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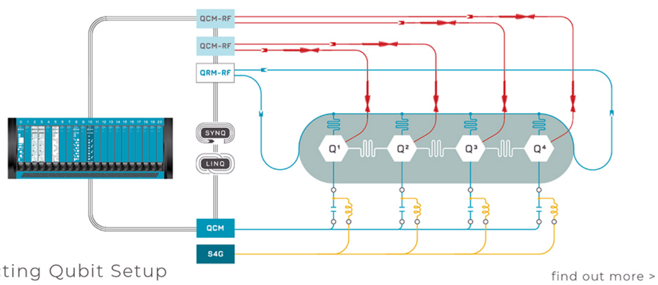

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Sagar Silva Pratapsi,[1,2,a)] Patrick H. Huber,[3,a)] Patrick Barthel,[3] Sougato Bose,[4] Christof Wunderlich,[3,a)] and Yasser Omar[1,5,6,a)]

AFFILIATIONS

[1]Instituto Superior Técnico, University of Lisbon, Lisbon 1049-001, Portugal
[2]Instituto de Telecomunicações, Lisbon 1049-001, Portugal
[3]Department of Physics, School of Science and Technology, University of Siegen, 57068 Siegen, Germany
[4]Department of Physics and Astronomy, University College London, London WC1E 6BT, United Kingdom
[5]Physics of Information and Quantum Technologies Group, Center of Physics and Engineering of Advanced Materials (CeFEMA), Lisbon 1049-001, Portugal
[6]PQI—Portuguese Quantum Institute, Lisbon 1600-531, Portugal

[a)]Authors to whom correspondence should be addressed: spratapsi@tecnico.ulisboa.pt; p.huber@physik.uni-siegen.de; Christof.Wunderlich@uni-siegen.de; and contact.yasser@pqi.pt

ABSTRACT

Reversible computation has been proposed as a future paradigm for energy efficient computation, but so far few implementations have been realized in practice. Quantum circuits, running on quantum computers, are one construct known to be reversible. In this work, we provide a proof-of-principle of classical logical gates running on quantum technologies. In particular, we propose and realize experimentally, Toffoli and Half-Adder circuits suitable for classical computation, using radio frequency-controlled $^{171}Yb^{+}$ ions in a macroscopic linear Paul-trap as qubits. We analyze the energy required to operate the logic gates, both theoretically and experimentally, with a focus on the control energy. We identify bottlenecks and possible improvements in future platforms for energetically efficient computation, e.g., trap chips with integrated antennas and cavity QED. Our experimentally verified energetic model also fills a gap in the literature of the energetics of quantum information and outlines the path for its detailed study, as well as its potential applications to classical computing.

Computational tasks are responsible for a non-negligible part of the world's energy consumption. It is estimated that computationally intensive data-centers represent 1% of the global energy budget.[1] So far, increases in energy efficiency have been able to offset the growing demand for computation: peak-usage energy efficiency has doubled every 1.5 years during the 1960–2000 period, while since the 2000s this figure is closer to 2.6 years.[1,2] However, processor efficiency gains cannot continue to grow forever. There is a fundamental limitation of the current paradigm of nonreversible computation, known as Landauer's principle,[3] where each irreversible bit operation dissipates $k_B T \ln 2$ of heat.

Reversible computation may, thus, become an important computation paradigm in the future. Reversible systems may also avoid the heat costs of contemporary CMOS processors, such as capacitor charging, switching, and current leakage,[4,5] which are ultimately responsible for the typical 40% energy cost for cooling in data centers;[6] they may also protect against external attacks, such as power usage analysis. It is, then, worthwhile to investigate how energy-efficient reversible platforms can become. Some proposals for reversible computing platforms have been billiard-ball models,[7,8] adiabatic circuits,[9–13] nano-machines,[14–18] superconducting devices,[19–21] quantum-dot cellular automata,[22] and others (see Ref. 23 for a review of reversible computation). However, so far, experimental realizations of reversible computation are lacking in practice. Quantum mechanical systems, which evolve unitarily, are also reversible by nature and are, thus, an attractive candidate for energetically efficient computation.[24,25] Although quantum platforms are limited by coherence time, we can reset the coherence for classical computations by measuring in the computational basis in-between logical operations. We may also exploit superselection rules to protect classical information, as was proposed recently in a quantum dot platform.[26] Can we then build energy efficient circuits for universal reversible computation using quantum computing platforms?

In this work, we explore an implementation of reversible computation using quantum technologies, by realizing a classical Half-Adder circuit—an important building block for arithmetic operations[27]—using quantum states of trapped ions. To do so, we implement a Toffoli gate, itself a universal gate for classical computation. We determine the energy to operate these gates, both theoretically and experimentally, with a special focus on the energy required to activate and control the logical gates, focusing on the power delivered to the quantum processing unit (QPU), as defined later. We point out possible improvements toward energy efficient computation. Some works[28] require realistic estimates for the energy consumed by quantum computers. Thus, our energetic analysis, supported by experimental measurements, also fills a gap in the literature, and establishes a baseline for additional research toward understanding the energetic impact of quantum technologies.[29]

A Half-Adder circuit is a fundamental component of arithmetic circuits. It computes the logical AND (multiplication modulo 2) and XOR (addition modulo 2) of two input bits. It is a building block for the Full-Adder circuit, addition circuits in their ripple-carry and carry-lookahead variants, multiplier circuits and other tasks in contemporary computer processors. The core operation behind our Half-Adder circuit is a quantum Toffoli gate, followed by the application of a CNOT to the two control qubits of the Toffoli (Fig. 1). A Toffoli gate, or a controlled-controlled-NOT gate, is a universal three-bit operation, i.e., it is sufficient to construct any classical reversible circuit. Antonio *et al.* proposed a Toffoli gate suitable for classical computation,[30] which can be realized on any three-qubit physical system with constant nearest-neighbour Ising couplings, via the Hamiltonian

$$H_{\mathrm{TOF}} = \frac{\hbar J}{2}\left(\sigma_1^z\sigma_2^z + \sigma_2^z\sigma_3^z\right) + \frac{\hbar\delta}{2}\sigma_2^z + \frac{\hbar\Omega}{2}\sigma_2^x. \qquad (1)$$

Here, σ_j^i is the σ^i Pauli operator acting on the jth qubit, appropriately tensored with the identity operators on the other qubits. The real constants J, δ, and Ω define interaction strengths. We simulated numerically the time evolution under the Hamiltonian (1) for a time of π/Ω and $\delta = 2J$. We found that $\Omega \approx 1.1J$ allows for a $\approx 99\%$ classical Toffoli gate fidelity while minimizing the gate time (see supplementary material).

Ions confined in a linear Paul trap are natural candidates to implement the Hamiltonian (1).[30,31] We use $^{171}\mathrm{Yb}^+$ ions confined in a linear Paul trap, with a superimposed static magnetic field gradient.[32] The qubit states $|0\rangle$ and $|1\rangle$ are the two hyperfine states of the electronic ground state $^2S_{1/2}$ with total angular momentum quantum number and magnetic quantum number $|F, m_F\rangle = |0,0\rangle$ and $|1,1\rangle$, connected by a magnetic dipole resonance near $2\pi \times 12.6$ GHz. The $|1\rangle$ state is sensitive to the magnetic field, which is position dependent, shifting individually the ions' resonances and, thus, allowing for individual addressing by tuning the microwave field driving the qubit resonance.[33] For high fidelity single qubit rotations, the ion crystal is cooled close to its motional groundstate using a sympathetic sideband cooling.[34]

When irradiating the ions with a microwave field with phase ϕ and frequency ω_x, nearly resonant with the frequency ω_2 of qubit 2, the ionic qubits are subject to the Hamiltonian

$$H^{(i)} = \underbrace{\sum_{i\neq j}\frac{\hbar J_{ij}}{2}\sigma_i^z\sigma_j^z}_{H_{zz}} + \sum_j \frac{\hbar\omega_j\sigma_j^z}{2} + \hbar\Omega\cos(\omega_x t + \phi)\sigma_2^x. \qquad (2)$$

Here, ω_i is the resonance frequency of the ith ion. The two-qubit couplings J_{ij} in a magnetic field gradient are mediated by the Coulomb interaction.[31,32] In the setup used here, the magnetic field gradient is 19.1 T/m at a secular axial trap frequency of $\omega_T = 2\pi \times 128.4(1)$ kHz, and $J_{12} = J_{23} = J \approx 2\pi \times 31$ Hz, which implies a gate time of $\pi/1.1J \approx 14.9$ ms. The additional $J_{13}\sigma_1^z\sigma_3^z$ coupling contributes with a complex phase in the computational basis, which is irrelevant for classical computation, so we choose to omit it. Finally, Ω is determined by the amplitude of the incident microwave radiation. H_{zz} is the Hamiltonian generating the required spin–spin interaction via magnetic gradient induced coupling (MAGIC). Cross-talk between qubits was neglected; its main source is the non-resonant excitation of neighboring qubits, which has been measured to be on the order 10^{-5}.[33] Choosing a detuning δ, such that $\omega_x = \omega_2 - \delta$, and in an appropriate rotating frame, H_I reads as

$$H_I^{(i)} \approx H_{zz} + \frac{\hbar\delta}{2}\sigma_2^z + \frac{\hbar\Omega}{2}\left(\cos(\phi)\sigma_2^x + \sin(\phi)\sigma_2^y\right), \qquad (3)$$

with an error of $O(\Omega/(2\omega_2 - 2\delta))$.[30] Choosing $\phi = 0$ recovers the Hamiltonian (1).

Fluctuations in the magnetic field dephase the qubits, which are first-order sensitive to them. Not using passive magnetic shielding and

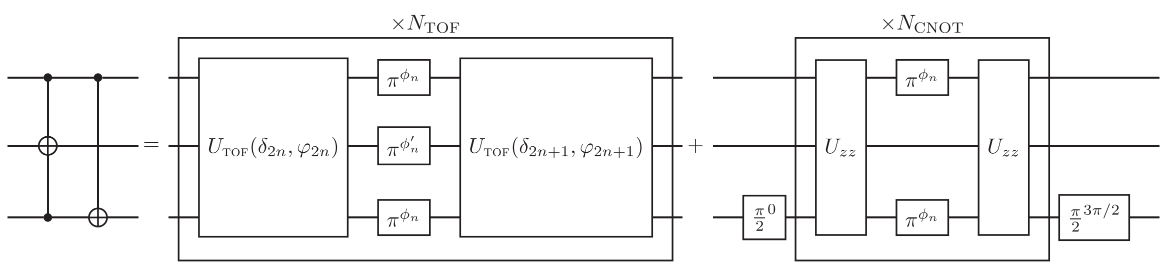

FIG. 1. A Half-adder circuit using a Toffoli followed by a CNOT gate. We choose the central qubit as the target of the Toffoli gate to fulfil the condition $J_{12} = J_{23}$ from Eq. (1). The Toffoli gate decomposes into a unitary $U_{\mathrm{TOF}}(\delta_n, \phi_n)$ [generated from Hamiltonian (3) for 14.9 ms/400] and single qubit π-pulses with some phase ϕ (π^ϕ, implementing Dynamical Decoupling). The block is repeated $N_{\mathrm{TOF}} = 200$ times with updated values of δ_n, φ_n and DD phases ϕ_n and ϕ_n'. The latter are chosen to implement a universal robust DD sequence on qubits 1 and 3 and a CPMGXY on qubit 2. The values $(\delta_{2n}, \delta_{2n+1})$ alternate between $(\delta, -\delta)$ and $(-\delta, \delta)$ for each π-pulse, while $(\varphi_{2n}, \varphi_{2n+1})$ alternates between $(0, \pi)$ and $(\pi, 0)$ for each $\pi^{\pi/2}$-pulse. The CNOT gate decomposes into a U_{zz} gate (implementing the zz coupling) and single qubit π-pulses. The block is repeated $N_{\mathrm{CNOT}} = 120$ times. The phases ϕ_n implement a UR DD sequence on the control and target qubits.

active compensation, the coherence time in this setup is $\approx 200\ \mu s$[35]—two orders of magnitude lower than our gate times. We, thus, employ dynamical decoupling (DD) to protect the qubits. For DD, we intersperse single-qubit π rotations in-between the Hamiltonian evolution, by periodically irradiating the qubits with a top hat-shaped pulse of Rabi frequency $2\pi \times 33$ kHz. The J_{ij} couplings are negligible during this time, since they are three orders of magnitude smaller than the Rabi frequency. We use the notation θ^{ϕ} for a Rabi rotation of angle θ with phase ϕ: π^0 and $\pi^{\pi/2}$ are, thus, σ^x and σ^y gates, respectively [see Eq. (3)]. Applying a π-pulse amounts to a change of basis, so we need to change the Hamiltonian evolution accordingly. The σ_2^z term acquires a relative minus sign, since $\sigma_i^z \sigma_i^{x,y} = -\sigma_i^{x,y} \sigma_i^z$, which we compensate by changing the microwave detuning as $\delta \to -\delta$. On the other hand, the $\sigma_i^z \sigma_j^z$ terms are left unchanged when both qubits are flipped simultaneously. The σ_2^x term acquires a minus sign when a σ_2^y pulse is applied, which we compensate by adding a phase of $\phi = \pi$ to the driving field's phase. This limits the applicable DD-sequences to either σ^x or σ^y pulses. For the Toffoli gate, we choose the Carr–Purcell–Meiboom–Gill pulse sequence[35,36] CPMGXY applied to the target qubit and a Universal Robust (UR) sequence[37] applied to the control qubits (see Fig. 1).

The CNOT gate implementation using the MAGIC scheme is realized by a $(\pi/2)^0$-pulse on the target qubit, and a unitary evolution generated by H_{zz} inducing a relative phase change of π conditioned on the logical state of the control qubit. Finally, a $(\pi/2)^{3\pi/2}$-pulse is applied to the target.[32] In a register exceeding size 2, it is necessary to decouple the spectator qubits from the qubits carrying out the CNOT.[32] This is achieved using a DD-sequence on the qubits participating in the CNOT gate and excluding spectator qubits.[35] The conditional evolution time used in this work is $T_{\text{cnot}} \approx 8.75$ ms. We applied a UR sequence of 120 DD pulses, with a pulse duration of 15 μs each to protect the qubits coherence.[38] The circuit diagram is shown in Fig. 1.

Using maximum-likelihood process tomography,[39] we characterized the CNOT, Toffoli, and Half-Adder gates with classical fidelities of $F_{CNOT} = 86.9\%$, $F_T = 58.8\%$, and $F_H = 60.6\%$, respectively (see Fig. 2). The 90% confidence intervals are, respectively, [0.665, 0.937], [0.221, 0.683], and [0.409, 0.662], i.e., the probability that the true fidelities lie in these intervals is guaranteed to be at least 90%.[40] The classical fidelity is the probability of obtaining the correct output given a uniformly random computational state. To better contextualize the fidelity we obtained for the Half-Adder, we note that a Half-Adder may alternatively be composed of single-qubit rotations and 11 CNOT gates instead of taking advantage of a Toffoli gate as in this work. To then achieve the same fidelity $F_H = 60.6\%$ as obtained here with such a decomposition of the Half-Adder and assuming nearest-neighbor couplings, each CNOT would require at least a fidelity $F_{11} = 95.5\%$ (see the supplementary material). Comparing the actual CNOT fidelity $F_{CNOT} = 86.9\%$ with $F_{11} = 95.5\%$ shows that a lower circuit depth obtained by using a multi-qubit gate not only speeds up the Half Adder implementation but also yields a higher fidelity for given hardware parameters.

We now turn to the question of energetic cost. In this work, we focus on the energy delivered to the quantum processing unit (QPU), which we define as the physical components housed inside the vacuum chamber. The energy required for the implementation of a Half Adder is supplied to the QPU in six steps: (I) Doppler Cooling, (II) Sideband Cooling, (III) State Preparation, (IV) Toffoli and (V) CNOT (information processing), and (VI) Readout. We are particularly interested in the cost of information processing, that is, steps IV and V. All steps, except IV and V, are implemented by applying laser light near 369 nm and near 935 nm, a microwave field near 12.6 GHz, and an RF field near 20 MHz that generates the RF trapping potential. During step IV and V, only the microwave field near 12.6 GHz is applied. The optical power of the laser beams, the microwave power, and the power of the RF signal near 20 MHz is measured using commercial devices. (More details on the experimental setup are given in the supplementary material.)

In addition to the field generating the Toffoli gate itself, there is energy necessary for the DD π-pulses. At a Rabi frequency of $2\pi \times 33$ kHz, microwave power of 0.58 W near 12.6. GHz is required in the current setup. The Toffoli is generated using a Rabi frequency of 34 Hz, a factor 10^3 less than the Rabi frequency used to implement single qubit π and $\pi/2$ rotations. Due to the low power, its cost was estimated assuming the law $P \propto \Omega^2$ [see Eq. (S14) of the supplementary material] as 9.2 nJ. The power consumption of 3×200 π-pulses dominates the energetic cost of the Toffoli gate.

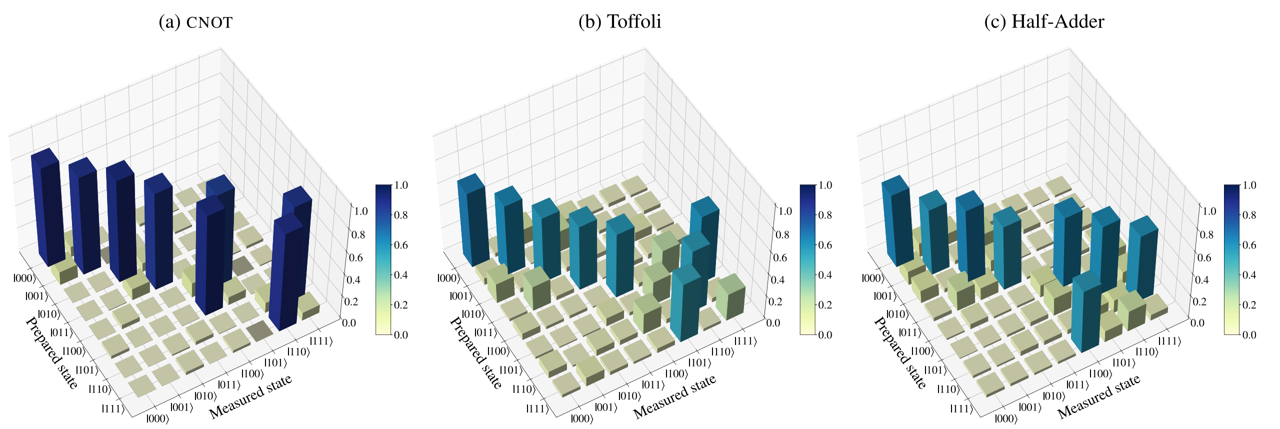

FIG. 2. Measurement probabilities for the CNOT, Toffoli and Half-Adder gates, reconstructed from Maximum-Likelihood Estimate tomography. The CNOT gate is controlled by qubit 1 and acts on qubit 3, while the Toffoli gate has qubit 2 as a target. The CNOT, Toffoli, and Half-adder gates have classical fidelities of 86.9%, 58.8%, and 60.6%, respectively.

TABLE I. Measured energy consumption of the experimental steps I–VI. The main contribution to the overall energy consumption comes from dynamical decoupling, apart from the RF trap's constant cost. The energy contribution of the CNOT gate consists of two $\pi/2$-pulses totaling up to a π-pulses energy contribution. All measured energies displayed here carry a relative error of 10^{-2}.

			Microwave			
				Dyn. decoupling		
Operation	Laser 369 nm	Laser 935 nm	Pulse	# π-pulses (8.8 μJ)	Cost	Total
I. Doppler c.	380 nJ	11 μJ	4.6 mJ	⋯	⋯	4.6 mJ
II. Sideband c.	10 nJ	81 μJ	35 mJ	⋯	⋯	35 mJ
III. State prep.	7 nJ	0.3 μJ	⋯	⋯	⋯	0.3 μJ
IV. Toffoli	⋯	⋯	9.2 nJ	3×200	1.1 mJ	1.1 mJ
V. CNOT	⋯	⋯	8.8 μJ	2×120	0.44 mJ	0.44 mJ
VI. Readout	140 nJ	4 μJ	⋯	⋯	⋯	4.2 μJ
Half-Adder (IV. + V.)						1.5 mJ

In Table I, we summarize the energy estimates for all experimental steps and gates, and in Fig. 3, we present the power delivered by each component as a function of time. The table includes the number of π-pulses, equivalent to NOT gates, used in the DD scheme.

The most efficient supercomputer today, Frontier TDS, requires $\approx e5$ eV per bit operation (see details in Ref. 26)—13 orders of magnitude lower than the NOT gate reported here. We emphasize that this work presents a baseline for systematically investigating energy consumption and efficiency of quantum platforms, in particular ion-trap setups. There is vast room for improvement. We exemplify this statement by showing how a next-generation ion trap will be five orders of magnitude more efficient.

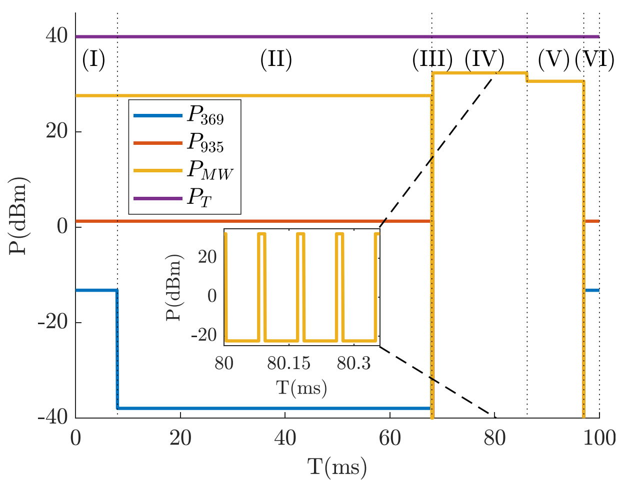

FIG. 3. Power delivered to the QPU: P_{369} is the power delivered by the cooling, preparation, and readout laser; P_{935}, repumping laser; P_{MW}, microwave to close the cooling cycle and coherent qubit control; P_T, RF trapping potential. The time stamps are, in order, (I) 8 ms Doppler cooling, (II) 60 ms sideband cooling,[34] (III) 0.2 ms state preparation, (IV) 18.2 ms Toffoli gate including dynamical decoupling, (V) 11 ms CNOT gate including dynamical decoupling, and (VI) 3 ms readout. In the inset, we can see a detail of the DD pulses for the Toffoli gate. See Table ST1 in the supplementary material for the power measurements.

Based on this energetic analysis, we identify three main sources of energy consumption. First, the microwave wave guide produces a wavefront [A_{rad} in Eq. (S14) of the supplementary material] that is orders of magnitude larger than the ions' interaction cross section. For a single NOT gate, $\approx 10^{17}$ photons are irreversibly lost. Second, the conditional gate times are much longer than the qubits' coherence times, making it necessary to use 10^3 DD pulses, where most of the energy is spent (see Table I). Third, the RF drive of the Paul trap currently produces the highest energy consumption of the order of 3 W/ion or 50 mJ/ion for the duration of a Toffoli gate.

Future planar ion traps can address these issues, and we now provide representative figures for illustration. First, such traps integrate microwave antennae and resonators closer to the ions into a planar setup,[41] which can greatly reduce the irreversible loss of microwave photons. We estimate, from preliminary results, that NOT gate is performed in 1.7 μs at an applied power of 10 mW, consuming 17 nJ of energy per gate, as opposed to 8.8 μJ in this work. In addition, the J-coupling, necessary for conditional gates, will be increased by about two orders of magnitude, thus reducing the time needed for CNOT, Toffoli, and Half-Adder by the same factor. Furthermore, the coherence time is prolonged by about two orders of magnitude, mainly due to the use of magnetic field shielding. Although faster gates imply higher energy consumption (for a given geometry), the energy consumption decreases when using fewer DD pulses, or completely omitting them. We expect a Toffoli gate to require 4 pJ and a gate time of 125 μs, which eliminates the need for DD. Implementing the CNOT gate still requires two $\pi/2$ pulses on the target qubit as well as two π-pulses on the target and control to decouple them from qubit 2, resulting in 5π-pulses. In total, 85 nJ will be required for the Half-Adder. This is approximately 10^5 times more efficient than the current setup (Table I), due to $1000\times$ more efficient pulses and $100\times$ fewer DD pulses. Still, this trap was not built with the specific goal of energy efficiency. Finally, shorter distances between the electrodes generating the trapping potential and the ion in a planar ion trap reduce the RF power necessary to run such a trap. Current planar traps require less than 1 W of RF power to maintain trapping. Furthermore, since this cost is fixed, we can increase the trap efficiency by increasing the number of ions per trap, or possibly combining traps efficiently using ion shuttling.

Cavity QED setups[42] can bring the radiation area (A_{rad}) closer to the ions' effective dipole area (A_{dip}). In that limit, and considering the

GHz operation regime, Eq. (S14) predicts a cost of on the order of μeV per NOT gate, which is close to the photon energy, 8 μeV. This is 10 orders of magnitude lower than the estimate for Frontier TDS.

Efficient control protocols will become necessary in the "one-photon limit,"[43] which is still an open problem.[44,45] Nevertheless, it was recently observed that, indeed, typically one quanta of energy is used from control fields.[46] Other strategies to reduce energetic costs may be reusing control energy, manipulating several qubits at once and/or recycling unused energy. Reducing the trap cost is also important, possibly using Penning traps, tighter geometries, and packing more ions per trap.

In conclusion, we implemented classical logic—NOT, CNOT, Toffoli, and Half-Adder circuits—using the quantum states of trapped ions. Our work opens the door for future implementations of classical logical on quantum technologies, with the potential energy savings of reversible computation, and presents a benchmark for future platforms. Our work presents a path toward low energy computation beyond CMOS, whose efficiency is decelerating. We show possible short-term improvements and long-term solutions, and hope to encourage the community to fulfill the vision of energy-efficient computing.

See the supplementary material for further details on the Toffoli gate, experimental setup, power measurements and estimation, and fidelity discussion.

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AUTHOR DECLARATIONS

Conflict of Interest

The authors have no conflicts to disclose.

Author Contributions

Sagar Silva Pratapsi and Patrick H. Huber contributed equally to this work.

Sagar Silva Pratapsi: Conceptualization (equal); Formal analysis (equal); Investigation (equal); Methodology (equal); Project administration (equal); Software (equal); Visualization (equal); Writing – original draft (equal); Writing – review & editing (equal). **Patrick H. Hüber:** Conceptualization (equal); Data curation (equal); Formal analysis (equal); Investigation (equal); Methodology (equal); Software (equal); Validation (equal); Visualization (equal); Writing – original draft (equal); Writing – review & editing (equal). **Patrick Barthel:** Data curation (equal); Investigation (equal); Methodology (equal). **Sougato Bose:** Conceptualization (equal); Formal analysis (equal); Investigation (equal); Methodology (equal); Project administration (equal); Supervision (equal); Validation (equal); Visualization (equal); Writing – review & editing (equal). **Christof Wunderlich:** Conceptualization (equal); Data curation (equal); Formal analysis (equal); Funding acquisition (equal); Investigation (equal); Methodology (equal); Project administration (equal); Resources (equal); Supervision (equal); Validation (equal); Writing – review & editing (equal). **Yasser Omar:** Conceptualization (equal); Funding acquisition (equal); Investigation (equal); Methodology (equal); Project administration (equal); Resources (equal); Supervision (equal); Validation (equal); Visualization (equal); Writing – review & editing (equal).

DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding authors upon reasonable request.

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