# Should an Incumbent Store Deter Entry of a Socially Responsible Retailer? 

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Should an incumbent for-profit retailer deter a "socially responsible" store from entering the market? As a differentiation strategy to avoid direct price competition with well-established retailers, some socially responsible stores (or brands) enter the market with a "pre-commitment" to donate a certain proportion of their (A) profits or (B) revenues to charities. Because these charitable donations generate a "warm glow" effect for consumers, these socially responsible stores can use pre-committed donations to gain market access, pressuring the incumbent retailer to lower its price to deter their entry.

In this paper, we present a game-theoretic model in which a socially responsible retailer enters the market to compete with an incumbent for-profit retailer and heterogeneous consumers. We determine and compare the incumbent retailer's deterrence strategies (i.e., deter or tolerate) across different types of socially responsible stores. Our equilibrium analysis generates the following insights. First, the incumbent retailer's deterrence strategy depends on its cost advantage over the socially responsible store, and this deterrence strategy hinges upon the socially responsible store's entry cost, pre-commitment level, and its warm-glow effect. Second, even if the incumbent retailer can profitably deter the socially responsible retailer's entry, the incumbent retailer can actually be better off by tolerating instead of deterring its entrance when the socially responsible store's entry cost is low and when the incumbent store's cost advantage is not significant. Third, relatively speaking, a type (B) store that donates a portion of its revenue is more vulnerable than a type (A) store that donates a portion of its profit unless a type (B) store can generate a much higher warm-glow effect. Thus, if the warm-glow effects generated by the two stores are the same, then it is more likely for the incumbent retailer to deter the type (B) store's entry. We extend our analysis numerically to examine the case when the pre-committed proportion is endogenously determined by the socially responsible stores, and obtain similar structural results.

Key words: Competition, Incumbent, Socially Responsible Operations.

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## 1 Introduction

New generations are more socially conscious: 73\% of Americans consider companies' social causes when making purchasing decisions (Mintel 2018). Because social causes generate the "warm-glow" effect for socially conscious consumers, a socially responsible store (hereafter, social retailer) can use "pre-committed" charitable donations (besides lower selling prices to gain market access) to create a new threat for the incumbent for-profit retailer (hereafter, for-profit retailer).

Social retailers can pre-commit to donating a certain proportion of their (A) profits or (B) revenues to charities (see Chen 2021 for a list of 35 such retailers). ${ }^{1}$ Two examples of type (A) social retailers are Toms and Ivory Ella. ${ }^{2}$ Toms donates $1 / 3$ of its profits for grassroots good, including cash grants and partnerships with community organizations, to drive sustainable change, whereas Ivory Ella donates $10 \%$ of its profits towards saving elephants. Two examples of type (B) social retailers are Cotopaxi and Judy. ${ }^{3}$ Cotopaxi donates $1 \%$ of its yearly revenue to nonprofits making sustainable changes in poverty alleviation, while Judy donates $1 \%$ of its annual revenue to the Los Angeles Fire Department Foundation, which provides essential equipment and training to supplement city resources. These charitable donations generate the "warm-glow" effect for consumers who shop at social retailers (cf. Andreoni 1990 and Harbaugh 1998).

While consumers welcome social retailers to enter the market and thrive, this movement can also trigger incumbent for-profit retailers to proactively reduce their prices to "deter" the entry of social retailers. Therefore, our intent is to examine and compare the incumbent store's deterrence strategies between type ( $A$ ) and type ( $B$ ) stores. We choose to compare these two types of socially responsible stores by design because they are commonly seen. More importantly, because of their similarity (i.e., donation based on profit versus revenue), their warm-glow effects are analytically comparable so that we can compare the deterrence strategies of the incumbent store across different types by focusing on one variation. ${ }^{4}$ In particular, we aim to answer the following research questions:

[^1]1. What is the entry and pricing strategy of a social retailer in the presence of an incumbent forprofit retailer?
2. Should the incumbent retailer deter or tolerate the entry of the social retailer?
3. How does the incumbent retailer's deterrence strategy differ between different types of social retailers?

In this paper, we present a two-stage Stackelberg model of a social retailer who enters a market to compete with an incumbent for-profit retailer. The social retailer incurs an entry cost and pre-commits to donating a certain proportion of its profit (if type (A)) or revenue (if type (B)). Consumers in the market are heterogeneous in their utility from shopping at both retailers and they obtain a warm-glow effect from shopping at the social retailer.

To answer our first research question, we first analyze the strategic interactions among utilitymaximizing consumers, a profit-maximizing retailer, and a type (A) social retailer. We find that a type (A) social retailer's optimal price depends on the incumbent retailer's price; and its profit and its entry condition depend on its cost of entry. Next, regarding our second research question, we discover that the deterrence strategy employed by the incumbent retailer depends on its unit-cost advantage over the social retailer, as well as the social retailer's entry cost. Interestingly, even if the incumbent retailer can profitably deter the social retailer's entry, the incumbent retailer can be better off by tolerating this entry unless it has a substantial competitive advantage over the social retailer (due to the incumbent's significantly lower unit cost or the social retailer's high entry cost).

We also examine the entry of a type (B) social retailer. We find that the aforementioned results associated with our first and second research questions remain valid in the case of a type (B) store. Additionally, we derive additional insights concerning a social retailer of type (B). Specifically, the donation proportion plays a more important role in the pricing decisions of a type (B) retailer. Specifically, to cover the donations, a type (B) social retailer needs to charge a higher price and obtain a smaller market share. Due to this challenge, ceteris paribus, a type (B) social retailer's entry poses a smaller threat to the incumbent retailer. For this reason, one may expect that the incumbent retailer is more tolerant towards a type (B) social retailer. Interestingly, we find an opposite result: the incumbent store is actually more likely to deter the entry of a type (B) social retailer that commits to donating a certain proportion of its revenue.

Regarding the third question, we analytically demonstrate that even if the two stores generate the same level of warm-glow effect, the incumbent store R may adopt different deterrence strategies against them. Specifically, we find that the incumbent is more aggressive in deterring a type (B) store (that donates a proportion of its revenue) than a type (A) store (that donates a proportion of its profit), unless a type (B) store can generate a significantly higher warm-glow effect. Our results
are informative for policymakers and entrepreneurs aiming to establish social retailers as they show how an incumbent retailer reacts to entry threats made by different types of social retailers.

By extending our analysis to the case when the pre-committed proportion is "endogenously determined" by each type of social retailer. Through our extensive numerical analysis, we find the same structural results continue to hold: even when both social retailers set the pre-committed proportion optimally, the incumbent store is more likely to tolerate the entrance of a type (A) store than a type (B) store. Also, we examine how the social store's entry cost and the incumbent store's cost advantage affect the optimal proportion, and the corresponding equilibrium outcomes (such as the social store's selling price, and the profit of the social store and the incumbent store). We find that these quantities behave in an intuitive manner in most instances. However, one may conjecture that a type (B) store, which donates a portion of its revenue, would always donate more than a type (A) store that donates a portion of its profit. Interestingly, this conjecture is not necessarily true: we find that it is also possible for a type (A) store to donate a larger amount than a type (B) store when the proportion is "endogenously determined".

This paper is organized as follows. We review the relevant literature in $\S 2$. After we define our model preliminaries in $\S 3$, we analyze the potential entry of a type (A) store and its impact in $\S 4$. We analyze the implications of the potential entry of a type (B) store in $\S 5$. We then compare the two types of stores in $\S 6$. In $\S 7$, we expand the analysis by considering endogenously determined donating proportion to show the robustness of our structural results. Finally, we conclude in $\S 8$.

## 2 Literature Review

Our study is related to studies on market entry, mixed oligopoly, and socially responsible retailers. The market-entry literature establishes the notion of an incumbent's decision to lower its price below the profit-maximizing price to "deter" the entry of a for-profit competitor. This literature mainly focuses on how a for-profit firm can deter the entry of a for-profit competitor of the same type, and suggests deterrence tools such as pricing (Bain 1949), strategic commitment (Spence 1977, 1979), long-term contracts (Aghion and Bolton 1987), cost signaling (Srinivasan 1991), bundle pricing (Nalebuff 2004), or discount contracts (Ide et al. 2016). Overall, these papers focus on the deterrence tools or the market structure rather than focusing on the entrant characteristics. (We refer the reader to Hall (2008) for a review of the market entry literature.) More recently, Gao et al. (2017) examine the entry of copycats and show that the incumbent firm can deter the copycat from entering by selling a higher quality product. There are also some papers in the supply chain competition literature (e.g., Corbett and Karmarkar 2001, Korpeoglu et al. 2020) that analyze the market entry and competition of identical for-profit firms. Our work contributes to the market-entry literature on several fronts. First, unlike the literature that studies the entry
of for-profit retailers, we examine the entry of a social retailer that pre-commits to donating a certain proportion of its profit (type A) or revenue (type B). This social commitment adds a new dimension to price competition because it also creates the warm-glow effect for the social retailer's customers. Our work also compares the entry of the two types of social retailers.

Our work is also related to mixed oligopoly in which firms compete with different objectives (De Fraja and Delbono 1990). (We refer the reader to Zhou et al. 2023 for a review of the recent literature). Our model differs from the literature on several fronts. First, our work considers the market entry of a social retailer that maximizes its profit but subject to donating a certain proportion of its profit or revenue. Indeed, a major contribution of our paper is to compare the entry of these two types of social retailers. Second, we consider the warm-glow effect that consumers receive from shopping at the social retailer.

Our work also contributes to the literature on socially responsible retailers. ${ }^{5}$ There is a body of work that recognizes the "warm glow" that consumers receive from shopping at socially responsible retailers (e.g., Strahilevitz 1999, Bloom et al. 2006). Some more recent papers study the impact of this warm glow on operational decisions. Arya and Mittendorf (2015) investigate the impact of a government subsidy in an environment with one supplier and one socially responsible retailer that commits to donating a certain number of goods. Gao (2020) studies the pricing decisions of a firm that commits to donating a proportion of its revenue to charity without considering any competition. Our work contributes to this literature by incorporating the warm-glow effect in a new context so that we can explore the entry strategy of a socially responsible retailer and the deterrence strategy of an incumbent retailer.

## 3 Model Preliminaries

We consider a Stackelberg competition model that involves an incumbent for-profit retailer (store R) and an entrant social retailer (store S ). For a consumer who shops at the incumbent store R, we assume that the consumer utility satisfies:

$$
\begin{equation*}
U_{r}=v-p_{r} \tag{1}
\end{equation*}
$$

where $v \sim U[0,1]$ captures the heterogenous consumer valuation for a certain product, and $p_{r}(<1)$ is the selling price set by store $R$.

For store R and store S , let $c_{r}$ and $c_{s}$ be their unit costs; respectively. (Throughout this paper, subscripts $r$ and $s$ are used to denote stores R and S , respectively.) Because store R has sourced

[^2]from its suppliers with a proven sales record over an extended period of time, we assume that store R has a "cost advantage" over store S so that the parameter $\alpha \equiv c_{r} / c_{s}<1$. ${ }^{6}$ Due to store S 's cost disadvantage (as $c_{s}>c_{r}$ ), store S (the new entrant) has to differentiate itself from the incumbent by creating the warm-glow effect so that consumers can obtain a higher valuation when shopping at store $S$. To create the warm-glow effect, we focus on the case when store $S$ is committed to differentiate itself from store R by donating a proportion $\gamma \in(0,1)$ of its profit to charities as a type (A) store, or by donating a proportion $\gamma \in(0,1)$ of its revenue to charities as a type (B) store. ${ }^{7}$ Here, $\gamma$ corresponds to a "generic" donating proportion of profit/revenue for a type $(A) /(B)$ store.

To obtain tractable analytical results, we first treat $\gamma \in(0,1)$ as an exogenously given parameter in our base model. In doing so, we establish structural results that serve as "benchmarks" for further examination. For instance, different types of store $S$ would commit to donating different proportions (i.e., $\gamma^{A} \neq \gamma^{B}$ ) especially when $\gamma^{j}, j=A, B$, is determined endogenously by each type $j$ store. This observation motivates us to consider $\gamma$ as an endogenous decision for each type of store S in $\S 7$. Specifically, in $\S 7$, we conduct a comprehensive numerical study for the case when $\gamma$ is endogenously determined, and we find that our structural (benchmark) results of the base model continue to hold.

For any exogenously given $\gamma>0$ in our base model, we assume that the consumer utility for shopping at store S takes the following form:

$$
\begin{equation*}
U_{s}^{j}=\beta^{j}(\gamma) \cdot v-p_{s}, j \in\{A, B\} \tag{2}
\end{equation*}
$$

where $p_{s}$ is the price set by store S . Here, the "warm-glow effect" is captured by a function $\beta^{j}(\gamma) \geq 1$, where $\beta^{j}(\gamma)$ is an increasing function of $\gamma$. For example, the warm glow effect can take a linear form (i.e., $\beta^{j}(\gamma)=\left(1+b^{j} \cdot \gamma\right)$ with $b^{j}>0$ so that consumers have a higher valuation when shopping at a type $(\mathrm{A}) /(\mathrm{B})$ store S$)$. Hence, by donating a proportion $\gamma \in(0,1)$ of its profit or revenue to charities, store S can leverage its higher consumer valuation $\beta^{j}(\gamma) \cdot v \equiv\left(1+b^{j} \cdot \gamma\right) v>v$ to compete against store R despite of its cost disadvantage $\alpha \equiv c_{r} / c_{s}<1$.

If one considers (2) in isolation, then store $S$ can be interpreted as the store that offers a vertically differentiated product that is better than store R . Therefore, our market entry game is similar to those examined in the literature (e.g., Hall (2008)). However, if one considers (2) in conjunction with store S's profit function, then a fundamental difference emerges. In a traditional market entry game, one store makes a "separate" investment to develop a vertically differentiated product (e.g.,

[^3]invest in quality for a higher quality product). However, in our context, store S's investment is the proportion $\gamma$ that is "embedded" in the profit (for type (A)) or in the revenue (for type (B)) to create the differentiation through the warm-glow effect $\beta^{j}(\gamma)$. Consequently, the analysis and store R's deterrence strategy is different from the traditional market entry game considered in the literature (e.g., Gao et al. 2017, Corbett and Karmarkar 2001, Korpeoglu et al. 2020).

Certainly, the exact function form of the warm-glow effect $\beta^{j}(\gamma)$ would depend on the type of charity and the consumer segment that store $S$ will be focusing on. For example, a store $S$ may be owned by a female who pre-commits to donate a proportion of its profit (or revenue) to the National Breast Cancer Foundation by selling a product targeting female consumers. As a generic model, we do not model a specific charity and a specific consumer segment in our analytical analysis; instead, we consider an implicit function form $\beta^{j}(\gamma)$ in the base model of exogenously given $\gamma$ so that our model can be applied to different charities. Later on in $\S 7$, we capture this issue succinctly by considering a general function form $\beta^{j}(\gamma)=1+b^{j} \cdot \gamma^{t}$, where the parameters $b^{j}, t>0$; and we conduct a comprehensive numerical study by varying the parameters $b^{j}$ and $t$.

### 3.1 A Sequential Market Entry Game: Should Store R deter or tolerate Store S's entry?

The dynamic decisions to be made by the incumbent store R and the new entrant store S can be modeled as a 2-period sequential game as depicted in Figure 1. In period 1, store $S$ has not yet


Figure 1 A market-entry game between the incumbent store $R$ and a "generic" store $S$ (of type (A) or (B)).
entered and incumbent store R can choose its price $p_{r}$ to either deter or tolerate store S's entry (the blue box in Figure 1). To deter, store R can set $p_{r}$ below a threshold $\tau$ (to be determined) so that store S cannot afford to enter. Consequently, store R operates as a monopoly with the set price
$p_{r}$ in period 2 and the game ends. If store R chooses to tolerate by setting $p_{r}$ above the threshold $\tau$, then store S can afford to enter in period 2 by incurring an entry cost $k$. (This entry cost $k$ can represent the present value of loan repayments store $S$ has to make using its future earnings to cover its initial investment.) Upon entry, store S observes $p_{r}$ and competes with store R in a duopoly in period 2 by choosing its own price $p_{s}$ that takes its unit cost $c_{s}$ into consideration (the green box in Figure 1). ${ }^{8}$

### 3.2 Backward Induction Steps for Determining Equilibrium Strategies

We now describe how we solve the sequential game that involves the potential entry of a generic store S (of type $j, j=A, B$ ) via backward induction. We shall use this approach to examine the potential entry of a type (A) and a type (B) store $S$ in $\S 4$ and $\S 5$, respectively. In preparation, let us first describe consumer demand. Then, we formulate store S's problem in period 2 (the green box in Figure 1) and then store R's problem in period 1 (the blue box in Figure 1).

### 3.2.1 Consumer Demand

Each retailer's consumer demand depends on store R's deterrence strategy. First, if store R chooses to deter any type of store S 's entry by setting $p_{r}<\tau$, it operates as a monopoly so that only consumers with utility $U_{r}=v-p_{r} \geq 0$ will buy the product from store R . It follows from (1), the consumer demand $q_{r}$ for store R is:

$$
\begin{equation*}
q_{r}=1-p_{r} \tag{3}
\end{equation*}
$$

Next, if store R sets its price $p_{r} \geq \tau$ in period 1 to tolerate a type $j=A, B$ store S 's entry and store S sets its price $p_{s}^{j}$ in period 2 , then a consumer will purchase from store R only when $U_{r} \geq 0$ and $U_{r} \geq U_{s}^{j}$; or purchase from store S only when $U_{s}^{j} \geq 0$ and $U_{s}^{j} \geq U_{r}$. By considering $U_{r}$ and $U_{s}^{j}$ given in (1) and (2), the consumer demand $q_{r}^{j}$ for store R and $q_{s}^{j}$ for a type $j$ store S satisfy:

$$
\begin{align*}
& q_{r}^{j}= \begin{cases}0 & \text { if } p_{s} \leq \beta^{j}(\gamma) \cdot p_{r} \\
\frac{p_{s}-p_{r}}{\beta^{j}(\gamma)-1}-p_{r} & \text { if } \beta^{j}(\gamma) \cdot p_{r}<p_{s}<\beta^{j}(\gamma)-1+p_{r} \\
1-p_{r} & \text { if } p_{s} \geq \beta^{j}(\gamma)-1+p_{r}\end{cases}  \tag{4}\\
& q_{s}^{j}= \begin{cases}1-\frac{p_{s}}{\beta^{j}(\gamma)} & \text { if } p_{s} \leq \beta^{j}(\gamma) \cdot p_{r} \\
1-\frac{p_{s}-p_{r}}{\beta^{j}(\gamma)-1} & \text { if } \beta^{j}(\gamma) \cdot p_{r}<p_{s}<\beta^{j}(\gamma)-1+p_{r} \\
0 & \text { if } p_{s} \geq \beta^{j}(\gamma)-1+p_{r}\end{cases} \tag{5}
\end{align*}
$$

Because store S's warm-glow effect $\beta^{j}(\gamma)$ depends on $\gamma, q_{r}^{j}$ and $q_{s}^{j}$ also depend on $\gamma$ through $\beta^{j}(\gamma)$.
${ }^{8}$ Throughout this paper, we adopt two standard assumptions in the market-entry literature reviewed in §2. First, store R's retail price $p_{r}$ is irreversible in the sense that store R does not change $p_{r}$ after store S's entry. Spence (1977) articulates that irreversibility is a way for a firm to commit itself to issue a credible threat to potential entry. This is also consistent with the notion of price stickiness (e.g. Chen et al. 2017). Second, store S's cost parameters $c_{s}$ and $k$ are known by the incumbent store R. This may be a reasonable assumption given that the incumbent retailer has been in business for a while and can roughly gauge costs of a newcomer.

### 3.2.2 Store S's Problem in Period 2

Using the consumer demand functions as stated above, we now formulate store S's problem in period 2 (the green box in Figure 1) in $\S 3.2 .2$, followed by Store R's problem in period 1 (the blue box in Figure 1) in §3.2.3. To begin, if store R deters store S's entry, then store S's problem is moot. Hence, store S's problem is only relevant when store R sets $p_{r} \geq \tau$ in period 1 to tolerate store S's entry. In period 2, store S can set its best response price $p_{s}$ to compete with store R in a duopoly upon observing $p_{r}$ set by store R in period 1 . By considering its unit cost $c_{s}$ ( $\leq \beta$ to eliminate trivial cases where store S can never make profit) and the entry cost $k$, store S can determine its best-response price $p_{s}$ that maximizes its net profit. For any given $p_{r}$, store S can use the consumer demand $q_{s}$ given by (5) to formulate its problem as follows.

First, for a type (A) store S who commits to donating a proportion $\gamma$ of its profit, its problem in period 2 is to set its price $p_{s}^{A}\left(p_{r}\right)$ as a best response by solving:

$$
\begin{align*}
\Pi_{s}^{A}\left(p_{r}\right)=\max _{p_{s} \geq c_{s}} \Pi_{s}= & (1-\gamma) \cdot\left[\left(p_{s}-c_{s}\right) \cdot q_{s}^{A}-k\right], \\
& \text { s.t. } q_{s}^{A}= \begin{cases}1-\frac{p_{s}}{\beta^{A}(\gamma)} & \text { if } p_{s} \leq \beta^{A}(\gamma) \cdot p_{r} \\
1-\frac{p_{s}-p_{r}}{\beta^{A}(\gamma)-1} & \text { if } \beta^{A}(\gamma) \cdot p_{r}<p_{s}<\beta^{A}(\gamma)-1+p_{r}, \\
0 & \text { if } p_{s} \geq \beta^{A}(\gamma)-1+p_{r}\end{cases} \tag{6}
\end{align*}
$$

where $q_{s}^{A}$ is given in (5). Hence, a type (A) store S will enter the market if and only if $\Pi_{s}^{A}\left(p_{r}\right) \geq 0$.
Similarly, for a type (B) store S who commits to donating a proportion $\gamma$ of its revenue to charities, its problem in period 2 is to choose its price $p_{s}^{B}\left(p_{r}\right)$ as a best response by solving:

$$
\begin{align*}
\Pi_{s}^{B}\left(p_{r}\right)=\max _{p_{s} \geq c_{s}} \Pi_{s}= & (1-\gamma) \cdot p_{s} \cdot q_{s}^{B}-\left(c_{s} \cdot q_{s}^{B}+k\right)=(1-\gamma) \cdot\left[\left(p_{s}-\frac{c_{s}}{1-\gamma}\right) \cdot q_{s}^{B}-\frac{k}{1-\gamma}\right], \\
& \text { s.t. } q_{s}^{B}= \begin{cases}1-\frac{p_{s}}{\beta^{B}(\gamma)} & \text { if } p_{s} \leq \beta^{B}(\gamma) \cdot p_{r} \\
1-\frac{p_{s}-p_{r}}{\beta^{B}(\gamma)-1} & \text { if } \beta^{B}(\gamma) \cdot p_{r}<p_{s}<\beta^{B}(\gamma)-1+p_{r} . \\
0 & \text { if } p_{s} \geq \beta^{B}(\gamma)-1+p_{r}\end{cases} \tag{7}
\end{align*}
$$

Hence, a type (B) store S will enter the market if and only if $\Pi_{s}^{B}\left(p_{r}\right) \geq 0$. From (7), we note that store S needs to charge $p_{s} \geq \frac{c_{s}}{1-\gamma}$ to ensure a positive gross margin and $p_{s}<\beta^{B}(\gamma)-1+p_{r}$ to ensure a positive demand. By noting that $p_{r} \leq 1$, we shall assume $c_{s}<(1-\gamma) \beta^{B}(\gamma)$ to rule out the trivial cases in which store S can never afford to enter the market.

By considering consumer utility given in (2) along with store S's profit functions given in (6) and (7), it becomes clear that store $S$ sacrifices a proportion $\gamma$ of its profit (or revenue) in exchange for the warm-glow effect as captured in (2). Also, because the proportion $\gamma$ generates the warm-glow effect $\beta^{j}(\gamma)$, it creates a different dynamics than those traditional market entry games considered in the literature as explained earlier. It is likely that different types of store $S$ would commit to donating different proportions (i.e., $\gamma^{A} \neq \gamma^{B}$ ) as illustrated in the examples of $\S 1$, especially when these proportions are determined endogenously as examined in $\S 7$.

### 3.2.3 Store R's Problem in Period 1

In period 1 , store R can anticipate a type j store S 's best-response entry decision and price $p_{s}^{j}\left(p_{r}\right)$ (i.e., the optimal solution to problem (6) for $j=A$ and problem (7) for $j=B$ ). Then, we can formulate store R's "deterrence strategy" problem (highlighted in blue box of Figure 1) according to store R's decision to deter or tolerate a type j store S 's entry.

If store R chooses to deter a type $j$ store S 's entry by choosing $p_{r}$ to ensure $\Pi_{s}^{j}\left(p_{s}^{j}\left(p_{r}\right)\right)<0$, then store R can operate as a monopoly. Hence, by considering store R's monopoly demand $q_{r}$ given in (3), store R can determine its optimal price $p_{r}^{d, j}$ by solving:

$$
\begin{align*}
\Pi_{r}^{d, j}= & \sup _{p_{r} \geq c_{r}} \Pi_{r}=\left(p_{r}-c_{r}\right) \cdot\left(1-p_{r}\right), \\
& \text { s.t. } \Pi_{s}^{j}\left(p_{s}^{j}\left(p_{r}\right)\right)<0 . \tag{8}
\end{align*}
$$

Similarly, if store R tolerates store S's entry by choosing $p_{r}$ to ensure $\Pi_{s}^{j}\left(p_{s}^{j}\left(p_{r}\right)\right) \geq 0$, then by considering store R's demand $q_{r}^{j}$ given in (4), store R determines its optimal price $p_{r}^{t, j}$ by solving:

$$
\begin{align*}
& \Pi_{r}^{t, j}=\max _{p_{r} \geq c_{r}} \Pi_{r}=\left(p_{r}-c_{r}\right) \cdot q_{r}^{j}, \\
& \text { s.t. } \Pi_{s}^{j}\left(p_{s}^{j}\left(p_{r}\right)\right) \geq 0, \\
& \quad q_{r}^{j}= \begin{cases}0 & \text { if } p_{s}^{j}\left(p_{r}\right) \leq \beta^{j}(\gamma) \cdot p_{r} \\
\frac{p_{s}^{j}\left(p_{r}\right)-p_{r}}{\beta^{j}(\gamma)-1}-p_{r} & \text { if } \beta^{j}(\gamma) \cdot p_{r}<p_{s}^{j}\left(p_{r}\right)<\beta^{j}(\gamma)-1+p_{r} \\
1-p_{r} & \text { if } p_{s}^{j}\left(p_{r}\right) \geq \beta^{j}(\gamma)-1+p_{r} .\end{cases} \tag{9}
\end{align*}
$$

Here, superscripts $d$ and $t$ denote store R's deterrence and tolerance strategies, respectively. Observe from (8), (9) that store R's problems depend on the store type $j$ and its warm-glow effect $\beta^{j}(\gamma)$.

### 3.2.4 Store R's Deterrence Strategy

By solving store R's problems (8) and (9), we can determine store R's deterrence strategy against a type $j$ store S as follows. First, by comparing store R 's optimal profit $\Pi_{r}^{d, j}$ when deterring store S as given in (8) and $\Pi_{r}^{t, j}$ when tolerating store S as given in (9), it is optimal for store R to deter the entry of a type $j$ store S if $\Pi_{r}^{d, j}>\Pi_{r}^{t, j}$, and tolerate a type $j$ store S's entry; otherwise. Meanwhile, store R chooses its equilibrium price $p_{r}^{j}$ that maximizes its profit (i.e., $\Pi_{r}^{j}=\max \left\{\Pi_{r}^{d, j}, \Pi_{r}^{t, j}\right\}$ ). Second, we characterize a type $j$ store S's equilibrium entry and pricing decisions in period 2. Specifically, if store R chooses to tolerate a type $j$ store S 's entry, we can retrieve a type $j$ store S's corresponding equilibrium price $p_{s}^{j}\left(p_{r}^{j}\right)$ through substitution (otherwise, store S cannot enter).

In this section, we have described the process by which we solve the market-entry game between store $R$ and a generic type of store $S$ as depicted in Figure 1. We now proceed to delve into our analysis for each type of store $S$ as follows. In $\S 4$, we analyze the case of a type (A) store $S$. Then, in $\S 5$, we analyze the case of a type (B) store S , followed by a comparison of our results associated with these two types of store S in $\S 6$.

## 4 Market Entry Game between a Type (A) Store S and an Incumbent Store R

We now analyze the market entry game between an incumbent store $R$ and a potential entry of a type (A) store $S$. To do so, we solve a type (A) store S's problem (6) in period 2 for any given store R's price $p_{r}$ in $\S 4.1$. Then, in $\S 4.2$, we solve store R's problems in period 1 as described in $\S 3.2 .3$, followed by store R's deterrence strategy as explained in §3.2.4.

### 4.1 Type (A) Store S's Best-Response Pricing Strategy in Period 2

For any store R's retail price $p_{r}$ (set in period 1), a type (A) store S solves problem (6) in period 2 as described in $\S 3.2 .2$ by determining the optimal solution $p_{s}^{A}$ that represents the best-response price for a type (A) store S . From (6), we can rewrite the effective maximum profit $\Pi_{s}^{A}\left(p_{r}\right)=$ $(1-\gamma) \cdot\left(\tilde{\Pi}_{s}^{A}\left(p_{r}\right)-k\right)$, where $\tilde{\Pi}_{s}^{A}\left(p_{r}\right) \equiv\left(p_{s}^{A}-c_{s}\right) \cdot q_{s}^{A}$ represents the maximum gross profit of store S without considering the donation proportion $\gamma$ or the entry cost $k$. By solving (6), we get:

Lemma 1. Given any store $R$ 's price $p_{r}$, a type (A) store $S$ 's maximum gross profit $\tilde{\Pi}_{s}^{A}\left(p_{r}\right)$ satisfies:

$$
\tilde{\Pi}_{s}^{A}\left(p_{r}\right)=\left\{\begin{array}{ll}
0 & p_{r} \leq c_{s}+1-\beta^{A}(\gamma)  \tag{10}\\
\frac{\left(\beta^{A}(\gamma)-1+p_{r}-c_{s}\right)^{2}}{4\left(\beta^{A}(\gamma)-1\right)} & p_{r} \in\left(c_{s}+1-\beta^{A}(\gamma), \frac{\beta^{A}(\gamma)-1+c_{s}}{2 \beta^{A}(\gamma)-1}\right) \\
{\left[\beta^{A}(\gamma) \cdot p_{r}-c_{s}\right]\left(1-p_{r}\right)} & p_{r} \in\left[\frac{\beta^{A}(\gamma)-1+c_{s}}{2 \beta^{A}(\gamma)-1}, \frac{\beta^{A}(\gamma)+c_{s}}{2 \beta^{A}(\gamma)}\right] \\
\frac{\left[\beta^{A}(\gamma)-c_{s}\right]^{2}}{4 \beta^{A}(\gamma)} & p_{r}>\frac{\beta^{A}(\gamma)+c_{s}}{2 \beta^{A}(\gamma)},
\end{array} .\right.
$$

and $\tilde{\Pi}_{s}^{A}\left(p_{r}\right)$ is non-decreasing in $p_{r}$.


Figure 2 Type (A) store S's best-response pricing strategy.

Figure 2(a) illustrates store S's maximum gross profit $\tilde{\Pi}_{s}^{A}\left(p_{r}\right)$ for those 4 cases stated in (10). Because store S's entry condition is $\tilde{\Pi}_{s}^{A}\left(p_{r}\right) \geq k$ (so that store S's net profit $\Pi_{s}^{A}\left(p_{r}\right)=(1-\gamma)$.
$\left[\tilde{\Pi}_{s}^{A}\left(p_{r}\right)-k\right] \geq 0$ ) and because $\tilde{\Pi}_{s}^{A}\left(p_{r}\right)$ is non-decreasing in store R's price $p_{r}$, this entry condition holds only when store R sets its price $p_{r}$ sufficiently high. This explicit entry condition for store S will be used in $\S 4.2$ for determining store R's deterrence strategy along with those thresholds $K_{1}^{A}$ and $K_{2}^{A}$ as explained later.

Next, observe from Lemma 1 that store S's entry condition is violated if store R sets its price $p_{r}$ sufficiently low so that $p_{r} \leq c_{s}+1-\beta^{A}(\gamma)$. By ruling out this trivial case, we can determine the best response price $p_{s}^{A}$ that depends on store R's price $p_{r}$, the warm-glow effect $\beta^{A}(\gamma)$ along with the proportion $\gamma$ as stated in the following proposition. Also, we can use the optimal price $p_{s}^{A}$ associated with the remaining 3 cases to retrieve the corresponding optimal quantity $q_{s}^{A}$ and the optimal profit $\Pi_{s}^{A}\left(p_{r}\right)$ from (6) as follows.

Proposition 1. Suppose store $R$ sets its price at $p_{r}$ so that $p_{r}>c_{s}+1-\beta^{A}(\gamma)$ and store $S$ 's entry condition holds (i.e., $k \leq \tilde{\Pi}_{s}^{A}\left(p_{r}\right)$, where $\tilde{\Pi}_{s}^{A}\left(p_{r}\right)$ is given in Lemma 1). Then a type (A) store $S$ 's best-response price $p_{s}^{A}$, consumer demand $q_{s}^{A}$, and retained profit $\Pi_{s}^{A}=(1-\gamma) \cdot\left(\tilde{\Pi}_{s}^{A}-k\right)$ satisfy:
(i) If $p_{r} \in\left(c_{s}+1-\beta^{A}(\gamma), \frac{\beta^{A}(\gamma)-1+c_{s}}{2 \beta^{A}(\gamma)-1}\right)$, then the best-response $p_{s}^{A}=\frac{\beta^{A}(\gamma)-1+p_{r}+c_{s}}{2}>\beta^{A}(\gamma) \cdot p_{r}$, so the corresponding $q_{s}^{A}=\frac{\beta^{A}(\gamma)-1+p_{r}-c_{s}}{2\left(\beta^{A}(\gamma)-1\right)}$ and $\Pi_{s}^{A}=(1-\gamma) \cdot\left[\frac{\left(\beta^{A}(\gamma)-1+p_{r}-c_{s}\right)^{2}}{4\left(\beta^{A}(\gamma)-1\right)}-k\right]$.
(ii) If $p_{r} \in\left[\frac{\beta^{A}(\gamma)-1+c_{s}}{2 \beta^{A}(\gamma)-1}, \frac{\beta^{A}(\gamma)+c_{s}}{2 \beta^{A}(\gamma)}\right]$, then the best-response $p_{s}^{A}=\beta^{A}(\gamma) \cdot p_{r}$, so the corresponding $q_{s}^{A}=$ $1-p_{r}$ and $\Pi_{s}^{A}=(1-\gamma) \cdot\left[\left(\beta^{A}(\gamma) \cdot p_{r}-c_{s}\right)\left(1-p_{r}\right)-k\right]$.
(iii) If $p_{r}>\frac{\beta^{A}(\gamma)+c_{s}}{2 \beta^{A}(\gamma)}$, then the best-response $p_{s}^{A}=\frac{\beta^{A}(\gamma)+c_{s}}{2}<\beta^{A}(\gamma) \cdot p_{r}$, so the corresponding $q_{s}^{A}=$ $\frac{\beta^{A}(\gamma)-c_{s}}{2 \beta^{A}(\gamma)}$, and $\Pi_{s}^{A}=(1-\gamma) \cdot\left[\frac{\left(\beta^{A}(\gamma)-c_{s}\right)^{2}}{4 \beta^{A}(\gamma)}-k\right]$.

By substituting the best-response price $p_{s}^{A}\left(p_{r}\right)$ in Proposition 1 into (4), we can retrieve the corresponding consumer demand for store R . As noted before, When $p_{r} \leq c_{s}+1-\beta^{A}(\gamma)$, store S cannot afford to enter the market so that store S's demand $q_{s}^{A}=0$ and store R's demand $q_{r}^{A}=1-p_{r}$ (as a monopoly), which is decreasing in $p_{r}$. Hence, as before, it suffices to focus on the remaining 3 cases in the following corollary that deals with the comparative statics of the subgame that has $p_{r}>c_{s}+1-\beta^{A}(\gamma)$.

Corollary 1. When $p_{r}>c_{s}+1-\beta^{A}(\gamma)$ and $k \leq \tilde{\Pi}_{s}^{A}\left(p_{r}\right)$, the demand for each store upon the entry of store $S$ depends on $p_{r}$ as follows.
(i) When $p_{r} \in\left(c_{s}+1-\beta^{A}(\gamma), \frac{\beta^{A}(\gamma)-1+c_{s}}{2 \beta^{A}(\gamma)-1}\right)$, store $R$ 's demand $q_{r}^{A}=\frac{\beta^{A}(\gamma)-1+c_{s}-\left(2 \beta^{A}(\gamma)-1\right) p_{r}}{2\left(\beta^{A}(\gamma)-1\right)}$, which is increasing in $c_{s}$ and decreasing in $p_{r}$. Also, the corresponding store $S$ 's demand $q_{s}^{A}$ as given in (i) of Proposition 1 is increasing in $p_{r}$ and decreasing in $c_{s}$.
(ii) When $p_{r} \geq \frac{\beta^{A}(\gamma)-1+c_{s}}{2 \beta^{A}(\gamma)-1}$, store $R$ 's demand $q_{r}^{A}=0$ and store $S$ 's demand $q_{s}^{A}$ as given in (ii) and (iii) of Proposition 1 is non-increasing in $p_{r}$ and $c_{s}$.

We now interpret Proposition 1 and Corollary 1 via Figure 2(b). Recall that store S's entry condition is violated when $p_{r} \leq c_{s}+1-\beta^{A}(\gamma)$ (Figure 2(b) zone (1)), it suffices to characterize store S's best response price $p_{s}^{A}$ in Figure 2(b) for the remaining 3 cases as stated in Proposition 1. First, when $p_{r}$ is moderate (i.e., $\left.p_{r} \in\left(c_{s}+1-\beta^{A}(\gamma), \frac{\beta^{A}(\gamma)-1+c_{s}}{2 \beta^{A}(\gamma)-1}\right)\right)$, Proposition 1(i) suggests that store S will charge $p_{s}^{A}=\frac{\beta^{A}(\gamma)-1+p_{r}+c_{s}}{2}$. As such, store S and R can co-exist in the market (see Figure 2(b) zone (2)), and the corresponding consumer demand $q_{s}^{A}$ for store S is increasing in $p_{r}$ and decreasing in $c_{s}$, while the consumer demand $q_{r}^{A}$ for store R is increasing in $c_{s}$ and decreasing in $p_{r}$.

Next, when store R's retail price $p_{r}$ is high (i.e., $p_{r} \geq \frac{\beta^{A}(\gamma)-1+c_{s}}{2 \beta^{A}(\gamma)-1}$ ), compared with $p_{r}$, store S can afford to charge a competitive price $p_{s}^{A}$ that is no larger than $\beta^{A}(\gamma) \cdot p_{r}$. Then as shown in Corollary 1(ii), upon store S's entry, store R's market share will be squeezed out. Specifically, Proposition 1(ii) implies that when $p_{r}$ is high but still lower than $\frac{\beta^{A}(\gamma)+c_{s}}{2 \beta^{A}(\gamma)}$ (i.e., $\left.p_{r} \in\left[\frac{\beta^{A}(\gamma)-1+c_{s}}{2 \beta^{A}(\gamma)-1}, \frac{\beta^{A}(\gamma)+c_{s}}{2 \beta^{A}(\gamma)}\right]\right)$, it is optimal for store S to charge $p_{s}^{A}=\beta^{A}(\gamma) \cdot p_{r}$, which is increasing in $p_{r}$ and independent of $c_{s}$ (see Figure 2(b) zone (3)). As such, the corresponding consumer demand $q_{s}^{A}$ is decreasing in $p_{r}$ and independent of $c_{s}$. Proposition 1(iii) suggests that if $p_{r}$ is very high (i.e., $\left.p_{r}>\frac{\beta^{A}(\gamma)+c_{s}}{2 \beta^{A}(\gamma)}\right)$, it is optimal for store S to charge $p_{s}^{A}=\frac{\beta^{A}(\gamma)+c_{s}}{2}<\beta^{A}(\gamma) \cdot p_{r}$, which is independent of $p_{r}$ and is increasing in $c_{s}$ (see Figure 2(b) zone (4)). As a result, the corresponding consumer demand $q_{s}^{A}$ is independent of $p_{r}$ and is decreasing in $c_{s}$.

### 4.2 Store R's Equilibrium Deterrence Strategy in Period 1

Observe that, for any given price $p_{r}$, store R can anticipate store S's best response price $p_{s}^{A}\left(p_{r}\right)$ stated in Proposition 1 and its corresponding gross profit $\tilde{\Pi}_{s}^{A}\left(p_{r}\right)$ stated in Lemma 1. Hence, store R can use these quantities to decide whether to: (i) deter a type A store S's entry by solving problem (8) and earn $\Pi_{r}^{d, A}$, or (ii) tolerate store S's entry by solving problem (9) and earn $\Pi_{r}^{t, A}$. Then by comparing $\Pi_{r}^{d, A}$ against $\Pi_{r}^{t, A}$, store R can determine its equilibrium price and deterrence strategy in period 1 as explained in $\S 3.2 .4$.

### 4.2.1 Store R's Deterrence Price Threshold $\tau^{A}$

To begin, recall from Lemma 1 that, for any given store R's price $p_{r}$, store S's gross profit $\tilde{\Pi}_{s}^{A}\left(p_{r}\right)$ given in (10) (Figure 2(a)) is increasing in $p_{r}$. Then, note that store S's entry condition is: $\tilde{\Pi}_{s}^{A}\left(p_{r}\right) \geq$ $k$ (so that store S's net profit $\left.\Pi_{s}^{A}\left(p_{r}\right)=(1-\gamma) \cdot\left[\tilde{\Pi}_{s}^{A}\left(p_{r}\right)-k\right] \geq 0\right)$. These two observations imply that there exists a threshold $\tau^{A}$ that solves $\tilde{\Pi}_{s}^{A}\left(\tau^{A}\right)=k$ in the following lemma so that store S's entry condition holds (i.e., $\tilde{\Pi}_{s}^{A}\left(p_{r}\right) \geq k$ ) if and only if $p_{r} \geq \tau^{A}$ (the blue box in Figure 1). Before we present the expression for $\tau^{A}$, let us define two terms for ease of exposition. Let:

$$
\begin{equation*}
K_{1}^{A} \equiv \tilde{\Pi}_{s}^{A}\left(\frac{\beta^{A}(\gamma)+c_{s}}{2 \beta^{A}(\gamma)}\right)=\frac{\left(\beta^{A}(\gamma)-c_{s}\right)^{2}}{4 \beta^{A}(\gamma)}, K_{2}^{A} \equiv \tilde{\Pi}_{s}^{A}\left(\frac{\beta^{A}(\gamma)-1+c_{s}}{2 \beta^{A}(\gamma)-1}\right)=\frac{\left(\beta^{A}(\gamma)-c_{s}\right)^{2}\left(\beta^{A}(\gamma)-1\right)}{\left(2 \beta^{A}(\gamma)-1\right)^{2}}, \tag{11}
\end{equation*}
$$

where $K_{1}^{A} \geq K_{2}^{A}$ as depicted in Figure 2(a).

Lemma 2. In period 1, store $R$ can deter store $S$ 's entry by setting $p_{r}<\tau^{A}$ or tolerate store $S$ 's entry by setting $p_{r} \geq \tau^{A}$, where:

$$
\tau^{A}= \begin{cases}\sqrt{4 k\left(\beta^{A}(\gamma)-1\right)}+c_{s}+1-\beta^{A}(\gamma) & k \leq K_{2}^{A}  \tag{12}\\ \frac{c_{s}+\beta^{A}(\gamma)-\sqrt{\left(\beta^{A}(\gamma)-c_{s}\right)^{2}-4 k \beta^{A}(\gamma)}}{2 \beta^{A}(\gamma)} & k \in\left(K_{2}^{A}, K_{1}^{A}\right] \\ \infty & k>K_{1}^{A}\end{cases}
$$

Lemma 2 implies that store S's entry condition $\tilde{\Pi}_{s}^{A}\left(p_{r}\right) \geq k$ (or equivalently, $\Pi_{s}^{A}\left(p_{r}\right) \geq 0$ ) is equivalent to the condition $p_{r} \geq \tau^{A}$. Hence, if store R aims to deter, it can afford to do so when its unit cost $c_{r}<\tau^{A}$. Hence, store R's problem (8) can be simplified as:

$$
\begin{equation*}
\Pi_{r}^{d, A}=\sup _{p_{r} \in\left[c_{r}, \tau^{A}\right)}\left(p_{r}-c_{r}\right) \cdot\left(1-p_{r}\right) . \tag{13}
\end{equation*}
$$

On the other hand, if store R aims to tolerate store S's entry, it sets a price $p_{r} \geq \tau^{A}$ (with $\tau^{A}>$ $c_{s}+1-\beta^{A}(\gamma)$ as observed from Lemma 2). In this case, by using the consumer demand $q_{r}^{A}$ for store R as stated Corollary 1, store R's problem (9) can be simplified as:

$$
\begin{align*}
\Pi_{r}^{t, A}= & \max _{p_{r} \geq \max \left\{c_{r}, \tau^{A}\right\}}\left(p_{r}-c_{r}\right) \cdot q_{r}^{A}, \\
\text { s.t. } & q_{r}^{A}= \begin{cases}0 & \text { if } p_{r} \geq \frac{\beta^{A}(\gamma)-1+c_{s}}{2 \beta^{A}(\gamma)-1} \\
\frac{\beta^{A}(\gamma)-1+c_{s}-\left(2 \beta^{A}(\gamma)-1\right) p_{r}}{2\left(\beta^{A}(\gamma)-1\right)} & \text { if } p_{r} \in\left(c_{s}+1-\beta^{A}(\gamma), \frac{\beta^{A}(\gamma)-1+c_{s}}{2 \beta^{A}(\gamma)-1}\right) .\end{cases} \tag{14}
\end{align*}
$$

By solving problems (13) and (14), we can identify Store R's deterrence strategy: deter store S's entry by setting a price $p_{r}$ below $\tau^{A}$ if $\Pi_{r}^{d, A}>\Pi_{r}^{t, A}$, and tolerate it; otherwise. Hence, we can determine store R's equilibrium deterrence strategy and its equilibrium retail price $p_{r}^{A}$ that yields:

$$
\begin{equation*}
\Pi_{r}^{A}\left(p_{r}^{A}\right)=\max _{p_{r} \geq c_{r}}\left\{\Pi_{r}^{d, A}, \Pi_{r}^{t, A}\right\} \tag{15}
\end{equation*}
$$

### 4.2.2 Store R's Deterrence Strategy: Cost Advantage $\alpha$

We now determine store R's deterrence strategy by solving problem (15) that hinges upon store S's entry cost $k$ and store R's cost advantage $\alpha \equiv c_{r} / c_{s}<1$. (As explained in $\S 3$, store R has a higher cost advantage when $\alpha$ is smaller.) In preparation, we define $\Theta_{1}^{A}(k)$ and $\Theta_{2}^{A}(k)$ as two thresholds for the cost advantage $\alpha$ so that: ${ }^{9}$

$$
\Theta_{1}^{A}(k)= \begin{cases}\frac{c_{s}+\beta^{A}(\gamma)-\sqrt{\left(\beta^{A}(\gamma)-c_{s}\right)^{2}-4 k \cdot \beta^{A}(\gamma)}}{2 \beta^{A}(\gamma) \cdot c_{s}} & k \in\left(K_{2}^{A}, K_{1}^{A}\right]  \tag{16}\\ \frac{c_{s}\left(4 \beta^{A}(\gamma)-3\right)+\left(\beta^{A}(\gamma)-1\right) \cdot\left[1-4 \beta^{A}(\gamma)+8 \sqrt{k\left(\beta^{A}(\gamma)-1\right)}+4 \sqrt{k\left(\frac{\beta^{A}(\gamma)-c_{s}}{\sqrt{k\left(\beta^{A}(\gamma)-1\right)}}-2\right)}\right]}{c_{s}\left(2 \beta^{A}(\gamma)-1\right)} & k \leq K_{2}^{A}\end{cases}
$$

[^4] $\Theta_{1}^{A}(k)$ is increasing in $k$, while $\Theta_{2}^{A}(k)$ is independent of $k$.
\[

\Theta_{2}^{A}(k)= $$
\begin{cases}\frac{c_{s}+\beta^{A}(\gamma)-\sqrt{\left(\beta^{A}(\gamma)-c_{s}\right)^{2}-4 k \beta^{A}(\gamma)}}{2 \beta^{A}(\gamma) \cdot c_{s}} & k \in\left(K_{2}^{A}, K_{1}^{A}\right]  \tag{17}\\ \frac{\beta^{A}(\gamma)-1+c_{s}}{\left(2 \beta^{A}(\gamma)-1\right) \cdot c_{s}} & k \leq K_{2}^{A}\end{cases}
$$
\]

Proposition 2. When facing a type (A) store $S$ 's potential entry, store $R$ 's deterrence strategy can be described as follows:

1. High entry cost: $k>K_{1}^{A}$. Suppose store $S$ 's entry cost $k>K_{1}^{A}$. Then store $S$ cannot afford to enter the market and store $R$ can operate as a monopoly.
2. Medium entry cost: $k \in\left(K_{2}^{A}, K_{1}^{A}\right]$. Suppose store $S^{\prime}$ 's entry cost $k \in\left(K_{2}^{A}, K_{1}^{A}\right]$. Then:
(a) if store $R$ 's cost advantage is high (as $\alpha<\Theta_{1}^{A}(k)=\Theta_{2}^{A}(k)$ ), then store $R$ should deter store $S$ 's entry. ${ }^{10}$
(b) if store $R$ 's cost advantage is low (as $\alpha \geq \Theta_{2}^{A}(k)=\Theta_{1}^{A}(k)$ ), then store $R$ cannot deter the inevitable entry of store $S .{ }^{11}$
3. Low entry cost: $k \in\left(0, K_{2}^{A}\right]$. Suppose store $S$ 's entry cost $k \in\left(0, K_{2}^{A}\right]$. Then:
(a) if store $R$ 's cost advantage is high (as $\alpha<\Theta_{1}^{A}(k)$ ), then store $R$ should deter store $S$ 's entry. ${ }^{12}$
(b) if store $R$ 's cost advantage is medium (as $\alpha \in\left[\Theta_{1}^{A}(k), \Theta_{2}^{A}(k)\right)$ ), then it is optimal for store $R$ to tolerate store $S$ 's entry. ${ }^{13}$
(c) if store $R$ 's cost advantage is low (as $\alpha \geq \Theta_{2}^{A}(k)$ ), then store $R$ cannot deter the inevitable entry of store $S .^{14}$

To interpret Proposition 2, we map out store R's equilibrium deterrence strategy as stated in Proposition 2 based on store S's entry cost $k$ and store R's cost advantage factor $\alpha$ in Figure 3.

First, when store S's entry cost is high: $k>K_{1}^{A}$, statement 1 of Proposition 2 is depicted in zone (E) in Figure 3, highlighting the market condition is untenable for store S to enter.

Second, when store S's entry cost is medium: $k \in\left(K_{2}^{A}, K_{1}^{A}\right]$, store S's potential entry is plausible. As such, store R's deterrence strategy hinges upon its cost advantage over store S via $\alpha$. Specifically,
${ }^{10}$ Specifically, (i) when $\alpha \in\left[\frac{c_{s}-\sqrt{\left(c_{s}-\beta^{A}(\gamma)\right)^{2}-4 k \beta^{A}(\gamma)}}{\beta^{A}(\gamma) \cdot c_{s}}, \Theta_{1}^{A}(k)\right)$, store R's equilibrium deterrence price $p_{r}^{A}=\tau^{A}-\epsilon=$ $\frac{c_{s}+\beta^{A}(\gamma)-\sqrt{\left(c_{s}-\beta^{A}(\gamma)\right)^{2}-4 k \beta^{A}(\gamma)}}{2 \beta^{A}(\gamma)}-\epsilon$, where $\epsilon \rightarrow 0^{+}$; (ii) when $\alpha<\frac{c_{s}-\sqrt{\left(\beta^{A}(\gamma)-c_{s}\right)^{2}-4 k \cdot \beta^{A}(\gamma)}}{\beta^{A}(\gamma) \cdot c_{s}}$, store R's equilibrium deterrence price $p_{r}^{A}=p_{r}^{0}=\frac{1+c_{r}}{2}$. Furthermore, store R's demand $q_{r}^{A}=1-p_{r}^{A}$.
${ }^{11}$ upon store S's entry, $q_{r}^{A}=0$ so that store R earns nothing.
${ }^{12}$ Specifically, (i) when $\alpha \in\left[\frac{2 c_{s}+1-2 \beta^{A}(\gamma)+4 \sqrt{k \cdot\left(\beta^{A}(\gamma)-1\right)}}{c_{s}}, \Theta_{1}^{A}(k)\right)$, store R's equilibrium deterrence price $p_{r}^{A}=\tau^{A}-$ $\epsilon=\sqrt{4 k \cdot\left(\beta^{A}(\gamma)-1\right)}+c_{s}+1-\beta^{A}(\gamma)-\epsilon$, where $\epsilon \rightarrow 0^{+}$; (ii) when $\alpha<\frac{2 c_{s}+1-2 \beta^{A}(\gamma)+4 \sqrt{k\left(\beta^{A}(\gamma)-1\right)}}{c_{s}}$, then store R's equilibrium deterrence price $p_{r}^{A}=p_{r}^{0}=\frac{1+c_{r}}{2}$. Furthermore, store R's demand $q_{r}^{A}=1-p_{r}^{A}$.
${ }^{13}$ Here, store R should set the equilibrium tolerating price $p_{r}^{A}=\frac{\beta^{A}(\gamma)-1+c_{s}+\left(2 \beta^{A}(\gamma)-1\right) \cdot c_{r}}{2\left(2 \beta^{A}(\gamma)-1\right)}$. In this case, store R's demand $q_{r}^{A}=\frac{\beta^{A}(\gamma)-1+c_{s}+c_{r}-2 \beta^{A}(\gamma) \cdot c_{r}}{4\left(\beta^{A}(\gamma)-1\right)}$.
${ }^{14}$ After store S enters, $q_{r}^{A}=0$ and store R earns nothing.


Figure 3 Store R's deterrence strategy in terms of store S's entry cost $k$ and store R's cost advantage $\alpha$.
when store R 's cost advantage over store S is sufficiently high (as $\alpha<\Theta_{1}^{A}(k)$ ), store R can afford to deter store S's entry as stated in statement 2 (a) of Proposition 2 and depicted in zones (C) and (D) within the range $k \in\left(K_{2}^{A}, K_{1}^{A}\right]$. However, when store R's cost advantage over store S is low (as $\alpha \geq \Theta_{1}^{A}(k)$ ), store R cannot afford to deter store S 's entry as depicted in zone (A) within the range $k \in\left(K_{2}^{A}, K_{1}^{A}\right]$.

Third, when store S's entry cost is low: $k \leq K_{2}^{A}$, store S's potential entry is now imminent. Specifically, the zones (C), (D), and (A) within the range $k \in\left(0, K_{2}^{A}\right]$ as depicted in Figure 3 are based on statements 3 (a) and 3(c), and they can be interpreted in the same manner as above. Interestingly, there is a new zone (B) corresponding to Statement 3(b) that deserves our attention. Specifically, when store S's entry cost is low, store R should tolerate store S's entry when its cost advantage is medium, even if it can profitably deter store S's entry. Faced with the threat from store S , the incumbent store R faces a trade-off between the loss of profit due to charging a lower deterrence price or the loss of profit due to losing some of its market share to store S . In zone (B), the incumbent store R can be "better off" by tolerating store S's entry and sharing the market with it, rather than lowering the price to a very low level to deter store S's entry.

By substituting the equilibrium price $p_{r}^{A}$ given by Proposition 2 for different regions of ( $k, \alpha$ ) into Proposition 1 and Corollary 1, we can derive store S's equilibrium price $p_{s}^{A}$, equilibrium profit $\tilde{\Pi}_{s}^{A}$, and demand $q_{s}^{A}$ (and store R's demand $q_{r}^{A}$ ). Because our focus is on the deterrence strategy, we shall omit these tedious expressions under different conditions.

In summary, our analytical results formalize our understanding regarding how a type (A) store S's entry cost $k$ and store R's cost advantage $\alpha$ affect store R's deterrence strategy as stated in Proposition 2 and Figure 3. How would these results change if the potential entrant is a type (B) store S who donates a proportion of its revenue (instead of profit) to charity? We shall examine this question next.

## 5 Market Entry Game between a Type (B) Store S and an Incumbent Store R

We now use the same approach as before to examine the market entry game between store R and a type (B) store S who donates a proportion $\gamma$ of its revenue (instead of profit), creating the warmglow effect $\beta^{B}(\gamma)$. Recognizing this difference, we characterize a type (B) store S's best-response pricing strategy in $\S 5.1$, followed by store R's deterrence strategy in $\S 5.2$.

### 5.1 Type (B) Store S's Best-Response Pricing Strategy in Period 2

Given store R's retail price $p_{r}$, a type (B) store S determines its best-response price $p_{s}^{B}\left(p_{r}\right)$ by solving (7). Before we present $p_{s}^{B}\left(p_{r}\right)$ as stated in Proposition 3, let us first rewrite the effective maximum profit given in (7) as: $\Pi_{s}^{B}\left(p_{r}\right)=(1-\gamma) p_{s}^{B} q_{s}^{B}-c_{s} q_{s}^{B}-k=\tilde{\Pi}_{s}^{B}-k$, where $\tilde{\Pi}_{s}^{B} \equiv\left((1-\gamma) p_{s}^{B}-c_{s}\right) \cdot q_{s}^{B}$ represents the maximum gross profit without considering the entry cost $k$. Hence, a type (B) store S can enter the market if and only if $\tilde{\Pi}_{s}^{B} \geq k$ (or equivalently, $\Pi_{s}^{B}\left(p_{r}\right) \geq 0$ ). By solving (7), we get:

Lemma 3. Given any store $R$ 's price $p_{r}$, a type ( $B$ ) store $S$ 's gross profit $\tilde{\Pi}_{s}^{B}\left(p_{r}\right)$ satisfies:

$$
\tilde{\Pi}_{s}^{B}\left(p_{r}\right)=\left\{\begin{array}{ll}
0 & p_{r} \leq \frac{c_{s}}{11-\gamma}+1-\beta^{B}(\gamma)  \tag{18}\\
\frac{\left[(1-\gamma)\left(\beta^{B}(\gamma)-1+p_{r}\right)-c_{s}\right]^{2}}{4\left(\beta^{B}(\gamma)-1\right)(1-\gamma)} & p_{r} \in\left(\frac{c_{s}}{1-\gamma}+1-\beta^{B}(\gamma), \frac{c_{s}}{(1-\gamma) \cdot\left(2 \beta^{B}(\gamma)-1\right)}+\frac{\beta^{B}(\gamma)-1}{2 \beta^{B}(\gamma)-1}\right) . \\
{\left[(1-\gamma) \cdot \beta^{B}(\gamma) \cdot p_{r}-c_{s}\right]\left(1-p_{r}\right)} & p_{r} \in\left[\frac{c_{s}}{(1-\gamma) \cdot\left(2 \beta^{B}(\gamma)-1\right)}+\frac{\beta^{B}(\gamma)-1}{2 \beta^{B}(\gamma)-1}, \frac{c_{s}}{2 \beta^{B}(\gamma)(1-\gamma)}+\frac{1}{2}\right] . \\
\frac{\left[\beta^{B}(\gamma) \cdot(1-\gamma)-c_{s}\right]^{2}}{4 \beta^{B}(\gamma) \cdot(1-\gamma)} & p_{r}>\frac{c_{s}}{2 \beta^{B}(\gamma)(1-\gamma)}+\frac{1}{2}
\end{array} .\right.
$$

Also, $\tilde{\Pi}_{s}^{B}\left(p_{r}\right)$ is non-decreasing in $p_{r}$.
We illustrate the gross profit $\tilde{\Pi}_{s}^{B}\left(p_{r}\right)$ in Figure 4(a) that resembles Figure 2(a) in §4.1, and it has the same interpretation. Also, observe from Lemma 3 that store $S$ 's entry condition is violated if store R set $p_{r}$ sufficiently low so that $p_{r} \leq \frac{c_{s}}{1-\gamma}+1-\beta^{B}(\gamma)$. By ruling out this trivial case, we get:

Proposition 3. Suppose store $R$ sets its price at $p_{r}$ that satisfies $p_{r}>\frac{c_{s}}{1-\gamma}+1-\beta^{B}(\gamma)$ and store S's entry condition $k \leq \tilde{\Pi}_{s}^{B}\left(p_{r}\right)$ holds, where $\tilde{\Pi}_{s}^{B}\left(p_{r}\right)$ is given in Lemma 3. Then a type ( $B$ ) store S's best-response price $p_{s}^{B}$, consumer demand $q_{s}^{B}$, and its retained profit $\Pi_{s}^{B}=\tilde{\Pi}_{s}^{B}-k$ satisfy:
(i) If $p_{r} \in\left(\frac{c_{s}}{1-\gamma}+1-\beta^{B}(\gamma), \frac{c_{s}}{(1-\gamma) \cdot\left(2 \beta^{B}(\gamma)-1\right)}+\frac{\beta^{B}(\gamma)-1}{2 \beta^{B}(\gamma)-1}\right)$, then the best-response $p_{s}^{B}=\frac{1}{2}\left(\beta^{B}(\gamma)-\right.$ $\left.1+p_{r}+\frac{c_{s}}{1-\gamma}\right)>\beta^{B}(\gamma) \cdot p_{r}$, so the corresponding $q_{s}^{B}=\frac{\beta^{B}(\gamma)-1+p_{r}}{2\left(\beta^{B}(\gamma)-1\right)}-\frac{c_{s}}{2(1-\gamma) \cdot\left(\beta^{B}(\gamma)-1\right)}$ and $\Pi_{s}^{B}=$ $\frac{\left[(1-\gamma)\left(\beta^{B}(\gamma)-1+p_{r}\right)-c_{s}\right]^{2}}{4\left(\beta^{B}(\gamma)-1\right)(1-\gamma)}-k$.
(ii) If $p_{r} \in\left[\frac{c_{s}}{(1-\gamma)\left(2 \beta^{B}(\gamma)-1\right)}+\frac{\beta^{B}(\gamma)-1}{2 \beta^{B}(\gamma)-1}, \frac{c_{s}}{2 \beta^{B}(\gamma) \cdot(1-\gamma)}+\frac{1}{2}\right]$, then the best-response $p_{s}^{B}=\beta^{B}(\gamma) \cdot p_{r}$, so the corresponding $q_{s}^{B}=1-p_{r}$ and $\Pi_{s}^{B}=\left[(1-\gamma) \cdot \beta^{B}(\gamma) \cdot p_{r}-c_{s}\right] \cdot\left(1-p_{r}\right)-k$.
(iii) If $p_{r}>\frac{c_{s}}{2 \beta^{B}(\gamma) \cdot(1-\gamma)}+\frac{1}{2}$, then the best-response $p_{s}^{B}=\frac{\beta^{B}(\gamma)}{2^{2}}+\frac{c_{s}}{2(1-\gamma)}<\beta^{B}(\gamma) \cdot p_{r}$, so the corresponding $q_{s}^{B}=\frac{1}{2}-\frac{c_{s}}{2 \beta^{B}(\gamma) \cdot(1-\gamma)}$, and $\Pi_{s}^{B}=\frac{\left[\beta^{B}(\gamma)(1-\gamma)-c s\right]^{2}}{4 \beta^{B}(\gamma) \cdot(1-\gamma)}-k$.

(a) Type (B) store S's gross profit $\tilde{\Pi}_{s}^{B}\left(p_{r}\right)$.

(b) Type (B) Store S's best-response price $p_{s}^{B}$.

Figure 4 Type (B) store S's best-response pricing strategy.

Observe that Lemma 3 and Proposition 3 illustrated in Figure 4 resemble Lemma 1 and Proposition 1 depicted in Figure 2. Next, by substituting $p_{s}^{B}\left(p_{r}\right)$ stated in Proposition 3 into (3) and (4), we can derive the consumer demand for store $R$ upon the entry of a type (B) store $S$ as follows.

Corollary 2. When $p_{r}>\frac{c_{s}}{1-\gamma}+1-\beta^{B}(\gamma)$ and $k \leq \tilde{\Pi}_{s}^{B}\left(p_{r}\right)$, the demand for each store upon the entry of a type (B) store $S$ depends on $p_{r}$ as follows.
(i) When $\quad p_{r} \in\left(\frac{c_{s}}{1-\gamma}+1-\beta^{B}(\gamma), \frac{c_{s}}{(1-\gamma)\left(2 \beta^{B}(\gamma)-1\right)}+\frac{\beta^{B}(\gamma)-1}{2 \beta^{B}(\gamma)-1}\right)$, store $\quad$ 's demand $q_{r}^{B}=$ $\frac{\beta^{B}(\gamma)-1-\left(2 \beta^{B}(\gamma)-1\right) \cdot p_{r}}{2\left(\beta^{B}(\gamma)-1\right)}+\frac{c_{s}}{2(1-\gamma) \cdot\left(\beta^{B}(\gamma)-1\right)}$, which is increasing in $c_{s}$ and decreasing in $p_{r}$. Also, the corresponding store $S$ 's demand $q_{s}^{B}$ as given in (i) of Proposition 3 is increasing in $p_{r}$ and decreasing in $c_{s}$.
(ii) When $p_{r} \geq \frac{c_{s}}{(1-\gamma) \cdot\left(2 \beta^{B}(\gamma)-1\right)}+\frac{\beta^{B}(\gamma)-1}{2 \beta^{B}(\gamma)-1}$, store $R$ 's demand $q_{r}^{B}=0$ and store $S^{\prime}$ 's demand $q_{s}^{B}$ as given in (ii) and (iii) of Proposition 3 is non-increasing in $p_{r}$ and $c_{s}$.

### 5.2 Store R's Equilibrium Deterrence Strategy in Period 1

By anticipating store (B)'s best response as stated in Proposition 3, we now characterize store R's deterrence strategy (by using the same approach as stated in §4.2).

### 5.2.1 Store R's Deterrence Price Threshold $\tau^{B}$

To begin, we characterize the deterrence threshold for $p_{r}$ (denoted as $\tau^{B}$ ) in Lemma 4 so that store R can deter a type (B) store S's entry by choosing a retail price $p_{r}<\tau^{B}$ or tolerate its entry by setting $p_{r} \geq \tau^{B}$. Analogous to thresholds $K_{1}^{A}$ and $K_{2}^{A}$ as defined in $\S 4.2$, let:

$$
K_{1}^{B}=\tilde{\Pi}_{s}^{B}\left(\frac{c_{s}}{2 \beta^{B}(\gamma)(1-\gamma)}+\frac{1}{2}\right)=\frac{\left[\beta^{B}(\gamma) \cdot(1-\gamma)-c_{s}\right]^{2}}{4 \beta^{B}(\gamma) \cdot(1-\gamma)},
$$

$$
\begin{equation*}
K_{2}^{B}=\tilde{\Pi}_{s}^{B}\left(\frac{c_{s}}{(1-\gamma)\left(2 \beta^{B}(\gamma)-1\right)}+\frac{\beta^{B}(\gamma)-1}{2 \beta^{B}(\gamma)-1}\right)=\frac{\left[\beta^{B}(\gamma) \cdot(1-\gamma)-c_{s}\right]^{2} \cdot\left(\beta^{B}(\gamma)-1\right)}{\left(2 \beta^{B}(\gamma)-1\right)^{2} \cdot(1-\gamma)}, \tag{19}
\end{equation*}
$$

where $K_{1}^{B} \geq K_{2}^{B}$. Observe from Proposition 3 and Figure 4(a) that a type (B) store S can never enter the market when its entry cost $k>K_{1}^{B}$ regardless of the value of $p_{r}$ (i.e., $\tau^{B}=\infty$ ). Hence, it suffices to focus on the case when $k \leq K_{1}^{B}$ by focusing on the deterrence threshold $\tau^{B}$ as follows.

Lemma 4. Store $R$ can either deter a type ( $B$ ) store's entry by setting $p_{r}<\tau^{B}$, or tolerate its entry by setting $p_{r} \geq \tau^{B}$, where:

$$
\tau^{B}=\left\{\begin{array}{ll}
\frac{\sqrt{4 k\left(\beta^{B}(\gamma)-1\right) \cdot(1-\gamma)}+c_{s}}{1-\gamma}+1-\beta^{B}(\gamma) & k \leq K_{2}^{B}  \tag{20}\\
\frac{c_{s}+\beta^{B}(\gamma) \cdot(1-\gamma)-\sqrt{\left[\beta^{B}(\gamma) \cdot(1-\gamma)-c_{s}\right]^{2}-4 k \beta^{B}(\gamma) \cdot(1-\gamma)}}{2 \beta^{B}(\gamma) \cdot(1-\gamma)} & k \in\left(K_{2}^{B}, K_{1}^{B}\right] \\
\infty & k>K_{1}^{B}
\end{array} .\right.
$$

Applying Lemma 4 in the same way as in $\S 4.2 .1$, we can examine type (B) store S's entry condition (i.e., $\tilde{\Pi}_{s}^{B}\left(p_{r}\right) \geq k$ ) from store R's perspective; i.e., $p_{r} \geq \tau^{B}$. Hence, we can simplify store R's problem (8) under deterrence that is analogous to (13) and obtain store R's corresponding profit $\Pi_{r}^{d, B}$. We can also simplify store R's problem (9) under tolerance that is analogous to (14) and obtain store R's corresponding profit $\Pi_{r}^{t, B}$. To avoid repetition, we omit the details.

### 5.2.2 Store R's Deterrence Strategy: Cost Advantage $\alpha$

Once we determine store R's profit $\Pi_{r}^{d, B}$ (or $\Pi_{r}^{t, B}$ ) when it chooses to deter (or tolerate) store S's entry, we can determine store R's deterrence strategy by comparing these two quantities as explained in §3.2.4. Before we characterize store R's equilibrium deterrence strategy in Proposition 4 , let us recall from $\S 4.2$ that store R's deterrence strategy is based on its cost competitiveness measured by $\alpha$ (because $c_{r}=\alpha \cdot c_{s}$ ) and store S's entry cost $k$. Analogous to the thresholds $\Theta_{1}^{A}(k)$ and $\Theta_{2}^{A}(k)$ as defined in $\S 4.2$, we define two thresholds for store R's cost advantage $\alpha=c_{r} / c_{s}$. Let:
$\Theta_{1}^{B}(k)=\left\{\begin{array}{ll}\frac{c_{s}+\beta^{B}(\gamma)(1-\gamma)-\sqrt{\left[\beta^{B}(\gamma) \cdot(1-\gamma)-c_{s}\right]^{2}-4 k \beta^{B}(\gamma) \cdot(1-\gamma)}}{2 \beta^{B}(\gamma) \cdot(1-\gamma) c_{s}} & k \in\left(K_{2}^{B}, K_{1}^{B}\right] \\ \frac{c_{s}\left(4 \beta^{B}(\gamma)-3\right)+\left(\beta^{B}(\gamma)-1\right) \cdot\left[1-\gamma-4 \beta^{B}(\gamma) \cdot(1-\gamma)+8 \sqrt{k\left(\beta^{B}(\gamma)-1\right) \cdot(1-\gamma)}+4 \sqrt{\left.k(1-\gamma)\left(\frac{\beta^{B}(\gamma) \cdot(1-\gamma)-c_{s}}{\sqrt{k\left(\beta^{B}(\gamma)-1\right)(1-\gamma)}-2}\right)\right]}\right.}{c_{s}\left(2 \beta^{B}(\gamma)-1\right)(1-\gamma)} & k \leq K_{2}^{B}\end{array}\right.$,
$\Theta_{2}^{B}(k)=\left\{\begin{array}{ll}\frac{c_{s}+\beta^{B}(\gamma)(1-\gamma)-\sqrt{\left[\beta^{B}(\gamma) \cdot(1-\gamma)-c_{s}\right]^{2}-4 k \beta^{B}(\gamma) \cdot(1-\gamma)}}{2 \beta^{B}(\gamma) \cdot(1-\gamma) c_{s}} & k \in\left(K_{2}^{B}, K_{1}^{B}\right] . \\ \frac{\beta^{B}(\gamma)-1}{\left(2 \beta^{B}(\gamma)-1\right) \cdot c_{s}}+\frac{1}{\left(2 \beta^{B}(\gamma)-1\right)(1-\gamma)} & k \leq K_{2}^{B}\end{array}\right.$.

Proposition 4. When facing the potential entry of a type (B) store $S$, store $R$ 's deterrence strategy can be described as follows:

1. High entry cost: $k>K_{1}^{B}$. Suppose a type ( $B$ ) store $S$ 's entry cost $k>K_{1}^{B}$. Then store $S$ cannot afford to enter the market and store $R$ can operate as a monopoly.
2. Medium entry cost: $k \in\left(K_{2}^{B}, K_{1}^{B}\right]$. Suppose a type $(B)$ store $S$ 's entry cost $k \in\left(K_{2}^{B}, K_{1}^{B}\right]$. Then:
(a) if store $R$ 's cost advantage is high (as $\alpha<\Theta_{1}^{B}(k)=\Theta_{2}^{B}(k)$ ), then store $R$ should deter store $S$ 's entry. ${ }^{15}$
(b) if store $R$ 's cost advantage is low (as $\alpha \geq \Theta_{2}^{B}(k)=\Theta_{1}^{B}(k)$ ), then store $R$ cannot deter the inevitable entry of store $S .{ }^{16}$
3. Low entry cost: $k \in\left(0, K_{2}^{B}\right]$. Suppose store $S$ 's entry cost $k \in\left(0, K_{2}^{B}\right]$. Then:
(a) if store $R$ 's cost advantage is high (as $\left.\alpha<\Theta_{1}^{B}(k)\right)$, then store $R$ should deter store $S$ 's entry. ${ }^{17}$
(b) if store $R$ 's cost advantage is medium (as $\alpha \in\left[\Theta_{1}^{B}(k), \Theta_{2}^{B}(k)\right)$ ), then it is optimal for store $R$ to tolerate store $S$ 's entry. ${ }^{18}$
(c) if store $R$ 's cost advantage is low (as $\alpha \geq \Theta_{2}^{B}(k)$ ), then store $R$ cannot deter the inevitable entry of store $S .{ }^{19}$

Proposition 4 shows that the deterrence strategy for store $R$ against a type (B) store S's entry possesses the same structure as that against a type (A) store $S$ (see Proposition 2 and Figure 3). As such, it can be interpreted in the same manner as in $\S 4.2 .2$. We omit the details.

## 6 Store R's Deterrence Strategies Across Different Types of Store S

While store R's deterrence strategies against both types of store S follow the same structure as presented in Proposition 2 (for type $(A)$ ) and Proposition 4 (for type (B)), store $R$ may be more willing to tolerate or deter the entry of one type than the other types. This is because, as shown in Propositions 2 and 4, store R's deterrence strategy depends on those thresholds $K_{1}^{j}$ and $K_{2}^{j}$ for
${ }^{15}$ Specifically, (i) When $\alpha \in\left[\frac{c_{s}-\sqrt{\left[\beta^{B}(\gamma) \cdot(1-\gamma)-c_{s}\right]^{2}-4 k \cdot \beta^{B}(\gamma) \cdot(1-\gamma)}}{\beta^{B}(\gamma) \cdot(1-\gamma) c_{s}}, \Theta_{1}^{B}(k)\right)$, store R's equilibrium deterrence price $p_{r}^{B}=\tau^{B}-\epsilon=\frac{c_{s}+\beta^{B}(\gamma) \cdot(1-\gamma)-\sqrt{\left[\beta^{B}(\gamma)(1-\gamma)-c_{s}\right]^{2}-4 k \cdot \beta^{B}(\gamma) \cdot(1-\gamma)}}{2 \beta^{B}(\gamma) \cdot(1-\gamma)}-\epsilon$, where $\epsilon \rightarrow 0^{+}$; (ii) when $\alpha<$ $\frac{c_{s}-\sqrt{\left[\beta^{B}(\gamma) \cdot(1-\gamma)-c_{s}\right]^{2}-4 k \cdot \beta^{B}(\gamma) \cdot(1-\gamma)}}{\beta^{B}(\gamma) \cdot(1-\gamma) c_{s}}$, store R's equilibrium deterrence price $p_{r}^{B}=p_{r}^{0}=\frac{1+c_{r}}{2}$. Furthermore, store R's demand $q_{r}^{B}=1-p_{r}^{B}$.
${ }^{16}$ Upon store S's entry, $q_{r}^{B}=0$ so that store R earns nothing.
${ }^{17}$ Specifically, (i) when $\alpha \in\left[\frac{2\left[c_{s}+\sqrt{4 k\left(\beta^{B}(\gamma)-1\right) \cdot(1-\gamma)}\right]-\left(2 \beta^{B}(\gamma)-1\right) \cdot(1-\gamma)}{c_{s}(1-\gamma)}, \Theta_{1}^{B}(k)\right)$, store R's equilibrium deterrence price $p_{r}^{B}=\tau^{B}-\epsilon=\frac{\sqrt{4 k\left(\beta^{B}(\gamma)-1\right)(1-\gamma)+c_{s}}}{1-\gamma}+1-\beta^{B}(\gamma)-\epsilon$, where $\epsilon \rightarrow 0^{+}$; (ii) when $\alpha<$ $\frac{2\left[c_{s}+\sqrt{4 k\left(\beta^{B}(\gamma)-1\right) \cdot(1-\gamma)}\right]-\left(2 \beta^{B}(\gamma)-1\right) \cdot(1-\gamma)}{c_{s}(1-\gamma)}$, store R's equilibrium deterrence price $p_{r}^{B}=p_{r}^{0}=\frac{1+c_{r}}{2}$. Furthermore, store R's demand $q_{r}^{B}=1-p_{r}^{B}$.
${ }^{18}$ In this case, store $R$ should set the equilibrium tolerating price $p_{r}^{B}=\frac{1}{2} \cdot\left[c_{r}+\frac{\left(\beta^{B}(\gamma)-1\right)(1-\gamma)+c_{s}}{\left(2 \beta^{B}(\gamma)-1\right) \cdot(1-\gamma)}\right]$ so that store R's corresponding demand $q_{r}^{B}=\frac{c_{s}+\left(\beta^{B}(\gamma)-1+c_{r}-2 \beta^{B}(\gamma) \cdot c_{r}\right)(1-\gamma)}{4\left(\beta^{B}(\gamma)-1\right) \cdot(1-\gamma)}$.
${ }^{19}$ After store S enters, $q_{r}^{B}=0$ and store R earns nothing.
type $j$ store S entry cost given in (11) and (19), and those thresholds $\Theta_{1}^{j}(k)$ and $\Theta_{2}^{j}(k)$ for store R's cost advantage $\alpha$ given in (16), (17), (21), and (22). Also, observe from (16), (17), (21), and (22) that these thresholds depend on those type-specific pre-committed donating proportions $\gamma^{A}$ and $\gamma^{B}$ along with those warm-glow effects $\beta^{A}\left(\gamma^{A}\right)$ and $\beta^{B}\left(\gamma^{B}\right)$.

These observations motivate us to compare store R's deterrence strategies when facing two different types of store S's entry for the case when both types of stores generate the same level of warm-glow effect $\beta^{A}\left(\gamma^{A}\right)=\beta^{B}\left(\gamma^{B}\right)$ even though the proportions $\gamma^{A} \neq \gamma^{B}$. Before we conduct the comparison analytically, we first use Figure 5 to numerically illustrate store R's optimal deterrence strategies against two types of store S by setting $c_{s}=0.8, \gamma^{A}=0.1$, and $\gamma^{B}=0.05$. While $\gamma^{A}>\gamma^{B}$ in this example, we consider a linear warm-glow effect. Specifically, we set $\beta^{A}\left(\gamma^{A}\right)=1+b^{A} \cdot \gamma^{A}=$ $1+5 \cdot \gamma^{A}=1.5$ and $\beta^{B}\left(\gamma^{B}\right)=1+b^{B} \cdot \gamma^{B}=1+10 \cdot \gamma^{B}=1.5$ so that $\beta^{A}\left(\gamma^{A}\right)=\beta^{B}\left(\gamma^{B}\right)$.


Figure 5 Store R's equilibrium deterrence strategy against both types of store S . (Setting: $c_{s}=0.8, \gamma^{A}=0.1$,

$$
\left.\gamma^{B}=0.05, \beta^{A}\left(\gamma^{A}\right)=1+5 \cdot \gamma^{A}=1.5, \text { and } \beta^{B}\left(\gamma^{B}\right)=1+10 \cdot \gamma^{B}=1.5 .\right)
$$

First, let us examine the entry conditions for both types of store S through $K_{1}^{j}$ and $K_{2}^{j}$; i.e., the thresholds with respect to the entry cost $k$ of a type $j$ store, $j=A, B$. Observe from Figures 5 (a) and (b) that, because $K_{1}^{B}<K_{1}^{A}$, the (blue) area $k>K_{1}^{B}$ is larger than the (blue) area $k>K_{1}^{A}$ so that a type (B) store will find it more difficult to enter the market relative to a type (A) store. More formally, when the entry cost $k \in\left[K_{1}^{B}, K_{1}^{A}\right)$ for both types of store S , a type (B) store S cannot afford to enter the market (because $k>K_{1}^{B}$ as stated in statement 1 of Proposition 4) even though a type (A) store S may be able to do so (because $k<K_{1}^{A}$ as stated in Proposition 2). Hence, from store S's vantage point of entry cost $k$, a type (B) store S is less affordable to enter the market than a type (A) store $S$.

Second, let us examine the thresholds $\Theta_{2}^{j}(k)$ in relation to store R's cost advantage $\alpha=c_{r} / c_{s}<1$. As shown in Figure $5(\mathrm{a})$ and (b), $\Theta_{2}^{B}(k)>\Theta_{2}^{A}(k)$, which implies that the (pink) area $\alpha>\Theta_{2}^{A}(k)$ is larger than the (pink) area $\alpha>\Theta_{2}^{B}(k)$. To be more precise, when store R's cost advantage
$\alpha \in\left(\Theta_{2}^{A}(k), \Theta_{2}^{B}(k)\right]$ for both types of store S , store R cannot deter the type (A) store S's entry (because $\alpha \geq \Theta_{2}^{A}(k)$ as stated in statements 2(b) and 3(c) of Proposition 2); however, store R can deter the type (B) store S's entry (because $\alpha<\Theta_{2}^{B}(k)$ as stated in statements 2(a), 3(a) and 3(b) of Proposition 4). Hence, from store R's vantage point of its cost advantage $\alpha$, it is relatively easier for store $R$ to deter a type (B) store than a type (A) store.

Third, observe from Figure 5(a) and (b) that $K_{2}^{B}<K_{2}^{A}$ and $\Theta_{1}^{A}(k)<\Theta_{1}^{B}(k)$. Thus, when $k<$ $K_{2}^{B}<K_{2}^{A}, \alpha \in\left(\Theta_{1}^{A}(k), \Theta_{2}^{A}(k)\right]$ and $\alpha<\Theta_{1}^{B}(k)$, store R would deter a type (B) store, but tolerate the entry of a type (A) store. In Figure 5, it can be observed that the (blue) area (in which store R will deter and/or store $S$ cannot enter) is larger for a type (B) store than that of a type (A) store, whereas the (yellow and pink) area (in which store $S$ can enter) is larger for a type (A) store $S$ than that of a type (B) store $S$. This implies that store $R$ tends to take a more aggressive deterrence strategy against the entry of a type (B) store $S$ than a type (A) store $S$.

To conclude, the numerical example shown in Figure 5 suggests that the incumbent store R is more aggressive in deterring a type (B) than a type (A) store $S$, and it is relatively easier for a type (A) store S to enter the market. However, is this result always true? Due to the complexity of the expressions for the thresholds mentioned earlier, it becomes analytically intractable to analyze a general case where the proportion $\gamma^{j}$ is endogenously determined by each type of store S . Therefore, in order to establish a hypothesis, we focus on analytically examining a benchmark case where both stores generate an identical warm-glow effect (i.e., $\beta^{A}\left(\gamma^{A}\right)=\beta^{B}\left(\gamma^{B}\right)$ ) in $\S 6$ (also note that the precommitted donating proportions of two types of store S can be different, i.e., $\gamma^{A} \neq \gamma^{B}$ ). We shall numerically examine this hypothesis by considering the case where $\gamma^{j}$ is endogenously determined by each type $j$ of store S in $\S 7$ so that the donation proportion along with the warm-glow effect generated by store S will become type-specific; i.e., $\gamma^{A} \neq \gamma^{B}$ and $\beta^{A}\left(\gamma^{A}\right) \neq \beta^{B}\left(\gamma^{B}\right)$.

### 6.1 Best-Response Pricing Strategies for Type (A) and Type (B) Store S

First, by comparing the results given in Propositions 1 and 3 together with Corollaries 1 and 2, we obtain Corollary 3 that compares entry conditions, best-response pricing strategies, and consumer demand for type (A) and type (B) store $S$.

Corollary 3. Suppose both types of store $S$ generate the same level of warm-glow effect (i.e., $\left.\beta^{A}\left(\gamma^{A}\right)=\beta^{B}\left(\gamma^{B}\right)\right)$. Then, given store $R$ 's price $p_{r}$, the entry conditions and best-response prices for both types of store $S$, and the corresponding consumer demand satisfy:
(a)Best-response pricing strategy comparison. A type ( $B$ ) store $S$ would charge a higher price than a type (A) $S$ store upon entering the market; i.e., $p_{s}^{B} \geq p_{s}^{A}$.
(b) Entry condition comparison. The entry condition for a type (A) store (i.e., $\tilde{\Pi}_{s}^{A} \geq k$ ) is less stringent than that of type (B) because $\tilde{\Pi}_{s}^{A} \geq \tilde{\Pi}_{s}^{B}$.
(c) Consumer demand comparison. The consumer demand for a type (A) store is higher than that of a type ( $B$ ) store; i.e., $q_{s}^{A} \geq q_{s}^{B}$. Accordingly, the consumer demand for store $R$ is lower upon a type (A) store $S$ 's entry than a type (B) store S's entry; i.e., $q_{r}^{A} \leq q_{r}^{B}$.

To motivate Corollary 3 , let us consider a general case where the warm-glow effect $\beta^{j}\left(\gamma^{j}\right)$ is store "type-specific" with $j=A, B$. Suppose the donating proportion is the same so that $\gamma^{A}=\gamma^{B}=\gamma$. Then a type (B) store may generate a higher warm-glow effect than a type (A) store (i.e., $\beta^{B}(\gamma)>$ $\left.\beta^{A}(\gamma)\right)$. This is because consumers have a stronger understanding of the direct relationship between their purchases and a type (B) store's donation that is based on a proportion of its revenue. ${ }^{20}$ In this context, the warm-glow effect for a type (B) store would be higher. Consequently, a type (B) store can afford to set a lower proportion that has $\gamma^{B}<\gamma^{A}$ in order to create the same level of warm-glow effect $\beta^{A}\left(\gamma^{A}\right)=\beta^{B}\left(\gamma^{B}\right)$. The numerical study illustrated in Figure 5 provides an example of the case where $0.05=\gamma^{B}<\gamma^{A}=0.1$ and $\beta^{B}\left(\gamma^{A}\right)=\beta^{B}\left(\gamma^{B}\right)=1.5$.

Corollary 3 examines the case when both types of store S generate the same level of warmglow effect $\beta^{A}\left(\gamma^{A}\right)=\beta^{B}\left(\gamma^{B}\right)$ (even though a type (B) store may donate a lower proportion, i.e., $\gamma^{B}<\gamma^{A}$ ). First, Corollary $3(\mathrm{a})$ implies that the best response price set by a type (B) store S is higher than a type (A) store $S$, given that both types of store $S$ generate the same level of warmglow effect. In particular, this holds true even if a type (B) store $S$ may contribute a much smaller proportion $\gamma^{B}$ compared to $\gamma^{A}$ from a type (A) store S . This is because a type (B) store donates its revenue (not profit as for a type (A) store). Consequently, a type (B) store $S$ has to charge a higher price than a type (A) store $S$ in order to cover its cost. Second, statement (b) states that it is easier for a type (A) store $S$ to enter the market than a type (B) store $S$ for the same entry cost $k$ because the gross profit $\tilde{\Pi}_{s}^{A} \geq \tilde{\Pi}_{s}^{B}$. Finally, Corollary 3(c) implies that, from store R's perspective, a type (A) store $S$ poses a higher threat than a type (B) store $S$ because the former can siphon off more demand from store R after entering the market than the latter. This is because a type (A) store $S$ can afford to charge a lower price than a type (B) store $S$ as shown in statement (a).

### 6.2 Store R's Deterrence Strategies Against a Type (A) and Type (B) Store S

In view of the differences in terms of entry conditions, best response price, and consumer demand across two types of stores as stated in Corollary 3, we now compare store R's deterrence strategies across different types of store $S$ in two ways. First, recall from Lemma 2 (Lemma 4) that store $R$ can deter a type (A) (type (B)) store S by setting a price $p_{r}<\tau^{A}\left(p_{r}<\tau^{B}\right)$. As such, by directly comparing $\tau^{A}$ and $\tau^{B}$, we can derive the relative difficulty of deterring different types of store S .

[^5]Second, because $\alpha=c_{r} / c_{s}<1$ captures store R's cost advantage over store S and $k$ captures store S's entry barrier, we can compare the store R's deterrence strategies across different types of store S by comparing the corresponding thresholds for $\alpha$ and $k$ as presented in Propositions 2 and 4. The following corollary compares store R's deterrence strategy across different types of store $S$.

Corollary 4. Suppose both types of store $S$ generate the same level of warm-glow effect (i.e., $\beta^{A}\left(\gamma^{A}\right)=\beta^{B}\left(\gamma^{B}\right)$ ). Then store $R$ 's deterrence strategies across different types of store $S$ satisfy:
(a) The price deterrence thresholds. The price deterrence thresholds for store $R$ 's retail price against two types of store $S$ satisfy $\tau^{A}<\tau^{B}$.
(b) The cost deterrence thresholds. Store R's deterrence strategies as stated in Propositions 2 and 4 hinge on whether $k$ lies within a certain region and whether $\alpha$ is above or below certain thresholds: $K_{i}^{j}$ and $\Theta_{i}^{j}(k), i \in\{1,2\}$ and $j \in\{A, B\}$. Specifically, these thresholds satisfy: $K_{1}^{A}>$ $K_{1}^{B}, K_{2}^{A}>K_{2}^{B}, \Theta_{1}^{A}(k)<\Theta_{1}^{B}(k)$, and $\Theta_{2}^{A}(k)<\Theta_{2}^{B}(k)$.

Corollary 4 implies that, given that both types of store $S$ generate the same level of warm-glow effect (i.e., $\beta^{A}\left(\gamma^{A}\right)=\beta^{B}\left(\gamma^{B}\right)$ ), store R is more likely to deter a type (B) store S than a type (A) store S , even though a type (B) store S may donate a lower proportion $\gamma^{B}$ than $\gamma^{A}$. Specifically, Corollary 4(a) indicates that, because $\tau^{A}<\tau^{B}$, the condition for store R to deter a type (A) store S (i.e., $p_{r}<\tau^{A}$ ) is more "stringent" than that of type (B) (i.e., $p_{r}<\tau^{B}$ ). Hence, relatively speaking, it is more affordable for store R to deter a type (B) store S without setting a much lower price $p_{r}$. Next, Corollary 4(b) demonstrates the same results as depicted in Figure 5.

To summarize, the analysis of the benchmark case, where two types of store $S$ generates the same level of warm-glow effect (i.e., $\beta^{A}\left(\gamma^{A}\right)=\beta^{B}\left(\gamma^{B}\right)$ ), indicates that the incumbent store R is more likely to deter a type (B) store $S$ than a type (A) store $S$. The question remains whether this hypothesis holds true when $\gamma$ is determined endogenously by each type of store S . We will address this through numerical examination in $\$ 7$.

## 7 Endogenously Determined Donating Proportion $\gamma^{j}$ for store type $j=A, B$

We now expand our analysis to the case when the donating proportion is "endogenously determined" by a type (A) (or (B)) store S . We consider a similar decision sequence described in §3.1. In period 1, store R chooses its price $p_{r}$ to either deter or tolerate store S's entry; and if store R chooses to tolerate store S's entry, then in period 2 a type $j$ store $S$ determines its donating proportion $\gamma^{j}$ along with its price $p_{s}^{j}$ to compete with store R , where $j=A, B$. Since the market entry game is not analytically solvable when $\gamma^{j}$ is endogenously determined, we numerically search for the optimal $\gamma^{j}$ in the range of $(0,1)$ that maximizes the profit of a type $j$ store S . To capture the warm-glow effect $\beta^{j}\left(\gamma^{j}\right) \geq 1$ is an increasing function of $\gamma^{j}$ (a decision variable for a type $j$ store S ), we shall assume:

$$
\begin{equation*}
\beta^{j}\left(\gamma^{j}\right)=1+b^{j} \cdot\left(\gamma^{j}\right)^{t}, j \in\{A, B\}, \tag{23}
\end{equation*}
$$

where the parameters $t>0{ }^{21}$ and $b^{j}>0$. In this section, we focus on the scenario where $t=1$ so that the warm-glow effect function given in (23) takes on a linear form. In the Online Appendix B, we present our results for the case when $t \neq 1$ so that $\beta^{j}\left(\gamma^{j}\right)$ is a non-linear function of $\gamma^{j}$. Overall, we find similar structural results as presented earlier in $\S 4$, $\S 5$, and $\S 6$.

### 7.1 Store R's Deterrence Strategies Against Both Types Store S.

Because the key focus of the paper is to analyze store R's deterrence strategies in response to store S's entry, we begin by presenting our numerical results related to store R's optimal deterrence strategy for the case when $\gamma$ is endogenously determined by each type of store S . Figure 6 and 7 depict store R's optimal deterrence strategies against a type (A) and a type (B) store $S$, respectively.

First, recall from Proposition 2 and 4 that the optimal deterrence strategy of store R against store S depends on both store S 's entry cost $k$ and store R 's cost advantage $\alpha$. Observe that Figures 6 and 7 resemble Figure 3, indicating that the optimal deterrence strategies of store R against store S for the case when the donating proportion $\gamma$ is endogenously determined by store S , possess a similar structure as stated in Proposition 2 and 4 for the base model when $\gamma$ is exogenously given.


(a) Store R's deterrence strategy against a type (A) (b) Store R's deterrence strategy against a type (A) store S with $\beta^{A}\left(\gamma^{A}\right)=1+3 \cdot \gamma^{A} \quad$ store S with $\beta^{A}\left(\gamma^{A}\right)=1+8 \cdot \gamma^{A}$
Figure 6 Store R's equilibrium deterrence strategy against a type (A) store S. Setting: $c_{s}=0.7$.

Next, observe from Figure 6 that, as the warm-glow factor for a type (A) store S, denoted as $b^{A}$, increases from 3 (Figure $6(\mathrm{a})$ ) to 8 (Figure $6(\mathrm{~b})$ ), consumers can derive more utility by shopping at a type (A) store $S$, making it difficult for store $R$ to deter. For this reason, store $R$ is more

[^6]

(a) Store R's deterrence strategy against a type (B) (b) Store R's deterrence strategy against a type (B) store S with $\beta^{B}\left(\gamma^{B}\right)=1+3 \cdot \gamma^{B} \quad$ store S with $\beta^{B}\left(\gamma^{B}\right)=1+8 \cdot \gamma^{B}$

Figure 7 Store R's equilibrium deterrence strategy against a type (B) store S. Setting: $c_{s}=0.7$.
likely to tolerate its entry. This is evident as the yellow "tolerance" region becomes larger in Figure 6(b). This line of logic continues to hold for a type (B) store S. Specifically, when the warm-glow factor for a type (B) store $S$, denoted as $b^{B}$, increases from 3 (Figure 7(a)) to 8 (Figure 7(b)), it becomes easier for a type (B) store $S$ to enter the market. This can be seen from the larger yellow "tolerance" region in Figure 7(b) compared to Figure 7(a).

Finally, observe from Figure 6(a) and Figure 7(b) that, when the warm-glow factor $8=b^{B}>$ $b^{A}=3$ (so that a type (B) store S can generate a stronger warm-glow effect), it can be seen that the sizes of both the yellow and the pink regions (i.e., regions within which a store S can enter) are larger in Figure 6(a) than in Figure 7(b). However, the size of the "deterrence" region, along with the region where store S can never enter the market (colored in blue), is smaller in Figure 6(a). It reveals that it is more likely for a type (A) store $S$ to enter the market than a type (B) store $S$ even when a type (B) store may generate a stronger warm-glow effect.

Remark 1. In the case when the donating proportion is endogenously determined by each type of store S , the incumbent store R's optimal deterrence strategy against store S yields the same structural results as when the donation proportion is exogenously given. Moreover, as the warm-glow factor increases, the incumbent store $R$ becomes less aggressive in deterring store S . Additionally, the incumbent store $R$ is generally more likely to deter a type (B) store $S$ (that donates revenue) rather than a type (A) store $S$ (that donates profit) unless a type (B) store $S$ can generate a significantly stronger warm-glow effect than a type (A) store $S$.

### 7.2 Equilibrium Results Analysis.

Next, we shall numerically analyze how the parameters would affect the optimal $\gamma^{j}, j \in\{A, B\}$ set by two types of store S along with other corresponding equilibrium outcomes.

### 7.2.1 Impact of store S's entry cost $k$ on the equilibrium results

First, we examine the impact of store S's entry cost $k$ by setting $c_{s}=0.7, \alpha=0.5$ and $b^{A}=b^{B}=3$. Observe from Figure 8(a) that a type (A) store S can enter the market more easily (when $k<0.591$ ) than a type (B) store $S$ (who can only enter when $k<0.026$ ). Also, observe from Figure 8(a) and (b) that, when store $S$ can enter the market, as the entry cost $k$ increases, a type (A) store S will increase its optimal donating proportion $\gamma^{A}$ to boost consumer utility and increase its price $p_{s}^{A}$ to offset the higher entry cost $k$. Interestingly, this behavior is not present for a type (B) store S . Specifically, the optimal $\gamma^{B}$ and $p_{s}^{B}$ set by a type (B) store S remain unchanged regardless of $k$. This is because a type (A) store S deducts its entry cost $k$ from its profit before donating, whereas a type (B) store S donates a portion of its revenue, which is independent of $k$.


Figure 8 Equilibrium results when $k$ varies. Setting: $c_{s}=0.7, \alpha=0.5, \beta^{j}\left(\gamma^{j}\right)=1+3 \cdot \gamma^{j}$ with $j \in\{A, B\}$.
Figure 8(c) and (d) depict the corresponding profits of store S and store R , respectively. It is intuitive that both types of store S's optimal profit will decrease when its entry cost $k$ increases. However, the impact of $k$ on the incumbent store R's profit is more nuanced. First, let us examine store R's profit in the face of the entry threat from a type (A) store S. Interestingly, when $k<0.591$ so that store $R$ chooses to tolerate the entry of a type (A) store $S$, store R's profit (in red) decreases
slightly in $k$. This is because as $k$ increases, a type (A) store S also increases its donating proportion $\gamma^{A}$, thus by tolerating its entry, store R needs to lower its price further to recapture market share, thereby squeezing its profit. However, when $0.591<k<K_{1}^{A}=0.681$ so that store R chooses to deter a type (A) store's entry, which will boost store R's profit as $k$ increases. This is because a higher entry cost makes it easier for store R to deter store S . Finally, when $k>K_{1}^{A}=0.681$ so that store S can never enter the market, store R's profit remains unaffected by $k$. Next, in the face of the potential entry of a type (B) store S , when store R chooses to tolerate its entry when $k<0.026$, store R's optimal profit will not be affected by the entry cost $k$. This is because a type (B) store S's optimal $\gamma^{B}$ and $p_{s}^{B}$ both remain unchanged when $k$ as shown in Figure 8(a) and (b). Also, similarly to a type (A) store S , when $0.026<k<K_{1}^{B}=0.066$ so that store R chooses to deter the entry of a type (B) store S, store R's profit increases with the entry cost $k$. When $k>K_{1}^{B}=0.066$ so that store $S$ can never enter the market, store R's profit will remain unchanged.

Figure 8 also indicates that when $b^{A}=b^{B}$, upon entry, a type (A) store S set a higher donating proportion $\gamma^{A}$ along with a higher price $p_{s}^{A}$ compared to a type (B) store S (as shown in Figures $8(\mathrm{a})$ and (b)). Additionally, upon entry, a type(A) store S attains a higher profit $\Pi_{s}^{A}$ (Figure 8(c)), resulting in a more significant squeeze on store R's profit (Figure 8(d)).

### 7.2.2 Impact of store $\mathbf{R}$ 's cost advantage $\alpha$ on the equilibrium results

Next, we numerically examine the impact of store R's cost advantage $\alpha \equiv \frac{c_{r}}{c_{s}}<1$ on the equilibrium results for the case when $\gamma$ is endogenously determined by store S . We set $c_{s}=0.7, k=0.03$ and $b^{A}=b^{B}=3$. Recall that $\alpha \equiv \frac{c_{r}}{c_{s}}$ so that as $\alpha$ increases, store R's cost advantage over store S is lower. Observe from Figure 6(a) and 7(a) that when $k=0.03$, a type (A) store S can always enter the market, while whether a type (B) store S can enter or not depends on $\alpha$. Observe from Figure 9 (a) that, when $\alpha>0.572$, a type (B) store S can enter the market. Moreover, Figure 6(a) and 7(a) also show that when $\alpha$ is very large so that store R 's cost advantage is low, not only can store S enter the market, but it can also drive store R out of the market. This is also supported by Figure $9(\mathrm{~d})$ that when $\alpha>0.780$, store R's profit becomes 0 upon a type (A) store S's entry; and when $\alpha>0.977$, store R will be squeezed out upon a type (B) store S's entry.

Observe from Figure 9(a) that, upon entry, both types of store S will reduce their donating proportion $\gamma^{j}$ as $\alpha$ increases. This is due to the fact that store S can enter the market even with a lower warm-glow effect when store R's cost advantage is sufficiently low (i.e., when $\alpha$ is high). Therefore, as $\alpha$ increases, both types of store S can afford to lower its optimal donating proportion in order to maximize its profit. Additionally, when store R's cost advantage decreases (i.e., $\alpha$ increases), a type (A) store $S$ can also afford to charge a higher price, whereas a type (B) store $S$ will lower its price upon entry. Hence, the change in store S's optimal price $p_{s}^{j}$ with respect to $\alpha$


Figure 9 Equilibrium results when $\alpha$ varies. Setting: $c_{s}=0.7, k=0.03, \beta^{j}\left(\gamma^{j}\right)=1+3 \cdot \gamma^{j}$ with $j \in\{A, B\}$.
(whether increasing or decreasing) is type-specific. Finally, we observe from Figure 9(c) that the optimal profits of both types of store S increase with $\alpha$. Consequently, store R's profit decreases as $\alpha$ increases due to lower cost advantage (Figure 9(d)).

### 7.2.3 Impact of relative warm-glow factor $\frac{b^{B}}{b^{A}}$ on the equilibrium results

We now examine the case where the warm-glow factors $b^{A}$ and $b^{B}$ are store "type-specific". As before, we set $c_{s}=0.7, k=0.03, \alpha=0.5$, and fix $b^{A}=3$ so that $\beta^{A}\left(\gamma^{A}\right)=1+3 \cdot \gamma^{A}$. In this case, store R chooses to tolerate a type (A) store S's entry, which can be observed from Figure 6(a). Here, we vary $b^{B}$ so that the corresponding ratio $\frac{b^{B}}{b^{A}}$ can vary between 1 to 4 . In doing so, we can examine the impact of the warm-glow factor ratio $\frac{b^{B}}{b^{A}}$ on the equilibrium results. Observe from Figure 10 that, only when $\frac{b^{B}}{b^{A}}>1.035$ can a type (B) store $S$ enter the market.

From Figure 10(a), observe that the optimal $\gamma^{B}$ set by a type (B) store S increases with the warm-glow factor $b^{B}$ (i.e., as $\frac{b^{B}}{b^{A}}=\frac{b^{B}}{3}$ increases). Interestingly, even when the ratio $\frac{b^{B}}{b^{A}}$ becomes very high so that $b^{B}$ is much higher than $b^{A}=3$, the optimal $\gamma^{B}$ remains lower than the optimal $\gamma^{A}$. This finding suggests that, as the warm-glow factor $b^{B}$ increases, although a type (B) store S will increase its donation proportion, it still cannot afford to set $\gamma^{B}$ as large as $\gamma^{A}$ because the

(a) Optimal donation proportion $\gamma^{j}$ when $b^{B}\left(\frac{b^{B}}{b^{A}}\right)$ varies

(b) Optimal absolute donation (b) Optimal absolute
amount when $b^{B}\left(\frac{b^{B}}{b^{A}}\right)$ varies

(c) Optimal $p_{s}^{j}$ when $b^{B}\left(\frac{b^{B}}{b^{A}}\right)$ varies

(d) $\Pi_{s}^{j}$ in equilibrium when $b^{B}\left(\frac{b^{B}}{b^{A}}\right)$ varies

(e) $\Pi_{r}^{j}$ in equilibrium when $b^{B}\left(\frac{b^{B}}{b^{A}}\right)$ varies

Figure 10 Equilibrium results when $b^{B}\left(\frac{b^{B}}{b^{A}}\right)$ varies. Setting: $c_{s}=0.7, k=0.03, \alpha=0.5, b^{A}=3$ so that $\beta^{A}(\gamma)=1+3 \cdot \gamma$.
donation is based on revenue and its profitability needs to be ensured. This observation is consistent with practical implications. However, upon examining the absolute donation amount, Figure 10(b) suggests that either a type (A) or a type (B) store $S$ would donate a greater amount. Next, it can be observed from Figure 10 (c) that the optimal $p_{s}^{B}$ also increases with the warm-glow factor $b^{B}$. Furthermore, as $b^{B}$ increases, a type (B) store S would charge a higher $p_{s}^{B}$ compared to $p_{s}^{A}$ in order to ensure profitability. Figure $10(\mathrm{~d})$ shows that when the warm-glow factor $b^{B}$ increases, a type (B) store can also earn a higher profit, which can be even higher than a type (A) store's profit. Nevertheless, it is worth noting that, even when $b^{B}>b^{A}$, a type ( B ) store S may still get a lower profit than a type (A) store $S$ unless it can generate a much stronger warm-glow effect than a type (A) store S. Figure 10 (e) presents the profit of the incumbent store R, which decreases with $b^{B}$.

## 8 Conclusion

In recent years, there is a strong shift in consumer preferences towards social responsibility, and this shift creates a suitable environment for new socially responsible retailers to enter the market. This observation motivated us to study entry conditions of a commonly observed class of socially responsible retailers that pre-commit to donating a certain proportion of their profits (type (A))
or revenues (type (B)). Our equilibrium analysis revealed that the incumbent retailer's deterrence strategy depends on its cost advantage (captured by $\alpha$ ) and the social retailer's entry cost (captured by $k$ ). An interesting finding is that even when the incumbent retailer has the power to deter the entry of the social retailer, it may still choose to tolerate its entry. We also compare the two types of social retailers. We find that a type (A) social retailer poses a higher entry threat for the incumbent than type (B) social retailer, yet interestingly, the incumbent is more aggressive to deter the entry of type (B) social retailer. Thus, it is easier for a type (A) social retailer to enter the market unless a type (B) social retailer can generate a sufficiently higher warm-glow effect than a type (A) store. This managerial insight may guide entrepreneurs who aim to establish social retailers to pre-commit to donating a certain proportion of their profits rather than revenues.

Our paper is the first attempt to understand the market dynamics between an incumbent forprofit retailer and a common class of socially responsible retailers. There are several avenues for further research. For instance, we have examined a common class of social retailers, but it is of interest to compare very different classes of social retailers (e.g., one type donates one unit to charity when a consumer buys one unit, and the other type donates the revenue to charity). Also, there are other classes of social retailers such as food cooperatives. Studying the entry of such cooperatives would be an interesting research avenue to pursue.

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## Online Appendix

## Online Appendix A: Proofs

Proof of Lemma 1 and Proposition 1. We first analyze the best-response pricing strategy $p_{s}$ for any given $p_{r}$ in the event that store S can enter the market. Based on the consumer demand for store S as given by (5), store S can either set (1) $p_{s} \leq \beta^{A}(\gamma) \cdot p_{r}$ or (2) $p_{s}>\beta^{A}(\gamma) \cdot p_{r}$.
Case 1: $p_{s} \leq \beta^{A}(\gamma) \cdot p_{r}$. In this case, after store $S$ enters the market, store S's demand $q_{s}=1-\frac{p_{s}}{\beta^{A}(\gamma)}$. As such, store S's problem given by (6) can be written as:

$$
\begin{equation*}
\max _{c_{s} \leq p_{s} \leq \beta^{A}(\gamma) \cdot p_{r}}(1-\gamma) \cdot\left[\left(p_{s}-c_{s}\right) \cdot\left(1-\frac{p_{s}}{\beta^{A}(\gamma)}\right)-k\right] . \tag{EC.1}
\end{equation*}
$$

When $p_{r}<\frac{c_{s}}{\beta^{A}(\gamma)}$, (EC.1) is infeasible and it is impossible for store S to set $p_{s} \leq \beta^{A}(\gamma) \cdot p_{r}$. When $p_{r} \geq \frac{c_{s}}{\beta^{A}(\gamma)}$, according to Store S's first-order condition, we can obtain the extreme point $p_{s}^{*}=$ $\frac{\beta^{A}(\gamma)+c_{s}}{2}$. By considering the boundary cases, we obtain: (1) when $p_{r} \in\left[\frac{c_{s}}{\beta^{A}(\gamma)}, \frac{\beta^{A}(\gamma)+c_{s}}{2 \beta^{A}(\gamma)}\right)$, the optimal solution to (EC.1) is $p_{s 1}^{A}=\beta^{A}(\gamma) \cdot p_{r}$; however, (2) when $p_{r} \geq \frac{\beta^{A}(\gamma)+c_{s}}{2 \beta^{A}(\gamma)}$, the optimal solution to (EC.1) is $p_{s 1}^{A}=\frac{\beta^{A}(\gamma)+c_{s}}{2}$.
Case 2: $p_{s} \in\left[\beta^{A}(\gamma) \cdot p_{r}, \beta^{A}(\gamma)-1+p_{r}\right)$. In this case, after store S enters the market, the demand for store $\mathrm{S} q_{s}=1-\frac{p_{s}-p_{r}}{\beta^{A}(\gamma)-1}$. As such, store S's problem given by (6) can be written as:

$$
\begin{equation*}
\max _{p_{s} \geq \max \left\{\beta^{A}(\gamma) \cdot p_{r}, c_{s}\right\}}(1-\gamma) \cdot\left[\left(p_{s}-c_{s}\right) \cdot\left(1-\frac{p_{s}-p_{r}}{\beta^{A}(\gamma)-1}\right)-k\right] \tag{EC.2}
\end{equation*}
$$

By checking the first-order condition, we obtain the extreme point $p_{s}^{*}=\frac{\beta^{A}(\gamma)-1+p_{r}+c_{s}}{2}$. By considering the boundary cases, we obtain: (1) when $p_{r} \leq c_{s}+1-\beta^{A}(\gamma)$, (EC.2) is infeasible because $p_{s} \geq$ $c_{s} \geq p_{r}+\beta^{A}(\gamma)-1$; (2) when $p_{r} \in\left(c_{s}-\beta^{A}(\gamma)+1, \frac{\beta^{A}(\gamma)-1+c_{s}}{2 \beta^{A}(\gamma)-1}\right)$, then the optimal solution to (EC.2) is $p_{s 2}^{A}=\frac{\beta^{A}(\gamma)-1+p_{r}+c_{s}}{2}$; (3) when $p_{r} \geq \frac{\beta^{A}(\gamma)-1+c_{s}}{2 \beta^{A}(\gamma)-1}$, the optimal solution to (EC.2) is $p_{s 2}^{A}=\beta^{A}(\gamma) \cdot p_{r}$. Case 3: $p_{s} \geq \beta^{A}(\gamma)-1+p_{r}$. In this case, as $q_{s}=0$, store S's profit $\Pi_{s}^{A}=-(1-\gamma) k \leq 0$.

We obtain store S's best-response pricing strategy $p_{s}^{A}$ as stated in Proposition 1 based on the optimal solution of $p_{s}$ under case 1 and 2 and by comparing the optimal profit of store S by either setting $p_{s} \leq \beta^{A}(\gamma) \cdot p_{r}$ or $p_{s} \geq \beta^{A}(\gamma) \cdot p_{r}$. As such, we can also obtain store S's corresponding demand $q_{s}^{A}$ and profit $\Pi_{s}^{A}$ via substitution. Next, by considering $\Pi_{s}^{A} \geq 0$, we can obtain the entry condition as stated in Proposition 1.

Proof of Corollary 1. By substituting the best-response price $p_{s}^{A}$ as given in Proposition 1 into the consumer demand for store R and store S as given by (4) and (5), we can obtain the corresponding $q_{r}^{A}$ and $q_{s}^{A}$ together with the comparative statistics as given in Corollary 1.

Proof of Lemma 2. Recall from Proposition 1 that store S's entry condition is $k \leq \tilde{\Pi}_{s}^{A}\left(p_{r}\right)$, where $\tilde{\Pi}_{s}^{A}$ is as given by (10) and is increasing in $p_{r}$. As such, by solving $k=\tilde{\Pi}_{s}^{A}\left(p_{r}\right)$, we can obtain the solution $p_{r}=\tau^{A}$, where $\tau^{A}$ as given by (12).

Proof of Proposition 2. From Proposition 1, we know that $\tilde{\Pi}_{s}^{A} \leq \frac{\left[\beta^{A}(\gamma)-c_{s}\right]^{2}}{4 \beta^{A}(\gamma)}=K_{1}^{A}$ so that when $k>K_{1}^{A}$, store S can never enter the market, which is as stated in first statement of Proposition 2. As such, we focus on the deterrence strategy of store R for the case when $k \leq K_{1}^{A}$ so that store S can have a chance to enter the market. In particular, we will consider the case when (1) $k \in\left(K_{2}^{A}, K_{1}^{A}\right]$ and (2) $k \leq K_{2}^{A}$.
Case 1: $k \in\left(K_{2}^{A}, K_{1}^{A}\right]$. According to Lemma 2, we can obtain $\tau^{A}=\frac{c_{s}+\beta^{A}(\gamma)-\sqrt{\left[\beta^{A}(\gamma)-c_{s}\right]^{2}-4 k \beta^{A}(\gamma)}}{2 \beta^{A}(\gamma)}>$ $\frac{\beta^{A}(\gamma)-1+c_{s}}{2 \beta^{A}(\gamma)-1}$ so that store R can deter store S's entry by setting $p_{r}<\tau^{A}$, while tolerate store S's entry by setting $p_{r} \geq \tau^{A}$. First, as store R needs to charge $p_{r} \geq c_{r}$, then if $\tau^{A}=$ $\frac{c_{s}+\beta^{A}(\gamma)-\sqrt{\left(\beta^{A}(\gamma)-c_{s}\right)^{2}-4 k \cdot \beta^{A}(\gamma)}}{2 \beta^{A}(\gamma)} \leq c_{r}$, store R cannot deter the entry of store S. If $c_{r}>\tau^{A}$, then store R can either choose to deter or tolerate store S's entry. Recall from corollary 1 that if $p_{r} \geq \frac{\beta^{A}(\gamma)-1+c_{s}}{2 \beta^{A}(\gamma)-1}$, then after store S enters, the consumer demand for store R in equilibrium $q_{r}^{A}=0$. Hence, if store R chooses to tolerate store S's entry by setting $p_{r} \geq \tau^{A}>\frac{\beta^{A}(\gamma)-1+c_{s}}{2 \beta^{A}(\gamma)-1}$, store R's profit will be zero. Hence, it is optimal for store R to deter store S 's entry when $c_{r}<\tau^{A}=\frac{c_{s}+\beta^{A}(\gamma)-\sqrt{\left(\beta^{A}(\gamma)-c_{s}\right)^{2}-4 k \beta^{A}(\gamma)}}{2 \beta^{A}(\gamma)}$. Hence, store R's deterrence problem given by (13) can be rewritten as $\Pi_{r}^{d, A}=\max _{p_{r} \in\left[c_{r}, \tau^{A}\right)} \Pi_{r}=$ $\left(p_{r}-c_{r}\right)\left(1-p_{r}\right)$. By considering the first-order condition together with the boundary cases, we obtain that store R's equilibrium deterrence strategy together with the equilibrium price as given by the second statement of Proposition 2.
Case 2: $k \in\left(0, K_{2}^{A}\right]$. Based on Lemma 2, $\tau^{A}=\sqrt{4 k \cdot\left(\beta^{A}(\gamma)-1\right)}+c_{s}+1-\beta^{A}(\gamma)$ when $k \leq K_{2}^{A}$. To analyze store R's optimal price $p_{r}^{d, A}$ when it chooses to deter store S's entry, we rewrite (13) as:

$$
\begin{equation*}
\Pi_{r}^{d, A}=\sup _{c_{r} \leq p_{r}<\tau^{A}} \Pi_{r}=\left(p_{r}-c_{r}\right) \cdot\left(1-p_{r}\right) \tag{EC.3}
\end{equation*}
$$

By considering the first-order condition together with the boundary cases, we obtain store R's optimal price $p_{r}^{d, A}$ that deters store S's entry as follows:

- If $c_{r}<2 c_{s}+1-2 \beta^{A}(\gamma)+4 \sqrt{k \cdot\left(\beta^{A}(\gamma)-1\right)}$, then it is optimal for store R to set $p_{r}^{d}=\frac{1+c_{r}}{2}$;
- If $c_{r} \in\left[2 c_{s}+1-2 \beta^{A}(\gamma)+4 \sqrt{k \cdot\left(\beta^{A}(\gamma)-1\right)}, \tau^{A}\right)$, then it is optimal for store R to set $p_{r}^{d, A}=$ $\tau^{A}-\epsilon$, where $\epsilon \rightarrow 0$;
- If $c_{r} \geq \tau^{A}$, then (EC.3) is infeasible and store R cannot deter store S's entry.

Next, to analyze store R's optimal price $p_{r}^{t}$ when it chooses to tolerate store S's entry, we rewrite (14) as:

$$
\begin{equation*}
\Pi_{r}^{t, A}=\max _{p_{r} \geq \max \left\{c_{r}, \tau^{A}\right\}}\left(p_{r}-c_{r}\right) \cdot q_{r}^{A} \tag{EC.4}
\end{equation*}
$$

where $q_{r}^{A}$ is as given in (14). By considering the first-order condition together with the boundary cases, we obtain store R's optimal price $p_{r}^{t, A}$ that tolerates store S's entry as follows:

- if $c_{r} \geq \frac{\beta^{A}(\gamma)-1+c_{s}}{2 \beta^{A}(\gamma)-1}$, then $p_{r}^{t, A}=c_{r}$ and according to Corollary 1, after store S enters, the consumer demand for store $\mathrm{R} q_{r}^{A}=0$;
- if $c_{r} \in\left(\frac{1}{2}\left[3+4 c_{s}+8 \sqrt{k \cdot\left(\beta^{A}(\gamma)-1\right)}-4 \beta^{A}(\gamma)+\frac{1-2 c_{s}}{2 \beta^{A}(\gamma)-1}\right], \frac{\beta^{A}(\gamma)-1+c_{s}}{2 \beta^{A}(\gamma)-1}\right)$, then it is optimal for store R to set $p_{r}^{t, A}=\frac{\beta^{A}(\gamma)-1+c_{s}+\left(2 \beta^{A}(\gamma)-1\right) c_{r}}{2\left(2 \beta^{A}(\gamma)-1\right)}$;
- if $c_{r} \leq \frac{1}{2}\left[3+4 c_{s}+8 \sqrt{k \cdot\left(\beta^{A}(\gamma)-1\right)}-4 \beta^{A}(\gamma)+\frac{1-2 c_{s}}{2 \beta^{A}(\gamma)-1}\right]$, then it is optimal for store R to set $p_{r}^{t, A}=\tau^{A}$.
Hence, we can obtain the corresponding $\Pi_{r}^{d, A}$ and $\Pi_{r}^{t, A}$ via substitution. By comparing $\Pi_{r}^{d, A}$ and $\Pi_{r}^{t, A}$, we can solve (15) as follows. First, if $c_{r} \geq \tau^{A}$ (and $\tau^{A}<\frac{\beta^{A}(\gamma)-1+c_{s}}{2 \beta^{A}(\gamma)-1}$, store R cannot deter store S's entry, so $p_{r}^{A}=p_{r}^{t, A}$. Second, if $c_{r} \leq \frac{1}{2}\left[3+4 c_{s}+8 \sqrt{k \cdot\left(\beta^{A}(\gamma)-1\right)}-4 \beta^{A}(\gamma)+\frac{1-2 c_{s}}{2 \beta^{A}(\gamma)-1}\right]$ (and $\frac{1}{2}\left[3+4 c_{s}+8 \sqrt{k \cdot\left(\beta^{A}(\gamma)-1\right)}-4 \beta^{A}(\gamma)+\frac{1-2 c_{s}}{2 \beta^{A}(\gamma)-1}>2 c_{s}+1-2 \beta^{A}(\gamma)+4 \sqrt{k \cdot\left(\beta^{A}(\gamma)-1\right)}\right.$ ), store R has to set $p_{r}^{t, A}=\tau^{A}$ to tolerate store S 's entry; however, by setting a slightly lower price $\tau^{A}-\epsilon$, store R can deter store S and get a higher consumer demand. Hence, in this case $p_{r}^{A}=p_{r}^{d, A}$. Finally, when $c_{r} \in\left(\frac{1}{2}\left[3+4 c_{s}+8 \sqrt{k \cdot\left(\beta^{A}(\gamma)-1\right)}-4 \beta^{A}(\gamma)+\frac{1-2 c_{s}}{2 \beta^{A}(\gamma)-1}\right], \tau^{A}\right)$, we compare store R's two strategies: tolerating store S by setting $p_{r}^{t, A}=\frac{\beta^{A}(\gamma)-1+c_{s}+\left(2 \beta^{A}(\gamma)-1\right) c_{r}}{2\left(2 \beta^{A}(\gamma)-1\right)}$ or deterring store S by setting $p_{r}^{d, A}=\tau^{A}-\epsilon$, and we obtain the threshold for $c_{r}$ as $\theta^{A} \equiv$ $\frac{c_{s}\left(4 \beta^{A}(\gamma)-3\right)+\left(\beta^{A}(\gamma)-1\right)\left(1-4 \beta^{A}(\gamma)+8 \sqrt{k \cdot\left(\beta^{A}(\gamma)-1\right)}+4 \sqrt{k\left(\frac{\beta^{A}(\gamma)-c_{s}}{\sqrt{k \cdot\left(\beta^{A}(\gamma)-1\right)}}-2\right)}\right)}{2 \beta^{A}(\gamma)-1}$ . As such, in this case when $c_{r} \geq$ $\theta^{A}, \Pi_{r}^{t, A} \geq \Pi_{r}^{d, A}$ so that it is optimal for store R to tolerate store S 's entry by setting $p_{r}^{t, A}=$ $\frac{\beta^{A}(\gamma)-1+c_{s}+\left(2 \beta^{A}(\gamma)-1\right) c_{r}}{2\left(2 \beta^{A}(\gamma)-1\right)}$, while when $c_{r}<\theta^{A}, \Pi_{r}^{t, A}<\Pi_{r}^{d, A}$ so that it is optimal for store R to deter store S's entry by setting $p_{r}^{d, A}=\tau^{A}-\epsilon$. By also considering $c_{r}=\alpha c_{s}$ and rearranging the results, we obtain store R's equilibrium deterrence strategy when $k \in\left(0, K_{2}^{A}\right]$ as given in the third statement of Proposition 2.

Proof of Lemma 3 and Proposition 3. Recall from (6) and (7) that the objective function of a type (B) store S resembles that of a type (A) store S by replacing $c_{s}$ with $\frac{c_{s}}{1-\gamma}$ and replacing $k$ with $\frac{k}{1-\gamma}$. As such, by using the same approach as we used to prove Lemma 1 and Proposition 1, we can prove that the best-response pricing strategy of a type (B) store $S$ together with its entry condition is as given by Lemma 3 and Proposition 3. Also, by checking the first order derivative of $p_{s}^{B}$ with respect to $\gamma$, it is easy to verify that $p_{s}^{B}$ is increasing in $\gamma$.

Proof of Corollary 2. By substituting the best-response price $p_{s}^{B}$ as given in Proposition 3 into the consumer demand for store R and store S as given by (4) and (5), we can obtain the corresponding $q_{r}^{B}$ and $q_{s}^{B}$ together with the comparative statistics as given in Corollary 2.

Proof of Lemma 4. Recall from Proposition 3 that the entry condition for store S is $k \leq \tilde{\Pi}_{s}^{B}\left(p_{r}\right)$, where $\tilde{\Pi}_{s}^{B}$ is as given by (18) and is increasing in $p_{r}$. As such, by solving $k=\tilde{\Pi}_{s}^{B}\left(p_{r}\right)$, we can obtain the solution $p_{r}=\tau^{B}$, where $\tau^{B}$ is as given by (20).

Proof of Proposition 4. Armed with store S's entry condition and best-response pricing strategy as given by Proposition 3 and by using the same approach as shown in the proof of Proposition 2,
we can derive store R's equilibrium deterrence strategy against a type (B) store $S$ together with its equilibrium price $p_{r}$ as given by Proposition 4.

Proof of Corollary 3. We now consider the case when the warm-glow effect $\beta^{A}\left(\gamma^{A}\right)=\beta^{B}\left(\gamma^{B}\right)$. By denoting $\beta \equiv \beta^{A}\left(\gamma^{A}\right)=\beta^{B}\left(\gamma^{B}\right)$, we can re-write $p_{s}^{A}$ and $p_{s}^{B}$ as:

$$
p_{s}^{A}=\left\{\begin{array}{ll}
\frac{\beta-1+p_{r}+c_{s}}{2} & p_{r} \in\left(c_{s}+1-\beta, \frac{\beta-1+c_{s}}{2 \beta-1}\right) \\
\beta \cdot p_{r} & p_{r} \in\left[\frac{\beta-1+c_{s}}{2 \beta-1}, \frac{\beta+c_{s}}{2 \beta}\right] \\
\frac{\beta+c_{s}}{2} & p_{r}>\frac{\beta+c_{s}}{2 \beta}
\end{array}, p_{s}^{B}= \begin{cases}\frac{\beta-1+p_{r}+\frac{c_{s}}{1-\gamma^{B}}}{2} & p_{r} \in\left(\frac{c_{s}}{1-\gamma^{B}}+1-\beta, \frac{\beta-1+\frac{c_{s}}{1-\gamma^{B}}}{1-\frac{c_{s}}{2 \beta-1}}\right) \\
\beta \cdot p_{r} & p_{r} \in\left[\frac{\beta-1+\frac{c_{s}}{1-\gamma^{B}}}{\left.\beta+\frac{\beta+\frac{c_{s}}{1-\gamma^{B}}}{2 \beta}\right]}\right. \\
\frac{\beta+\frac{c_{s}}{1-\gamma^{B}}}{2} & p_{r}>\frac{\beta+\frac{2 \varepsilon_{s}}{1-\gamma^{B}}}{2 \beta}\end{cases}\right.
$$

As $p_{s}^{B}$ is increasing in $\gamma^{B}$, we can verify that $p_{s}^{B} \geq p_{s}^{A}$ (and $p_{s}^{B} \rightarrow p_{s}^{A}$ when $\gamma^{B} \rightarrow 0$ ). By rewriting $\tilde{\Pi}_{s}^{A}$ and $\tilde{\Pi}_{s}^{B}$, together with the consumer demand $q_{s}^{A}, q_{r}^{A}, q_{s}^{B}, q_{r}^{B}$ in a same manner as showed above, we can also easily verify that $\tilde{\Pi}_{s}^{A} \geq \tilde{\Pi}_{s}^{B}, q_{s}^{A} \geq q_{s}^{B}$ and $q_{r}^{A} \leq q_{r}^{B}$ (and $\tilde{\Pi}_{s}^{B} \rightarrow \tilde{\Pi}_{s}^{A}, q_{s}^{B} \rightarrow q_{s}^{A}, q_{r}^{B} \rightarrow q_{r}^{A}$ when $\gamma^{B} \rightarrow 0$ ).

Proof of Corollary 4. By denoting $\beta \equiv \beta^{A}\left(\gamma^{A}\right)=\beta^{B}\left(\gamma^{B}\right)$, we can rewrite $K_{1}^{A}$ and $K_{1}^{B}$ given in (11) and (19) as follows:

$$
K_{1}^{A}=\frac{\left(\beta-c_{s}\right)^{2}}{4 \beta}, K_{1}^{B}=\frac{\left[\beta\left(1-\gamma^{B}\right)-c_{s}\right]^{2}}{4 \beta\left(1-\gamma^{B}\right)}=\left(1-\gamma^{B}\right) \cdot \frac{\left[\beta-\frac{c_{s}}{1-\gamma^{B}}\right]^{2}}{4 \beta} .
$$

As $\beta>\frac{c_{s}}{1-\gamma^{B}}, K_{1}^{B}$ showed above is decreasing in $\gamma^{B}$. Therefore, it is easy to verify that $K_{1}^{B}<K_{1}^{A}$ (and $K_{1}^{B} \rightarrow K_{1}^{A}$ when $\gamma^{B} \rightarrow 0$ ). By using the same approach, we can also show that $\tau^{B}>\tau^{A}$, $\Theta_{i}^{B}>\Theta_{i}^{A}(i \in\{1,2\})$, and $K_{2}^{B}<K_{2}^{A}\left(\right.$ and $\tau^{B} \rightarrow \tau^{A}, \Theta_{i}^{B} \rightarrow \Theta_{i}^{A}, K_{2}^{B} \rightarrow K_{2}^{A}$ when $\left.\gamma^{B} \rightarrow 0\right)$.

## Online Appendix B. Endogenous $\gamma^{j}$ : Non-Linear Warm-Glow Function Form

Like $\S 7$, we consider the proportion $\gamma^{j}$ is endogenously determined by each type of store S . However, unlike $\S 7$, we examine the case when the warm-glow effect function $\beta^{j}\left(\gamma^{j}\right)$ with $j \in\{A, B\}$ is nonlinear as stated in (23). While we consider the case when $t=1$ in $\S 7$, we now consider the case when $t \neq 1$ to examine the robustness of our results. Specifically, we consider two settings: (1) $t=0.5$ so that $\beta^{j}\left(\gamma^{j}\right)=1+b^{j} \cdot \sqrt{\gamma^{j}}$; and (2) $t=2$ so that $\beta^{j}\left(\gamma^{j}\right)=1+b^{j} \cdot\left(\gamma^{j}\right)^{2}$.

## Setting (1): Increasing-concave warm-glow effect function: $\beta^{j}\left(\gamma^{j}\right)=1+b^{j} \cdot \sqrt{\gamma^{j}}$

By setting $t=0.5$, the warm-glow effect function $\beta^{j}\left(\gamma^{j}\right)=1+b^{j} \cdot \sqrt{\gamma^{j}}$ is increasing and concave. In our numerical analysis, we set $c_{s}=0.7$ and $b^{A}=b^{B}=3$. Figure EC. 1 illustrates store R's optimal deterrence strategies against both types of store S. It can be observed that Figure EC. 1 exhibits the same characteristics as shown in Figures 3 and 5 for the base model with exogenously pre-committed $\gamma^{j}$ and Figures 6 and 7 for the case with endogenously determined $\gamma^{j}$ and linear warm-glow effect function. Therefore, we can conclude that store R's optimal deterrence strategy against store S always possesses a similar structure as given in Propositions 2 and 4.


Figure EC. 1 Store R's equilibrium deterrence strategy against both types of store $S$ when the warm-glow effect function takes an increasing-concave form.. Setting: $c_{s}=0.7$.


Figure EC. 2 Equilibrium results when $k$ varies. Setting: $c_{s}=0.7, \alpha=0.5, \beta^{j}\left(\gamma^{j}\right)=1+3 \cdot \sqrt{\gamma^{j}}, j \in\{A, B\}$.

Next, we conduct our numerical analysis by setting $\alpha=0.5$. Figure EC. 2 illustrates the impact of store S's entry cost $k$ on the optimal $\gamma^{j}$ (Figure EC.2(a)) and $p_{s}^{j}$ (Figure EC.2(b)) set by both types of store S , as well as the profits of store S (Figure EC.2(c)) and store R (Figure EC.2(d)). It is worth noting that Figure EC. 2 resembles Figure 8, suggesting that the impact of store S's entry
cost $k$ is similar when the warm-glow effect function is concave compared to when it is linear: (1) Upon entry, both $\gamma^{A}$ and $p_{s}^{A}$ set by a type (A) store S increase with $k$, while $\gamma^{B}$ and $p_{s}^{B}$ of a type (B) store S are independent of $k$. (2) The profits of both types of store S decrease with $k$ upon entry. (3) By choosing to tolerate store S's entry, store R's profit decreases with $k$ when tolerating the entry of a type (A) store $S$, while it remains independent of $k$ when tolerating a type (B) store S's entry. However, by choosing to deter store S's entry, store R's profit increases with $k$ regardless of the type of store S . (4) In the case when $b^{A}=b^{B}$, a type ( B ) store S is more vulnerable than a type (A) store $S$. This can be observed from Figure EC.2, which shows that a type (B) store $S$ can enter the market only when $k<0.114$, while a type (A) store S can enter the market as long as $k<0.591$.


Figure EC. 3 Equilibrium results when $\alpha$ varies. Setting: $c_{s}=0.7, k=0.1, \beta^{j}\left(\gamma^{j}\right)=1+3 \cdot \sqrt{\gamma^{j}}, j \in\{A, B\}$.

Figure EC. 3 depicts the impact of store R's cost advantage $\alpha \equiv \frac{c_{r}}{c_{s}}$. We set $c_{s}=0.7, k=0.1$ and $b^{A}=b^{B}=3$, and $t=0.5$. Notably, Figure EC. 3 resembles Figure 9 presented in §7.2.2, indicating that the effect of $\alpha$ on the equilibrium results remains consistent regardless of whether the warmglow effect function is increasing-linear or increasing-concave. From Figure EC.3, we observe that
when $\beta^{j}\left(\gamma^{j}\right)=1+3 \cdot \sqrt{\gamma^{j}}$ and $k=0.1$, a type (A) store S can always enter the market, while a type B store S can only enter when $\alpha>0.380$. Moreover, store R's profit will be squeezed out by a type (A) store S when $\alpha>0.860$ and by a type (B) store S when $\alpha>0.773$. Similar to the case of a linear warm-glow function, as $\alpha$ increases, indicating a decrease in store R's cost advantage, both types of store S will reduce the donating proportion $\gamma^{j}$ upon entry when the warm-glow function takes an increasing-concave form. Next, it can be observed from EC.3(b) that $p_{s}^{A}$ set by a type (A) store S always increases with $\alpha$, which is consistent with the linear case shown in Figure 9(b). However, when $\alpha \in(0.380,0.773)$ so that store R chooses to tolerate the entry of a type (B) store $\mathrm{S}, p_{s}^{B}$ increases with $\alpha$, which is different from the linear case as shown in Figure 9 (b). This is because when the warm-glow function takes on an concave form, similar to a type (A) store S , a type (B) store $S$ can also afford to increase its price and get a higher profit margin when competing with store R. Finally, Figure EC.3(c) and (d) show that both types of store S will experience an increase in profits with $\alpha$, while store R's profit will decrease with $\alpha$.

Figures EC.1, EC.2, and EC. 3 depict a scenario where both types of store $S$ share the same warm-glow factor (i.e., $b^{A}=b^{B}=3$ ). Similar to the linear warm-glow effect scenario, it remains relatively easier for a type (A) store $S$ to enter the market compared to a type (B) store $S$ with the same warm-glow factor, even when the warm-glow effect function takes on an increasing-concave form. However, it is worth noting that with the same warm-glow factor $b^{B}=3$, the yellow tolerance region becomes larger in Figure EC.1(b) compared to Figure 7(a), indicating that, relative to the linear case, a type (B) store $S$ is less susceptible in the concave case. On the other hand, it is observed that with the same warm-glow factor $b^{A}=3$, Figures 6(a) and EC.1(a) are quite similar, suggesting that store $R$ tends to employ a similar deterrence strategy against a type (A) store $S$ regardless of the form of the warm-glow function.

Next, let's examine the scenario where the warm-glow factors $b^{A}$ and $b^{B}$ are store-specific. As before, we set $c_{s}=0.7, k=0.1, \alpha=0.5$, and fix $b^{A}=3$ so that $\beta^{A}\left(\gamma^{A}\right)=1+3 \cdot \sqrt{\gamma^{A}}$. In this case, based on Figure EC.1(a), store R will tolerate the entry of a type (A) store S. By varying $b^{B}$ from 1.5 to 13.5 , which results in the ratio $\frac{b^{B}}{b^{A}}$ ranging from 0.5 to 4.5 , we can examine the impact of the warm-glow factor ratio $\frac{b^{B}}{b^{A}}$ on the equilibrium outcomes. From Figure EC.4, we can observe that when $\frac{b^{B}}{b^{A}}>0.945$ (i.e., $b^{B}=2.835$ ), a type (B) store S can also enter the market. Recall from Figure 10 that given the warm-glow effect function takes a linear form, only when $\frac{b^{B}}{b^{A}}>1.570$ can a type (B) store $S$ enter the market. Therefore, it further supports the notion that a type (B) store S becomes less vulnerable when the warm-glow function takes an increasing-concave form. In addition to the aforementioned finding, Figure EC. 4 exhibits a similar pattern to Figure 10: (1) upon entry, the optimal donating proportion $\gamma^{B}$ and price $p_{s}^{B}$, along with the corresponding store S's profit $\Pi_{s}^{B}$, increase with the warm-glow factor $b^{B}$ (or the ratio $\frac{b^{B}}{b^{A}}$ ); (2) the optimal $\gamma^{B}$

(a) Optimal $\gamma^{j}$ set by store S when $b^{B}\left(\frac{b^{B}}{b^{A}}\right)$ varies (b) Optimal $p_{s}^{j}$ set by store S when $b^{B}\left(\frac{b^{B}}{b^{A}}\right)$ varies

(c) $\Pi_{s}^{j}$ in equilibrium when $b^{B}\left(\frac{b^{B}}{b^{A}}\right)$ varies


(d) $\Pi_{r}^{j}$ in equilibrium when $b^{B}\left(\frac{b^{B}}{b^{A}}\right)$ varies

Figure EC. 4 Equilibrium results when $b^{B}\left(\frac{b^{B}}{b^{A}}\right)$ varies. Setting: $c_{s}=0.7, k=0.1, \alpha=0.5, b^{A}=3$ so that

$$
\beta^{A}\left(\gamma^{A}\right)=1+3 \cdot \sqrt{\gamma^{A}}
$$

remains lower than $\gamma^{A}$ even when a type (B) store S has a much higher warm-glow factor than a type (A) store $S$; (3) a type (B) store $S$ can generate a higher profit than a type (A) store $S$ when it can generate a much stronger warm-glow effect; (4) store R's profit $\Pi_{r}^{B}$ decreases with the warm-glow factor $b^{B}$, and notably, $\Pi_{r}^{B}$ decrease sharply when $0.643<\frac{b^{B}}{b^{A}}<0.945$ as store R chooses to significantly lower its price to deter a type (B) store S.

Setting (2): Increasing-convex warm-glow effect function: $\beta^{j}\left(\gamma^{j}\right)=1+b^{j} \cdot\left(\gamma^{j}\right)^{2}$
In the second scenario, we examine an increasing-convex warm-glow function by setting $t=2$, resulting in the warm-glow function becoming $\beta^{j}\left(\gamma^{j}\right)=1+b^{j} \cdot\left(\gamma^{j}\right)^{2}$. By setting $c_{s}=0.7$ and $b^{A}=$ $b^{B}=8$, we utilize Figure EC. 5 to illustrate store R's optimal deterrence strategies against both types of store S. Once again, Figure EC. 5 resembles Figures 3 and 5 for the case with exogenously given $\gamma^{j}$, and Figures 6 and 7 for the case with endogenously determined $\gamma^{j}$ and linear warm-glow effect function. Therefore, we demonstrate the robustness of our structural results for the incumbent store R's optimal deterrence strategy. We summarize our findings in the following remark:

Remark EC.1. When facing the potential entry of a social store (i.e., store $S$ ), regardless of whether it donates a proportion of its profit (or revenue) and whether the donating proportion is pre-committed (or endogenously determined), and regardless of the function form of the warm-glow effect, the incumbent store R's optimal deterrence strategy always possesses the following structure (Figures 3, 6, 7, EC.1, EC.5):

1. High entry cost: if store S's entry cost $k$ is relatively high, then store S cannot afford to enter the market, and store R can operate as a monopoly.
2. Medium entry cost: if store S's entry cost $k$ is medium, then store R may either deter store S 's entry or cannot deter the inevitable entry of store S, depending on store R's cost advantage $\alpha$.
3. Low entry cost: if store $S$ 's entry cost $k$ is relatively low, then store R should: (1) deter store S's entry when its cost advantage is high (i.e., when $\alpha$ is low); (2) tolerate store S's entry when its cost advantage is medium; or (3) cannot deter the inevitable entry of store S when its cost advantage is low.


Figure EC. 5 Store R's equilibrium deterrence strategy against both types of store $S$ when the warm-glow effect function takes an increasing-convex form. Setting: $c_{s}=0.7$.

We then examine the impact of parameters $k, \alpha$ and $\frac{b^{B}}{b^{A}}$ on the equilibrium outcomes when the warm-glow effect exhibits an increasing-convex shape. As before, by setting $\alpha=0.5$ and $b^{A}=b^{B}=8$, we utilize Figure EC. 6 to show how store S's entry cost $k$ affects the equilibrium results. In this numerical case, a type (B) store S can only enter the market when $k<0.072$, while a type (A) store S can enter as long as $k<1.788$. Once again, Figure EC. 6 exhibits the same characteristics as Figures 8 and EC.2, indicating that the impact of the entry cost $k$ on the equilibrium results always possesses similar characteristics regardless of the warm-glow function form. To formalize this observation, we present the following remark:

Remark EC.2. When store S endogenously determines its donating proportion $\gamma^{j}$ along with the price $p_{s}^{j}, j \in\{A, B\}$, the impact of its entry cost $k$ possesses the following characteristics regardless of the function form of the warm-glow effect (Figures 8, EC.2, EC.6):

1. Upon entry, a type (A) store S will increase both of the donating proportion $\gamma^{A}$ and its price $p_{s}^{A}$ when its entry cost $k$ increases. However, a type (B) store S's donating proportion $\gamma^{B}$ and price $p_{s}^{B}$ are independent of its entry cost $k$.
2. Upon entry, both types of store S's profits $\Pi_{s}^{j}$ will decrease with the entry cost $k$.
3. By tolerating the entry of store S , store R 's profit $\Pi_{r}^{A}$ will decrease with $k$ when tolerating a type (A) store S's entry; however, $\Pi_{r}^{B}$ remains independent of $k$ when tolerating a type (B) store S's entry. On the other hand, by deterring the entry of store S., store R's profit $\Pi_{r}^{j}$ will increase with $k$ regardless of the store type.


Figure EC. 6 Equilibrium results when $k$ varies. Setting: $c_{s}=0.7, \alpha=0.5, \beta^{j}\left(\gamma^{j}\right)=1+8 \cdot\left(\gamma^{j}\right)^{2}, j \in\{A, B\}$.

By setting $k=0.1$ and $b^{A}=b^{B}=8$, we illustrate the impact of store R's cost advantage $\alpha \equiv \frac{c_{r}}{c_{s}}$ on the equilibrium results using Figure EC.7. Generally speaking, Figure EC. 7 also resembles Figures 9 and EC.3, indicating similar insights associated with the impact of $\alpha$. Observing from Figure

EC. 7 that a type (A) store $S$ can always enter the market in this numerical scenario, while a type (B) store S can only enter the market when $\alpha>0.788$. Additionally, store R will be squeezed out by a type (A) store S when $\alpha>0.746$ and by a type (B) store S when $\alpha>0.974$, as shown in Figure EC.7(d). Interestingly, it is worth noting from Figure EC.7(b) that when the warm-glow function takes on an increasing-convex form, as $\alpha$ increases, both types of store S will lower its price $p_{s}^{j}$ when co-existing with store R . This is because in the convex case, as $\alpha$ increases, even though both types of store $S$ can afford to lower $\gamma^{j}$, but they also have to lower the price in order to compete with store R.


Figure EC. 7 Equilibrium results when $\alpha$ varies. Setting: $c_{s}=0.7, k=0.1, \beta^{j}\left(\gamma^{j}\right)=1+8 \cdot\left(\gamma^{j}\right)^{2}, j \in\{A, B\}$.

To summarize the impact of store R's cost advantage $\alpha$, we include the following remark:
Remark EC.3. When store S endogenously determines its donating proportion $\gamma^{j}$ along with the price $p_{s}^{j}, j \in\{A, B\}$, as store R's cost advantage becomes lower (i.e., $\alpha \equiv \frac{c_{r}}{c_{s}}$ increases), the equilibrium results exhibit the following changes (Figures 9, EC.3, EC.7):

1. Upon entry, both types of store $S$ will lower the donating proportion $\gamma^{j}$. Consequently, the profits of both types of store $\mathrm{S} \Pi_{s}^{j}$ will increase, and store R's profit $\Pi_{r}^{j}$ will decrease.
2. Whether the optimal price $p_{s}^{j}$ set by store S is increasing (or decreasing) with $\alpha$ depends on both the function form of the warm-glow effect and the store type. In general, as the power parameter $t$ in (23) increases (i.e., the warm-glow function form switches from increasing-concave to increasing-convex), store S's corresponding optimal price $p_{s}^{j}$ tends to switch from increasing with $\alpha$ to decreasing with $\alpha$ when co-existing with store R .

As before, Figures EC.5, EC. 6 and EC. 7 illustrate a scenario in which both types of store S have the same warm-glow factor (i.e., $b^{A}=b^{B}=8$ so that $\left.\beta^{j}\left(\gamma^{j}\right)=1+8 \cdot\left(\gamma^{j}\right)^{2}\right)$ ). Similar to the linear and increasing-concave cases, we observe that in the convex warm-flow function scenario, it is also easier for a type (A) store $S$ to enter the market than a type (B) store $S$ when they have identical warm-glow factors. Next, when we compare Figure EC.5(b) with Figure 7(b) where the warm-glow factor $b^{B}$ is set as 8 , we observe that the yellow tolerance region becomes smaller in Figure EC.5(b), indicating that relative to the linear case, a type (B) store $S$ is more vulnerable in the convex case. However, with the same $b^{A}=8$, Figures $6(\mathrm{~b})$ and EC.5(a) are quite similar, which suggests that store $R$ will adopt a similar deterrence strategy against a type (A) store $S$ regardless of the warm-glow function form. The aforementioned findings can be summarized as follows:

REmark EC.4. When facing the potential entry of two types of store S with the same warm-glow factor (i.e., $b^{A}=b^{B}$ ), store R's optimal deterrence strategy exhibits the following characteristics (Figures 6, 7, EC.1, EC.5):

1. Store $R$ tends to adopt a more aggressive strategy against a type (B) store $S$ than a type (A) store $S$, regardless of the function form of the warm-glow effect.
2. Store R's deterrence strategy against a type (B) store $S$ is more sensitive to the function form of the warm-glow effect compared to a type (A) store S. Specifically, store $R$ tends to adopt a more aggressive deterrence strategy against a type (B) store $S$ when the power parameter $t$ in (23) increases (i.e., the warm-glow function form switches from increasing-concave to increasingconvex), while adopting a similar deterrence strategy against a type (A) store $S$ regardless of the function form of the warm-glow effect.

Finally, we examine the scenario where the warm-glow factors $b^{A}$ and $b^{B}$ are store-specific. We set $c_{s}=0.7, k=0.1, \alpha=0.5, b^{A}=8$ so that a type (A) store $S$ can enter the market and compete with store R . We vary $b^{B}$ from 4 to 36 , resulting in the ratio $\frac{b^{B}}{b^{A}}$ ranging from 0.5 to 4.5 . From Figure EC.8, we observe that only when $\frac{b^{B}}{b^{A}}>1.077$ (so that $b^{B}=8.616$ ) can a type (B) store $S$ enter the market. Figure EC. 8 resembles Figures 10 and EC.4, which also supports our previous findings associated with the impact of $\frac{b^{B}}{b^{A}}$ on the equilibrium outcomes. Formally, we add the following remark:

Remark EC.5. Suppose store $S$ endogenously determines its donating proportion $\gamma^{j}$ along with the price $p_{s}^{j}, j \in\{A, B\}$. When the warm-glow factor $b^{j}$ is store-specific, and a type (A) store S with a certain fixed $b^{A}$ can enter the market to compete with store R , then the equilibrium outcomes associated with a type (B) store $S$ exhibit the following characteristics (Figures 10, EC.4, EC.8):

1. Upon entry, a type (B) store S will increase its donating proportion $\gamma^{B}$ along with the price $p_{s}^{B}$ as the warm-glow factor $b^{B}$ increases. However, a type (B) store S tends to set a lower donating proportion than a type (A) store S even when it has a higher warm-glow factor $b^{B}$.
2. Upon entry, a type (B) store S's profit $\Pi_{s}^{B}$ will increase with the warm-glow factor $b^{B}$. However, a type (B) store $S$ can only generate a higher profit than a type (A) store $S$ when it can generate a much stronger warm-glow effect than a type (A) store $S$.
3. To deter a type (B) store S's entry, store R's profit $\Pi_{r}^{B}$ decreases sharply with the warm-glow factor $b^{B}$. By tolerating a type (B) store S's entry, $\Pi_{r}^{B}$ decreases further as $b^{B}$ increases.


Figure EC. 8 Equilibrium results when $b^{B}\left(\frac{b^{B}}{b^{A}}\right)$ varies. Setting: $c_{s}=0.7, k=0.1, \alpha=0.5, b^{A}=8$ so that

$$
\beta^{A}\left(\gamma^{A}\right)=1+8 \cdot\left(\gamma^{A}\right)^{2}
$$


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[^1]:    ${ }^{1}$ There are other types of stores that create social values but are beyond the scope of our study. First, there are other retailers that make charitable donations "without pre-commitments," but we do not classify them as "socially responsible" retailers in our context. This is because, without pre-commitments, a consumer cannot take a firm's "future potential charitable donations" into consideration when she makes purchasing decisions. Second, there are forprofit neighborhood stores that create access to fresh produce as well as non-profit cooperative stores that support the local economy. However, consumers would react to these stores differently than those with pre-committed charitable donations. We shall discuss this issue in $\S 8$.
    ${ }^{2}$ Founded in 2006, Toms.com is a for-profit company that designs and sells shoes, eyewear, coffee, apparel, and handbags. Founded in 2015, ivoryella.com is an online for-profit retailer that sells clothing and accessories.
    ${ }^{3}$ Founded in 2014, Cotopaxi is a Utah-based B-corp that sells outdoor gear and apparel with a social-focused mission of eradicating extreme poverty. Founded in 2020, Judy.co sells emergency preparedness kits.
    ${ }^{4}$ If we were to compare very different types of socially responsible stores (e.g., type (A) store donates its profit and type (B) resells used products), then the warm-glow effects will be very different. The warm-glow effect of type (A) is generated from the charitable donations; however, the warm-glow effect of type ( B ) is not well understood because it is motivated by environmental sustainability and social responsibility. Consequently, the comparison will involve different factors and modeling parameters (such as cost), which is beyond the scope of this paper.

[^2]:    ${ }^{5}$ The literature on socially responsible enterprises is quite broad. We refer the reader to Lee and Tang (2018) for a discussion. There is also literature on producer cooperatives (e.g., An et al. 2015, Ayvaz-Cavdaroğlu et al. 2020) and consumer cooperatives (e.g., Sexton and Sexton 1987). As cooperatives significantly differ from social retailers (e.g., are owned by members, have different objectives, charge membership fees), they are beyond the scope of this paper.

[^3]:    ${ }^{6}$ The same approach can be used to examine the case when $\alpha \geq 1$.
    ${ }^{7}$ If store S does not differentiate itself from store R by setting $\gamma=0$, then store S cannot generate the warm-glow effect. In this case, store R can easily deter store S from entering the market by setting $p_{r}=c_{s}>c_{r}$ due to cost advantage. To rule out this trite case, we shall focus our analysis for the case when $\gamma>0$.

[^4]:    ${ }^{9}$ It is easy to verify that when $k \in\left(K_{2}^{A}, K_{1}^{A}\right], \Theta_{1}^{A}(k)=\Theta_{2}^{A}(k)$, which are increasing in $k$. However, when $k \leq K_{2}^{A}$,

[^5]:    ${ }^{20}$ On the contrary, consumers cannot fully understand how their purchases are related to a type (A) store's donation that is based on a proportion of its profit, which involves other cost factors that consumers have no control or visibility.

[^6]:    ${ }^{21}$ If $t=1$, then the warm-glow effect $\beta^{j}\left(\gamma^{j}\right)$ increases linearly in the donating proportion $\gamma^{j}$; if $0<t<1$, then the warm-glow effect is an increasing-concave function of $\gamma^{j}$; and if $t>1$, then the warm-glow effect is an increasing-convex function of $\gamma^{j}$.

