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A tentative framework for students' mathematical digital competencies

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Abstract: New digital trends have found a place in the mathematics classroom and there is a potentially "hidden" demand for students to acquire both digital and mathematical competencies. Current frameworks often talk about one or the other. In this paper, we propose a combined framework for mathematical digital competencies based on two existing frameworks: the KOM framework for mathematical competencies, and the DigComp framework for digital competencies. We discuss the potential value of such a framework for the mathematics education community, i.e. researchers, mathematics educators and practitioners.

Keywords: Mathematical competencies; digital competencies; mathematical digital competencies.

Introduction

Although it is surely possible to distinguish between mathematical and digital competencies, it appears productive to "coin" the two in order to be able to talk about *mathematical digital competencies*, or MDCs (Geraniou & Jankvist, under review) – not least taking into consideration the large-scale embedment of digital technologies in mathematics education today. Of course, tools to do mathematics come in different forms, e.g., physical tools such as centicubes, abacuses, Cuisenaire rods, etc. not to mention rulers, compasses, spirographs, specially ruled paper and so on and so forth. Surely, technology is only *one* tool amongst many. But while several other tools serve one, or a few, purposes, a technological software such as a Dynamic Geometry System (DGS) or a Computer Algebra System (CAS) serves a multitude of purposes. As mathematical digital technologies advance, so do the demands to the competencies of their uses, both inside and outside mathematics educational contexts. However, one should not be blind to the potential pitfalls of the increasing use of technology in mathematics education (e.g., Geraniou & Mavrikis, 2015; Jankvist & Misfeldt, 2015; Jankvist, Misfeldt, & Marcussen, 2016). As well-known, digital tools can perform many of the mathematical tasks that students traditionally are expected to do. For example, the *GeoGebra* feature for constructing regular polygons. As pointed out by Niss (2016), digital technologies should not be a substitution for competencies, but an amplifier of capacities. Enforcing mathematical capacity is the positive idea of using technology as a lever potential (Dreyfus, 1994), i.e. that students may save time on tedious routine work and instead focus their mathematical efforts and increase their capacity. The pragmatic outsourcing of the lever potential, however, also black boxes the underlying mathematical processes, and may leave students dependant on the digital tool for carrying out even basic mathematical exercises (Lagrange, 2005). Surely such scenarios are not what we aim at with the notion of MDCs. Rather we are concerned with those situations where neither mathematical nor digital competencies are replaced by technology, but where the digital tools actually enforce students' capacities in an epistemic sense (e.g., Geraniou & Mavrikis, 2017).

The Danish KOM framework: mathematical competencies

In relation to mathematics and competencies, Kilpatrick (2014) states that school mathematics sometimes "is portrayed as a simple contest between knowledge and skill" while "Competency frameworks are designed to demonstrate to the user that learning mathematics is more than acquiring an array of facts and that doing mathematics is more than carrying out well-rehearsed procedures" (p. 87). As examples of such frameworks, Kilpatrick mention three: the five strands of mathematical proficiency as identified by the Mathematics Learning Study of the US National Research Council; the five components of mathematical problem-solving ability identified in the Singapore mathematics framework; and the Danish KOM project[1], which lists eight distinct yet mutually related mathematical competencies. Of these three, the KOM framework appears to be the more elaborated one concerning mathematical competencies, but also that which so far has had the most widespread influence in other countries (Niss & Højgaard, in progress). Furthermore, KOM's competencies description was implemented as the basis of the PISA mathematical framework in the years from 2000 through 2018 (e.g., see OECD, 2013).

The Danish KOM defines *mathematical competency* as (an individual's) "…well-informed readiness to act appropriately in situations involving a certain type of mathematical challenge" (Niss & Højgaard, 2011, p. 49). By addressing the question of what it means to master mathematics, KOM identified eight competencies, each possessing both an analytic side and a productive side. The competencies fall into two groups (see Table 1 below).

The ability to ask and answer questions in and with mathematics	(1) mathematical thinking competency (2) problem tackling competency (3) modelling competency (4) reasoning competency
The ability to deal with mathematical language and tools	(5) representing competency (6) symbol and formalism competency (7) communicating competency (8) aids and tools competency

Table 1. The eight mathematical competencies of the KOM framework (see Niss & Højgaard, 2011).

Each of the eight competencies has both an analytic side involving understanding and examining mathematics, and a productive side involving carrying it out. For instance, the aids and tools competency, firstly consists of having knowledge of the existence and properties of the diverse sorts of relevant aids and tools employed in mathematics and of having an insight into their capabilities and limitations within different kinds of contexts. Secondly, it comprises the ability to reflectively use such

[1] KOM is short for "Kompetencer Og Matematiklæring" which is Competencies and Mathematical Learning.

aids and tools. In KOM, the general description of the aids and tools competency also covers the use of digital tools. As a consequence the digital aspects of this competency are not very elaborated.

Digital Competencies frameworks and the European DigComp framework

Living in the digital era, we are witnesses of an increasingly digitalised society, in which digital competencies are becoming 'life skills' and can be compared to skills, such as mathematics and literacy (Ferrari, 2013). A *digital competency* is "the set of knowledge, skills, attitudes […] required when using ICT and digital media to perform tasks; solve problems; communicate; manage information; collaborate; create and share content; and build knowledge effectively, efficiently, appropriately, critically, creatively, autonomously, flexibly, ethically, reflectively for work, leisure, participation, learning, socialising, consuming, and empowerment" (Ferrari, 2012, p. 43). There is a plethora of terms used to refer to digital competencies. For example, digital literacies, which in essence are the information, media and communication skills (Hockly, 2012) or media literacy or ICT literacy as identified and cited by Hatlevik and Christophersen (2013). Hague and Payton (2010) describe digital literacy across the curriculum as: "the skills, knowledge and understanding that enables critical, creative, discerning and safe practices when engaging with digital technologies in all areas of life" (p.19). Regarding the terms digital competency and digital literacy, some authors use them interchangeably (Hockly, 2012). However, referring to school students in particular, Hatlevik and Christophersen (2013) claim that there are differences:

> A concept such as digital skills focuses on dealing with the technical conditions, whereas digital competence and literacy are broader terms that emphasise what kind of skills, understandings, and critical reflections students are able to use. When analysing and discussing the terminology, the concepts seem to have gradually shifted focus from the simple use of digital tools, often linked to concepts such as digital skills, to broader terms, including the students' digital competence and literacy (p. 241).

In fact, many countries include into their curriculum digital literacies, although there is disagreement in terminology: e.g., "digital competency" (Norway); "digital media literacy" (Australia); "media literacy" (UK) (Hatlevik & Christophersen, 2013).

Digital Competencies have been used to characterise people's certain skills in different contexts; these being the workplace, everyday responsibilities or in education and schools in particular. For example, Kent et al. (2005) introduced the term techno-mathematical literacies "as a way of thinking about mathematics as it exists as part of modern, increasingly IT-based workplace practices" (p.1). Focusing though in the school context, there are certain digital literacies which we expect school students to acquire and these are referred to as school-based digital literacies:

> Students' mastery of basic tools and computer programs is only a first step towards the development of advanced knowledge, skills, and attitudes [...]. Often the development of digital competency is considered a continuum from instrumental skills into productive and strategic personal competency and cognitive skills [...]. Therefore, digital competency includes students' ability to use technology in order to consume and access information. Moreover, digital competency also includes how students make use of technology to process, acquire, and evaluate gathered information. Finally, digital competency means that students can produce and communicate information with digital tools or media (Hatlevik & Christophersen, 2013, p. 241).

There are various digital competencies frameworks currently used at schools (e.g. Hague & Payton, 2010; in wales: learning.gov.wales/resources/browse-all/digital-competence-framework/). All these different digital competencies frameworks have similarities in what skills students are required to gain. The main difference is that for each framework, these skills are grouped in different overarching categories. Upon reviewing the above mentioned different digital competencies frameworks, any of these frameworks could have been chosen as a basis for our investigation on a potential framework on MDCs. The counterargument though is that these are produced to be used to the schools in these specific countries, Norway, UK and Wales, and to our knowledge have not been used outside these countries in different contexts. We have therefore decided to choose the most internationally recognised framework on digital competencies, the DigComp Framework for Citizens by Ferrari (2013).

Like the KOM framework, the DigComp framework is structured around a number of main areas, each encompassing a number of digital competencies as shown in table 2. These though are not directly linked to the mathematical context. The digital competencies not deemed to be of relevance in relation to the development and possession of mathematical competencies have been omitted. Of the remaining ones, we briefly elaborate on those digital competencies, which are less self-explanatory than the rest. One such is (3.2) which encompasses to "modify, refine and mash-up existing resources to create new, original and relevant content and knowledge" (Ferrari, 2013, p.5). Another one is (5.1) which comprises to "identify possible problems and solve them (from trouble-shooting to solving more complex problems) with the help of digital means" (Ferrari, 2013, p.6), and not least (5.4) which has the nature of a kind of meta-competency: "To understand where [one's] own competence needs to be improved or updated, to support others in the development of their digital competence, to keep up-to-date with new developments" (Ferrari, 2013, p.6).

(1) Information	(1.1) Browsing, searching and filtering information (1.2) Evaluating information (1.3) Storing and retrieving information
(2) Communication	(2.1) Interacting through technologies (2.2) Sharing information and content […] (2.4) Collaborating through digital channels […]
(3) Content criterion	(3.1) Developing content (3.2) Integrating and re-elaborating […] (3.4) Programming
(4) Safety	[…]
(5) Problem-solving	(5.1) Solving technical problems (5.2) Identifying needs and technological responses (5.3) Innovating and creatively using technology (5.4) Identifying digital competency gaps

Table 2. The DigComp Framework for Citizens with its five main areas (Ferrari, 2013, p.12).

Exploring the potential interplay between mathematical and digital competencies

In our experiences as educators, we have noticed in several occasions what appears to be a simultaneous activation of mathematical competencies and digital competencies. From the KOM framework perspective, digital competency might fit as a minor part of the tools and aids competency, i.e. in terms of the *reflective* use of ICT, referring also to having an understanding of ICT's capabilities and limitations in given contexts. However, from a digital competency perspective, this would constitute too narrow a point of view. Considering the relevance of the eight mathematical competencies for each of the 21 digital competencies of the DigComp framework and vice versa, we have identified *two overarching themes for interplay*, which may provide structure to a potential framework for MDCs: "communication and collaboration" and "problem handling and modelling" (Table 3).

Starting from *communication and collaboration* it seems somewhat straightforward to expect learners to acquire competencies, mathematical and digital, so as to apply both and use them effectively. Digital resources for mathematical learning are designed to incorporate and map mathematical language, but for example students would not be able to share their mathematical answers in a given digital resource or medium if they did not know how to type their answers and use the keyboard effectively, save their answers, upload them on a sharing forum, etc. Being literate in both domains, the mathematical and the digital, seems necessary to achieve in either one of them. Also, in mathematics being able to represent mathematical concepts, entities, etc. is an integral part of communication and so is being able to interpret other's representations, digital or not.

Moving onto the second overarching theme for interplay, "problem handling and modelling", the ability to ask and answer questions in and with mathematics is in fact a problem handling and/or modelling capability. The digital competencies area of problem solving (cf. Table 2) involves identifying what is needed to provide technological responses or identifying one's gaps in technical knowledge. Indeed, both these competencies can reasonably be placed under the overarching umbrella of "problem handling and modelling". But, in our view, thinking about someone who possesses MDCs in terms of problem handling and modelling in the context of (educational) technologies, we have in mind those individuals who have the competencies to (i) address a mathematical problem using digital resources and media creatively and effectively; (ii) use digital resources and media to solve mathematical problems or model extra-mathematical situations, which they were unable to handle or found it more difficult to deal with without the support digital technologies offer; (iii) interpret the instant feedback given by digital technologies and decide upon the next step or action to take. "Problem handling and modelling" also involves the interplay between mathematical thinking and computational thinking, e.g., algorithms, recursion, programming, etc. (for a description of computational thinking, see e.g., Weintrop et al. 2016). Of course, one should bear in mind that "problem", whether it be handled by means of digital or mathematical competencies or an interplay of both, is still relative to the individual (cf. the KOM framework).

Suggesting a tentative framework for mathematical digital competencies

For each of the two overarching themes for interplay between mathematical and digital competencies we now attempt to "flesh out" a set of MDCs (Table 3). Of course, the division of two types of interplays into MDCs should not be thought of as a strict division. As with the KOM framework, overlap of competencies may occur. The placement and description of the MDCs has been made according to what we conceive as the competency's "center of gravity".

Communication and collaboration	(1) Mathematical digital literacy (2) Mathematical digital collaboration (3) Mathematical digital representation (4) Mathematical digital interpretation
Problem handling and modelling	(5) Mathematical digital thinking (6) Mathematical digital reasoning (7) Mathematical digital manipulation

Table 3. Two main areas and seven mathematical digital competencies.

(1) Mathematical digital literacy – Being literate digitally, but mathematically too, in order to take a critical stance to the integration of digital technologies in mathematical activities (in particular in teaching and learning situations). It involves knowing which digital tools are most applicable for different kinds of mathematics as well as different mathematical problems and modelling situations. The competency involves also being able to interpret mathematical tasks presented within a digital environment, use the mathematical language to share answers and justifications within the digital environment, but also save, revisit, edit, submit one's work.
(2) Mathematical digital collaboration – Being able to collaborate verbally and/or digitally with peers. Having the ability to build upon one's peers' contributions with the aim of producing shared problem solutions or mathematical models. Within a digital environment being able to articulate mathematical ideas accurately as well as carry out discussions using mathematically valid arguments with peers. Also ensuring that the language used is appropriate and relevant to the given task. *(3) Mathematical digital representation* – Choosing the most appropriate functionality/feature of the digital tool/medium to represent and solve a mathematical problem or build a mathematical model. Also, being creative when representing mathematical entities involved in the given task, or the task itself. And knowing how to use mathematical notation in a digital environment. *(4) Mathematical digital interpretation* – Reading and interpreting mathematically the instant (usually dynamic) feedback – this includes recognising a mathematical error and fixing it (e.g., when you get an "*x*" instead of a tick) including also being able to interpret the digital media's feedback (e.g., digital responses such as "true", "false", "undefined" etc.). Observing the animation/simulation of any constructed models and interpreting mathematically such simulations. *(5) Mathematical digital thinking* – Being able to think mathematically as well as computationally, e.g., algorithmically and/or recursively. Knowing what kinds of mathematical and extra-mathematical problems that may be dealt with by means of digital tools and which may not. Understanding and being able to apply principles of programming, and to understand what is behind the programme. *(6) Mathematical digital reasoning* – Verifying solutions and validating mathematical models with the support of the digital technology by being able to provide mathematically valid justifications (not only rely on the tool's instant feedback, e.g., getting a tick or "looking" at an image). Knowing what constitutes a valid mathematical argument or proof, and make reflective decisions about when to outsource (e.g., black box)

processes of a mathematical reasoning (i.e. a chain of arguments) to a digital tool and knowing when not to. *(7) Mathematical Digital Manipulation* – Manipulating constructed mathematical representations or features of the digital tool and identifying the mathematical rules/connections within these. Being able to manipulate mathematical expressions using a digital tool, while at the same time knowing and understanding why such manipulations are both possible and correct.

Exemplifying and discussing the tentative framework for MDCs

Taking as an example some of the embedded affordances of a widespread DGS like GeoGebra, allows us to briefly exemplify the above described combined framework for MDCs. Recall the mentioning of 'regular polygon' in the introduction. Surely, if students are to create a regular polygon in GeoGebra using the 'regular polygon' feature of the DGS, not much mathematics may be activated. However, if students are to construct a regular polygon equivalent to GeoGebra's regular polygon, i.e. one which keeps its internal structure when dragged, then the activation of both mathematical and digital competencies may be so intertwined that it no longer makes sense to distinguish the two.

For example, students may revisit their existing knowledge of mathematics and/or digital technologies, gather information while interacting with GeoGebra and decide upon a sequence of actions, which potentially changes or gets adapted based on the instant dynamic feedback they receive from the tool and their inferences of that feedback. They may decide that GeoGebra is the ideal digital tool to construct a regular polygon, which indicates the activation of the *mathematical digital literacy MDC*; or they may choose to use the GeoGebra's affordances, such as constructing line segments and circles to make their chosen regular polygon, which indicates the *mathematical digital representation MDC*; or they may decide to use their constructed polygon to construct a different polygon or solve another mathematical problem, which indicates the *mathematical digital manipulation MDC*; or they interpret GeoGebra's feedback, which indicates the *mathematical digital interpretation MDC*; and they may argue for the correctness of their construction considering their mathematical knowledge of the properties of the chosen polygon as well as its mathematical definition, which indicates the *mathematical digital reasoning MDC*.

To conclude, our argument is that there seems to be a potential in the fruitful *interplay* between mathematical and digital competencies, which perhaps is not captured efficiently using two separate frameworks, and that this interplay might be better articulated through one framework for MDCs. In a sense the sum of the whole is greater than its parts.

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