The development of socio-mathematical agency

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In this paper we report on the development of 'socio-mathematical agency' (SMA), a concept which was introduced at the previous CERME12 conference. We summarise our initial conceptualisation of SMA, justifying its importance and highlighting its three components: powerful mathematical knowledge; critical understanding of mathematics; and collective mathematical agency. We report on a collaborative research project with six teacher researchers that aimed to operationalise SMA in the primary school classroom setting. We present findings from statistical analyses of responses to a survey designed to measure sense of agency relating to SMA and from thematic analyses of teacher researchers' reflections on their students' agentic behaviour relating to SMA during interviews and research team meetings. We conclude that the concept of SMA warrants serious consideration and further attention should be given to refining and developing its conceptualisation.

Keywords: Socio-mathematical agency, powerful knowledge, student agency, social justice.

Introduction

In this paper we report on the development of 'socio-mathematical agency' (SMA), a concept which was introduced by the first author at the CERME12 conference (Wright, 2022). We define socio-mathematical agency (SMA) as the ability to use mathematics effectively to argue collectively for social change. In the title of this paper, we assign dual meaning to the term 'development'. Firstly, it refers to primary school students' development of SMA, informed by the findings from the recent Primary Maths and Social Justice (PMSJ) research project conducted by the authors along with six teacher researchers. Secondly, it refers to our own development of the concept of SMA. Elliott (2009) highlights the importance of involving teachers in applying theoretical knowledge to classroom situations, in order to generate practical insights that will help generate new knowledge. We begin, in the next section, by summarising our initial conceptualisation of SMA. This served as a useful starting point for the PMSJ project, which aimed to operationalise SMA in the primary school classroom. In subsequent sections, we present relevant findings from the PMSJ project and consider in turn how these will help us to refine and develop our conceptualisation of SMA in future.

Initial conceptualisation of socio-mathematical agency

In an increasingly information and data-rich world, individuals are expected to understand complex mathematical ideas to make sense of their own situations and take decisions that will promote their own well-being and the 'public good'. It is questionable how well schooling currently prepares many learners to do this. This was highlighted during the recent Covid-19 pandemic when many people in England were baffled by terms such as 'moving averages' and 'exponential growth', which were referred to daily in news broadcasts in England in the hope that individuals would be able to make sense of data relating to the growth of infections and adapt their behaviour accordingly. This lack of preparedness reflects the calls of several inter-governmental policy-making organisations for a more humanistic school curriculum that cultivates the critical understanding and collective knowledge needed to address the social, economic and environmental challenges facing global society (United Nations Educational, Scientific and Cultural Organization [UNESCO], 2015). We argue that cultivating students' SMA, i.e. their ability to use mathematics effectively to argue collectively for social change, is the most significant contribution the mathematics curriculum can make towards addressing the social, economic and environmental challenges facing global society (Wright, 2022).

Muller and Young (2019) highlight the importance of 'powerful knowledge' (which they describe as abstract, formal, specialised and coherent) for enabling individuals and societies to extend their horizons and to think the 'not-yet-thought'. Manyukhina & Wyse (2019), however, argue that the power of knowledge depends on the agency of the knower, and abstract knowledge alone cannot be powerful. Given its importance in solving real life problems, Skovsmose (2021) argues that mathematics has a critical role to play in tackling the current crises facing global society, however, in the wrong hands, it can be used to create or format crises. Muller and Young (2019) acknowledge the importance of developing an appreciation of 'disciplinary meaning', i.e. how new knowledge is generated within the discipline, for fostering powerful knowledge. For this to happen in mathematics, students need to experience processes that mathematicians go through in generating new knowledge including conjecturing, reasoning, explaining, justifying and (since they generally operate in teams) working collaboratively (Mason et al., 1985; Schoenfeld, 2012). Gutstein (2006) highlights the need for a mathematics curriculum that emphasises reasoning, problem-solving and communication, and enables learners to develop agency in solving real world problems.

Drawing on the above, our initial conceptualisation of SMA was comprised of three components:

1) The development of 'powerful mathematical knowledge' that includes:

An appreciation of disciplinary meaning (how new knowledge is generated) in mathematics.
An ability to apply abstract mathematical concepts in solving meaningful real-life problems.

2) The development of a 'critical understanding of mathematics' that includes:

A readiness to use mathematics to explore and develop understanding of social justice issues.
A disposition towards using mathematics to expose/challenge exploitation/social injustice.

3) The development of 'collective mathematical agency' that includes:

A willingness to work with others in using mathematics to construct an argument for change.
Confidence that it is possible to influence society through mathematical argument and action.

We sought to refine and develop our conceptualisation of SMA through working with teachers to explore how it might be operationalised in the classroom setting.

The Primary Maths and Social Justice research project

The PMSJ research project was a collaboration between the three authors and six teacher researchers (TRs) based in two London primary schools. The TRs shared an interest in addressing social justice issues and the aim of the project was to explore how to maintain and build upon this interest through the teaching of mathematics. There were three specific aims of the project: to develop effective strategies that can be used to enhance primary school students' SMA; to explore how SMA can be measured, assessed and developed; and to investigate how teachers' agency in tackling social justice issues through teaching mathematics can be enhanced (Wright at al., 2023). This paper focuses on the first two aims which are most relevant to developing SMA.

We adopted a participatory action research methodology (Wright, 2021), which involved coming together to establish a research team to meet regularly throughout the duration of the project. The participatory principles of the project meant that the TRs played an integral role in the project design. Meetings of the research team included engaging with theories relating to the conceptualisation of SMA, jointly planning and evaluating two research lessons aimed at operationalising SMA, co-designing and planning for the implementation of a survey aimed at measuring/assessing students' SMA (see next section). Discussions continued between meetings with TRs working in pairs to complete the final planning and organisation of the research lessons, with each pair being supported by one of the authors. The research lessons were taught by the TRs to their own classes across the primary age range (5 to 11), with students from Year 1 (Emma and Kate's classes), Year 2 (David and Aidan' classes), Year 5 (Rose's class) and Year 6 (Layla's class). Note the TRs real names have been replaced here by pseudonyms of their choice.

The TRs decided that there were more opportunities to apply mathematics in tackling real life social justice issues outside of dedicated 'mathematics' lessons (for which the schemes of work were considered to be too prescriptive). They sought to apply mathematical ideas to the project-based work that they commonly undertook at other times with their classes. Examples included debating which voting systems (simple majority or using preferences or allocating a given number of cubes) might be fairest in choosing between various fun activities/games on offer during class 'golden time' (used as a reward for achievement), and how to interpret the outcomes of such votes (considering only the winner or allocating class time proportionally). Other projects included considering different options for addressing various environmental issues (low emission zones, deforestation, litter, pollution, climate change) and using statistics in arguing which of these solutions should be adopted. Sometimes the issues around voting methods were also used to decide which argument was most convincing.

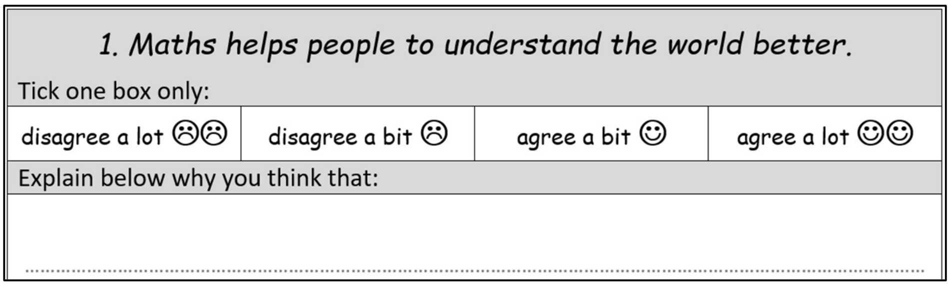
Measuring socio-mathematical agency

Manyukhina and Wyse (2019) characterise learner agency as incorporating two aspects: a 'sense of agency' and 'agentic behaviour'. The first aspect involves a feeling of control over your own learning and the second aspect involves exercising control through the actions you take. We sought to apply these two aspects of learner agency to measuring/assessing SMA. We considered student surveys to be the most appropriate tool for measuring students' sense of agency in relation to SMA, since these involve students self-reporting their own views and beliefs. To measure students' agentic behaviour in relation to SMA, we felt it was more appropriate to draw on the TRs' evaluations and reflections following each research lesson, through analysing discussions held during meetings and interviews.

In developing a survey designed to measure students' sense of agency in relation to SMA, we sought to explore the extent to which the concept of SMA was measurable by testing the efficacy of such a survey. We also believed that attempting to measure SMA would help us to refine and develop our conceptualisation. We searched the research literature to see what previous research findings might help us in this endeavour but found little to help us. We did come across one study by Pampaka et al. (2011) which tested the validity of an original survey designed to measure the ‘mathematics self-efficacy’ of students in post-compulsory (beyond age 16) schooling in England. Note that Pampaka’s team defined ‘mathematics self-efficacy’ as “self-belief in one’s ability to use mathematics effectively in the future” (ibid., p.170). Their survey was designed to measure perceived self-efficacy in applying mathematics to real life problems in general, and not those specific to social justice issues. We found their approach useful in designing our own survey, although the items and language needed substantial adaptation for use with younger children.

We decided to limit the number of items on our survey as we would ultimately like to produce a tool that would be readily accessible to primary students and easy for teachers to administer. Since we had identified three components of SMA, we decided that six items would be appropriate and that these should ask students to rank the extent to which they agreed with a series of statements. Choosing two items for each of the three components would allow us to test for internal reliability by comparing responses to items corresponding to the same component. We decided to use a 4-point Likert scale (strongly disagree, disagree, agree, strongly agree) to avoid any tendency to opt for the middle value. We also decided to start with ‘strongly disagree’ to balance participants’ tendency to agree with a statement with their tendency to choose an earlier option on the scale. Students were asked to rank the extent to which they agreed with each statement and then invited to provide a comment explaining why they chose this option. This enabled us to test the validity of the survey by comparing the actual rankings with the authors’ predictions for students’ rankings based on looking at the comments alone.

We came up with three initial statements, corresponding to the three components of SMA (identified earlier), which we shared with the TRs at an early research team meeting. We drew on the experience of TRs and their knowledge of students (a relatively high proportion of whom were bilingual learners) to agree on accessible statements, which we piloted with a group of Year 2 students who were not involved in the project. This enabled us to finalise the six items to be included in the survey and agree protocols for how these would be administered by TRs. These protocols included allowing teachers and teacher helpers to read statements to students and scribe their responses (for Years 1 and 2 only). We also developed an appropriate layout for presenting the items to students (see figure 1 below).


1. Maths helps people to understand the world better.

Tick one box only:

disagree a lot ☹☹	disagree a bit ☹	agree a bit ☺	agree a lot ☺☺

Explain below why you think that:

Figure 1: Example of survey item

The final six statements included in the survey are shown below. Statements 1 and 5 relate to component 1 of SMA, statements 3 and 6 to component 2, and statements 2 and 4 to component 3:

1. Maths helps people to understand the world better.
2. It is good to solve maths problems with other people.
3. Maths can be used to make the world a better place.

4. You can do more in maths when you work with others.
5. Maths can be used to mislead people.
6. You can use maths to help you explain something.

The surveys were designed to measure sense of agency in relation to SMA and the extent to which it increased over the course of the project. It was administered by the TRs to all students in their class who consented to take part, on two occasions, before the first research lesson and after the second research lesson. The surveys were anonymous and unique identifiers were used to match the responses for those students who completed both surveys. We used statistical analyses to test for validity and reliability of the survey, and to look for what these analyses might tell us about the development of students' SMA over the duration of the project.

In order to measure/assess the development of students' agentic behaviour, we conducted and audio-recorded two interviews with each TR, near the start and near the end of the project. During these interviews, we invited TRs to reflect on the research lessons and their experiences of participating in the research project. Questions posed by the authors included how TRs thought social justice and student agency related to their teaching of mathematics and what they thought the impact of the project had been on their students and on their own thinking and classroom practice. We also audio-recorded those discussions held during research team meetings 3 and 5 which focused on evaluating the two research lessons. All recordings were transcribed, then a thematic analysis (Braun & Clarke, 2022) was carried out on the transcripts by making use of Nvivo software to apply a combination of deductive codes (derived from the theory) and inductive codes (derived from the data).

Findings from the statistical analysis of surveys

We report here on selected findings from the statistical analyses. More detailed findings from the analysis can be found in the project report (Wright et al., 2023). First, we used the Spearman's Correlation Coefficient test (appropriate for our ordinal data) to look for correlations between students' rankings of the six different statements on Survey 2. For each pair of variables, we compared students who had ranked both statements. Strong positive correlations between variables, particularly those relating to the same components of SMA, would indicate a high level of internal reliability for the survey. The results showed no significant correlation between the rankings for statement 5 and those for each of statements 1, 3, 4 and 6. There was a significant negative correlation for statements 5 and 2 ($r_s = -0.200$, $p = .010$, $N = 133$), the opposite of what would be expected if there were internal reliability. However, if statement 5 is excluded from the analysis, all but one of the remaining 10 pairs of variables showed significant positive correlations. There were strong correlations ($r_s > 0.4$) for statements 1 and 3 ($r_s = 0.544$, $p < .001$, $N = 134$), statements 3 and 6 ($r_s = 0.511$, $p < .001$, $N = 129$), statements 2 and 4 ($r_s = 0.482$, $p < .001$, $N = 134$) and statements 1 and 6 ($r_s = 0.442$, $p < .001$, $N = 132$). Note the second and third pair of statements relate to the same components of SMA. This indicates a high level of internal reliability if Question 5 is excluded.

Next, we tested for validity by looking for correlations for each statement between actual rankings and the authors' predictions of students' rankings based on looking at their comments alone. We chose to focus on responses from Aidan's Year 2 class, as a relatively large number of students had made comments explaining their choice of rankings for all statements (largely due to significant support from teachers and teacher helpers in completing the survey). There were 45 completed surveys from this class (both surveys combined) and all students ranked every statement. For some responses, it was not possible to make a prediction because either: no comment was made; or the comment suggested the student had misunderstood the statement; or the comment did not allow for a prediction to be made. For statement 1 ('Maths can be used to make the world a better place'), the following are examples of the last two cases above: "Because maths is really good for your brain" and "I'm not sure". For statement 5, only 12 predictions (from the 45 responses) were possible, suggesting that students experienced serious difficulties in interpreting the language in this statement. For statements 1, 3 and 6, it was possible to make 36, 24 and 19 predictions, suggesting students had some difficulties in interpreting the language. For statements 2 and 4, we were able to make 43 and 38 predictions, suggesting the language was relatively straightforward for students to interpret.

Results from applying the Spearman's Correlation Coefficient test showed a strong ($r_s > 0.4$) and significant (p, one tailed < .05) corelation between predicted and actual rankings for all six statements: Statement 1 (r_s=0.575, p< .001, N=36); Statement 2 (r_s=0.686, p< .001, N=43); Statement 3 (r_s=0.787, p< .001, N=24); Statement 4 (r_s=0.524, p< .001, N=38); Statement 5 (r_s=0.872, p< .001, N=12); Statement 6 (r_s=0.406, p= .042, N=19). Given the above caveats, this suggests a reasonably high level of criterion validity within the survey, particularly for items 1 to 4 (less so for statements 5 and 6 due to the small sample size). In other words, for those students who were able to access the language, the survey appeared to measure what was it was intended to measure.

Given the reliability and validity of 5 out of the 6 items on the survey, we felt reasonably confident in using the survey (without item 5) to measure the development of SMA over the course of the project. We used the Wilcoxon Matched-Pairs Signed-Rank Test (appropriate for ordinal data) to look for significant differences between students' rankings on the two surveys for each of the five remaining statements. There were no significant differences when we aggregated the responses for all classes, or when we looked at five of the classes separately. However, when we looked at results for Aidan's class, the test showed a significant increase in rankings for the three statements, all with large effect sizes: Statement 2 (r = 0.568, p = .011, N = 20); Statement 3 (r = 0.592, p = .010, N = 19); Statement 6 (r = 0.734, p = .002, N = 17). There were also increases in rankings for statements 1 and 4, although these fell slightly below the significance threshold (p = .100 and p = .052).

Findings from the thematic analysis of interviews and meetings

Given the apparent increase in SMA amongst students in Aidan's Year 2 class over the duration of the project, we chose to restrict the evidence presented below to comments made by Aidan during interviews and meetings. However, it should be noted that there was sufficient evidence from the thematic analysis to generalise the findings below to all TRs and their classes. More substantial evidence from the analysis can be found in the project report (Wright et al., 2023).

There was a general appreciation amongst TRs of how providing students with meaningful activities, over which they feel greater ownership, leads to increasing levels of engagement:

> It's really important … at the very start to make sure that whatever we do choose is meaningful for the children and makes them feel empowered and want to contribute. (Aidan, Interview 1)

The TRs noticed how the activities tried out during research lessons resulted in students becoming more willing and able to apply mathematics to solve real life problems:

> There was a lot more children talking about the scientific aspects of how that affects … you know how we live, whether that be through, you know, energy … and mathematical calculations are so important to this and that without this we can't evolve. (Aidan, Interview 2)

The TRS articulated how engaging with meaningful problems enhanced students' learning of mathematics and fostered more positive attitudes towards the subject:

> I think it's had a really positive impact on the students and in how their understanding of mathematics and the importance of mathematics, not just in the context of a maths lesson, but in the wider world. (Aidan, Interview 2)

The TRs described how students became more competent in using mathematics to strengthen their arguments relating to social justice issues:

> So, the children collated the results of how they commuted to school. We used a tally chart, and they were quite shocked to realise that … the majority of my class, anyway, came to school by the car. … And I think they were quite interested to see how … the maths was actually quite important there to help us, not only just work things out, but interpret the data as well. (Aidan, Meeting 3)

The TRs reported their surprise at the extent to which students became more willing to engage in collaborative tasks involving mathematics and problem solving:

> But there was some really good conversation. I was quite surprised. Again, similar to the other groups, certain children really engaging in the task. (Aidan, Meeting 5)

Implications for the development of socio-mathematical agency

The statistical analyses described above suggested that the survey exhibited high levels of internal reliability and validity in seeking to measure students' sense of agency in relation to SMA. The analysis did however reveal some issues relating to the accessibility of language for primary school students, particularly for statement 5 and younger children. It is clear that more attention needs to be given to the support provided to students and the language used when revising and using the existing survey. When discounting survey item 5, it appeared that students in Aidan's class made significant developments in their sense of agency in relation to SMA during the project. Whilst there was no evidence of significant increases in SMA in other classes, this does not necessarily mean that no such changes took place. It may be that the project was too short for any changes to be measurable (with less than five months between the two surveys). It is worth noting that students in Aidan's class provided more comments to explain their rankings than those in other classes. This may be because they received more support in completing the surveys, enabling them to overcome the issues of accessing the language and to engage with the substance of the statements. Alternatively, it may be possible that an increasing familiarity with the concept of SMA amongst the teacher and teacher helpers may have inadvertently influenced students' responses in survey 2, leading to biased results.

The thematic analysis of interviews and research team meetings provides evidence to suggest that the strategy of focusing on applying mathematical skills to cross curricular projects and engaging with social justice issues were responsible for parallel developments in students' agentic behaviour in relation to SMA. Whilst the evidence in this paper relating to the development of agentic behaviour is limited to students in Aidan's Year 2 class, similar developments were reported for students across all classes in all year groups in both schools. We believe that there is sufficient evidence presented in this paper to conclude that serious credibility should be attached to, and more attention devoted to, the concept of SMA, including its three components and its dual aspects of sense of agency and agentic behaviour. We intend to explore the concept of SMA further and carry out more research on operationalising SMA in the classroom, in order to refine and develop our initial conceptualisation.

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