Multi-Objective Optimization of Robotically Bent In-Situ Reinforcement System

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ABSTRACT

This paper describes a novel process towards the application of multi-objective optimization as the form-finding process for the integration of computational design, fabrication, and construction sequences. The design and construction of a doubly curved large-scale prototype made of textile-reinforced GRC shotcrete with a robotically fabricated in-situ reinforcement system serves as the case study for the proposed methodology. Global geometry form-finding process takes into consideration the location and geometrical properties of the in-situ reinforcement rebar system, robotic rod-bending constraints, structural performance, and functional objectives. These criteria are integrated through the application of a multi-objective optimization method in order to formulate multiple trade-off solutions that possess multiple constraints (fitness objectives) which are primarily in conflict with each other and are intended towards automation in fabrication. The primary contribution of the research is the demonstration of a multi-objective optimization methodology that incorporates geometrical form-finding, material and fabrication constraints, and FEA as design drivers during the early stages of design. This optimization method can be further extended and utilized across a multitude of scales in order to save energy, materials, and cost in architectural projects.

Author Keywords

Form-finding; Multi-objective optimization; Data Driven decision; Robotic Fabrication; Robotic rod bending; Glass-reinforced concrete (GRC); Stay-in-place textile formwork

ACM Classification Keywords

I.2.8 Problem Solving, Control Methods, and Search (Heuristic methods); I.2.9 Robotics (Commercial robots and applications); J.5 Arts and Humanities (Architecture); J.6 Computer-aided design; J.6 Computer-aided manufacturing (CAM).

1 INTRODUCTION

The research presented in this paper outlines a novel methodology for the application of multi-objective optimization as the form-finding process towards the integration of computational design, fabrication, and construction sequences. The design and construction of a doubly curved large-scale prototype made of textile-reinforced GRC shotcrete with a robotically fabricated in-situ reinforcement system serves as the case study for the proposed methodology. One of the primary considerations of the research is to allow for the geometrical freedom to create a range of complex doubly curved geometries with the employment of functional, fabrication-related, and structural constraints.

Concrete, being one of the most extensive building materials in construction, can be utilized to create architectural forms with complex geometry. The formal flexibility of concrete raises the question of the application of formwork during the construction process. Furthermore, custom-made reinforcement strategies need to be developed both for the formwork and the reinforcement of the structure. While it is viable to produce formwork with complex geometries via advanced digital and robotic fabrication tools, a key consideration area is the reduction of form-work waste material in manufacturing methods. The potential to incorporate this constraint in the preliminary design process as a design driver will pose advantages in waste optimization as well as production costs. Recent investigations on textile stay-in-place formwork [6] and 3d-printed stay-in-place formwork demonstrate the advantages of employing this method by correlating the geometric flexibility of the formwork with the structural capacity of concrete [15].

Advances in the biological sciences and computation in recent years paved the way to mimic the principles of evolutionary science to solve common real-world problems. This problem-solving methodology comprises search and optimization procedures of single or multiple objectives. Evolutionary multi-objective optimization strategies have been utilized widely since the late 20th century as problem-solving methods. In the 1930s, Sewell Wright was one of the first figures who attempted to apply biological evolutionary principles as optimization processes in solving complex problems [20]. Halfway through the 20th century, John Holland [7], Rechenberg and Schwefel [17] and Fogel et al. [5] respectively developed genetic algorithms (GA),

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evolutionary strategies (ES), and evolutionary programming (EP) independently from one another. Their findings eventually led to the creation of a unified field of Evolutionary Computation in the late 20th century [1].

A multi-objective optimization methodology that incorporates geometrical form-finding, material and fabrication constraints, and FEA as design drivers during the early stages of design is demonstrated through in this paper. This optimization method can further be utilized across different scales in order to save energy, materials, and cost in architectural projects. The incorporation of geometrical, material, and fabrication constraints with criteria related to robotic fabrication presents an integrative morphogenetic design methodology [16].

The one-to-one scale prototype presented in this paper is a case study to test the proposed methodology with the design and construction of a complex doubly curved prototype. The prototype, an urban furniture piece that accommodates seating areas, is made of textile reinforced GRC shotcrete with an integrated solution for a reinforcement system that is fabricated via robotic rod bending [3]. The dimensions of the structure are 1,580 mm. width, 3,850 mm. length, with a height that varies between 750 – 1,500 mm. The location of the case study is the outdoor area of Santral Istanbul Campus of Istanbul Bilgi University.

2 EVOLUTION AS A DESIGN METHODOLOGY

In the biological sciences, Genotype is a set of genes or instructions (codes) that performs as a blueprint for the development of the Phenotype. Phenotype is the physical expression (formal and behavioral manifestation) of the Genotype. In the field of evolutionary computation, and more specifically in the context of architecture and design disciplines, a genotype is equivalent to a set of instructions or codes that will produce the geometry, namely the phenotype.

An evolutionary model, as described by Ernst Mayr, includes a two-step process; random variation within the genotype of a phenotype, and the selection of the phenotype through environmental pressures [14]. In line with evolutionary processes in nature, the application of evolutionary computation in design is founded upon these two primary components of variation in the code responsible for generating the geometry (genotype) and the selection of the geometry (phenotype) that fits better in the environmental conditions. In the context of the application of evolutionary computation in design, the environment is equivalent to a set of fitness objectives (e.g. design constraints to be met). Evolutionary algorithm goes through a basic loop. It starts with the generation of an initial random population of solutions. It continues with modifications of genomes through random variations, followed by the evaluation of solutions based on their objective performance. The algorithm concludes with the selection of a group of solutions that correspond to a predefined selection mechanism [4] (Figure1). Through this iterative process of generation, evaluation and selection, each phenotype will be evaluated based on a set of fitness objectives. Therefore, the formulation of the environment, that is the design problem and the calculation of the fitness objectives, in this process is essential for constructing a successful evolutionary model to produce meaningful design options.

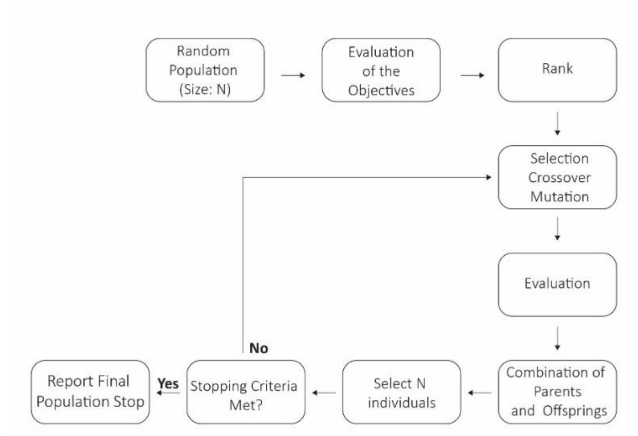


Figure 1. WallaceiX Core Algorithm (NSGA-II) pseudocode. (developed by [2]).

In recent years, evolutionary optimization processes have gained recognition in architecture and related design disciplines both in academia and practice.

Research conducted by Ayman Hassaan et al. explored the application of evolutionary optimization in the design phase by studying geometric formations of a facade at the early stages of design [10]. Yun Kyu Yi implemented NSGA-II algorithm in optimizing building facades with a similar objective to the previous research, nevertheless with a different design methodology [21]. In the building scale, the studies carried out by Machairas et al. employ genetic algorithms to address a set of conflicting objectives such as energy, comfort, and cost in early design phase [9]. In larger and more complex design domains, Makki et al. applied an evolutionary model to generate variations of urban forms that address a set of conflicting objectives [13].

Bionic Partition is amongst other projects carried out by the Living, serving as an example of the application of evolutionary optimization techniques in practice. Functional and structural constraints have been amongst objectives that they sought to address [19].

3 EXPERIMENT SETUP

The presented experiment utilizes multi-objective Non-Dominated Sorting Genetic Algorithm II (NSGA-II) developed by Deb. et al. [2] as the base algorithm upon which the evolutionary simulation is developed. Rhinoceros3D, Grasshopper3D and its plugin ‘Wallacei’ [11] are used to run the simulation, analyze the results thoroughly, and select the option to be further developed and fabricated. In the conducted experiment, the algorithm parameters within the evolutionary simulation have been set to the following values. (Table 1) (For a thorough description of the terminology used in the simulation see [12]).

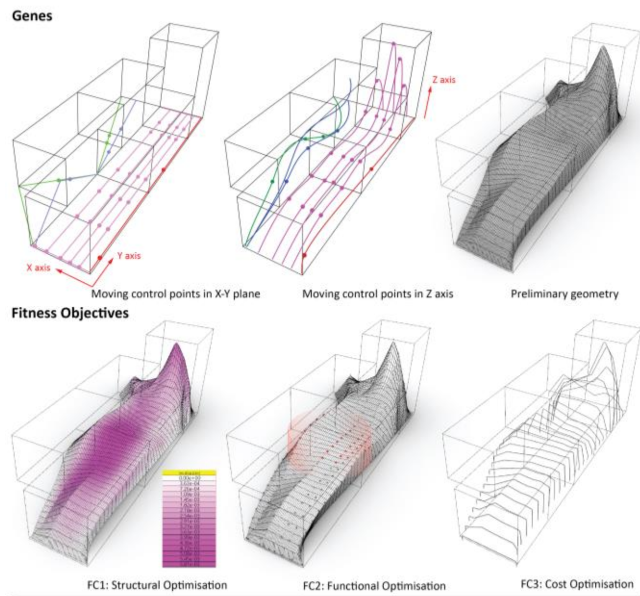
Parameter	Short Description	Value
Generation Size	*Number of individuals per generation*	30
Generation Count	*Number of generations in the simulation*	100
Crossover Probability	*Percentage of solutions that reproduce in each generation*	0.9
Mutation Probability	*The percentage of mutation 1/ (number of variables)*	1/n
Crossover distribution index	*Probability of similarity of the offspring to the parents*	20
Mutation Distribution Index	*Probability of similarity of the offspring to the parents*	20
Random Seed	*Random seed in the simulation*	1

Table 1. The algorithm parameters can be modified in WallaceiX UI first tab. Full description of each setting can be obtained from Wallacei Primer [12].

In order to test the construction of a complex doubly curved prototype, which is made through the application of textile-reinforced GRC shotcrete on an in-situ skeleton acting as the reinforcement system, a preliminary design idea was proposed by the students of the AAVS Istanbul 2019 workshop to serve as a piece of urban furniture with seating areas. In order to successfully fabricate the proposed design idea, a set of design objectives were addressed to calibrate the initial designed form and adjust it for fabrication and construction purposes. Three design objectives were specified to regulate and optimize the initial form-finding process in three categories. These include structural optimization to minimize structural displacements, functional optimization to accommodate for suitable seating areas, and cost optimization to regulate the amount of material used for the in-situ skeleton. The objective of the experiment is to integrate the above stated objectives in the early stages of design in order to realize a full-scale prototype with the given design choices and constraints.

The preliminary design idea was built algorithmically in Grasshopper 3D (visual scripting platform of Rhino3D) to establish the foundation of modelling the *environment* and subsequently the calculation of the fitness objectives (Figure 2). The predefined domains extracted from the fabrication constraints, such as the required curvature of the surface for shotcrete process and the accessibility of the surface by the shotcrete operator, were assigned to the motion vector of the control points of the curves by which the overall morphology is generated. Accordingly, the genotype of the experiment was formed of 49 genes which modify the motion vectors of the control points of the phenotype. As the result, the size of the design space is calculated to be $9.6 * 10^{75}$. Three fitness objectives were defined to drive the optimization process to calibrate the preliminary design idea. They are:

- Structural Optimization: Minimizing the structural displacement through FEA.
- Functional Optimization: Creating a flat surface to serve as a seating area on the form.
- Cost Optimization: Minimizing the length of the rods to be used for creating the in-situ skeleton.



Fitness Objectives	Genes	
	Moving control points in X-Y plane	Moving control points in Z axis
FO1: Structural Optimisation: Minimizing displacement	✔	✔
FO2: Functional Optimisation: Creating flat surfaces	✔	✔
FO3: Cost Optimisation: Minimizing the length of rods	✔	✔

Figure 2. The genotype and the fitness objectives of the experiment.

The first fitness objective was set to minimize the structural displacement of the global geometry through FEA (Finite Element Analysis). As the final prototype was set to be fabricated with textile-reinforced GRC shotcrete application on an in-situ skeleton, the structural model of the phenotype was built as a concrete shell with an approximately 40 mm. thickness. In order to calculate the structural displacement of the generated phenotypes, an FEA analysis was conducted through Karamba3d, a plugin for Grasshopper3d [8]. The numerical value of the calculated structural displacement (in centimeters) was set to be the first fitness objective of the optimization process (Figure 2).

The second fitness objective was determined to evolve the phenotype towards developing flat surfaces to serve as seating areas. A set of control points on the surface of the geometry in a specified spatial domain of the phenotype was selected. The relative differences between the Z coordinates of the selected points was calculated. The fitness objective was computed to minimize this calculated difference. Thereby, the points on the surface can be moved towards the same horizontal plane, and accordingly the phenotype will develop areas with a flat region suitable for seating (Figure 2).

The third fitness objective was assigned to drive the optimization process to the direction of evolving phenotypes to use as fewer metal rods as possible for the in-situ reinforcement system. The length of the bent rods (in meters) was computed as the third design objective to be minimized (Figure 2).

Due to the complexity of the form, robotic rod bending technique was selected for fabricating the in-situ skeleton onto which textile reinforcement would be placed, followed by the application of GRC shotcrete. Therefore, the angle of the rods to be bent by the industrial robotic arm was an essential factor to consider in the design process. In the iterative process of evolutionary optimization, fitness objective two (developing flat areas) and fitness objective three (minimizing the length of rods) would alter the angle of the rods freely if no constraint had been put in place. In order to automate and streamline the robotic rod bending process, the genes that were controlling the control points of the constructive lines of the phenotype (Figure 2), as well as their location were restricted in a domain which could only produce desirable bending angles. The angles that were divisible by 15 (i.e. 15°, 30°, 45°, 60° etc.) were set as the possible options that the rods could bend and evolve. The algorithmic approach that was used to constrain the generated angles through the iterative optimization process falls outside of the domain of this paper and will be explained in the context of another publication thoroughly.

Given the time constraints of the workshop, the optimization problem was limited to 30 individuals per generation with a total number of 100 generations (in total 3,000 generated solutions). The primary purpose of the conducted experiment was to calibrate the preliminary design proposed by the AA Istanbul visiting school 2019 students, structurally, functionally and materially, all independently from each other through a multi-objective optimization process.

4 EXPERIMENT RESULT AND SELECTED OPTION

By running the multi-objective evolutionary simulation for the main design problem, 3,000 genotypes/phenotypes with three fitness values per solution were produced. The visual analysis and recognition of the fitness performance of each individual, and the selection of the candidate solution for fabrication can be a highly inefficient process, subject to the visual preferences of the user. Therefore, analysis of the data associated with each phenotype plays a crucial role in the selection of the candidate solution to be further developed and fabricated.

The evaluation of the multi-objective optimization process commenced with a set of analyses to examine how the evolutionary simulation addressed the fitness objectives and how successful it performed in its entirety. Figure 3 illustrates the Parallel Coordinate Plot where each line represents an individual in the simulation and each axis is indicative of one objective.

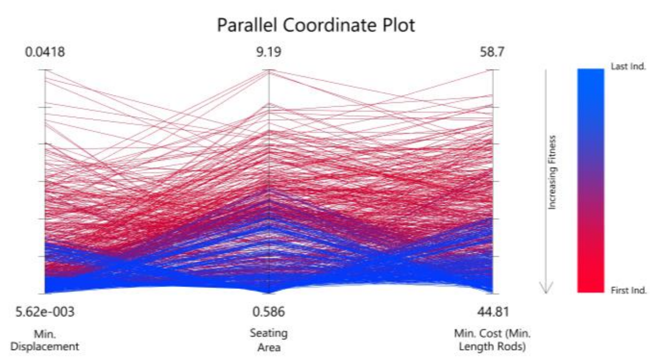


Figure 3. Parallel Coordinate Plot represents the overall performance of the optimization simulation. The graph was drawn by Wallacei Analytics Component.

The color gradient from red to blue illustrates the progression from former to later generations. Wallacei X, the evolutionary solver used for the experiment presented in this paper (the evolutionary solver of Wallacei plugin) optimizes the fitness objectives by minimizing the inputted values through search and optimization based on evolutionary principles. By introducing variations in the genotype and random mutations, individuals with lower numerical fitness values are considered the fittest in each iteration of the evolutionary simulation. These individuals are selected for the next round of the iteration by variation through crossovers, mutations, and selection. Figure 4 demonstrates that the fitness objectives have been successfully optimized towards the end of the simulation, as can be depicted by the blue lines appearing towards the bottom section of the plot. It is understood that the evolutionary process could generate individuals with high performance in all three fitness objectives in comparison to the preliminary design idea proposed.

Figure 4 compares the performance of the optimization run for all three fitness objectives independently and side by side. It comprises four graphs, Standard Deviation Graphs, Fitness Value Graphs, Standard Deviation Trendline (per generation) and Mean Fitness Value Trendline (per generation). Standard Deviation graphs illustrate that as the simulation advances, the population starts to improve for all fitness objectives. The progression of the SD graphs towards left is an indication of the mean fitness value's decrease per generation, thereby illustrating that the overall fitness objective is optimized. This can be cross-checked with the Mean Value Trend Line graphs on the bottom right which demonstrate the decrease in the mean fitness values per generation and subsequently increase in fitness performance by generating optimized solutions. SD value trendlines describe the decrease in the value of standard deviation per generation which shows that the population is converging towards the optimized solutions by the end of the simulation run. Fitness Values Graphs illustrate that the later generations, illustrated in blue, are accumulated towards the bottom of the graphs, since they have less numerical values, and therefore describe the success of the evolutionary simulation run for all fitness objectives.

One of the challenges of the application of evolutionary principles through multi-objective optimization processes in design is the selection phase where a candidate or a set of candidate solutions need to be chosen for the subsequent stages of project development. In the context of this paper, one single solution was set to be selected to be further developed and fabricated accordingly.

For the purpose of selecting a candidate solution amongst the population of generated options, the population was filtered down into a smaller solution set to examine methodically and select the candidate design to be further developed and constructed. The population of 3,000 individuals was filtered to a sub-set of 86 individuals. These individuals comprised Pareto front solutions of the entire simulation (81 individuals), best performing individuals for each fitness objectives (3 individuals), an individual that perform as equal as possible for all three objectives (1 individual), and finally the individual that has the highest average rank amongst all population (1 individual) (For further description on the last two selection strategies, please refer to [12] and [13]). Due to the morphological similarities of these individuals, their comparison had to be accompanied by the data associated with each individual (Figure 5).

Individual number 24 in generation 89 which is the solution with the highest average rank amongst all population was selected to be further developed and fabricated (Figure 6). Table 2 shows the information of 10 out of 86 chosen individuals in detail. The selected individual, highlighted in orange, is amongst the highest performing individuals in the population. The diamond charts plotted next to each individual in the table demonstrate how well the individual performs for each fitness objective. In the diamond chart, each fitness objective is plotted on one axis where the center indicates best performance and the edges designate worst performance respectively. The selected induvial is ranked high for all objectives equally.

5 POST-RATIONALIZATION FOR ROBOTIC ROD BENDING

There were several post-rationalization adjustments that required to be applied to the selected individual to streamline the fabrication process through automated robotic rod bending, such as the application of longitudinal rods, the calculation of spring-back in the rods, and a computational process to include the spring-back values in the algorithm in order to reach the target angles without tolerances.

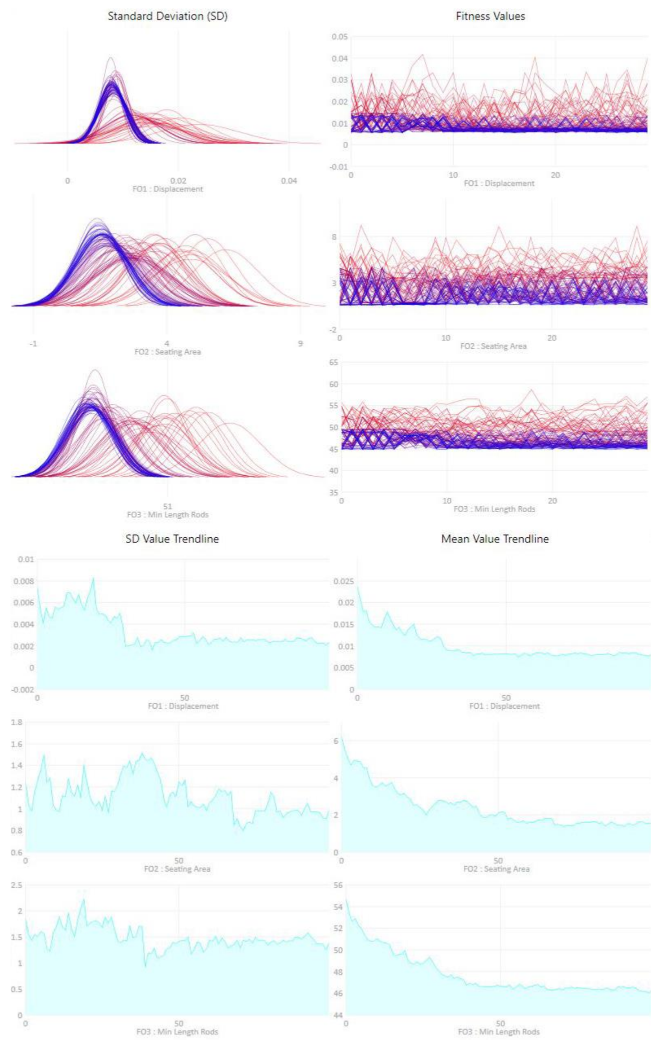


Figure 4. The graphs were obtained from the second tab of WallaceiX user interface.

Figure 5. The subset of the population that was exported from WallaceiX. All the solutions are laid out on a grid containing the data associated with each individual. All solutions are compared morphologically and numerically side by side. The figure shows only 18 out of 81 exported individuals.

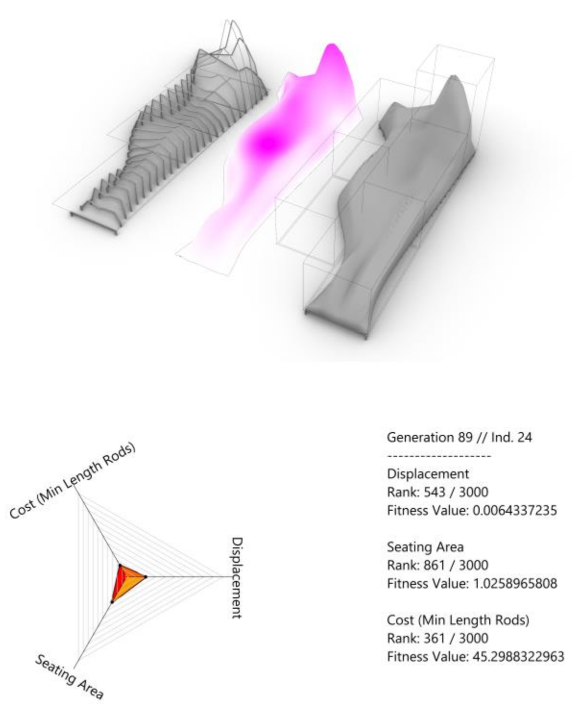


Figure 6. The selected individual with associated data pertaining to three fitness criteria.

Firstly, a set of physical experiments have been conducted in order to test the spring-back of rods with a fixed diameter, 6 mm., fixed length, 2,000 mm., and a fixed jig radius, X mm. The angles that have been tested for spring-back are multiples of 15 (i.e. 15°, 30°, 45°, 60° etc.) (See Section 3). These experiments have been carried out iteratively to accurately record the resulting bending angles against the target angles. It was observed that the deviation of each bending angle due to spring-back was approximately 12%. Hence, a linear regression analysis, which is a supervised machine learning algorithm, was opted for in order to derive a linear function between the resulting bending angles and target angles. The linear regression analysis was carried out in Lunchbox, a plugin for Grasshopper3d [22].

Subsequently, robotic control for rod bending was developed by using a general robotic control software, Robots for Grasshopper [18], and Rhino 6. The computational process was streamlined with all the constraints related to rod qualities, namely the working area, the final bending shape of the rod, and spring-back values. A fully integrated algorithm was developed, defining every step, from the rod division in sets, order of manufacturing and bending angles. Every rod was then identified by the algorithm and this information was used for the correct position and angle of each rod.

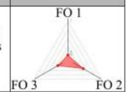
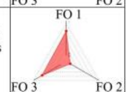
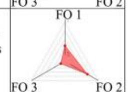
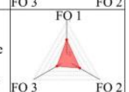
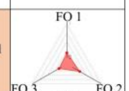
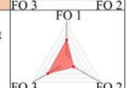
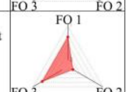
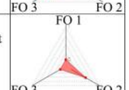
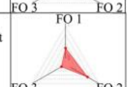
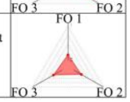
Name	Fitness Objective 1 Displacement (cm)	Fitness Objective 2 Seating Area (m^2)	Fitness Objective 3 Cost, Rod Length (m)	Description	
Generation 81 Individual 3	0.0056200186 *Rank: 0/2999*	1.382345094 *Rank: 1115/2999*	45.6481656375 *Rank: 609/2999*	Best Performance for Fitness objective 1	
Generation 94 Individual 1	0.0127958266 *Rank: 2312/2999*	0.5856362427 *Rank: 0/2999*	48.781808684 *Rank: 2091/2999*	Best Performance for Fitness objective 2	
Generation 99 Individual 2	0.0070189687 *Rank: 930/2999*	2.9710719237 *Rank: 1871/2999*	44.8078991025 *Rank: 0/2999*	Best Performance for Fitness objective 3	
Generation 59 Individual 29	0.0077716913 *Rank: 1444/2999*	1.1270681489 *Rank: 927/2999*	45.5707707246 *Rank: 573/2999*	Solution with performance as equal as possible for all objectives	
Generation 89 Individual 24	**0.0064337235** ***Rank: 543/2999***	**1.0258965808** ***Rank: 861/2999***	**45.2988322963** ***Rank: 361/2999***	**Solution with the best overall rank**	
Generation 99 Individual 5	0.0080857542 *Rank: 1571/2999*	0.7297315064 *Rank: 283/2999*	46.9864998716 *Rank: 1465/2999*	In the Pareto Front of the last generation	
Generation 99 Individual 10	0.0110894113 *Rank: 2087/2999*	0.6450605679 *Rank: 72/2999*	48.7768083345 *Rank: 2089/2999*	In the Pareto Front of the last generation	
Generation 99 Individual 15	0.0059566409 *Rank: 158/2999*	2.2183824048 *Rank: 1585/2999*	45.069855818 *Rank: 128/2999*	In the Pareto Front of the last generation	
Generation 99 Individual 20	0.0071210202 *Rank: 1038/2999*	2.9499949217 *Rank: 1858/2999*	44.8891545929 *Rank: 2/2999*	In the Pareto Front of the last generation	
Generation 99 Individual 25	0.0064912887 *Rank: 576/2999*	1.0362837197 *Rank: 864/2999*	46.0962877867 *Rank: 984/2999*	In the Pareto Front of the last generation	

Table 2. Detailed comparison of 10 selected individuals.

6 DISCUSSION

The research presented in this paper demonstrates the advantages of the application of an adaptable computational model, driven by an evolutionary multi-objective optimization process, to generate morphological variation in response to multiple design objectives. The success of the implemented method in rationalizing a free-form design idea into a manufacturable prototype proves to be an advantageous approach for solving complex design problems. Contrary to the conventional design process of improving a single design solution, the proposed method utilizes an iterative process of developing a population of design candidates, thereby allowing for greater morphological variation within the specific design domain.

The adoption of fabrication-related and structural parameters, coupled with the geometrical freedom and constraints of a selected research agenda, in this case robotic rod bending, has the potential to address multiple performance criteria embedded in the high level of complexity of various design processes. Limitations related to the selected fabrication methods are not within the scope of this paper and will be further described in a separate publication.

Figure 7. In-situ reinforcement system built with robotic rod bending, and the final prototype.

The successful implementation of multi-objective optimization to address geometrical, structural, and material-oriented constraints simultaneously as well as independently in the optimization process can be streamlined in various stages of design, from early design exploration as demonstrated in this paper through to fabrication and construction practices. This optimization method can be further extended and utilized across a multitude of scales in order to save energy, materials, and cost in architectural projects.

The capacity to evolve a population of design candidates that vary in morphological diversity and performance is essential for design problems that cannot have a single optimal solution. This is particularly important when the design problem poses multiple conflicting fitness objectives, thereby presenting the necessity for a population of variable design solutions rather than a single individual or group of individuals. The degree of variation generated through the employment of multi-objective optimization allows for greater flexibility when addressing the fitness objectives. Nevertheless, infinite variation serves little to no purpose, and informed design decisions need to be made when analyzing and selecting the final set of design candidates. Hence, coupling the geometry and data simultaneously during the analysis and evaluation stages of multi-objective optimization is a key step for attaining objectivity in this design methodology. Through the understanding of data that accompanies the geometry, the evolved solution set can become a robust and powerful alternative to a single, preference-based approach.

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