$

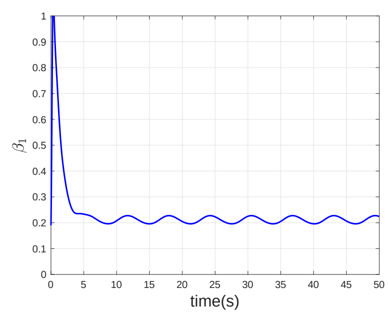


(d) The time response of σ_1 and its estimate $\hat{\sigma}_1$

Fig. 18: The system states z_1 with σ_1 and their estimates under the parameters given in (85)

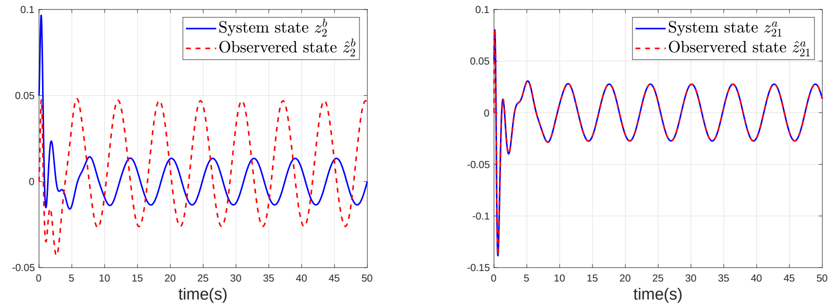


(a) The time response of the system control signal u_1

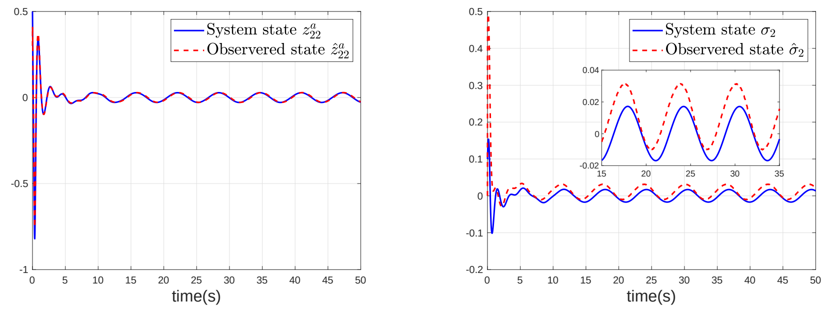


(b) The time response of the adaptation gain β_1

Fig. 19: The time response of the system control signal u_1 and the adaptation gain β_1 under the parameters given in (85)

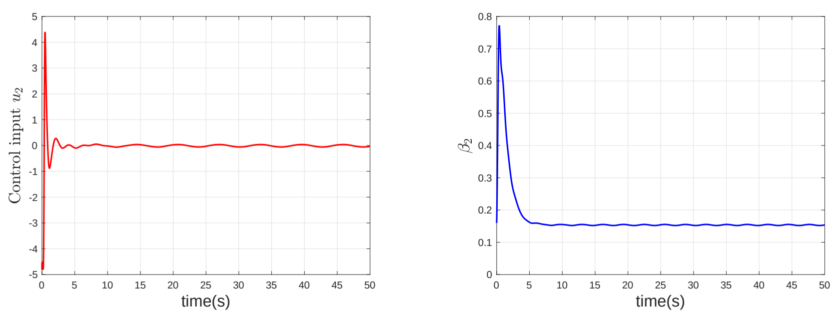


(a) The system state z_2^b and its estimate $\hat{z}_2^b$ (b) The system state z_{21}^a and its estimate $\hat{z}_{21}^a$



(c) The system state z_{22}^a and its estimate $\hat{z}_{22}^a$ (d) The time response of σ_2 and its estimate $\hat{\sigma}_2$

Fig. 20: The system states z_2 with σ_2 and their estimates under the parameters given in (85)



(a) The time response of the system control signal u_2 (b) The time response of the adaptation gain β_2

Fig. 21: The time response of the system control signal u_2 and the adaptation gain β_2 under the parameters given in (85)

V. Conclusion

A decentralized EKF-EHGO based robust adaptive backstepping control method has been designed for a class of interconnected systems with non-minimum phase and nonlinear nominal isolated systems. An adaptive nonlinear damping strategy is used to handle the nonlinear interconnections and uncertainties, which are allowed to be unmatched and have higher-order nonlinear bounds. Simulation test results are given to show the effectiveness of the proposed control scheme. Future work will focus on how to obtain global results and handle some strong interconnections.

VI. Declaration of conflicting interests

The authors declare no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

VII. Funding

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Appendix

By expanding $\omega_i\left(z_i^b, \hat{z}_{i1}^a\right)$ and $\xi_i\left(z_i^b, \hat{z}_{i1}^a\right)$ into power series in the domain $\tilde{z}_i^b \in \aleph_i$, $\chi_i \in R^{r+1}$ and $z_i \in \mathrm{Z}_i$, it follows that

$$\begin{aligned}\Theta_i\left(\hat{z}_i^b, \hat{z}_{i1}^a, \tilde{z}_i^b, 0, t\right) &= \omega_i\left(z_i^b, \hat{z}_{i1}^a\right) - \omega_i\left(\hat{z}_i^b, \hat{z}_{i1}^a\right) - L_i(t)\left(\xi_i\left(z_i^b, \hat{z}_{i1}^a\right) - \xi_i\left(\hat{z}_i^b, \hat{z}_{i1}^a\right)\right) \\ &= \bar{A}_i(t)\,\tilde{z}_i^b + \Gamma_{\omega_i} - L_i(t)\,\bar{C}_i(t)\,\tilde{z}_i^b - L_i(t)\,\Gamma_{\xi_i}\end{aligned} \tag{86}$$

where

$$\begin{aligned}\Gamma_{\omega_i} &= \omega_i\left(z_i^b, \hat{z}_{i1}^a\right) - \omega_i\left(\hat{z}_i^b, \hat{z}_{i1}^a\right) - \frac{\partial \omega_i}{\partial z_i^b}\left(\hat{z}_i^b, \hat{z}_{i1}^a\right)\tilde{z}_i^b \\ \Gamma_{\xi_i} &= \xi_i\left(z_i^b, \hat{z}_{i1}^a\right) - \xi_i\left(\hat{z}_i^b, \hat{z}_{i1}^a\right) - \frac{\partial \xi_i}{\partial z_i^b}\left(\hat{z}_i^b, \hat{z}_{i1}^a\right)\tilde{z}_i^b\end{aligned} \tag{87}$$

represent the corresponding terms of second and higher order in $\tilde{z}_i^b$.

The following lemma is listed below.

Lemma 4: [24], [25] Suppose Assumptions 4-5 are satisfied. When $\left\|\bar{C}_i(t)\right\|$ and $P_i(t)$ are all bounded, there exist positive constants k_{i1} and k_{i2} such that

$$\left\|\Gamma_{\omega_i} - L_i\Gamma_{\xi_i}\right\| \le k_{i1}\left\|\tilde{z}_i^b\right\|^2 \tag{88}$$

$$\left\|\Theta_i\left(\hat{z}_i^b, \hat{z}_{i1}^a, \tilde{z}_i^b, \chi_i, t\right) - \Theta_i\left(\hat{z}_i^b, \hat{z}_{i1}^a, \tilde{z}_i^b, 0, t\right)\right\| \le k_{i2}\left\|\chi_i\right\| \tag{89}$$

It follows from Lemma 4 and (86) that

$$\begin{aligned}2\left(\tilde{z}_i^b\right)^T P_i^{-1}\Theta_i\left(\hat{z}_i^b, \hat{z}_{i1}^a, \tilde{z}_i^b, \chi_i, t\right) + \left(\tilde{z}_i^b\right)^T \dot{P}_i^{-1}\tilde{z}_i^b &= 2\left(\tilde{z}_i^b\right)^T P_i^{-1}\Theta_i\left(\hat{z}_i^b, \hat{z}_{i1}^a, \tilde{z}_i^b, 0, t\right) \\ &+ 2\left(\tilde{z}_i^b\right)^T P_i^{-1}\left\{\Theta_i\left(\hat{z}_i^b, \hat{z}_{i1}^a, \tilde{z}_i^b, \chi_i, t\right) - \Theta_i\left(\hat{z}_i^b, \hat{z}_{i1}^a, \tilde{z}_i^b, 0, t\right)\right\} \\ &+ \left(\tilde{z}_i^b\right)^T\left\{-P_i{}^{-1}\left(\bar{A}_iP_i + P\bar{A}_i^T + Q_i - 2P_i\bar{C}_i^TR_i^{-1}\bar{C}_iP_i\right)P_i{}^{-1}\right\}\tilde{z}_i^b \\ &\le 2\left(\tilde{z}_i^b\right)^T P_i^{-1}\left(\bar{A}_i - L_i\bar{C}_i\right)\tilde{z}_i^b + 2\left\|\tilde{z}_i^b\right\|\left\|P_i^{-1}\right\|\left\|\Gamma_{\omega_i} - L_i\Gamma_{\xi_i}\right\| \\ &+ 2\left\|\tilde{z}_i^b\right\|\left\|P_i^{-1}\right\|\left\|\Theta_i\left(\hat{z}_i^b, \hat{z}_{i1}^a, \tilde{z}_i^b, \chi_i, t\right) - \Theta_i\left(\hat{z}_i^b, \hat{z}_{i1}^a, \tilde{z}_i^b, 0, t\right)\right\| \\ &+ \left(\tilde{z}_i^b\right)^T\left\{-P_i{}^{-1}\left(\bar{A}_iP_i + P\bar{A}_i^T + Q_i - 2P_i\bar{C}_i^TR_i^{-1}\bar{C}_iP_i\right)P_i{}^{-1}\right\}\tilde{z}_i^b \\ &\le -\left(\tilde{z}_i^b\right)^T P_i{}^{-1}Q_iP_i{}^{-1}\tilde{z}_i^b + 2\bar{p}_ik_{i1}\left\|\tilde{z}_i^b\right\|^3 + 2\bar{p}_ik_{i2}\left\|\tilde{z}_i^b\right\|\left\|\chi_i\right\|\end{aligned} \tag{90}$$

Assume in the considered domain $\tilde{z}_i^b \in \aleph_i$ there exists a positive constant k_{i3} such that $\left\|\tilde{z}_i^b\right\| \le k_{i3}$, then the inequality (89) can be further expressed as

$$2\left(\tilde{z}_i^b\right)^T P_i^{-1}\Theta_i\left(\hat{z}_i^b, \hat{z}_{i1}^a, \tilde{z}_i^b, \chi_i, t\right) + \left(\tilde{z}_i^b\right)^T \dot{P}_i^{-1}\tilde{z}_i^b \le -\alpha_{i3}\left\|\tilde{z}_i^b\right\|^2 + \alpha_{i4}\left\|\chi_i\right\|^2 \tag{91}$$

where $\alpha_{i3} = \underline{p}_i^2\lambda_{\min}\left(Q_i\right) - 2\bar{p}_ik_{i1}k_{i3} - 1$ and $\alpha_{i4} = \bar{p}_i^2k_{i2}^2$.

Hence, the inequality (29) follows as long as appropriate parameters are selected to guarantee $\alpha_{i3} > 0$.

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